# Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production

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**ABSTRACT**

This paper uses factor methods to decompose industrial production (IP) into components arising from aggregate and idiosyncratic sector-specific shocks. An approximate factor model finds that nearly all (90%) of the variability in IP growth rates are associated with common factors during our 1972-2007 sample period. Because common factors may reflect idiosyncratic shocks that have propagated through sectoral linkages, we use a multisector growth model to adjust for the effects of these linkages in the factor analysis. In particular, we show that neoclassical multisector models of the type first introduced by Long and Plosser (1983) produce an approximate factor model as a reduced form. A structural factor analysis then indicates that the Great Moderation was characterized by a fall in the importance of aggregate shocks. In contrast, the volatility of sectoral shocks was essentially unchanged throughout the 1972-2007 period. Consequently, idiosyncratic shocks took on a relatively more important role during the Great Moderation explaining approximately half of the quarterly variation in IP.

Key Words: Input-Output Matrix, Great Moderation, Approximate Factor Model

JEL Codes:  E32, E23, C32

## 1. Introduction

The Federal Reserve Board's Index of Industrial Production (IP) is an important indicator of aggregate economy activity in the United States. Month-to-month and quarter-to-quarter variations in the index are large. Monthly and quarterly growth rates for the seasonally adjusted IP index over 1972-2007 are plotted in Figure 1. Over this sample period, the standard deviation of monthly growth rates was over eight percentage points (at an annual rate), and quarterly growth rates had a standard deviation of nearly six percentage points. Also evident in the figure is the large fall in volatility associated with the Great Moderation; the standard deviation in the 1984-2007 period is roughly half its pre-1984 value for both the monthly and quarterly series.

Because the IP index is constructed as a weighted average of production indices across many sectors, the large volatility in the IP index is somewhat puzzling. Simply put, while production in an individual sector (for example, "Motor Vehicle Parts") may vary substantially from month-to-month or quarter-to-quarter, apparently much of this variability does not "average out" in the index of economy-wide production. There are three leading explanations for this puzzle. The first relies on aggregate shocks that affect all industrial sectors. Since these shocks are common across sectors, they do not average out and thus become the dominant source of variation in aggregate economic activity. The other two explanations rely on uncorrelated sector-specific shocks. The first of these, carefully articulated in Gabaix (2010), postulates a handful of very large sectors in the economy and because these large sectors receive a large weight in aggregate IP, shocks to these sectors do not average out.

The second sector-specific explanation postulates complementarities in production, such as input-output linkages, that propagate sector-specific shocks throughout the economy in a way that generates substantial aggregate variability. The purpose of this paper is to determine the quantitative importance of these three explanations for explaining the variability in aggregate U.S. industrial activity as measured by the IP index.

The related literature analyzing sector-specific versus aggregate sources of variations in the business cycle has followed two main approaches. Long and Plosser (1987), Forni and Reichlin (1998), and Shea (2002) among others rely on factor analytic methods, coupled with broad identifying restrictions, to assess the relative contributions of aggregate and sector-specific shocks to aggregate variability. These papers generally find that sector-specific shocks contribute a non-trivial fraction of aggregate fluctuations (e.g. approximately 50 percent in Long and Plosser (1987)). A second strand of literature is rooted in more structural calibrated multisector models, such as Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999) and Carvalho (2007), that explicitly take into account sectoral linkages across sectors. In these models, whether sectoral linkages are sufficiently strong to generate substantial aggregate variability from idiosyncratic shocks depends in part on the exact structure of these linkages.

This paper bridges these reduced form and structural approaches and sorts through the leading explanations underlying both the volatility of IP and its decline in the Great Moderation period. In particular, it describes conditions under which neoclassical multisector models that explicitly consider sectoral linkages, such as those of Long and Plosser (1983) or Horvath (1998), produce an approximate factor model as a reduced form. Aggregate shocks to sectoral productivity emerge as the common output factors in the approximate factor model. The "uniquenesses" in the factor model are associated with sector-specific shocks. However, because sectoral linkages induce correlation across the uniquenesses, factors estimated from reduced form models may be biased and reflect not only aggregate shocks but also idiosyncratic shocks that propagate across sectors by way of these sectoral linkages.

We begin our analysis in section 2 by describing data on 117 sectors that make up aggregate IP and that form the basis of our empirical analysis. The reduced form analysis that we carry out in this section suggests that, in contrast to Gabaix (2010), sectoral weights play little role in explaining the variability of the aggregate IP index. As in Shea (2002), aggregate variability is driven mainly by covariability across sectors and not sector-specific variability. This leads us to rule out the "few-large-sectors" explanation in favor of explanations that rely on covariability across sectors.

In section 3 we focus on this covariability. Consistent with Quah and Sargent (1993), who study comovement in employment across 60 industries, and Forni and Reichlin (1998), who study annual U.S. output and productivity data over 1958-1986, we find that much of the covariability in sectoral production can be explained by a small number of common factors. These common factors are the leading source of variation in the IP index, and a decrease in the variability of these common factors explains virtually all of the 1984-2007 decline in aggregate volatility.

Because common factors may reflect not only aggregate shocks but also the propagation of idiosyncratic shocks by way of sectoral linkages, section 4 develops a generalized version of the multisector growth model introduced by Long and Plosser (1983), and later extended by Horvath (1998), to filter out the effects of these linkages. The generalized model we present is detailed enough to accommodate our decomposition of IP into 117 sectors while still allowing these sectors to interact via input-output linkages in both intermediate materials and capital goods. Empirical results from the model suggest that the onset of the Great Moderation is characterized by a fall in the importance of aggregate shocks. In contrast, the volatility of sectoral shocks remained approximately unchanged throughout the 1972-2007 sample period. As a result, sectoral shocks acquired a more prominent role in the latter part of our sample period. More specifically, the model implies that while sector-specific shocks accounted for 20 percent of the variability in aggregate IP during 1972-1983, this share increased to roughly 50 percent during 1984-2007.

The model developed in section 4 is sufficiently complicated that we must solve parts of it using numerical methods, and this hides some of the key mechanisms that are responsible for its empirical predictions. Hence, in section 5, we use insights obtained from analytic solutions to the models of Long and Plosser (1983), Horvath (1998) and Dupor (1999), and Carvalho (2007), together with numerical experiments, to elucidate these mechanisms. In particular we show that the model's predictions depend on the nature of contemporaneous input-output interactions associated with intermediate materials as well as intertemporal input-output interactions associated with capital goods. For example, in the original work of Long and Plosser (1983), the structural model imposes a one-period lag in the delivery of materials and abstracts from capital accumulation. Consequently, when confronted to U.S. data using factor analytic methods, that model predicts a relatively limited role for idiosyncratic sectoral shocks. In contrast, the propagation of sectoral shocks is enhanced when materials are used contemporaneously in production and capital goods in a sector are potentially produced in other sectors.

Section 6 carries out some robustness analysis and shows, for example, that the empirical conclusions in section 3 are robust to changes in sectoral input-output relationships over our 35-year sample period. Section 7 offers some concluding remarks.

## 2. A First Look at the Sectoral IP Data

### 2.1 Overview of the Data

This paper uses sectoral data on Industrial Production over the period 1972-2007. The data are for 117 sectors that make up aggregate IP, where each sector roughly corresponds to a 4-digit industry using the North American Industry Classification System (NAICS). As discussed in Appendix A, where the data are described in detail, this is the finest level of disaggregation that allows us to match the sectors to the input-output and capital-use tables used to calibrate the structural models in Section 4.

Let $IP_t$ denote the value of the aggregate IP index at date $t$, and $IP_{it}$ denote the index for the $i^{\text{th}}$ sector. These indices are available monthly. Quarterly values of the indices are constructed as averages of the months in the quarter. Figure 1 indicates that the quarterly and monthly data share many features, except that monthly data exhibit more high frequency variability. Because we are less interested in this high frequency variability, we will focus our attention on the quarterly data. Growth rates (in percentage points) are denoted by $g_t$ for aggregate IP and by $x_{it}$ for sectoral IP, where $g_t = 400 \times \ln(IP_t/IP_{t-1})$ and similarly for $x_{it}$. We will let $N$ denote the number of sectors in our sample (so that $N$=117 sectors) and $T$ denote the number of time periods ($T$ = 144 quarters).

The sectoral growth rates are more volatile than the aggregate growth rates plotted in Figure 1. Figure 2 shows the distributions of standard deviations of growth rates for the 117 IP sectors over the sample periods 1972-1983 and 1984-2007. The Great Moderation is evident in these distributions, where the median sectoral growth rate standard deviation fell from 17.4 in the early sample period to 11.3 in the latter period.

### 2.2 Sectoral Size, Covariability and Aggregate Variability

Let $w_{it}$ denote the share of the $i^{\text{th}}$ sector in the aggregate index, so that the growth rate of aggregate IP can be written as $g_t = \sum_{i=1}^{N} w_{it} x_{it}$ . This section investigates the role of the sectoral weights, $w_{it}$ , and that of the covariance of $x_{it}$ across sectors in explaining the variability in $g_t$.

Our focus on the role of $w_{it}$ is motivated by recent work by Gabaix (2010), who considers an economy in which $N$ firms operate independently. The growth rate in aggregate output is then a share-weighted average of the firms' growth rates, which are assumed to be mutually uncorrelated. Gabaix derives conditions on the distribution of firm size that guarantee that the variability of the aggregate growth rate remains bounded away from zero as the number of firms grows large. The result relies on a size distribution that places sufficient weight on relatively large firms so that shocks to these firms have a non-negligible effect on aggregate output.

In our context, Gabaix's analysis suggests that the volatility of aggregate IP may arise from idiosyncratic shocks to IP sectors with large shares. In our data, the empirical distribution of the 117 share weights suggests that this could indeed be an important source of variability in aggregate IP: with 117 sectors, the average share is less than one percent, but the largest 4 sectors account for 20 percent of aggregate IP, and the largest 20 sectors account for more than 50 percent. Thus, the volatility of aggregate IP may arise from shocks to a relatively small number of sectors that receive a large weight in the aggregate average.

To quantify the share effect, consider the following decomposition of $g_t$:

(1) $$g_t = \sum_{i=1}^{N} w_{it} x_{it} = \frac{1}{N}\sum_{i=1}^{N} x_{it} + \sum_{i=1}^{N}\left( w_{it} - \frac{1}{N}\right) x_{it}$$

where the first equality defines the aggregate growth rate, and the second decomposes $g_t$ into a component associated with equal shares ($1/N$) and a component associated with the deviation of shares from ($1/N$).[1] The first term, $(1/N)\sum_{i=1}^{N} x_{it}$, weights each sector equally, and when $x_{it}$ are uncorrelated, this component has a variance proportional to $N^{-1}$, and so will be small when $N$ is large. On the other hand, the variance of the second term, $\sum_{i=1}^{N}\left( w_{it} - \frac{1}{N}\right) x_{it}$, may be large if the cross-sectional variance of shares is large at date $t$. Letting $\bar{w}_i = (1/T)\sum_{t=1}^{T} w_{it}$ denote the average share of sector $i$, this second term may be large because some sectors have large average shares over the sample period (so $\bar{w}_i$ is large for some $i$), or have very unstable shares (so for each date $t$, the value of $w_{it} - \bar{w}_i$ is large for some sector $i$). To capture these two distinct effects, it useful to further decompose $g_t$ as

[1] Gabaix (2010) refers to the term $\sum_{i=1}^{N}\left( w_{it} - \frac{1}{N}\right) x_{it}$ as the "granular residual."

(2) $$g_t = \frac{1}{N}\sum_{i=1}^{N} x_{it} + \sum_{i=1}^{N}\left(\bar{w}_i - \frac{1}{N}\right)x_{it} + \sum_{i=1}^{N}\left(w_{it} - \bar{w}_i\right)x_{it}$$

Figure 3 plots this decomposition of $g_t$, and Table 1 shows the standard deviation of each of the components over the full sample and the two subsamples. Examination of the figure and table indicates that the equally-weighted component, $N^{-1}\sum_{i=1}^{N} x_{it}$, tracks the aggregate IP series closely. (Indeed, from Table 1, this component has a variance slightly larger than the aggregate, $g_t$.) In contrast, the unequal share components are far less important. Therefore, this decomposition leads us to conclude that although aggregate IP is composed in part of a few large sectors, the mechanism highlighted by Gabaix (2010) is not an important factor in explaining the variability of aggregate IP.

While the distribution of sectoral shares (i.e. differences in the size of sectors) is not a crucial consideration for the variability of IP, the covariability of sectoral growth rates is an important factor in this calculation. Tables 2 and 3 summarize key results to clarify this point.

Table 2 shows that average pairwise correlations of the sectoral growth rates was positive and large (0.27) in the 1972-83 sample period and fell substantially (to 0.11) in the 1984-2007 sample period. Table 3 summarizes a calculation suggested by Shea (2002). In particular, because $g_t$ is a sum ($=\sum_{i=1}^{N} w_{it}x_{it}$), the variance of $g_t$ is the sum of the variances of the summands plus all of the pairwise covariances. Thus, Table 2 compares the standard deviation of $g_t$ computed using the usual formula (which includes all of the covariances) to the value from the formula that considers only the individual variances of $x_{it}$. The results indicate that the covariance terms are the dominant source of variation in the growth rate of aggregate IP. For example, in Table 3A, the full-sample standard deviation of $g_t$ falls by a factor of 3 (from 5.8 percent to 1.9 percent) when the covariance terms are set equal to zero. This result also holds for a version of aggregate IP that weights each sector equally and, therefore, is not driven by a relatively few covariances associated with large sectors (Table 3B).

To sum up, the results in this section allow us to discount the "large shocks in sectors with large shares" explanation for the variability of IP growth. If anything, the fact that, in Table 3, the standard deviation of IP growth using uniform $1/N$ shares is slightly larger than that computed using actual share weights suggests that larger sectors are associated with smaller shocks. In addition, the variability of aggregate IP growth is associated with shocks that lead to covariability in sectoral output, not shocks that lead to large idiosyncratic sectoral volatility. Therefore, the remaining challenge is to measure and understand the shocks that lead to covariability, and this challenge is taken up in the remainder of the paper.

