Appendix for

RELATIVE GOODS' PRICES, PURE INFLATION, AND THE PHILLIPS CORRELATION

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A.1 Data

All price series are from NIPA Table 2.4.4U available from http://www.bea.gov/national/nipaweb/nipa_underlying/SelectTable.asp. Quarterly inflation rates were computed using the first difference of logarithms of the price indices for the last month of the quarter. Inflation observations that differed from the series median by more than six times the interquartile range were replaced by the local median computed using the six adjacent observations. The table below shows the price index from the NIPA table, the series description, the standard deviation of the (outlier-adjusted) series over 1959:2-2006:2 and the 2005 PCE expenditure share. To save space, the final four columns of this table are used to show the estimated parameters from the benchmark 3-factor model.

Table A1: Series Descriptions, Summary Statistics,
and Parameter Estimates from the Benchmark 3-factor Model

Num.	Label	Description	s_π	2005 Share	Benchmark Model Parameters			
					γ_1	γ_2	β	σ_e
001	P1NFCG D	New foreign autos	4.5	0.5	1.14	0.00	-0.13	0.88
002	P1NETG D	Net transactions in used autos	1.8	0.4	2.35	0.42	0.15	2.71
003	P1MARG D	Used auto margin	6.9	0.3	1.09	0.18	0.02	4.22
004	P1REEG D	Employee reimbursement	7.5	0.0	1.11	0.15	-0.19	1.68
005	P1TRUG D	Trucks, new and net used	4.8	2.4	1.25	-0.09	-0.12	0.96
006	P1TATG D	Tires and tubes	5.8	0.3	0.15	0.57	0.12	1.27
007	P1PAAG D	Accessories and parts	5.5	0.4	-0.21	-0.04	0.26	1.15
008	P1FNRG C	Furniture, incl. matt. and bedsprings	4.1	0.9	0.53	0.30	-0.29	0.77
009	P1MHAG D	Major household appliances	4.0	0.4	0.84	0.13	0.09	0.73
010	P1SEAG D	Small electric appliances	5.0	0.1	1.06	0.35	0.12	0.93
011	P1CHNG C	China, glassware, tableware, and utensil	6.7	0.4	1.32	0.93	-0.28	1.25
012	P1TVSG D	Television receivers	5.4	0.2	1.16	0.47	0.42	0.99
013	P1AUDG D	Audio equipment	5.2	0.3	0.57	0.06	-0.17	1.17
014	P1RTDG D	Records, tapes, and disks	4.9	0.2	-0.21	0.07	-0.06	1.17
015	P1MSCG D	Musical instruments	4.0	0.1	0.41	0.22	-0.13	0.85
016	P1FLRG D	Floor coverings	5.8	0.2	0.60	0.09	-0.24	1.27
017	P1CLFG D	Clocks, lamps, and furnishings	6.0	0.4	1.22	0.45	-0.04	1.29
018	P1TEXG D	Blinds, rods, and other	8.6	0.1	1.54	1.07	-0.28	1.81
019	P1WTRG D	Writing equipment	5.1	0.0	0.18	-1.01	-0.28	1.06
020	P1HDWG D	Tools, hardware, and supplies	4.7	0.1	0.56	0.14	-0.04	1.05
021	P1LWNG D	Outdoor equipment and supplies	5.1	0.0	0.73	0.13	-0.16	1.11
022	P1OPTG C	Ophth. prd, and orthopedic appliances	2.8	0.3	0.29	-0.05	-0.07	0.55
023	P1CAMG D	Photographic equipment	6.0	0.1	1.26	0.04	0.34	1.25
024	P1BCYG D	Bicycles	4.3	0.1	-0.09	0.30	-0.15	0.90
025	P1MCYG D	Motorcycles	4.7	0.2	1.18	-0.11	0.01	1.00
026	P1AIRG D	Pleasure aircraft	7.2	0.0	0.05	0.57	0.06	1.64
027	P1JRYG C	Jewelry and watches (18)	7.3	0.7	0.15	0.33	-0.21	1.67
028	P1BKSG C	Books and maps (87)	5.8	0.5	1.00	-0.37	-0.25	1.23
029	P1GRAG D	Cereals	6.3	0.4	-1.34	-0.19	0.45	1.34
030	P1BAKG D	Bakery products	4.6	0.6	-0.22	0.25	0.14	1.01
031	P1BEEG D	Beef and veal	13.0	0.4	-4.16	-0.28	-0.16	2.88
032	P1PORG D	Pork	6.9	0.3	-3.52	-0.91	0.19	3.96
033	P1MEAG D	Other meats	8.3	0.2	-2.72	-0.74	0.17	1.84
034	P1POUG D	Poultry	7.0	0.5	-2.23	0.03	-0.20	4.06
035	P1FISG D	Fish and seafood	5.7	0.2	-0.69	0.01	0.18	1.22
036	P1GGSG D	Eggs	7.4	0.1	-5.34	-0.42	-0.03	6.63

