

Aggregate Implications of Changing Sectoral Trends*

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Abstract

We find disparate trend variation in TFP and labor growth across major U.S. production sectors over the post-WWII period. When aggregated, these sector-specific trends imply secular declines in the growth rate of aggregate labor and TFP. We embed this sectoral trend variation into a dynamic multi-sector framework in which materials and capital used in each sector are produced by other sectors. The presence of capital induces important network effects from production linkages that amplify the consequences of changing sectoral trends on GDP growth. Thus, in some sectors, changes in TFP and labor growth lead to changes in GDP growth that may be as large as three times these sectors' share in the economy. We find that trend GDP growth has declined by more than 2 percentage points since 1950, and that this decline has been primarily shaped by sector-specific rather than aggregate factors. Sustained contractions in growth specific to Construction, Nondurable Goods, and Professional and Business and Services make up close to sixty percent of the estimated trend decrease in GDP growth. In addition, the slow process of capital accumulation means that structural changes have endogenously persistent effects. We estimate that trend GDP growth will continue to decline for the next 10 years absent persistent increases in TFP and labor growth.

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1 Introduction

The U.S. economy is currently on track for the longest expansion on record in the aftermath of the Great Recession. However, it has also become evident that output has been growing conspicuously slowly during this expansion. [Fernald et al. \(2017\)](#) find that slow growth in total factor productivity (TFP) and a fall in labor force participation are the main culprits behind this weak recovery. Importantly, the authors also find that these adverse forces are mostly unrelated to the financial crisis associated with the Great Recession. [Cette et al. \(2016\)](#) suggest that a slowdown in productivity growth that began prior to the Great Recession reflects in part the fading gains from the Information Technology (IT) revolution.¹ This view is consistent with the long lags associated with the productivity effects of IT adoption found by [Basu and Fernald \(2001\)](#), and the collapse of the dot-com boom in the early 2000s. Moreover, [Decker et al. \(2016\)](#) point to a decline in business dynamism that began in the 1980s as an additional force underlying slowing economic activity.²

This paper highlights the steady decline in trend GDP growth over the post-war period, 1950 – 2016. Building on [Fernald et al. \(2017\)](#), we explore the roles played by TFP and labor inputs in explaining this secular decline, but we do so at a disaggregated sectoral level. We estimate an empirical model where, in each industry, TFP growth and labor growth have unobserved persistent and transitory components, and where each component can itself stem from either aggregate or idiosyncratic forces. The estimates reveal that trends in TFP and labor growth have steadily decreased across a majority of U.S. sectors since 1950. Interestingly, more than 2/3 of the secular decline in aggregate TFP growth results from the combination of sector-specific disturbances, thus leaving only an ancillary role for aggregate TFP. Therefore, if technical progress in general purpose technologies has helped drive trends in sectoral TFP growth, this thrust has not been shared widely enough across sectors to generate comovement in TFP growth. This finding aligns with the observation in [Decker et al. \(2016\)](#) that over the last 30 years, manufacturing has gradually shifted from producing “general purpose” technologies to producing “special purpose” technologies. Similarly to TFP growth, trend labor growth has generally been dominated by sector-specific factors, especially after 1980 and the latter part of the post-war period. The decline in trend labor growth is especially large in the Durable Goods sector, though even in that sector trend growth always remains positive. In general, we find that secular changes in TFP and labor growth have been mostly driven by sector-specific rather than common components.

We define the process of structural change in different sectors as concurrently determined by the observed low frequency behavior of TFP and labor growth in those sectors. We then embed those changes into a dynamic multi-sector framework in which materials and capital used by different sectors in the economy are produced by other sectors. We use balanced-growth accounting to determine the aggregate effects of sectoral

¹From a measurement standpoint, [Byrne et al. \(2016\)](#) also argue that the slowdown in TFP growth that preceded the last recession is not likely the result of mismeasurement of IT related goods and services. [Aghion et al. \(2017\)](#) find that the process of creative destruction does lead growth to be understated when inflation is imputed from surviving products. However, this missing growth did not accelerate much after 2005, and was roughly constant before then.

²[Fernald and Jones \(2014\)](#) more generally make the case that diminishing marginal returns to the discovery of ideas ultimately curbs economic expansion.

changes in trend rates of growth in labor and TFP. This paper, therefore, falls partially within the literature on equilibrium multi-sector models first developed by [Long and Plosser \(1983\)](#), and later [Horvath \(1998, 2000\)](#), and [Dupor \(1999\)](#). Since then, a large body of work including [Gabaix \(2011\)](#), [Foerster et al. \(2011\)](#), [Acemoglu et al. \(2012\)](#), [di Giovanni et al. \(2014\)](#), [Atalay \(2017\)](#), [Baqaee and Farhi \(2017b\)](#), [Miranda-Pinto \(2018\)](#), and others have worked out important features of those models for generating aggregate fluctuations from idiosyncratic shocks.³ In contrast to this literature, the focus herein centers on the implications of production linkages for secular dynamics and the determination of both sectoral and aggregate trend growth rates.⁴ While recent work has suggested a somewhat muted role for aggregate shocks in explaining cyclical variations in GDP growth, we now find that sector-specific disturbances also explain most of the trend variations in U.S. GDP growth.

Our paper returns to the original multi-sector model of [Long and Plosser \(1983\)](#) and maintains the original assumptions of competitive input and product markets as well as constant-returns-to-scale technologies. However, we explicitly allow different industries to produce investment goods for other industries. Unlike [Horvath \(1998\)](#) or [Dupor \(1999\)](#), capital is not constrained to be sector-specific and is allowed to depreciate only partially within the period. We assume unit elastic preferences and technologies that allow us to derive analytical expressions for the model’s sectoral and aggregate balanced growth paths. These expressions highlight how changes in trend TFP or labor growth in different sectors affect value added growth in every other sector and, therefore, GDP growth. The implied elasticities reflect induced changes in capital trend growth rates across sectors. Thus, our analysis in part extends [Greenwood et al. \(1997\)](#) to a multi-sector environment.⁵

The fact that changes in TFP or labor growth in a sector affect value added growth in every other sector hinges critically on the presence of capital. This feature of the environment leads to quantitatively important multiplier effects from sectoral linkages to GDP growth. The size of this multiplier for a sector depends on its importance as a supplier of capital or materials to other sectors. The density of production linkages more generally determines the degree to which the sectoral network propagates structural changes in a sector to the rest of the economy. The U.S. Capital Flow tables produced by the Bureau of Economic Analysis (BEA) indicate that the Construction and Durable Goods sectors produce roughly 80 percent of the capital used in almost every industry. The strength of these linkages results in GDP growth multipliers for those sectors that are almost 3 times their share in the economy. Professional and Business Services, as well as Finance, Insurance, and Real Estate (FIRE), and Wholesale Trade are also associated with relatively

³An additional dimension of this work explores the implications of frictions for aggregate fluctuations in these models, including [Bigio and La’O \(2016\)](#), [Baqaee \(2018\)](#), [Grassi \(2018\)](#), and [Baqaee and Farhi \(2017a\)](#). Other recent work has also investigated the implications of production linkages for higher order moments, for instance [Acemoglu et al. \(2017\)](#), and [Atalay et al. \(2018\)](#)

⁴[Ngai and Pissarides \(2007\)](#) provide a seminal study of balanced growth in a multi-sector environment. They consider both multiple intermediates and multiple capital-producing sectors, but not at the same time. More importantly, the analysis abstracts from pairwise linkages in both intermediates and capital-producing sectors that play a key role in this paper.

