Spatial Unit Roots

--- Supplementary Material ---

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The following tables show the rejection frequency (size) and average length of confidence intervals for each method and DGP.

Method: OLS (C-SCPC)

Size: k = 1

DGP	
Levy-BM	0.226 (0.203,0.266)
I(1) c=0.01	0.245 (0.214,0.279)
I(1) c=0.03	0.271 (0.235,0.310)
I(1) Matern	0.251 (0.226,0.287)
J c=0.03	0.036 (0.032,0.039)
J c = 0.50	0.146 (0.130,0.166)
Br. Sheet	0.256 (0.212,0.294)

Size: k = 5

DGP	
Levy-BM	0.197 (0.186,0.212)
I(1) c=0.01	0.199 (0.184,0.209)
I(1) c=0.03	0.225 (0.210,0.240)
I(1) Matern	0.204 (0.194,0.217)
J c=0.03	0.038 (0.034,0.041)
J c = 0.50	0.144 (0.134,0.156)
Br. Sheet	0.234 (0.208,0.252)

Average Length: k = 1

DGP	
Levy-BM	1.133 (1.071,1.204)
I(1) c=0.01	1.332 (1.268,1.407)
I(1) c=0.03	1.425 (1.356,1.519)
I(1) Matern	1.389 (1.317,1.449)
J c=0.03	0.498 (0.491,0.504)
J c = 0.50	1.030 (0.993,1.087)
Br. Sheet	1.060 (0.977,1.151)

Average Length: k = 5

DGP	
Levy-BM	0.853 (0.825,0.887)
I(1) c=0.01	1.099 (1.058,1.131)
I(1) c=0.03	1.185 (1.142,1.245)
I(1) Matern	1.161 (1.111,1.200)
J c=0.03	0.483 (0.476,0.488)
J c = 0.50	0.832 (0.812,0.870)
Br. Sheet	0.798 (0.769,0.854)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: Isotropic difference (C-SCPC)

Size: k = 1

DGP	b =0.030	b =0.060	b =0.090	b =0.120	b =0.150
Levy-BM	0.020 (0.016,0.024)	0.022 (0.017,0.027)	0.028 (0.023,0.035)	0.034 (0.027,0.044)	0.040 (0.033,0.055)
I(1) c=0.01	0.056 (0.046,0.065)	0.045 (0.041,0.051)	0.045 (0.039,0.056)	0.049 (0.042,0.065)	0.056 (0.045,0.074)
I(1) c=0.03	0.097 (0.080,0.112)	0.079 (0.069,0.089)	0.071 (0.062,0.083)	0.072 (0.060,0.093)	0.076 (0.063,0.105)
I(1) Matern	0.079 (0.067,0.089)	0.065 (0.057,0.073)	0.059 (0.054,0.065)	0.060 (0.053,0.073)	0.065 (0.056,0.086)
J c=0.03	0.019 (0.015,0.024)	0.021 (0.017,0.026)	0.026 (0.021,0.031)	0.029 (0.024,0.035)	0.033 (0.027,0.038)
J c = 0.50	0.020 (0.016,0.024)	0.022 (0.018,0.028)	0.027 (0.022,0.034)	0.033 (0.026,0.045)	0.038 (0.031,0.055)
Br. Sheet	0.042 (0.033,0.066)	0.067 (0.050,0.117)	0.092 (0.071,0.153)	0.109 (0.086,0.175)	0.120 (0.096,0.185)

