Aggregate Implications of Changing Sectoral Trends

Technical Appendix and Supplementary Material

Andrew Foerster
Federal Reserve Bank of San Francisco

Andreas Hornstein
Federal Reserve Bank of Richmond

Pierre-Daniel Sarte
Federal Reserve Bank of Richmond

Mark Watson
Princeton University

April 23, 2021

Contents

1 Statistical Model of Trend Growth

1.1 Low-Frequency Transformation of the Annual Growth Rates

As discussed in the text, we follow the methods discussed in Müller and Watson (2020) to extract low frequency trends in the growth rates of GDP, TFP, and labor input. This process is summarized in Figure A1 which shows results for the growth rate of aggregate GDP. Panel (a) shows the raw data, that is, the cyclically adjusted annual growth rates of GDP. Panel (b) plots nine regressors, a constant (in blue) and eight cosine functions, $\Psi_j(s) = \sqrt{2}\cos(js\pi)$, with $s = (t-1/2)/T$, for $j = 1, ..., 8$, where $T = 69$ years is the sample size. Note that $\Psi_j(s)$ has period $2T/j$ so that the first cosine function has a period of 138 years, the second has a period 69 years, and so forth. The last cosine function, $j = 8$, has period 17.25 years. Panel (c) shows the fitted values from the regression of the data from panel (a) onto the regressors in panel (b). The solid line is from the regression onto the constant and all $q = 8$ cosine functions; this captures periodicities longer than 17.25 years and is the trend used in the body of the paper. The figure also shows the fitted values using only the first $q = 6$ cosine functions corresponding to periods longer than 23 years. This trend was used in the robustness analysis reported in Figure 11 in the paper. Panel (d) plots the OLS regression coefficients from the regressions of the data on the cosine functions. These eight regression coefficients are denoted by $\mathbf{X}$ and summarize the variation in the low-frequency trends plotted in panel (c). The values of $\mathbf{X}$ are called the cosine transforms of the raw data.

1.2 Factor Model

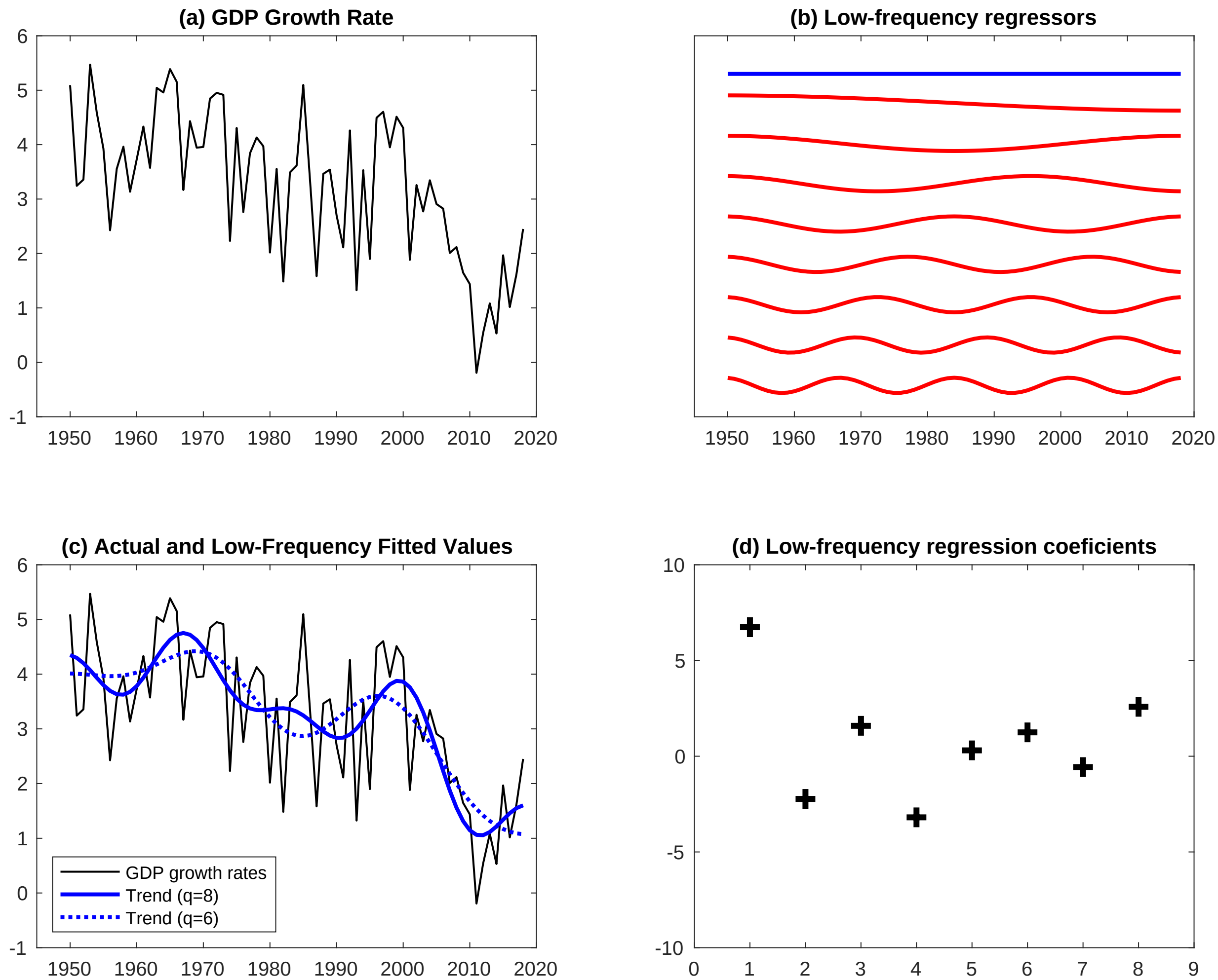
We use a low-frequency factor model. Written in terms of the growth rates of labor and TFP, the model is

$$\begin{bmatrix} \Delta \ln \ell_{i,t} \\ \Delta \ln z_{i,t} \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell & 0 \\ 0 & \lambda_i^z \end{bmatrix} \begin{bmatrix} f_t^\ell \\ f_t^z \end{bmatrix} + \begin{bmatrix} u_{i,t}^\ell \\ u_{i,t}^z \end{bmatrix}, \tag{1}$$

where $f_t = (f_t^\ell \ f_t^z)'$ are unobserved common factors, $\lambda_i = (\lambda_i^\ell \ \lambda_i^z)'$ are factor loadings, and $u_{i,t} = (u_{i,t}^\ell \ u_{it}^z)'$ are sector-specific disturbances. Written in terms of the cosine transforms $(\mathbf{X}_i^\ell, \mathbf{X}_i^z, \mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$, the factor model is:

$$\begin{bmatrix} \mathbf{X}_i^\ell \\ \mathbf{X}_i^z \end{bmatrix} = \begin{bmatrix} \lambda_i^\ell I_q & 0 \\ 0 & \lambda_i^z I_q \end{bmatrix} \begin{bmatrix} \mathbf{F}^\ell \\ \mathbf{F}^z \end{bmatrix} + \begin{bmatrix} \mathbf{U}_i^\ell \\ \mathbf{U}_i^z \end{bmatrix}, \tag{2}$$

Figure A1: Computing low-frequency transformations



which characterizes the low-frequency variation and covariation in the data. See Müller and Watson (2020) for a detailed *Handbook* discussion related to low-frequency factor models. A key result from that analysis is that $(\mathbf{X}_i^\ell, \mathbf{X}_i^z, \mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$ is approximately normally distributed in large T samples, so that (2) can be estimated by standard factor analysis methods after parameterizing the various covariance matrices.

As discussed in the text, we use a 'local-level model' parameterizations for each of the covariance matrices. This model parameterizes the low frequency spectrum (that is, the trend variation) by linearly combining a flat spectrum (from an $I(0)$ component) and a steeply decreasing spectrum (from an $I(1)$ component). In this model, the growth rate time series behave like the sum of independent $I(0)$ and $I(1)$ processes over the long run. The resulting covariance matrix of the cosine transforms depends on two parameters, (σ^2, γ), where σ is an overall scale parameter and γ governs the relative importance of the $I(0)$ and

$I(1)$ components; larger values of σ produce a more variable low frequency trend and larger values of γ produce a more persistent trend. Let $\mathbf{X}$ denote one of the $q \times 1$ vector of cosine transforms in $(\mathbf{F}^\ell, \mathbf{F}^z, \mathbf{U}_i^\ell, \mathbf{U}_i^z)$. The resulting covariance matrix has the form:

$$\text{Var}(\mathbf{X}) = \sigma_X^2 D(\gamma_X), \tag{3}$$

where the notation emphasizes that each component has its own (σ, γ) parameter. As shown in Müller and Watson (2020), the matrix $D(\gamma)$ is diagonal with $D_{jj}(\gamma) = 1 + \gamma^2(j\pi)^{-2}$.

We use this local-level parameterization to characterize the covariance matrix of each of the components in $(\mathbf{F}^\ell, \mathbf{F}^z, \{\mathbf{U}_i^\ell, \mathbf{U}_i^z\}_{i=1}^{16})$, where each component has its own value of (σ, γ); thus, for example, $\text{Var}(\mathbf{F}^\ell) = \sigma_{F,\ell}^2 D(\gamma_F^\ell)$, and similarly for $\mathbf{F}^z$ and each of the $\mathbf{U}_i^\ell$ and $\mathbf{U}_i^z$ components. As in a standard factor model, we assume that $\mathbf{F}$ and $\mathbf{U}$ are uncorrelated, as are $\mathbf{U}_i$ and $\mathbf{U}_j$ for $i \neq j$. We allow $\mathbf{F}^\ell$ and $\mathbf{F}^z$ to be correlated by introducing a covariance parameter $\sigma_{F,\ell z}$ and letting $\text{Cov}(\mathbf{F}^\ell, \mathbf{F}^z) = \sigma_{F,\ell z} D(\gamma_F^\ell)^{1/2} D(\gamma_F^z)^{1/2}$. We use an analogous parameterization for the covariance between each of the sectoral values of $\mathbf{U}_i^\ell$ and $\mathbf{U}_i^z$. Thus, each pair $(\mathbf{F}^\ell, \mathbf{F}^z)$ or $(\mathbf{U}_i^\ell, \mathbf{U}_i^z)$ is characterized by a parameter pair, $\gamma = (\gamma^\ell, \gamma^z)$, that governs persistence and a 2×2 covariance matrix, say Σ, that includes $(\sigma_\ell^2, \sigma_z^2, \sigma_{\ell z})$.

1.3 Bayes Estimation

Our ultimate goal is to decompose the various sectoral and aggregate trends into components associated with the common factors (f^l, f^z) and sector specific terms $\{u_i^\ell, u_i^z\}_{i=1}^{16}$. As shown in Figure A1, the trends in these factors are deterministic functions of $(\mathbf{F}^\ell, \mathbf{F}^z, \{\mathbf{U}_i^\ell, \mathbf{U}_i^z\}_{i=1}^{16})$. The probability distribution of $(\mathbf{F}^\ell, \mathbf{F}^z, \{\mathbf{U}_i^\ell, \mathbf{U}_i^z\}_{i=1}^{16})$ given $\{\mathbf{X}_i^\ell, \mathbf{X}_i^z\}_{i=1}^{16}$ can be computed using standard signal extraction formulae, given values for the various parameters of the model. The probability distribution of the parameters given $\{\mathbf{X}_i^\ell, \mathbf{X}_i^z\}_{i=1}^{16}$ can be computed using standard Bayes methods. As discussed in Müller and Watson (2020), the likelihood is Gaussian and estimation is facilitated by using standard conjugate priors for some of the parameters and multinomial priors for others.

1.3.1 Priors

There are three sets of parameters in the model:

- The factor loadings, $(\lambda^\ell, \lambda^z)$,

- The covariance matrices,

$$\Sigma_F = \begin{bmatrix} \sigma^2_{F,\ell} & \sigma_{F,\ell z} \\ \sigma_{F,z\ell} & \sigma^2_{F,z} \end{bmatrix} \text{ and } \Sigma_{U,i} = \begin{bmatrix} \sigma^2_{U,i,\ell} & \sigma_{U,i,\ell z} \\ \sigma_{U,i,z\ell} & \sigma^2_{U,i,z} \end{bmatrix} \text{ for } i = 1, ..., 16,$$

- The persistence parameters, $\gamma_F = (\gamma_F^\ell, \gamma_F^z)$ and $\gamma_{U,i} = (\gamma_{U,i}^\ell, \gamma_{U,i}^z)$ for $i = 1, ..., 16$.

As discussed in the text, we use the following independent priors for these parameters:

- $\lambda^\ell \sim N(\mathbf{1}, P_\ell)$, where $\mathbf{1}$ is a $q \times 1$ vector of ones and $\mathbf{P}_\ell = \eta^2(\mathbf{I}_n - s_\ell(s_\ell' s_\ell)^{-1} s_\ell')$, where s_ℓ is the vector of sectoral labor shares. The prior for λ^z is analogous, but uses s_z, the sectoral TFP shares. The parameter η governs the tightness of the prior. The benchmark specification uses $\eta = 1$, and results using $\eta = 0.5$ and $\eta = 2.0$ are summarized in the text and Tables A2-A3.

- For each Σ matrix, we use an inverse-Wishart with $\nu = 0.01$ degrees of freedom and scale $\nu \mathbf{I}_2$. The small value for ν makes this prior nearly uninformative.

- Unlike for λ and Σ, there isn't a conjugate prior for γ. We use a prior with $\ln(\gamma) \sim U(0, \ln(500))$. This puts relatively more weight on small values of γ, i.e., small weight on the $I(1)$ component of the local-level model (consistent with a body of evidence beginning in Stock and Watson (1998)) but allows for low-frequency behavior dominated by $I(1)$ dynamics. We appoximate this prior by a 15-point equally-spaced discrete grid on $(0, \ln(500))$, with equal prior weight on each of the grid points.

1.3.2 MCMC Algorithm

Müller and Watson (2020) discusses an MCMC algorithm for a closely related model. The steps are standard and are outlined here. Let $\mathbf{X} = \{\mathbf{X}_i^\ell, \mathbf{X}_i^z\}_{i=1}^{16}$, $\mathbf{F} = (\mathbf{F}^\ell, \mathbf{F}^z)$, and $\mathbf{U} = \left\{\mathbf{U}_i^\ell, \mathbf{U}_i^z\right\}_{i=1}^{16}$. Let $\theta_1 = (\Sigma_F, \{\Sigma_{U,i}\}_{i=1}^{16})$, $\theta_2 = ((\gamma_F^\ell, \gamma_T^z), \left\{\gamma_{U,i}^\ell, \gamma_{U,i}^z\right\}_{i=1}^{16})$, $\theta_3 = (\lambda^\ell, \lambda^z)$, and $\theta = (\theta_1, \theta_2, \theta_3)$. The MCMC algorithm uses is a Gibbs algorithm with four steps:

1. Draw $\mathbf{F}$ from the distribution $\mathbf{F}|(\mathbf{X}, \theta)$. The distribution of $\mathbf{F}|(\mathbf{X}, \theta)$ is normal, and the draw uses standard multivariate normal formulae.

2. Draw θ_1 from the distribution of $\theta_1|(\mathbf{X}, \mathbf{F}, \theta_2, \theta_3)$. Given $\mathbf{F}$, $D(\gamma_F^\ell)$ and $D(\gamma_F^z)$, the draw of Σ_F is a draw from the inverse Wishart distribution. $\Sigma_{U,i}$ is drawn analogously given $\mathbf{U}_i^\ell = \mathbf{X}_i^\ell - \lambda_i^\ell \mathbf{F}^\ell$ and $\mathbf{U}_i^z = \mathbf{X}_i^z - \lambda_i^z \mathbf{F}^z$.

3. Draw θ_2 from the distribution of $\theta_2|(\mathbf{X}, \mathbf{F}, \theta_1, \theta_3)$. Given $\mathbf{F}^\ell$ and σ_F^ℓ, the likelihood for γ_F^ℓ can be computed at each of the grid points making up the support of γ_F^ℓ; the distribution of $\gamma_F^\ell|\mathbf{F}^\ell, \sigma_F^\ell$ is multinomial. Draws for the other γ parameters are similarly obtained.

4. Draw θ_3 from the the distribution of $\theta_3|(\mathbf{X}, \mathbf{F}, \theta_1, \theta_2)$. Note that this distribution is normal and corresponds to drawing linear regression coefficients in a regression model with a known covariance matrix for the regressions errors.

The results shown in the text were computed from 550,000 draws from this algorithm. The first 50k draws were discarded and every 200th draw of the remaining 500k draws were saved. The code was tested using the procedure outlined in Geweke (2004).

1.4 Results

Table A1 summarizes the posterior for the benchmark model ($q = 8$, $\eta = 1.0$). The entries in the table show the posterior median, 68% and 90% equal-tail credible intervals (in parentheses and brackets, respectively) for each of the model parameters. Also shown are the fraction of the variability of the trends in labor and TFP explained by the common factors, R_ℓ^2 and R_z^2, and the correlation of the sector-specific labor and TFP trends (labeled $corr(\ell, z)$ in the table). The final row of the table shows the parameters associated with the factors, where in the this case the R^2 measures the fraction of variance in aggregate labor and TFP explained by the common factors and $corr(\ell, z)$ shows the correlation of the trends in the common factors. Tables A2-A4 show results for the alternative models presented in Figures 11 in the text.