⁵[Basu et al. \(2013\)](#) also construct a multi-sector extension of the [Greenwood et al. \(1997\)](#) environment, but they work with an aggregate capital stock and an aggregate labor endowment with each factor being perfectly mobile across sectors. In contrast to this paper, the authors study short-run responses to TFP shocks.

large GDP growth multipliers because of their central role as suppliers of materials. We find that changing sectoral trends in the last 6 decades, translated through the economy’s production network, have on net lowered trend GDP growth by roughly 2.3 percentage points. Construction more than any other sector stands out by a considerable margin for its contribution to the trend decline in GDP growth since 1950, accounting for close to 1/3 of this decline. Structural changes in Professional and Business Services and Nondurable Goods together account for another 25 percent.

This paper is organized as follows. Section 2 gives an overview of the behavior of trend GDP growth over the past 60 years. Section 3 provides an empirical description of TFP and labor growth by industry that allows for persistent and transitory components, where each component itself may be driven by idiosyncratic or aggregate forces. Section 4 develops the implications of these structural changes at the sector level in the context of a dynamic multi-sector model with production linkages in materials and investment. This model serves as the balanced growth accounting framework that we use to determine the aggregate implications of changes in the sectoral trend growth rates of labor and TFP. Section 5 presents our quantitative findings. Section 6 concludes and discusses possible directions for future research.

2 The Long-Run Decline in U.S. GDP Growth

Figure 1 shows the behavior of U.S. GDP growth over the post-WWII period. Here, GDP is measured annually as the share-weighted value added from 15 sectors comprising the private U.S. economy; details are provided in the next section.

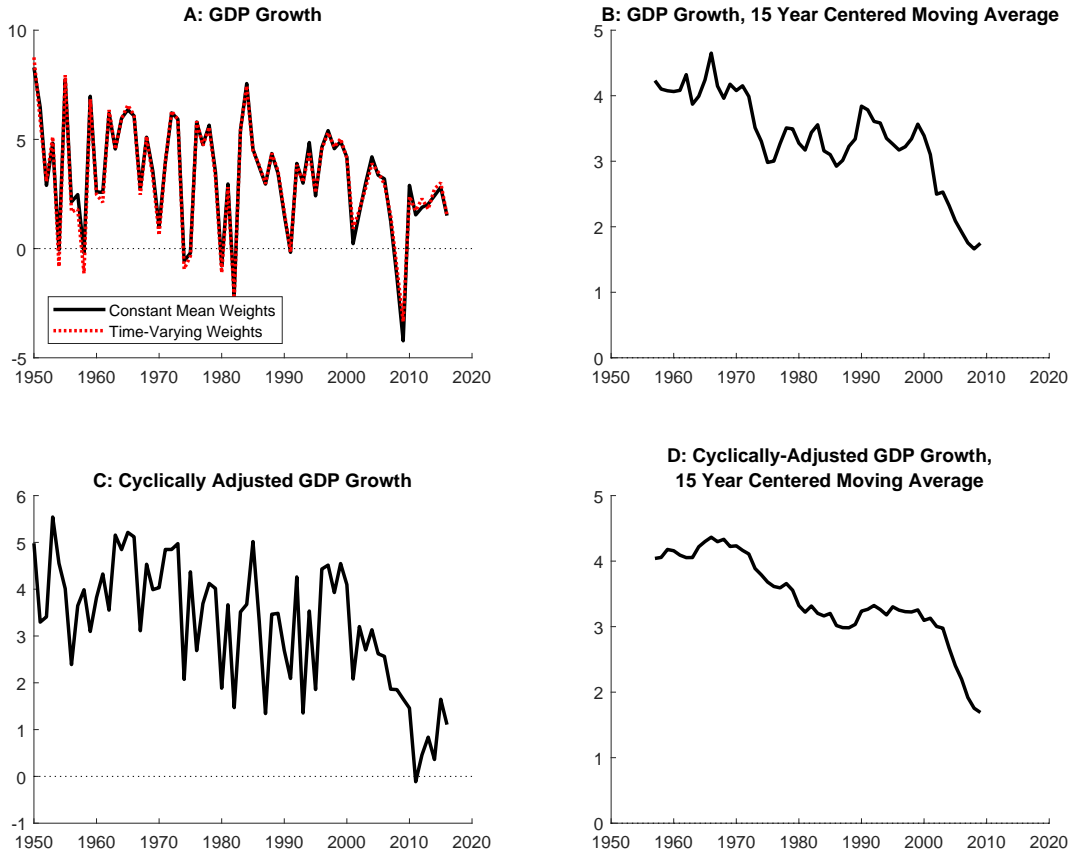
Panel A shows aggregate private-sector growth rates computed using time-varying shares (i.e., chain-weighting) and using average shares (fixed-weights), with virtually no difference between the two calculations. Panel A shows large variation in GDP growth rates – the standard deviation is 2.5 percent over the period 1950 – 2016 – but much of this variation is relatively short-lived and is associated with business cycles and other relatively transitory phenomena. Our interest here is in longer-run variation.

Panel B, therefore, plots centered 15–year moving averages of the annual growth rates. Here too there is variability. In the 1950s and early 1960s average annual growth exceeded 4 percent. This fell to 3 percent in the 1970s, rebounded to nearly 4 percent in the 1990s, but plummeted to less than 2 percent in the 2000s (See Table 1).

Panels C and D refine these calculations by eliminating the cyclical variation using an Okun’s law regression in GDP growth rates as in Fernald et al. (2017).⁶ Thus, panel C plots the residuals from a regression of GDP growth rates onto a short distributed lead and lag of changes in the unemployment rate ($\Delta u_{t+1}, \Delta u_t, \Delta u_{t-1}$). This cyclical adjustment eliminates much of the cyclical variability evident in panel A. In addition, the 15–year moving average in Panel D now produces a more focused picture of the trend variation in the growth rate of private GDP.

⁶Compared to other measures of cyclical slack or resource utilization, Fernald et al. (2017) point out that the civilian unemployment rate has two key advantages. First, it has been measured using essentially the same survey instrument since 1948. Second, changes in the unemployment rate have nearly a mean of zero over long periods.

Figure 1: US GDP growth rates (Percentage points at an annual rate)



Notes: GDP growth rates are in percentage points at an annual rate. Growth rates are share-weighted value added from 15 sectors making up the private U.S. economy. Cyclical adjustment uses a regression on leads and lags of the first-difference in the unemployment rate.

The numbers reported in Table 1 frame the key question of this paper: why did the average growth rate of GDP fall from 4 percent per year in the 1950s to just over 3 percent in the 1980s and 1990s, and then further decline precipitously in the 2000s? We look to inputs – specifically TFP and labor at the sectoral level – for the answer. That is, interpreting the data as variations around a balanced growth path, changes in GDP growth are primarily determined by changes in the growth rates of those two inputs. However, as the analysis in Section 4 makes clear, not all sectoral inputs are created equal. Some sectors have a large value-added share in GDP and also provide a large share of materials or capital to other sectors. Put another way, input variation across sectors is a particularly important driver of low frequency movements in aggregate GDP growth.