037	P1MILG D	Fresh milk and cream	6.9	0.2	-1.10	0.04	-0.03	1.63
038	P1DAIG D	Processed dairy products	6.2	0.5	-1.19	0.08	0.28	1.32
039	P1FRUG D	Fresh fruits	4.5	0.3	-0.89	0.21	-0.07	3.55
040	P1VEGG D	Fresh vegetables	9.3	0.4	-2.70	-0.21	-0.41	6.59
041	P1PFVG D	Processed fruits and vegetables	5.7	0.2	0.40	0.15	0.38	1.21
042	P1JNBG D	Juices and nonalcoholic drinks	6.4	0.8	0.16	0.64	0.32	1.22
043	P1CTMG D	Coffee, tea and beverage materials	1.8	0.2	1.49	0.89	0.58	2.31
044	P1FATG D	Fats and oils	9.3	0.1	-0.60	1.33	0.52	1.71
045	P1SWEG D	Sugar and sweets	6.3	0.5	-0.97	0.36	0.27	1.37
046	P1OFDG D	Other foods	4.1	1.3	0.11	0.05	0.11	0.76
047	P1PEFG D	Pet food	3.9	0.3	-0.19	0.04	-0.04	0.79
048	P1MLTG D	Beer and ale, at home	3.6	0.7	0.42	0.18	0.13	0.66
049	P1WING D	Wine and brandy, at home	3.9	0.2	-0.51	0.14	-0.02	0.79
050	P1LIQG D	Distilled spirits, at home	2.1	0.2	-0.17	-0.40	0.25	0.54
051	P1OPMG D	Other purchased meals	2.8	4.5	-0.15	0.09	0.30	0.32
052	P1APMG C	Alcohol in purchased meals	3.7	0.6	0.45	-0.06	-0.16	0.79
053	P1MFDG D	Food supplied military	3.0	0.0	-0.20	0.10	0.25	0.40
054	P1FFDG C	Food produced and consumed on farms	0.9	0.0	-4.86	-1.37	-0.09	4.98
055	P1SHUG C	Shoes (12)	3.8	0.6	-0.01	0.41	0.01	0.78
056	P1WGCG D	Clothing for females	4.5	1.8	-0.14	0.30	0.02	1.10
057	P1WICG D	Clothing for infants	8.9	0.1	1.40	0.58	-0.33	1.88
058	P1MBCG D	Clothing for males	3.5	1.2	0.30	0.34	0.11	0.74
059	P1MSGG D	Sewing goods for males	6.4	0.0	0.28	0.25	-0.29	1.46
060	P1MUGG D	Luggage for males	2.6	0.0	1.29	1.25	-0.21	2.82
061	P1MICG C	Std. clothing issued to military personnel	2.8	0.0	0.28	0.16	0.15	0.43
062	P1GASG D	Gasoline and other motor fuel	4.2	3.2	-6.30	1.54	-0.13	5.37
063	P1LUBG D	Lubricants	5.5	0.0	-0.37	0.47	0.37	1.09
064	P1OILG D	Fuel oil	3.7	0.1	-7.75	2.55	0.21	4.84
065	P1FFWG D	Farm fuel	6.0	0.0	-3.91	1.84	0.14	3.38
066	P1TOBG C	Tobacco products	7.5	1.0	0.36	-0.70	0.06	1.83
067	P1SOAG D	Soap	4.9	0.1	1.21	0.25	-0.13	0.92
068	P1CSMG D	Cosmetics and perfumes	4.3	0.2	1.07	0.17	-0.24	0.78
069	P1SDHG C	Semidurable house furnishings	7.4	0.5	1.76	0.64	-0.44	1.40
070	P1CLEG D	Cleaning preparations	4.2	0.4	0.66	0.13	0.09	0.75
071	P1LIGG D	Lighting supplies	7.2	0.1	0.87	0.53	-0.13	1.59
072	P1PAPG D	Paper products	5.6	0.3	0.36	0.40	0.04	1.17
073	P1RXDG D	Prescription drugs	4.0	2.6	0.33	-0.62	0.67	0.55
074	P1NRXG D	Nonprescription drugs	4.0	0.3	0.91	-0.45	0.10	0.64
075	P1MDSG D	Medical supplies	3.7	0.1	0.77	-0.58	-0.13	0.64
076	P1GYNG D	Gynecological goods	4.2	0.0	1.02	0.24	-0.08	0.68
077	P1DOLG D	Toys, dolls, and games	5.4	0.6	1.04	0.47	0.10	1.08
078	P1AMMG D	Sport supplies, including ammunition	4.7	0.2	0.35	0.15	-0.16	1.06
079	P1FLMG D	Film and photo supplies	4.6	0.0	0.62	-0.25	0.10	1.06
080	P1STSG D	Stationery and school supplies	4.7	0.1	0.91	0.50	-0.04	0.95
081	P1GREG D	Greeting cards	4.8	0.1	0.92	0.50	-0.04	0.97
082	P1ABDG C	Expenditures abroad by U.S. residents	16.8	0.1	0.28	0.54	0.18	4.02
083	P1MGZG D	Magazines and sheet music	5.5	0.3	0.66	-0.44	-0.31	1.17
084	P1NWPG D	Newspapers	3.8	0.2	0.87	0.24	0.14	0.78
085	P1FLOG C	Flowers, seeds, and potted plants	6.7	0.2	0.57	0.29	-0.12	1.54
086	P1OMHG D	Owner occupied mobile homes	2.5	0.4	0.03	-0.74	-0.30	0.24
087	P1OSTG D	Owner occupied stationary homes	2.4	10.7	0.00	-0.75	-0.17	0.19
088	P1TMHG D	Tenant occupied mobile homes	3.8	0.1	0.07	-0.75	-0.26	0.77
089	P1TSPG D	Tenant occupied stationary homes	2.4	2.8	-0.04	-0.77	-0.31	0.17
090	P1TLDG D	Tenant landlord durables	3.8	0.1	0.45	-0.51	0.25	0.66
091	P1FARG C	Rental value of farm dwellings (26)	4.3	0.2	-0.27	-0.15	0.70	0.84
092	P1HOTG D	Hotels and motels	6.3	0.6	0.19	-0.01	-0.10	1.38
093	P1HFRG D	Clubs and fraternity housing	2.9	0.0	0.03	-0.65	-0.33	0.43
094	P1HHEG D	Higher education housing	3.0	0.2	-0.15	-0.78	0.04	0.54
095	P1HESG D	El. and secondary education housing	8.9	0.0	0.16	-0.84	-0.36	2.01
096	P1TGRG D	Tenant group room and board	3.4	0.0	-0.12	-0.70	-0.38	0.60
097	P1ELCG C	Electricity (37)	5.7	1.5	0.43	-0.16	0.23	1.15
098	P1NGSG C	Gas (38)	2.6	0.8	0.35	0.19	0.44	2.71
099	P1WSMG D	Water and sewerage maintenance	3.9	0.6	0.88	-0.50	0.20	0.75
100	P1REFG D	Refuse collection	4.1	0.2	1.02	-0.56	0.29	0.75
101	P1LOCG D	Local and cellular telephone	4.5	1.3	0.41	-0.84	0.05	0.98
102	P1OLCG D	Local telephone	4.4	0.6	0.05	-1.00	0.00	1.00