## 3. Statistical Factor Analysis

As discussed in Forni and Reichlin (1998), the approximate factor model is a natural way to model the covariance matrix of sectoral production. Letting $X_t$ represent the $N\times 1$ vector of sectoral growth rates, this model represents $X_t$ as

$$X_t = \Lambda F_t + u_t \tag{3}$$

where $F_t$ is a $k\times 1$ vector of latent factors, $\Lambda$ is an $N\times k$ matrix of coefficients called factor loadings, and $u_t$ is an $N\times 1$ vector of sector-specific idiosyncratic disturbances. In classical factor analysis (Anderson (1984)), $F_t$ and $u_t$ are mutually uncorrelated i.i.d. sequences of random variables, and $u_t$ has a diagonal covariance matrix. Thus, $X_t$ is an i.i.d. sequence of random variables with covariance matrix $\Sigma_{XX} = \Lambda\Sigma_{FF}\Lambda' + \Sigma_{uu}$, where $\Sigma_{FF}$ and $\Sigma_{uu}$, are the covariance matrices of $F_t$ and $u_t$ respectively. Because $\Sigma_{uu}$, is diagonal, any covariance between the elements of $X_t$ arises from the common factors $F_t$.

Approximate factor models (see Chamberlain and Rothschild (1983), Connor and Korackzyk (1986), Forni, Halli, Lippi, and Reichlin (2000), and Stock and Watson (2000)), weaken the classical factor model assumptions by (essentially) requiring that, in large samples, key sample moments involving $F_t$ and $u_t$ mimic the behavior of sample moments in classical factor analysis. This allows for weak cross sectional and temporal dependence in the series, subject to the constraint that sample averages satisfy laws of large numbers with the same limits as those that would obtain in classical factor analysis. When $N$ and $T$ are large, as they are in this paper's application, the approximate factor model has proven useful because relatively simple methods can be used for estimation and inference. For example, penalized least squares criteria can be used to consistently estimate the number of factors (Bai and Ng (2002)), principal components can be used to consistently estimate factors (Stock and Watson (2000)), and the estimation error in the estimated factors is sufficiently small that it can be ignored when estimating variance decompositions or conducting inference about Λ (Stock and Watson (2000), Bai (2003)).

Tables 4-5 and Figures 4-5 summarize the results from applying these methods to the sectoral IP growth rates. To begin, we estimated the number of factors using the Bai and Ng (2002) ICP1 and ICP2 estimators. These estimators yielded 2 factors in the full sample period (1972-2007), and first sample period (1972-1983). They yielded 1 factor in the second sample period (1984-2007). The results shown in Tables 4-5 and Figures 4-5 are based on estimated 2-factor models. For robustness, we also carried out our analysis using 1- and 3-factor models. The results (not shown) were similar to those we report for the 2-factor model.

We gauge the importance of common shocks, $F$, relative to the idiosyncratic shocks, $u$, in two related ways. First, we calculate the fraction of variability in the growth rates of aggregate IP explained by $F$. Second, we calculate the fraction of the variability in each sector's growth rate that is explained by $F$. The fraction of variability in $x_{it}$ associated with the common factors is given by $R_i^2(F) = \lambda_i'\Sigma_{FF}\lambda_i/\sigma_i^2$, where $\lambda_i'$ denotes the $i^{th}$ row of Λ and $\sigma_i^2$ = var($x_{it}$). To compute the analogous $R^2$ for aggregate IP, we ignore time variation in $w_{it}$ and use the approximation $g_t \approx \bar{w}' X_t$ where $\bar{w}$ is the vector of mean sectoral shares. The fraction of the variability of $g_t$ attributable to the factors is then $R^2(F) = \bar{w}'\Lambda\Sigma_{FF}\Lambda'\bar{w}/\sigma_g^2$.

Table 4 shows the 2-factor model's implied standard deviation of aggregate IP (computed using constant shares, $\bar{w}$) together with $R^2(F)$. The factor model implies an aggregate IP index with volatility that is essentially identical to that found in the data, and the common factors explain nearly all of the variability in growth rates of the aggregate IP index over both sample periods. Because the common factors explain roughly 90 percent of the variability in both sample periods, they are responsible for 90 percent of the decrease in aggregate volatility across the two time periods. Figure 4 plots the growth rate of aggregate IP and its fitted value from the factor model ($\bar{w}'\hat{\Lambda}\hat{F}_t$). The fitted value closely tracks the actual value during the entire sample period. Thus, viewed through the lens of the statistical factor model, the Great Moderation is explained by a decrease in the variance of the common shocks that affect IP.

At a disaggregated level, Figure 5 shows the distribution $R_i^2(F)$ for the two sample periods. Common shocks were an important source of variability in sectoral growth rates prior to

1984 (the median value of $R_i^2(F)$ is 0.41), but were less important post-1984 (the median value of $R_i^2(F)$ falls to 0.19). While the importance of common shocks for the variability of sectoral growth rates declines after 1984, Figure 5 indicates that these shocks nevertheless explain a large fraction of output growth volatility in several individual series in both sample periods. Table 5 lists the ten sectors with the largest fraction of variability accounted for by common shocks. Prior to 1984, for example, idiosyncratic shocks played virtually no role in the variability of output growth in the sectors related to "Fabricated Metals: Forging and Stamping," or "Other Fabricated Metal Products."

Because the sectors in Table 5 move mainly with common shocks, and movements in the aggregate IP index are associated with these shocks, the sectors listed in Table 5 turn out to be particularly informative about the IP index. Consider, for instance, the problem of tracking movements in IP in real time using only a subset $M$ of the IP sectors, say the five highest ranked sectors in Table 5.[2] Let $X_{M,t}$ represent the vector of output growth rates associated with these $M$ sectors. The minimum mean square error linear predictor of $g_t$ based on $X_{M,t}$ is $\psi' X_{M,t}$ where $\psi$ are the linear regression coefficients. Table 6 shows the fraction of variability of $g_t$ explained by $\psi' X_{M,t}$ using the five highest ranked sectors in Table 5. Prior to 1984, these five sectors alone account for 83 percent of the variation in the growth rate of the aggregate index. In addition, 96 percent of the variability in IP growth rates is captured by considering only the twenty highest ranked sectors (out of 117) over 1972-1983. This fraction is somewhat smaller (83 percent) over the Great Moderation period. In either case, however, it is apparent that information about movements in IP turns out to be concentrated in a small number of sectors. Contrary to conventional wisdom, these sectors are not necessarily those with the largest weights, the most volatile output growth, nor the sectors with the most links to other sectors (e.g. "Electric Power Generation"). Since aggregate IP is driven mainly by common shocks, what matters is that those sectors also move with common shocks.

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[2] IP numbers are typically released with a one month lag, revised up to three months after their initial release, and further subject to an annual revision. Both to confirm initial releases and to independently track economic activity, the Institute for Supply Management constructs an index of manufacturing production based on nationwide surveys. In addition, several Federal Reserve Banks including Dallas, Kansas City, New York, Philadelphia, and Richmond, produce similar indices that are meant to capture real time changes in activity at a more regional level. Of course, a central issue pertaining to these surveys is that gathering information on a large number of sectors in a timely fashion is costly, so that the scope of the surveys is generally limited.

## 4. Structural Factor Analysis

An important assumption underlying the estimation of factors in the previous section was that the covariance matrix of the uniquenesses, $u_t$, in equation (3) satisfies weak cross sectional dependence. However, as discussed in Long and Plosser (1983), Horvath (1998), Carvalho (2007), and elsewhere, sectoral linkages between the industrial sectors may lead to the propagation of sector-specific shocks throughout the economy in a way that generates comovement across sectors. In other words, these linkages may effectively transform shocks that are specific to particular sectors into common shocks, and thereby explain in part the variability of aggregate output. As discussed in Horvath (1998) and Dupor (1999), the strength of this amplification mechanism depends importantly on the structure of production linkages between sectors.

In this section, we use the Bureau of Economic Analysis (BEA) estimates of the input-output and capital use matrices describing U.S. sectoral production to quantify the effects of shock propagation on the volatility of the aggregate IP index. Because this calculation requires a model that incorporates linkages between sectors, the first subsection describes a generalization of the framework introduced in Long and Plosser (1983) and extended by Horvath (1998). In the model of Horvath (1998), each sector uses intermediate materials produced in other sectors. However, for analytical tractability, capital in a given sector can only be produced within that sector and depreciates entirely within the period. In addition, Horvath (1998) abstracts from labor supply considerations. Our generalization relaxes these assumptions by allowing for a conventional labor supply decision and a standard depreciation rate on capital. More important, it allows for capital used in a sector to be produced by other sectors in a manner consistent with BEA estimates of capital use. Therefore, shocks to an individual sector may be disseminated to other sectors, because of interlinkages in materials and capital use, and over time, because the effects of interlinkages on capital are persistent under a standard depreciation rate, in a way that potentially contributes to aggregate fluctuations.

This section also bridges the reduced form factor approach of section 3 with the more structural approaches to sectoral analysis introduced by Long and Plosser (1983). In particular, it describes the conditions under which the factor model in (3) may, in fact, be interpreted as the reduced form of a structural model with explicit sectoral linkages. It then illustrates how the structural model may be used to filter out the effects of these linkages and presents quantitative findings based on a variety of experiments.

### 4. 1 A Canonical Model with Sectoral Linkages

Consider an economy composed of $N$ distinct sectors of production indexed by $j = 1, \ldots, N$. Each sector produces a quantity $Y_{jt}$ of good $j$ at date $t$ using capital, $K_{jt}$, labor, $L_{jt}$, and materials that are produced in the other sectors, $M_{ijt}$, according to the technology

$$(4) \qquad Y_{jt} = A_{jt} K_{jt}^{\alpha_j} \left( \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}} \right) L_{jt}^{1-\alpha_j - \sum_{i=1}^{N} \gamma_{ij}}$$

where $A_{jt}$ is a productivity index for sector $j$.

The fact that each sector uses materials from other sectors represents a source of interconnectedness in the model. An input-output matrix for this economy is an $N \times N$ matrix $\Gamma$ with typical element $\gamma_{ij}$. The column sums of $\Gamma$ give the degree of returns to scale in materials in each sector. The row sums of $\Gamma$ measure the importance of each sector's output as materials to all other sectors. Put simply, we can think of the rows and columns of $\Gamma$ as "sell to" and "buy from" shares for each sector, respectively.

Denote the vector of capital shares by $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_N)'$, let $A_t = (A_{1t} \ldots, A_{NT})'$ be the vector of productivity indices, and suppose that $\ln(A_t)$ follows the random walk process,

$$(5) \qquad \ln(A_t) = \ln(A_{t-1}) + \varepsilon_t,$$

where $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})'$ is a vector-valued martingale difference process with covariance matrix $\Sigma_{\varepsilon\varepsilon}$. The degree to which sectoral productivity is influenced by aggregate shocks will be reflected in the matrix $\Sigma_{\varepsilon\varepsilon}$. When $\Sigma_{\varepsilon\varepsilon}$ is diagonal, sectoral productivity is driven only by idiosyncratic shocks, while common shocks are reflected in non-zero off-diagonal elements of $\Sigma_{\varepsilon\varepsilon}$.

In each sector $j$, the capital stock evolves according to the law of motion

$$(6) \qquad K_{jt+1} = Z_{jt} + (1-\delta)K_{jt},$$

where $Z_{jt}$ denotes investment in sector $j$ and $\delta$ is the depreciation rate. Investment in sector $j$ is produced using the amount $Q_{ijt}$ of sector $i$ output by way of the constant-returns to scale technology

(7) $$Z_{ji} = \prod_{i=1}^{N} Q_{ijt}^{\theta_{ij}}, \quad \sum_{i=1}^{N} \theta_{ij} = 1$$

The fact that new capital goods in a given sector are produced using other sectors' output represents an additional source of interconnectedness in the model. A capital use matrix for this economy is an $N \times N$ matrix $\Theta$ with typical element, $\theta_{ij}$. The row sums of $\Theta$ measure the importance of each sector's output as a source of investment to all other sectors.

A representative agent derives utility from the consumption of these $N$ goods according to

(8) $$E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{N} \left( \frac{C_{jt}^{1-\sigma} - 1}{1-\sigma} - \psi L_{jt} \right)$$

where the labor specification follows Hansen's (1985) indivisible labor model. In addition, each sector is subject to the following resource constraint,

(9) $$C_{jt} + \sum_{i=1}^{N} M_{jit} + \sum_{i=1}^{N} Q_{jit} = Y_{jt}$$

The model is closely related to previous work including Long and Plosser (1983), Horvath (1998), and the benchmark model in Carvalho (2007), all of which admit analytical solutions for the competitive equilibrium. In a technical appendix (Foerster, Sarte, and Watson (2010)), we show that the deterministic steady state of the model continues to be analytically tractable, which is key for our disaggregation of the data into 117 sectors, and that a linear approximation of the model's first-order conditions and resource constraints around that steady state yields a vector ARMA(1,1) model for sectoral output growth, $X_t = [\Delta\ln(Y_{1t}), \ldots, \Delta\ln(Y_{Nt})]'$ :

$$(10) \qquad (\mathrm{I} - \Phi \mathrm{L})X_t = (\Pi_0 + \Pi_1 \mathrm{L})\varepsilon_t$$

where L is the lag operator and $\Phi$, $\Pi_0$, and $\Pi_1$ are $N{\times}N$ matrices that depend only on the model parameters, $\alpha$, $\Gamma$, $\Theta$, $\beta$, $\sigma$, $\psi$, and $\delta$.

