Studienzentrum Gerzensee Doctoral Program in Economics
Econometrics 2017 Week 4 Take-Home Empirical Exercise

1. The spreadsheet EX2_DTFP.xlsx contains quarterly values on the growth rate of total factor productivity (TFP) for the United States in percentage points at annual rate from 1955:Q1 – 2016:Q3. These data have been adjusted for "utilization adjusted" to eliminate variation associated with the business cycle. (These data are from John Fernald. You can read about how they constructed here: http://www.frbsf.org/economic-research/files/wp12-19bk.pdf).

An important parameter governing the growth of the U.S. macroeconomy is the average growth rate of TFP. Some have argued that this average has varied over time. You will investigate this in this exercise.

Let y_t denote the growth rate of TFP. Consider the model $y_t = \mu + \varepsilon_t$, where μ denotes the mean growth rate and ε_t is white noise.

(a) Estimate μ and σ_ε using the standard method of moments estimators and verify that $\hat{\mu} = 1.16$ and $\hat{\sigma}_\varepsilon = 3.18$.

(b) Break the sample into three equally sized sample periods. Compute the mean growth rate for each subsample. (i) Are there quantitatively large differences in the sample means? (What would you consider to be a quantitatively large change in the sample mean of TFP? Why?) (ii) Are the differences you found in (i) statistically significant?

(c) Consider the model $y_t = \mu_t + \varepsilon_t$ where now $\mu_t = \mu_{t-1} + \eta_t$. Assume that

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \right)$$

Note that with $\sigma_\eta = 0$, $\mu_t = \mu_{t-1}$, so that μ is constant.

(i) Using $\sigma_\varepsilon = 3.18$, $\mu_0 \sim$ Normal with mean 1.0 and variance 4.0, and $\sigma_\eta = 0.15$, use the Kalman filter to compute $\mu_{t|t}$. Plot y_t and $\mu_{t|t}$ and discuss the estimated evolution of μ_t over the sample period.

(ii) Using $\sigma_\varepsilon = 3.18$, $\mu_0 \sim$ Normal with mean 1.0 and variance 4.0, compute the log-likelihood for a grid of values of σ_η with $0 \le \sigma_\eta \le 1.0$. Does the log-likelihood suggest that μ_t is constant or time-varying? Explain.

(iii) Implement the Kalman smoother to construct $\mu_{t|T}$ over the sample period. Compare the smoothed estimates to the filtered estimates that you computed in (i). Are the smoothed estimates more precise estimates of μ_t? Explain (and quantify the difference in precision of the two estimators.)

2. Go to the FRB St. Louis FRED website and download quarterly data on U.S. Real Gross Domestic Product, Billions of chained \$2009, quarterly, seasonally adjusted annual rate. (The series name is GDPC1). Compute $x_t = 400\times\ln(GDP_t/GDP_{t-1})$, the growth rate of real GDP in percentage points at an annual rate.

(a) Using the sample period 1960:Q1 through 2017:Q1, estimate an AR(1) for x_t by regressing x_t on a constant and x_{t-1}.

(i) What is the implied mean and variance of x_t from the estimated model?

(ii) What is the implied standard deviation of the 1-quarter-ahead forecast error? Of the 4-quarter-ahead forecast error.

(b) Use the estimated AR model to estimate the spectrum for x_t. Plot the estimated spectrum and describe its shape.

(c) Consider the stationary AR(1) model: $y_t = \alpha + \phi y_{t-1} + \varepsilon_t$. Suppose that $\varepsilon_t \sim i.i.d.$ $N(0, \sigma^2)$. You have a sample of length T. Let ^'s denote the usual OLS/Method of moments estimators.

(i) Show that

$$\sqrt{T}\begin{bmatrix} \hat{\alpha}-\alpha \\ \hat{\phi}-\phi \\ \hat{\sigma}^2-\sigma^2 \end{bmatrix} \xrightarrow{d} N(0,V)$$

and derive an expression for V. (Where did you use the normality assumption? What would change if ε_t was $i.i.d.$ but not normally distributed?)

(d) Use your expression in (c) to estimate V for the AR model you estimated in (a). (You may assume normality.)

(e) The spectrum for x_t depends on the AR parameters. In (b) you "plugged" in estimated AR parameters to estimate the spectrum. Use your results from (b), (c) and (d) and the δ-method to compute an approximated 95% confidence interval for the spectrum of x at frequency $\omega = 2\pi/32$.