103	P1LDTG D	Long distance telephone	5.3	0.3	0.15	-0.31	0.33	1.24
104	P1INCG D	Intrastate toll calls	5.1	0.1	-0.08	-0.66	0.36	1.17
105	P1ITCG D	Interstate toll calls	6.3	0.2	0.38	0.09	0.23	1.52
106	P1DMCG D	Domestic service, cash	4.3	0.2	0.27	0.10	0.24	0.98
107	P1DMIG D	Domestic service, in kind	6.0	0.0	-1.76	-0.21	-0.03	1.24
108	P1MSEG D	Moving and storage	3.7	0.2	0.15	0.09	-0.03	0.69
109	P1FIPG D	Household insurance premiums	3.7	0.2	0.13	-0.49	0.32	0.84
110	P1FIBG D	Less: Household insurance benefits paid	3.3	0.1	0.86	0.38	-0.28	0.40
111	P1RCLG D	Rug and furniture cleaning	4.4	0.0	0.33	0.06	-0.36	0.79
112	P1EREG D	Electrical repair	3.8	0.1	0.06	0.12	0.17	0.79
113	P1FREG D	Reupholstery and furniture repair	3.2	0.0	-0.11	-0.20	0.13	0.74
114	P1MHOG D	Household operation services, n.e.c.	3.7	0.2	0.03	0.09	-0.02	0.73
115	P1ARPG D	Motor vehicle repair	2.9	1.7	0.17	0.06	0.30	0.34
116	P1RLOG D	Motor vehicle rental, leasing, and other	4.9	0.6	0.82	0.15	-0.16	0.96
117	P1TOLG C	Bridge, tunnel, ferry, and road tolls	6.2	0.1	0.00	-0.75	-0.19	1.42
118	P1AING C	Insurance	4.2	0.7	0.84	-0.73	0.13	3.61
119	P1IMTG C	Mass transit systems	5.4	0.1	0.09	-0.45	0.09	1.35
120	P1TAXG C	Taxicab	5.7	0.0	0.05	0.22	0.02	1.27
121	P1IBUG C	Bus	9.2	0.0	-0.10	-0.37	-0.20	2.13
122	P1IAIG C	Airline	15.0	0.4	-0.64	0.75	-0.04	3.60
123	P1TROG C	Other	9.1	0.1	-0.23	-0.04	-0.05	2.11
124	P1PHYG C	Physicians	3.3	4.0	0.63	-0.09	0.50	0.42
125	P1DENG C	Dentists	2.7	1.0	0.39	-0.22	0.17	0.48
126	P1OPSG C	Other professional services	3.2	2.7	0.61	0.04	0.25	0.50
127	P1NPHG C	Nonprofit	3.1	4.4	0.05	-0.02	0.03	0.48
128	P1GVHG C	Government	4.3	1.4	-0.10	-0.06	0.51	0.76
129	P1NRSG C	Nursing homes	3.3	1.3	0.05	0.11	-0.30	0.62
130	P1MING C	Medical care and hospitalization	0.3	1.4	-0.90	-0.95	0.29	4.89
131	P1IING C	Income loss	5.7	0.0	0.70	-1.74	0.64	4.86
132	P1PWCG C	Workers' compensation	8.1	0.2	-0.55	0.26	0.80	1.16
133	P1MOVG C	Motion picture theaters	4.1	0.1	0.05	0.08	0.15	1.07
134	P1LEGG C	Leg. theaters and opera,	4.2	0.1	0.13	0.11	0.16	1.10
135	P1SPEG C	Spectator sports	4.1	0.2	-0.15	-0.34	-0.08	1.03
136	P1RTVG C	Radio and television repair	3.1	0.1	0.28	-0.52	0.33	0.62
137	P1CLUG C	Clubs and fraternal organizations	4.2	0.3	-0.13	0.42	-0.27	0.77
138	P1SIGG D	Sightseeing	5.3	0.1	0.04	0.00	-0.07	1.21
139	P1FLYG D	Private flying	9.8	0.0	0.48	0.19	-0.28	2.27
140	P1BILG D	Bowling and billiards	4.1	0.0	0.46	-0.31	0.05	0.96
141	P1CASG D	Casino gambling	2.9	0.9	-0.28	0.10	-0.22	0.32
142	P1OPAG D	Other com. participant amusements	2.8	0.3	0.27	0.06	0.16	0.59
143	P1PARG C	Pari-mutuel net receipts	4.8	0.1	-0.66	-0.09	0.51	0.99
144	P1PETG D	Pets and pets services excl. vet.	3.6	0.1	-0.12	-0.07	0.00	0.76
145	P1VETG D	Veterinarians	3.0	0.2	-0.18	-0.23	0.13	0.67
146	P1CTVG D	Cable television	7.0	0.7	0.18	-0.21	0.08	1.76
147	P1FDVG D	Film developing	3.8	0.1	0.76	-0.08	0.39	0.85
148	P1PICG D	Photo studios	3.8	0.1	0.12	-0.12	0.09	0.89
149	P1CMPG D	Sporting and recreational camps	3.4	0.0	0.09	-0.04	-0.07	0.81
150	P1HREG D	High school recreation	4.7	0.0	0.05	-0.14	-0.22	1.12
151	P1NECG D	Commercial amusements n.e.c.	3.4	0.6	0.25	0.00	-0.05	0.80
152	P1NISG D	Com. amusements n.e.c. except ISPs	3.3	0.4	0.12	-0.05	-0.04	0.80
153	P1SCLG D	Shoe repair	3.3	0.0	0.04	-0.27	0.12	0.64
154	P1DRYG D	Drycleaning	3.6	0.1	0.30	0.18	0.24	0.52
155	P1LGRG D	Laundry and garment repair	3.6	0.1	-0.03	0.07	0.12	0.57
156	P1BEAG D	Beauty shops, including combination	3.9	0.5	0.08	-0.09	0.17	0.76
157	P1BARG D	Barber shops	2.8	0.0	0.01	0.08	0.11	0.56
158	P1WCRG D	Watch, clock, and jewelry repair	3.3	0.0	-0.01	-0.30	-0.03	0.66
159	P1CRPG D	Miscellaneous personal services	3.8	0.5	0.17	0.11	-0.02	0.62
160	P1BROG C	Brokerage charges and inv. couns.	1.2	1.0	0.30	0.50	0.01	5.18
161	P1BNKG C	Bnk srv. chges, trust serv., s-d box rental	5.7	1.2	1.81	-0.70	0.39	1.02
162	P1IMCG D	Commercial banks	2.4	1.0	-0.18	0.76	0.18	2.93
163	P1IMNG D	Other financial institutions	15.0	1.4	0.19	-0.32	0.58	3.05
164	P1LIFG C	Exp. of handl. life ins. and pension plans	2.3	1.2	-0.37	-0.24	0.49	0.45
165	P1GALG C	Legal services (65)	4.4	1.0	0.60	-0.41	0.14	0.91
166	P1FUNG C	Funeral and burial expenses	3.2	0.2	0.47	-0.61	0.35	0.57
167	P1UNSG D	Labor union expenses	4.1	0.2	-0.32	0.29	0.07	0.74
168	P1ASSG D	Profession association expenses	6.5	0.1	-0.23	0.03	-0.37	1.33