Suppose that innovations to sectoral productivity, $\varepsilon_t$, reflect both aggregate shocks, represented by the $k{\times}1$ vector $S_t$, and idiosyncratic shocks, represented by the vector $v_t$,

$$(11) \qquad \varepsilon_t = \Lambda_S S_t + v_t,$$

where $\Lambda_S$ is a matrix of coefficients that determine how the aggregate disturbances affect productivity in individual sectors. We assume that $(S_t, v_t)$ is serially uncorrelated, $S_t$ and $v_t$ are mutually uncorrelated, and that $\Sigma_{vv} = \mathrm{E}(v_t v_t')$ is a diagonal matrix (so the idiosyncratic shocks are uncorrelated).

Given equations (10) and (11), the evolution of sectoral output growth can then be written as the dynamic factor model

$$(12) \qquad X_t = \Lambda(\mathrm{L})F_t + u_t$$

where $\Lambda(\mathrm{L}) = (\mathrm{I} - \Phi\mathrm{L})^{-1}(\Pi_0 + \Pi_1\mathrm{L})\Lambda_S$, $F_t = S_t$, and $u_t = (\mathrm{I} - \Phi\mathrm{L})^{-1}(\Pi_0 + \Pi_1\mathrm{L})v_t$. In other words, the vector of sectoral output growth rates in this multisector extension of the standard growth model produces an approximate factor model as a reduced form. The common factors in this reduced form, $F_t$, are associated with aggregate shocks to sectoral productivity, $S_t$. The reduced-form idiosyncratic shocks, $u_t$, reflect linear combinations of the underlying structural sector-specific shocks, $v_t$. Importantly, even though the elements of $v_t$ are uncorrelated, sectoral linkages will induce some cross-sectional dependence among the elements of $u_t$ by way of the matrix $(\mathrm{I} - \Phi\mathrm{L})^{-1}(\Pi_0 + \Pi_1\mathrm{L})$. This dependence may in turn lead the reduced form factor model to overestimate the importance of aggregate shocks. Said differently, by ignoring the covariance in $u_t$ that arises from production linkages, the reduced form factor model may incorrectly attribute any resulting comovement in sectoral output growth to aggregate shocks.

To eliminate the propagation effects of idiosyncratic shocks induced by production linkages, we use equation (10) and construct $\varepsilon_t$ as a filtered version of the observed growth rate data $X$:[3]

$$\varepsilon_t = (\Pi_0 + \Pi_1 L)^{-1}(I - \Phi L)X_t. \quad (13)$$

Factor analytic methods can then be applied directly to $\varepsilon_t$ to estimate the relative contribution of aggregate shocks, $S_t$, and sector specific shocks, $v_t$, for the variability of aggregate output.

**4. 2 Choosing Values for the Model Parameters**

We interpret the model as describing the sectoral production indices analyzed in sections 2 and 3. Thus, we abstract from output of the agriculture, public, and service sectors. In order to construct the filtered series described in equation (13), we must first choose values for the model's parameters. A subset of these parameters is standard and chosen in accordance with previous work on business cycles: $\sigma = 1$, $\psi = 1$, $\beta = 0.99$ and $\delta = 0.025$. The choice of input-output matrix, $\Gamma$, and capital use matrix, $\Theta$, requires more discussion.

Our choice of parameter values describing the final goods technology ($\gamma_{ij}$ and $\alpha_i$) derives from estimates of the "use tables" constructed by the BEA. We use the BEA's use tables from 1997 (although in our robustness analysis in Section 6 we show results using the 1977 use table). The BEA input use tables measures the value of inputs (given by commodity codes) used by each industry (given by industry codes). By matching commodity and industry codes for the 117 industries in our data, we obtain the value of inputs from each industry used by every other industry. The use tables also include compensation of employees (wages) and other value added (rents on capital). A column sum in the use table represents total payments from a given sector to all other sectors (i.e. material inputs, labor, and capital) and defines the value of output in that sector. A row sum in the use table gives the importance of a given sector as an input supplier to all other sectors, measured as the value of inputs in other sectors' production. Hence, input shares, $\gamma_{ij}$, are estimated as dollar payments from industry $j$ to industry $i$ expressed as a fraction of the value of production in sector $j$.

[3] In order to recover the innovations to productivity, $\varepsilon_t$, from current and lagged values of $X_t$, the process must be invertible; that is, the roots of $|\Pi_0+\Pi_1 z|$ must lie outside the unit circle.

The parameters that describe the production of investment goods ($\theta_{ij}$) derive from estimates of capital flow tables provided by the BEA, again for 1997. The capital flow table shows the destination of new investment in equipment, software, and structures, in terms of the industries purchasing or leasing the new investment. Similar to the use table, the capital flow table provides the most detailed view of investment by commodity and by industry. By matching commodity and industry codes, we obtain the value of investment goods from each industry purchased by all other industries. A column sum in the capital flow table represents total payments for investment goods from a given industry to all other industries. A row sum in the capital flow table gives the importance of a given sector as a source of investment to all other sectors. Thus investment shares, $\theta_{ij}$ , are estimated as dollar payments from industry *j* to industry *i* expressed as a fraction of the total investment expenditures of sector *j*. Appendix A discusses measurement issues associated with the input use and capital use tables.

The remaining set of parameters that need to be estimated are those making up $\Sigma_{\varepsilon\varepsilon}$, the covariance matrix of the structural productivity shocks, $\varepsilon_t$. We choose two calibrations for $\Sigma_{\varepsilon\varepsilon}$ that help highlight the degree to which the model is able propagate purely idiosyncratic shocks, and thus effectively transform these shocks into common shocks. In the first calibration, $\Sigma_{\varepsilon\varepsilon}$ is a diagonal matrix with entries given by the sample variance of $\varepsilon_t$ in the different IP sectors. The sample variance of $\varepsilon_t$ is computed over different sample periods (e.g. 1972-1983, 1984-2007) to account for heteroskedasticity that may be important for the Great Moderation. Because this calibration uses uncorrelated sectoral shocks, it allows us to determine whether sectoral linkages per se can explain the covariance in sectoral production that is necessary to generate the variability in aggregate output.

In the second calibration, we use a factor model to represent $\Sigma_{\varepsilon\varepsilon}$. That is, we model $\varepsilon_t$ as shown in equation (11), where $S_t$ is a $k\times1$ vector of common factors and $v_t$ is an $N\times1$ vector of mutually uncorrelated sector-specific idiosyncratic shocks. In this model $\Sigma_{\varepsilon\varepsilon} = \Lambda_S\Sigma_{SS}\Lambda_S' + \Sigma_{vv}$, where $\Lambda_S$, and the covariance matrices $\Sigma_{SS}$ and $\Sigma_{vv}$, are estimated using the principal components estimator of $S_t$ constructed from the sample values of $\varepsilon_t$. This second calibration allows two sources of covariance in sectoral output: a structural component arising from sectoral linkages and a statistical component arising from aggregate shocks affecting sectoral productivity.[4]

[4] We do not attempt to identify the source of the common factors affecting sectoral productivity. These factors may reflect aggregate shocks affecting all sectors in the economy or, given that our data comprise only the industrial

### 4.3 Results from the Structural Analysis

***Comovement from Sectoral Linkages***

Table 7 summarizes the results. The first four columns of Table 7 show the average pairwise correlation of sectoral growth rates, $x_{it}$, and the standard deviation of aggregate IP growth, $\sigma_g$, for the data and as implied by the model with uncorrelated $\varepsilon$ shocks ($\Sigma_{\varepsilon\varepsilon}$ diagonal). The last three columns show the average pairwise correlation, standard deviation of aggregate IP growth, and the fraction of IP variability explained by aggregate shocks in the model where $\varepsilon$ depends on two common factors ($\Sigma_{\varepsilon\varepsilon} = \Lambda_S \Sigma_{SS} \Lambda_S' + \Sigma_{vv}$).

Four results are evident in Table 7. First, the model with input-output linkages alone and uncorrelated sector-specific shocks implies noticeably less co-movement across sectors than in U.S. data. For example, in the 1972-83 sample period, the average pairwise correlation in the data is 0.27, but the model with uncorrelated shocks implies an average pairwise correlation of only 0.05. Because the model with uncorrelated shocks under-predicts the correlation in sectoral growth rates, it is unable to replicate all of the variability in aggregate IP growth. The second result is that the model with two common factors closely matches the data's sectoral covariability and the variability in aggregate IP (although the model over-predicts the variability somewhat in the 1984-2007 sample period). Third, although the model's internal propagation mechanism with purely idiosyncratic shocks falls short of reproducing the sectoral comovement seen in the data, it does not mean that this mechanism is inoperative. Thus, the fraction of IP variability explained by aggregate shocks is smaller than that explained by the reduced form factors in both sample periods. For example, over 1984-2007, the reduced form model attributes 87 percent of the variability in aggregate IP to the common factors, $F$, while the structural model attributes only 50 percent to the common shocks, $S$. Finally aggregate shocks explain a smaller fraction of the variability in aggregate IP in the 1984-2007 period than in the 1972-83 period. The final column of the table shows that this fraction fell from 0.81 to 0.50. Of course, this implies that the contribution of idiosyncratic shocks to IP variability increased from 19 in the early period to 50 percent in the latter period. In Section 6, we list the sectors with the most important idiosyncratic shocks and discuss how this has changed over time.

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sector, may reflect idiosyncratic shocks arising in the service, financial, public or agricultural sectors. See Carvalho and Gabaix (2010) for an interesting discussion of the role these sectors have played in the Great Moderation and the increase in volatility during the 2008-2009 recession.

## 5. Deconstructing the Empirical Results

The previous literature on multisector models has at times argued that, despite sectoral linkages, idiosyncratic shocks should average out in aggregation. Most notably, Dupor (1999) provides a careful analysis of the conditions under which this averaging out occurs in the framework studied by Horvath (1998). It is instructive to examine our empirical conclusions in the context of this previous literature. We do this using three models, the model in Long and Plosser (1983), the benchmark model used by Carvalho (2007), and the model used by Horvath (1998) and Dupor (1999). Each of these models admits an analytic solution that fits into our generic solution (10).

In the economy of Long and Plosser (1983), households have log preferences over consumption and leisure, materials are delivered with a one-period lag, and there is no capital. In that economy, the evolution of sectoral output growth follows the generic solution (10) with $\Phi = \Gamma'$, $\Pi_0 = I$, and $\Pi_1 = 0$,

$$X_t = \Gamma' X_{t-1} + \varepsilon_t. \tag{14}$$

In his benchmark economy, Carvalho (2007) studies a version of our environment with no capital, the same preferences over leisure, and log preferences over consumption. In that economy, sectoral output growth evolves as in (10) with $\Phi = 0$, $\Pi_0 = (I - \Gamma')^{-1}$, and $\Pi_1 = 0$,

$$X_t = (I - \Gamma')^{-1}\varepsilon_t. \tag{15}$$

Finally, the economic environment in Horvath (1998) and Dupor (1999) is a version of our economy with sector-specific capital subject to full depreciation within the period (so that $\Theta = I$ and $\delta=1$), no labor, and log preferences over consumption. In that economy, sectoral output follows (10) with $\Phi = (I-\Gamma')^{-1}\alpha_d$, $\Pi_0 = (I-\Gamma')^{-1}$, and $\Pi_1 = 0$, where $\alpha_d$ is diagonal matrix with the values $\alpha_i$ on the diagonal,

$$X_t = (I - \Gamma')^{-1}\ \alpha_d X_{t-1} + (I - \Gamma')^{-1}\varepsilon_t. \tag{16}$$

The solution to each of these models, (14), (15), and (16), yields the following expressions for the covariance matrix of sectoral growth rates respectively

(17)
$$\Sigma_{XX}^{Long-Plosser} = \sum_{i=0}^{\infty} (\Gamma')^i \Sigma_{\varepsilon\varepsilon} \Gamma^i$$
$$\Sigma_{XX}^{Carvalho} = (\mathrm{I}-\Gamma')^{-1} \Sigma_{\varepsilon\varepsilon} (\mathrm{I}-\Gamma)^{-1}$$
$$\Sigma_{XX}^{Horvath-Dupor} = \sum_{i=1}^{\infty} \left[ (\mathrm{I}-\Gamma')^{-1}\alpha_d \right]^i (\mathrm{I}-\Gamma')^{-1} \Sigma_{\varepsilon\varepsilon} (\mathrm{I}-\Gamma)^{-1} \left[ \alpha_d (\mathrm{I}-\Gamma)^{-1} \right]^i$$

In each of the models, off-diagonal elements in the input-output matrix $\Gamma$ propagate sector specific $\varepsilon$ shocks to other sectors, although the exact form of the propagation depends on the specifics of the model. Said differently, in each of the models, diagonal $\Sigma_{\varepsilon\varepsilon}$ matrices are transformed in non-diagonal $\Sigma_{XX}$ matrices through input-output linkages. These expressions show that the variance of aggregate output depends on $\Sigma_{\varepsilon\varepsilon}$, $\Gamma$, and (in the Horvath-Dupor economy) $\alpha_d$ in a fairly complicated, and model-specific way.

Dupor (1999) studies a version of the Horvath (1998) model under two key restrictions:

i) $\Gamma$ has a unit eigenvector, so that $\Gamma l = \kappa l$, where $l$ is the unit vector and $\kappa$ is a scalar,

ii) all capital shares are equal, so that $\alpha_d = \alpha \mathrm{I}$, where $\alpha$ is a scalar.

Under these restrictions, it is possible to derive simple expressions for the variance of the equally weighted aggregate growth rate, denoted with a superscript "*ew*", $g_t^{ew} = N^{-1}\sum_{i=1}^{t} x_{it}$ = $N^{-1}l'X_t$. In particular, the variance of $g_t^{ew}$ in each of the models is given by

(18)
$$\sigma_{g^{ew}}^2(Long-Plosser) = (1-\kappa^2)^{-1}\bar{\sigma}_{ij}$$
$$\sigma_{g^{ew}}^2(Carvalho) = (1-\kappa)^{-2}\bar{\sigma}_{ij}$$
$$\sigma_{g^{ew}}^2(Horvath-Dupor) = [(1-\kappa-\alpha)(1-\kappa+\alpha)]^{-1}\bar{\sigma}_{ij}$$

where $\bar{\sigma}_{ij} = N^{-2}\sum_i \sum_j E(\varepsilon_{it}\varepsilon_{jt})$ denotes the value of the average element of $\Sigma_{\varepsilon\varepsilon}$.[5] When idiosyncratic shocks are uncorrelated so that $\Sigma_{\varepsilon\varepsilon}$ is diagonal, $\bar{\sigma}_{ij}$ denotes the average variance of sectoral shocks divided by *N*. Moreover, under the restrictions on $\Gamma$ and $\alpha_d$ described by i) and ii), the variance of the aggregate growth rate is proportional to $\bar{\sigma}_{ij}$ in not only in Horvath (1998) but in all the models, where the details of the model give rise to different factors of proportionality.

