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THE CONVERGENCE OF MULTIVARIATE 'UNIT ROOT' DISTRIBUTIONS TO THEIR ASYMPTOTIC LIMITS
The Case of Money–Income Causality

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We examine the quality of recently developed asymptotic approximations to the sampling distributions of various statistics in levels regressions when the regressors have unit roots. The calculations were performed using a bivariate probability model typical of some considered in applied macroeconomic research: the parameters of the model were obtained by estimating a VAR using postwar U.S. money and industrial production growth rates, resulting in pseudo-data that are I(1) with drifts. With 100 observations the asymptotic approximations are often found to be adequate; with 400 observations they are generally good. In addition, when the statistics have nonstandard distributions, both the asymptotic and exact distributions differ substantially from the usual normal or χ^2 distributions that would apply were the regressors stationary.

1. Introduction

There has been considerable theoretical progress towards understanding the asymptotic behavior of regression statistics when some or all of the variables are integrated. A central objective of this research is to provide guidance in approximating the sampling distributions of estimators and test statistics in data sets in which there are one or more unit roots in the multivariate representation of the series. But do these asymptotic approximations provide a

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good guide to sampling distributions in sample sizes and probability models 'typical' of those found in empirical macroeconomic research? Previous researchers have used Monte Carlo techniques to check the validity of asymptotic approximations in related problems with moderate sample sizes; e.g., tests of univariate or multivariate unit roots [e.g., Dickey and Fuller (1979), Phillips and Ouliaris (1988), and Schwert (1987)] and least squares estimators of cointegrating vectors [e.g., Banerjee et al. (1986), Stock (1987)]. In many contexts, however, the question of interest is different, often reducing to the behavior of certain test statistics – such as F-tests of exclusion restrictions – in linear time series models.

In this paper we examine the rate of convergence of the sampling distribution to the asymptotic distribution using a Monte Carlo experiment involving two variables, both constructed to be integrated of order one. The major difficulty in designing such an experiment is determining a probability model to generate the data that is 'realistic': typical linear time series models encountered in applied research involve multiple variables and many lags, which in turn requires specifying many parameters in developing the experimental design. Our solution to this problem is to consider a particular linear probability model (or 'data generation process') that has received widespread attention in the macroeconomic literature: a bivariate vector autoregression (VAR) with money and output, estimated using U.S. postwar data.[1] This model is estimated imposing the assumption that each series is integrated of order one, that the series are not cointegrated, that each series contains a nonzero drift, and that money does not enter the output equation. This final assumption permits the generation of the distribution of the Granger causality test statistic under the null hypothesis.

We examine the asymptotic and Monte Carlo distributions of four statistics in a OLS regression of income on lags of income and money, one of which is the usual 'Granger causality' F-statistic testing the hypothesis that money does not enter the income equation. As is discussed below, three of these – including the Granger causality statistic – have nonstandard asymptotic distributions. These statistics – one point estimate and three test statistics – are examined for two regressions: one in which the only deterministic term is a constant and a second in which a linear time trend is added as well.[2]

The details of the experimental design and a brief discussion of the relevant asymptotics are presented in section 2. The results are discussed in section 3, and we conclude with section 4.

[1]For recent reviews of the literature on the money–output relation, see Eichenbaum and Singleton (1986), Blanchard (1987), Christiano and Ljungqvist (1987), and Stock and Watson (1987).

[2]These specifications are of interest for macroeconomic as well as econometric reasons: as Bernanke (1986) and Runkle (1987) point out, different macroeconomic conclusions can obtain when time is added as a regressor in similar levels specifications.

2. Experimental design

2.1. The probability model and asymptotic behavior

We adopt the probability model

$$\begin{bmatrix} \Delta y_t \\ \Delta m_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_m \end{bmatrix} + \begin{bmatrix} A_{yy}(L) & 0 \\ A_{my}(L) & A_{mm}(L) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \Delta m_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{mt} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{mt} \end{bmatrix} \text{ i.i.d. } \mathrm{N}(0, \Sigma), \tag{1}$$

in which the first through the fifth lags of Δy_t and Δm_t appear on the right-hand side. The parameters of (1) were estimated by OLS using 460 observations on postwar U.S. monthly industrial production and M1 growth.[3] The resulting parameters are presented in the appendix.