169	P1GENG D	Employment agency fees	5.5	0.0	1.40	-0.11	-0.04	1.03
170	P1AMOG D	Money orders	5.3	0.0	1.12	-0.24	-0.21	1.09
171	P1CLAG D	Classified ads	5.4	0.0	1.15	-0.23	-0.16	1.09
172	P1ACCG D	Tax return preparation services	5.2	0.1	0.97	-0.31	-0.11	1.12
173	P1THEG D	Personal business services, n.e.c.	7.1	0.1	0.61	-0.55	-0.03	1.66
174	P1PEDG D	Private higher education	4.4	0.7	-0.25	-0.13	0.02	0.89
175	P1GEDG D	Public higher education	4.1	0.7	0.52	-0.27	0.07	0.89
176	P1ESCG D	Elementary and secondary schools	4.3	0.4	-0.47	0.20	-0.02	0.84
177	P1NSCG D	Nursery schools	4.8	0.1	-0.63	0.01	0.02	1.05
178	P1VEDG D	Commercial and vocational schools	4.1	0.4	-0.96	-0.38	0.20	0.88
179	P1REDG D	Foundations and nonprofit research	4.5	0.2	-0.37	-0.27	-0.03	1.05
180	P1POLG D	Political organizations	8.2	0.0	0.04	0.39	-0.32	1.83
181	P1MUSG D	Museums and libraries	5.7	0.1	-0.70	0.08	-0.13	1.18
182	P1FOUG D	Foundations to religion and welfare	5.4	0.2	-0.54	0.09	0.01	1.11
183	P1WELG D	Social welfare	3.3	1.7	-0.39	0.12	-0.01	0.54
184	P1RELG D	Religion	5.0	0.7	0.17	0.19	-0.09	1.11
185	P1AFTG D	Passenger fares for foreign travel	9.8	0.5	-0.95	0.39	-0.08	2.32
186	P1USTG D	U.S. travel outside the U.S.	9.6	0.6	-2.04	0.50	0.15	2.16
187	P1FTUG D	Foreign travel in U.S.	3.6	1.0	-0.20	0.00	0.04	0.62