The expressions in (18) lead to three conclusions. First, if the idiosyncratic errors are uncorrelated (or weakly correlated), so that $\lim_{N\to\infty} \bar{\sigma}_{ij} = 0$, then $\lim_{N\to\infty} \sigma^2_{g^{ew}} = 0$. This is Dupor's well-known result on the irrelevance of idiosyncratic shocks for aggregate volatility applied to all three models. The second result is that $\Sigma_{\varepsilon\varepsilon}$ affects $\sigma^2_{g^{ew}}$ only through the value of its average element, $\bar{\sigma}_{ij}$. Thus, for example, severe heteroskedasticity associated with a handful of sectors being subject to large shocks only affect $\sigma^2_{g^{ew}}$ through its effect on the average element in $\Sigma_{\varepsilon\varepsilon}$. This effect will be small when the number of sectors is large. Finally, when $\varepsilon$ depends on common shocks *S*, the fraction of the variability in $\sigma^2_{g^{ew}}$ associated with the common shocks (denoted $R^2(S)$ in the last section) is the fraction of $\bar{\sigma}_{ij}$ associated with *S*, which the same for each of the models.

All of these conclusions rest on the restriction that $\Gamma l = \kappa l$. In other words, the elements in each row of the input-output matrix sum to the same constant. Recall that the $i^{\text{th}}$ row sum of $\Gamma$ represents the intensity of sector *i* as a material provider to the economy. Therefore, the row-sum restriction implies that all sectors are equally important in this regard. Figure 6A shows the row sums for the input-output matrix for IP. Evidently, in contrast to restriction i) above, the row sums of the input-output matrix for IP differ considerably across sectors. In particular, the "skewness" of the row sums is consistent with the notion emphasized by Carvalho (2007) that a few sectors play a key role as input providers. This suggests that the conclusions that follow

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[5] When $\Gamma l = \kappa l$, $\alpha_d l = \alpha l$, and $g_t^{ew} = N^{-1} l' X_t$, equations (14)-(16) imply simple representations for $g_t^{ew}$ as a function of $\bar{\varepsilon}_t = N^{-1} l' \varepsilon_t$. Specifically $g_t^{ew} = \kappa g_{t-1}^{ew} + \bar{\varepsilon}_t$ for the Long-Plosser economy, $g_t^{ew} = (1-\kappa)^{-1}\bar{\varepsilon}_t$ for the Carvalho economy, and $g_t^{ew} = (1-\kappa)^{-1}\alpha g_{t-1}^{ew} + (1-\kappa)^{-1}\bar{\varepsilon}_t$ for the Horvath-Dupor economy. The expressions given in (18) follow directly from these representations.

from Dupor's equal-row-sum restriction may not be relevant for the IP data, or more to the point, they must be investigated by evaluating the general expressions in (17) and their generalizations for our model, which is done in Table 8.

Table 8 summarizes the implied variability in aggregate IP and sectoral covariability for various versions of the model. It investigates the role of the input-output matrix $\Gamma$, which has been the focus of much of the previous literature, and also the role of the parameters that govern sectoral interactions in investment ($\Theta$ and $\delta$) which have not been investigated previously. The analysis is carried out using models in which $\Sigma_{\varepsilon\varepsilon}$ is a diagonal matrix (so that all shocks are idiosyncratic) using the value from the last section, which we denote by $\Sigma_{\varepsilon\varepsilon}^{Benchmark}$. However, because the various models imply different scaling for the shocks (that is, the diagonal elements of $\Pi_0$ differ in the models), we also show results for $\Sigma_{\varepsilon\varepsilon} = \eta\Sigma_{\varepsilon\varepsilon}^{Benchmark}$, where $\eta$ is a model-specific scale factor described below. Thus, scaled comparisons capture only the internal mechanics of the different models with $\varepsilon$ shocks that are normalized across models.

The first row of Table 8 shows the average correlation of sectoral growth rates ($\bar{\rho}_{ij}$ = 0.19, from Table 2), the standard deviation of IP growth rates ($\sigma_g$ = 5.8 percent from Table 1), and the standard deviation of IP growth associated with the diagonal elements of $\Sigma_{XX}$ ($\sigma_g(diag)$ = 1.9 percent from Table 3), computed over the full sample (1972-2007) period. The other entries show the values implied by the various models, where the entry for "$\sigma_g$" (the standard deviation of IP growth rates) uses $\Sigma_{\varepsilon\varepsilon} = \Sigma_{\varepsilon\varepsilon}^{Benchmark}$, the entry for "$\sigma_g$ (*scaled*)" uses $\Sigma_{\varepsilon\varepsilon} = \eta\Sigma_{\varepsilon\varepsilon}^{Benchmark}$, and where $\eta$ is chosen so that the model-implied variance of IP growth associated with the diagonal elements of $\Sigma_{XX}$, $\sigma(diag)$, corresponds to the value in the data (which is 1.9 percent from the first row in the table).

The second row of the table shows results for the benchmark model considered in the last section. As discussed in that section, idiosyncratic shocks in the benchmark model under predict the correlation between sectoral growth rates (the model's average value is 0.04 compared to 0.19 in the data) and under predict the variability in the aggregate growth rate (the model predicts a scaled standard deviation of 3.8 percent compared to 5.8 percent in the data).

Rows 3-5 show results for the Long and Plosser, Carvalho, and Horvath-Dupor economies described above, solved using the same parameter values as in our benchmark model.

In Long and Plosser, materials are subject to a one-period delivery lag. Therefore, output growth initially reacts to shocks as though there were no spillovers (i.e. the coefficient on $\varepsilon_t$ in (14) is I), and input-output linkages only propagate this initial "neutral" response. Moreover, the Long and Plosser model does not have capital. Taken together, these features lead to a lower correlation among the sectors and a smaller aggregate variance than in the benchmark model shown in row 2. While Carvalho also abstracts from capital, materials are used within the quarter so that idiosyncratic shocks are contemporaneously amplified through propagation to other sectors (i.e. $\varepsilon_t$ is immediately amplified by $(I - \Gamma')^{-1}$ in equation (15)). As shown in row 4, this change increases the sectoral correlation and aggregate variability relative to Long and Plosser (1983), although the scaled aggregate variability is still somewhat smaller than the benchmark model.

The Horvath-Dupor model adds sector-specific capital (so that $\Theta = I$) but imposes full depreciation within the period (so that $\delta = 1$), shown in row 5 of Table 8. This model produces results much in line with the benchmark model. In the Horvath-Dupor environment, not only are idiosyncratic shocks propagated to other sectors contemporaneously, as in Carvalho, but these shocks are then further propagated through their effects on capital accumulation, by way of $(I - \Gamma')^{-1} \alpha_d$ in (16), where $\alpha_d$ is the matrix of capital shares. Therefore, aggregate variability is more pronounced in Horvath-Dupor than in the models of either Long and Plosser or Carvalho.

To understand why the Horvath-Dupor model produces results similar to our benchmark model, Rows 6 and 7 parse the two key restrictions imposed by Horvath-Dupor relative to our benchmark in section 4. The first, that $\Theta = I$, is shown in Row 6. This restriction reduces the covariability (by eliminating a channel in which the sectors interact) and the variability of aggregate output. Figure 6B shows the row sums of $\Theta$ and suggests why imposing $\Theta = I$ results in a reduction of correlation and volatility. The distribution of the row sums of $\Theta$ is highly skewed. Roughly 85 percent of the sectors provide no capital to other sectors, and only a handful of sectors are responsible for the bulk of inter-sector trade in capital goods.[6] The effect of the second restriction, $\delta = 1$, is shown in Row 7. This restriction increases sectoral covariability, by reducing the intertemporal smoothing from capital accumulation, and the variance of output.

[6] Six sectors (Commercial and Service Industry Machinery, Metalworking Machinery, Industrial Machinery, Computer and Peripheral Equipment, Navigational/Measuring/Electromedical/Control Instruments, and Automobiles and Light Duty Motor Vehicle s), account for 78 percent of the off-diagonal elements in $\Theta$.

Thus, the Horvath-Dupor model produces numerical results similar to the benchmark because it eliminates two features that have countervailing effects.

Rows 8-11 of Table 8 modify $\Gamma$ and $\alpha_d$ so that they (counterfactually) satisfy the restrictions used by Dupor (1999). In particular, the actual input-output matrix is replaced by a symmetric version with common diagonal and common off-diagonal elements, that is $\gamma_{ii} = \gamma_{Diag}$ for all $i$ and $\gamma_{ij} = \gamma_{OffDiag}$ for all $i \neq j$, and where $\gamma_{Diag}$ and $\gamma_{OffDiag}$ were chosen as the average values in the actual input-output matrix. Similarly $\alpha_d$ was replaced by $\alpha$I, where $\alpha$ was chosen as the average capital share. Rows 8-11 show that these restrictions lead to a reduction in the sectoral correlation and aggregate IP volatility relative to the corresponding models that do not impose these restrictions. This holds not only for the Long-Plosser, Carvalho, and Horvath-Dupor models (shown in rows 8-10), but also for the more general model with capital interactions and realistic depreciation (shown in row 11).

The final row of the table investigates the effect of heteroskesdasticity on sectoral output. Recall that in the restricted models of this section, aggregate variability depends only on $\bar{\sigma}_{ij}$, the value of the average element in $\Sigma_{\varepsilon\varepsilon}$, and not on the distribution of elements within $\Sigma_{\varepsilon\varepsilon}$. Therefore, even if a few sectors were to have large shocks, this will have only a small effect on the aggregate variance because the number of sectors is large. However, this argument does not necessarily hold in the unrestricted model of section 4. In row 12, the model is solved using $\Sigma_{\varepsilon\varepsilon} = \sigma^2$I, where $\sigma^2$ is chosen as the average sample variance (so that $\bar{\sigma}_{ij}$ is unchanged). The result is a slight reduction in covariability and (scaled) variance of the aggregate relative the benchmark model shown in row 2. Put another way, the fact that some sectors are subject to larger idiosyncratic shocks than other sectors leads to a somewhat larger variance in the aggregate series (3.82 in row 2 versus 3.55 in row 12).

The final column of the table shows the aggregate volatility for each of the models relative to the values in the benchmark model. The entries range from 0.32 (for the Long-Plosser model using the counterfactual value of the input-output matrix) to 1.12 percent (for the benchmark model with full depreciation). This highlights the fact that, in the context of the Long-Plosser model and its descendants, the importance of idiosyncratic shocks can vary widely, and depends critically on details of the specification and the values of parameters that govern intra- and intertemporal linkages.

## 6. Level of Aggregation and Time Variation in the Input-Output Matrix

This section addresses the robustness of our findings along two dimensions. First, we assess how our findings are affected by the level of sectoral disaggregation. Second, we explore whether changes in the structure of sectoral linkages in U.S. production over time have changed the propagation of idiosyncratic shocks.

The results summarized thus far have used IP disaggregated to (essentially) the four-digit level. As discussed above, this yields 117 sectors and is the finest level of disaggregation that allows us to compute the input-output and capital-use matrices Γ and Θ. Table 9 compares results from models estimated at coarser levels of aggregation (88 three-digit and 26 two-digit sectors). The table shows that the average pairwise correlation of sectoral growth rates ($\bar{\rho}_{ij}$) increases as the level of aggregation becomes more coarse, but that the main conclusions for the model estimated using the four-digit data continue to hold for the three-digit and two-digit data. In particular, the models with uncorrelated shocks are unable to capture all of the covariability in sectoral growth rates that is estimated from the data. Because of this, these models underpredict the variability in aggregate IP. That said, the fraction of variability associated with aggregate shocks ($R^2(S)$) is remarkably stable across the different levels of aggregation. In particular, the conclusions that the post-1983 decline in the aggregate volatility is associated with decline in the variability of the common factors and that roughly 50 percent of the variability of IP was associated with sector-specific shocks during the 1984-2007 period are robust to the level of disaggregation.

Another important issue involves the evolution of sectoral production linkages in the U.S. economy over the 1972-2007 sample period. To investigate the effect of these changes before and after the Great Moderation, we consider two BEA benchmark years, 1977 and 1997. BEA benchmark input use tables are available only every five years. However, the use tables for the years prior to 1997 are not updated by the BEA to reflect the reclassification of industries by NAICS definitions, and are broken down instead according to SIC codes. For consistency, therefore, we extend our analysis to vintage IP data provided by the Board of Governors where sectors are disaggregated by SIC codes, and which cover the period 1967-2002. Unfortunately, capital flow tables broken down according to SIC codes were never constructed for the level of disaggregation considered here. Therefore, in this exercise, we are limited to the model with $\Theta$ = I.

Table 10 summarizes results for models estimated over pre- and post-1983 periods using the 1997 input-output matrix ($\Gamma^{1997}$) and sectors classified by NAICS, and using the 1977 input-output matrix ($\Gamma^{1977}$) and sectors classified by SIC. The table shows the average pairwise correlation of sectoral growth rates ($\bar{\rho}_{ij}$) computed from the data along with the standard deviation of IP growth rates ($\sigma_g$). The corresponding values are shown for the version of the model driven by sector-specific shocks (that is, using a diagonal $\Sigma_{\varepsilon\varepsilon}$ matrix). Also shown are the fraction of variability of aggregate variability associated with the common factors from the structural model ($R^2(S)$) and from the reduced form model ($R^2(F)$). Examination of the table entries suggests that our key conclusions are unaffected by changes in the input-output matrix.