Size: k = 5

DGP	b =0.030	b =0.060	b =0.090	b =0.120	b =0.150
Levy-BM	0.023 (0.019,0.028)	0.024 (0.020,0.030)	0.029 (0.025,0.038)	0.035 (0.029,0.049)	0.042 (0.032,0.058)
I(1) c=0.01	0.059 (0.050,0.069)	0.047 (0.042,0.052)	0.045 (0.039,0.053)	0.048 (0.042,0.064)	0.053 (0.045,0.076)
I(1) c=0.03	0.096 (0.082,0.105)	0.077 (0.069,0.088)	0.068 (0.062,0.075)	0.067 (0.060,0.081)	0.071 (0.062,0.092)
I(1) Matern	0.080 (0.069,0.089)	0.064 (0.057,0.072)	0.058 (0.051,0.065)	0.058 (0.050,0.071)	0.063 (0.053,0.081)
J c=0.03	0.022 (0.017,0.025)	0.023 (0.019,0.028)	0.026 (0.022,0.032)	0.030 (0.025,0.037)	0.032 (0.028,0.040)
J c = 0.50	0.022 (0.019,0.026)	0.024 (0.019,0.028)	0.028 (0.023,0.036)	0.033 (0.028,0.045)	0.039 (0.032,0.056)
Br. Sheet	0.047 (0.037,0.079)	0.072 (0.055,0.131)	0.090 (0.072,0.162)	0.108 (0.086,0.175)	0.120 (0.097,0.182)

Average Length: k = 1

DGP	b =0.030	b =0.060	b =0.090	b =0.120	b =0.150
Levy-BM	0.465 (0.410,0.533)	0.415 (0.384,0.454)	0.428 (0.400,0.483)	0.473 (0.433,0.563)	0.531 (0.475,0.625)
I(1) c=0.01	0.705 (0.640,0.783)	0.636 (0.588,0.686)	0.644 (0.599,0.720)	0.701 (0.634,0.809)	0.762 (0.690,0.893)
I(1) c=0.03	0.824 (0.772,0.932)	0.736 (0.683,0.822)	0.729 (0.680,0.791)	0.770 (0.712,0.858)	0.838 (0.770,0.947)
I(1) Matern	0.843 (0.779,0.928)	0.746 (0.699,0.837)	0.733 (0.689,0.803)	0.764 (0.723,0.868)	0.819 (0.766,0.958)
J c=0.03	0.465 (0.405,0.517)	0.403 (0.377,0.435)	0.404 (0.374,0.436)	0.418 (0.389,0.463)	0.436 (0.405,0.483)
J c = 0.50	0.462 (0.417,0.541)	0.411 (0.382,0.441)	0.426 (0.399,0.472)	0.467 (0.431,0.552)	0.518 (0.473,0.620)
Br. Sheet	0.536 (0.478,0.595)	0.498 (0.468,0.543)	0.517 (0.486,0.569)	0.542 (0.510,0.610)	0.575 (0.543,0.661)

Average Length: k = 5

DGP	b =0.030	b =0.060	b =0.090	b =0.120	b =0.150
Levy-BM	0.449 (0.405,0.502)	0.402 (0.376,0.435)	0.425 (0.394,0.472)	0.468 (0.427,0.534)	0.514 (0.471,0.600)
I(1) c=0.01	0.661 (0.607,0.711)	0.606 (0.570,0.656)	0.633 (0.591,0.706)	0.691 (0.641,0.797)	0.756 (0.703,0.868)
I(1) c=0.03	0.779 (0.716,0.817)	0.705 (0.658,0.745)	0.717 (0.676,0.785)	0.774 (0.723,0.885)	0.844 (0.775,0.965)
I(1) Matern	0.786 (0.738,0.859)	0.721 (0.684,0.772)	0.728 (0.690,0.785)	0.777 (0.728,0.868)	0.839 (0.778,0.942)
J c=0.03	0.456 (0.408,0.506)	0.393 (0.372,0.425)	0.397 (0.374,0.429)	0.415 (0.387,0.450)	0.433 (0.403,0.471)
J c = 0.50	0.449 (0.408,0.495)	0.403 (0.377,0.430)	0.422 (0.391,0.476)	0.464 (0.430,0.542)	0.512 (0.471,0.605)
Br. Sheet	0.506 (0.464,0.562)	0.480 (0.452,0.517)	0.498 (0.464,0.535)	0.527 (0.489,0.576)	0.557 (0.519,0.624)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: Cluster fixed-effects (clustered standard error)

