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## Bubbles, Rational Expectations, and Financial Markets

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Economists and financial-market participants often hold quite different views about the pricing of assets. Economists usually believe that given the assumption of rational behavior and of rational expectations, the price of an asset must simply reflect market fundamentals, that is to say, can only depend on information about current and future returns from this asset. Deviations from this market fundamental value are taken as *prima facie* evidence of irrationality. Market participants on the other hand, often believe that fundamentals are only part of what determines the prices of assets. Extraneous events may well influence the price, if believed by other participants to do so—*crowd psychology* becomes an important determinant of prices.

It turns out that economists have overstated their case. Rationality both of behavior and also of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value—rational bubbles.

The purpose of the chapter is twofold. The first is to characterize the conditions under which such a deviation may appear, the shape it may take and the potential implications of such deviations. The second is to investigate how we can discover such deviations empirically. Some of the chapter is a review of recent work, but much of it is exploratory in nature and will appear a bit tentative. Although this is no doubt due to shortcomings in the authors' thinking, it may also be due to the nature of these bubbles. They present economists and econometricians with many questions to which they may have little to say.

Some may object to our dealing with rational bubbles only. There is little question that most large historical bubbles have elements of irrationality; Charles Kindleberger (1978) gives a fascinating description of many

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historical bubbles. Our justification is the standard one: it is hard to analyze rational bubbles. It would be much harder to deal with irrational bubbles.

### Rationality, Arbitrage, and Bubbles

Rationality of behavior and of expectations, together with market clearing, implies that assets are voluntarily held and that no agent can, given his private information and the information revealed by prices, increase his expected utility by reallocating his portfolio.

With many more assumptions, this leads to the standard *efficient market* or *no arbitrage* condition.

Let:

$$R_t = \frac{p_{t+1} - p_t + x_t}{p_t}$$

then

$$E(R_t | \Omega_t) = r$$

or equivalently

$$E(p_{t+1} | \Omega_t) - p_t + x_t = rp_t \quad (11.1)$$

$p_t$  is the price of the asset,  $x_t$  the direct return. We shall refer to  $x_t$  as the *dividend*, although it may take, depending on the asset, pecuniary or nonpecuniary forms.  $R_t$  is therefore the rate of return on holding the asset, which is the sum of the dividend price ratio and the capital gain.  $\Omega_t$  is the information set at time  $t$ , assumed common to all agents. The condition therefore states that the expected rate of return on the asset is equal to the interest rate  $r$ , assumed constant.

Among the assumptions needed to get equation 11.1 some are inessential and could be relaxed at the cost of increased notational complexity. These are the assumptions of a constant interest rate, no constraints on short sales, and risk neutrality. One assumption is, however, of more consequence: it is that, at least after having observed the price, all agents have the same information. As we shall show, bubbles can exist even in this case and these bubbles would remain even if agents had differential information. The question is, however, whether differential information allows for a larger class of bubbles and whether some aspects of real world bubbles involve differential information. We shall return to this issue—with not much to say—after we define bubbles. (Note also that because of the common information assumption, equation 11.1 is stronger than the usual efficient market formulation, which is that, *for the subset of information common to all agents*,  $\omega_t$ , the following relation holds:

$$E(p_{t+1} | \omega_t) - p_t + x_t = rp_t$$

Given the assumption of rational expectations and that agents do not forget, so that  $\Omega_t \subseteq \Omega_{t+1}$ , we can solve equation 11.1 recursively forward, using :

$$E(E(\cdot | \Omega_{t+i}) | \Omega_t) = E(\cdot | \Omega_t), \quad \forall i \geq 0$$

Thus the following  $p_t^*$  is a solution to equation 11.1:

$$p_t^* = \sum_{i=0}^{\infty} \theta^{i+1} E(x_{t+i} | \Omega_t), \quad \theta \equiv (1+r)^{-1} < 1 \quad (11.2)$$

$p^*$  is the present value of expected dividends and thus can be called the *market fundamental* value of the asset. (The term is standard in financial markets. It was introduced in economics in a similar context by Robert Flood and Peter Garber [1980]).  $p^*$  is, however, not the only solution to 11.1. Any  $p_t$  of the following form is a solution as well:

$$p_t = \sum_{i=0}^{\infty} \theta^{i+1} E(x_{t+i} | \Omega_t) + c_t = p_t^* + c_t,$$

with

$$E(c_{t+1} | \Omega_t) = \theta^{-1} c_t \quad (11.3)$$

Thus the market price can deviate from its market fundamental value without violating the arbitrage condition. As  $\theta^{-1} > 1$ , this deviation  $c_t$  must, however be expected to grow over time.<sup>1</sup>

Can this deviation  $c_t$  embody the popular notion of a bubble, namely movements in the price, apparently unjustified by information available at the time—taking the form of a rapid increase followed by a burst or at least a sharp decline? The following three examples give paths of  $c_t$  that satisfy equation 11.3 and seem to fit this notion.

The simplest is that of a deterministic bubble,  $c_t = c_0 \theta^{-t}$ . In this case the higher price is justified by the higher capital gain and the deviations grow exponentially. To be rational, such an increase in the price must continue forever, making such a deterministic bubble implausible. Consider, therefore, the second example:

$$\begin{aligned} c_t &= (\pi \theta)^{-1} c_{t-1} + \mu_t && \text{with probability } \pi \\ &= \mu_t && \text{with probability } 1 - \pi \end{aligned} \quad (11.4)$$



where

$$E(\mu_t | \Omega_{t-1}) = 0$$

How will such a bubble look? In each period, the bubble will remain, with probability  $\pi$ , or crash, with probability  $1 - \pi$ . While the bubble lasts, the actual average return is higher than  $r$ , so as to compensate for the risk of a crash. The average duration will be of  $(1 - \pi)^{-1}$ . There can be many minor extensions of this example, which also appear to capture certain aspects of bubbles. The probability that the bubble ends may well be a function of how long the bubble has lasted, or of how far the price is from market fundamentals. If  $\pi$  increases for some period of time,  $c_t$  will be growing at a decreasing exponential expected rate; if  $\pi$  decreases, the higher probability of a crash leads to an acceleration while the bubble lasts.

In these two examples, the bubble proceeds independently of the fundamental value. There is no reason for this to be true as the last example shows. Consider a war-related stock that pays 1 every period if there is a war, and 0 if there is no war. Suppose a war starts and in each period there is a probability  $\pi$  that the war goes on, a probability  $(1 - \pi)$  that it stops forever. The fundamental value is therefore equal to:

$$p_t^* = \sum_{i=0}^{\infty} \theta^{i+1} E(x_{t+i} | \Omega_t) = \sum_{i=0}^{\infty} \theta^{i+1} \pi^i = \theta(1 - \theta\pi)^{-1}$$

Furthermore, it is constant during the duration of the war. The price may, however, increase above  $p^*$  in anticipation of future increases during the war. For example, the following bubble might arise:

$$\begin{aligned} c_t &= c_0 \\ c_{t+i} &= (\theta\pi)^{-1} c_{t+i-1} && \text{if there is war at } t+i, \\ &= 0 && \text{if there is no war at } t+i. \end{aligned}$$

This will lead to an increase in the price above its fundamental value initially, a further increase during the war, and a crash in both the fundamental value and the bubble when the war ends.

Now that a definition and examples of bubbles have been given, we may return to the simplifying assumptions made to obtain equation 11.1. What if agents were risk averse? As the last two examples show, bubbles are likely to increase the risk associated with holding the asset. If agents are risk