A.2 Solving the restricted least squares problem in (6)

To solve the least squares problem in (6), notice that (i) if a_t were known, the least squares problem could be solved by computing principal components for variables $z_{it} = \sqrt{w_i}(\pi_{it} - a_t)$ and (ii) if $\boldsymbol{\gamma}_i'\mathbf{R}_t$ were known, the least squares estimator of a_t could be computed from the weighted least squares of $\pi_{it} - \boldsymbol{\gamma}_i'\mathbf{R}_t$ onto a constant. We iterated between these two steps to solve (6).

A.3 State-space representation of the dynamic factor model, the log-likelihood function, and the EM algorithm.

Let $\mathbf{B}$ be an NxN diagonal matrix with β_i on the diagonal, let p be the order of the VAR, and let the $N\times 1$ vector $\mathbf{y_t} = \boldsymbol{\pi}_t - \mathbf{B}\boldsymbol{\pi}_{t-1} - \mathbf{c}$. Then, the unobserved-components model in (7)-(9) can be written in state-space form as:

$$\mathbf{y_t} = \mathbf{H}\mathbf{s_t} + \mathbf{e_t} \tag{A.1}$$

$$\mathbf{s_t} = \mathbf{F}\mathbf{s_{t-1}} + \mathbf{G}\boldsymbol{\varepsilon}_t \tag{A.2}$$

where, $\mathbf{s_t} = (\mathbf{x_t}' \mathbf{x_{t-1}}' \ldots \mathbf{x_{t-p+1}}')'$ with $\mathbf{x_t} = (a_t \ \mathbf{R_t}')'$ a $k\times 1$ vector, and:

$\mathbf{H} = \begin{bmatrix} \mathbf{I} & \mathbf{\Gamma} & -\mathbf{B}\mathbf{I} & -\mathbf{B}\mathbf{\Gamma} & \mathbf{0}_{(N,(p-2)*(k+1))} \end{bmatrix}$, $\mathbf{F} = \begin{pmatrix} \mathbf{\Phi}_1,\ldots,\mathbf{\Phi}_{p-1} & \mathbf{\Phi}_p \\ \mathbf{I}_{(p-1)(k+1)} & \mathbf{0}_{(p-1)(k+1),k+1} \end{pmatrix}$, and $\mathbf{G} = \begin{pmatrix} \mathbf{I}_{k+1} \\ \mathbf{0}_{(p-1)(k+1)} \end{pmatrix}$. The Gaussian log-likelihood for the unknown parameters conditional on $\{\mathbf{y_t}\}_{t=2}^{T}$ can be computed using the Kalman filter innovations and their variances as described in Hamilton (1993, Chapter 13).

The EM algorithm is a well-known approach (Watson and Engle, 1983, Shumway and Stoffer, 1982) to maximize the Gaussian log-likelihood function for state-space problems. The method is convenient here because it straightforward to compute the expected value of the "complete data" ($\{\mathbf{y_t}, \mathbf{s_t}\}$) sufficient statistics conditional on the observed data ($\{\mathbf{y_t}\}$), and because maximization of the complete data Gaussian likelihood follows from familiar regression formulae. The standard linear regression formulae are modified in two ways to estimate the parameters in (A.1)-(A.2). First, Gauss-Seidel/Cochrane-Orcutt iterations are used to estimate **B** conditional on **c** and $\mathbf{\Gamma}$, and **c** and $\mathbf{\Gamma}$ conditional on **B**. Second, $\mathbf{\Gamma}$ is estimated subject to the constraint $\mathbf{l}'\mathbf{\Gamma} = 0$ using the standard restricted least squares formula, in order to impose the normalization that we used.