We study four statistics computed in two specifications of the output equation in the unconstrained levels VAR(6) implied by (1). In the first, the level of income is regressed against lagged income, lagged money, and a constant:

Regression C: Regress y_t on $\{1;\ y_{t-1}, y_{t-2}, \ldots, y_{t-6};\ m_{t-1}, m_{t-2}, \ldots, m_{t-6}\}$.

Sims, Stock and Watson (1986) examine the asymptotic properties of OLS regressions when the data are generated by a probability model such as (1). Using their arguments, the estimated sum of the coefficients on lagged money $[\hat{\beta}_{ym}(1)]$ will have a nonstandard asymptotic distribution; so will the usual t-ratio produced by OLS regression packages testing the (correct) hypothesis that this sum is zero [call this $t_{\beta_{ym}(1)}$]. The usual Granger-causality F-test [call this $F(m_{t-1}, \ldots, m_{t-6})$] will also have a nonstandard limiting distribution. However, a F-test of linear restrictions on any proper subset of the coefficients will have the usual asymptotic χ^2 distribution, since such restrictions can be

[3]The entire industrial production (IP) series and the M1 data since 1959 were taken from the Citibase data base. The pre-1959 M1 data are taken from Christiano and Ljungqvist. The IP data were transformed by taking first differences of their logarithm [so that $\Delta y_t = \Delta \log(IP_t)$]. The transformation applied to M1 was somewhat more involved, since money growth appears to contain a linear time trend over this sample period [see Stock and Watson (1987)]. Accordingly, money growth was first detrended by taking the residuals of a regression (from 1948:1 to 1985:12) of $\Delta \log(M1_t)$ against a constant and time. Since this series has mean zero by construction, the average postwar drift in money was restored to the data by adding the postwar average of $\Delta \log(M1_t)$ to these residuals. These transformed data were used to estimate the constrained VAR(5) in (1) over 1948:7 to 1985:12, using earlier observations as initial values in the regressions.

rewritten as restrictions on mean zero, stationary variables. In particular, this will be true of the F-test of the hypothesis that the coefficients on all lags of money except the first are zero [call this $F(m_{t-2},\dots,m_{t-6})$]. We study the quality of the asymptotic approximation to the sampling distribution by considering these four statistics.[4]

Because both variables contain nonzero drifts, y_t is in effect 'detrending' m_t in regression C. Since the resultant 'detrended' series is integrated, this detrending affects the asymptotic distribution of the first three statistics. Consequently, we also consider the quality of the approximation of the asymptotic distribution when a linear time trend is included as a regressor, in which case these three statistics will have different asymptotic distributions. This will be referred to as:

Regression T: Regress y_t on $\{1, t;\ y_{t-1}, y_{t-2},\dots, y_{t-6};\ m_{t-1}, m_{t-2},\dots, m_{t-6}\}$.

2.2. Numerical issues

Evaluation of the asymptotic distributions. Sims, Stock and Watson (1986) provided explicit expressions for the weak limits of the three statistics with nonstandard distributions. These expressions – which are general enough to apply to the statistics from both regressions C and T – depend on certain functionals of Brownian motion over the unit interval and on the parameters of the VAR. In general, all the parameters of the VAR in (1) will enter into these expressions; we henceforth refer to the vector of these parameters as θ. We evaluate these asymptotic distributions numerically by first generating and storing sample path equivalents of these functionals, computed using driftless Gaussian random walks with 1000 observations. Given θ (which is of course known in this Monte Carlo experiment), the percentiles of the asymptotic distributions are approximated by the percentiles of the empirical distribution constructed using 4000 draws of these previously computed functionals; the multiple draws of the random functionals need to be computed only once. The details of this procedure are discussed in Stock (1987) and Stock and Watson (1987).

Evaluation of the sampling distributions. The Monte Carlo simulations involve generating $n+18$ observations of Δy_t and Δm_t according to the Gaussian probability model (1), where $n = 100$, 200, and 400. The 18 additional observations were judged sufficient to provide a stationary initial

[4]An alternative interpretation of the $F(m_{t-2},\dots,m_{t-6})$-statistic comes from recognizing that it is equivalent to the F-statistic that tests whether the growth rate of money belongs in the income equation. That is, rewriting the lagged money regressors to be m_{t-1}, $\Delta m_{t-1},\dots,\Delta m_{t-5}$, $F(m_{t-2},\dots,m_{t-6}) = F(\Delta m_{t-1},\dots,\Delta m_{t-5})$.

distribution for a regression with n observations, given the low dependence evident in the parameters in the appendix. The pseudo-observations were cumulated and the levels regressions were run using n observations on the dependent variable; the previous six observations were used as initial conditions.