Finally, while the empirical results suggest that much of the variability in IP is associated with the aggregate shocks, *S*, sector-specific shocks explain 20 percent of the variability in IP during the 1972-1983 period and 50 percent of its variability in the 1984-2007 period. Table 11 lists the 10 sectors that are most responsible for this variability in each of the sample periods computed using the models with two factors. To account for changes in the structure of U.S. production, panel (A) shows results for the idiosyncratic shocks constructed using the 1977 input-output matrix for the period 1967-1983 (using the SIC classification) and panel (B) uses the 1997 input-output matrix over 1984-2007 (using the NAICS classification). As in Table 11, because the capital flow table is not available for the SIC data, these results are calculated using the model with $\Theta$ = I. Panel (C) shows the results corresponding to panel (B), but using $\Theta^{1997}$ constructed from the BEA's 1997 Capital use table.

We make three comments about the sectors highlighted in the table. First, comparing panels (A) and (B), the highest ranked sectors tend to be those that serve as inputs to many other sectors. This is the case, for example, of "Basic Steel and Mill Products" in the pre-1984 period (6.4 percent), which corresponds to "Iron and Steel Products" after 1984 (4.2 percent). It is also true of "Utilities" prior to 1984, which corresponds approximately to "Electric Power Generation and Distribution" in the second sample period. Second, comparing panels (B) and (C), the ranking of industries in the two panels is very similar (9 industries appear in both panels and the top 6 industries coincide), although in panel (C) the fraction of variability of IP associated with the sector-specific shocks is higher. Thus, while ignoring production linkages through capital investment affects the estimate of the fraction of aggregate variability attributed to idiosyncratic shocks (biasing it down because of the neglected linkages), it does not affect the ranking of the most important sector-specific shocks. Finally, Table 11 points to some changes in the structure of U.S. production. For example, "Coal Mining," a traditional industry ranked second prior to 1984 (at 3.4 percent), is no longer among the highest ten ranked sectors in the second sample period.

## 7. Conclusions

In this paper, we explore various leading explanations underlying the volatility of industrial production and its decline since 1984. We find that neither time variation in the sectoral shares of IP nor their distribution are important factors in determining the variability of the aggregate IP index. Instead, the analysis reveals that aggregate shocks largely explain changes in aggregate IP, and a decrease in the volatility of these shocks explains why aggregate IP is considerably less variable after 1984. Because of this decline in the variability of aggregate shocks, the relative importance of sector-specific shocks has increased substantially over the Great Moderation period. Taking into account sectoral linkages that characterize U.S. production, both in materials and investment, sector-specific shocks explain approximately 20 percent of the variation in IP growth prior to 1984 and 50 percent of IP fluctuations after the onset of the Great Moderation. We also provide evidence that changes in the structure of the input-output matrix between 1977 and 1997 have not lead to a more pronounced propagation of sectoral shocks.

The analysis also highlights the conditions under which a class of neoclassical multisector growth models first studied by Long and Plosser (1983) admit an approximate factor model as a reduced form. In doing so, it bridges two literatures, one that has relied on factor analytic methods to assess the relative importance of aggregate and idiosyncratic shocks, and the other rooted in more structural calibrated models that explicitly take into account sectoral linkages in production. In the reduced form factor model, aggregate shocks emerge as the common output factors. The idiosyncratic shocks in the reduced form are associated with sector specific shocks but, because of sectoral linkages, these can be cross sectionally correlated.

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## Appendix A

Data on Industrial Production is obtained from the Board of Governors of the Federal Reserve System and disaggregated according to the North American Industry Classification System (NAICS). The raw data are indices of real output and sectoral shares. While data on the vast majority of sectors is directly available from the Board of Governors, data are missing for some sectors, and these missing are approximated using the Board's recommended methodology. For example, if industry C is composed of industry A and industry B, then the growth rate of C's output is approximated by

$$\frac{IP_{Ct}}{IP_{Ct-1}} = \frac{w_{At-1}\dfrac{IP_{At}}{IP_{At-1}} + w_{Bt-1}\dfrac{IP_{Bt}}{IP_{Bt-1}}}{w_{At-1} + w_{Bt-1}}$$

where $w_{it}$ is the share of industry $i$ at date $t$, and $w_{Ct} = w_{At} + w_{Bt}$. Alternatively, if industry C is made up of industry A less industry B, then

$$\frac{IP_{Ct}}{IP_{Ct-1}} = \frac{w_{At-1}\dfrac{IP_{At}}{IP_{At-1}} - w_{Bt-1}\dfrac{IP_{Bt}}{IP_{Bt-1}}}{w_{At-1} - w_{Bt-1}}$$

and $w_{Ct} = w_{At} - w_{Bt}$. As mentioned in the text, we also make use of vintage IP data, provided by the Board of Governors, which are disaggregated according to Standard Industry Classification (SIC) codes. Growth rates and shares for missing sectors are computed in the manner we have just described. A summary of the NAICS IP data is provided in Table A1.

Benchmark Input-Output tables, available every five years, are obtained from the Bureau of Economic Analysis. The "Use Table" measures the dollar value of inputs given by commodity codes, used by each industry given by industry codes, as well as payments to other factors such as labor and capital. Commodity codes are matched to industries to provide a matrix showing the dollar value of inputs purchased by each industry from each industry. We consider benchmark tables for 1977 and 1997 which are broken down according SIC and NAICS industry definitions respectively.

In matching the input-output (IO) matrices with IP data we find the smallest set of common industries for which we can match the IP data to the IO data. Because the two data sources are originally disaggregated to different levels, the resulting sectoral breakdown represents collections of either NAICS or SIC industry levels (depending on whether we are using current or vintage IP data). The approach is as follows: taking the finest available partition of industries from the benchmark input-output tables, we match as many industries as possible; the remaining industries with no matches are aggregated until a match is found. The result are collections of industries whose level of disaggregation ranges from 2-digit to 4-digit levels. Results reported in the text are for the highest level of disaggregation available, the 4-digit level, unless otherwise stated.

Data from the 1997 input-output table are matched with IP data disaggregated using NAICS definitions over the period 1972Q1-2007Q4. Similarly, data from the 1977 input-output Table are matched with IP data broken down by SIC code from 1967Q1-2002Q3. The reclassification of industries from the SIC system to the NAICS system, and the fact that older input-output tables are not updated according to NAICS definitions, makes the use of vintage IP data necessary since there is no easy mapping from SIC to NAICS definitions.

As a practical matter, the distinction between what constitutes materials, as captured by $\Gamma$, and investment goods, as captured by $\Theta$, is not always unambiguous. The BEA distinguishes between materials and capital goods by estimating the service life of different commodities. Thus, commodities expected to be used in production within the year are defined as materials.

There are two notable measurement issues that arise with the construction of the capital flow table. First, the table accounts for the purchases of new capital goods and does not include the purchases of used assets. This means, for example, that a firm's purchases of used trucks in a given industry from another industry will not be recorded as investment in the capital flow table even when the trucks' remaining service life is well in excess of a year.[7] Second, as discussed in McGrattan and Schmitz (1999), maintenance expenditures are largely absent from the table. The absence of data on purchases of used capital goods leads us to ignore the first measurement problem. However, we do adjust the capital flow table for maintenance expenditures. McGrattan and Schmitz (1999) provide evidence that in aggregate manufacturing,

[7] The difficulty of the problem lies in having to keep track of the entire age distribution of the different types of capital that switch industries. It is not an issue for the overall measurement of investment in IP since the capital is always within IP but can matter for tracking investment at a more disaggregated level.

maintenance and repair payments may be as large as 50 percent of total investment. If sector *j*'s missing expenditures are proportional to the capital expenditures to each sector *i* reported in the capital flow table, then the missing expenditures will have no effect on the shares $\theta_{ij}$. However, there is reason to suspect that this is not the case. Specifically, much maintenance and repair takes place using within-sector resources, and yet many of the diagonal elements of the capital flow table are very small or zero. It is difficult to know exactly what to do about this bias. We have taken a rough-and-ready approach and added 25 percent of the reported total expenditures from the table to the diagonal. Essentially, this assumes that the published table underestimates total expenditures by roughly 50 percent, that half of this is associated with within-sector expenditures on maintenance and repair, and that the other half is proportional to capital expenditures reported in the table. In our numerical experiments, we found that some of our results were sensitive to this adjustment. In particular, adjusting the diagonal elements by values much less than 25 percent (say, using 15 percent) led to a non-invertible model, so that $\varepsilon_t$ could not be recovered from current lagged values of $X_t$. On the other hand, using larger values closer to that of McGrattan and Schmitz (1999) (say, using 35 percent) led to results much like those obtained using 25 percent.

# Table A.1
# 117 Sectors NAICS Industry Classification

| Sector | Weight | Standard Deviation of Quarterly Growth Rates (PAAR) | |
|---|---|---|---|
| | | 1972-1983 | 1984-2007 |
| Logging | 0.27 | 17.9 | 14.2 |
| Oil and Gas Extraction | 5.79 | 3.8 | 5.7 |
| Coal Mining | 1.06 | 79.7 | 15.4 |
| Iron Ore Mining | 0.09 | 132.0 | 37.9 |
| Gold, Silver, and Other Ore Mining | 0.16 | 30.3 | 22.0 |
| Copper, Nickel, Lead, and Zinc Mining | 0.15 | 61.4 | 14.2 |
| Nonmetallic Mineral Mining and Quarrying | 0.63 | 14.3 | 10.4 |
| Support Activities for Mining | 1.06 | 20.6 | 28.6 |
| Electric Power Generation, Transmission and Distribution | 7.65 | 8.0 | 6.9 |
| Natural Gas Distribution | 1.65 | 16.0 | 16.7 |
| Animal Food | 0.41 | 8.1 | 10.3 |
| Grain and Oilseed Milling | 0.77 | 12.4 | 8.9 |
| Sugar and Confectionery Products | 0.55 | 23.4 | 12.0 |
| Fruit and Vegetable Preserving and Specialty Foods | 1.03 | 9.6 | 11.3 |
| Dairy Products Ex Frozen | 0.72 | 4.8 | 7.4 |
| Ice Cream and Frozen Desserts | 0.11 | 9.7 | 12.8 |
| Animal Slaughtering and Processing | 1.29 | 10.4 | 6.3 |
| Seafood Product Preparation and Packaging | 0.14 | 28.6 | 24.6 |
| Bakeries and Tortilla | 1.23 | 4.3 | 4.2 |
| Coffee and Tea | 0.18 | 39.0 | 17.7 |
| Other Food Except Coffee and Tea | 0.95 | 11.1 | 7.1 |
| Soft Drinks and Ice | 0.60 | 7.5 | 8.9 |
| Breweries | 0.45 | 14.7 | 9.1 |
| Wineries and Distilleries | 0.26 | 26.6 | 20.5 |
| Tobacco | 1.07 | 12.6 | 19.3 |
| Fiber, Yarn, and Thread Mills | 0.22 | 23.6 | 15.3 |
| Fabric Mills | 0.69 | 16.5 | 9.9 |
| Textile and Fabric Finishing and Fabric Coating Mills | 0.31 | 17.5 | 11.6 |
| Carpet and Rug Mills | 0.20 | 26.9 | 16.3 |
| Curtain and Linen Mills | 0.16 | 18.3 | 12.6 |
| Other Textile Product Mills | 0.20 | 16.4 | 8.7 |
| Apparel | 1.83 | 10.8 | 8.9 |
| Leather and Allied Products | 0.34 | 12.3 | 13.2 |
| Sawmills and Wood Preservation | 0.44 | 25.8 | 11.3 |
| Veneer, Plywood, and Engineered Wood Products | 0.33 | 26.9 | 12.1 |
| Millwork | 0.34 | 21.1 | 9.0 |
| Wood Containers and Pallets | 0.08 | 12.2 | 8.7 |
| All Other Wood Products | 0.29 | 26.8 | 15.9 |
| Pulp Mills | 0.08 | 16.7 | 10.8 |
| Paper and Paperboard Mills | 1.59 | 16.0 | 6.2 |
| Paperboard Containers | 0.71 | 15.7 | 6.6 |
| Paper Bags and Coated and Treated Paper | 0.39 | 11.4 | 10.0 |
| Other Converted Paper Products | 0.37 | 15.1 | 8.3 |
| Printing and Related Support Activities | 2.30 | 7.2 | 4.9 |
| Petroleum Refineries | 1.66 | 13.9 | 7.6 |
| Paving, Roofing, and Other Petroleum and Coal Products | 0.33 | 17.8 | 11.3 |
| Organic Chemicals | 1.39 | 19.0 | 12.8 |
| Industrial Gas | 0.21 | 19.9 | 17.7 |
| Synthetic Dyes and Pigments | 0.15 | 30.9 | 16.8 |
| Other Basic Inorganic Chemicals | 0.57 | 26.7 | 21.7 |
| Resins and Synthetic Rubber | 0.79 | 32.9 | 11.4 |
| Artificial and Synthetic Fibers and Filaments | 0.34 | 38.4 | 12.9 |
| Pesticides, Fertilizers, and Other Agricultural Chemicals | 0.49 | 12.9 | 10.1 |
| Pharmaceuticals and Medicines | 2.52 | 5.6 | 6.7 |
| Paints and Coatings | 0.40 | 18.3 | 13.7 |
| Adhesives | 0.13 | 16.7 | 11.2 |
| Soap, Cleaning Compounds, and Toilet Preparation | 1.42 | 9.8 | 9.3 |
| Other Chemical Product and Preparation | 0.94 | 10.4 | 9.6 |