While there are many parameters to estimate (971 in the benchmark model), there are two features of the model that make estimation feasible. First, while N is large, because **R** is diagonal, the sufficient statistics for the complete data likelihood can be computed in $O(Tm)$ calculations, where m is the dimension of the state vector s. Second, because N and T are large, the principal component estimators of (a_t $\mathbf{R_t}'$) are reasonably accurate and regression based estimators of the model parameters can be constructed using these estimates of the factors. These principal component based estimates serve as useful initial values for the MLE algorithm. (See Doz, Giannone and Reichlin, 2008, for further discussion.) Results reported in the text are based on 40,000 EM iterations, although results using 5,000 iterations are essentially identical.

A.4 MLEs for the benchmark model

Table A1 includes the estimates of $\mathbf{\Gamma}$, **B**, and σ_e for the benchmark 3-factor model. The estimated parameters in the VAR(4) state transition equation are

$$\mathbf{\Phi_1} = \begin{bmatrix} 0.40 & -0.10 & 0.35 \\ 0.44 & 0.63 & -0.01 \\ -0.72 & -0.25 & 1.33 \end{bmatrix}, \mathbf{\Phi_2} = \begin{bmatrix} 0.73 & 0.06 & -0.28 \\ -0.19 & 0.06 & 0.06 \\ 1.14 & 0.21 & -0.71 \end{bmatrix}, \mathbf{\Phi_2} = \begin{bmatrix} 0.00 & -0.13 & -0.05 \\ -0.45 & 0.16 & -0.10 \\ -0.30 & -0.36 & 0.36 \end{bmatrix}$$

$$\mathbf{\Phi_4} = \begin{bmatrix} -0.13 & 0.17 & -0.01 \\ 0.20 & 0.15 & 0.12 \\ -0.11 & 0.39 & -0.11 \end{bmatrix}, \; Var(\boldsymbol{\varepsilon}) = \begin{bmatrix} 0.40 & -0.16 & 0.45 \\ -0.16 & 1.0 & 0 \\ 0.45 & 0 & 1.0 \end{bmatrix}$$

A.5 Estimating υ_t and $\boldsymbol{\rho_t}$

Recall that $\upsilon_t = a_t - E(a_t \mid \{\mathbf{R_\tau}\}_{\tau=1}^{T})$. We estimate υ_t using the inflation data, that is we construct $E(\upsilon_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T})$. One way to construct this estimate is to note that the projection $E(a_t \mid \{\mathbf{R_\tau}\}_{\tau=1}^{T})$ can be computed from the Kalman smoother of a_t from a state space system with state equation given by (A.2) and observation equation given by $\mathbf{R_t} = [0 \; \mathbf{I}_k \; \mathbf{0}_{(k,\,(k+1)p)}]\mathbf{s_t}$. That is, the Kalman smoother implicitly computes the projection coefficients, say $\boldsymbol{\beta}$, for the equation $E(a_t \mid \{\mathbf{R_\tau}\}_{\tau=1}^{T}) = \sum_{\tau=1}^{T} \boldsymbol{\beta}_{\mathbf{t},\boldsymbol{\tau}} \mathbf{R_\tau}$, so that $\upsilon_t = a_t - \sum_{\tau=1}^{T} \boldsymbol{\beta}_{\mathbf{t},\boldsymbol{\tau}} \mathbf{R_\tau}$. From the law of iterated expectations $E(\upsilon_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T}) = E(a_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T}) - \sum_{\tau=1}^{T} \boldsymbol{\beta}_{\mathbf{t},\boldsymbol{\tau}} E(\mathbf{R_\tau} \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T})$. As a practical matter this can be computed in two steps:

Step 1: Use the Kalman smoother applied to (A.1) and (A.2) to compute the smoothed estimates of a_t and $\mathbf{R_t}$ given $\{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T}$. Call these estimates $a_{t/T}$ and $\mathbf{R_{t/T}}$.

Step 2: Construct $\sum_{\tau=1}^{T} \boldsymbol{\beta}_{\mathbf{t},\boldsymbol{\tau}} \mathbf{R}_{\boldsymbol{\tau}/\mathbf{T}}$ as the smoothed estimate of a_t from a state-space model with observation equation $\mathbf{R_{t/T}} = [0 \; \mathbf{I}_k \; \mathbf{0}_{(k,\,(k+1)p)}]\mathbf{s_t}$ and state transition equation (A.2). Then $E(\upsilon_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T})$ is the smoothed estimate of a_t from Step 1 minus its smoothed estimate from Step 2.