Before investigating these input-output interactions, we begin by describing the sectoral data, both how these data are measured and how sectoral value-added as well as labor and TFP inputs have evolved over

Table 1: 15-Year averages of GDP growth rates

Dates	Cyclically Adjusted	
	Growth Rates	Growth Rates
1950 – 2016	3.3	3.3
1950 – 1965	4.5	4.2
1966 – 1982	3.1	3.6
1983 – 1999	3.9	3.4
2000 – 2016	1.7	1.7

Notes: The values shown are averages of the series plotted in Figure 1, panels (A) and (C), over the periods shown.

the post-WWII period.

3 An Empirical Description of Trend Growth in TFP and Labor

We begin by estimating an empirical model of TFP and labor growth for different sectors of the U.S. economy. As a benchmark, our paper applies the insights of [Hulten \(1978\)](#) on the interpretation of aggregate productivity (TFP) changes as a weighted average of sector-specific value-added TFP changes. In particular, under constant-returns-to-scale and perfect competition in product and input markets, the sectors’ weights are the ratios of their valued added to GDP.⁷ We calculate standard TFP growth rates at the sectoral level following [Jorgenson et al. \(2017\)](#) among others, and estimate permanent and transitory components in these growth rates.

3.1 Data

Sectoral TFP growth rates are calculated using the KLEMS dataset constructed by Jorgenson and his collaborators, as well as its recently updated version in the form of the BEA’s Integrated Industry-Level Production Accounts (ILPA). These datasets are attractive for our purposes because they provide a unified approach to the construction of gross output, the primary inputs capital and labor, as well as intermediate inputs (‘materials’) for a large number of industries. The KLEMS data are based on U.S. National Income and Product Accounts (NIPA) and consistently integrate industry data with Input-Output tables and Fixed Asset tables. The KLEMS dataset contains quantity and price indices for inputs and outputs across 65 industries. The growth rate of any one industry’s aggregate is defined as a Divisia index given by the value-share weighted average of its disaggregated component growth rates. Labor input is differentiated

⁷In the absence of constant-returns-to-scale or perfect competition, [Basu and Fernald \(1997, 2001\)](#) and [Baqaee and Farhi \(2018\)](#) show that aggregate TFP changes also incorporate reallocation effects. These effects reflect the movement of inputs between low and high returns to scale sectors stemming from changes in relative sectoral TFP.

by gender, age, education, and labor status. Labor input growth is then defined as a weighted average of growth in annual hours worked across all labor types using labor compensation shares of each type as weights. Similarly, intermediate input growth reflects a weighted average of the growth rate of all intermediate inputs averaged using payments to those inputs as weights. Finally, capital input growth reflects a weighted average of growth rates across 53 capital types using payments to each type of capital as weights. Capital payments are based on implicit rental rates consistent with a user-cost-of-capital approach. Total payments to capital are the residuals after deducting payments to labor and intermediate inputs from the value of production. Put another way, there are no economic profits.

An industry’s TFP growth rate is defined in terms of its Solow residual, specifically output growth less the revenue-share weighted average of input growth rates. This calculation is consistent with the canonical theoretical framework we adopt in Section 4 where all markets operate under perfect competition and production is constant-returns-to-scale. For earlier versions of Jorgenson’s KLEMS data up to 1990, [Basu and Fernald \(1997, 2001\)](#) compute total payments to capital as the sum of rental rates implied by the user-cost-of-capital and find small industry profits on average that amount at most to three percent of gross output. In the presence of close to zero profits, elasticities to scale and markups are equivalent. [Basu and Fernald \(2001\)](#) estimate returns-to-scale across industries and find few significant deviations from constant returns or, alternatively, little evidence of markups. More recently, an active debate has emerged on the extent to which the competitive environment has changed in the U.S. over the last two decades. On the one hand, [Barkai \(2017\)](#), also applying the user-cost-of-capital framework but using post 1990 data, finds substantial profit shares over that period. Moreover, [De Loecker and Eeckhout \(2017\)](#), estimating industry production functions from corporate balance sheets, present evidence of rising markups and returns to scale since the 1980s. On the other hand, [Karabarbounis and Neiman \(2018\)](#) argue that the user-cost-of-capital framework, to the extent that it implies high profit shares starting in the 1990s, also implies unreasonably high profit shares in the 1950s. In addition, [Traina \(2018\)](#) argues that the evidence on rising markups from corporate balance sheets depends crucially on the measurement of variable costs and weights in aggregation.⁸ In this paper, we maintain the assumptions of competitive markets and constant-returns-to-scale as a benchmark from which to study the aggregate implications of sectoral changes in labor and TFP inputs.

Our calculations rely on the 2017 version of the Jorgenson KLEMS dataset which covers the period 1946 – 2014, and the ILPA KLEMS dataset which covers the period 1987 – 2016.⁹ For ease of presentation, and in order to consider the role of structural change in individual sectors separately, we carry out the

⁸Similarly, [Rossi-Hansberg et al. \(2018\)](#) show while sales concentration has unambiguously risen at the national level since the 1980s, concentration has steadily declined at the Core-Based Statistical Area, county, and ZIP code levels over the same period. While these facts can seem conflicting, the authors present evidence that large firms have become bigger through the opening of more establishments or stores in new local markets, but this process has lowered concentration in those markets.

⁹The Jorgenson dataset is downloaded from <http://www.worldklems.net/data.htm> and the IPPA is downloaded from <https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems>. A detailed description of the Jorgenson data can be found in [Jorgenson et al. \(2014\)](#), as well as [Jorgenson et al. \(2017\)](#), and the BEA dataset is described in [Fleck et al. \(2014\)](#).

empirical analysis using private industries at the two-digit level. In particular, we aggregate the 65 industries included in the two KLEMS datasets into 15 two-digit private industries following the procedure in [Hulten \(1978\)](#).¹⁰ Another advantage of the aggregation into two-digit industries is that any differences between the two KLEMS datasets are attenuated and we feel comfortable splicing the levels of the two datasets in 1987.¹¹ That is, we use the growth rates calculated using the Jorgenson KLEMS data before 1987 and using ILPA data after that date.

Finally, we note that the KLEMS data rely on U.S. NIPA measures for fixed assets. Specifically, these measures are based in part on estimates of capital goods prices. To the degree that these prices perennially understate quality growth in capital goods, then KLEMS data understate productivity improvements in the investment goods sector.¹² If capital accumulation is an important driver of growth, our results would then provide a lower bound for the growth contributions of the capital goods producing sectors.

Table 2 lists the 15 sectors we consider. For each sector, the table shows average cyclically adjusted growth rates of value-added, labor, and TFP over 1950 – 2016, and it also shows their average shares in aggregate value-added and labor input. The aggregate growth rates in the bottom row are the value-weighted averages of the sectoral growth rates with average value added and labor shares used as fixed weights.