Size: k = 1

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.168 (0.155,0.178)	0.139 (0.130,0.148)	0.105 (0.098,0.111)	0.076 (0.072,0.082)
I(1) c=0.01	0.263 (0.239,0.277)	0.281 (0.263,0.296)	0.285 (0.261,0.310)	0.238 (0.217,0.257)
I(1) c=0.03	0.350 (0.331,0.367)	0.390 (0.363,0.412)	0.412 (0.391,0.435)	0.369 (0.336,0.391)
I(1) Matern	0.305 (0.284,0.322)	0.339 (0.318,0.360)	0.364 (0.337,0.390)	0.326 (0.296,0.347)
J c=0.03	0.092 (0.086,0.097)	0.080 (0.076,0.085)	0.070 (0.066,0.075)	0.066 (0.061,0.070)
J c = 0.50	0.140 (0.132,0.149)	0.117 (0.109,0.124)	0.093 (0.087,0.100)	0.075 (0.070,0.081)
Br. Sheet	0.282 (0.243,0.339)	0.258 (0.219,0.310)	0.213 (0.185,0.262)	0.133 (0.116,0.162)

Size: k = 5

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.175 (0.164,0.185)	0.142 (0.130,0.151)	0.109 (0.101,0.116)	0.083 (0.078,0.088)
I(1) c=0.01	0.271 (0.255,0.287)	0.283 (0.265,0.297)	0.284 (0.268,0.298)	0.243 (0.230,0.265)
I(1) c=0.03	0.348 (0.326,0.366)	0.378 (0.356,0.399)	0.398 (0.374,0.414)	0.356 (0.338,0.375)
I(1) Matern	0.311 (0.288,0.328)	0.338 (0.315,0.355)	0.363 (0.339,0.378)	0.327 (0.306,0.342)
J c=0.03	0.097 (0.092,0.104)	0.084 (0.079,0.090)	0.074 (0.071,0.079)	0.072 (0.068,0.076)
J c = 0.50	0.149 (0.142,0.160)	0.123 (0.115,0.133)	0.098 (0.092,0.105)	0.079 (0.074,0.084)
Br. Sheet	0.295 (0.256,0.340)	0.266 (0.231,0.312)	0.221 (0.188,0.285)	0.141 (0.124,0.178)

Average Length: k = 1

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.353 (0.342,0.364)	0.307 (0.299,0.317)	0.294 (0.288,0.301)	0.355 (0.347,0.364)
I(1) c=0.01	0.474 (0.458,0.495)	0.412 (0.396,0.431)	0.382 (0.365,0.408)	0.442 (0.420,0.474)
I(1) c=0.03	0.501 (0.480,0.520)	0.424 (0.406,0.448)	0.389 (0.363,0.410)	0.441 (0.415,0.469)
I(1) Matern	0.497 (0.481,0.515)	0.430 (0.407,0.453)	0.392 (0.369,0.414)	0.450 (0.422,0.478)
J c=0.03	0.275 (0.271,0.280)	0.264 (0.260,0.269)	0.272 (0.267,0.276)	0.342 (0.337,0.348)
J c = 0.50	0.343 (0.335,0.352)	0.302 (0.295,0.312)	0.291 (0.287,0.297)	0.354 (0.347,0.361)
Br. Sheet	0.376 (0.358,0.402)	0.329 (0.312,0.351)	0.314 (0.303,0.328)	0.380 (0.361,0.398)

Average Length: k = 5

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.334 (0.327,0.344)	0.297 (0.289,0.307)	0.288 (0.283,0.295)	0.349 (0.343,0.356)
I(1) c=0.01	0.448 (0.438,0.463)	0.395 (0.386,0.411)	0.371 (0.358,0.383)	0.424 (0.407,0.444)
I(1) c=0.03	0.476 (0.463,0.492)	0.411 (0.399,0.426)	0.382 (0.369,0.395)	0.431 (0.412,0.450)
I(1) Matern	0.475 (0.463,0.491)	0.414 (0.400,0.429)	0.384 (0.366,0.398)	0.433 (0.413,0.454)
J c=0.03	0.269 (0.264,0.274)	0.260 (0.255,0.265)	0.269 (0.264,0.273)	0.338 (0.333,0.344)
J c = 0.50	0.327 (0.320,0.336)	0.293 (0.287,0.300)	0.286 (0.282,0.292)	0.347 (0.342,0.352)
Br. Sheet	0.347 (0.337,0.362)	0.313 (0.303,0.326)	0.305 (0.294,0.316)	0.370 (0.355,0.381)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: Cluster fixed-effects (C-SCPC)