| | | | |
|---|---|---|---|
| Plastics Products | 2.27 | 16.8 | 6.0 |
| Tires | 0.44 | 44.9 | 14.0 |
| Rubber Products Ex Tires | 0.40 | 20.0 | 7.8 |
| Pottery, Ceramics, and Plumbing Fixtures | 0.12 | 14.7 | 12.5 |
| Clay Building Materials and Refractories | 0.16 | 22.7 | 16.9 |
| Glass and Glass Products | 0.63 | 11.7 | 6.9 |
| Cement | 0.19 | 28.0 | 16.8 |
| Concrete and Products | 0.68 | 13.6 | 9.2 |
| Lime and Gypsum Products | 0.11 | 19.1 | 17.1 |
| Other Nonmetallic Mineral Products | 0.37 | 16.3 | 8.2 |
| Iron and Steel Products | 1.70 | 41.5 | 19.5 |
| Alumina and Aluminum Production and Processing | 0.52 | 17.6 | 13.4 |
| Nonferrous Metal Smelting and Refining [Ex Aluminum] | 0.13 | 50.6 | 19.0 |
| Copper and Nonferrous Metal Rolling, Drawing, Extruding, and Alloying | 0.35 | 41.2 | 17.4 |
| Foundries | 0.79 | 19.5 | 9.1 |
| Fabricated Metals: Forging and Stamping | 0.50 | 16.7 | 8.0 |
| Fabricated Metals: Cutlery and Handtools | 0.35 | 15.0 | 8.0 |
| Architectural and Structural Metal Products | 1.15 | 11.7 | 6.0 |
| Boiler, Tank, and Shipping Containers | 0.59 | 9.1 | 6.6 |
| Fabricated Metals: Hardware | 0.29 | 18.4 | 9.5 |
| Fabricated Metals: Spring and Wire Products | 0.20 | 16.5 | 8.8 |
| Machine Shops; Turned Products; and Screws, Nuts, and Bolts | 1.05 | 15.5 | 8.8 |
| Coating, Engraving, Heat Treating, and Allied Activities | 0.40 | 11.6 | 9.4 |
| Other Fabricated Metal Products | 1.33 | 10.9 | 5.8 |
| Agricultural Implements | 0.50 | 22.0 | 27.6 |
| Construction Machinery | 0.44 | 46.7 | 32.0 |
| Mining and Oil and Gas Field Machinery | 0.29 | 28.4 | 21.9 |
| Industrial Machinery | 0.74 | 11.9 | 20.5 |
| Commercial and Service Industry Mach/Other Gen Purpose Mach | 2.19 | 11.8 | 6.2 |
| Ventilation, Heating, Air-cond & Commercial Refrigeration eq | 0.72 | 25.0 | 19.2 |
| Metalworking Machinery | 0.86 | 19.0 | 9.7 |
| Engine, Turbine, and Power Transmission Equipment | 0.80 | 20.2 | 14.9 |
| Computer and Peripheral Equipment | 1.52 | 17.4 | 16.4 |
| Communications Equipment | 1.55 | 10.7 | 16.6 |
| Audio and Video Equipment | 0.19 | 37.7 | 43.4 |
| Semiconductors and Other Electronic Components | 2.34 | 18.8 | 16.5 |
| Navigational/Measuring/Electromedical/Control Instruments | 2.33 | 10.7 | 6.2 |
| Magnetic and Optical Media | 0.20 | 28.0 | 21.6 |
| Electric Lighting Equipment | 0.34 | 15.2 | 8.0 |
| Small Electrical Household Appliances | 0.15 | 19.4 | 18.4 |
| Major Electrical Household Appliances | 0.37 | 36.3 | 15.4 |
| Electrical Equipment | 0.89 | 17.0 | 8.7 |
| Batteries | 0.16 | 22.0 | 16.6 |
| Communication and Energy Wires and Cables | 0.21 | 19.1 | 14.8 |
| Other Electrical Equipment | 0.47 | 20.7 | 8.0 |
| Automobiles and Light Duty Motor Vehicles | 2.32 | 51.2 | 24.8 |
| Heavy Duty Trucks | 0.16 | 50.3 | 35.3 |
| Motor Vehicle Bodies and Trailers | 0.41 | 29.9 | 17.1 |
| Motor Vehicle Parts | 3.09 | 28.2 | 13.3 |
| Aerospace Products and Parts | 3.17 | 14.1 | 13.4 |
| Railroad Rolling Stock | 0.23 | 30.2 | 22.4 |
| Ship and Boat Building | 0.51 | 11.3 | 10.8 |
| Other Transportation Equipment | 0.16 | 25.4 | 17.1 |
| Household and Institutional Furniture and Kitchen Cabinets | 0.87 | 18.5 | 7.1 |
| Office and Other Furniture | 0.62 | 13.4 | 8.7 |
| Medical Equipment and Supplies | 1.18 | 6.7 | 5.4 |
| Other Miscellaneous Manufacturing | 1.36 | 11.3 | 5.8 |
| Newspaper Publishers | 1.47 | 6.0 | 6.7 |
| Periodical, Book, and Other Publishers | 1.99 | 9.0 | 7.3 |

Notes: The column labeled "Weight" shows the sector's percentage point weight in aggregate IP.

**Figure 1: Growth Rates of Industrial Production**
Monthly: thin line, Quarterly: thick line
Percentage points at an annual rate

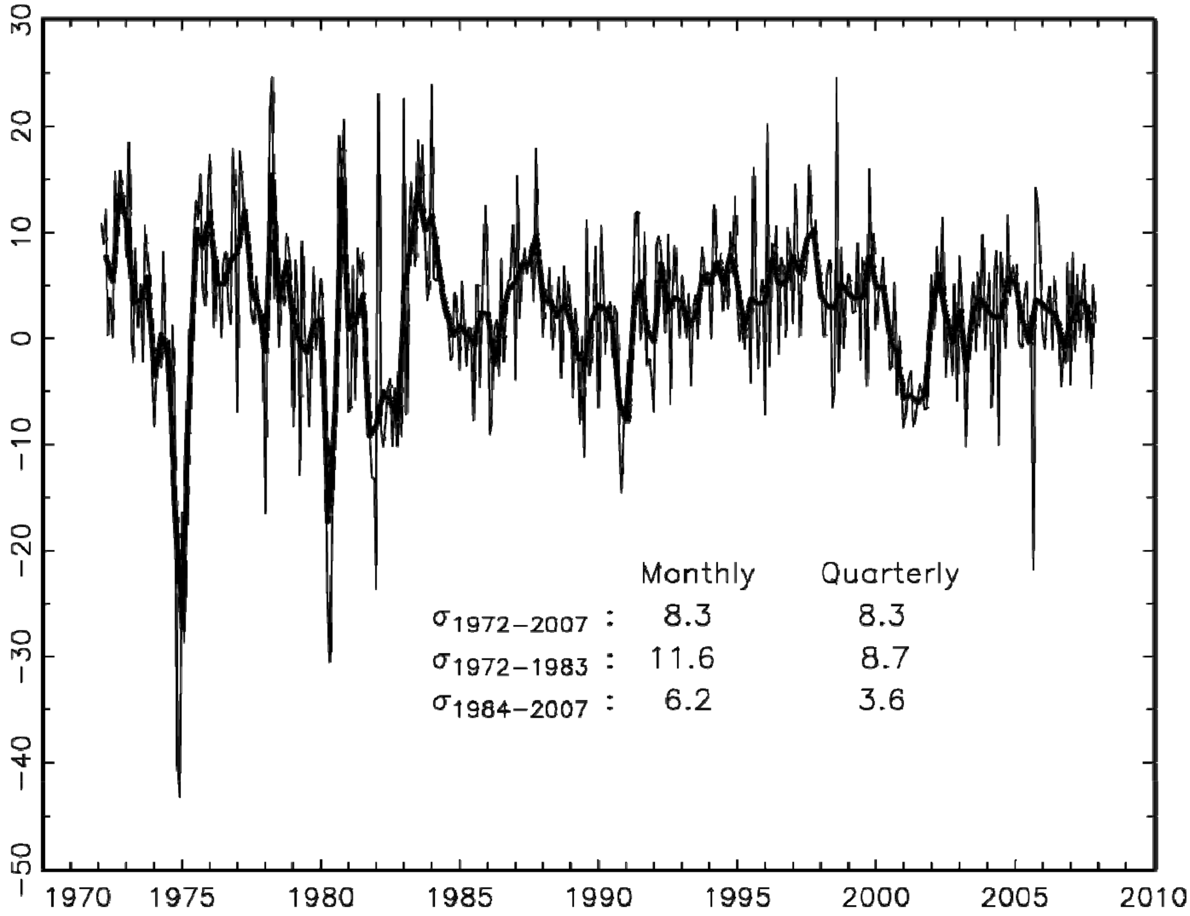
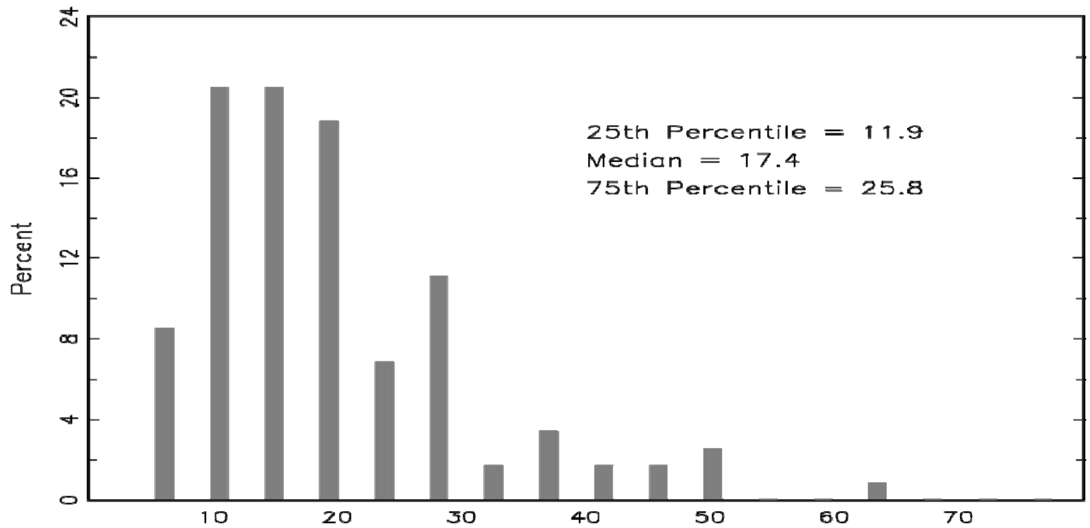
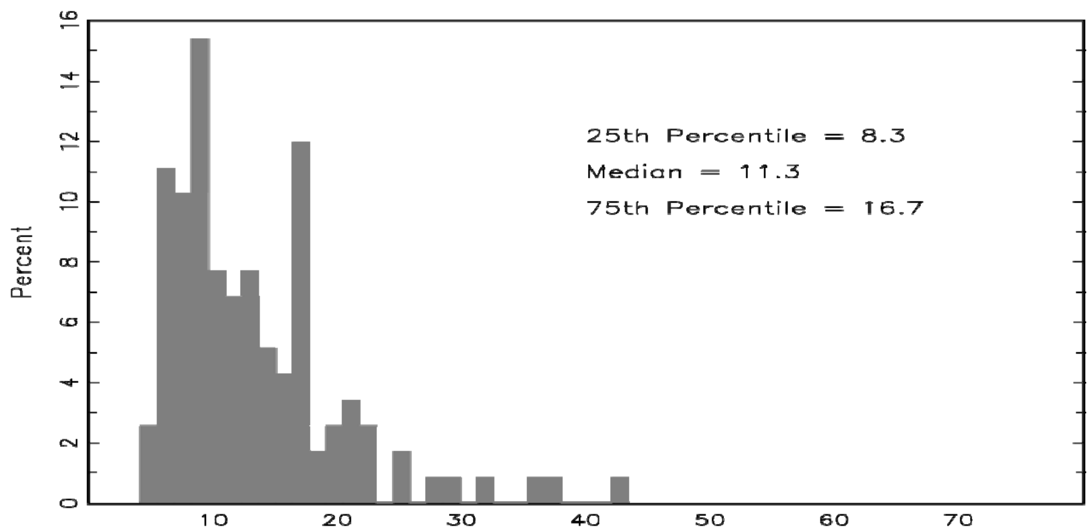