Table A1: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 1.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	2.01	0.59	1.14	5.88	-5.35	45.80	1.00	0.21	0.02	-0.32
	(1.24,2.71)	(-0.59, 1.64)	(0.38, 2.41)	(4.46, 7.99)	(-13.22, -1.62)	(17.61, 119.16)	(0.00, 2.60)	(0.06, 0.44)	(0.00, 0.13)	(-0.52, -0.15)
	[0.51, 3.15]	[-1.30, 2.23]	[0.18, 3.57]	[3.65, 9.95]	[-24.62, -0.62]	[10.92, 310.00]	[0.00, 6.77]	[0.01, 0.60]	[0.00, 0.34]	[-0.69, -0.05]
Min	0.73	1.10	10.36	11.08	-38.63	1.61	4.20	0.01	0.01	-0.35
	(-0.17, 1.64)	(0.10, 2.09)	(7.58, 14.29)	(5.94, 15.82)	(-111.87, -3.35)	(0.00, 6.77)	(1.00, 28.40)	(0.00, 0.07)	(0.00, 0.04)	(-0.63, -0.06)
	[-0.83, 2.35]	[-0.61, 2.81]	[3.71, 18.26]	[1.05, 20.14]	[-220.92, 13.71]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.18]	[0.00, 0.10]	[-0.78, 0.11]
Utl	1.13	1.36	2.01	4.28	1.17	6.77	2.60	0.24	0.05	0.22
	(0.41, 1.82)	(0.36, 2.35)	(0.48, 3.13)	(2.08, 6.12)	(-0.33, 6.46)	(1.00, 73.88)	(1.00, 28.40)	(0.04, 0.58)	(0.00, 0.29)	(-0.06, 0.58)
	[-0.16, 2.31]	[-0.32, 3.08]	[0.16, 4.18]	[0.41, 8.08]	[-3.71, 14.71]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.78]	[0.00, 0.63]	[-0.33, 0.77]
Con	1.55	1.26	3.45	1.22	-2.05	1.61	45.80	0.33	0.02	-0.25
	(0.95, 2.08)	(0.21, 2.66)	(2.57, 4.73)	(0.30, 3.80)	(-8.83, -0.26)	(0.00, 4.20)	(10.92, 192.20)	(0.10, 0.61)	(0.00, 0.19)	(-0.55, -0.04)
	[0.42, 2.49]	[-0.50, 3.24]	[1.95, 6.09]	[0.15, 6.71]	[-19.53, 0.10]	[0.00, 17.61]	[4.20, 500.00]	[0.02, 0.77]	[0.00, 0.42]	[-0.72, 0.02]
DurG	0.40	1.31	2.15	4.80	-2.16	10.92	6.77	0.03	0.03	-0.35
	(-0.23, 1.03)	(0.44, 2.17)	(0.59, 3.72)	(1.17, 7.44)	(-12.28, -0.10)	(4.20, 73.88)	(1.00, 73.88)	(0.00, 0.18)	(0.00, 0.15)	(-0.63, -0.05)
	[-0.67, 1.45]	[-0.10, 2.72]	[0.17, 5.38]	[0.29, 9.61]	[-26.63, 0.75]	[1.00, 310.00]	[0.00, 310.00]	[0.00, 0.39]	[0.00, 0.38]	[-0.77, 0.16]
NdG	0.59	1.22	2.48	3.97	-2.24	6.77	6.77	0.06	0.04	-0.36
	(-0.20, 1.38)	(0.36, 2.13)	(0.49, 4.01)	(2.40, 5.79)	(-9.03, -0.17)	(1.00, 119.16)	(1.61, 17.61)	(0.01, 0.29)	(0.00, 0.23)	(-0.65, -0.06)
	[-0.75, 1.86]	[-0.26, 2.80]	[0.17, 5.29]	[0.62, 7.50]	[-18.98, 1.77]	[0.00, 310.00]	[0.00, 119.16]	[0.00, 0.54]	[0.00, 0.52]	[-0.80, 0.15]
WT	1.09	0.88	1.20	2.05	0.34	4.20	10.92	0.53	0.04	0.20
	(0.62, 1.49)	(0.06, 1.74)	(0.65, 1.81)	(0.37, 3.71)	(-0.06, 2.08)	(1.00, 28.40)	(1.61, 119.16)	(0.17, 0.81)	(0.00, 0.20)	(-0.06, 0.53)
	[0.22, 1.82]	[-0.55, 2.35]	[0.18, 2.44]	[0.15, 4.97]	[-0.84, 4.65]	[0.00, 119.16]	[0.00, 310.00]	[0.03, 0.90]	[0.00, 0.44]	[-0.30, 0.72]
RT	0.80	1.14	1.76	3.42	0.24	2.60	2.60	0.26	0.05	0.06
	(0.26, 1.29)	(0.17, 2.82)	(1.02, 2.51)	(0.92, 5.05)	(-1.85, 2.42)	(1.00, 17.61)	(1.00, 10.92)	(0.04, 0.60)	(0.00, 0.85)	(-0.25, 0.62)
	[-0.19, 1.65]	[-0.49, 3.48]	[0.26, 3.29]	[0.31, 6.57]	[-5.40, 5.41]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.77]	[0.00, 0.98]	[-0.51, 0.94]
TW	-0.04	0.88	2.91	3.04	0.35	4.20	2.60	0.05	0.06	0.06
	(-0.75, 0.72)	(-0.02, 1.79)	(1.91, 4.12)	(2.15, 4.26)	(-2.73, 3.67)	(1.00, 10.92)	(1.00, 10.92)	(0.00, 0.23)	(0.00, 0.28)	(-0.25, 0.36)
	[-1.23, 1.33]	[-0.62, 2.40]	[0.61, 5.29]	[0.90, 5.49]	[-7.43, 8.65]	[0.00, 73.88]	[0.00, 45.80]	[0.00, 0.43]	[0.00, 0.54]	[-0.49, 0.59]
Inf	1.34	0.77	3.81	3.05	-2.54	1.61	4.20	0.22	0.03	-0.25
	(0.69, 2.01)	(-0.18, 1.81)	(2.80, 5.31)	(0.70, 4.58)	(-9.15, -0.03)	(0.00, 6.77)	(1.00, 73.88)	(0.04, 0.51)	(0.00, 0.19)	(-0.56, -0.00)
	[0.19, 2.47]	[-0.75, 2.79]	[1.48, 7.03]	[0.20, 6.02]	[-19.33, 3.42]	[0.00, 45.80]	[0.00, 310.00]	[0.01, 0.70]	[0.00, 0.49]	[-0.74, 0.21]
F(x-H)	1.92	0.35	1.59	1.59	0.02	2.60	2.60	0.76	0.08	0.01
	(1.34, 2.48)	(-0.42, 1.34)	(1.01, 2.54)	(0.90, 2.30)	(-1.02, 1.20)	(1.00, 10.92)	(0.00, 10.92)	(0.35, 0.92)	(0.01, 0.40)	(-0.41, 0.40)
	[0.76, 2.90]	[-0.92, 2.18]	[0.47, 3.44]	[0.27, 2.97]	[-2.72, 2.89]	[0.00, 28.40]	[0.00, 73.88]	[0.08, 0.97]	[0.00, 0.80]	[-0.73, 0.62]
PBS	1.87	0.90	1.42	1.65	-1.90	6.77	10.92	0.64	0.06	-0.92
	(1.48, 2.29)	(-0.01, 1.80)	(0.52, 2.65)	(0.59, 3.20)	(-7.57, -0.31)	(1.61, 45.80)	(2.60, 45.80)	(0.31, 0.87)	(0.00, 0.39)	(-0.98, -0.67)
	[1.15, 2.63]	[-0.40, 2.42]	[0.24, 3.66]	[0.25, 4.41]	[-14.38, -0.07]	[0.00, 73.88]	[1.00, 119.16]	[0.12, 0.96]	[0.00, 0.73]	[-1.00, -0.32]
EdHe	0.59	1.36	1.42	2.23	-1.84	6.77	4.20	0.16	0.10	-0.63
	(-0.06, 1.05)	(0.24, 2.49)	(0.56, 2.18)	(0.66, 3.68)	(-5.17, -0.38)	(1.00, 45.80)	(1.00, 45.80)	(0.01, 0.56)	(0.01, 0.54)	(-0.88, -0.26)
	[-0.49, 1.40]	[-0.41, 3.09]	[0.23, 3.03]	[0.22, 4.92]	[-10.12, -0.09]	[0.00, 119.16]	[0.00, 192.20]	[0.00, 0.77]	[0.00, 0.84]	[-0.96, -0.10]
AEFS	1.19	0.37	0.53	2.04	-0.22	45.80	2.60	0.37	0.05	-0.18
	(0.69, 1.75)	(-0.39, 1.31)	(0.17, 1.39)	(1.16, 2.89)	(-1.20, 0.02)	(10.92, 119.16)	(1.00, 17.61)	(0.11, 0.67)	(0.00, 0.26)	(-0.51, 0.02)
	[0.27, 2.12]	[-0.93, 2.06]	[0.09, 2.36]	[0.28, 3.73]	[-2.64, 0.56]	[2.60, 192.20]	[0.00, 119.16]	[0.02, 0.83]	[0.00, 0.51]	[-0.75, 0.24]
OthS	0.68	0.74	3.10	0.65	-0.13	6.77	73.88	0.06	0.02	-0.07
	(-0.10, 1.48)	(-0.10, 1.63)	(0.66, 4.83)	(0.19, 2.22)	(-1.78, 0.23)	(1.00, 73.88)	(17.61, 192.20)	(0.01, 0.23)	(0.00, 0.10)	(-0.35, 0.17)
	[-0.61, 2.05]	[-0.65, 2.26]	[0.20, 6.31]	[0.11, 4.30]	[-7.00, 1.22]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.42]	[0.00, 0.26]	[-0.56, 0.45]
Hous	0.82	0.75	10.38	1.31	0.32	6.77	6.77	0.01	0.10	0.07
	(-0.13, 1.74)	(0.08, 1.50)	(1.82, 15.95)	(0.43, 2.10)	(-2.98, 5.61)	(1.00, 119.16)	(1.00, 45.80)	(0.00, 0.04)	(0.01, 0.44)	(-0.21, 0.40)
	[-0.77, 2.34]	[-0.38, 2.17]	[0.53, 20.39]	[0.15, 2.85]	[-11.58, 16.28]	[0.00, 310.00]	[0.00, 192.20]	[0.00, 0.10]	[0.00, 0.74]	[-0.44, 0.64]
Fac			0.48	0.69	-0.08	28.40	4.20	0.67	0.30	-0.29
			(0.18, 1.05)	(0.28, 1.33)	(-0.46, 0.05)	(10.92, 73.88)	(1.00, 28.40)	(0.48, 0.82)	(0.10, 0.58)	(-0.35, 0.22)
			[0.10, 1.58]	[0.13, 1.85]	[-1.17, 0.36]	[2.60, 119.16]	[0.00, 73.88]	[0.34, 0.89]	[0.03, 0.75]	[-0.25, -0.35]

Notes: For each sector, the entries are the posterior median and (68%) and [90%] credible intervals for each parameter in the model. Also shown are the fraction of variance explained by the common factor (R_ℓ^2 and R_z^2) and the correlation between between the trends associated with the u components (labeled $corr(\ell, z)$). The row labeled *Fac* shows the parameter values for the factors, the fraction of variance for aggregate labor and TFP explained by the common factors (R_ℓ^2 and R_z^2), and the correlation of factor trends (labeled $corr(\ell, z)$).

Table A2: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 0.5$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	1.38	0.84	1.19	6.06	-5.32	45.80	1.61	0.15	0.01	-0.32
	(0.90, 1.84)	(0.36, 1.35)	(0.36, 2.48)	(4.68, 8.17)	(-13.50, -1.48)	(17.61, 192.20)	(0.00, 4.20)	(0.05, 0.36)	(0.00, 0.05)	(-0.54, -0.14)
	[0.54, 2.11]	[-0.01, 1.68]	[0.17, 3.73]	[3.82, 10.37]	[-25.68, -0.57]	[10.92, 310.00]	[0.00, 6.77]	[0.02, 0.54]	[0.00, 0.13]	[-0.70, -0.04]
Min	0.91	1.04	10.34	11.06	-39.25	2.60	4.20	0.02	0.00	-0.35
	(0.45, 1.37)	(0.55, 1.49)	(7.51, 14.22)	(6.22, 15.77)	(-108.08, -3.10)	(0.00, 6.77)	(1.00, 17.61)	(0.00, 0.08)	(0.00, 0.02)	(-0.62, -0.06)
	[0.14, 1.72]	[0.18, 1.80]	[3.37, 18.06]	[1.02, 19.90]	[-209.93, 12.34]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.16]	[0.00, 0.05]	[-0.76, 0.11]
Utl	1.09	1.12	1.95	4.24	1.35	4.20	2.60	0.33	0.02	0.26
	(0.66, 1.51)	(0.61, 1.57)	(0.58, 3.01)	(2.11, 6.07)	(-0.03, 6.51)	(1.00, 73.88)	(1.00, 28.40)	(0.10, 0.64)	(0.00, 0.14)	(-0.00, 0.60)
	[0.35, 1.81]	[0.27, 1.90]	[0.19, 3.98]	[0.38, 7.82]	[-2.98, 14.31]	[0.00, 192.20]	[0.00, 192.20]	[0.02, 0.83]	[0.00, 0.34]	[-0.25, 0.79]
Con	1.15	1.02	3.61	0.98	-1.43	1.61	73.88	0.27	0.01	-0.20
	(0.78, 1.52)	(0.52, 1.45)	(2.69, 4.93)	(0.26, 3.75)	(-7.47, -0.24)	(0.00, 4.20)	(10.92, 310.00)	(0.08, 0.55)	(0.00, 0.05)	(-0.45, -0.03)
	[0.47, 1.75]	[0.19, 1.76]	[2.10, 6.32]	[0.14, 6.62]	[-17.67, 0.12]	[0.00, 10.92]	[4.20, 500.00]	[0.03, 0.70]	[0.00, 0.12]	[-0.64, 0.02]
DurG	0.78	1.09	2.31	4.61	-2.02	10.92	6.77	0.11	0.01	-0.29
	(0.41, 1.15)	(0.66, 1.50)	(0.67, 3.88)	(1.11, 7.13)	(-11.07, -0.02)	(2.60, 73.88)	(1.00, 73.88)	(0.02, 0.31)	(0.00, 0.08)	(-0.61, -0.01)
	[0.16, 1.42]	[0.35, 1.81]	[0.18, 5.46]	[0.26, 9.33]	[-25.20, 1.70]	[1.00, 310.00]	[0.00, 310.00]	[0.00, 0.51]	[0.00, 0.22]	[-0.75, 0.21]
NdG	0.89	1.06	2.78	4.09	-3.05	4.20	6.77	0.16	0.02	-0.37
	(0.48, 1.32)	(0.59, 1.52)	(0.79, 4.12)	(2.62, 5.88)	(-10.78, -0.29)	(1.00, 73.88)	(1.61, 17.61)	(0.03, 0.41)	(0.00, 0.10)	(-0.65, -0.07)
	[0.16, 1.58]	[0.29, 1.87]	[0.20, 5.44]	[0.85, 7.69]	[-20.14, 1.54]	[0.00, 310.00]	[0.00, 119.16]	[0.01, 0.60]	[0.00, 0.29]	[-0.78, 0.13]
WT	0.94	0.94	1.21	2.12	0.19	4.20	10.92	0.55	0.03	0.13
	(0.64, 1.24)	(0.48, 1.43)	(0.59, 1.83)	(0.37, 3.74)	(-0.20, 1.77)	(1.00, 28.40)	(1.61, 119.16)	(0.22, 0.80)	(0.00, 0.12)	(-0.12, 0.44)
	[0.41, 1.46]	[0.15, 1.73]	[0.17, 2.53]	[0.14, 5.03]	[-1.55, 4.45]	[0.00, 119.16]	[0.00, 310.00]	[0.07, 0.90]	[0.00, 0.32]	[-0.36, 0.65]
RT	0.84	0.99	1.75	3.81	0.00	2.60	2.60	0.39	0.03	0.00
	(0.53, 1.16)	(0.53, 1.43)	(1.14, 2.44)	(2.56, 5.27)	(-2.36, 2.23)	(1.00, 10.92)	(1.00, 10.92)	(0.14, 0.68)	(0.00, 0.13)	(-0.30, 0.29)
	[0.31, 1.40]	[0.22, 1.75]	[0.31, 3.24]	[0.54, 6.68]	[-6.20, 5.31]	[0.00, 119.16]	[0.00, 119.16]	[0.03, 0.81]	[0.00, 0.30]	[-0.54, 0.51]
TW	0.61	0.97	3.04	3.11	0.11	4.20	2.60	0.07	0.04	0.01
	(0.14, 1.06)	(0.50, 1.45)	(1.85, 4.43)	(2.24, 4.26)	(-3.84, 3.19)	(1.00, 17.61)	(1.00, 10.92)	(0.01, 0.23)	(0.00, 0.19)	(-0.31, 0.31)
	[-0.17, 1.41]	[0.19, 1.76]	[0.42, 5.87]	[1.42, 5.70]	[-10.11, 7.66]	[0.00, 119.16]	[0.00, 28.40]	[0.00, 0.42]	[0.00, 0.41]	[-0.56, 0.52]
Inf	1.14	0.96	3.78	3.11	-2.26	1.61	4.20	0.23	0.02	-0.22
	(0.72, 1.56)	(0.51, 1.44)	(2.85, 5.21)	(0.75, 4.59)	(-9.08, 0.16)	(0.00, 6.77)	(1.00, 73.88)	(0.07, 0.51)	(0.00, 0.15)	(-0.53, 0.02)
	[0.40, 1.82]	[0.19, 1.76]	[1.99, 6.88]	[0.20, 5.95]	[-18.68, 3.29]	[0.00, 28.40]	[0.00, 310.00]	[0.02, 0.68]	[0.00, 0.36]	[-0.72, 0.22]
F(x-H)	1.40	0.79	1.83	1.70	0.13	2.60	1.61	0.64	0.08	0.05
	(0.99, 1.76)	(0.38, 1.24)	(1.23, 2.84)	(1.18, 2.40)	(-1.01, 1.62)	(1.00, 10.92)	(0.00, 10.92)	(0.26, 0.87)	(0.01, 0.33)	(-0.30, 0.40)
	[0.67, 2.02]	[0.08, 1.57]	[0.85, 3.81]	[0.45, 3.12]	[-2.81, 3.77]	[0.00, 28.40]	[0.00, 45.80]	[0.08, 0.94]	[0.00, 0.58]	[-0.55, 0.62]
PBS	1.49	1.08	1.50	1.56	-1.79	6.77	10.92	0.61	0.06	-0.87
	(1.20, 1.77)	(0.61, 1.52)	(0.51, 2.84)	(0.53, 3.05)	(-6.90, -0.28)	(1.00, 28.40)	(2.60, 45.80)	(0.29, 0.86)	(0.01, 0.37)	(-0.98, -0.54)
	[0.95, 1.96]	[0.26, 1.84]	[0.22, 3.90]	[0.22, 4.42]	[-14.13, -0.07]	[0.00, 73.88]	[1.00, 119.16]	[0.12, 0.96]	[0.00, 0.68]	[-1.00, -0.23]
EdHe	0.73	1.33	1.51	2.02	-1.66	2.60	6.77	0.39	0.08	-0.58
	(0.42, 1.02)	(0.80, 1.87)	(0.87, 2.17)	(0.52, 3.39)	(-4.69, -0.34)	(1.00, 17.61)	(1.00, 73.88)	(0.10, 0.70)	(0.01, 0.50)	(-0.85, -0.21)
	[0.11, 1.29]	[0.40, 2.20]	[0.32, 2.89]	[0.19, 4.54]	[-8.49, -0.07]	[0.00, 73.88]	[0.00, 192.20]	[0.01, 0.87]	[0.00, 0.88]	[-0.95, -0.05]
AEFS	1.09	0.83	0.48	2.08	-0.19	45.80	2.60	0.43	0.04	-0.16
	(0.75, 1.45)	(0.31, 1.33)	(0.17, 1.27)	(1.25, 2.91)	(-1.12, 0.05)	(10.92, 119.16)	(1.00, 17.61)	(0.17, 0.71)	(0.00, 0.23)	(-0.48, 0.04)
	[0.49, 1.70]	[-0.02, 1.68]	[0.09, 2.13]	[0.31, 3.80]	[-2.56, 0.76]	[4.20, 192.20]	[0.00, 119.16]	[0.07, 0.83]	[0.00, 0.48]	[-0.71, 0.27]
OthS	0.88	0.91	2.99	0.60	-0.13	10.92	73.88	0.10	0.02	-0.09
	(0.42, 1.32)	(0.43, 1.42)	(0.52, 4.88)	(0.18, 2.16)	(-1.89, 0.15)	(1.61, 119.16)	(17.61, 192.20)	(0.02, 0.26)	(0.00, 0.08)	(-0.37, 0.15)
	[0.12, 1.62]	[0.10, 1.73]	[0.19, 6.50]	[0.10, 3.97]	[-7.00, 0.95]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.43]	[0.00, 0.20]	[-0.59, 0.43]
Hous	0.96	0.94	10.14	1.31	0.41	6.77	6.77	0.01	0.11	0.08
	(0.48, 1.46)	(0.50, 1.38)	(1.59, 15.88)	(0.45, 2.02)	(-2.00, 5.67)	(1.00, 119.16)	(1.00, 45.80)	(0.00, 0.04)	(0.01, 0.45)	(-0.16, 0.41)
	[0.14, 1.76]	[0.20, 1.75]	[0.52, 20.41]	[0.15, 2.63]	[-8.94, 15.43]	[0.00, 310.00]	[0.00, 119.16]	[0.00, 0.09]	[0.00, 0.74]	[-0.40, 0.66]
Fac			0.48	0.54	-0.11	28.40	4.20	0.75	0.22	-0.37
			(0.19, 1.16)	(0.20, 1.13)	(-0.60, 0.00)	(10.92, 73.88)	(1.00, 28.40)	(0.58, 0.87)	(0.04, 0.53)	(-0.35, 0.26)
			[0.11, 1.80]	[0.10, 1.70]	[-1.58, 0.18]	[4.20, 192.20]	[0.00, 73.88]	[0.46, 0.92]	[0.01, 0.72]	[-0.20, -0.29]

Notes: See Notes for Table A1.