Similarly, recall that $\rho_t = E[\mathbf{F_t} \mid \{\boldsymbol{R_\tau}\}_{\tau=1}^{T}]$, which we estimate as $E(\rho_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T})$. From (2) $\mathbf{F_t} = (\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\mathbf{l}a_t + (\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}\mathbf{T}\mathbf{R_t}$. The component of the projection of $\mathbf{F_t}$ onto $\{\boldsymbol{R_\tau}\}_{\tau=1}^{T}$ that depends on a_t can be computed from the smoothed estimate of a_t in step 2. The component that depends directly on $\mathbf{R_t}$ can be computed from the smoothed estimate of $\mathbf{R_t}$ from step 1.

A.6 Calculating the Average Squared Coherences shown in the tables

Consider a VAR for a vector of variables $\mathbf{X_t}$ written as $\mathbf{\Phi}(L)\mathbf{X_t} = \boldsymbol{\varepsilon_t}$, where var($\boldsymbol{\varepsilon_t}$) = $\mathbf{\Omega}$. The spectral density of $\mathbf{X}$ at frequency ω is given by $\boldsymbol{S}(\omega) = \mathbf{\Phi}(e^{-i\omega})^{-1}\mathbf{\Omega}\mathbf{\Phi}(e^{i\omega})'^{-1}$. The squared coherence between the X_{it} and X_{jt} at frequency ω is $coh_{ij}(\omega) = \dfrac{\left|S_{ij}(\omega)\right|^2}{S_{ii}(\omega)S_{jj}(\omega)}$, which is recognized at the frequency domain analogue of the squared correlation between the variables. Similarly, the squared coherence between X_{it} and X_{jt}, controlling from X_{kt} at frequency ω is $coh_{ij\bullet k}(\omega) = \dfrac{\left|S_{ij\bullet k}(\omega)\right|^2}{S_{ii\bullet k}(\omega)S_{jj\bullet k}(\omega)}$, where $S_{ij\bullet k}(\omega) = S_{ij}(\omega) - S_{ik}(\omega)S_{kk}(\omega)^{-1}S_{kj}(\omega)$, and $S_{ii\bullet k}(\omega)$ and $S_{jj\bullet k}(\omega)$ are defined analogously.

Estimates of these coherences were computed by estimating the VAR parameters in $\mathbf{\Phi}(L)$ and $\mathbf{\Omega}$, and then plugging these estimates into the formula above. The average coherences reported in the tables are averages of the coherences over a fine grid of frequencies in the desired frequency band. Finally, standard errors were computed using the delta method and the asymptotic covariance matrix of the estimated VAR parameters.

A.7 Solution of the model in section 3

The representative agent's satisfies the conditions for Gorman aggregation, so it can be split into two stages. The optimal choice of how much of each variety to consume implies the optimality conditions:

(A.3) $$C_{it} = C_t(P_{it}/P_t)^{-\gamma} \quad \text{and} \quad C_{it}(j) = C_{it}(P_{it}(j)/P_{it})^{-\gamma},$$

(A.4) $$P_{it} = \left(\int_0^1 P_{it}(j)^{1-\gamma}dj\right)^{1/(1-\gamma)} \quad \text{and} \quad P_t = \left(N^{-1}\sum_{i=1}^{N} P_{it}^{1-\gamma}\right)^{1/(1-\gamma)}.$$

These imply that $S_t = P_tC_t$. Log-linearizing the static cost-of-living price indices around the steady state where all the prices are the same leads to:

(A.5) $$p_t = N^{-1}\sum_{i=1}^{N} p_{it} \quad \text{and} \quad p_{it} = \int_0^1 p_{it}(j)dj$$

The second-stage optimality conditions for the representative consumer are:

$$
\begin{aligned}
&\frac{1}{P_t C_t} = \frac{L_t^{\psi}}{(1-T_t)W_t} + z_t \\
&\frac{L_t^{\psi}}{(1-T_t)W_t} = \delta E_t\left[\frac{L_{t+1}^{\psi}}{(1-T_{t+1})W_{t+1}} + z_{t+1}\right] \\
&z_t(M_{t-1} + H_t - P_t C_t) = 0 \\
&\lim_{i\to\infty} \delta^{t+i} E_t\left(\frac{M_{t+1} L_{t+i}^{\psi}}{(1-T_{t+i})W_{t+i}}\right) = 0
\end{aligned}
\tag{A.6}
$$

where z_t is the Lagrange multiplier on the cash-in-advance constraint (15). The first condition is the static labor supply condition equating the marginal utility of consumption divided by its price to the marginal disutility of labor divided by its after-tax wage plus z_t reflecting the tightening of the cash-in-advance constraint that comes with consuming more. The second condition is the standard Euler equation, equating the marginal utility of an extra dollar today from working to its discounted expected value tomorrow, which also includes the relaxation of the budget constraint that comes with holding money as savings. Third, we have the complementary slackness condition associated with the constraint, and fourth the transversality condition.