All computations were done on a 16 MHz Compaq 80386/80387 desktop computer using the GAUSS programming language. The total computation time for the asymptotic distributions reported here was $2\frac{1}{2}$ minutes, exclusive of the one-time computation of the multiple draws of the random functionals of Brownian motion. The Monte Carlo simulations (4000 draws) required 25 hours.

3. Results

The finite sample and asymptotic distributions of the four statistics computed in regression C are presented in figs. 1–4; the corresponding distributions for regression T are shown in figs. 5–8. Selected percentiles of the asymptotic distributions are tabulated in table 1. For purposes of comparison, the 'usual' asymptotic distributions (that would apply were the regressors all stationary) are also presented for the $t_{\beta_{ym}(1)}$- and $F(m_{t-1},\ldots,m_{t-6})$-statistics.

Focusing first on the results for regression C, inspection of figs. 1–4 and table 1 indicate that $n\hat{\beta}_{ym}(1)$ is sharply skewed and has a nonzero mean, both in finite samples and in the limit. While this nonzero mean seems substantial,

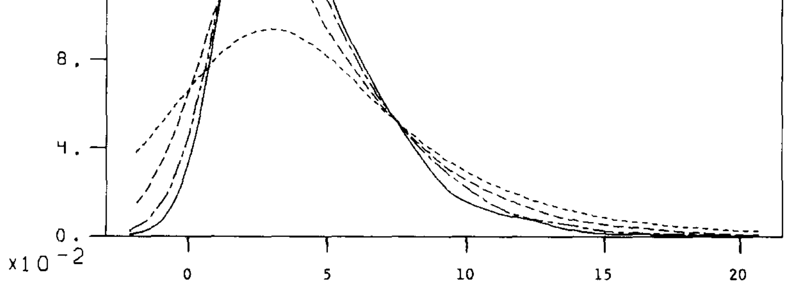

Fig. 1. Distributions for regression C: $n\hat{\beta}_{ym}(1)$.

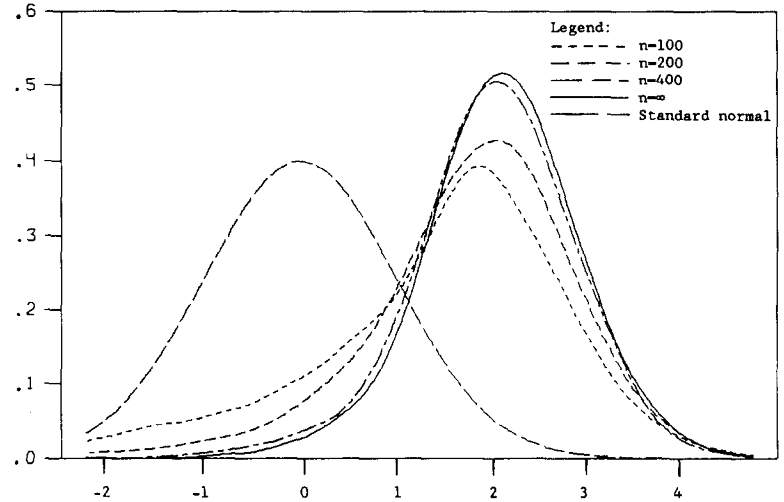

Fig. 2. Distributions for regression C: $t_{\beta_{ym}(1)}$.

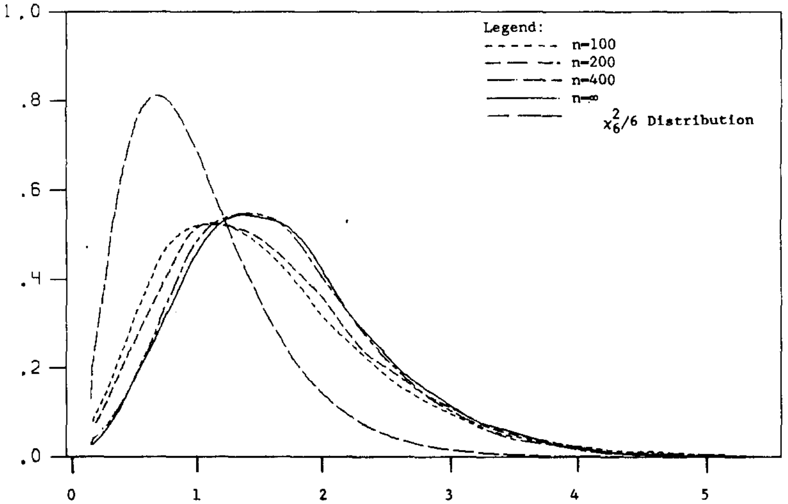

Fig. 3. Distributions for regression C: $F_{y,m}(m_{t-1}, \ldots, m_{t-6})$.