Size: k = 1

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.053 (0.045,0.062)	0.056 (0.044,0.073)	0.056 (0.046,0.065)	0.047 (0.040,0.061)
I(1) c=0.01	0.084 (0.076,0.094)	0.097 (0.080,0.129)	0.112 (0.091,0.132)	0.102 (0.081,0.133)
I(1) c=0.03	0.122 (0.112,0.134)	0.132 (0.116,0.175)	0.157 (0.134,0.183)	0.150 (0.126,0.173)
I(1) Matern	0.098 (0.089,0.112)	0.114 (0.097,0.149)	0.134 (0.118,0.166)	0.127 (0.107,0.150)
J c=0.03	0.030 (0.026,0.035)	0.034 (0.029,0.044)	0.041 (0.034,0.049)	0.042 (0.035,0.049)
J c = 0.50	0.043 (0.039,0.051)	0.048 (0.039,0.064)	0.050 (0.041,0.062)	0.046 (0.040,0.059)
Br. Sheet	0.104 (0.082,0.145)	0.106 (0.080,0.150)	0.105 (0.081,0.150)	0.076 (0.058,0.103)

Size: k = 5

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.053 (0.048,0.061)	0.056 (0.046,0.073)	0.060 (0.048,0.076)	0.051 (0.041,0.063)
I(1) c=0.01	0.080 (0.073,0.090)	0.098 (0.078,0.139)	0.122 (0.104,0.147)	0.116 (0.097,0.136)
I(1) c=0.03	0.107 (0.096,0.118)	0.129 (0.107,0.178)	0.177 (0.153,0.202)	0.165 (0.146,0.188)
I(1) Matern	0.090 (0.081,0.103)	0.102 (0.088,0.157)	0.149 (0.129,0.179)	0.144 (0.130,0.163)
J c=0.03	0.030 (0.026,0.035)	0.036 (0.030,0.047)	0.043 (0.034,0.054)	0.046 (0.040,0.058)
J c = 0.50	0.044 (0.038,0.051)	0.050 (0.039,0.067)	0.055 (0.047,0.067)	0.050 (0.042,0.059)
Br. Sheet	0.106 (0.090,0.145)	0.115 (0.085,0.163)	0.109 (0.086,0.152)	0.083 (0.064,0.114)

Average Length: k = 1

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.550 (0.530,0.572)	0.447 (0.420,0.468)	0.393 (0.373,0.411)	0.453 (0.431,0.471)
I(1) c=0.01	0.809 (0.774,0.841)	0.697 (0.648,0.744)	0.627 (0.588,0.664)	0.683 (0.632,0.723)
I(1) c=0.03	0.907 (0.876,0.942)	0.813 (0.745,0.854)	0.737 (0.683,0.775)	0.773 (0.721,0.820)
I(1) Matern	0.878 (0.848,0.916)	0.773 (0.712,0.822)	0.708 (0.647,0.747)	0.763 (0.716,0.806)
J c=0.03	0.405 (0.392,0.418)	0.370 (0.348,0.382)	0.352 (0.337,0.365)	0.433 (0.409,0.458)
J c = 0.50	0.527 (0.510,0.546)	0.433 (0.411,0.451)	0.386 (0.368,0.401)	0.449 (0.424,0.466)
Br. Sheet	0.620 (0.576,0.675)	0.514 (0.476,0.559)	0.455 (0.419,0.485)	0.502 (0.454,0.532)

Average Length: k = 5

DGP	m = 30.000	m = 60.000	m = 120.000	m = 240.000
Levy-BM	0.530 (0.513,0.548)	0.433 (0.410,0.455)	0.379 (0.363,0.397)	0.444 (0.424,0.467)
I(1) c=0.01	0.795 (0.765,0.820)	0.680 (0.595,0.722)	0.586 (0.552,0.615)	0.626 (0.596,0.672)
I(1) c=0.03	0.901 (0.872,0.922)	0.778 (0.687,0.824)	0.659 (0.622,0.692)	0.702 (0.651,0.741)
I(1) Matern	0.878 (0.842,0.906)	0.780 (0.661,0.814)	0.655 (0.618,0.685)	0.699 (0.665,0.731)
J c=0.03	0.401 (0.390,0.412)	0.363 (0.344,0.378)	0.346 (0.330,0.362)	0.427 (0.404,0.444)
J c = 0.50	0.513 (0.500,0.527)	0.420 (0.394,0.445)	0.375 (0.356,0.390)	0.439 (0.421,0.459)
Br. Sheet	0.583 (0.556,0.619)	0.493 (0.454,0.532)	0.435 (0.408,0.461)	0.483 (0.450,0.516)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: LBM-GLS