(f) Use calculations like you carried out in (e) to construct (point-wise) 95% confidence bands for the spectrum for each value of ω and plot these along with the estimated spectrum from (b).

3. Let $c(\mathrm{L}) = \sum_{j=-r}^{r} c_j \mathrm{L}^j$, where

$$c_j = \frac{d_j}{\sum_{k=-r}^{r} d_k} \text{ with } d_j = \left(\frac{15}{16}\right)\left(1 - \left(\frac{j}{r+1}\right)^2\right)^2 .$$

This is called a "biweight" moving average with bandwidth r.

(a) Plot the c_j weights for a filter with bandwidth $r = 30$ and a filter with bandwidth $r = 60$. How do the filter weights differ?

(b) Compute the gain of the filter with $r = 30$ and plot the gain for $0 \leq \omega \leq \pi$. Carry out the same calculation for the filter with $r = 60$. Explain how the gains differ.

(c) Using data on the growth rate of GDP, x_t, from the problem above from 1947:Q2 – 2017:Q1:

> (i) Compute $y_t = c(\mathrm{L})x_t$ using the filter with $r = 30$. (Note: because of the leads and lags, you won't be able to compute y_t for all dates).
>
> (ii) Compute $y_t = c(\mathrm{L})x_t$ using the filter with $r = 60$.
>
> (iii) Plot x_t and y_t for the two different bandwidths. Use you calculations from (b) to discuss the properties of the three time series.

4. Real GDP growth following the 2008-09 recession has been slower than recoveries from previous recessions. Standard growth accounting attributes a large share of this slowdown to a decrease in the rate of growth of TFP. In this exercise I want you to examine the evolution of TFP growth over the past 60 years using the data on TFP growth that you used in Exercise 1.

(a)

> (a.1) The sample ranges from 1955:Q1-2016:Q3. Break the data into six (approximately) equal segments and compute the sample mean in each segment. Discuss the evolution of average TFP growth.
>
> (a.2) In exercise 3 you computed a "biweight" moving average filter, and showed that filter captured low-frequency variation in a time series.. I want you to apply this filter to the TFP growth rate data using $r = 60$. (*Note: The filter required r leads and lags of the data and thus can't be used for the last r period in the sample. To overcome this problem, use a truncated version of the filter for the first and last r periods of the sample, and renormalize the weights so they add to unity.*) Plot the resulting filtered version of the growth of TFP. Discuss the evolution of the filtered series over 1955-2016 sample.

(b) Let y_t denote the growth rate of TFP. Suppose that $y_t = \mu + u_t$, where u_t is a zero-mean stationary stochastic process, so that y_t is stationary with mean μ.

(b.1) Using the full sample:

(i) Compute the sample mean. (Call this estimate $\bar{y}_{1:T}$.)

(ii) Estimate the variance and first 4 autocovariances of y.

(iii) Suppose the k'th autocovarinace, λ_k, satisfies $\lambda_k = 0$ for $|k| > 4$. Use the result in (ii) to estimate the long-run variance of u. Call this estimate $\hat{\Omega}_1$.

(iv) Compute the long-run variance of u using a Newey-West estimator with truncation parameter $m = 4$. Call this estimate $\hat{\Omega}_2$.

(v) Use $\bar{y}_{1:T}$ and $\hat{\Omega}_1$ to construct a 90% confidence interval for μ. Repeat the exercise using $\hat{\Omega}_2$. Are there substantive differences between the two confidence sets?

(b.2) Using $\hat{\Omega}_2$ as the estimate of the long-run variance:

(i) Consider a sample mean computed over a decade (40 quarters), say $\bar{y}_{1:40}$. How large is the standard error of $\bar{y}_{1:40}$?

(ii) Consider the change in average growth rates over two non-overlapping decades, say $\bar{y}_{1:40}$ - $\bar{y}_{41:80}$. What is the standard error for the change?

(iii) Suppose $\bar{y}$ changed by 0.50 from one decade to the next. Would you be surprised? What if the change was 0.75? 1.00? 1.25?

(You might want to read more about low-frequency variations in TFP and how they contributed to recent slowdown in GDP growth in the U.S. See:

http://www.princeton.edu/~mwatson/papers/FHSW.05.02.2017%20post-conf%20draft_Final.pdf)