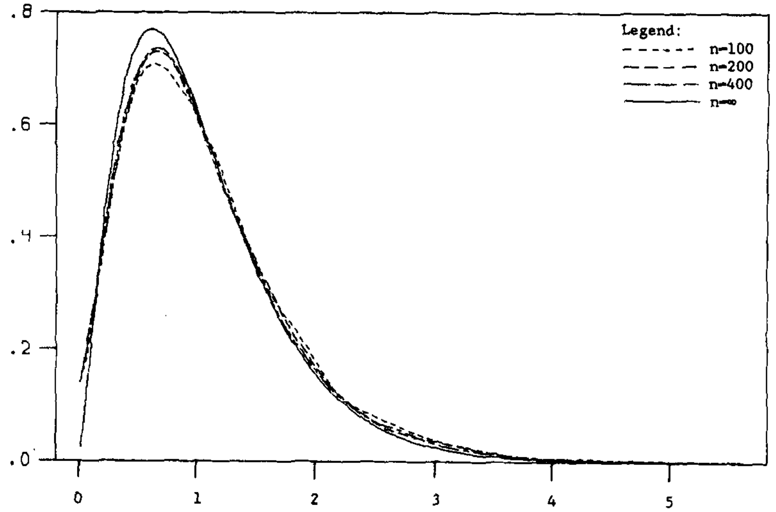

Fig. 4. Distributions for regression C: $F_{y,m}(m_{t-2},\ldots,m_{t-6})$.

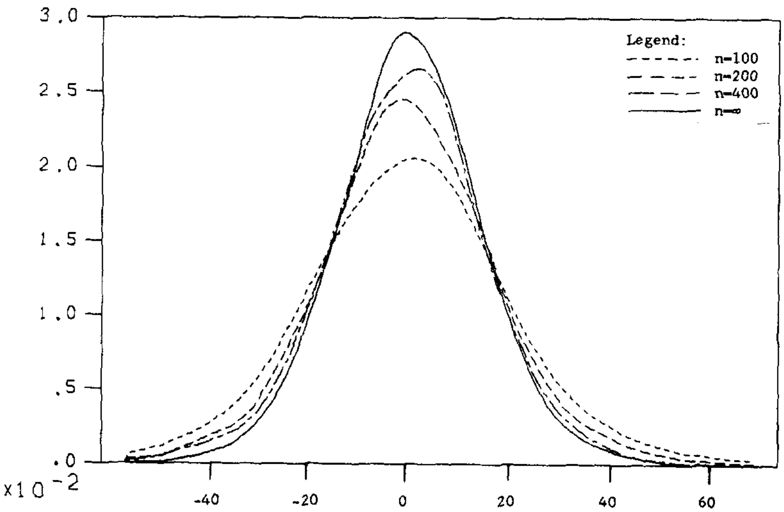

Fig. 5. Distributions for regression T: $n\hat{\beta}_{ym}(1)$.

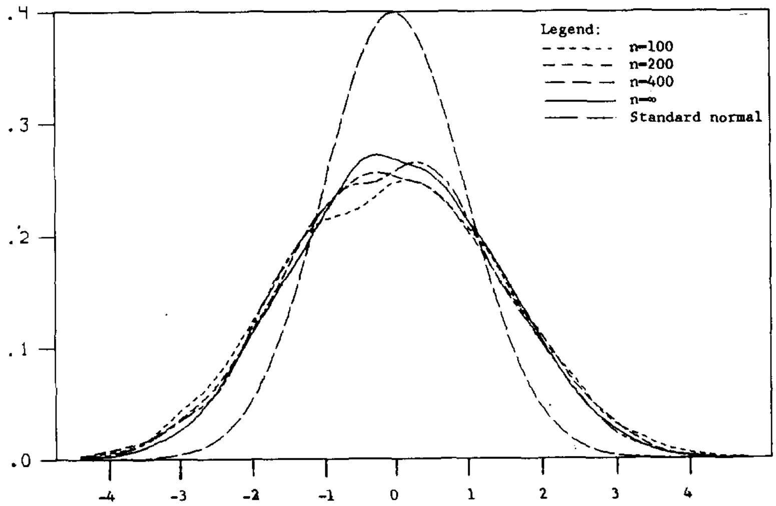

Fig. 6. Distributions for regression T: $t_{\beta_{ym}(1)}$.