Clearly sectors grow at different rates and this disparity is hidden in studies that only consider aggregates. Average real value added growth rates range from 1.1 percent in Mining to 4.7 percent in Wholesale Trade, bracketing the aggregate value added growth rate of 3.3 percent. With the exception of the Durable Goods sector, most sectors with growth rates that exceed the aggregate growth rate provide services. Similarly, labor input growth rates range from a negative 1.4 percent in Agriculture to 3.3 percent in Professional and Business Services, bracketing the average aggregate growth rate of 1.5 percent. Again, most sectors with labor input growth rates that exceed the aggregate growth rate provide services. Finally, TFP growth rates range from -0.7 percent in Utilities to 3.2 percent in Agriculture, bracketing the average aggregate TFP growth rate of 0.7 percent. Sectoral TFP growth rates are less aligned with either value added or labor input growth rates. There are three sectors with notable TFP declines, namely Utilities, Other Services, and Construction, as well as a number of sectors with stagnant TFP levels. Negative TFP growth rates are a counter-intuitive but well known feature of disaggregated industry data. To the degree that they occur in service industries, they are in part attributed to measurement issues with respect to output.

To a first approximation, the contributions of the different sectors to aggregate outcomes are given by the nominal value added and labor input shares in the last two columns. The two largest contributors to value added and TFP are Durable Goods and Finance, Insurance, and Real Estate (FIRE), where the latter reflects the large contribution of the service flow from residential housing. The two largest contributors

¹⁰ The procedure is described in detail in the online-only Technical Appendix and Supplementary Material to this paper, Foerster, Hornstein, Sarte, Watson (2019).

¹¹ While the ILPA builds on the Jorgenson KLEMS data, the two datasets are not exactly identical for the time period in which they overlap. Since both datasets are constructed to be consistent with the BEA’s input-output tables, they mostly agree on industry details and both cover the same 65 private industries. Nevertheless, there remain differences but these are reflected mostly in the levels of variables and not their growth rates.

¹² See [Gordon \(1990\)](#) or [Cummins and Violante \(2002\)](#).

Table 2: 15 Sector Decomposition of the U.S. Private Economy
(1950-2016)

Sectors	Average Growth Rate Cyclically Adjusted Data (Percentage points at an annual rate)			Average Share (Percentage points)	
	Value Added	Labor	TFP	Value Added	Labor
1 Agriculture	2.63	-1.42	3.16	2.70	3.25
2 Mining	1.06	0.27	-0.06	2.15	1.66
3 Utilities	1.87	0.98	-0.74	2.42	1.05
4 Construction	1.67	1.60	-0.10	5.04	7.64
5 Durable Goods	3.36	0.42	1.88	13.48	15.59
6 Nondurable Goods	2.29	0.07	0.79	9.31	8.84
7 Wholesale Trade	4.65	1.68	1.13	6.92	6.50
8 Retail Trade	3.16	1.08	1.07	8.44	9.86
9 Trans. & Ware.	2.43	0.85	1.35	4.19	4.91
10 Information	4.58	1.30	0.86	4.90	3.68
11 FIRE	3.76	2.73	-0.03	19.18	7.66
12 PBS	4.36	3.26	0.30	8.59	10.88
13 Educ. & Health	3.32	2.75	0.04	6.01	9.28
14 Ent. & FS	2.44	2.00	0.22	3.70	4.67
15 Oth. Serv.	2.02	2.38	-0.60	2.98	4.50
Aggregate	3.25	1.49	0.67	100	100

Notes: The values shown are average annual growth rates for the 15 sectors. The row labelled “Aggregate” is the share-weighted average of the 15 sectors.

to labor payments are Durable Goods and Professional and Business Services. Over time, the shares of goods-producing sectors has declined while the shares of services-producing sectors has increased. However, despite these changes, aggregating sectoral outputs and inputs using constant mean shares, as opposed to time-varying shares, has little effect on the measurement of aggregate outputs and inputs (Figure 1A).

3.2 Aggregate Balanced Growth Implications in a Canonical Model without Linkages

Before describing the secular evolution of sectoral labor growth and TFP growth in more detail, we briefly consider the implications of the long-run averages shown in Table 2 through the lens of the standard one-sector growth model. In particular, let $\overline{\Delta \ln z}$ denote the average growth rate of aggregate TFP from 1950 to

2016, $\overline{\Delta \ln z} = \sum_{j=1}^n s_j^v \overline{\Delta \ln z_j}$, where n , s_j^v , and $\overline{\Delta \ln z_j}$ denote respectively the number of sectors, constant mean shares of sectoral value added in GDP, and the average growth rates of sectoral TFP that are shown in Table 2. Similarly, let $\overline{\Delta \ln \ell}$ represent the post-war average growth rate of aggregate labor. Therefore, $\overline{\Delta \ln \ell} = \sum_{j=1}^n s_j^\ell \overline{\Delta \ln \ell_j}$ where s_j^ℓ and $\overline{\Delta \ln \ell_j}$ represent respectively average sectoral labor shares and average sectoral labor growth rates in Table 2. Suppose that the economy admits an aggregate production function such that at any date t , $V_t = z_t k_t^\alpha \ell_t^{1-\alpha}$, where V_t is aggregate value added or GDP and k_t represents aggregate capital. Then, along a balanced growth path, the capital-output ratio is constant and

$$\Delta \ln V = \frac{1}{1-\alpha} \Delta \ln z + \Delta \ln \ell. \quad (1)$$

Over the period 1950–2016, $\overline{\Delta \ln z}$ is 0.67 percent while $\overline{\Delta \ln \ell}$ is 1.49 percent in Table 2. Assuming a share of aggregate labor in GDP, $1-\alpha$, of 2/3, equation (1) then implies that GDP would have grown by around 2.49 percent on average over the same period. In other words, the predicted growth rate from equation (1) falls short of actual average GDP growth in Table 2 by more than 3/4 of a percentage point. Motivated in part by this discrepancy, we explore and highlight below the key role of an economy’s sectoral network in determining its aggregate growth rate. In particular, we show that linkages between sectors give rise to powerful sectoral multipliers that amplify the role of idiosyncratic structural changes in the economy. This effect also accounts for much of the inconsistency between the long-run growth rate of GDP implied by equation (1) and that in Table 2.