Size: k = 1

DGP	
Levy-BM	0.053 (0.049,0.057)
I(1) c=0.01	0.256 (0.244,0.267)
I(1) c=0.03	0.392 (0.374,0.412)
I(1) Matern	0.379 (0.359,0.396)
J c=0.03	0.058 (0.055,0.062)
J c = 0.50	0.053 (0.050,0.056)
Br. Sheet	0.234 (0.204,0.298)

Size: k = 5

DGP	
Levy-BM	0.054 (0.051,0.058)
I(1) c=0.01	0.257 (0.243,0.268)
I(1) c=0.03	0.392 (0.377,0.408)
I(1) Matern	0.380 (0.363,0.400)
J c=0.03	0.060 (0.056,0.063)
J c = 0.50	0.054 (0.051,0.057)
Br. Sheet	0.234 (0.206,0.300)

Average Length: k = 1

DGP	
Levy-BM	0.195 (0.195,0.195)
I(1) c=0.01	0.212 (0.209,0.215)
I(1) c=0.03	0.224 (0.219,0.231)
I(1) Matern	0.222 (0.215,0.229)
J c=0.03	0.196 (0.195,0.196)
J c = 0.50	0.195 (0.195,0.196)
Br. Sheet	0.208 (0.199,0.213)

Average Length: k = 5

DGP	
Levy-BM	0.195 (0.195,0.195)
I(1) c=0.01	0.212 (0.208,0.214)
I(1) c=0.03	0.224 (0.218,0.229)
I(1) Matern	0.223 (0.218,0.228)
J c=0.03	0.196 (0.195,0.196)
J c = 0.50	0.195 (0.195,0.195)
Br. Sheet	0.208 (0.199,0.212)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: LBM-GLS (C-SCPC)

Size: k = 1

DGP	
Levy-BM	0.030 (0.027,0.035)
I(1) c=0.01	0.049 (0.043,0.055)
I(1) c=0.03	0.069 (0.060,0.076)
I(1) Matern	0.059 (0.051,0.066)
J c=0.03	0.029 (0.025,0.033)
J c = 0.50	0.030 (0.027,0.035)
Br. Sheet	0.088 (0.072,0.125)

Size: k = 5

DGP	
Levy-BM	0.031 (0.027,0.035)
I(1) c=0.01	0.050 (0.043,0.056)
I(1) c=0.03	0.069 (0.061,0.078)
I(1) Matern	0.059 (0.052,0.067)
J c=0.03	0.029 (0.025,0.033)
J c = 0.50	0.030 (0.027,0.034)
Br. Sheet	0.085 (0.072,0.132)

Average Length: k = 1

DGP	
Levy-BM	0.254 (0.251,0.257)
I(1) c=0.01	0.419 (0.408,0.430)
I(1) c=0.03	0.541 (0.524,0.559)
I(1) Matern	0.545 (0.523,0.562)
J c=0.03	0.264 (0.260,0.266)
J c = 0.50	0.255 (0.252,0.258)
Br. Sheet	0.333 (0.319,0.349)

Average Length: k = 5

DGP	
Levy-BM	0.256 (0.253,0.258)
I(1) c=0.01	0.419 (0.408,0.430)
I(1) c=0.03	0.536 (0.517,0.553)
I(1) Matern	0.547 (0.528,0.565)
J c=0.03	0.266 (0.262,0.268)
J c = 0.50	0.257 (0.253,0.259)
Br. Sheet	0.335 (0.320,0.347)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: Low-pass eigenvector