We conjecture that, in equilibrium, the cash-in-advance constraint holds at all dates and states, so $z_t > 0$ always. To verify the conjecture, note that it implies that in equilibrium $P_t C_t = M_t$. Combining the first two optimality conditions in (A.6), we obtain an expression for $z_t = \left[1 - \delta E_t\left(M_t / M_{t+1}\right)\right] / M_t$, which given the assumption on $\bar{m}$ verifies the conjecture. Using the result for z_t on the first optimality condition gives an expression for the real wage, which after taking logs and ignoring constants is:

$$
w_t - p_t = y_t + \psi l_t - \ln(1 - T_t)\,. \tag{A.7}
$$

Turning to the problem of the firm, using the production function to replace out $L_{it}(j)$, the demand for each variety in (A.3) to substitute out $C_{it}(j)$, and the market clearing condition in the goods market to replace C_t for Y_t, real profits are:

(A.8) $$\left(\frac{P_{it}(j)}{P_t}\right)^{1-\gamma} Y_t - \left(\frac{P_{it}(j)}{P_t}\right)^{-\gamma/\eta}\left(\frac{W_t Y_t^{1/\eta}}{P_t X_{it}(j)^{1/\eta}}\right)$$

Maximizing this expression, taking logs and ignoring constants, the price charged by an attentive firm in sector *i* is:

(A.9) $$p_{it}^*(j) = p_t + \left(\frac{1}{\eta+\gamma(1-\eta)}\right)\left[\eta(w_t - p_t) + (1-\eta)y_t - x_{it}(j)\right]$$

The desired price rises one-to-one with the price index, and increases with marginal costs, which rise with the price of the labor input, rise with output because of diminishing returns to scale, and fall with higher productivity. Only ϕ_i of the firms actually set $p_{it}(j) = p_{it}^*(j)$ with the remaining $1-\phi_i$ choosing, up to a first-order log-linear approximation, the certainty-equivalent $p_{it}(j) = \hat{E}\left(p_{it}^*(j)\right)$.

Finally, integrating over *j* the prices set by the firms, using the definition of p_{it} in (A.5), and substituting out wages using equation (A.7), we obtain the solution in (20), where the parameters are: $\kappa^{-1} = \eta + \gamma(1-\eta)$ and $\alpha = (1+\eta\psi)\kappa$. The cash-in-advance constraint combined with market clearing in the goods market implies equation (22), and the first equation in (A.5) is (21).

Turning to the solutions for π_{it} and y_t, start by defining a new variable $q_t = p_t + \alpha y_t$. Taking the $\hat{E}(.)$ operator over both sides of (A.9) and substituting out for real wages gives: $\hat{E}\left(p_{it}\right) = \hat{E}\left[q_t - \alpha x_t + \kappa\eta\tau_t - \kappa\left(x_{it} - x_t\right)\right]$. Taking the average over *N*, using the process for $x_{it}(j)$ in (17), using the price index equation in (21), and ignoring constants: $\hat{E}\left(p_t\right) = \hat{E}\left(q_t\right) + \kappa\eta\tau_t - \alpha\bar{\theta}\zeta_{t-1}$. Then, using the definition of q_t and (22), $q_t = \alpha m_t + (1-\alpha)p_t$, so substituting out for p_t in the previous expression, we find: $\hat{E}\left(q_t\right) = \omega_t + \mu_{t-1} + \varpi\zeta_{t-1} + \left(1/\alpha - 1\right)\left(\kappa\eta\tau_t - \alpha\bar{\theta}\zeta_{t-1}\right)$. Now, using the quantity theory relation (22), it follows that: $\hat{E}(y_t) = -(\kappa\eta/\alpha)\tau_t + \bar{\theta}\zeta_{t-1}$. Moreover, one gets:

(A.10) $$\hat{E}(p_{it}) = \omega_t + \varpi\zeta_{t-1} + \mu_{t-1} + \left(\frac{\kappa\eta}{\alpha}\right)\tau_t - \left[\bar{\theta} + \kappa\left(\theta_i - \bar{\theta}\right)\right]\zeta_{t-1} - \kappa\chi_{it-1}$$

By going through the same steps, one solves for the "news" part $q_t - \hat{E}(q_t)$, and again taking the same steps find:

$$p_{it} - \hat{E}(p_{it}) = \left(\frac{\alpha\phi_i}{1-(1-\alpha)\bar{\phi}}\right)(\Delta\mu_t + \varpi\Delta\zeta_t) - \phi_i\left(\frac{\alpha\bar{\theta} + \bar{\phi}(1-\alpha)(1-\bar{\theta})(\alpha-\kappa)}{1-(1-\alpha)\bar{\phi}}\right)\Delta\zeta_t \tag{A.11}$$
$$-\kappa\phi_i(\theta_i - \bar{\theta})\Delta\zeta_t - \kappa\phi_i\Delta\chi_{it}$$

$$y_t - \hat{E}(y_t) = \left[\frac{1-\bar{\phi}}{1-(1-\alpha)\bar{\phi}}\right]\Delta\mu_t + \bar{\phi}\left[\frac{\alpha+\kappa(\bar{\theta}-1)}{1-(1-\alpha)\bar{\phi}}\right]\Delta\zeta_t . \tag{A.12}$$

Adding the two parts of each solution, taking first-differences of the solution for prices to obtain π_{it}, and using the definitions of the three components of price changes gives the expressions in (23)-(26).

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