3.3 Empirical Framework

Let $\Delta \ln \tilde{x}_{j,t}$ denote the growth rate ($100 \times$ the first difference of the logarithm) of annual measurements of labor or TFP in sector j at time t . These sectoral growth rates are volatile and, in many sectors, much of the variability is associated with the business cycle. Our interest is in trend (i.e., low-frequency) variation, which is more easily measured after cyclically adjusting the raw growth rates. Thus, as with the cyclically adjusted measure of GDP shown in Figure 1, we follow Fernald et al. (2017) and cyclically adjust these growth rates using the change in the unemployment rate, Δu_t , as a measure of cyclical resource utilization. That is, we estimate

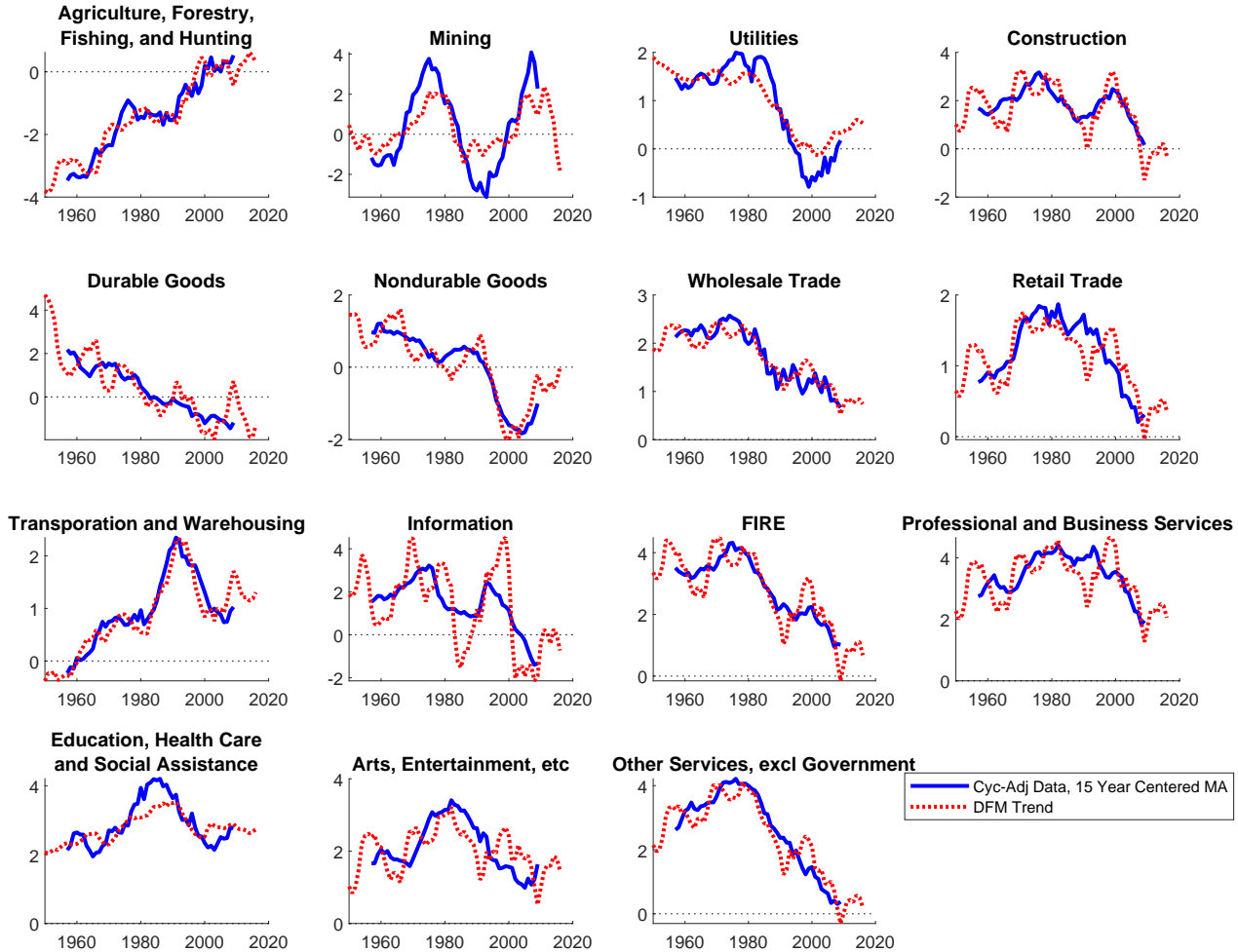
$$\Delta \ln \tilde{x}_{j,t} = \mu_j + \beta_j(L) \Delta u_t + e_{j,t}, \quad (2)$$

where $\beta_j(L) = \beta_{j,1}L + \beta_{j,0} + \beta_{j,-1}L^{-1}$ and the leads and lags of Δu_t captures much of the business-cycle variability in the the data. Throughout the remainder of the paper, we use $\Delta \ln x_{j,t} = \Delta \ln \tilde{x}_{j,t} - \hat{\beta}_j(L) \Delta u_t$, where $\hat{\beta}_j(L)$ denotes the OLS estimator, and where $x_{j,t}$ represents the implied cyclically adjusted value of sector TFP (denoted $z_{j,t}$) or labor input (denoted $\ell_{j,t}$) growth rates.¹³

Figures 2 and 3 plot centered 15–year moving averages of the cyclically adjusted growth rates of labor

¹³Other measures of variable utilization are estimated in Kimball et al. (2006). These are generally found to be stationary so that any differences across measures of utilization will likely affect the transitory components of TFP or labor rather than their trends.

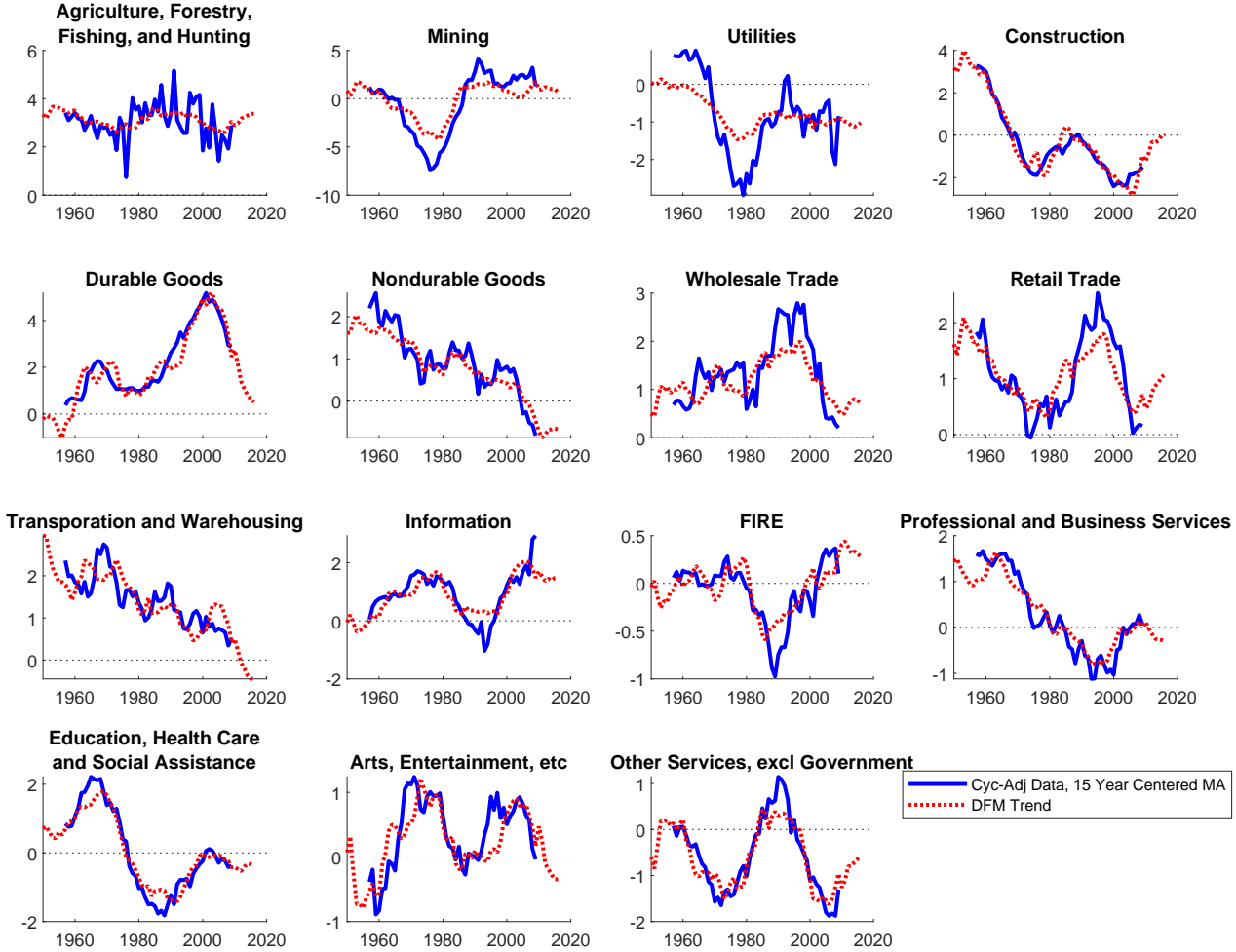
Figure 2: Trend growth rates in labor by sector (Percentage points at an annual rate)