Size: k = 1

DGP	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.050 (0.046,0.054)	0.050 (0.047,0.054)	0.050 (0.046,0.053)
I(1) c=0.01	0.051 (0.047,0.054)	0.052 (0.049,0.056)	0.064 (0.060,0.068)
I(1) c=0.03	0.053 (0.050,0.057)	0.063 (0.058,0.067)	0.105 (0.099,0.110)
I(1) Matern	0.051 (0.047,0.055)	0.055 (0.052,0.059)	0.082 (0.077,0.087)
J c=0.03	0.100 (0.093,0.107)	0.094 (0.088,0.099)	0.078 (0.074,0.083)
J c = 0.50	0.056 (0.052,0.060)	0.054 (0.050,0.059)	0.052 (0.048,0.055)
Br. Sheet	0.128 (0.095,0.171)	0.160 (0.120,0.209)	0.210 (0.170,0.272)

Size: k = 5

DGP	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.050 (0.046,0.054)	0.050 (0.046,0.054)	0.050 (0.048,0.054)
I(1) c=0.01	0.050 (0.047,0.054)	0.051 (0.048,0.056)	0.062 (0.059,0.066)
I(1) c=0.03	0.052 (0.048,0.055)	0.060 (0.057,0.063)	0.101 (0.096,0.107)
I(1) Matern	0.050 (0.046,0.053)	0.054 (0.050,0.058)	0.080 (0.074,0.085)
J c=0.03	0.095 (0.089,0.100)	0.095 (0.088,0.099)	0.079 (0.075,0.083)
J c = 0.50	0.054 (0.050,0.057)	0.054 (0.050,0.057)	0.052 (0.048,0.055)
Br. Sheet	0.104 (0.080,0.135)	0.147 (0.119,0.180)	0.201 (0.168,0.243)

Average Length: k = 1

DGP	q = 10.000	q = 20.000	q = 50.000
Levy-BM	1.507 (1.499,1.515)	0.960 (0.957,0.963)	0.574 (0.573,0.575)
I(1) c=0.01	1.508 (1.500,1.515)	0.960 (0.956,0.964)	0.574 (0.573,0.575)
I(1) c=0.03	1.507 (1.500,1.516)	0.960 (0.957,0.964)	0.574 (0.573,0.576)
I(1) Matern	1.508 (1.499,1.518)	0.961 (0.956,0.964)	0.574 (0.572,0.576)
J c=0.03	1.509 (1.496,1.517)	0.960 (0.956,0.964)	0.574 (0.573,0.576)
J c = 0.50	1.508 (1.499,1.513)	0.960 (0.957,0.963)	0.574 (0.573,0.575)
Br. Sheet	1.507 (1.498,1.518)	0.959 (0.956,0.967)	0.574 (0.572,0.576)

Average Length: k = 5

DGP	q = 10.000	q = 20.000	q = 50.000
Levy-BM	2.299 (2.279,2.317)	1.101 (1.095,1.106)	0.601 (0.599,0.602)
I(1) c=0.01	2.297 (2.280,2.313)	1.100 (1.096,1.105)	0.600 (0.599,0.602)
I(1) c=0.03	2.297 (2.276,2.317)	1.100 (1.095,1.105)	0.600 (0.599,0.602)
I(1) Matern	2.298 (2.283,2.316)	1.101 (1.095,1.105)	0.600 (0.599,0.602)
J c=0.03	2.297 (2.274,2.318)	1.100 (1.095,1.104)	0.600 (0.599,0.602)
J c = 0.50	2.300 (2.282,2.320)	1.101 (1.096,1.106)	0.600 (0.599,0.602)
Br. Sheet	2.300 (2.283,2.322)	1.101 (1.095,1.106)	0.600 (0.598,0.602)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: High-pass eigenvector (C-SCPC)

