Exercises For Wednesday Evening

1. Presented by: Christian Stettler

(a) $Z \sim$ N(0,1). Use the moment generating function to derive the first 4 moments of Z.

(b) $Y \sim \chi_1^2$. Use the result in (a) to derive $E(Y)$, $E(Y^2)$, and the variance of Y.

(c) $W \sim \chi_k^2$. Use the result in (b) to derive $E(W)$, the variance of W, and $E(W^2)$.

2. Presented by: Ling Zhou

$X \sim N(1, 4)$ and $Y = e^X$.

(a) What is the density of Y?
(b) Use Jensen's inequality to show that $E(Y) \geq e^{E(X)}$.
(c) Compute $E(Y)$. (Hint: What is the MGF for X?) Is the inequality in (b) strict?

3. Presented by: Anna B. Kis

Suppose that X and Y are two random variables with a joint normal distribution. Further suppose $var(X) = var(Y)$. Let U=X+Y and V =X −Y.

(a) Prove that U and V are jointly normally distributed.
(b) Prove that U and V are independent.

4. Presented by: Jonas Meier

Suppose that the 3×1 vector X is distributed $N(0, I_3)$, and let V be a 2×3 non-random matrix that satisfies $VV' = I$. Let $Y = VX$.

(a) Show that $Y'Y \sim \chi_2^2$.

(b) Use Chebyshev's inequality to construct an upper bound for $Pr(Y'Y>6)$. (You will need some moments for a χ_2^2 random variable. Use the results from the question 1.)

(c) Find the exact value of $Pr(Y'Y>6)$. (You might find it useful to use Matlab, Stata, Excel, etc., to evaluate the χ_2^2 cdf.)

5. Presented by: Andrea Berlanda

$X \sim$ N(3,10), $Y \sim$ N(3,100), X and Y are independent, and $X_n = X + 1/n$.

(a) Show that $X_n \xrightarrow{p} X$.

(b) Show that $X_n \xrightarrow{d} X$.

(c) Show that $X_n \xrightarrow{d} Y$.

(d) Does $X_n \xrightarrow{p} X$? Explain.