Notes: The solid line is the centered 15-year moving average of the annual rates of growth of labor in each of the sectors shown. The dotted line is the estimated trend component (common + idiosyncratic) for sectoral growth rates estimated from the DFM.

and TFP. These are shown as the thick blue lines in the figures (ignore the thin dotted red line for now). The disparity in experiences across different sectors stands out. In particular, the moving averages show large sector-specific variation through time. For example, labor input was contracting at nearly 4 percent per year in agriculture in the 1950s, but stabilized near the end of the sample. In contrast, labor input in the Durables and Nondurable goods sectors was increasing in the 1950s, but has been contracting since the mid-1980s. At the same time, the rate of growth of labor in several service sectors are shown to exhibit large ups and downs over the sample. Looking at TFP, there are important differences across sectors as well. In Sections 4 and 5, we quantify the aggregate implications of these sectoral variations in labor and

Figure 3: Trend growth rates in TFP by sector (Percentage points at an annual rate)



Notes: See notes to Figure 2.

TFP inputs.

In the economic model presented in Section 4, we treat $z_{j,t}$ and $\ell_{j,t}$ as exogenous processes and study the implied values of output and value-added that arise from realizations of these processes. We do so in a dynamic model that features input-output and capital flow linkages between the sectors. This multi-sector accounting exercise requires joint stochastic processes for the sectoral values of $z_{j,t}$ and $\ell_{j,t}$. For this purpose, we use a reduced-form statistical model that captures the salient cross-sectional and dynamic correlations in the data. Cross-correlations and autocorrelations summarized in the Technical Appendix suggest a reduced-form model with three features. The sectoral growth rates of labor ($\Delta\ell_{j,t}$) are correlated across sectors; there is also cross-sector correlation in the sectoral TFP growth rates ($\Delta z_{j,t}$). Second, both

$\Delta \ln \ell_{j,t}$ and $\Delta \ln z_{j,t}$ exhibit substantial year-to-year variation around slowly varying levels. Finally, labor and TFP growth rates are only weakly correlated within or across sectors, that is $cor(\Delta \ell_{j,t}, \Delta z_{k,t})$ is small for all j and k .

These features lead us to consider independent stochastic processes for $\Delta \ln \ell_{j,t}$ and $\Delta \ln z_{j,t}$, where the processes have a structure that includes factors common to all sectors together with sector-specific factors, and where these factors include both slowly-varying level terms (modeled as martingales) and terms capturing more transitory variation (modeled as white noise). Specifically, we consider a dynamic factor model (DFM) of the form,

$$\Delta \ln x_{j,t} = \lambda_{j,\tau}^x \tau_{c,t}^x + \lambda_{j,\varepsilon}^x \varepsilon_{c,t}^x + \tau_{j,t}^x + \varepsilon_{j,t}^x, \quad (3)$$

where $x = z$ or ℓ and $(\Delta \tau_{c,t}^x, \varepsilon_{c,t}^x, \{\Delta \tau_{j,t}^x, \varepsilon_{j,t}^x\}_{j=1}^n)$ are *i.i.d.* Gaussian random variables with mean zero and variable-specific variances. The τ -terms are random walks and describe the slowly varying (or ‘local’) levels in the growth rate of $x_{j,t}$. Some of this variation is common, through $\tau_{c,t}^x$, and some is sector-specific, through $\tau_{j,t}^x$. Deviations of the data from their local levels are represented by the ε -terms, part of which is common, $\varepsilon_{c,t}^x$, and part of which is sector-specific, $\varepsilon_{j,t}^x$.

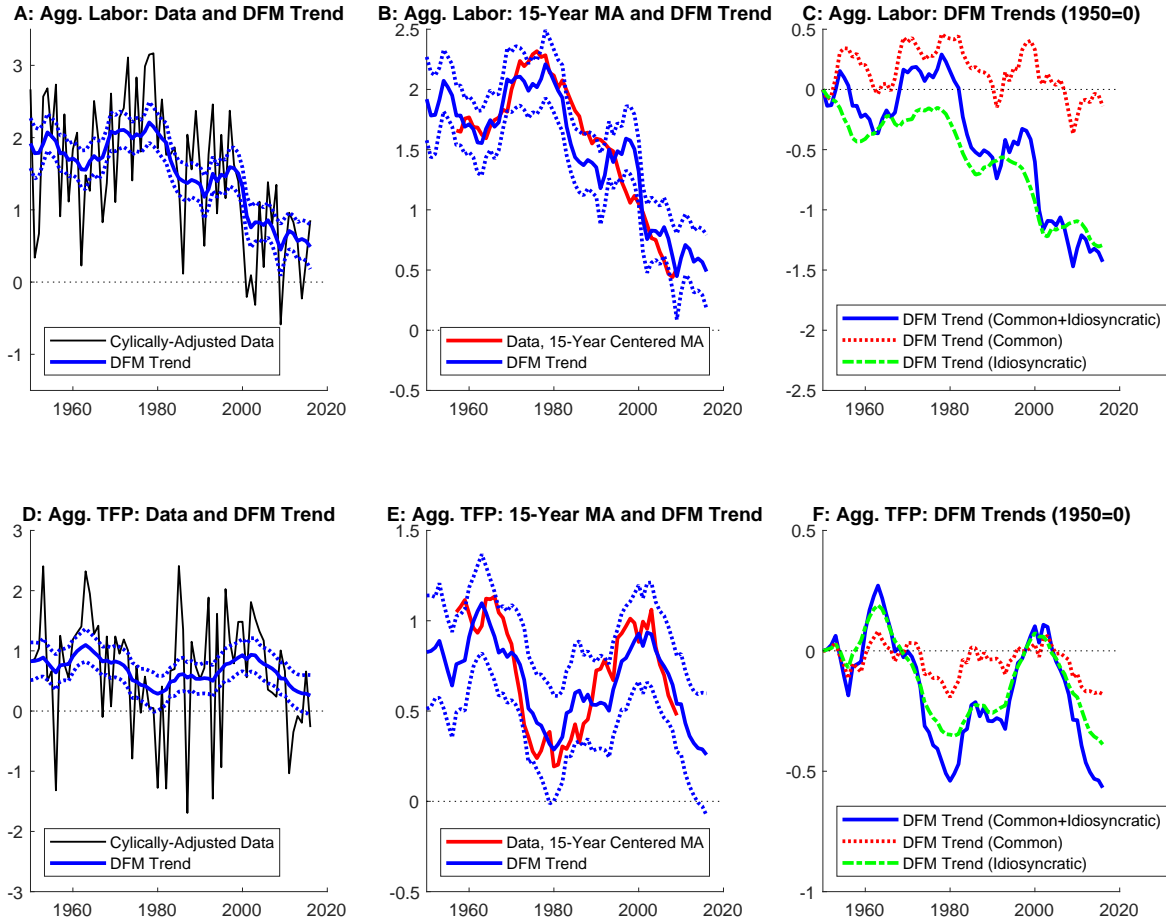
The sectoral model produces aggregate versions of ℓ_t and z_t that also have random-walk-plus-white-noise representations. In particular, let s_j denote the (time-invariant) share weight of sector j . The aggregate value of x_t then satisfies $\Delta \ln x_t = \sum_{j=1}^n s_j \Delta \ln x_{j,t} = \tau_t + \varepsilon_t$. Here, $\tau_t = \tau_{c,t}^x \sum_{j=1}^n s_j \lambda_{j,\tau}^x + \sum_{j=1}^n s_j \tau_{j,t}^x$ is a random walk that represents the ‘local’ level of the aggregate growth rate and ε_t , defined analogously, is white noise.

