Thursday Evening

1. Lukas Heim (SNB)

A time series x_t exhibits substantial persistence. To eliminate the persistence, a researcher transforms x as $y_t = (1-L)x_t$.

(a) Derive the gain of the filter (1–L) and discuss how the filter has altered x. (Has the filter attenuated the long-period components (persistence) of x? What has it done to the other periodic components of x?)

(b) Another researcher transforms x as $w_t = (1+L)(1-L)x_t$. Explain what this filter does to the periodic components of x_t?

2. Florian Eckert (ETH)

Suppose that y and x follow the process

$$y_t = \alpha x_{t-1} + \varepsilon_t$$
$$x_t = \phi x_{t-1} + v_t$$

where $v_t = \varepsilon_t + e_t$, where ε and e are mutually independent iid$(0,1)$ processes. Suppose that $|\phi| < 1$. Let $\hat{\alpha}$ and $\hat{\phi}$ denote the OLS estimators of α and ϕ.

(a)

(i) Derive asymptotic distribution of $\hat{\alpha}$.

(ii) Derive asymptotic distribution of $\hat{\phi}$.

(iii) Derive asymptotic distribution of the 2×1 vector $(\hat{\alpha}, \hat{\phi})$.

(b) In a sample of 100 observations $\hat{\alpha} = 1.3$ and $\hat{\phi} = 0.72$. Derive a 95% confidence interval for α.

3. Olaf Verbeek (EPFI) … (2nd time)

Suppose that $y_t = x_t\beta + u_t$, where $u_t = \phi u_{t-1} + \varepsilon_t$, $x_t = e_t + \theta e_{t-1}$, where ε_t and e_t are both i.i.d. with mean zero and variance σ_ε^2 and σ_e^2, and ε_t and e_τ are independent for all t and τ. Let $\hat{\beta}$ denote the OLS estimator of β based on a sample of size T, and let $\hat{u}_t$ denote the OLS residual.

(a) Show that $\sqrt{T}(\hat{\beta}-\beta) \xrightarrow{d} N(0,V)$ $\sqrt{T}(\hat{\beta}-\beta) \xrightarrow{d} N(0,V)$ and derive an expression for V.

(b) Suppose that $T = 100$, $\hat{\beta} = 2.1$, $\frac{1}{100}\sum_{t=1}^{100} x_t^2 = 5$, $\frac{1}{99}\sum_{t=2}^{100} x_t x_{t-1} = 2.5$, $\frac{1}{98}\sum_{t=3}^{100} x_t x_{t-2} = 1.0$, $\frac{1}{100}\sum_{t=1}^{100} \hat{u}_t^{\,2} = 4$, $\frac{1}{99}\sum_{t=2}^{100} \hat{u}_t \hat{u}_{t-1} = 3.6$, $\frac{1}{98}\sum_{t=3}^{100} \hat{u}_t \hat{u}_{t-2} = 3.1$, $\frac{1}{99}\sum_{t=2}^{100} x_t \hat{u}_{t-1} = 0.8$, $\frac{1}{99}\sum_{t=2}^{100} \hat{u}_t x_{t-1} = 0.2$. Use your result in (a) to construct an approximate 95% confidence interval for β.

4. Wild Card

Suppose that $y_t = x_t\beta + u_t$, with $u_t = \varepsilon_t - 0.8\varepsilon_{t-1}$, and $x_t = \varepsilon_{t+1}$, where ε_t is iid(0,1). Let $\hat{\beta}$ denote the OLS estimator of β.

(a) Show that $\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0, V)$, and derive an expression for V.

(b) Noting that $(1-0.8\text{L})^{-1}u_t = \varepsilon_t$, the GLS estimator of β can be formed by regressing $(1-0.8\text{L})^{-1}y_t$ onto $(1-0.8\text{L})^{-1}x_t$. Derive the probability limit of the GLS estimator.