Exercises For Tuesday Evening

1. Presented by: David Torun

Suppose X has probability density function (pdf) $f(x) = 1/x^2$ for $x \geq 1$, and $f(x) = 0$ elsewhere.

(a) Compute $P(X \geq 5)$.
(b) Derive the CDF of X.
(c) Show that the mean of X does not exist.

2. Presented by: Nadia Ceschi

(a) X has probability density $f(x) = 1/4$ for $2 \leq x \leq 6$. Let $Y = X^2$.
(i) What is the CDF of Y?
(ii) What is the probability density function for Y?

(b) X has probability density $f(x) = 1/4$ for $-2 \leq x \leq 2$. Let $Y = X^2$.
(i) What is the CDF of Y?
(ii) What is the probability density function for Y?

3. Presented by: Oliver Kalsbach

X is a continuous random variable with density $f(x)$ and CDF $F(x)$ where F is 1-to-1. Let $Y = F(X)$. Show that $Y \sim U[0,1]$. (Y is called the "probability integral transform" (PIT) of X).

4. Presented by: Seda Basihos

(a) Suppose X and Y are independent discrete random variables. X can take on the values 0, 1, 2, 3 each with probability 1/4. Y can take on the values 10, 11, 12, each with probability 1/3. $Z = X+Y$. What is the pdf of Z? (Jargon: the pdf of Z is called the *convolution* of the pdfs of X and Y.)

(b) Consider the same setup as (a), so X can take on the values 0, 1, 2, 3 and Y can take on the values 10,11,12. The pdf of Y is as in (a). The pdf of X is different and unknown. Suppose you know the pdf of Z. Show how to compute the pdf of X. (Jargon: This is called *deconvolution*.)

(c) Now suppose X and Y are independent and continuously distributed with densities f_X and f_Y. $Z = X + Y$. Write an expression for the pdf of Z in terms of the pdfs of X and Y.

5. Presented by: Simon Tiechi

The joint density of X and Y is given by $f_{X,Y}(x,y) = c(x^2 + y)$ for $0 < x < 2$ and $0 < y < 1$, and is equal to zero elsewhere, where $c > 0$ is a constant. You are told that $X = 1.2$

(a) Compute the minimum mean square error forecast of Y.

(c) Compute the mean squared error of your forecast.