Table A3: Posterior Summary for Benchmark Model, $q = 8$ and $\eta = 2.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	2.90	1.18	1.34	5.50	-5.61	45.80	1.61	0.17	0.05	-0.35
	(1.56, 4.11)	(-0.95, 2.69)	(0.42, 2.72)	(4.17, 7.61)	(-14.63, -1.60)	(17.61, 119.16)	(0.00, 4.20)	(0.05, 0.39)	(0.00, 0.24)	(-0.55, -0.15)
	[0.38, 4.94]	[-2.52, 3.64]	[0.19, 3.97]	[3.38, 9.81]	[-26.65, -0.60]	[10.92, 310.00]	[0.00, 10.92]	[0.01, 0.57]	[0.00, 0.48]	[-0.72, -0.05]
Min	0.18	0.97	10.51	10.99	-45.62	1.61	4.20	0.01	0.01	-0.39
	(-1.58, 1.81)	(-1.13, 3.19)	(7.81, 14.55)	(5.84, 15.76)	(-121.76, -4.81)	(0.00, 6.77)	(1.00, 17.61)	(0.00, 0.07)	(0.00, 0.06)	(-0.67, -0.08)
	[-2.80, 3.20]	[-2.67, 5.10]	[4.31, 18.58]	[0.93, 20.14]	[-229.55, 11.08]	[0.00, 45.80]	[0.00, 192.20]	[0.00, 0.18]	[0.00, 0.21]	[-0.80, 0.11]
Utl	1.32	1.52	1.99	4.42	0.68	6.77	2.60	0.16	0.05	0.14
	(-0.11, 2.47)	(-0.53, 3.72)	(0.45, 3.13)	(1.72, 6.45)	(-1.01, 6.38)	(1.00, 73.88)	(1.00, 17.61)	(0.02, 0.51)	(0.00, 0.35)	(-0.17, 0.53)
	[-1.24, 3.38]	[-1.86, 5.47]	[0.16, 4.13]	[0.37, 8.50]	[-5.21, 13.89]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.73]	[0.00, 0.90]	[-0.55, 0.73]
Con	2.53	3.49	3.25	1.60	-3.86	1.61	45.80	0.43	0.13	-0.37
	(1.67, 3.39)	(0.14, 4.69)	(2.42, 4.49)	(0.47, 3.75)	(-10.93, -0.48)	(0.00, 4.20)	(10.92, 119.16)	(0.16, 0.69)	(0.01, 0.35)	(-0.70, -0.10)
	[0.56, 4.03]	[-1.60, 5.46]	[1.85, 5.76]	[0.19, 5.81]	[-19.45, -0.01]	[0.00, 10.92]	[6.77, 310.00]	[0.02, 0.83]	[0.00, 0.58]	[-0.84, -0.00]
DurG	-0.42	1.15	1.70	5.17	-2.10	17.61	4.20	0.04	0.02	-0.32
	(-1.56, 0.63)	(-0.17, 2.53)	(0.35, 3.40)	(1.71, 7.64)	(-11.00, -0.10)	(6.77, 119.16)	(1.00, 45.80)	(0.00, 0.16)	(0.00, 0.11)	(-0.61, -0.06)
	[-2.50, 1.39]	[-1.15, 3.56]	[0.13, 4.80]	[0.33, 9.75]	[-23.25, 0.45]	[1.61, 310.00]	[0.00, 310.00]	[0.00, 0.32]	[0.00, 0.26]	[-0.75, 0.10]
NdG	-0.21	1.75	1.50	3.85	-1.13	28.40	6.77	0.06	0.06	-0.32
	(-1.64, 1.30)	(0.19, 3.17)	(0.28, 3.46)	(2.26, 5.63)	(-5.97, -0.03)	(2.60, 119.16)	(1.61, 17.61)	(0.01, 0.27)	(0.00, 0.27)	(-0.63, -0.01)
	[-2.55, 2.29]	[-1.04, 4.30]	[0.13, 4.88]	[0.65, 7.39]	[-13.61, 2.50]	[1.00, 310.00]	[0.00, 119.16]	[0.00, 0.50]	[0.00, 0.53]	[-0.81, 0.20]
WT	1.62	1.49	1.30	1.88	0.73	4.20	17.61	0.56	0.08	0.30
	(0.94, 2.26)	(-0.05, 2.72)	(0.84, 1.90)	(0.40, 3.71)	(0.05, 2.78)	(1.00, 17.61)	(1.61, 119.16)	(0.18, 0.83)	(0.01, 0.28)	(0.03, 0.62)
	[0.19, 2.79]	[-1.24, 3.71]	[0.30, 2.51]	[0.16, 4.98]	[-0.37, 6.03]	[0.00, 73.88]	[0.00, 310.00]	[0.02, 0.90]	[0.00, 0.51]	[-0.18, 0.79]
RT	0.72	3.87	1.73	1.35	0.54	4.20	2.60	0.13	0.79	0.33
	(-0.32, 1.59)	(0.24, 5.23)	(0.73, 2.53)	(0.45, 4.16)	(-0.40, 2.45)	(1.00, 28.40)	(1.00, 10.92)	(0.01, 0.47)	(0.02, 0.98)	(-0.10, 0.90)
	[-1.06, 2.22]	[-1.37, 6.05]	[0.21, 3.30]	[0.19, 5.66]	[-2.90, 5.05]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.70]	[0.00, 1.00]	[-0.43, 0.98]
TW	-1.07	0.29	2.63	3.04	0.25	4.20	2.60	0.13	0.06	0.05
	(-2.15, 0.03)	(-1.22, 2.23)	(1.62, 3.74)	(2.07, 4.28)	(-2.18, 3.40)	(1.00, 17.61)	(1.00, 10.92)	(0.01, 0.42)	(0.00, 0.28)	(-0.24, 0.36)
	[-3.02, 1.05]	[-2.17, 3.42]	[0.40, 4.94]	[0.69, 5.44]	[-6.31, 8.21]	[0.00, 119.16]	[0.00, 73.88]	[0.00, 0.64]	[0.00, 0.55]	[-0.47, 0.57]
Inf	1.85	-0.18	3.85	2.68	-2.37	1.61	4.20	0.18	0.06	-0.26
	(0.69, 3.01)	(-1.81, 2.31)	(2.81, 5.36)	(0.58, 4.29)	(-8.82, -0.02)	(0.00, 6.77)	(1.00, 73.88)	(0.03, 0.49)	(0.00, 0.38)	(-0.60, -0.00)
	[-0.38, 3.86]	[-2.92, 5.37]	[1.42, 7.01]	[0.19, 5.75]	[-18.15, 2.99]	[0.00, 45.80]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.81]	[-0.82, 0.22]
F(x-H)	3.26	-0.53	1.16	1.42	-0.08	2.60	2.60	0.89	0.20	-0.07
	(2.35, 4.21)	(-1.54, 1.33)	(0.55, 2.02)	(0.54, 2.08)	(-0.91, 0.89)	(1.00, 10.92)	(1.00, 17.61)	(0.60, 0.98)	(0.01, 0.71)	(-0.71, 0.46)
	[1.51, 4.93]	[-2.33, 3.19]	[0.26, 2.92]	[0.21, 2.73]	[-2.26, 2.28]	[0.00, 28.40]	[0.00, 73.88]	[0.17, 0.99]	[0.00, 0.95]	[-0.94, 0.74]
PBS	2.42	-0.05	1.36	1.57	-1.65	17.61	17.61	0.47	0.02	-0.95
	(1.81, 3.10)	(-0.73, 1.24)	(0.50, 2.74)	(0.58, 3.21)	(-8.16, -0.30)	(1.61, 45.80)	(4.20, 45.80)	(0.19, 0.75)	(0.00, 0.12)	(-0.99, -0.82)
	[1.35, 3.66]	[-1.19, 2.36]	[0.25, 3.83]	[0.27, 4.44]	[-15.10, -0.10]	[0.00, 119.16]	[1.00, 119.16]	[0.08, 0.89]	[0.00, 0.34]	[-1.00, -0.56]
EdHe	0.16	0.33	1.09	2.68	-1.47	17.61	4.20	0.06	0.03	-0.57
	(-0.69, 1.13)	(-0.81, 2.05)	(0.32, 2.03)	(0.74, 4.10)	(-5.00, -0.30)	(1.61, 73.88)	(1.00, 45.80)	(0.00, 0.36)	(0.00, 0.20)	(-0.84, -0.24)
	[-1.29, 1.66]	[-1.59, 3.40]	[0.16, 2.82]	[0.22, 5.31]	[-9.97, -0.08]	[0.00, 192.20]	[0.00, 192.20]	[0.00, 0.61]	[0.00, 0.51]	[-0.92, -0.08]
AEFS	1.30	-0.16	0.55	2.05	-0.20	45.80	2.60	0.20	0.07	-0.14
	(0.39, 2.14)	(-1.31, 1.38)	(0.17, 1.57)	(1.20, 2.91)	(-1.17, 0.05)	(10.92, 119.16)	(1.00, 17.61)	(0.03, 0.51)	(0.01, 0.35)	(-0.45, 0.03)
	[-0.23, 2.87]	[-2.02, 2.63]	[0.09, 2.67]	[0.36, 3.77]	[-2.91, 0.76]	[2.60, 310.00]	[0.00, 73.88]	[0.00, 0.73]	[0.00, 0.60]	[-0.69, 0.24]
OthS	0.20	0.31	3.30	0.64	-0.12	4.20	73.88	0.04	0.02	-0.05
	(-0.98, 1.50)	(-0.86, 1.66)	(0.90, 4.93)	(0.19, 2.37)	(-1.89, 0.30)	(1.00, 73.88)	(17.61, 192.20)	(0.00, 0.18)	(0.00, 0.08)	(-0.34, 0.15)
	[-1.89, 2.51]	[-1.79, 2.66]	[0.23, 6.31]	[0.11, 4.43]	[-7.35, 1.50]	[0.00, 310.00]	[6.77, 500.00]	[0.00, 0.37]	[0.00, 0.20]	[-0.55, 0.41]
Hous	0.48	0.35	10.62	1.32	0.12	4.20	10.92	0.01	0.05	0.03
	(-1.35, 2.27)	(-0.49, 1.39)	(2.21, 15.90)	(0.38, 2.15)	(-4.27, 4.45)	(1.00, 73.88)	(1.61, 73.88)	(0.00, 0.04)	(0.00, 0.27)	(-0.25, 0.36)
	[-2.61, 3.53]	[-1.14, 2.37]	[0.62, 20.75]	[0.14, 2.78]	[-13.11, 14.19]	[0.00, 310.00]	[0.00, 192.20]	[0.00, 0.12]	[0.00, 0.53]	[-0.49, 0.61]
Fac			0.44	0.71	-0.00	17.61	2.60	0.46	0.22	-0.00
			(0.17, 0.85)	(0.38, 1.06)	(-0.18, 0.14)	(6.77, 73.88)	(1.00, 10.92)	(0.28, 0.66)	(0.09, 0.44)	(-0.39, 0.14)
			[0.10, 1.21]	[0.19, 1.45]	[-0.51, 0.40]	[1.61, 119.16]	[0.00, 45.80]	[0.17, 0.80]	[0.04, 0.61]	[-0.37, -0.32]

Notes: See Notes for Table A1.

Table A4: Posterior Summary for Benchmark Model, $q = 6$ and $\eta = 1.0$

Sector	λ_l	λ_z	σ_l	σ_z	σ_{lz}	g_l	g_z	R_l^2	R_z^2	$corr(l,z)$
Agr	1.20	0.72	0.60	3.04	-0.56	73.88	4.20	0.18	0.08	-0.28
	(0.51, 1.87)	(0.07, 1.41)	(0.20, 1.86)	(0.48, 5.27)	(-3.66, -0.02)	(17.61, 192.20)	(1.00, 73.88)	(0.03, 0.48)	(0.01, 0.32)	(-0.60, -0.03)
	[-0.11, 2.32]	[-0.44, 1.96]	[0.11, 3.89]	[0.15, 7.39]	[-11.80, 0.23]	[10.92, 310.00]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.57]	[-0.76, 0.23]
Min	0.56	1.63	10.71	2.79	-14.47	2.60	45.80	0.02	0.04	-0.37
	(-0.32, 1.47)	(0.61, 2.58)	(7.10, 16.05)	(0.52, 10.01)	(-73.84, -2.13)	(0.00, 10.92)	(4.20, 192.20)	(0.00, 0.09)	(0.00, 0.19)	(-0.71, -0.11)
	[-0.96, 2.15]	[-0.03, 3.22]	[1.19, 21.76]	[0.26, 15.41]	[-179.11, -0.18]	[0.00, 119.16]	[1.00, 500.00]	[0.00, 0.24]	[0.00, 0.42]	[-0.85, -0.01]
Utl	1.27	2.87	2.35	2.47	-3.71	4.20	4.20	0.44	0.72	-0.79
	(0.71, 1.68)	(1.99, 3.50)	(1.14, 3.62)	(0.98, 4.06)	(-11.52, -0.39)	(1.00, 28.40)	(1.00, 28.40)	(0.12, 0.75)	(0.30, 0.91)	(-0.97, -0.24)
	[0.22, 2.00]	[1.12, 3.94]	[0.31, 5.18]	[0.25, 5.96]	[-24.64, 0.19]	[0.00, 119.16]	[0.00, 119.16]	[0.02, 0.86]	[0.05, 0.97]	[-0.99, 0.05]
Con	1.29	1.87	3.05	1.09	-1.83	1.61	45.80	0.42	0.20	-0.33
	(0.85, 1.73)	(1.05, 2.58)	(2.08, 4.61)	(0.28, 3.50)	(-8.59, -0.25)	(0.00, 6.77)	(10.92, 192.20)	(0.14, 0.71)	(0.03, 0.58)	(-0.66, -0.09)
	[0.43, 2.07]	[0.46, 3.10]	[0.92, 6.23]	[0.13, 6.45]	[-21.87, 0.02]	[0.00, 45.80]	[4.20, 310.00]	[0.03, 0.85]	[0.00, 0.79]	[-0.83, 0.01]
DurG	0.70	1.25	2.21	6.15	-3.18	10.92	4.20	0.10	0.06	-0.34
	(0.10, 1.21)	(0.49, 1.98)	(0.54, 4.19)	(1.36, 9.78)	(-19.27, -0.06)	(2.60, 73.88)	(1.00, 73.88)	(0.01, 0.40)	(0.01, 0.22)	(-0.67, -0.03)
	[-0.43, 1.57]	[-0.01, 2.51]	[0.16, 6.11]	[0.30, 13.84]	[-45.52, 1.49]	[0.00, 310.00]	[0.00, 310.00]	[0.00, 0.68]	[0.00, 0.43]	[-0.82, 0.17]
NdG	1.56	2.10	4.30	1.10	-3.77	1.61	45.80	0.37	0.21	-0.33
	(0.86, 1.98)	(1.32, 2.67)	(2.94, 6.43)	(0.39, 2.53)	(-11.65, -0.78)	(0.00, 4.20)	(17.61, 119.16)	(0.11, 0.63)	(0.07, 0.46)	(-0.57, -0.13)
	[0.06, 2.29]	[0.58, 3.06]	[1.28, 8.66]	[0.19, 4.07]	[-22.17, -0.07]	[0.00, 28.40]	[6.77, 192.20]	[0.01, 0.80]	[0.01, 0.65]	[-0.78, -0.02]
WT	1.09	0.68	0.82	3.36	0.29	4.20	6.77	0.82	0.06	0.18
	(0.80, 1.38)	(-0.09, 1.48)	(0.33, 1.40)	(0.72, 5.67)	(-0.21, 2.44)	(1.00, 28.40)	(1.00, 73.88)	(0.48, 0.94)	(0.01, 0.25)	(-0.12, 0.55)
	[0.56, 1.63]	[-0.57, 2.13]	[0.13, 2.07]	[0.20, 7.99]	[-2.10, 6.02]	[0.00, 119.16]	[0.00, 310.00]	[0.19, 0.98]	[0.00, 0.49]	[-0.44, 0.74]
RT	0.68	1.63	1.83	3.40	2.49	4.20	4.20	0.28	0.27	0.48
	(0.29, 1.05)	(0.92, 2.40)	(1.01, 2.87)	(1.12, 5.43)	(0.31, 8.43)	(1.00, 28.40)	(1.00, 45.80)	(0.04, 0.63)	(0.05, 0.64)	(0.13, 0.79)
	[-0.05, 1.38]	[0.39, 2.96]	[0.27, 4.08]	[0.31, 7.82]	[-0.70, 19.24]	[0.00, 119.16]	[0.00, 192.20]	[0.00, 0.80]	[0.01, 0.85]	[-0.10, 0.90]
TW	-0.14	-0.17	2.55	0.41	-0.01	4.20	28.40	0.08	0.08	-0.02
	(-0.78, 0.62)	(-0.50, 0.34)	(0.98, 4.25)	(0.15, 1.26)	(-0.94, 0.35)	(1.00, 28.40)	(10.92, 119.16)	(0.01, 0.32)	(0.01, 0.34)	(-0.39, 0.33)
	[-1.25, 1.19]	[-0.74, 1.00]	[0.24, 6.00]	[0.08, 2.64]	[-4.34, 1.68]	[0.00, 192.20]	[2.60, 192.20]	[0.00, 0.57]	[0.00, 0.61]	[-0.66, 0.63]
Inf	1.09	0.04	4.26	3.51	-5.17	1.61	4.20	0.22	0.03	-0.38
	(0.49, 1.68)	(-0.73, 0.94)	(3.11, 6.25)	(0.72, 5.71)	(-17.51, -0.48)	(0.00, 6.77)	(1.00, 73.88)	(0.03, 0.53)	(0.00, 0.18)	(-0.68, -0.06)
	[0.01, 2.12]	[-1.25, 1.61]	[2.00, 8.37]	[0.23, 8.06]	[-37.44, 1.86]	[0.00, 17.61]	[0.00, 192.20]	[0.00, 0.74]	[0.00, 0.42]	[-0.84, 0.13]
F(x-H)	1.74	-0.11	0.43	1.57	-0.08	4.20	2.60	0.97	0.10	-0.16
	(1.45, 2.05)	(-0.60, 0.47)	(0.17, 1.02)	(0.70, 2.58)	(-0.87, 0.30)	(1.00, 28.40)	(1.00, 28.40)	(0.83, 1.00)	(0.01, 0.42)	(-0.71, 0.48)
	[1.22, 2.32]	[-0.94, 1.00]	[0.10, 1.66]	[0.21, 3.73]	[-2.40, 1.06]	[0.00, 73.88]	[0.00, 119.16]	[0.56, 1.00]	[0.00, 0.69]	[-0.90, 0.88]
PBS	1.82	0.64	1.68	3.01	-4.71	10.92	4.20	0.63	0.09	-0.89
	(1.52, 2.15)	(0.27, 1.04)	(0.68, 3.38)	(1.76, 4.66)	(-13.32, -1.20)	(2.60, 28.40)	(1.00, 10.92)	(0.35, 0.83)	(0.01, 0.35)	(-0.98, -0.65)
	[1.29, 2.42]	[-0.05, 1.40]	[0.38, 4.85]	[0.74, 6.53]	[-26.17, -0.44]	[1.00, 45.80]	[0.00, 45.80]	[0.20, 0.92]	[0.00, 0.61]	[-1.00, -0.37]
EdHe	0.75	0.94	1.70	3.28	-3.28	4.20	4.20	0.35	0.11	-0.67
	(0.43, 1.03)	(0.35, 1.51)	(0.71, 2.63)	(0.98, 4.99)	(-9.61, -0.70)	(1.00, 28.40)	(1.00, 45.80)	(0.08, 0.66)	(0.01, 0.35)	(-0.88, -0.30)
	[0.09, 1.27]	[-0.09, 2.02]	[0.25, 3.50]	[0.29, 6.82]	[-18.18, -0.18]	[0.00, 119.16]	[0.00, 192.20]	[0.01, 0.82]	[0.00, 0.61]	[-0.94, -0.12]
AEFS	0.63	-0.97	0.71	1.87	-0.91	45.80	1.61	0.13	0.46	-0.37
	(0.14, 1.17)	(-1.37, -0.28)	(0.26, 1.84)	(1.23, 2.80)	(-2.87, -0.20)	(10.92, 119.16)	(0.00, 6.77)	(0.01, 0.41)	(0.07, 0.78)	(-0.69, -0.13)
	[-0.17, 1.58]	[-1.68, 0.67]	[0.14, 3.11]	[0.55, 4.15]	[-6.25, -0.01]	[2.60, 192.20]	[0.00, 45.80]	[0.00, 0.65]	[0.01, 0.90]	[-0.87, -0.01]
OthS	0.16	1.14	2.88	0.84	-0.43	6.77	73.88	0.04	0.08	-0.24
	(-0.49, 0.88)	(0.41, 1.89)	(0.40, 5.18)	(0.23, 3.39)	(-4.26, 0.01)	(1.00, 119.16)	(10.92, 192.20)	(0.00, 0.20)	(0.01, 0.27)	(-0.61, 0.01)
	[-0.95, 1.49]	[-0.18, 2.44]	[0.16, 7.67]	[0.12, 7.14]	[-15.19, 0.89]	[0.00, 310.00]	[2.60, 500.00]	[0.00, 0.40]	[0.00, 0.50]	[-0.79, 0.29]
Hous	0.36	1.50	10.92	1.88	-14.68	10.92	2.60	0.01	0.63	-0.74
	(-0.59, 1.33)	(1.18, 1.89)	(3.12, 18.23)	(1.13, 2.87)	(-41.27, -2.72)	(1.00, 73.88)	(1.00, 10.92)	(0.00, 0.04)	(0.30, 0.86)	(-0.97, -0.29)
	[-1.25, 2.04]	[0.80, 2.28]	[1.01, 25.59]	[0.39, 4.04]	[-87.76, -0.40]	[0.00, 192.20]	[0.00, 45.80]	[0.00, 0.10]	[0.09, 0.94]	[-0.99, -0.05]
Fac			0.49	1.29	-0.31	28.40	4.20	0.77	0.49	-0.47
			(0.19, 1.19)	(0.50, 2.13)	(-1.30, -0.05)	(10.92, 73.88)	(1.00, 28.40)	(0.58, 0.89)	(0.27, 0.72)	(-0.37, -0.79)
			[0.11, 1.92]	[0.19, 3.09]	[-3.13, 0.01]	[6.77, 192.20]	[0.00, 119.16]	[0.43, 0.94]	[0.14, 0.85]	[-0.33, -0.34]

Notes: See Notes for Table A1.

2 A Growth Model with Sectoral Linkages in Materials and Investment

In this section, we describe an economy where different sectors produce materials and investment goods for other sectors in a way that mimics the U.S. make-use and capital flow tables. These production linkages give rise to *sectoral multipliers* that summarize the influence that different sectors have on the aggregate economy. In general, a sector that has a significant role in producing capital goods and intermediate inputs for other sectors will be associated with a large sectoral multiplier.

2.1 Economic Environment

The representative household has preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t C_t,$$

$$C_t = \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \ \sum_{j=1}^{n} \theta_j = 1, \ \theta_j \geq 0,$$

where C_t represents an aggregate consumption bundle taken to be the numéraire good. The production side of the economy is described as follows:

Gross output in sector j is produced according to the technology,

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{(1-\gamma_j)}, \ \gamma_j \in [0,1].$$

Materials and value added in sector j are produced respectively with the technologies,

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \ \sum_{i=1}^{n} \phi_{ij} = 1, \ \phi_{ij} \geq 0,$$

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j}, \ \alpha_j \in [0,1],$$

where the definitions of variables are those given in the main text.

