Wednesday Evening

1. Jorma Juhan Schaublin (Basel)

Suppose x_t evolves as $x_t = 0.9x_{t-1} + u_t$ and $y_t = x_t + v_t$ where $(u_t, v_t) \sim$ i.i.d. $N(0,I_2)$. You learn that $x_{t-1} = 0.0$ and $y_{t-1} = 2.0$

(a) Derive the probability density of $y_t \mid (x_{t-1} = 0.0$ and $y_{t-1} = 2.0)$.

(b) You learn that $y_t = 1.0$. Derive the probability density of $x_t \mid (x_{t-1} = 0.0, y_{t-1} = 2.0,$ and $y_t = 1.0)$.

2. Preetha Kalambaden (Bern)

Suppose x_t is a binary random variable with $P(x_t = 1 \mid x_{t-1} = 0) = 0.2$ and $P(x_t = 1 \mid x_{t-1} = 1) = 0.9$. The random variable y_t is related to x_t by the equation $y_t = x_t + v_t$ where $v_t \sim$ i.i.d. $N(0,1)$ and is independent of x_j for all t and j. You learn that $x_{t-1} = 0.0$ and $y_{t-1} = 2.0$

(a) Derive the probability density of $y_t \mid (x_{t-1} = 0.0$ and $y_{t-1} = 2.0)$.

(b) You learn that $y_t = 1.0$. Derive the probability density of $x_t \mid (x_{t-1} = 0.0, y_{t-1} = 2.0,$ and $y_t = 1.0))$.

3. Fabrizio Colella (Lausanne)

For each of the stationary stochastic processes given below:

(a) Generate a realization of length $T = 500$. (Use the stationary distribution for any initial conditions, so the realization is a draw from the stationary process.) Plot the time series.

(b) Compute the spectrum of the stochastic process.

(c) Discuss what the spectrum in (b) tells you about the characteristics of the realization plotted in (a).

Processes to use: Let $\varepsilon_t \sim i.i.d.\ N(0,1)$

(i) (White noise) $y_t = \varepsilon_t$

(ii) (AR(1)) $y_t = 0.95y_{t-1} + \varepsilon_t$

(iii) (MA(1)) $y_t = \varepsilon_t - 0.95\varepsilon_{t-1}$

(iv) (MA(4) $y_t = \varepsilon_t + 0.8\varepsilon_{t-4}$

4. Helena Xin Ting (Graduate Institute)

Suppose that X_t follows the MA(1) process: $X_t = \mu + \varepsilon_t - \theta\varepsilon_{t-1}$ where μ is a constant and ε_t is $i.i.d.$ with mean 0 and variance σ^2.

(a) Show that $\bar{X} = \mu + (1-\theta)T^{-1}\sum_{t=1}^{T}\varepsilon_t + R$ (where R is a "remainder term") and derive an expression for R.

(b) Show that $\sqrt{T}R \xrightarrow{p} 0$.

(c) Show that $T^{-1/2}\sum_{t=1}^{T}\varepsilon_t \xrightarrow{d} N(0,\sigma^2)$

(d) Use the result in (a)-(c) to show that $\sqrt{T}(\bar{X}-\mu) \xrightarrow{d} N(0,V)$, and derive V.

(e) (b) In a sample of T = 100, $\bar{X}$ = 31.2, $T^{-1}\sum_{t=1}^{100}(X_t-\bar{X})^2$ = 5 and $T^{-1}\sum_{t=2}^{100}(X_t-\bar{X})(X_{t-1}-\bar{X})$ = 1.5. Construct a 95% confidence interval for μ.