**Figure 2: Standard Deviation of Sectoral Growth Rates**
Percentage points at annual rates




A. 1972 – 1983




B. 1984-2007

**Figure 3: Share Weight Decomposition of Industrial Production**
(Percentage points at annual rates)

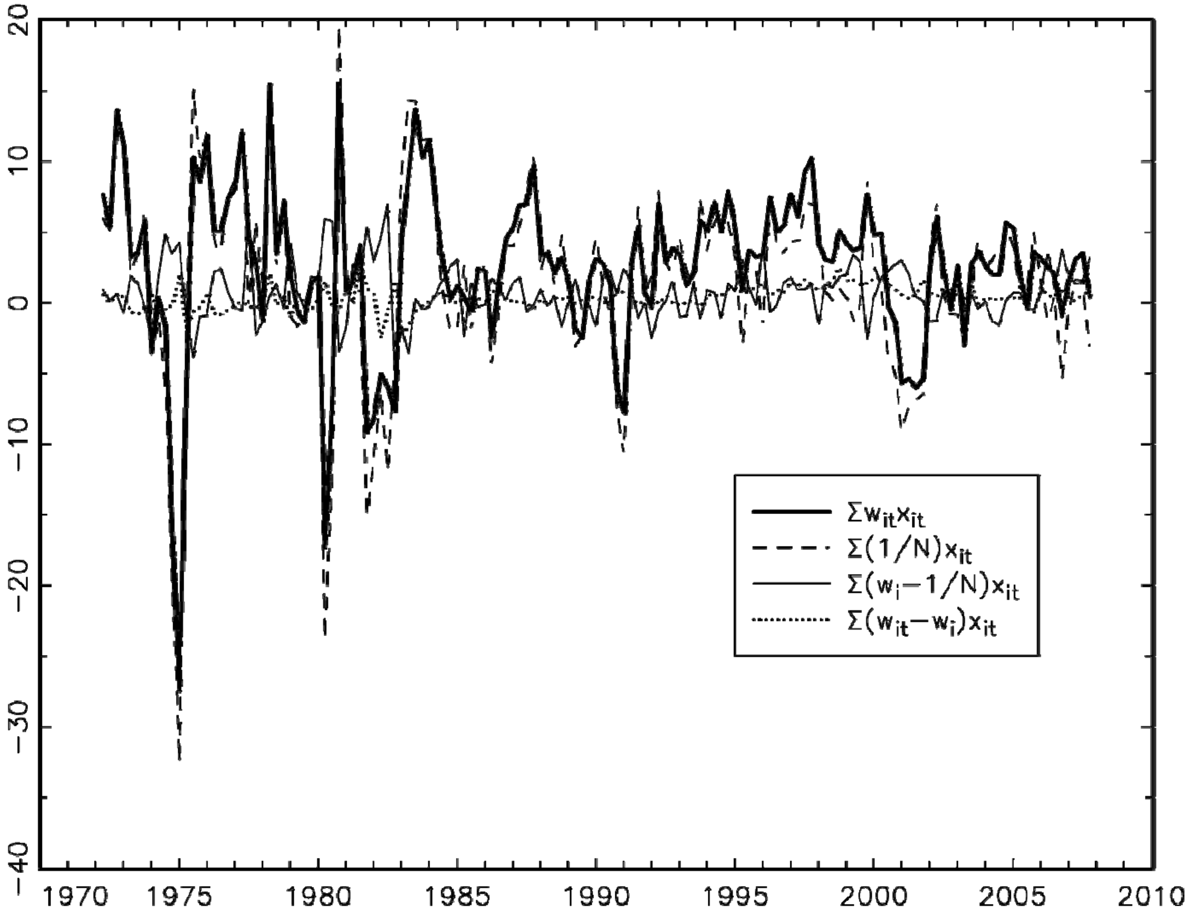


**Figure 4: Factor Decomposition of Industrial Production**
(Percentage points at annual rates)

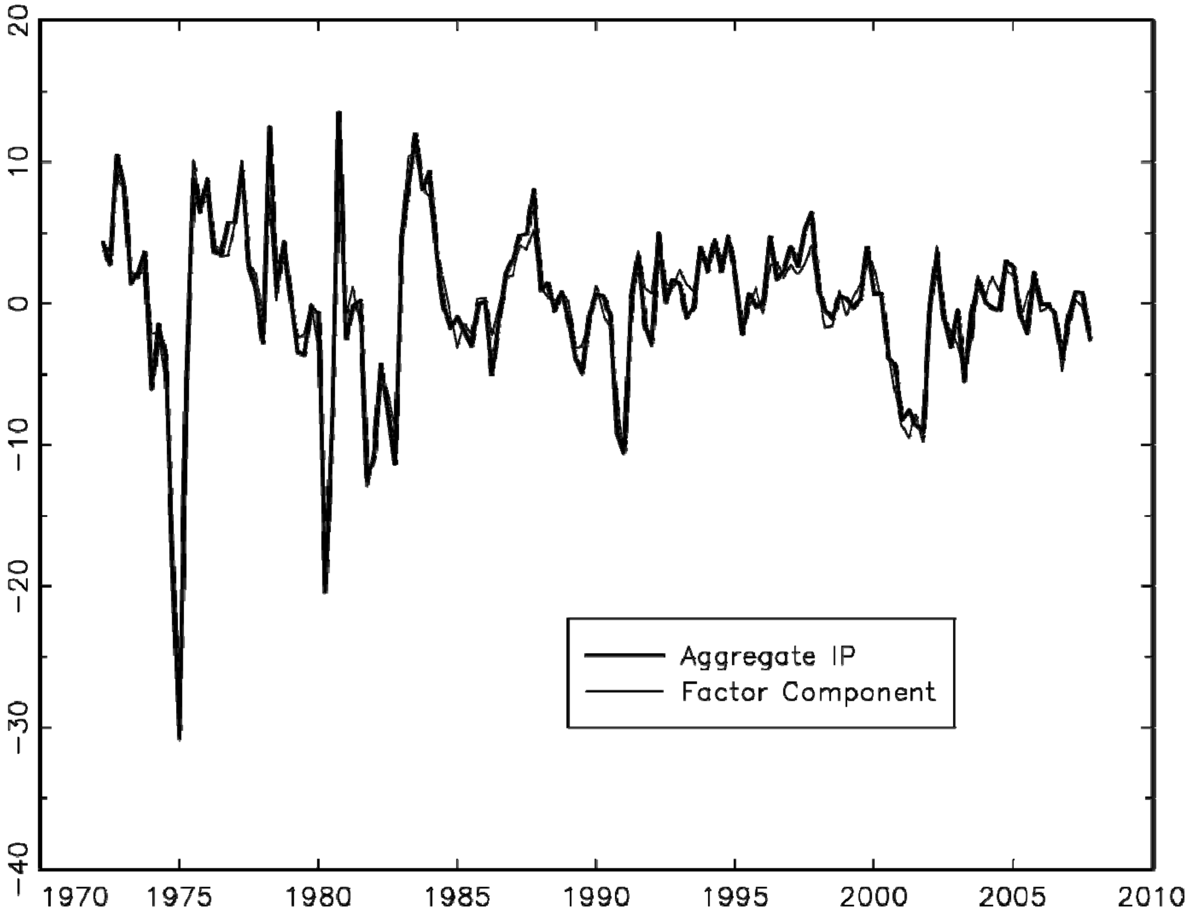
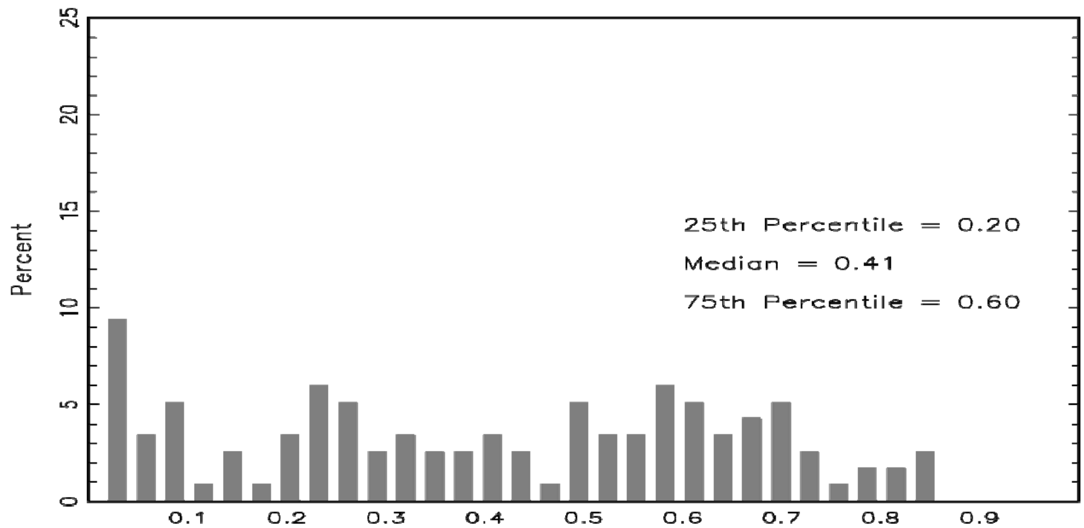
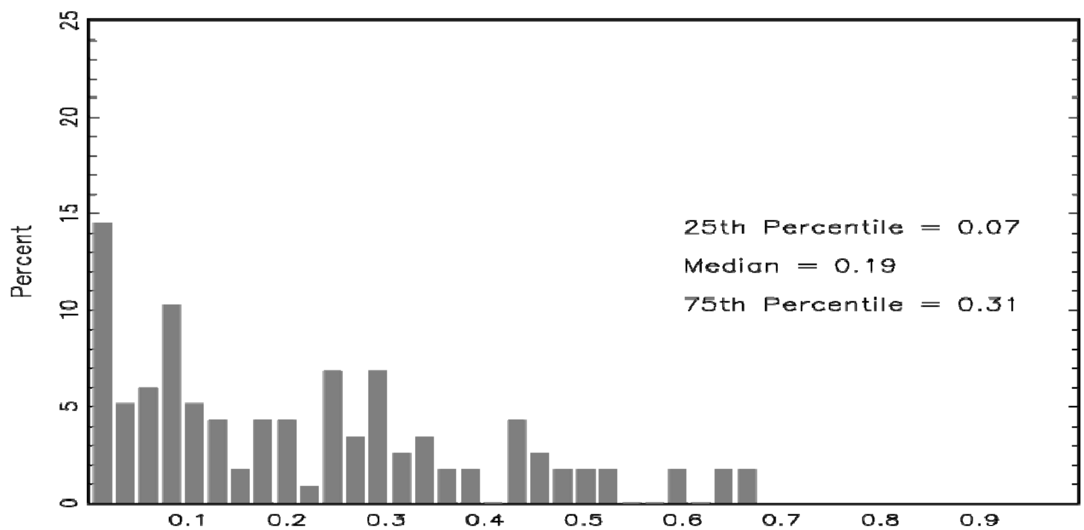


**Figure 5: Distribution of $R_i^2(F)$ of Sectoral Growth Rates**

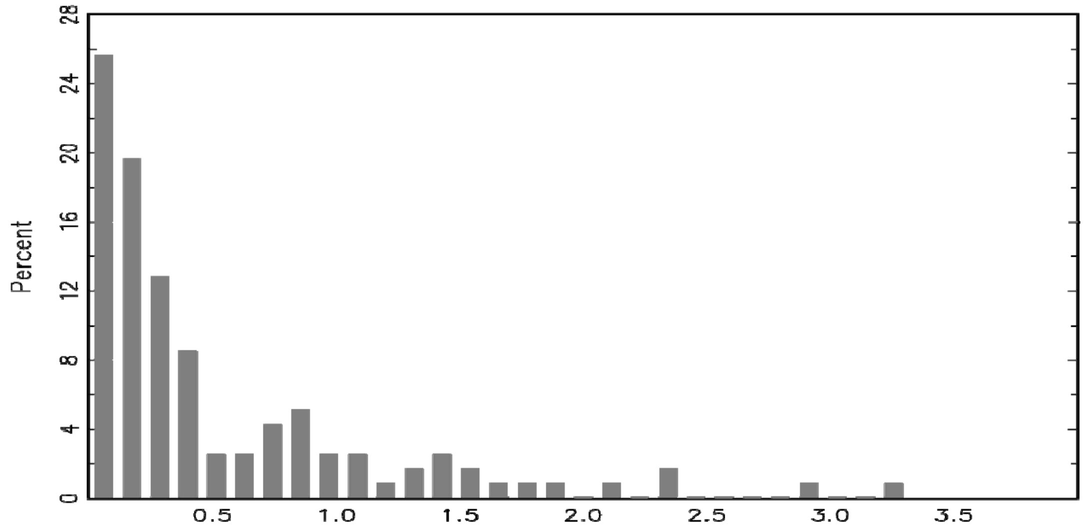



A. 1972 - 1983

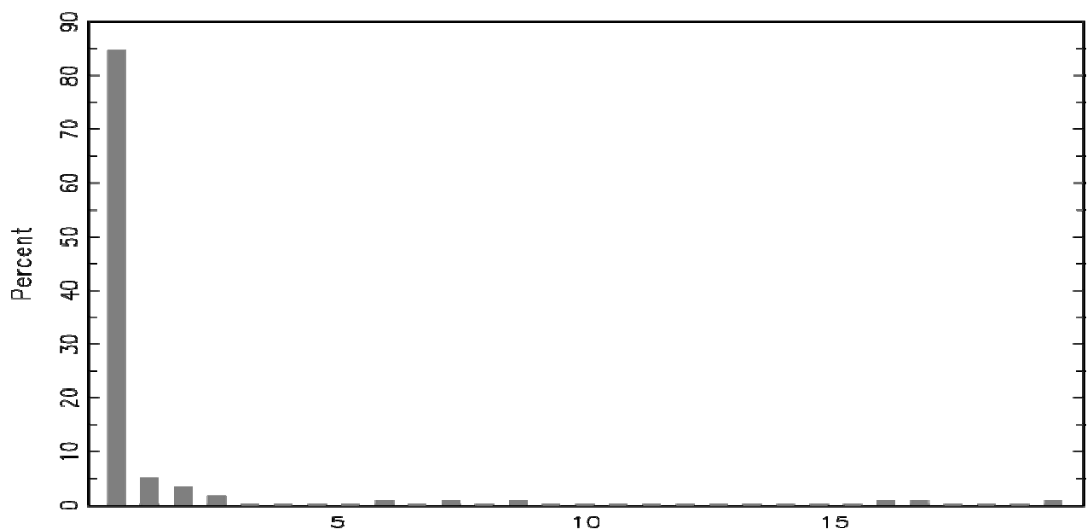



B. 1984-2007

**Figure 6: Some Characteristics of Sectoral Production**




A. Intensity of Sectors as Material Input Providers to Other Sectors ($\sum_{j=1}^{N} \gamma_{ij}$)




B. Importance of Capital in Sectoral Production ($\sum_{j=1}^{N} \theta_{ij}$)

**Table 1: Share Weight Decomposition of Industrial Production**

Standard Deviations of Growth Rates: Percentage points at annual rates

| Series | 1972-2007 | 1972-1983 | 1984-2007 |
|---|---|---|---|
| $g_t = \sum w_{it} x_{it}$ | 5.8 | 8.7 | 3.6 |
| Components | | | |
| $\frac{1}{N}\sum x_{it}$ | 6.9 | 10.5 | 4.2 |
| $\sum\left(\overline{w}_i - \frac{1}{N}\right)x_{it}$ | 2.0 | 2.8 | 1.5 |
| $\sum\left(w_{it} - \overline{w}_i\right)x_{it}$ | 0.8 | 1.0 | 0.6 |

Notes: Entries are the sample standard deviations of the growth rate of IP, $g_t$, and its components.