Capital accumulation in each sector follows

$$k_{j,t+1} = x_{j,t} + (1-\delta_j) k_{j,t},$$

$$x_{j,t} = \prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}}, \ \sum_{i=1}^{n} \omega_{ij} = 1, \ \omega_{ij} \geq 0.$$

Goods market clearing requires that

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + \sum_{i=1}^{n} x_{ji,t} = y_{j,t}.$$

Finally, for now observed labor input is taken to be exogenous so that we define

$$A_{j,t} = z_{j,t} \left(\frac{\ell_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j},$$

and express value added in sector j as

$$v_{j,t} = A_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j}.$$

We then express the driving process for $A_{j,t}$ as

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1-\alpha_j) \Delta \ln \ell_{j,t},$$

where $\Delta \ln z_{j,t}$ and $\Delta \ln \ell_{j,t}$ grow at exogenous rates.

Throughout this appendix, we use the following notation: $\Theta = (\theta_1, ..., \theta_n)_{1 \times n}$, $\Gamma_d = diag\{\gamma_j\}_{n \times n}$, $\Phi = \{\phi_{ij}\}_{n \times n}$, $\Omega = \{\omega_{ij}\}_{n \times n}$, $\alpha_d = diag\{\alpha_j\}_{n \times n}$, $\delta_d = diag\{\delta_j\}_{n \times n}$.

2.2 The Planner's Problem

Because the economy we have just described has no explicit frictions, the equilibrium and first-best allocations coincide. Lagrange multipliers in the solution to the planner's problem will correspond to prices in the decentralized equilibrium. Hence, we denote the price of gross output in sector j by $p^y_{j,t}$, the price of the composite materials bundle in sector j by $p^m_{j,t}$, etc.

The planner then solves

$$\max \ \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j} + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^y_{j,t} \left[\left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{(1-\gamma_j)} - c_{j,t} - \sum_{i=1}^{n} m_{ji,t} - \sum_{i=1}^{n} x_{ji,t} \right]$$

$$
+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{m}\left[\prod_{i=1}^{n}\left(\frac{m_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}}-m_{j,t}\right]
+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{v}\left[A_{j,t}\left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j}-v_{j,t}\right]
+\sum_{t=0}^{\infty}\beta^t\sum_{j=1}^{n}p_{j,t}^{x}\left[\prod_{i=1}^{n}\left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}}+(1-\delta_j)k_{j,t}-k_{j,t+1}\right].
$$

The first-order conditions from the planner's problem give a solution described by:

$$
\frac{\theta_j C_t}{c_{j,t}}=p_{j,t}^{y},
$$

which also defines the ideal price index,

$$
1=\prod_{j=1}^{n}\left(p_{j,t}^{y}\right)^{\theta_j}.
$$

Moreover, we have that

$$
\gamma_j\frac{p_{j,t}^{y}y_{j,t}}{v_{j,t}}=p_{j,t}^{v},
$$

and

$$
(1-\gamma_j)\frac{p_{j,t}^{y}y_{j,t}}{m_{j,t}}=p_{j,t}^{m},
$$

which define a price index for gross output,

$$
p_{j,t}^{y}=\left(p_{j,t}^{v}\right)^{\gamma_j}\left(p_{j,t}^{m}\right)^{1-\gamma_j}.
$$

In addition,

$$
\phi_{ij}\frac{p_{j,t}^{m}m_{j,t}}{m_{ij,t}}=p_{i,t}^{y},
$$

which gives material prices in terms of gross output prices,

$$
p_{j,t}^{m}=\prod_{i=1}^{n}\left(p_{i,t}^{y}\right)^{\phi_{ij}},
$$

and

$$
\omega_{ij}\frac{p_{j,t}^{x}x_{j,t}}{x_{ij,t}}=p_{i,t}^{y},
$$

which gives prices for capital in each sector in terms of gross output prices,

$$p^x_{j,t} = \prod_{i=1}^{n} \left(p^y_{i,t}\right)^{\omega_{ij}}.$$

Finally, we have an Euler equation associated with optimal investment in each sector j,

$$p^x_{j,t} = \beta \mathbb{E}_t \left[\alpha_j \frac{p^v_{j,t+1} v_{j,t+1}}{k_{j,t+1}} + p^x_{j,t+1}(1-\delta_j) \right].$$

Value added in sector j in this economy is $p^y_{j,t} y_{j,t} - \sum_i p^y_{i,t} m_{ij,t} = p^y_{j,t} y_{j,t} - \sum_i (1 - \gamma_j)\phi_{ij} p^y_{j,t} y_{j,t} = \gamma_j p^y_{j,t} y_{j,t} = p^v_{j,t} v_{j,t}$. GDP is then given by $\sum_j p^v_{j,t} v_{j,t}$. It is also the case that $p^y_{j,t} y_{j,t} - \sum_i p^y_{j,t} m_{ji,t} = p^y_{j,t} c_{j,t} + \sum_i p^y_{j,t} x_{ji,t}$.

2.3 The Full Set of Equilibrium Conditions

For clarity, we collect in this subsection the full set of equilibrium conditions,

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + \sum_{i=1}^{n} x_{ji,t} = y_{j,t}, \ \forall j,$$

$$x_{j,t} = \prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}}, \ \forall j,$$

$$k_{j,t+1} = x_{j,t} + (1-\delta) k_{j,t}, \ \forall j, \text{ and } k_{j,0} \text{ given},$$

$$v_{j,t} = A_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j}, \ \forall j,$$

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \ \forall j,$$

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{1-\gamma_j}, \ \forall j.$$

The first-order conditions from the planner's problem are,

$$\frac{\theta_j C_t}{c_{j,t}} = p^y_{j,t}, \ \forall j,$$

$$C_t = \prod_{j=1}^{n} \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j},$$

$$\gamma_j \frac{p^y_{j,t} y_{j,t}}{v_{j,t}} = p^v_{j,t},\ \forall j,$$

$$(1-\gamma_j) \frac{p^y_{j,t} y_{j,t}}{m_{j,t}} = p^m_{j,t},\ \forall j,$$

$$\phi_{ij} \frac{p^m_{j,t} m_{j,t}}{m_{ij,t}} = p^y_{i,t},\ \forall i, j,$$

$$\omega_{ij} \frac{p^x_{j,t} x_{j,t}}{x_{ij,t}} = p^y_{i,t},\ \forall i, j,$$

$$p^x_{j,t} = \beta \mathbb{E}_t \left[\alpha_j \frac{p^v_{j,t+1} v_{j,t+1}}{k_{j,t+1}} + p^x_{j,t+1}(1-\delta_j) \right]\ \forall j$$

The exogenous sectoral processes driving the scale of value added in sector j, $A_{j,t}$, are embedded in

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1-\alpha_j) \Delta \ln \ell_{j,t}. \tag{4}$$

Thus, we have $2n^2 + 11n + 1$ equations with unknowns given by: $y_{j,t}, c_{j,t}, m_{j,t}, x_{j,t}, v_{j,t}, k_{j,t+1}$, $A_{j,t}$, $p^y_{j,t}, p^v_{j,t}, p^m_{j,t}, p^x_{j,t}, j = 1, ..., n$; $m_{ij,t}, x_{ij,t}, i, j = 1, ..., n$; and C_t.

2.4 Balanced Growth and Sectoral Multipliers

This section describes how, in the long run, changes in the growth rates of TFP or labor in different sectors affect GDP (aggregate value added) growth. We describe how this effect may be summarized in the form of *sectoral multipliers* for different sectors.

Consider a balanced growth path where the growth rates of TFP and labor in sector j are given by g^z_j and g^ℓ_j respectively. From equation (4), it follows that along that path,

$$\Delta \ln A_{j,t} = g^a_j = g^z_j + (1-\alpha_j)\, g^\ell_j.$$

Furthermore, as highlighted in the empirical section of the main text, we let

$$g^z_j = \lambda^z_j g^z_f + g^z_{u,j} \text{ and } g^\ell_j = \lambda^\ell_j g^\ell_f + g^\ell_{u,j}. \tag{5}$$

In other words, composite sources of sectoral growth in the steady state, g^a_j, reflect steady state sectoral TFP growth, g^z_j, and sectoral labor growth, g^ℓ_j. The growth rates of these inputs in turn reflect both common (aggregate) factors, $(\lambda^z_j g^z_f, \lambda^\ell_{j,} g^\ell_f)$, and unique idiosyncratic components, $(g^z_{u,j}, g^\ell_{u,j})$.

Then,

$$\Delta \ln A_{j,t} \equiv g_j^a = \lambda_j^z g_f^z + g_{u,j}^z + (1-\alpha_j)\left(\lambda_j^\ell g_f^\ell + g_{u,j}^\ell\right), \tag{6}$$

and we denote by $\widetilde{A}_{j,t}$ the gross growth rate of $A_{j,t}$,

$$\widetilde{A}_{j,t} = \frac{A_{j,t}}{A_{j,t-1}} = e^{g_j^a} \approx 1 + g_j^a.$$

The balanced growth path of the economy is one in which, given the constant growth rates of TFP, $\lambda_j^z g_f^z + g_{u,j}^z$, and labor input, $\lambda_j^\ell g_f^\ell + g_{u,j}^\ell$, all other variables grow at constant rates and all shares are constant. Thus, to derive the aggregate balanced growth path, we need to normalize the model's variables in such a way that these 'detrended' variables (generally denoted by a '$\sim$' over the variable) are constant along that path. Because different sectors will generally grow at different rates along the balanced growth path, the factors used to normalize variables will be sector-specific. Hence, we generally denote these normalizing factors by $\mu_{j,t}$ (or functions thereof). Solving for those factors below will yield a system of equations that is stationary in the normalized variables along the economy's steady state growth path and, importantly, the growth rates of all variables along that path.

2.4.1 Making the Model Stationary

If all growth rates are constant, the resource constraint in any individual sector implies that all the variables in that constraint must grow at the same rate. Thus, define $\widetilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$, $\widetilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$, $\widetilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\widetilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$. The goal in this subsection is to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters only (and in particular the constant growth rates of TFP and labor input).

The economy's resource constraint becomes

$$\widetilde{c}_{j,t} + \sum_{i=1}^{n} \widetilde{m}_{ji,t} + \sum_{i=1}^{n} \widetilde{x}_{ji,t} = \widetilde{y}_{j,t}.$$

Given the above definitions, the production of investment goods may be re-written as

$$\widetilde{x}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{x}_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}},$$

where $\widetilde{x}_{j,t} = x_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$. Under this normalization, the capital accumulation equation is

$$k_{j,t+1} = \widetilde{x}_{j,t} \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}} + (1 - \delta_j) k_{j,t},$$

and so becomes

$$\widetilde{k}_{j,t+1} = \widetilde{x}_{j,t} + (1 - \delta_j) \widetilde{k}_{j,t} \prod_{i=1}^{n} \left(\frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where $\widetilde{k}_{j,t+1} = k_{j,t+1} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$.

The expression for value added may be written as

$$v_{j,t} = A_{j,t} \left(\frac{\widetilde{k}_{j,t} \prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}}}{\alpha_j} \right)^{\alpha_j},$$

so that defining

$$\widetilde{v}_{j,t} = \frac{v_{j,t}}{A_{j,t} \left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}}, \tag{7}$$

where $A_{j,t} \left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$ is the scaling factor that makes normalized value added, $\widetilde{v}_{j,t}$, constant along the balanced growth path, we have

$$\widetilde{v}_{j,t} = \left(\frac{\widetilde{k}_{j,t}}{\alpha_j} \right)^{\alpha_j}.$$

The composite bundle of materials used in sector j may be expressed as

$$\widetilde{m}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{m}_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}},$$

with $\widetilde{m}_{j,t} = m_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\phi_{ij}}$.

Under our normalization, gross output may be written as

$$\widetilde{y}_{j,t}\mu_{j,t} = \left(\frac{\widetilde{v}_{j,t}A_{j,t}\prod_{i=1}^{n}\mu_{i,t-1}^{\alpha_j\omega_{ij}}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}}{1-\gamma_j}\right)^{1-\gamma_j},$$

which, collecting terms, gives

$$\widetilde{y}_{j,t} = \left(\frac{\widetilde{v}_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}\left[\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\right].$$

We can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\left(\prod_{i=1}^{n}\frac{\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}}{\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}}\right)\left(\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}\right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Thus, without loss of generality with respect to growth rates, we choose $\mu_{j,t}$ in every sector such that, on the steady state growth path,[1]

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}+(1-\gamma_j)\phi_{ij}} = 1.$$

2.4.2 Sectoral Value Added Growth

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln A_{j,t} - \ln\mu_{j,t} + \sum_{i=1}^{n}\left(\gamma_j\alpha_j\omega_{ij} + (1-\gamma_j)\phi_{ij}\right)\ln\mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d \ln A_t - \ln\mu_t + \Gamma_d\alpha_d\Omega'\ln\mu_t + (I-\Gamma_d)\Phi'\ln\mu_t = 0,$$

which gives us

$$\ln\mu_t = \Xi'\ln A_t, \tag{8}$$

[1]This is without loss of generality since in the derivations of growth rates below, any constant κ may be used instead of 1.

where

$$\Xi' = \left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d,$$

with $\Xi = \{\xi_{ij}\}$ is the generalized Leontief inverse.

Going back to equation (6), and writing the vector of productivity growth rates as $\Delta \ln A_t = g^a$ where $g^a = (g_1^a, ..., g_n^a)$, it follows that[2]

$$\Delta \ln \mu_t = \Xi' g^a = \Xi' \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d)\left(\lambda^\ell g_f^\ell + g_u^\ell\right)\right),$$

where, given equation (5), g_f^z and g_f^ℓ are common sources of TFP and labor growth, λ^z and λ^ℓ are loading vectors and g_u^z and g_u^ℓ are vectors of (unique) idiosyncratic TFP and labor growth rates.

Recall from equation (7) above that the normalizing factor for value added in sector j is $A_{j,t}\left(\prod_{i=1}^{n} \mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}$. Thus, define this factor by $\mu_{j,t}^v$,

$$\mu_{j,t}^v = A_{j,t}\left(\prod_{j=1}^{n} \mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}.$$

In particular, since $\mu_{j,t}^v$ is the normalizing factor that makes value added in sector j constant along the balanced growth path, it follows that $\mu_{j,t}^v$ grows at the same rate as j's value added along that path, denoted g_j^v. Then, using equation (8), we have that

$$\ln \mu_t^v = \ln A_t + \alpha_d \Omega' \Xi' \ln A_{t-1},$$

or

$$g^v = \left[I + \alpha_d \Omega' \Xi'\right]\left(\lambda^z g_f^z + g_u^z + (I - \alpha_d)\left(\lambda^\ell g_f^\ell + g_u^\ell\right)\right), \tag{9}$$

where $g^v = (g_1^v, ..., g_n^v)$ is a vector that summarizes value added growth in every sector.

2.4.3 GDP Growth and Sectoral Multipliers

In equation (9), the generalized Leontief inverse, $\Xi' = \left(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d$, captures the importance of production linkages across sectors. In particular, changes in the growth rates of productivity or labor in sectors that produce capital and materials for a wide range of other sectors will have a greater influence on the path of GDP growth compared to sectors that have few linkages to the rest of the economy.

[2] Note: $\Gamma_d^{-1}\left(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d)\Phi'\right)\mathbf{1} = (I - \alpha_d)\mathbf{1}$.

The Divisia index describing aggregate GDP growth is

$$\Delta \ln V_t = \sum_{j=1}^{n} s_j^v \Delta \ln v_{j,t},$$

where $\Delta \ln v_{j,t}$ denotes the growth rate of real value added in sector j and s_j^v is the (constant) share of j's value added in GDP,

$$s_j^v = \frac{p_j^v v_j}{\sum_{j=1}^{n} p_j^v v_j}.$$

Hence, from equation (9), the balanced growth rate of real aggregate GDP, denoted g^V, is

$$g^V = s^{v\prime} \left[I + \alpha_d \Omega' \Xi' \right] \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d) \left(\lambda^\ell g_f^\ell + g_u^\ell \right) \right),$$

where s^v is the vector of value added sectoral shares, $(s_1^v, ..., s_1^v)$. Alternatively, we have that

$$g^V = \sum_{j=1}^{n} s_j^v \left[g_j^a + \sum_{i=1}^{n} \alpha_j \omega_{ij} \sum_{k=1}^{n} \xi_{ki} g_k^a \right],$$

where $g_j^a = \lambda_j^z g_f^z + g_{u,j}^z + (1 - \alpha_j) \left(\lambda_j^\ell g_f^\ell + g_{u,j}^\ell \right), \ j = 1, ..., n$. This implies that

$$\frac{\partial g^V}{\partial g_j^a} = s_j^v + s_j^v \alpha_j \sum_{k=1}^{n} \omega_{kj} \xi_{jk} + \sum_{i \neq j} s_i^v \alpha_i \sum_{k=1}^{n} \omega_{ki} \xi_{jk}. \tag{10}$$

In equation (10), the first term captures the direct effects of changes in sources of input growth in sector j by way of productivity or labor on GDP growth. This direct effect is simply j's value added share in GDP reminiscent of Hulten (1978) but now in growth rates. The second and third terms reflect the indirect effects of changes in j's productivity or labor growth on aggregate value added through j's production linkages to other sectors. In other words, disturbances in sector j percolate to sectors that rely on it for inputs and thus amplify j's effects on the aggregate economy.

We define sector j's direct and indirect effects on GDP growth in (10) as j's *sectoral multiplier* given by the j^{th} element of

$$s^{v\prime} \left[I + \alpha_d \Omega' \Xi' \right].$$

3 Examples and Relationship to Previous Work

This section provides examples of sectoral multipliers by relating our analysis to previous work, in particular Greenwood, Hercowitz, and Krusell (1997) (henceforth GHK) and variations thereof. We also discuss briefly Ngai and Pissarides (2007). These examples help underscore the role of capital-producing sectors for the strength of sectoral multipliers. In these examples, goods and factor markets are perfectly competitive and factors of production are freely mobile across sectors. However, as we also make clear, the way in which sectoral sources of growth are amplified at the aggregate level is invariant to the assumption of factor mobility.

3.1 Greenwood, Hercowitz, and Krusell (1997)

To relate the economic environment in GHK (1997) to that of Section 2 above, note first that the one-sector environment with an aggregate production function in GHK (1997) also has an interpretation as a two-sector economy. As highlighted in Section V. A. of GHK (1997), under that interpretation, one sector produces consumption goods (sector 1) and the other investment goods (sector 2), and each sector's production function has the same capital elasticity, α. For simplicity, we focus on the discussion in Section III of GHK (1997) which abstracts from the distinction between equipment and structures.

3.1.1 Interpretation of GHK (1997) as a Two-Sector Economy

Consider an economy whose production structure is described by

$$c_t = y_{1,t} = z_{1,t} k_{1,t}^{\alpha} \ell_{1,t}^{1-\alpha},$$

$$x_t = y_{2,t} = z_{2,t} k_{2,t}^{\alpha} \ell_{2,t}^{1-\alpha},$$

$$k_t = k_{1,t} + k_{2,t},$$

$$\ell_t = \ell_{1,t} + \ell_{2,t},$$

$$k_{t+1} = x_t + (1 - \delta) k_t,$$

where the constant scale factors in the production functions (which simplify the algebra) in the main text have been dropped. We now show that under the maintained assumptions, this two-sector environment indeed allows for an aggregate production function and the one-sector framework of GHK (1997).

To see this, observe that the FOCs for optimal production in the two-sector economy are,

$$p_{2,t} = (1 + r_t)^{-1} \left[u_{t+1} + (1 - \delta) p_{2,t+1} \right],$$

$$w_t = (1 - \alpha) p_{1,t} z_{1,t} \left(\frac{k_{1,t}}{\ell_{1,t}} \right)^{\alpha} = (1 - \alpha) p_{2,t} z_{2,t} \left(\frac{k_{2,t}}{\ell_{2,t}} \right)^{\alpha},$$

$$u_t = \alpha p_{1,t} z_{1,t} \left(\frac{k_{1,t}}{\ell_{1,t}} \right)^{\alpha - 1} = \alpha p_{2,t}, z_{2,t} \left(\frac{k_{2,t}}{\ell_{2,t}} \right)^{\alpha - 1},$$

Equality of factor rentals then implies equal capital-labor ratios across sectors,

$$\frac{w_t}{u_t} = \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{k_{i,t}}{\ell_{i,t}} \right) \rightarrow \frac{k_{i,t}}{\ell_{i,t}} = \frac{k_t}{\ell_t}.$$

Production of the two goods can then be rewritten as

$$c_t = z_{1,t} \left(\frac{k_t}{\ell_t} \right)^{\alpha} \ell_{1,t} \text{ and } x_t = z_{2,t} \left(\frac{k_t}{\ell_t} \right)^{\alpha} \ell_{2,t} = \frac{z_{2,t}}{z_{1,t}} z_{1,t} \left(\frac{k_t}{\ell_t} \right)^{\alpha} \ell_{2,t} = \frac{z_{1,t}}{q_t} \left(\frac{k_t}{\ell_t} \right)^{\alpha} \ell_{2,t},$$

where $q_t = \frac{z_{1,t}}{z_{2,t}}$ is the relative price of investment goods. In particular, adding the two resource constraints, we have that

$$c_t + q_t x_t = z_{1,t} \left(\frac{k_t}{\ell_t} \right)^{\alpha} (\ell_{1,t} + \ell_{2,t}) = z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha}.$$

This gives us an expression for aggregate output (in units of consumption goods) as a function of total factor endowment only, which is also equation (24) in GHK (1997). Thus, to the extent that technical progress in the investment sector is generally more pronounced than in the consumption sector, the relative price of investment goods will decline over time as emphasized by GHK (1997).

