Exercises For Wednesday Evening

1. Presented by: Paolo Campli

(a) Suppose $Z \sim$ N(0,1). Use the moment generating function to derive the first four moments for Z.

(b) Suppose $W \sim \chi_5^2$. Use the result in (a) to derive the mean and variance of W.

(c) The distribution of X given $W = w$ is U[0, w] (that is uniform on 0 to w). Use the law of iterated expectations to find $E(X)$, $E(X^2)$ and var(X).

2. Presented by: Marius Faber

$X \sim N(1, 4)$ and $Y = e^X$.

(a) What is the density of Y?

(b) Use Jensen's inequality to show that $E(Y) = E(e^X) \geq e^{E(X)} = e^{\mu}$

(c) Compute $E(Y)$. (Hint: What is the MGF for X?)

3. Presented by: Ksenia Melnikova

$X \sim$ N(0,1). The distribution of $Y|X=x$ is N(x,1).

(a) Find

- (i) $E(Y)$
- (ii) Var(Y)
- (iii) Cov(X,Y)

(b) Prove that the joint distribution of X and Y is normal. (Hint: how do you find a joint distribution from a marginal and a conditional ?)

4. Presented by: Lea Wirth

Suppose X_i, $i = 1, \ldots, n$ are a sequence of iid random variables with $E(X_i) = 0$ and var(X_i) = 1.

Let $Y_i = iX_i$ and $\bar{Y} = n^{-2}\sum_{i=1}^{n} Y_i$. Prove $\bar{Y} \xrightarrow{p} 0$. (Hint – can you show mean square convergence?)