**Table 2: Average Pairwise Correlations of Sectoral Growth Rates**

| 1972-2007 | 1972-1983 | 1984-2007 |
|---|---|---|
| 0.19 | 0.27 | 0.11 |

Notes: Entries are the sample correlations of $x_{it}$ and $x_{jt}$ averaged over all $i \neq j$.

**Table 3: Standard Deviation of $g_t$ with and without sectoral covariance**

Standard Deviations (Percentage points at annual rate)

| | 1972-2007 | 1972-1983 | 1984-2007 |
|---|---|---|---|
| | A. Using actual ($w_{it}$) share weights | | |
| With Sectoral Covariation | 5.8 | 8.7 | 3.6 |
| Without Sectoral Covariation | 1.9 | 2.4 | 1.6 |
| | B. Using equal (1/$N$) share weights | | |
| With Sectoral Covariation | 6.9 | 10.5 | 4.2 |
| Without Sectoral Covariation | 1.8 | 2.5 | 1.4 |

Notes: The entries for rows labeled "With Sectoral Covariation" are the sample standard deviations of $\sum_i w_{it} x_{it}$ (panel A) and $N^{-1}\sum_i x_{it}$ (panel B). The entries labeled "Without Sectoral Covariation" are computed as $\sqrt{T^{-1}\sum_t \sum_i h_{it}^2 (x_{it} - \overline{x}_i)^2}$ where $h_{it} = w_{it}$ in panel A and $h_{it} = N^{-1}$ in panel B.

### Table 4: Decomposition of Variance from Statistical 2-Factor Model

| | 1972-1983 | 1984-2007 |
|---|---|---|
| Std. Deviation (Data) | 8.7 | 3.6 |
| Std. Deviation (Model) | 8.5 | 3.6 |
| $R^2(F)$ | 0.89 | 0.87 |

Notes: The first row shows the sample standard deviation of $g_t$, and the second row shows the implied standard deviation from the factor model with constant shares and uncorrelated $u_{it}$. The final row shows the fraction of the variance of $g_t$ accounted for by the common factors.

### Table 5: Fraction of Variability in Selected Sectoral Growth Rates Explained by Common Factors

| Sector | $R_i^2(F)$ |
|---|---|
| A. 1972-1983 | |
| Other Fabricated Metal Products | 0.86 |
| Fabricated Metals: Forging and Stamping | 0.85 |
| Machine Shops: Turned Products and Screws | 0.83 |
| Commercial and Service Industry Machinery/Other General Purpose Machinery | 0.83 |
| Foundries | 0.80 |
| Other Electrical Equipment | 0.79 |
| Metal Working Machinery | 0.78 |
| Fabricated Metals: Cutlery and Handtools | 0.76 |
| Electrical Equipment & | 0.73 |
| Architectural and Structural Metal Products | 0.72 |
| B. 1984-2007 | |
| Coating, Engraving, Heat Treating, and Allied Activities | 0.68 |
| Plastic Products | 0.67 |
| Commercial and Service Industry Machinery/Other General Purpose Machinery | 0.65 |
| Fabricated Metals:\ Forging and Stamping | 0.65 |
| Household and Institutional Furniture and Kitchen Cabinets | 0.59 |
| Veneer, Plywood, and Engineered Wood Products | 0.59 |
| Metal Working Machinery | 0.52 |
| Foundries | 0.52 |
| Millwork | 0.51 |
| Other Fabricated Metal Products | 0.50 |

Notes: The entries are the sectors ordered by $R_i^2(F) = \lambda_i'\Sigma_{FF}\lambda_i/\sigma_i^2$, where parameters and moments are estimated over the periods shown in the headings.

**Table 6: Information Content of IP Contained in Individual Sectors**
Fraction of Variability of IP Explained by a Subset of Sectors
(Percentage Points)

| Number of Sectors ordered by $R_i^2(F)$ | 1972-1983 | 1984-2007 |
|---|---|---|
| 5 | 83.2 | 74.1 |
| 10 | 87.6 | 78.1 |
| 20 | 96.2 | 82.8 |

Notes: The entries show var($\psi' X_{M,t}$)/var($g_t$), where $X_{M,}$ are the set of $M$ elements of $X_t$ with the largest values of $R_i^2(F)$. Results are shown for $M$=5,10, and 20. The vector $\psi$ is computed using the constant share approximation $g_t \approx \bar{w}' X_t$, which implies that $\psi = (S\Sigma_{XX}S')^{-1}(S\Sigma_{XX}\bar{w})$, where $S$ is a selection matrix that satisfies $X_{M,t} = SX_t$. The various average shares, parameters and moments are estimated over the periods shown in the column heading.

**Table 7: Sectoral Correlations and Volatility of IP Growth Rates Implied by Structural Model**

| | Data | | Model with Uncorrelated Shocks | | Model with 2 Factors | | |
|---|---|---|---|---|---|---|---|
| Sample Period | $\bar{\rho}_{ij}$ | $\sigma_g$ | $\bar{\rho}_{ij}$ | $\sigma_g$ | $\bar{\rho}_{ij}$ | $\sigma_g$ | $R^2(S)$ |
| 1972-1983 | 0.27 | 8.8 | 0.05 | 5.1 | 0.26 | 9.5 | 0.81 |
| 1984-2007 | 0.11 | 3.6 | 0.04 | 3.1 | 0.10 | 4.1 | 0.50 |

Notes: The columns labeled $\bar{\rho}_{ij}$ shows the average pairwise correlation of sectoral growth rates; the columns labeled $\sigma_g$ shows the standard deviation of IP growth rates, and the column labeled $R^2(S)$ shows the fraction of the variability in IP associated with the common factors $S$.

**Table 8: Selected Summary Statistics for Data and Various Models with Uncorrelated Shocks** (1972-2007 Sample Period)

| | | $\bar{\rho}_{ij}$ | $\sigma_g$ | $\sigma_g$ (*diag*) | $\sigma_g$ (*scaled*) | $\frac{\sigma_g^2(scaled)}{\sigma_{g,Benchmark}^2(scaled)}$ |
|---|---|---|---|---|---|---|
| 1 | Data | 0.19 | 5.80 | 1.85 | 5.80 | |
| | | | | | | |
| 2 | Benchmark Model | 0.04 | 3.87 | 1.88 | 3.82 | 1.00 |
| | | | | | | |
| 3 | Long-Plosser | 0.01 | 2.66 | 2.07 | 2.38 | 0.39 |
| 4 | Carvalho | 0.04 | 3.15 | 1.64 | 3.56 | 0.87 |
| 5 | Horvath-Dupor | 0.06 | 3.76 | 1.81 | 3.84 | 1.01 |
| | | | | | | |
| 6 | Benchmark, $\Theta = I$ | 0.02 | 3.86 | 2.43 | 2.94 | 0.59 |
| 7 | Benchmark, $\delta = 1$ | 0.06 | 3.74 | 1.72 | 4.04 | 1.12 |
| | | | | | | |
| 8 | Long-Plosser, $\Gamma$ average | 0.01 | 1.61 | 1.39 | 2.15 | 0.32 |
| 9 | Carvalho, $\Gamma$ average | 0.04 | 2.60 | 1.53 | 3.15 | 0.68 |
| 10 | Horvath-Dupor, $\Gamma$ average, $\alpha_d = \alpha I$ | 0.05 | 2.89 | 1.58 | 3.40 | 0.79 |
| 11 | Benchmark, $\Gamma$ average, $\alpha_d = \alpha I$ | 0.05 | 3.30 | 1.71 | 3.57 | 0.87 |
| | | | | | | |
| 12 | Benchmark, $\Sigma_{\varepsilon\varepsilon} = \sigma^2 I$ | 0.04 | 5.72 | 2.99 | 3.55 | 0.86 |

Notes: Entries for rows 2-12 correspond to models that are described in the text. $\bar{\rho}_{ij}$ denotes the average pairwise correlation of sectoral growth rates, $\sigma_g$ denotes the standard deviation of *IP* growth rates, $\sigma_g$ (*diag*) denotes the standard deviation computed using the diagonal elements of $\Sigma_{XX}$ (the covariance matrix of sector growth rates), $\sigma_g$ (*scaled*) denotes the standard deviation with shocks that are scaled so that the model's implied value for $\sigma_g$ (*diag*) is equal to the value for the data, and the final column shows $\sigma_g^2(scaled)$ for each model relative the value in the benchmark model.

**Table 9: Selected Summary Statistics with Different Levels of Sectoral Aggregation**

| | 1972-1983 | | | 1984-2007 | | |
|---|---|---|---|---|---|---|
| | $\bar{\rho}_{ij}$ | | $R^2(S)$ | $\bar{\rho}_{ij}$ | | $R^2(S)$ |
| | Data | Model with Diagonal $\Sigma_{\varepsilon\varepsilon}$ | | Data | Model with Diagonal $\Sigma_{\varepsilon\varepsilon}$ | |
| 2-Digit Level (26 Sectors) | 0.38 | 0.09 | 0.76 | 0.22 | 0.07 | 0.53 |
| 3-Digit Level (88 Sectors) | 0.29 | 0.05 | 0.85 | 0.13 | 0.05 | 0.53 |
| 4-Digit Level (117 Sectors) | 0.27 | 0.05 | 0.81 | 0.11 | 0.04 | 0.50 |

Notes: The table shows the average pairwise correlation of sectoral growth rates ($\bar{\rho}_{ij}$) for different levels of sectoral disaggregation for the data and for the model with uncorrelated shocks ($\Sigma_{\varepsilon\varepsilon}$ a diagonal matrix). The columns labeled $R^2(S)$ shows the fraction of variability in aggregate IP associated with two common factors ($S$) computed at the level of aggregation shown in the first column.

**Table 10: Comparing Results (Model with Θ = I) Sectoral Correlations and Volatility of IP Growth Rates Implied by Structural Model**

| | | Data | | Model with Uncorrelated Shocks | | Structural Model with 2 Factors | Reduced Form Model with 2 Factors |
|---|---|---|---|---|---|---|---|
| Sample Period | | $\bar{\rho}_{ij}$ | $\sigma_g$ | $\bar{\rho}_{ij}$ | $\sigma_g$ | $R^2(S)$ | $R^2(F)$ |
| 1972-1983 | $\Gamma^{1997}$ | 0.27 | 8.8 | 0.02 | 3.7 | 0.88 | 0.89 |
| 1984-2007 | $\Gamma^{1997}$ | 0.11 | 3.6 | 0.02 | 2.2 | 0.69 | 0.87 |
| 1967-1983 | $\Gamma^{1977}$ | 0.21 | 8.5 | 0.03 | 4.0 | 0.83 | 0.85 |
| 1984-2002 | $\Gamma^{1977}$ | 0.10 | 3.9 | 0.02 | 2.4 | 0.73 | 0.94 |

Notes: The columns labeled $\bar{\rho}_{ij}$ shows the average pairwise correlation of sectoral growth rates; the columns labeled $\sigma_{IP}$ shows the standard deviation of IP growth rates, and the columns labeled $R^2(S)$ shows the fraction of the variability in IP associated with the common factors $S$. Results are shown using the 1997 input-output matrix $\Gamma^{1997}$ and the sectors classified by NAICS and for the 1977 matrix $\Gamma^{1977}$ and the sectors classified by SIC.

## Table 11: Fraction of Variability of IP Explained by Sector-Specific Shocks

| Rank | Sector | Fraction |
|---|---|---|
| | A. 1972-1983 SIC ($\Theta = I$) | |
| 1 | Basic Steel and Mill Products | 0.064 |
| 2 | Coal Mining | 0.034 |
| 3 | Motor Vehicles, Trucks, and Buses | 0.008 |
| 4 | Utilites | 0.007 |
| 5 | Oil and Gas Extraction | 0.005 |
| 6 | Copper Ores | 0.004 |
| 7 | Iron and Other Ores | 0.003 |
| 8 | Petroleum Refining and Miscellaneous | 0.003 |
| 9 | Motor Vehicle Parts | 0.004 |
| 10 | Electronic Components | 0.002 |
| | B. 1984-2007 NAICS ($\Theta = I$) | |
| 1 | Iron and Steel Products | 0.042 |
| 2 | Electric Power Generation, Transmission and Distribution | 0.036 |
| 3 | Semiconductors and Other Electronic Components | 0.026 |
| 4 | Oil and Gas Extraction | 0.017 |
| 5 | Automobiles and Light Duty Motor Vehicles | 0.017 |
| 6 | Organic Chemicals | 0.017 |
| 7 | Aerospace Products and Parts | 0.015 |
| 8 | Motor Vehicle Parts | 0.013 |
| 9 | Natural Gas Distribution | 0.012 |
| 10 | Support Activities for Mining | 0.011 |
| | C. 1984-2007 NAICS ($\Theta = \Theta^{1997}$) | |
| 1 | Iron and Steel Products | 0.078 |
| 2 | Semiconductors and Other Electronic Components | 0.076 |
| 3 | Electric Power Generation, Transmission and Distribution | 0.046 |
| 4 | Oil and Gas Extraction | 0.031 |
| 5 | Organic Chemicals | 0.026 |
| 6 | Automobiles and Light Duty Motor Vehicles | 0.024 |
| 7 | Natural Gas Distribution | 0.020 |
| 8 | Motor Vehicle Parts | 0.018 |
| 9 | Support Activities for Mining | 0.014 |
| 10 | Other Basic Inorganic Chemicals | 0.013 |

Notes: The entries show the fraction of variability of IP growth rates explained by sector-specific shocks ($u_{it}$) from the estimated 2-factor models. Results are shown for the 10 sectors with the largest fractions. Results in panel A are for the model estimated using the 1977 value of the IO matrix ($\Gamma^{1977}$) , sectors classified by SIC, and with $\Theta = I$. Results in panel B are for the model estimated using the 1997 value of the IO matrix ($\Gamma^{1997}$), sectors classified by NAICS, and with $\Theta = I$. Results in panel C use the NAICS sectors with 1997 values of IO and Capital Use matrices and the ($\Gamma^{1997}$ and $\Theta^{1997}$).