3.1.2 Amplification of Sectoral Growth Along the Balanced Growth Path

We now derive the balanced growth path (BGP) in GHK (1997) and discuss its implications for sectoral multipliers. That is, we highlight how capital accumulation amplifies sectoral drivers of growth. Therefore, capital producing sectors will tend to have an outsize effect on the aggregate economy relative to sectors that produce mainly consumption goods.

Along the BGP, all variables grow at constant but potentially different rates. From the market clearing conditions and the form of production technologies, it follows that sectoral output growth rates, g_i^v, are given by (in terms of the notation introduced above),

$$g_i^v = g_i^z + (1-\alpha)g^\ell + \alpha g^k = g_i^a + \alpha g^k, \ i = 1, 2. \tag{11}$$

Equation (11) makes clear that any amplification of sectoral sources of growth, g_i^a, can only take place through capital accumulation. In this case, it follows from the capital accumulation equation that along the BGP, capital grows at the same rate as investment which, in the capital goods producing sector, is also that of output. Thus, we have that

$$g_2^v = g^k = \frac{1}{1-\alpha} g_2^a, \tag{12}$$

and

$$g_1^v = g_1^a + \frac{\alpha}{1-\alpha} g_2^a. \tag{13}$$

Note that the assumption of factor mobility across sectors has only minor implications for the characterization of the BGP. First, even with sector-specific investment, the resource constraint for investment implies that investment and capital grow at the same rate in each sector. Second, with sector-specific labor, the expression for output growth remains as in equation (11) with the only difference being that sector-specific labor growth rates, g_i^ℓ, now replace the aggregate labor growth rate, g^ℓ,

$$g_i^a = g_i^z + (1-\alpha) g_i^\ell.$$

Aggregate GDP growth is defined as the Divisia index of sectoral value-added growth rates weighted by their respective value added shares. Because GHK (1997) do not consider intermediate goods, there is no distinction between gross output and value added in equation (11). Thus, from equations (12) and (13), aggregate GDP growth is

$$g^V = s_1^v \left(g_1^a + \frac{\alpha}{1-\alpha} g_2^a \right) + s_2^v \frac{1}{1-\alpha} g_2^a, \tag{14}$$

or alternatively,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{\alpha}{1-\alpha} g_2^a, \tag{15}$$

where s_i^v is sector i's value-added share in GDP.

In this economy, sector 2 is the sole producer of capital for both sectors 1 and 2 and has both a direct and indirect effect on the aggregate economy. As explained above, the indirect effect stems from the fact that capital accumulation amplifies the role of sectoral sources of growth. In equation (14), sector 2 contributes $\frac{\alpha}{1-\alpha} g_2^a > 0$ to value added growth in sector 1 and scales its contributions from TFP and labor to its own value added growth by $\frac{1}{1-\alpha} > 1$. Thus, the direct effect of an expansion in sector 2, by way of TFP or labor growth, on GDP growth is simply its share, s_2^v, while its aggregate indirect effect is $\frac{\alpha}{1-\alpha} > 0$. Sector 2's sectoral multiplier, therefore, is $s_2^v + \frac{\alpha}{1-\alpha}$. In contrast, because sector 1 produces goods that are only fit for final consumption, it only has a direct effect on the aggregate economy. Its sectoral multiplier is then simply its share in GDP, s_1^v. The sum of sectoral multipliers is larger than 1 so that in principle, sectoral changes in TFP growth that leave aggregate TFP growth unchanged will nevertheless have an effect on GDP.

3.1.3 Relationship to FHSW (2021) with Two Sectors

We now show that a straightforward application of the framework laid out in Section 2 produces the same balanced growth path and sectoral multipliers for sectors 1 and 2 we have just discussed. In particular, the GHK (1997) economy is a special case of Section 2 where $n = 2$ and, since sector 2 is the only sector producing investment goods, $\omega_{2j} = 1, j = 1, 2$ (and $\omega_{1j} = 0, j = 1, 2$). In addition, each good is produced without intermediate inputs, $\gamma_j = 1, j = 1, 2$, and the sectors use the same production functions, $\alpha_j = \alpha, j = 1, 2$, except for the scale factors, $z_j, j = 1, 2$.

With these restrictions, the parameters of the model are summarized by $\Gamma_d = I$, $\alpha_d = \alpha I$ and

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

where the last matrix reflects the production structure whereby all capital in the economy is produced by sector 2.

Then, the generalized Leontief inverse is

$$\Xi' = (I - \alpha_d\Gamma_d\Omega' - (I - \Gamma_d)\Phi')^{-1}\,\Gamma_d = (I - \alpha\Omega')^{-1} = \begin{pmatrix} 1 & -\alpha \\ 0 & 1-\alpha \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{\alpha}{1-\alpha} \\ 0 & \frac{1}{1-\alpha} \end{pmatrix},$$

and the BGP equations reduce to

$$g^v = (I + \alpha\Omega'\Xi')g^a = \begin{pmatrix} 1 & \frac{\alpha}{1-\alpha} \\ 0 & \frac{1}{1-\alpha} \end{pmatrix} \begin{pmatrix} g_1^a \\ g_2^a \end{pmatrix},$$

where

$$g_i^a = g_i^z + (1-\alpha)g_i^\ell,\ i = 1, 2.$$

It follows that value added growth in sectors 1 and 2 are, respectively,

$$g_1^v = g_1^a + \frac{\alpha}{1-\alpha} g_2^a,$$

and

$$g_2^v = \frac{1}{1-\alpha} g_2^a.$$

The Divisia aggregate index of GDP growth is then $g^V = s^{v\prime} g^v = s^{v\prime} g^a + s^{v\prime} \alpha \Omega' \Xi' g^a$ or

$$\begin{aligned} g^V &= s_1^v (g_1^a + \alpha\omega_{21}(\xi_{12} g_1^a + \xi_{22} g_2^a)) + s_2^v (g_2^a + \alpha\omega_{22}(\xi_{12} g_1^a + \xi_{22} g_2^a)), \\ &= s_1^v g_1^a + s_2^v g_2^a + (s_1^v + s_2^v) \frac{\alpha}{1-\alpha} g_2^a, \\ &= s_1^v g_1^a + s_2^v g_2^a + \frac{\alpha}{1-\alpha} g_2^a. \end{aligned}$$

Observe that this last expression is exactly equation (15) above.

Moreover (holding shares constant), the sectoral multipliers, which summarize the effects of sectoral growth by way of TFP or labor, on GDP growth are given by

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v,$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{\alpha}{1-\alpha}.$$

Alternatively, sectoral multipliers are given by the elements of $s^{v\prime} [I + \alpha_d \Omega' \Xi']$. As discussed above, as the sole producer of capital goods, sector 2 has both a direct effect, s_2^v, and an indirect effect, $\frac{\alpha}{1-\alpha}$, on GDP growth. In contrast, sector 1 only has a direct effect on GDP growth, s_1^v.

3.2 Greenwood, Hercowitz, and Krusell (1997) with Different Factor Shares

Actual production linkages are generally more involved than those we have just discussed. Thus, consider the case where factor income shares differ, $\alpha_1 \neq \alpha_2$, while the rest of the production side of the economy remain as in Section 3.1. Importantly, even in the context of two sectors and no materials, the simple fact that factor income shares differ across sectors is enough to prohibit an aggregate production function and thus a one-sector interpretation of the economic environment.[3] The implications of unequal factor income shares, however, are

[3] See also GHK (1997), Section V. A.

relatively straightforward to work out in our framework. As before, it is still the case that sources of growth in the capital goods sector are amplified relative to sectors that mainly produce consumption goods. However, this amplification now depends on a value-added-share-weighted average of capital elasticities.

3.2.1 Amplification of Sectoral Growth along the Balanced Growth Path

With perfect factor mobility, sectoral output growth rates along the BGP are given by

$$g_i^v = g_i^z + (1-\alpha_i)g^\ell + \alpha_i g^k = g_i^a + \alpha_i g^k, \tag{16}$$

and the indirect effect of sectoral growth now depends on a sector's specific capital elasticity, cf. equation (11). As before, the investment goods producing sector determines capital accumulation

$$g_2^v = g^k = \frac{1}{1-\alpha_2} g_2^a,$$

and value added growth in sector 1 is

$$g_1^v = g_1^a + \frac{\alpha_1}{1-\alpha_2} g_2^a,$$

cf. equations (12) and (13). In the absence of factor mobility across sectors, the sector-specific labor growth rates, g_i^ℓ, simply replace the aggregate labor growth rate, g^ℓ,

$$g_i^a = g_i^z + (1-\alpha_i)g_i^\ell.$$

The long-run growth rate of GDP is now given by

$$g^V = s_1^v \left(g_1^a + \frac{\alpha_1}{1-\alpha_2} g_2^a \right) + s_2^v \frac{1}{1-\alpha_2} g_2^a,$$

or alternatively,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2} g_2^a, \tag{17}$$

where s_i^v is sector i's value-added share in GDP.

Thus, the sectoral multipliers for sectors 1 and 2 are respectively s_1^v and $s_2^v + \frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2}$. In this economy, the indirect effect of changes in sector 2 on GDP growth, $\frac{(s_1^v \alpha_1 + s_2^v \alpha_2)}{1-\alpha_2}$, depends for the most part on α_2. As $\alpha_2 \to 0$, this effect tends to $s_1^v \alpha_1 < 1$. Thus, even when sector 2 uses mostly labor in production, it nevertheless has an indirect effect on aggregate growth (over and above its direct effect through its own value added share, s_2^v) since it remains a supplier of (new) capital goods to sector 1. In this case, however, this indirect effect is entirely determined by the parameters of sector 1, specifically its importance in the economy as measured by its value added share in GDP, s_1^v, scaled by the intensity with which it uses capital to produce consumption goods, α_1.[4]

3.2.2 Relationship to FHSW (2021) with Two Sectors

With different factor shares, GHK (1997) may no longer be interpreted as a one-sector economy. Then, a direct application of our framework to the two-sector version of GHK (1997) with different factor shares,

$$\alpha_d = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix},$$

and all other parameters defined as in the previous section, immediately gives the generalized Leontief inverse as

$$\Xi' = \left(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d)\Phi'\right)^{-1} \Gamma_d = (I - \alpha_d \Omega')^{-1} = \begin{pmatrix} 1 & -\alpha_1 \\ 0 & 1-\alpha_2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{\alpha_1}{1-\alpha_2} \\ 0 & \frac{1}{1-\alpha_2} \end{pmatrix},$$

so that the BGP equations reduce to

$$g^v = (I + \alpha \Omega' \Xi') g^a = \begin{pmatrix} 1 & \frac{\alpha_1}{1-\alpha_2} \\ 0 & \frac{1}{1-\alpha_2} \end{pmatrix} \begin{pmatrix} g_1^a \\ g_2^a \end{pmatrix},$$

where

$$g_i^a = g_i^z + (1-\alpha_i) g_i^\ell, \ i = 1, 2.$$

It follows that value added growth in sectors 1 and 2 are, respectively,

$$g_1^v = g_1^a + \frac{\alpha_1}{1-\alpha_2} g_2^a,$$

and

$$g_2^v = \frac{1}{1-\alpha_2} g_2^a.$$

[4] As $\alpha_2 \to 1$, the indirect effect becomes ill-defined since our derivation of the BGP assumes exogenous forces, g_i^z and g_i^ℓ, whereas in the limit where the capital elasticity is one, the model becomes an AK-type endogenous growth model.

The Divisia aggregate index of GDP growth is then $g^V = s^{v\prime} g^v = s^{v\prime} g^a + s^{v\prime} \alpha \Omega' \Xi' g^a$ or

$$\begin{aligned} g^V &= s_1^v(g_1^a + \alpha_1\omega_{21}(\xi_{12}g_1^a + \xi_{22}g_2^a)) + s_2^v(g_2^a + \alpha_2\omega_{22}(\xi_{12}g_1^a + \xi_{22}g_2^a)), \\ &= s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2} g_2^a, \end{aligned}$$

which is indeed equation (17). The corresponding sectoral multipliers in $s^{v\prime}\left[I + \alpha_d \Omega' \Xi'\right]$ are,

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v,$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{(s_1^v\alpha_1 + s_2^v\alpha_2)}{1-\alpha_2},$$

discussed above.

3.3 Greenwood, Hercowitz, Krusell (1997) with Intermediate Inputs

Actual production linkages are more involved still in that they also reflect a network of materials between sectors. Thus, we now introduce intermediate goods into the GHK (1997) environment. With intermediate inputs, additional sectoral contributions to value-added growth continue to arise through the capital growth rate. However, when the consumption sector (sector 1) also produces materials for the investment goods sector (sector 2), the growth rate of capital depends on conditions in both sectors 1 and 2. This means that in contrast to the previous two examples, both sectors 1 and 2 will have indirect effects on long-run GDP growth over and above their share in the economy.

We illustrate these points via a simple network of intermediate goods. Here, sector 1 produces not only consumption goods but also materials, $m_{1,t}$, used by sector 2. Similarly, sector 2 still produces capital goods for both sectors but also materials, $m_{2,t}$ used by sector 1. Since sector 1 now produces consumption goods and intermediate goods, we refer to sector 1 as the non-durables sector. Thus, in terms of the notation used in Section 2, we have that $\gamma_i \neq 1$ and $\omega_{2,i} = 1$ for $i = 1, 2$. Moreover, the relevant resource constraints in sectors 1 and 2 are now

$$c_t + m_{1,t} = y_{1.t} = \left[z_{1,t} k_{1,t}^{\alpha_1} \ell_{1,t}^{1-\alpha_1}\right]^{\gamma_1} m_{2,t}^{1-\gamma_1}, \tag{18}$$

and

$$x_t + m_{2,t} = y_{2,t} = \left[z_{2,t} k_{2,t}^{\alpha_2} \ell_{2,t}^{1-\alpha_2}\right]^{\gamma_2} m_{1,t}^{1-\gamma_2}, \tag{19}$$

while the rest of the production side of the economy is as in Section 3.1.

3.3.1 Amplification of Sectoral Growth along the Balanced Growth Path

As before, with perfect factor mobility it follows that

$$g^x = g^k = g_1^k = g_2^k \text{ and } g^\ell = g_1^\ell = g_2^\ell,$$

while from the goods market clearing conditions, we have that

$$g_1^y = g^c = g_1^m \text{ and } g_2^y = g^x = g_2^m.$$

From the production of goods, which now uses intermediate inputs, it follows that gross output growth rates are

$$\begin{aligned} g_1^y &= \gamma_1 \left[g_1^z + \alpha_1 g_2^y + (1-\alpha_1) g^\ell \right] + (1-\gamma_1) g_2^y, \\ g_2^y &= \gamma_2 \left[g_2^z + \alpha_2 g_2^y + (1-\alpha_2) g^\ell \right] + (1-\gamma_2) g_1^y. \end{aligned}$$

Therefore, solving for the growth rate of new capital goods, we obtain

$$g_2^y = \frac{(1-\gamma_2)\gamma_1 g_1^a + \gamma_2 g_2^a}{\Delta} = g^k. \tag{20}$$

where $\Delta = 1 - \gamma_2\alpha_2 - (1-\gamma_2)\left[\gamma_1\alpha_1 + (1-\gamma_1)\right]$. With intermediate inputs, sectoral value added growth now differs from gross output growth. Using the definition of the value added index, sectoral value added growth is still determined as in the two previous examples without intermediate inputs, equations (11) and (16),

$$g_i^v = g_i^z + \alpha_i g^k + (1-\alpha_i) g^\ell = g_i^a + \alpha_i g^k,$$

Aggregate GDP growth is then given by

$$g^V = s_1^v g_1^a + s_2^v g_2^a + (s_1^v \alpha_1 + s_2^v \alpha_2) g^k, \tag{21}$$

where g^k follows from equation (20). Two important observations emerge relative to the previous examples. First, since the non-durable goods sector now produces intermediate inputs for the investment goods sector, the growth rate of (new) capital goods in equation (20) reflects sources of growth in both sectors, g_1^a and g_2^a. Unlike in the previous examples, therefore, both sectors 1 and 2 will have an indirect effect on long-run GDP growth in equation (21), $(s_1^v\alpha_1 + s_2^v\alpha_2)\frac{\partial g^k}{\partial g_1^a}$ and $(s_1^v\alpha_1 + s_2^v\alpha_2)\frac{\partial g^k}{\partial g_2^a}$ respectively, over and above their shares in the economy, s_1^v and s_2^v. Second, from equation (21), the indirect effect from sector 2 on GDP growth will dominate that from sector 1 if and only if its contributions to overall capital growth, $\frac{\partial g^k}{\partial g_2^a}$, are larger than the corresponding contributions from sector 1, $\frac{\partial g^k}{\partial g_1^a}$. Going back to equation (20), this condition holds if and only if

$$\gamma_2 > (1-\gamma_2)\gamma_1.$$

Put differently, this condition implies that the effect from a one percent change in TFP growth in sector 2 on overall capital growth, g^k, is larger than the corresponding effect from sector 1 transmitted through intermediate inputs. It will fail to hold, for example, in economies where the value added share in gross output of the capital sector, γ_2, is small. In that case, the main input into the production of capital goods in equation (19) are intermediate inputs from the non-durables sector. Therefore, it is that sector's conditions that matter most.

More generally, in a multi-sector environment, the amplification of a non-durable goods sector's sources of growth depends on how much that sector contributes intermediate inputs, however indirectly, to capital goods sectors.

3.3.2 Relationship to FHSW (2020) with Two Sectors

In the general framework we lay out, the relevant parameterization is now

$$\Omega' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix},$$

since sector 2 is still the sole producer of capital goods, while

$$\alpha_d = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix},\ \Gamma_d = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix},\ \text{and } \Phi' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

In this case, the general Leontief inverse, $\Xi' = (I - \alpha_d\Gamma_d\Omega' - (I-\Gamma_d)\Phi')^{-1}\,\Gamma_d$, reduces to

$$\Xi' = \frac{1}{\Delta}\begin{pmatrix} \gamma_1 - \alpha_2\gamma_1\gamma_2 & \gamma_2 - (1-\alpha_1)\gamma_1\gamma_2 \\ \gamma_1(1-\gamma_2) & \gamma_2 \end{pmatrix},$$

where $\Delta = 1-\gamma_2\alpha_2-(1-\gamma_2)(\gamma_1\alpha_1+1-\gamma_1)$ is the determinant of $(I - \alpha_d\Gamma_d\Omega' - (I - \Gamma_d)\Phi')$. Then the vector of value added growth is given by

$$g^v = I + \frac{1}{\Delta}\begin{pmatrix}\alpha_1\gamma_1(1-\gamma_2) & \alpha_1\gamma_2\\ \alpha_2\gamma_1(1-\gamma_2) & \alpha_2\gamma_2\end{pmatrix}\begin{pmatrix}g_1^a\\ g_2^a\end{pmatrix}.$$

so that

$$g_1^v = \frac{1}{\Delta}\left(\alpha_1\gamma_1(1-\gamma_2)g_1^a + \alpha_1\gamma_2 g_2^a\right) + g_1^a,$$

and

$$g_2^v = \frac{1}{\Delta}(\alpha_2\gamma_1(1-\gamma_2)g_1^a + \alpha_2\gamma_2 g_2^a) + g_2^a.$$

With intermediate inputs added to the GHK (1997) economy, GDP growth becomes,

$$g^V = s_1^v g_1^a + s_2^v g_2^a + \frac{(s_1^v\alpha_1\gamma_1(1-\gamma_2) + s_2^v\alpha_2\gamma_1(1-\gamma_2))}{\Delta}g_1^a + \frac{s_1^v\alpha_1\gamma_2 + s_2^v\alpha_2\gamma_2}{\Delta}g_2^a.$$

It follows that the sectoral multipliers, $s^{v\prime}(I + \alpha_d\Omega'\Xi')$, in this case are given by

$$\frac{\partial g^V}{\partial g_1^a} = s_1^v + \frac{(s_1^v\alpha_1\gamma_1(1-\gamma_2) + s_2^v\alpha_2\gamma_1(1-\gamma_2))}{\Delta},$$

and

$$\frac{\partial g^V}{\partial g_2^a} = s_2^v + \frac{s_1^v\alpha_1\gamma_2 + s_2^v\alpha_2\gamma_2}{\Delta},$$

which reproduces the intuition given above.