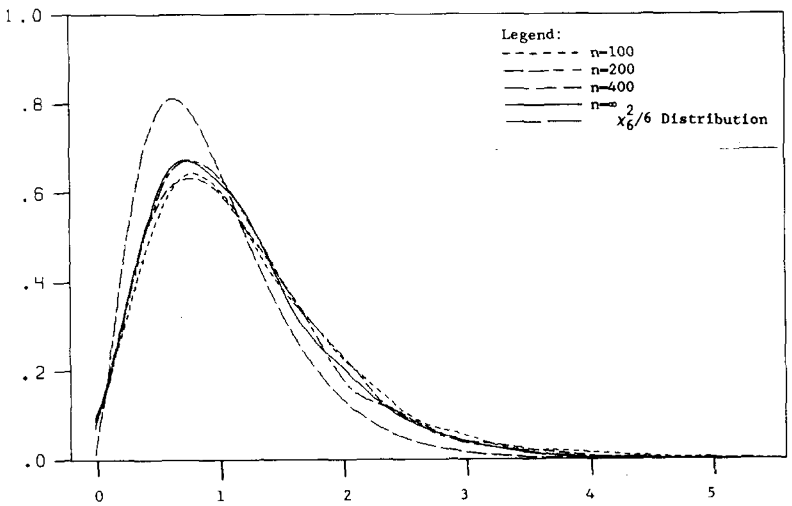

Fig. 7. Distributions for regression T: $F_{y,m}(m_{t-1},\ldots,m_{t-6})$.

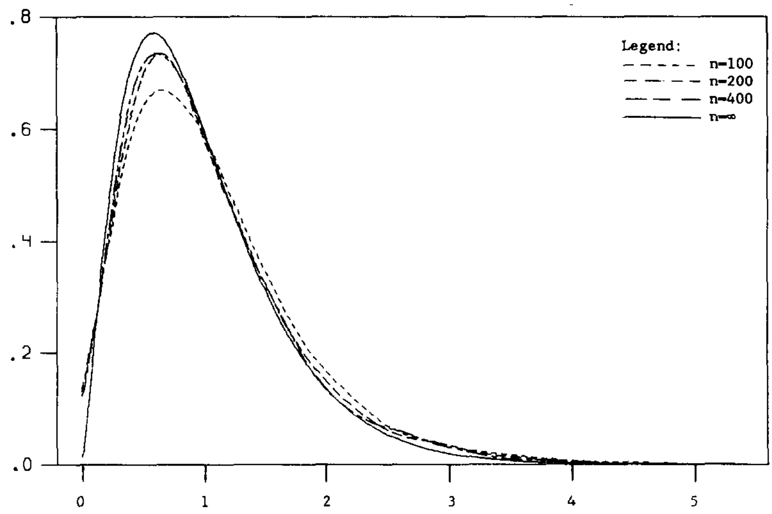

Fig. 8. Distributions for regression T: $F_{y,m}(m_{t-2},\ldots,m_{t-6})$.

since $\hat{\beta}_{ym}(1)$ converges at the rate n the bias is in fact small; for example, with $n = 100$, the bias implied by the asymptotic distribution is 0.0362. Comparing the finite sample and limiting distributions suggests that the convergence of $n\hat{\beta}_{ym}(1)$ is rather slow, with substantial differences in the left tail and center even with $n = 400$. The $t_{\hat{\beta}_{ym}(1)}$-statistic inherits the positive mean of $\hat{\beta}_{ym}(1)$. However, in contrast to $\hat{\beta}_{ym}(1)$, the convergence of the finite sample distributions of $t_{\beta_{ym}(1)}$ to their limit is sufficiently fast that the asymptotic and $n = 400$ distributions are very close. Inspection of fig. 3 suggests that, like $t_{\beta_{ym}(1)}$, the distribution of the $F(m_{t-1},\ldots,m_{t-6})$-statistic differs substantially from the 'usual' $\chi^2_6/6$ distribution. Again, the convergence of the finite sample distributions to the asymptotic limit is fast, in the sense that the $n = 400$ and asymptotic distributions are essentially the same. Finally, the finite sample distributions of the $F(m_{t-2},\ldots,m_{t-6})$-statistic are quite close to their $\chi^2_5/5$ asymptotic limit.