While the empirical model has a simple dynamic and cross-sectional structure, it fits the sectoral labor and TFP data well (details are provided in the Technical Appendix) and versions of the model have proved useful in describing sectoral data in other contexts (cf. [Stock and Watson \(2016\)](#)). The dynamic factor model is estimated using Bayesian methods together with a Gaussian likelihood for the various shocks. Priors are standard (normal priors for the λ -coefficients and inverse gamma priors for the variances). The scale of the common factors and the λ coefficients are not separately identified; we impose a normalization where $\Delta \tau_{c,t}^x$ and $\varepsilon_{c,t}^x$ have unit variance and the average value of λ is non-negative. The priors for the variance of the idiosyncratic terms are reasonably uninformative but we use more informative priors for λ . Details are provided in Appendix A.

3.4 Estimated Sectoral and Aggregate Trend Growth Rates in Labor and TFP

Appendix A and The Technical Appendix contain details of the estimation method and results for the empirical models. For our purposes, the key results are summarized in three figures and a table. Figures 2 and 3, introduced earlier, show the composite estimated trend component ($\lambda_{j,\tau}^x \tau_{c,t}^x + \tau_{j,t}^x$) as the red dotted line along with the 15-year moving averages of the cyclically adjusted growth rates. While the estimated trends from the dynamic factor model closely track the 15-year moving averages for most of the sectors, these trends now also allow for a decomposition into common and sector-specific components. Table 3 shows

Figure 4: Aggregate trend growth rates in labor and TFP (Percentage points at an annual rate)



Notes: The dotted lines in panels (A), (B), (D), and (E) are 68 percent credible intervals for the DFM trends. In panels (C) and (F) the estimated DFM trends are normalized to 0 in 1950.

the changes in trend growth for labor and TFP over different periods, as well as the decomposition of these changes into various components. In the table, common and sector-specific changes in trend growth rates add up to the aggregate change.

Figure 4 plots the aggregate values of the growth rates of labor and TFP along with their estimated trends from the sectoral empirical model. Panels A and D show the growth rates and the estimates of τ ; panels B and E show the 15-year moving averages of the data along with the estimate of τ and associated 68 percent credible intervals; and panels C and F decompose the estimate of τ into its common component, $\sum_{j=1}^n s_j \lambda_{j,\tau}^x \times \tau_{c,t}^x$, and its sector-specific component, $\sum_{j=1}^n s_j \tau_{j,t}^x$.¹⁴ As with the sectoral data, the implied

¹⁴Share weights for labor and TFP use labor compensation and value added weights respectively. The initial values for the common and sector-specific trend values are not separately identified - the data is only informative about their sum - so that

aggregate trends estimated from the dynamic factor model closely track the low-frequency movements in the aggregate data.

Panels A and B include error bands (68 percent posterior credible intervals) computed from the dynamic factor model. The width of these error bands (approximately 0.50 percentage points) highlights the inherent uncertainty in estimating the level of time series from noisy observations. This uncertainty carries over to the structural exercise in Section 4 and is amplified by uncertainty concerning the economic model postulated in that exercise, its calibrated parameters, as well as the quality of the data. However, to the degree that our sectoral trends estimates mimic the behavior of 15-year moving averages, the economic model with parameters informed by BEA estimates traces out how these trends propagate to the rest of the economy.

Panels C and F suggest that much of the low-frequency variation in aggregate labor and TFP, as identified by the dynamic factor model, is associated with sector-specific rather than common trends. In particular, only about 1/3 of the trend decline in TFP growth is the result of shocks common to all sectors, and only about 10 percent of the trend decline in labor is attributable to common shocks.

Panel C shows that aggregate trend labor growth fell by around 1.4 percentage points between 1950 and 2016. It also shows considerable variation over this period. In particular, from 1950 to 1980, the trend growth rate of aggregate labor increases as the common component of the trend more than offsets the decline in its sector-specific component. This early period coincides with the entrance of the Baby Boomers into the labor force, which is consistent with the idea of a common demographic change that is distributed among the different sectors. However, starting in 1980, the last of the Baby Boomers (those born in 1964) turn 16 years old and have become part of the labor force. The decline in the sector-specific component of trend labor growth now begins to dominate the aggregate trend. One interpretation of this finding is that in the latter period, idiosyncratic changes in the demand for labor absorb most of the demographic forces that now push towards a declining trend growth rate.¹⁵ Under this interpretation, shocks to the idiosyncratic factors are not strictly sector-specific – for example, some workers being hired at a lower rate because of ‘sector-specific’ forces in the Durable Goods sector presumably have labor opportunities in other sectors – but rather the dominant feature of these shocks is that they appear in a specific sector.¹⁶

Panel F focuses on TFP and displays considerable swings in the trend of aggregate TFP growth between 1950 and 2016, with long stretches of rising and falling growth over different decades. It also shows that through these swings, trend TFP growth has fallen by approximately 0.6 percentage points in the last 6 and a half decades. Furthermore, Panel F reveals that the secular behavior of aggregate TFP growth reflects to a large degree the (weighted) sum of idiosyncratic TFP trends rather than an aggregate common trend. By 2016, more than 2/3 of the secular decline in TFP growth is accounted for by its sector-specific component.

Panels C and F normalize the initial values in 1950 to be zero.

¹⁵Aside from a U.S. population that begins to age in 1980, the participation rate of prime-age working males sees a steady decline over the entire post-war period while that of females rises until 1999 and then begins to fall.