Size: k = 1

DGP	q = 5.000	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.129 (0.117,0.139)	0.095 (0.087,0.103)	0.070 (0.063,0.078)	0.050 (0.045,0.056)
I(1) c=0.01	0.174 (0.160,0.184)	0.141 (0.132,0.152)	0.118 (0.106,0.128)	0.090 (0.081,0.099)
I(1) c=0.03	0.215 (0.205,0.234)	0.183 (0.168,0.197)	0.150 (0.137,0.167)	0.111 (0.096,0.128)
I(1) Matern	0.193 (0.180,0.206)	0.165 (0.152,0.180)	0.146 (0.131,0.159)	0.118 (0.106,0.136)
J c=0.03	0.050 (0.045,0.054)	0.051 (0.046,0.056)	0.050 (0.045,0.055)	0.045 (0.040,0.049)
J c = 0.50	0.120 (0.112,0.133)	0.093 (0.086,0.099)	0.070 (0.064,0.076)	0.050 (0.045,0.055)
Br. Sheet	0.213 (0.186,0.270)	0.192 (0.163,0.246)	0.167 (0.141,0.221)	0.132 (0.113,0.174)

Size: k = 5

DGP	q = 5.000	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.125 (0.116,0.134)	0.093 (0.087,0.101)	0.070 (0.065,0.078)	0.051 (0.045,0.057)
I(1) c=0.01	0.161 (0.151,0.170)	0.135 (0.125,0.147)	0.114 (0.106,0.126)	0.089 (0.078,0.102)
I(1) c=0.03	0.200 (0.187,0.212)	0.173 (0.161,0.184)	0.144 (0.133,0.158)	0.108 (0.094,0.126)
I(1) Matern	0.179 (0.167,0.188)	0.157 (0.147,0.168)	0.139 (0.129,0.153)	0.118 (0.104,0.134)
J c=0.03	0.051 (0.046,0.054)	0.051 (0.048,0.056)	0.051 (0.046,0.054)	0.045 (0.042,0.051)
J c = 0.50	0.117 (0.108,0.128)	0.090 (0.085,0.096)	0.069 (0.063,0.074)	0.050 (0.045,0.057)
Br. Sheet	0.203 (0.182,0.249)	0.183 (0.161,0.232)	0.160 (0.140,0.214)	0.129 (0.108,0.174)

Average Length: k = 1

DGP	q = 5.000	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.565 (0.552,0.578)	0.467 (0.459,0.476)	0.391 (0.382,0.399)	0.328 (0.322,0.335)
I(1) c=0.01	0.744 (0.720,0.770)	0.647 (0.625,0.671)	0.558 (0.541,0.576)	0.464 (0.450,0.479)
I(1) c=0.03	0.789 (0.755,0.825)	0.690 (0.656,0.715)	0.587 (0.567,0.617)	0.489 (0.468,0.510)
I(1) Matern	0.788 (0.759,0.820)	0.690 (0.666,0.720)	0.607 (0.581,0.628)	0.521 (0.492,0.542)
J c=0.03	0.419 (0.412,0.425)	0.388 (0.381,0.394)	0.353 (0.349,0.359)	0.318 (0.314,0.324)
J c = 0.50	0.558 (0.542,0.575)	0.465 (0.455,0.475)	0.389 (0.383,0.399)	0.329 (0.322,0.334)
Br. Sheet	0.592 (0.571,0.614)	0.523 (0.498,0.551)	0.472 (0.449,0.498)	0.423 (0.401,0.447)

Average Length: k = 5

DGP	q = 5.000	q = 10.000	q = 20.000	q = 50.000
Levy-BM	0.535 (0.524,0.549)	0.456 (0.447,0.467)	0.386 (0.380,0.394)	0.329 (0.322,0.334)
I(1) c=0.01	0.735 (0.711,0.755)	0.647 (0.623,0.665)	0.557 (0.542,0.576)	0.465 (0.448,0.480)
I(1) c=0.03	0.792 (0.764,0.820)	0.693 (0.667,0.719)	0.594 (0.573,0.618)	0.491 (0.471,0.511)
I(1) Matern	0.786 (0.765,0.812)	0.697 (0.677,0.725)	0.613 (0.594,0.634)	0.526 (0.498,0.548)
J c=0.03	0.412 (0.406,0.418)	0.383 (0.378,0.389)	0.352 (0.346,0.357)	0.318 (0.314,0.324)
J c = 0.50	0.533 (0.520,0.545)	0.455 (0.446,0.462)	0.387 (0.380,0.393)	0.329 (0.323,0.335)
Br. Sheet	0.551 (0.529,0.575)	0.498 (0.476,0.514)	0.454 (0.436,0.471)	0.413 (0.398,0.430)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.