Including time as a regressor evidently makes a substantial difference in the distributions of some of these statistics. On the one hand, the asymptotic distributions seem to provide good approximations for $n = 400$ (and in some cases $n = 200$); on the other hand, the shapes of the nonstandard asymptotic distributions differ sharply between regressions C and T. The distributions of $\hat{\beta}_{ym}(1)$ and $t_{\beta_{ym}(1)}$ from regression T both have a mean close to zero and exhibit substantially less skewness than in regression C. It is worth noting, however, that the limiting distribution of $t_{\beta_{ym}(1)}$ still has substantially heavier tails than the standard normal limit.

Table 1

Percentiles of asymptotic distributions.[a]

Statistic	Percentile										
	1%	2.5%	5%	10%	25%	50%	75%	90%	95%	97.5%	99%
A. Percentiles for statistics estimated in regression C											
$n\hat{\beta}_{ym}(1)$	−0.10	0.43	0.86	1.32	2.23	3.62	5.56	7.78	9.33	11.12	12.82
$t_{\beta_{ym}(1)}$	−0.06	0.37	0.74	1.09	1.59	2.10	2.61	3.08	3.38	3.63	3.95
	(−2.32)	(−1.96)	(−1.64)	(−1.28)	(−0.67)	(0.00)	(0.67)	(1.28)	(1.64)	(1.96)	(2.32)
$F_{y,m}(m_{t-1},\dots,m_{t-6})$	0.34	0.47	0.61	0.77	1.12	1.55	2.11	2.68	3.05	3.43	3.84
	(0.15)	(0.21)	(0.27)	(0.37)	(0.58)	(0.89)	(1.31)	(1.77)	(2.10)	(2.41)	(2.80)
$F_{y,m}(m_{t-2},\dots,m_{t-6})$	0.11	0.17	0.23	0.32	0.53	0.87	1.33	1.85	2.21	2.56	3.00
B. Percentiles for statistics estimated in regression T											
$n\hat{\beta}_{ym}(1)$	−33.57	−27.35	−22.69	−17.22	−8.67	0.01	9.01	17.78	23.34	28.51	35.24
$t_{\beta_{ym}(1)}$	−3.07	−2.60	−2.23	−1.77	−0.95	0.00	0.99	1.83	2.30	2.68	3.11
	(−2.32)	(−1.96)	(−1.64)	(−1.28)	(−0.67)	(0.00)	(0.67)	(1.28)	(1.64)	(1.96)	(2.32)
$F_{y,m}(m_{t-1},\dots,m_{t-6})$	0.17	0.22	0.30	0.41	0.65	1.00	1.48	2.06	2.45	2.75	3.33
	(0.15)	(0.21)	(0.27)	(0.37)	(0.58)	(0.89)	(1.31)	(1.77)	(2.10)	(2.41)	(2.80)
$F_{y,m}(m_{t-2},\dots,m_{t-6})$	0.11	0.17	0.23	0.32	0.53	0.87	1.33	1.85	2.21	2.56	3.00

[a] The number of observations is denoted by n. The $F_{y,m}(m_{t-2},\dots,m_{t-6})$-statistic has an asymptotic $\chi_5^2/5$ distribution; the other statistics have nonstandard asymptotic distributions. The method for computing the nonstandard asymptotic distributions is described in the text. The percentiles in parentheses correspond to the 'usual' (incorrect) values obtained using Gaussian asymptotic theory: percentiles of a standard normal are given for the $t_{\beta_{ym}(1)}$-statistic, and $\chi_6^2/6$ percentiles are given for the $F_{y,m}(m_{t-1},\dots,m_{t-6})$-statistic.