3.4 Ngai and Pissarides (2007), or GHK with Multiple Consumption Goods

The economy described in Ngai and Pissarides (2007) extends the two-sector interpretation of GHK (1997) to multiple consumption goods, $i = 1, ..., m$. As before, all of the capital used by these m sectors is produced in a single sector, in this case sector m. In terms of our notation, this implies the following resource constraints,

$$c_{i,t} = y_{i,t} = z_{i,t}k_{i,t}^{\alpha_i}\ell_{i,t}^{1-\alpha_i},\ i = 1, ..., m-1,$$

and

$$x_t + c_{m,t} = y_{m,t} = z_{m,t}k_{m,t}^{\alpha_m}\ell_{m,t}^{1-\alpha_m}.$$

Thus, value added growth is determined as in the previous sections without intermediate inputs, with a similar sectoral multiplier for the capital goods sector. However, consumption goods may now grow at different rates. Therefore, a balanced growth path with constant shares now requires unitary elasticity of substitution between goods in preferences.[5] Put another way, constant differences in consumption growth need to be consistent with a market equilibrium.

To see this, observe that for a general utility function, $U(C_t)$, optimal consumer demand implies that the marginal rate of substitution between 2 goods i and j must be equal to their relative prices,

$$\frac{\partial U(C_t)/\partial c_{i,t}}{\partial U(C_t)\ \partial c_{j,t}} = \frac{p^y_{i,t}}{p^y_{j,t}}.$$

Therefore, on the balance growth path, we have that

$$\sigma_i g_{c_i} - \sigma_j g_{c_j} = g^y_{p_i} - g^y_{p_j},$$

where σ_i and σ_j denote the elasticity of utility with respect to the i^{th} and j^{th} consumption goods respectively. From the resource constraints, consumption and output must grow at the same rate in each sector along a BGP. Since nominal values of all sectors grow at the same rate, so that value shares remain constant, the price growth differential must be the negative of the output growth differential. It follows that

$$\sigma_i g_{y_i} - \sigma_j g_{y_j} = g_{y_j} - g_{y_i},$$

or

$$(1+\sigma_i)g_{y_i} = (1+\sigma_j)g_{y_j}.$$

This condition will hold for arbitrary growth rates if and only if the utility function is of the form,

$$U(c) = u\left(\sum_{i=1,2} \phi_i \ln c_i\right),$$

where $u(.)$ is an increasing concave function. This also means a unitary elasticity of substitution between different consumption goods which applies, for example, to the preferences

[5] Ngai and Pissarides (2007) work out conditions on preferences that allow for a more flexible balanced growth path but in a more restrictive model of production. They maintain the assumption of equal factor income shares across all goods which yields an aggregate production function as in Section 3.1. However, they then show the existence of a balanced growth path for aggregate output, consumption, and capital that can coincide with non-constant individual consumption shares. These changing consumption shares then lead to changing implicit labor shares in the production of the consumption goods.

in Section 2 of this appendix.

4 Endogenous Labor Supply

In this section, we extend the model described in Section 2 above to include more general preferences including an endogenous labor supply decision in each sector. A conventional treatment of labor supply produces a growth formula that is isomorphic to that presented in the main text. In particular, the way in which the network features of production and capital accumulation determine the influence of different sectors on aggregate growth is unchanged, as are the effects of long-run changes in TFP growth on GDP growth. The key difference is that with endogenous labor supply, the common and idiosyncratic components of labor input now carry a structural interpretation. Specifically, in the example below, the common component is associated with broad demographics such as population growth and how they affect labor input in each sector. The idiosyncratic component reflects sector-specific factors such as those which determine the disutility cost of working in different sectors, including a sector-specific Frisch elasticity, or sector-specific labor quality adjustments.

We underscore two observations in this context. First, the structural interpretation of variations in labor will necessarily be model dependent. In contrast, our focus in the main text is on growth accounting given the behavior of labor input whatever its underlying forces. Second, in this vein, we provide a historical decomposition of our findings but refrain from speculating on counterfactuals. As this section now makes clear, the upper and lower bound calculations provided in an earlier version of the paper, Foerster et al. (2019), reduce in part to making assumptions about sector-specific elasticities and other drivers of labor supply (which is not the focus of this paper).

4.1 A Model with Labor Choice

The observation in the 1980's that aggregate per capita hours worked in the post-WWII United States appeared more or less stationary motivated the traditional assumption that per capita employment should be constant along a BGP. This in turn motivated restrictions on preferences consistent with constant per capita employment along a BGP (see King, Plosser, and Rebelo (1988)). More recently, however, Boppart and Krusell (2020) have argued that over longer time periods, per capita hours worked are not actually stationary. In particular, they have declined over time in several advanced economies. Consequently, they propose preferences that generalize those in King et al. (1988) and allow for constant growth of per capita employment along the BGP.

Our data suggest variations in the trend growth rate of aggregate labor in the U.S., in part driven by quality improvements, as well as disparate trend variations in labor growth across sectors. Therefore, we introduce endogenous labor supply along the lines of Boppart and Krusell (2020) which allows for sectoral per capita labor to grow (or decline) at different rates in steady state.[6]

At each date t, the economy is populated by a continuum of identical households uniformly distributed on $[0,1]$ consisting of N_t family members. Each household supplies $h_{j,t}$ hours to sector j. Labor hours are quality adjusted according to a sector-specific factor, $q_{j,t}$. Therefore, total effective labor in sector j, $\ell_{j,t}$, is given by $\ell_{j,t} = q_{j,t}h_{j,t}$.

In each period, a family member derives utility from their share of an aggregate consumption bundle, C_t/N_t, and experiences a disutility cost from supplying labor to different sectors according to

$$\ln\left(\frac{C_t}{N_t}\right) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j},\ \nu_j \geq 0,$$

where $e_{j,t}$ scales the disutility of labor supplied to sector j and ν_j is a sector-specific Frisch elasticity of labor supply.

The planner then maximizes the utility of households,

$$\sum_{t=0}^{\infty} \beta^t N_t \left[\ln\left(\frac{C_t}{N_t}\right) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j}\right],$$

where

$$\ln(C_t) = \sum_{j=1}^{n} \theta_j \ln\left(\frac{c_{j,t}}{\theta_j}\right),\ \sum_{j=1}^{n} \theta_j = 1,\ \theta_j \geq 0.$$

We let N_t grow at exogenous rate g_t^N to account for common or aggregate demographic forces in the economy (that raise the overall working age population). Moreover, to the degree that $e_{j,t}$ grows or declines at a constant rate over time along a balanced growth path, per capita labor, $h_{j,t}/N_t$, will decline or grow at a rate inversely proportional to $e_{j,t}^{\frac{1}{1+\nu_j}}$ so as to leave $e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}$ constant along that path. Finally, the rate of quality adjustment of labor is given by $g_{j,t}^q$.

As before, we let

$$\Delta \ln z_{j,t} = \lambda_j^z g_{f,t}^z + g_{u,j,t}^z,$$

[6] Ngai and Pissarides (2007) explore an alternative framework where the reallocation of labor among consumption goods sectors is an outcome of unbalanced growth among those goods while preserving balanced growth at the aggregate level. Absent from their work, however, are the network considerations and the role of capital in determining network multipliers that are central to this paper.

where $g^z_{f,t}$ and $g^z_{u,j,t}$ denote respectively the common and sector-specific components of TFP growth in sector j, and λ^z_j is a loading that captures the effect of the common TFP component on sector j's productivity. We allow the exogenous drivers of labor supply in each sector, $\Delta \ln e_{j,t}$, to have their own (unique) idiosyncratic component, denoted $g^e_{u,j,t}$ and to differentially reflect the effects of demographics, $\lambda^N_j g^N_t$, so that

$$\Delta \ln e_{j,t} = \lambda^N_j g^N_t + g^e_{u,j,t}. \tag{22}$$

This specification allows common demographics in the working age population, such as the baby boom or the rising female labor force participation rate, to affect different sectors in different ways. In addition, it is also conceivable that labor quality adjustments in different sectors, $q_{j,t}$, are also driven by a common component reflecting, say, the overall state of education (which would introduce a second common factor that we abstract from for transparency). All other aspects of the economic environment are as described as in Section 2.

4.2 The Planner's Problem

The planner's problem now is

$$\begin{aligned}
\max\ \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t N_t \left[\sum_{j=1}^{n} \theta_j \ln\left(\frac{c_{j,t}}{\theta_j}\right) - \ln(N_t) - \sum_{j=1}^{n} \frac{e_{j,t}(h_{j,t}/N_t)^{1+\nu_j}}{1+\nu_j} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^y_{j,t} \left[\left(\frac{v_{j,t}}{\gamma_j}\right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j}\right)^{(1-\gamma_j)} - c_{j,t} - \sum_{i=1}^{n} m_{ji,t} - \sum_{i=1}^{n} x_{ji,t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^m_{j,t} \left[\prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}} - m_{j,t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^v_{j,t} \left[z_{j,t} \left(\frac{\ell_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j} \left(\frac{k_{j,t}}{\alpha_j}\right)^{\alpha_j} - v_{j,t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} p^x_{j,t} \left[\prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}}\right)^{\omega_{ij}} + (1-\delta_j)k_{j,t} - k_{j,t+1} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} w_{j,t} \left[q_{j,t} h_{j,t} - \ell_{j,t} \right].
\end{aligned}$$

The key differences with the first-order conditions presented in Section 2 are,

$$\frac{N_t \theta_j}{c_{j,t}} = p^y_{j,t},$$

which also defines the ideal price index,

$$1 = \frac{C_t}{N_t} \underbrace{\prod_{j=1}^{n} \left(p^y_{j,t}\right)^{\theta_j}}_{P_t}.$$

Labor supply and labor demand satisfy respectively,

$$w_{j,t} = \frac{e_{j,t}}{q_{j,t}} \left(\frac{h_{j,t}}{N_t}\right)^{\nu_j},$$

and

$$w_{j,t} = (1-\alpha_j)\,\frac{p^v_{j,t} v_{j,t}}{\ell_{j,t}},$$

while labor market clearing implies

$$\ell_{j,t} = q_{j,t} h_{j,t}.$$

Together these equations give:

$$\ell_{j,t} = q_{j,t}(1-\alpha_j)\frac{p^v_{j,t} v_{j,t}}{e_{j,t}} \left(\frac{h_{j,t}}{N_t}\right)^{-\nu_j},$$

It follows that

$$h_{j,t} = \left((1-\alpha_j)\frac{p^v_{j,t} v_{j,t}}{e_{j,t}} N_t^{\nu_j}\right)^{\frac{1}{1+\nu_j}} = \left((1-\alpha_j)\gamma_j \frac{p^y_{j,t} y_{j,t}}{e_{j,t}} N_t^{\nu_j}\right)^{\frac{1}{1+\nu_j}} = N_t \left((1-\alpha_j)\gamma_j \frac{\theta_j y_{j,t}}{c_{j,t} e_{j,t}}\right)^{\frac{1}{1+\nu_j}}$$

so that along a balanced growth path where $y_{j,t}$ and $c_{j,t}$ grow at the same rate, $\frac{h_{j,t}}{N_t}$ will grow at a rate inversely proportional to that of $e_{j,t}^{\frac{1}{1+\nu_j}}$ with $\frac{h_{j,t}}{N_t} e_{j,t}^{\frac{1}{1+\nu_j}}$ constant. The remaining equations are as in Section 2.

4.3 The Full Set of Equilibrium Conditions

The full set of equilibrium conditions includes the description of the economic environment,

$$c_{j,t} + \sum_{i=1}^{n} m_{ji,t} + \sum_{i=1}^{n} x_{ji,t} = y_{j,t}, \ \forall j,$$

$$x_{j,t} = \prod_{i=1}^{n} \left(\frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}}, \ \forall j,$$

$$k_{j,t+1} = x_{j,t} + (1-\delta_j) k_{j,t}, \ \forall j, \ \text{and } k_{j,0} \text{ given},$$

$$v_{j,t} = z_{j,t} \left(\frac{\ell_{j,t}}{1-\alpha_j} \right)^{1-\alpha_j} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j}, \ \forall j,$$

$$m_{j,t} = \prod_{i=1}^{n} \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \ \forall j,$$

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1-\gamma_j} \right)^{1-\gamma_j}, \ \forall j.$$

$$\ell_{j,t} = q_{j,t} h_{j,t}, \ \forall j.$$

From the planner's problem, we have

$$\frac{N_t \theta_j}{c_{j,t}} = p^y_{j,t}, \ \forall j,$$

$$\ln(C_t) = \sum_{j=1}^{n} \theta_j \ln \left(\frac{c_{j,t}}{\theta_j} \right)$$

$$\ell_{j,t} = q_{j,t} h_{j,t}, \ \forall j,$$

$$h_{j,t} = N_t \left((1-\alpha_j) \gamma_j \frac{\theta_j y_{j,t}}{c_{j,t} e_{j,t}} \right)^{\frac{1}{1+\nu_j}}, \ \forall j,$$

$$w_{j,t} = (1-\alpha_j) \frac{p^v_{j,t} v_{j,t}}{q_{j,t} h_{j,t}}, \ \forall j,$$

$$\gamma_j \frac{p^y_{j,t} y_{j,t}}{v_{j,t}} = p^v_{j,t}, \ \forall j,$$

$$(1-\gamma_j) \frac{p^y_{j,t} y_{j,t}}{m_{j,t}} = p^m_{j,t}, \ \forall j,$$

$$\phi_{ij} \frac{p^m_{j,t} m_{j,t}}{m_{ij,t}} = p^y_{i,t}, \ \forall i, j,$$

$$\omega_{ij} \frac{p^x_{j,t} x_{j,t}}{x_{ij,t}} = p^y_{i,t}, \ \forall i, j,$$

$$p_{j,t}^{x}=\beta\mathbb{E}_t\left[\alpha_j\frac{p_{j,t+1}^{v}v_{j,t+1}}{k_{j,t+1}}+p_{j,t+1}^{x}(1-\delta_j)\right]\ \forall j.$$

4.4 Balanced Growth

The derivation of the balanced growth path follows as in Section 2. As before, TFP growth is constant in the long run and allows for both common and sector-specific components,

$$\Delta\ln z_{j,t}\equiv\lambda_j^z g_f^z+g_{u,j}^z,$$

so that

$$\widetilde{z}_{j,t}=\frac{z_{j,t}}{z_{j,t-1}}=e^{g_j^z}\approx 1+g_j^z.$$

In addition, population growth and the rate of labor quality adjustment are given by

$$\Delta\ln N_t\equiv g^N\rightarrow\widetilde{N}_t=\frac{N_t}{N_{t-1}}=e^{g^N}\approx 1+g^N,$$

and

$$\Delta\ln q_{j,t}\equiv g_j^q\rightarrow\widetilde{q}_{j,t}=\frac{q_{j,t}}{q_{j,t-1}}=e^{g_j^q}\approx 1+g_j^q,$$

while the growth rates of the exogenous drivers of labor supply follow equation (22),

$$\Delta\ln e_{j,t}\equiv g_j^e=\lambda_j^N g^N+g_{u,j}^e$$

so that

$$\widetilde{e}_{j,t}=\frac{e_{j,t}}{e_{j,t-1}}=e^{g_j^e}\approx 1+g_j^e.$$

4.4.1 Making the Model Stationary

Following the steps in Section 2, we normalize the model's variables using sector-specific factors, $\mu_{j,t}$, that we need to solve for to characterize the BGP. As before, the resource constraint in any individual sector implies that all variables in that constraint must grow at the same rate. Thus, define $\widetilde{y}_{j,t}=y_{j,t}/\mu_{j,t}$, $\widetilde{c}_{j,t}=c_{j,t}/\mu_{j,t}$, $\widetilde{m}_{ji,t}=m_{ji,t}/\mu_{j,t}$, and $\widetilde{x}_{ji,t}=x_{ji,t}/\mu_{j,t}$. Then, the economy's resource constraint becomes

$$\widetilde{c}_{j,t}+\sum_{i=1}^{n}\widetilde{m}_{ji,t}+\sum_{i=1}^{n}\widetilde{x}_{ji,t}=\widetilde{y}_{j,t}.$$

The production of investment goods may be re-written as

$$\widetilde{x}_{j,t} = \prod_{i=1}^{n} \left(\frac{\widetilde{x}_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}},$$

where $\widetilde{x}_{j,t} = x_{j,t} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$. Under this normalization, the capital accumulation equation is

$$k_{j,t+1} = \widetilde{x}_{j,t} \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}} + (1-\delta_j) k_{j,t},$$

and so becomes

$$\widetilde{k}_{j,t+1} = \widetilde{x}_{j,t} + (1-\delta_j) \widetilde{k}_{j,t} \prod_{i=1}^{n} \left(\frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where $\widetilde{k}_{j,t+1} = k_{j,t+1} / \prod_{i=1}^{n} \mu_{i,t}^{\omega_{ij}}$.

For the normalization of the labor market equations when labor supply is endogenous, observe that using the normalized first-order demand condition,

$$\frac{\theta_j}{\widetilde{c}_{j,t}} = \widetilde{p}^{y}_{j,t} \equiv \frac{p^{y}_{j,t} \mu_{j,t}}{N_t},$$

with

$$\ell_{j,t} = q_{j,t} h_{j,t} = q_{j,t} N_t \left((1-\alpha_j) \gamma_j \frac{\theta_j y_{j,t}}{c_{j,t} e_{j,t}} \right)^{\frac{1}{1+\nu_j}} = q_{j,t} N_t \left((1-\alpha_j) \gamma_j \frac{\theta_j \widetilde{y}_{j,t}}{\widetilde{c}_{j,t} e_{j,t}} \right)^{\frac{1}{1+\nu_j}},$$

we obtain

$$\widetilde{\ell}_{j,t} = \frac{\ell_{j,t} e_{j,t}^{\frac{1}{1+\nu_j}}}{q_{j,t} N_t} = \left((1-\alpha_j) \gamma_j \widetilde{p}^{y}_{j,t} \widetilde{y}_{j,t} \right)^{\frac{1}{1+\nu_j}}.$$

$$\widetilde{h}_{j,t} = \frac{h_{j,t} e_{j,t}^{\frac{1}{1+\nu_j}}}{N_t} = \left((1-\alpha_j) \gamma_j \widetilde{p}^{y}_{j,t} \widetilde{y}_{j,t} \right)^{\frac{1}{1+\nu_j}}.$$

Labor market clearing thus gives us a solution for wages,

$$\begin{aligned} w_{j,t} &= (1-\alpha_j) \frac{p^{v}_{j,t} v_{j,t}}{q_{j,t} h_{j,t}} = (1-\alpha_j) \gamma_j \frac{p^{y}_{j,t} y_{j,t}}{\ell_{j,t}} \\ &= e_{j,t}^{\frac{1}{1+\nu_j}} ((1-\alpha_j) \gamma_j \frac{\widetilde{p}^{y}_{j,t} \widetilde{y}_{j,t}}{q_{j,t} \widetilde{\ell}_{j,t}} \end{aligned}$$

where normalized wages are then given by

$$\widetilde{w}_{j,t} = ((1-\alpha_j)\gamma_j \widetilde{p}^y_{j,t}\widetilde{y}_{j,t})^{\frac{\nu_j}{1+\nu_j}} = \frac{q_{j,t}w_{j,t}}{e_{j,t}^{\frac{1}{1+\nu_j}}}.$$

Following the steps in Section 2, the expression for value added may be written as

$$v_{j,t} = z_{j,t}\left(\frac{\widetilde{\ell}_{j,t}q_{j,t}N_t}{(1-\alpha_j)e_{j,t}^{\frac{1}{1+\nu_j}}}\right)^{1-\alpha_j}\left(\frac{\widetilde{k}_{j,t}\prod_{i=1}^{n}\mu_{i,t-1}^{\omega_{ij}}}{\alpha_j}\right)^{\alpha_j},$$

so that, defining $\widetilde{v}_{j,t} = v_{j,t}e_{j,t}^{\frac{1-\alpha_j}{1+\nu_j}}/z_{j,t}(q_{j,t}N_t)^{1-\alpha_j}\left(\prod_{i=1}^{n}\mu_{i,t-1}^{\omega_{ij}}\right)^{\alpha_j}$, it becomes

$$\widetilde{v}_{j,t} = \left(\frac{\widetilde{\ell}_{j,t}}{1-\alpha_j}\right)^{1-\alpha_j}\left(\frac{\widetilde{k}_{j,t}}{\alpha_j}\right)^{\alpha_j}.$$

The composite bundle of materials used in sector j may be expressed as

$$\widetilde{m}_{j,t} = \prod_{i=1}^{n}\left(\frac{\widetilde{m}_{ij,t}}{\phi_{ij}}\right)^{\phi_{ij}},$$

with $\widetilde{m}_{j,t} = m_{j,t}/\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}$.