Method: Ibragimov-Müller

Size: k = 1

DGP	m = 10.000	m = 20.000	m = 50.000
Levy-BM	0.105 (0.090,0.117)	0.105 (0.096,0.114)	0.080 (0.072,0.087)
I(1) c=0.01	0.125 (0.110,0.137)	0.144 (0.131,0.157)	0.154 (0.137,0.168)
I(1) c=0.03	0.152 (0.130,0.166)	0.193 (0.174,0.207)	0.235 (0.211,0.254)
I(1) Matern	0.134 (0.115,0.147)	0.163 (0.149,0.175)	0.193 (0.179,0.206)
J c=0.03	0.062 (0.058,0.067)	0.062 (0.056,0.067)	0.053 (0.047,0.058)
J c = 0.50	0.088 (0.081,0.095)	0.087 (0.082,0.094)	0.070 (0.063,0.077)
Br. Sheet	0.182 (0.132,0.223)	0.198 (0.156,0.234)	0.158 (0.131,0.193)

Size: k = 5

DGP	m = 10.000	m = 20.000	m = 50.000
Levy-BM	0.084 (0.077,0.091)	0.076 (0.068,0.082)	0.048 (0.043,0.052)
I(1) c=0.01	0.091 (0.082,0.098)	0.091 (0.081,0.098)	0.062 (0.057,0.069)
I(1) c=0.03	0.104 (0.092,0.114)	0.114 (0.102,0.121)	0.080 (0.072,0.087)
I(1) Matern	0.092 (0.082,0.101)	0.098 (0.088,0.105)	0.072 (0.064,0.079)
J c=0.03	0.060 (0.055,0.064)	0.055 (0.049,0.060)	0.043 (0.039,0.047)
J c = 0.50	0.075 (0.070,0.081)	0.068 (0.060,0.072)	0.045 (0.042,0.051)
Br. Sheet	0.142 (0.115,0.169)	0.135 (0.106,0.159)	0.070 (0.063,0.085)

Average Length: k = 1

DGP	m = 10.000	m = 20.000	m = 50.000
Levy-BM	0.587 (0.575,0.599)	0.442 (0.432,0.470)	0.418 (0.379,0.479)
I(1) c=0.01	0.871 (0.851,0.899)	0.696 (0.680,0.722)	0.627 (0.583,0.707)
I(1) c=0.03	1.004 (0.976,1.046)	0.798 (0.780,0.824)	0.701 (0.656,0.789)
I(1) Matern	0.964 (0.942,0.988)	0.782 (0.762,0.807)	0.709 (0.664,0.768)
J c=0.03	0.365 (0.357,0.376)	0.330 (0.320,0.345)	0.375 (0.329,0.438)
J c = 0.50	0.550 (0.537,0.564)	0.428 (0.417,0.442)	0.407 (0.376,0.455)
Br. Sheet	0.609 (0.590,0.643)	0.468 (0.454,0.488)	0.435 (0.397,0.473)

Average Length: k = 5

DGP	m = 10.000	m = 20.000	m = 50.000
Levy-BM	0.480 (0.472,0.491)	0.411 (0.393,0.481)	0.461 (0.385,0.539)
I(1) c=0.01	0.755 (0.740,0.770)	0.647 (0.614,0.727)	0.652 (0.552,0.767)
I(1) c=0.03	0.867 (0.851,0.886)	0.730 (0.708,0.805)	0.668 (0.577,0.821)
I(1) Matern	0.883 (0.863,0.910)	0.783 (0.761,0.873)	0.780 (0.617,0.935)
J c=0.03	0.359 (0.349,0.372)	0.357 (0.338,0.448)	0.454 (0.359,0.527)
J c = 0.50	0.467 (0.457,0.477)	0.404 (0.383,0.466)	0.478 (0.396,0.571)
Br. Sheet	0.503 (0.495,0.522)	0.435 (0.413,0.531)	0.487 (0.390,0.571)

Notes: Entries show the median across spatial locations and ( ) show the 5^{th} and 95^{th} percentiles.