Table 2

Monte Carlo rejection probabilities for tests of level α based on the correct asymptotic critical values.[a]

	n	α 1%	5%	10%	25%	50%
		A. $t_{\beta_{ym}(1)}$				
C	100	0.01	0.04	0.07	0.17	0.35
	200	0.01	0.04	0.08	0.20	0.42
	400	0.01	0.04	0.09	0.23	0.47
T	100	0.02	0.08	0.14	0.29	0.55
	200	0.02	0.07	0.13	0.28	0.52
	400	0.01	0.06	0.11	0.26	0.50
		B. $F_{y,m}(m_{t-1},\dots,m_{t-6})$				
C	100	0.02	0.05	0.09	0.21	0.41
	200	0.01	0.05	0.10	0.22	0.43
	400	0.01	0.05	0.09	0.23	0.48
T	100	0.02	0.08	0.14	0.30	0.54
	200	0.02	0.07	0.12	0.29	0.52
	400	0.01	0.06	0.11	0.26	0.51
		C. $F_{y,m}(m_{t-2},\dots,m_{t-6})$				
C	100	0.02	0.07	0.12	0.27	0.52
	200	0.01	0.06	0.11	0.26	0.51
	400	0.01	0.06	0.11	0.26	0.51
T	100	0.02	0.07	0.14	0.29	0.55
	200	0.02	0.07	0.12	0.27	0.51
	400	0.02	0.06	0.11	0.26	0.51

[a]The t-test is two-sided. Based on 4000 Monte Carlo simulations using data generated from model (1) as described in the text. The correct asymptotic percentiles are taken from table 1 (for regression C and regression T); nonstandard distribution theory was used to obtain the correct critical values for the tests in panels A and B.

A basic question motivating this investigation is whether tests calculated using the asymptotic critical values have the desired sizes in finite samples. To this end, the sizes of tests (based on the correct critical values) using $t_{\beta_{ym}(1)}$, $F(m_{t-1},\dots,m_{t-6})$ and $F(m_{t-2},\dots,m_{t-6})$ are given in table 2. These results suggest that for small samples the asymptotic approximation can be rather unsatisfactory; for example, with $n = 100$, a two-sided t-test based on the asymptotic 25% critical value would have a size of 17%.[5] However, in most cases the size is quite close to the level for $n = 200$; this is true in all cases for $n = 400$.

[5]The two-sided t-tests were performed using the square of the statistic and the corresponding asymptotic critical value.

Table 3

Monte Carlo rejection probabilities for tests of level α based on the incorrect (Gaussian and χ^2) asymptotic critical values.[a]

	n	1%	5%	10%	25%	50%
			A. $t_{\beta_{ym}(1)}$			
C	100	0.26	0.55	0.69	0.85	1.00
	200	0.31	0.62	0.75	0.90	1.00
	400	0.36	0.70	0.83	0.95	1.00
T	100	0.12	0.29	0.42	0.68	1.00
	200	0.11	0.27	0.39	0.66	1.00
	400	0.09	0.26	0.38	0.66	1.00
			B. $F_{y,m}(m_{t-1},\ldots,m_{t-6})$			
C	100	0.08	0.21	0.32	0.54	0.75
	200	0.08	0.23	0.35	0.57	0.80
	400	0.08	0.24	0.37	0.62	0.85
T	100	0.04	0.13	0.20	0.37	0.62
	200	0.03	0.11	0.19	0.37	0.62
	400	0.03	0.10	0.16	0.35	0.60

[a] The t-test is two-sided. Based on 4000 Monte Carlo simulations using data generated from model (1). 95% confidence intervals for these rejection probabilities range from ± 0.003 for the 1% level tests to ± 0.014 for the 50% level tests. The (incorrect) critical values used to perform the tests are the applicable values given in parentheses in table 1.

Finally, it is of some interest to consider what mistakes might be made were the 'usual' Gaussian critical values used to perform the tests. To this end, the sizes of tests calculated using the $t_{\beta_{ym}(1)}$- and $F(m_{t-1},\ldots,m_{t-6})$-statistics, based on the *incorrect* standard critical values, are presented in table 3. In almost all cases the size is substantially greater than the level; this is particularly true for tests based on regression C and on the $t_{\beta_{ym}(1)}$-statistic.

4. Conclusions

It is important to mention two cautionary notes about interpreting these results too broadly. First, this analysis has focused ona single probability model. While this made it possible to proceed with the analysis, and while this probability model may be typical of models studied when analyzing the relation between money and income, there is no reason that the same quantitative conclusions would obtain were we to analyze a different model. Second, we have considered a model that is linear, with known autoregressive order, and – most importantly – in which there are exact unit roots. It would be surprising indeed were these assumptions precisely to describe any extant macroeconomic time series, although they might be satisfactory approximations in the context of certain linear prediction problems with stochastically trending variables. Thus this investigation has proceeded on grounds most favorable to the asymptotic 'unit roots' distribution theory. In this light, satisfactory performance of the asymptotic theory in this experiment is but a minimal condition for thinking that it will provide a satisfactory approximation in practice.

