Exercises For Thursday Evening

1. Presented by: Janasch Weiss

X_i are distributed *i.i.d.* Bernouli, with $E(X_i) = p$, for $i = 1, \dots, n$. Let $\bar{X}$ denote the sample mean.

(a) Prove that $\sqrt{n}(\bar{X} - p) \xrightarrow{d} N(0,V)$ and derive an expression for V.

(b) Prove that $\sqrt{n}(\ln(\bar{X}) - \ln(p)) \xrightarrow{d} N(0,W)$ and derive an expression for W.

2. Presented by: Marc Brunner

A researcher wants to estimate the parameter p from a Bernoulli population. She plans to choose a random sample estimate p using $\bar{X}$. She wants to choose a sample size, n, large enough so that $P(|\bar{X} - p| > 0.02) \le 0.01$. Using the approximation based on Question 1a:

(a) How large should n be if $p = 0.3$?

(b) How large should n be if $p = 0.1$?

(c) What advice would you give the researcher that would work regardless of the value of p?

3. Presented by: Federica Braccioli

$X_i \sim$ *i.i.d.* with mean 0 and variance σ^2. Consider the method of moments estimator $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2$.

(a) Show that $\sqrt{n}(\hat{\sigma}^2 - \sigma^2) \xrightarrow{d} N(0,V)$ and derive an expression for V. (Hint and some notation: Let $a_i = X_i^2 - \sigma^2$. Show that $a_i \sim$ *i.i.d.* and derive the mean and variance of a_i. Write the problem in terms of a_i. Note: you will have to make additional assumptions about the existence of moments. State your assumptions carefully.)

(b) Propose an estimator for V and prove that your estimator is consistent.

4. Presented by: Armando Näf

Suppose $X_i = \varepsilon_i + \varepsilon_{i-1}$ where ε_i are $i.i.d.$ with mean 0 and variance 1 for $i = 0, \dots, n$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(a) Show that $\bar{X} = 2\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i + R_n$ (where R_n is a "remainder term") and derive an expression for R_n.

(b) Show that $R_n \xrightarrow{p} 0$.

(c) Show that $\bar{X} \xrightarrow{p} 0$.

(d) Show that $\sqrt{n}R_n \xrightarrow{p} 0$.

(e) Show that $\sqrt{n}\bar{X} \xrightarrow{d} N(0,V)$ and derive an expression for V.

5. Presented by: Alice Antunes

$Y \sim N(\mu,1)$. I have a prior on μ that puts weight of 1/3 on $\mu = 1$ and a weight of 2/3 on $\mu = 2$. I observe $Y = 1$.

(a) Derive the posterior for μ.

(b) Loss is quadratic: $L(\hat{\mu}, \mu) = (\hat{\mu} - \mu)^2$. What is the Bayes estimate of μ?