Under the normalization, gross output may be written as

$$\widetilde{y}_{j,t}\mu_{j,t} = \left(\frac{\widetilde{v}_{j,t}z_{j,t}(q_{j,t}N_t)^{1-\alpha_j}\prod_{i=1}^{n}\mu_{i,t-1}^{\alpha_j\omega_{ij}}}{e_{j,t}^{\frac{1-\alpha_j}{1+\nu_j}}\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}\prod_{i=1}^{n}\mu_{i,t}^{\phi_{ij}}}{1-\gamma_j}\right)^{1-\gamma_j},$$

which, collecting terms, gives

$$\widetilde{y}_{j,t} = \left(\frac{\widetilde{v}_{j,t}}{\gamma_j}\right)^{\gamma_j}\left(\frac{\widetilde{m}_{j,t}}{1-\gamma_j}\right)^{1-\gamma_j}\left[\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\right].$$

As before, we can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\left(\prod_{i=1}^{n}\frac{\mu_{i,t-1}^{\gamma_j\alpha_j\omega_{ij}}}{\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}}\right)\left(\prod_{i=1}^{n}\mu_{i,t}^{(1-\gamma_j)\phi_{ij}}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}}\right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Then, without loss of generality with respect to growth rates, we choose $\mu_{j,t}$ such that

$$\frac{(z_{j,t}(q_{j,t}N_t)^{1-\alpha_j})^{\gamma_j}}{e_{j,t}^{\frac{\gamma_j(1-\alpha_j)}{1+\nu_j}}\mu_{j,t}}\prod_{i=1}^{n}\mu_{i,t}^{\gamma_j\alpha_j\omega_{ij}+(1-\gamma_j)\phi_{ij}}=1.$$

4.4.2 Sectoral Value Added Growth

Taking logs of both sides of the above expression, we have

$$\gamma_j\ln z_{j,t}q_{j,t}^{1-\alpha_j}N_t^{1-\alpha_j}e_{j,t}^{-\frac{1-\alpha_j}{1+\nu_j}}-\ln\mu_{j,t}+\sum_{i=1}^{n}(\gamma_j\alpha_j\omega_{ij}+(1-\gamma_j)\phi_{ij})\ln\mu_{i,t}=0,$$

or in vector form,

$$\Gamma_d\ln z_t+\Gamma_d(1-\alpha_d)(\ln N_t+\ln q_t)-\Gamma_d(I-\alpha_d)(I+\nu_d)^{-1}\ln e_t-\ln\mu_t+\Gamma_d\alpha_d\Omega'\ln\mu_t+(I-\Gamma_d)\Phi'\ln\mu_t=0,$$

which gives us

$$\ln\mu_t=\Xi'(\ln z_t+(I-\alpha_d)(\ln N_t+\ln q_t)-(I-\alpha_d)(I+\nu_d)^{-1}\ln e_t),\tag{23}$$

where

$$\Xi'=(I-\Gamma_d\alpha_d\Omega'-(I-\Gamma_d)\Phi')^{-1}\Gamma_d,$$

with $\Xi=\{\xi_{ij}\}$ is the same generalized Leontief inverse as in Section 2.

Going back to equation (23), and writing the vector of productivity growth rates as $\Delta\ln z_t=g^z$, population growth as $\Delta\ln N_t=g^N$, the rates of labor quality adjustment as $\Delta\ln q_t=g^q$, and the growth rates of drivers of labor supply as $\Delta\ln e_t=g^e$, it follows that

$$\begin{aligned}\Delta\ln\mu_t&=\Xi'(g^z+(I-\alpha_d)g^N+(I-\alpha_d)g^q-(I-\alpha_d)(I+\nu_d)^{-1}g^e)\\&=\Xi'\left(\underbrace{\lambda^z g_f^z+g_u^z}_{g^z}+(I-\alpha_d)g^N+(I-\alpha_d)g^q-(I-\alpha_d)(I+\nu_d)^{-1}(\underbrace{\lambda^N g^N+g_u^e}_{g^e})\right).\end{aligned}$$

Let $\mu_{j,t}^v$ denote the normalizing factor for value added in sector j, and recall from the definition of normalized or detrended value added above that

$$\mu_{j,t}^v = z_{j,t}(q_{j,t}N_t)^{1-\alpha_j} e_{j,t}^{-\frac{1-\alpha_j}{1+\nu_j}} \left(\prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}.$$

In addition, since $\mu_{j,t}^v$ makes normalized value added in sector j constant along the steady state growth path, it must grow at the same rate as j's value added along that path, denoted g_j^v. Then, using equation (23), we have that

$$\begin{aligned} \ln \mu_t^v = {} & (\ln z_t + (I - \alpha_d) \ln N_t + (I - \alpha_d) \ln q_t - (I - \alpha_d)(I + \nu_d)^{-1} \ln e_t) \\ & + \alpha_d \Omega' \Xi' (\ln z_{t-1} + (I - \alpha_d) \ln N_{t-1} + (I - \alpha_d) \ln q_{t-1} - (I - \alpha_d)(I + \nu_d)^{-1} \ln e_{t-1}), \end{aligned}$$

or

$$g^v = [I + \alpha_d \Omega' \Xi'] \left(\lambda^z g_f^z + g_u^z + (I - \alpha_d) \left(g^N + g^q - (I + \nu_d)^{-1} (\lambda^N g^N + g_u^e) \right) \right). \tag{24}$$

Comparing equation (24) to (9), the basic expression is unchanged. In particular, the production features that determine the influence that different sectors have on GDP both directly, via I, and indirectly, via $\alpha_d \Omega' \Xi'$, are the same as before as are the effects of long-run TFP growth on GDP growth. The key difference is that with endogenous labor supply, the common and idiosyncratic components of sectoral labor input now have a structural interpretation. Thus, the common component of labor input growth in equation (9), g_f^ℓ, is associated with overall demographics, g^N, and individual sector loadings in equation (9), λ_j^ℓ, are given by $(1 - \alpha_j)(1 - (1 + \nu_j)^{-1} \lambda_j^N)$ which capture the way these demographics affect labor in individual sectors. Similarly, the idiosyncratic component of labor input growth in equation (9), g_j^ℓ, corresponds to $(1 - \alpha_j) g_j^q - (1 - \alpha_j)(1 + \nu_j)^{-1} g_j^e$. In other words, in each sector, our estimates of sector-specific labor input growth reflect sector-specific capital shares and Frisch elasticities as well as idiosyncratic labor quality adjustments and the disutility costs of providing labor to sector j (a lower disutility cost increases labor input in that sector).

5 Hulten (1978) in an Economy with Capital Goods

In this section, we relate our work to that of Hulten (1978) and, more recently, Baqaee and Farhi (2019). In its simplest form, Hulten (1978) states that the effects of a shock to productivity in a given sector on GDP is that sector's ratio of gross output to GDP, namely its Domar weight. This result hinges in part on interpreting TFP as scaling gross output. When TFP is instead interpreted as scaling value added, a sector's influence on GDP becomes its value added share in GDP.[7]

Baqaee and Farhi (2019) explore the role of non-linearities in generating aggregate effects from sectoral shocks over and above the linear approximations highlighted by Hulten (1978). Our work focuses instead on other key assumptions, unexplored in Hulten (1978) and related work, that nevertheless are central for understanding the aggregate implications of sectoral trends. One is the role that capital plays as part of a production network in amplifying the effects of sectoral changes on long-run GDP growth. Here, the long-run dynamics of capital accumulation are central to that role. Another is that while Hulten (1978) and subsequent work focus mostly on level effects, $\partial \ln V / \partial \ln A$, our motivation is the implications of sectoral (trend) growth rates for long-run GDP growth, $\partial \Delta \ln V / \partial \Delta \ln A$.

Importantly, Hulten (1978)'s basic insight remains nested by a static version of our economic environment without capital and where the focus is on levels rather than growth rates. Below, we show how this 'levels' insight changes in the steady state of a dynamic economy with capital.[8]

5.1 The Model with Capital Goods but No Steady State Growth

In the steady state of an economy with no growth, the full set of equilibrium conditions outlined in Section 2 above immediately imply a set of relationships (in levels) between the prices of materials, investment, and value added,

$$\Theta \ln p^y = 0,$$

$$\ln p^v = \Gamma_d^{-1} \left[I - (I - \Gamma_d)\Phi' \right] \ln p^y,$$

$$\ln p^x = \Omega' \ln p^y,$$

where $p^v = (p_1^v, ..., p_n^v)'$, $p^x = (p_1^x, ..., p_n^x)'$, and $p^y = (p_1^y, ..., p_n^y)'$, and where recall that Θ is a $1 \times n$ vector of consumption shares, Γ_d is an $n \times n$ matrix of value added shares in gross output, Φ is an $n \times n$ matrix of materials input shares and Ω is an $n \times n$ capital flow matrix.

[7] These results are evidently related. When sectoral TFP, z_j, is measured as scaling value added, $\widetilde{z}_{j,t} = z_{j,t}^{\gamma_j}$ becomes the relevant scalar for sectoral gross output, where γ_j is j's value added share in gross output, $\frac{p_j^v v_j}{p_j^y y_j}$. Hulten's (1978) theorem then states that $\frac{\partial \ln V_t}{\partial \ln \widetilde{z}_{j,t}} = \mathscr{D}_j$, where $\mathscr{D}_j$ is sector j's Domar weight or ratio of gross output to GDP, $\frac{p_j^y y_j}{V}$. It immediately follows from the definition of $\widetilde{z}_j$ that $\frac{\partial \ln V_t}{\partial \ln z_{j,t}} = \gamma_j \mathscr{D}_j$, where $\gamma_j \mathscr{D}_j$ is then simply sector j's value added share in GDP, s_j^v.

[8] Hulten (1978)'s original result then emerges as a special case where shares of capital in value added in every sector, α_d, are set to zero.

Furthermore, from the definition of value added, we have that

$$\ln v_j = \ln A_j + \alpha_j \ln \left(\frac{k_j}{\alpha_j} \right),$$

and from the Euler equation governing the optimal choice of capital in each sector,

$$\frac{k_j}{\alpha_j} = \left(\frac{p_j^v v_j}{p_j^x} \right) \left(\frac{\beta}{1-\beta(1-\delta_j)} \right).$$

Combining these expressions gives

$$\ln v_j = \ln A_j + \alpha_j \ln \left(p_j^v v_j \right) - \alpha_j \ln p_j^x + \alpha_j \ln \left(\frac{\beta}{1-\beta(1-\delta_j)} \right),$$

or in matrix form,

$$(I - \alpha_d) \ln (p^v . \times v) = \ln A + \ln p^v - \alpha_d \ln p^x + \alpha_d \ln \Delta_d,$$

where $(p^v . \times v)$ represents the vector of nominal value added, $\left\{ p_j^v v_j \right\}$, $\alpha_d = diag(\alpha_j)$, and $\Delta_d = diag \left(\frac{\beta}{1-\beta(1-\delta_j)} \right)$. Substituting for investment and value added prices on the right-hand-side of this last expression, we obtain

$$\ln p^y = \left(\Gamma_d^{-1} \left[I - (I - \Gamma_d) \Phi' \right] - \alpha_d \Omega' \right)^{-1} \left[(I - \alpha_d) \ln (p^v . \times v) - \ln A - \alpha_d \ln \Delta_d \right]. \quad (25)$$

From the resource constraints in each sector j, and the optimal allocation of materials and investment in the economy, it follows that

$$\frac{p_j^v v_j}{\gamma_j} = \theta_j C + \sum_{i=1}^n \phi_{ji} (1 - \gamma_j) \frac{p_i^v v_i}{\gamma_i} + \sum_{i=1}^n \omega_{ji} \frac{\beta \delta_j}{1-\beta(1-\delta_j)} \alpha_i p_i^v v_i,$$

or in matrix form,

$$\Gamma_d^{-1} (p^v . \times v) = \Theta' C + \Phi (I - \Gamma_d) \Gamma_d^{-1} (p^v . \times v) + \Omega \Delta_d \delta_d \alpha_d (p^v . \times v), \quad (26)$$

so that

$$\frac{(p^v . \times v)}{C} = \left(\left[I - \Phi (I - \Gamma_d) \right] \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d \right)^{-1} \Theta'. \quad (27)$$

Aggregate GDP (in units of the final consumption bundle) is then given by

$$V = \mathbf{1}' (p^v . \times v) = \mathbf{1}' \psi C, \quad (28)$$

where $\psi = \left([I - \Phi(I - \Gamma_d)] \, \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d \right)^{-1} \Theta'$.

Substituting the expression for value added in (26) into the equation for gross output prices, (25), and using the fact that the ideal price index for the final consumption bundle implies $\Theta \ln p^y = 0$, we obtain

$$\ln C = \frac{\Theta \left(\Gamma_d^{-1} \left[I - (I - \Gamma_d) \Phi' \right] - \alpha_d \Omega' \right)^{-1} \left[\ln A + \alpha_d \ln \Delta_d - (I - \alpha_d) \ln \psi \right]}{\Theta \left(\Gamma_d^{-1} \left[I - (I - \Gamma_d) \Phi' \right] - \alpha_d \Omega' \right)^{-1} (I - \alpha_d) \mathbf{1}},$$

Note: In the denominator, $\Theta \left(\Gamma_d^{-1} \left[I - (I - \Gamma_d) \Phi' \right] - \alpha_d \Omega' \right)^{-1} (I - \alpha_d) \mathbf{1} = 1$.

It follows that

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{ \left([I - \Phi(I - \Gamma_d)] \, \Gamma_d^{-1} - \Omega \alpha_d \right)^{-1} \Theta' \right\}_j . \tag{29}$$

5.1.1 Interpretation in Terms of Sectoral Shares

To give an an interpretation to this result, observe from equations (27) and (28) above that the vector of value added shares s^v, is simply given by

$$\frac{(p^v . \times v)}{V} = \frac{\left([I - \Phi(I - \Gamma_d)] \, \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d \right)^{-1} \Theta'}{\mathbf{1}' \left([I - \Phi(I - \Gamma_d)] \, \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d \right)^{-1} \Theta'}. \tag{30}$$

We now examine the limit case where $\beta \to 1$ so that $\Delta_d \delta_d \to I$. Then, we can write equation (29) as

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v,$$

where $\eta = \mathbf{1}' \left([I - \Phi(I - \Gamma_d)] \, \Gamma_d^{-1} - \Omega \alpha_d \right)^{-1} \Theta'$.

In particular, η is approximately the inverse of the mean labor share in value added across sectors. To see this, observe that η can also be expressed as $\eta = \frac{\mathbf{1}' \left([I - \Phi(I - \Gamma_d)] \Gamma_d^{-1} - \Omega \alpha_d \right)^{-1} \Theta'}{\mathbf{1}' (I - \alpha_d) \left([I - \Phi(I - \Gamma_d)] \Gamma_d^{-1} - \Omega \alpha_d \right)^{-1} \Theta'}$ since the denominator equals 1. Thus, when $\alpha_j = \alpha \; \forall j$, $\eta = \frac{1}{1 - \alpha}$.

Equation (29) then tells us that in the steady state of a dynamic economy with capital (and no growth of any kind), the effect of a change in productivity in a given sector on GDP is that sector's value added share in GDP scaled by the inverse of the mean labor share in the economy.

5.1.2 Recovering Hulten (1978) without Capital as a Special Case

To recover Hulten's original insight, it suffices to get rid of capital in the economy and set the corresponding shares, α_d, to zero across sectors. Then equation (29) becomes

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{\Gamma_d \left[I - \Phi(I - \Gamma_d)\right]^{-1} \Theta'\right\}_j .$$

At the same time, equation (30) which defines sectoral value added shares in GDP becomes

$$\frac{(p^v . \times v)}{V} = \frac{\Gamma_d \left[I - \Phi(I - \Gamma_d)\right]^{-1} \Theta'}{\mathbf{1}'\Gamma_d \left[I - \Phi(I - \Gamma_d)\right]^{-1} \Theta'},$$

where the denominator is simply equal to 1. In other words, in this case, the effect of a change in productivity in a given sector on GDP is simply that sector's value added share in GDP, $\partial \ln V / \partial \ln A_j = s_j^v$. Alternatively, Acemoglu et al. (2012) refer to the matrix, $\Gamma_d \left[I - \Phi(I - \Gamma_d)\right]^{-1} \Theta'$, in equation (29) as the *influence vector*. In that expression, the matrix $\left[I - \Phi(I - \Gamma_d)\right]^{-1}$ is the Leontief inverse, $\mathscr{L}$, which can also be expressed as the Neumann series $\mathscr{L} = I + \Phi(I - \Gamma_d) + \Phi(I - \Gamma_d)^2 + ...$. This series represents the successive rounds in which a disturbance in sector j percolates to other sectors that purchase inputs from it and, since those sectors may also sell inputs to it, back to sector j and so on.

6 Measurement Bias in the Contributions from Capital

While the framework we laid out allows for a relatively tractable characterization of long-run growth when intricate production linkages are present, both in materials and investment goods, it also contains a discrepancy in the way that capital aggregation is treated relative to NIPA. In our framework, multiple investment goods are aggregated into one investment good, and the capital stock reflects the undepreciated component of this aggregate investment good. In NIPA, the services of multiple capital stocks are instead aggregated into one aggregate capital service flow, and each type of capital evolves according to its corresponding investment decisions.[9] Gourio and Rognlie (2020) point out that the difference in setups can bias estimates of the contributions from capital to growth in the model with multiple investment goods. The importance of this bias hinges on different capital types having substantially different rates of depreciation and price changes, e.g., structures versus equipment.

[9] In an analysis of transition dynamics or impulse responses to disturbances, our approach has the dimension of the capital state vector increasing linearly with the number of sectors, rather than quadratically as in the NIPA procedure.

It is easiest to illustrate these points in the context of an aggregate model that nevertheless allows for multiple capital goods as well as one consumption good, similar to Greenwood, Hercowitz, and Krusell (1997) discussed above. Thus, consider the following model with aggregate resource constraint,

$$c_t + \sum_{j=2}^{n} q_{j,t} x_{j,t} = y_{1,t} = z_{1,t} k_t^{\alpha} \ell_t^{1-\alpha}, \tag{31}$$

$$q_{j,t} = z_{1,t}/z_{j,t},\ j = 2, \ldots, n.$$

Here, aggregate capital in the production function, k_t, reflects the services of multiple capital stocks, $k_{j,t}$ according to weights ϕ_j, and the evolution of each capital type is determined by a type-specific accumulation equation. In particular,

$$k_t = \prod_{j=2}^{n} k_{j,t}^{\phi_j},\ \text{with}\ \sum_{j=2}^{n} \phi_j = 1, \tag{32}$$

$$k_{j,t+1} = x_{j,t} + (1-\delta_j) k_{j,t},\ j = 2, \ldots, n.$$

We refer to this set up as the 'capital-aggregate' model. The capital pricing equations in this case are

$$(1+r_t) q_{j,t} = u_{j,t+1} + (1-\delta_j) q_{j,t+1}, \quad j = 2, \ldots, n, \tag{33}$$

and

$$u_{j,t} = \alpha z_{1,t} \left(\frac{k_t}{\ell_t}\right)^{\alpha-1} \phi_j \frac{k_t}{k_{j,t}}, \quad j = 2, \ldots, n.$$

Along a BGP, all variables grow at constant but potentially different rates. Specifically, the resource constraint implies that

$$g_1^y = g^c = g_j^q + g_j^x = g_1^z + \alpha g^k + (1-\alpha) g^{\ell} = g_1^a + \alpha g^k,$$

while from the capital aggregation and accumulation equations, it also follows that

$$g^k = \sum_{j=2}^{n} \phi_j g_j^k \text{ where } g_j^k = g_j^x.$$

Therefore, along the BGP, output growth (in terms of consumption units) is

$$g_1^y = \frac{g_1^a - \alpha \sum_{j=2}^{n} \phi_j g_j^q}{1-\alpha}. \tag{34}$$

6.1 Bias from Alternative Aggregation

The framework we exploit in the main text allows for a relatively tractable characterization of long-run growth in the presence of different types of production linkages. However, its aggregation properties with respect to capital differ somewhat from those we have just discussed. In particular, in the context of the multi-capital goods model just introduced, while the resource constraint is consistent with equation (31), multiple investment goods, x_t^j, are aggregated into one investment good, x_t, and the capital stock reflects the undepreciated component of this aggregate investment good. Thus, we have that

$$x_t = \prod_{j=2}^{n} x_{j,t}^{\omega_j}, \text{ with } \sum_{j=2}^{n} \omega_j = 1, \tag{35}$$

with

$$k_{t+1} = x_t - (1-\delta)k_t.$$

We refer to this set-up as the 'investment-aggregate' model.

Along the BGP, the growth rate of output (in consumption units) is now

$$g_1^y = \frac{g_1^a - \alpha \sum_{j=2}^{n} \omega_j g_j^q}{1-\alpha}. \tag{36}$$

Therefore, the extent to which the growth rate in equation (36) differs from (or is biased relative to) that in equation (34) depends on the degree to which changes in relative prices, g_j^q, are weighted differently. The weights in equation (34) refer to capital income shares in production, ϕ_j, while the weights in equation (36) refer to investment shares, ω_j.