Viewed within the context of this exercise, however, our results suggest four general conclusions. First, the nonstandard asymptotic distributions of certain estimators can be sharply skewed and can have a nonzero mean. Second, the rate of convergence of the finite sample distributions to their nonstandard limits is fast, in the practical sense that the sizes of tests performed using series of lengths typically found in empirical macroeconomic research are close to their asymptotic level. Third, substantial errors in inference can be made if the 'usual' critical values predicated on the Gaussian theory of stationary regressors are used in implementing t- and F-tests that have nonstandard limits. These three observations are consistent with earlier results both for univariate 'unit roots' distributions and for the distributions of estimators of cointegrating vectors. Finally, an additional lesson suggested by these results is that the nonstandard asymptotic (and finite sample) distributions can be very sensitive to seemingly minor changes in the specification, such as including a time trend.

Appendix

Table 4

Coefficients of probability model (1).[a]

Regressor	Δy equation	Δm equation
Constant	0.0015	0.0026
Δy_{t-1}	0.3919	0.0449
Δy_{t-2}	0.0822	−0.0104
Δy_{t-3}	0.0469	−0.0031
Δy_{t-4}	0.0386	−0.0258
Δy_{t-5}	−0.0679	0.0068
Δm_{t-1}	0.0000	0.2486
Δm_{t-2}	0.0000	−0.0407
Δm_{t-3}	0.0000	0.1557
Δm_{t-4}	0.0000	−0.2040
Δm_{t-5}	0.0000	0.1325

$$\Sigma = \begin{bmatrix} \Sigma_{yy} & \Sigma_{ym} \\ \Sigma_{my} & \Sigma_{mm} \end{bmatrix} = \begin{bmatrix} 113.12 & 2.44 \\ 2.44 & 13.21 \end{bmatrix} \times 10^{-6}$$

[a] These coefficients are the point estimates obtained by OLS using monthly industrial production growth and nominal M1 growth (detrended) as described in the text. The estimation period was 1948:7 to 1985:12, with earlier observations used for initial values.

References

Banerjee, A., J.J. Dolado, D.F. Hendry and G.W. Smith, 1986, Exploring equilibrium relationships in econometrics through static models: Some Monte Carlo evidence, Oxford Bulletin of Economics and Statistics 48, 253–277.

Blanchard, O.J., 1987, Why does money affect output? A survey, NBER working paper no. 2285.

Bernanke, B.S., 1986, Alternative explanations of the money-income correlation, Carnegie–Rochester Conference Series on Public Policy 25, 49–100.

Christiano, L.J. and L. Ljungqvist, 1987, Money does Granger-cause output in the bivariate output–money relation, Journal of Monetary Eonomics, forthcoming.

Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Society 74, 427–431.

Eichenbaum, M. and K.J. Singleton, 1986, Do equilibrium real business cycle theories explain postwar U.S. business cycles, NBER Macroeconomics Annual 1986, 91–134.

Fuller, W.A., 1976, Introduction to statistical time series (Wiley, New York).

Litterman, R. and L. Weiss, 1985, Money, real interest rates, and output: A reinterpretation of postwar U.S. data, Econometrica 53, 129–156.

Ohanian, L.E., 1986, The spurious effects of unit roots on vector autoregressions: A Monte Carlo study, Manuscript (University of Southern California, Los Angeles, CA).

Phillips, P.C.B. and S. Ouliaris, 1986, Testing for cointegration using principal component methods, Journal of Economic Dynamics and Control, this issue.

Runkle, D., 1987, Vector autoregressions and reality, Journal of Business and Economic Statistics 5, 437–442.

Schwert, G.W., 1987, Tests for unit roots: A Monte Carlo investigation, Manuscript (University of Rochester, Rochester, NY).

Sims, C.A., 1980, Comparison of interwar and postwar business cycles: Monetarism reconsidered, American Economic Review 70, 250–257.

Sims, C.A., J.H. Stock and M.W. Watson, 1986, Inference in linear time series models with some unit roots, Manuscript (Stanford University, Stanford, CA).

Stock, J.H., 1987, Asymptotic properties of least squares estimators of cointegrating vectors, Econometrica 55, 1035–1056.

Stock, J.H. and M.W. Watson, 1987, Interpreting the evidence on money-income causality, Journal of Econometrics, forthcoming.