Along the BGP, capital income share weights, ϕ_j, and investment expenditure share weights, ω_j, in the 'capital-aggregate' model are related through the capital rental and Euler equations as well as investment-capital ratios. In particular, from equation (33) we have that

$$u_j k_j = \left[\frac{1+r}{1+g_j^q} - (1-\delta_j)\right] q_j k_j \approx \left(r - g_j^q + \delta_j\right) q_j k_j \tag{37}$$

Dividing through by the aggregate capital income share and using the capital accumulation equation for each type, we obtain

$$\phi_j = \frac{r - g_j^q + \delta_j}{g_j^k + \delta_j} \cdot \frac{q_j x_j}{\alpha y_1} = \frac{r - g_j^q + \delta_j}{g_1^y - g_j^q + \delta_j} \cdot \frac{q_j x_j}{\alpha y_1}.$$

Therefore,

$$q_j x_j = \left[\alpha y_1 \frac{g_1^y - g_j^q + \delta_j}{r - g_j^q + \delta_j} \right] \phi_j = \psi_j \phi_j,$$

which establishes the relationship between sectoral investment shares, ω_j, and capital income shares, ϕ_j, conditional on sectoral depreciation rates and the steady state evolution of relative prices,

$$\omega_j = \frac{\phi_j \psi_j}{\sum_{s>1} \phi_s \psi_s}.$$

Note that when depreciation rates, δ_j, and the steady state evolution of relative prices, q_j, are close, the differences between ψ_j's are small which implies that differences between investment and capital income shares will also be small.

6.2 Quantitative Assessment

We now gauge how the potential for misspecification bias plays out in the non-financial corporate sector of the U.S. economy for the period 1950-2016.

The BEA's Fixed Asset Tables gives us information on the three main capital aggregates: structures, equipment and intellectual property products (IPP). For each capital type, we have information on nominal and real net-stocks, depreciation, and investment. To better highlight the effects of differences in depreciation rates and relative price changes on BGP calculations using investment expenditure shares rather than capital income shares, we aggregate the two high depreciation types, equipment and IPP, into a single category, 'E&I.' We then contrast this category with structures. Repeating the exercise with the three capital types yields essentially the same findings.

From the National Income Accounts (NIAs), we obtain nominal gross value added (GVA) and the price indices for non-durable consumption goods and services. We construct a joint price index for non-durable consumption goods and services which we use to deflate nominal GVA and investment goods prices to obtain aggregate output, y_1, and investment good prices, q_j, in units of consumption goods. From the BLS Productivity and Cost Tables, we obtain labor input as total hours worked and the labor compensation share.

Assuming zero profits, we first allocate non-labor compensation to the two capital types assuming that the rates of return are equalized. This is a standard procedure in productivity accounting, Organisation for Economic Co-operation and Development (2009). Summing the capital rental equation (37) across capital types, we have that

$$\alpha y_1 = \sum_{j \in \{S, E\&I\}} u_j k_j = \sum_{j \in \{S, E\&I\}} \left(r + \delta_j - g_j^q \right) q_j k_j$$

which allows us to solve for the implicit rate of return on capital,

$$r = \frac{\alpha y_1 - \sum_j \left(\delta_j - g_j^q\right) q_j k_j}{\sum_j q_j k_j}.$$

Given data on capital compensation, depreciation, and the value of the net-stock of capital, we can calculate the implicit rate of return on capital, r. Given r we can then calculate income shares for the different capital types.

For the full sample, the average capital income share and its allocation among the two capital types are

$$\alpha = 0.37,\ \phi_S = 0.44 \text{ and } \phi_{E\&I} = 0.56.$$

This compares with the average allocation of investment among the two capital types of

$$\omega_S = 0.26 \text{ and } \omega_{E\&I} = 0.74.$$

Because structures depreciate at a lower rate than does E&I, the net-stock of structures is relatively high despite the smaller investment share for structures. The higher net-stock of structures in turn implies a higher implicit share in capital income for structures.

Carrying out a growth accounting exercise similar to Greenwood, Hercowitz, and Krusell (1997), we calculate the average growth rates for employment, consumption-specific TFP, and the relative TFP for the production of investment goods,

$$g^\ell = 1.5\%,\ g_1^z = 0.6\%,\ g_S^q = 0.8\%,\ g_{E\&I}^q = -1.6\%$$

Using these primitives, we can calculate the implied growth rates along the BGP using either of the two average capital allocation shares,

$$\phi : g^k = 3.3\% \text{ and } g_1^y = 2.8\%$$
$$\omega : g^k = 4.0\% \text{ and } g_1^y = 3.0\%$$

Using investment shares instead of capital shares puts relatively more weight on the faster growing E&I TFP component and, therefore, overstates somewhat the BGP contributions from capital, g^k. However, the implied bias for output growth is about 0.2 ppts, 3.0 percent vs. 2.8 percent. This bias is noticeable but not overly so. Moreover, it would have immaterial effects on our findings regarding the relative importance of sector-specific factors in driving long-run aggregate growth rates.

7 Data

Our calculations rely on the official 2020 version of the ILPA KLEMS dataset which covers the period 1987-2018, and the experimental ILPA KLEMS dataset for the period $1947-2016$.[10] To simplify the presentation and analysis, we carry out the empirical work using private industries at the two-digit level. In particular, we aggregate the available private industries included in the two KLEMS datasets into 16 two-digit private industries following the procedure in Hulten (1978). Another advantage of the aggregation into two-digit industries is that any differences between the two KLEMS datasets are attenuated and we feel comfortable splicing the two datasets in 1987.

While the two ILPAs are related, they are not exactly identical for the time period in which they overlap. Since both data sets are constructed to be consistent with the BEA's input-output tables, they mostly agree on industry details and cover the same aggregated 16 private industries. Nevertheless, there remain minor differences but these are reflected mostly in the levels of the variables and not their growth rates. Hence, we use the growth rates calculated using the experimental ILPA data before 1987 and using the official ILPA data after that date.

The experimental ILPA data from 1947-1963 cover 42 SIC private industries while the experimental ILPA data from 1963-2016, and the official ILPA data from 1987-2018, cover 61 private NAICS industries. All data sets can be aggregated into the same 16 two-digit industries and contain nominal and real series for gross output, $Y_{j,t}$ and $y_{j,t}$ respectively, intermediate inputs, $M_{j,t}$ and $m_{j,t}$, capital, $K_{j,t}$ and $k_{j,t}$, and labor, $L_{j,t}$ and $l_{j,t}$.

The experimental ILPA data makes available an intermediate input aggregate and the official ILPA data give us separate series for nominal and real energy, materials, and services. Thus, we construct an intermediate input aggregate corresponding to the official ILPA series as a Divisia index from these three components. The Divisia quantity index for a series of nominal and real components respectively, $X_{j,t}$ and $x_{j,t}$, with $j \in J$, is defined as

$$100 \times \Delta \ln x_t = 100 \times \sum_{j \in J} \bar{S}^x_{j,t} \Delta \ln x_{j,t},$$

where $\bar{S}^x_{j,t} = \left(S^x_{j,t} + S^x_{j,t-1}\right)/2$ and $S^x_{j,t} = X_{j,t}/\sum_{s \in J} X_{s,t}$. Both ILPA datasets give us nominal and real series for two types of labor inputs: non-college and college labor; and five types

[10] The official ILPA dataset for 1987-2018 is downloaded from `https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems` and the experimental ILPA dataset for 1947-2016 is downloaded from `https://www.bls.gov/mfp/special_requests/tables_detail.xlsx`. See Fleck et al. (2014) and Corby et al. (2020) for a detailed description of the official ILPA data, and Eldridge et al. (2020) for the experimental ILPA data.

of capital inputs: IT equipment, software, R&D, entertainment related intellectual property, and others. We then construct aggregate series for labor and capital inputs by way of Divisa quantity indices using these different input types.

For each dataset, we construct growth rates of nominal and real value added, capital, labor, and value-added TFP at the level of sectoral detail available. The official ILPA data include measures of nominal and real value added, but the experimental ILPA data do not. We define the growth rates of real valued added in the experimental ILPA through the Divisia index definition of real gross output. The following definitions hold:

1. Nominal Value Added,
$$V_{j,t} = Y_{j,t} - M_{j,t},$$

2. Value Added Share in Gross Output,
$$S_{j,t}^{VY} = \frac{V_{j,t}}{Y_{j,t}},$$

3. Intermediate Input Share in Gross Output,
$$S_{j,t}^{MY} = \frac{M_{j,t}}{Y_{j,t}},$$

4. Value Added Growth Rates,
$$\Delta\tilde{v}_{j,t} = 100 \times \frac{\Delta \ln y_{j,t} - \bar{S}_{j,t}^{MY} \Delta \ln m_{j,t}}{\bar{S}_{j,t}^{VY}},$$

5. Capital Share in Value Added,
$$S_{j,t}^{KV} = \frac{K_{j,t}}{V_{j,t}},$$

6. Labor Share in Value Added,
$$S_{j,t}^{LV} = \frac{L_{j,t}}{V_{j,t}},$$

7. Capital Growth Rates,
$$\Delta\tilde{k}_{j,t} = 100 \times \Delta \ln k_{j,t},$$

8. Labor Growth Rates,
$$\Delta\tilde{\ell}_{j,t} = 100 \times \Delta \ln \ell_{j,t},$$

9. Value Added TFP Growth Rates,

$$\Delta \tilde{z}_{j,t} = 100 \times \left[\Delta \ln v_{j,t} - \bar{S}^{KV}_{j,t} \Delta \ln k_{j,t} - \bar{S}^{LV}_{j,t} \Delta \ln \ell_{j,t} \right].$$

7.1 Housing

The detailed ILPA industry data include a 'Real Estate' (RE) industry, which combines residential housing (HO), both tenant and owner occupied, and 'Other Real Estate' (ORE), and the detailed data also contain a separate 'Rental and Leasing Services' industry. We separate out residential housing, and include the other real estate related industries in Finance, Insurance, and Real Estate (FIRE) ex housing.

We separate out HO and ORE from the RE industry as follows. We obtain data on nominal and real gross output, value added, and intermediate inputs for residential housing from the NIPA Supplemental Tables 7.4. We construct real gross output, value added, and intermediate inputs for ORE as Divisia indices using real and nominal gross output, value added, and intermediate inputs for RE and HO, similar to our construction of real value added from gross output and intermediate input data described above. We construct real employment and capital services for HO and ORE by splitting total employment and capital services in RE according to the wage and capital shares of HO and ORE. This procedure assumes that the factor rentals in HO and ORE are the same. We end up with real and nominal inputs and outputs for HO and ORE. We treat HO as a separate industry, and we include ORE in the FIRE ex Housing industry aggregate.

7.2 Aggregating KLEMS into Consolidated Sectors

As mentioned, we combine the disaggreated KLEMS sectors above into broader consolidated sectors. For example, we might combine sectors $j \in \{1, ..., n\}$ into a single sector J. We use the following formulas to create consolidated sectors:

1. Nominal Value Added in Consolidated Value Added Shares,

$$S^{VV_J}_{j,t} = \frac{V_{j,t}}{\sum_{s \in J} V_{s,t}},$$

2. Nominal Labor in Consolidated Labor Shares,

$$S^{LL_J}_{j,t} = \frac{L_{j,t}}{\sum_{s \in J} L_{s,t}},$$

3. Nominal Capital in Consolidated Capital Shares,

$$S_{j,t}^{KK_J} = \frac{K_{j,t}}{\sum_{s\in J} K_{s,t}},$$

4. Value Added Growth Rates,

$$\Delta\tilde{v}_{J,t} = \sum_{j\in J} \bar{S}_{j,t}^{VV_J} \Delta\tilde{v}_{j,t},$$

5. Capital Growth Rates,

$$\Delta\tilde{k}_{J,t} = \sum_{j\in J} \bar{S}_{j,t}^{KK_J} \Delta\tilde{k}_{j,t},$$

6. Labor Growth Rates,

$$\Delta\tilde{\ell}_{J,t} = \sum_{j\in J} \bar{S}_{j,t}^{LL_J} \Delta\tilde{\ell}_{j,t},$$

7. TFP Growth Rates,

$$\Delta\tilde{z}_{J,t} = \Delta\tilde{v}_{J,t} - \bar{S}_{J,t}^{KV_J} \Delta\tilde{k}_{J,t} - \bar{S}_{J,t}^{LV_J} \Delta\tilde{\ell}_{J,t}, = \sum_{j\in J} \Delta\tilde{z}_{j,t}.$$

We obtain measures of nominal and real aggregate value added, that is, GDP, capital, labor, and TFP the same way we construct these measures for consolidated sectors. Frequently we replace the time-varying shares with constant sample averages of these shares.

8 Summary Tables of Sectoral Linkages

Below is the Capital Flow table for the U.S. economy.

Table A5: Ω, Capital Flow Table

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Min	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Util	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Const	0.12	0.07	0.47	0.03	0.15	0.14	0.14	0.52	0.14	0.20	0.15	0.12	0.38	0.57	0.41	0.91
Dur Gds	0.70	0.31	0.36	0.76	0.58	0.61	0.60	0.35	0.69	0.52	0.58	0.45	0.43	0.30	0.42	0.05
Nd Gds	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00
Wh Trd	0.10	0.05	0.04	0.08	0.07	0.08	0.07	0.04	0.06	0.06	0.06	0.07	0.09	0.05	0.05	0.00
Ret Trd	0.04	0.01	0.01	0.04	0.01	0.02	0.03	0.02	0.01	0.02	0.07	0.04	0.02	0.03	0.04	0.01
T&W	0.02	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.00
Inf	0.00	0.02	0.03	0.04	0.03	0.02	0.05	0.02	0.02	0.10	0.04	0.06	0.03	0.01	0.02	0.00
FIRE (x-Hous)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
PBS	0.02	0.04	0.04	0.04	0.16	0.12	0.09	0.03	0.06	0.10	0.08	0.25	0.06	0.03	0.04	0.00
Ed&H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A,E&FS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oth Serv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Ω_{ij} shows the share of investment goods used by sector j that originated in sector i. Entries are computed from the 1997 Capital Flow Tables.

Below is the Make-Use table for the U.S. economy.

Table A6: Φ, Make-Use Table

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.39	0.00	0.00	0.00	0.01	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Min	0.01	0.27	0.32	0.02	0.02	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Util	0.01	0.02	0.02	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.04	0.01	0.02	0.02	0.01	0.00
Const	0.01	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.04	0.00	0.00	0.00	0.01	0.40
Dur Gds	0.04	0.15	0.02	0.38	0.57	0.06	0.03	0.03	0.07	0.12	0.01	0.06	0.07	0.05	0.19	0.06
Nd Gds	0.26	0.10	0.11	0.13	0.09	0.37	0.04	0.05	0.21	0.04	0.01	0.05	0.12	0.22	0.07	0.01
Wh Trd	0.10	0.05	0.03	0.09	0.09	0.07	0.08	0.05	0.07	0.04	0.00	0.03	0.04	0.04	0.04	0.01
Ret Trd	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.03	0.03
T&W	0.05	0.08	0.17	0.04	0.04	0.05	0.13	0.14	0.27	0.03	0.02	0.04	0.03	0.03	0.02	0.00
Inf	0.00	0.02	0.02	0.02	0.02	0.01	0.06	0.06	0.02	0.37	0.05	0.08	0.04	0.04	0.04	0.00
FIRE (x-Hous)	0.07	0.05	0.08	0.03	0.02	0.01	0.18	0.27	0.12	0.07	0.52	0.19	0.31	0.18	0.36	0.42
PBS	0.04	0.22	0.16	0.13	0.13	0.11	0.42	0.33	0.19	0.25	0.25	0.47	0.28	0.30	0.18	0.07
Ed&H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.03	0.00	0.01	0.00
A,E&FS	0.00	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.06	0.03	0.05	0.03	0.06	0.02	0.00
Oth Serv	0.00	0.00	0.01	0.01	0.00	0.00	0.04	0.02	0.01	0.02	0.02	0.03	0.04	0.02	0.03	0.00
Housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Φ_{ij} shows the share of materials used by sector j that originated in sector i. Entries are computed from the 2015 BEA Make and Use Tables.

The implied generalized Leontief inverse from the Capital Flow and Make-Use tables above is as follows.

Table A7: Ξ', Generalized (Weighted) Leontief Inverse

	Agr	Min	Util	Const	Dur Gds	Nd Gds	Wh Trd	Ret Trd	T&W	Inf	FIRE x-H	PBS	Ed &H	A,E, FS	Oth Serv	Hous
Agr	0.53	0.08	0.02	0.09	0.43	0.14	0.19	0.03	0.07	0.05	0.10	0.28	0.00	0.01	0.01	0.00
Min	0.01	1.06	0.01	0.09	0.38	0.07	0.12	0.02	0.06	0.05	0.07	0.29	0.00	0.01	0.01	0.00
Util	0.01	0.16	0.67	0.18	0.35	0.07	0.11	0.03	0.07	0.05	0.07	0.27	0.00	0.02	0.01	0.00
Const	0.01	0.05	0.01	0.61	0.37	0.07	0.12	0.05	0.05	0.04	0.06	0.24	0.00	0.01	0.01	0.00
Dur Gds	0.02	0.06	0.01	0.07	0.87	0.08	0.15	0.02	0.05	0.05	0.07	0.31	0.00	0.02	0.01	0.00
Nd Gds	0.07	0.18	0.02	0.09	0.40	0.50	0.16	0.02	0.07	0.05	0.07	0.34	0.00	0.02	0.01	0.00
Wh Trd	0.01	0.03	0.01	0.07	0.29	0.04	0.76	0.02	0.06	0.06	0.09	0.30	0.00	0.01	0.02	0.00
Ret Trd	0.01	0.03	0.01	0.10	0.23	0.04	0.08	0.60	0.06	0.05	0.12	0.27	0.01	0.01	0.02	0.00
T&W	0.01	0.05	0.01	0.07	0.31	0.09	0.11	0.02	0.60	0.05	0.10	0.28	0.00	0.01	0.01	0.00
Inf	0.01	0.04	0.01	0.10	0.38	0.06	0.12	0.02	0.05	0.69	0.09	0.35	0.00	0.03	0.02	0.00
FIRE (x-Hous)	0.01	0.03	0.03	0.09	0.31	0.05	0.09	0.03	0.04	0.07	0.73	0.32	0.00	0.02	0.02	0.00
PBS	0.01	0.02	0.01	0.05	0.21	0.04	0.07	0.02	0.03	0.05	0.09	0.93	0.00	0.02	0.02	0.00
Ed&H	0.01	0.03	0.01	0.05	0.18	0.05	0.06	0.01	0.03	0.04	0.12	0.23	0.59	0.02	0.02	0.00
A,E&FS	0.02	0.04	0.01	0.10	0.23	0.08	0.09	0.02	0.04	0.05	0.11	0.28	0.00	0.55	0.02	0.00
Oth Serv	0.01	0.03	0.01	0.07	0.23	0.04	0.07	0.02	0.03	0.04	0.12	0.20	0.00	0.01	0.65	0.00
Housing	0.01	0.05	0.01	0.54	0.39	0.06	0.12	0.06	0.05	0.04	0.10	0.25	0.00	0.01	0.01	0.90

Notes: See text for definition of Ξ.

References

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The Network Origins of Aggregate Fluctuations. *Econometrica 80*(5), 1977–2016.

Baqaee, D. R. and E. Farhi (2019). The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem. *Econometrica 87*(4), 1155–1203.

Boppart, T. and P. Krusell (2020). Labor supply in the past, present, and future: A balanced-growth perspective. *Journal of Political Economy 128*(1), 118 – 157.

Corby, G., J. Harper, M. Russell, and J. Samuels (2020). Integrated Industry-Level Production Account for the United States and the Sources of U.S. Economic Growth Between 1987 and 2018. *Survey of Current Business 100*(4), 1–14.

Eldridge, L. P., C. Garner, T. F. Howells, B. C. Moyer, M. Russell, J. D. Samuels, E. H. Strassner, and D. B. Wasshausen (2020). *Toward a BEA-BLS Integrated Industry-level Production Account for 1947-2016*, pp. 221–249. Academic Press.

Fleck, S., S. Rosenthal, M. Russell, E. H. Strassner, and L. Usher (2014). *A Prototype BEA/BLS Industry-Level Production Account for the United States*, pp. 323–372. University of Chicago Press.

Foerster, A. T., A. Hornstein, P.-D. G. Sarte, and M. W. Watson (2019). "Aggregate Implications of Changing Sectoral Trends". NBER Working Papers 25867, National Bureau of Economic Research, Inc.

Geweke, J. (2004). Getting it right: Joint distribution tests of posterior simulators. *Journal of the American Statistical Association 99*(467), 799–804.

Gourio, F. and M. Rognlie (2020). Capital Heterogeneity and Investment Prices: How Much are Investment Prices Declining? Mimeo.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997). Long-Run Implications of Investment-Specific Technological Change. *American Economic Review 87*(3), 342–362.

Hulten, C. R. (1978). Growth Accounting with Intermediate Inputs. *Review of Economic Studies 45*(3), 511–518.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988). Production, Growth and Business Cycles. *Journal of Monetary Economics 21*(2-3), 195–232.

Müller, U. K. and M. W. Watson (2020). Low-frequency analysis of economic time series, *in preparation*. In *Handbook of Econometrics*. Elsevier.

Ngai, L. R. and C. A. Pissarides (2007). Structural Change in a Multisector Model of Growth. *American Economic Review 97*(1), 429–443.

Organisation for Economic Co-operation and Development (2009). *Measuring Capital: OECD Manual* (Second edition ed.). Organisation for Economic Co-operation and Development.

Stock, J. and M. Watson (1998). Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model. *Journal of the American Statistical Association 93*, 349–358.