¹⁶As measured in KLEMS, sector-specific labor represents an aggregate of a variety of labor types (i.e. gender, age, education, labor status, etc.) with different sectors employing different compositions of labor types. Thus, there likely is a limit to how substitutable different types of labor are across sectors. This limit means that declining trend labor growth in specific sectors also reflects idiosyncratic changes in the composition of labor.

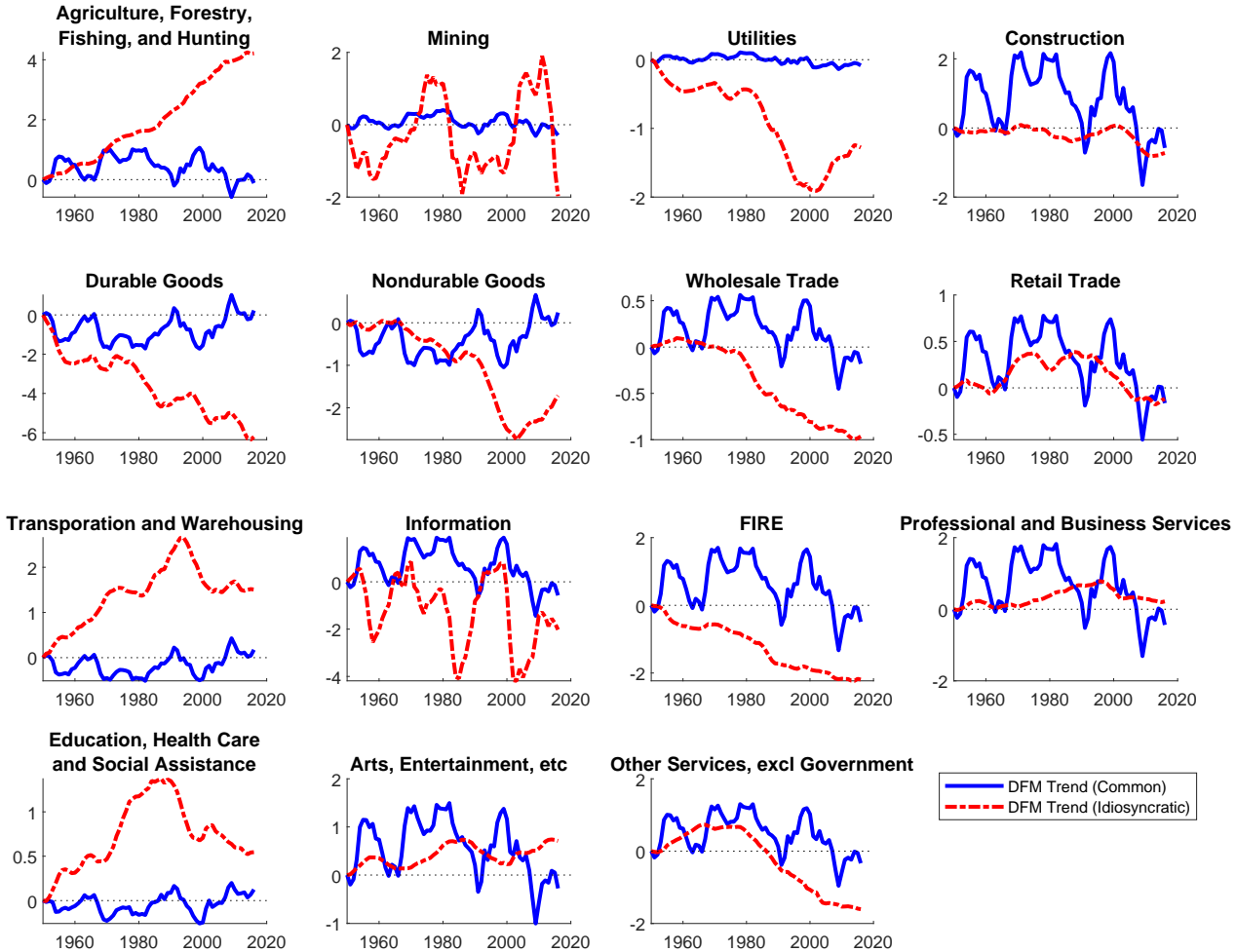
Table 3: Changes in Trend Value of Labor and TFP Growth Rates

	1950 – 2016		1950 – 1982		1982 – 1999		1999 – 2016	
	Labor	TFP	Labor	TFP	Labor	TFP	Labor	TFP
Aggregate	-1.43	-0.57	0.01	-0.47	-0.43	0.53	-1.02	-0.63
Common	-0.14	-0.18	0.44	-0.12	-0.04	0.14	-0.54	-0.19
Sector specific (total)	-1.29	-0.39	-0.43	-0.35	-0.39	0.39	-0.47	-0.44
<i>Sector specific (by sector)</i>								
Agriculture	0.14	-0.00	0.05	-0.00	0.05	-0.00	0.03	0.00
Mining	-0.03	0.00	0.00	-0.05	-0.02	0.06	-0.01	-0.01
Utilities	-0.01	-0.02	-0.01	-0.02	-0.01	0.01	0.01	-0.00
Construction	-0.06	-0.15	-0.02	-0.17	0.02	-0.05	-0.06	0.07
Durable Goods	-0.97	0.18	-0.54	0.20	-0.18	0.39	-0.26	-0.42
Nondurable Goods	-0.15	-0.19	-0.07	-0.05	-0.14	-0.06	0.06	-0.08
Whl. Trade	-0.06	0.02	-0.02	0.05	-0.03	0.00	-0.01	-0.04
Ret. Trade	-0.01	-0.04	0.02	-0.05	-0.01	0.04	-0.03	-0.03
Trans. & Ware.	0.07	-0.11	0.07	-0.06	0.02	-0.03	-0.02	-0.02
Information	-0.07	0.07	-0.05	0.03	0.08	0.00	-0.10	0.04
FIRE	-0.17	0.04	-0.08	-0.04	-0.06	0.05	-0.03	0.03
PBS	0.02	-0.12	0.05	-0.11	0.01	-0.04	-0.04	0.04
Ed. & Health	0.05	-0.06	0.12	-0.10	-0.05	0.04	-0.01	0.01
Ent. & FS	0.03	-0.00	0.03	0.01	-0.02	0.00	0.02	-0.01
Oth. Serv.	-0.07	-0.00	0.02	0.01	-0.06	-0.01	-0.03	-0.00

Notes: This table shows the change in the DFM trend growth rates over the period shown. For example, the first column shows the DFM estimates of $\tau_{2016} - \tau_{1950}$. The first row shows the results for the share-weighted aggregate; the following two rows decompose this aggregate change into the component associated with the common τ and the sector-specific τ 's, which is further decomposed by sector in the remaining rows.

This finding suggests that technical progress in general purpose technologies (e.g. personal computers, information technology, nanotechnology, etc.), to the extent that it has affected trends in sectoral TFP growth, has not generated pronounced comovement in TFP growth across sectors. In particular, the rapid technical progress associated with the 1990s IT boom was not accompanied by an increase in the common trend TFP growth rate. The finding is also consistent with [Decker et al. \(2016\)](#) who document a shift away from the production of “general purpose” technologies in manufacturing towards technologies that meet

Figure 5: Idiosyncratic and Common Labor Trends by Sector from DFM



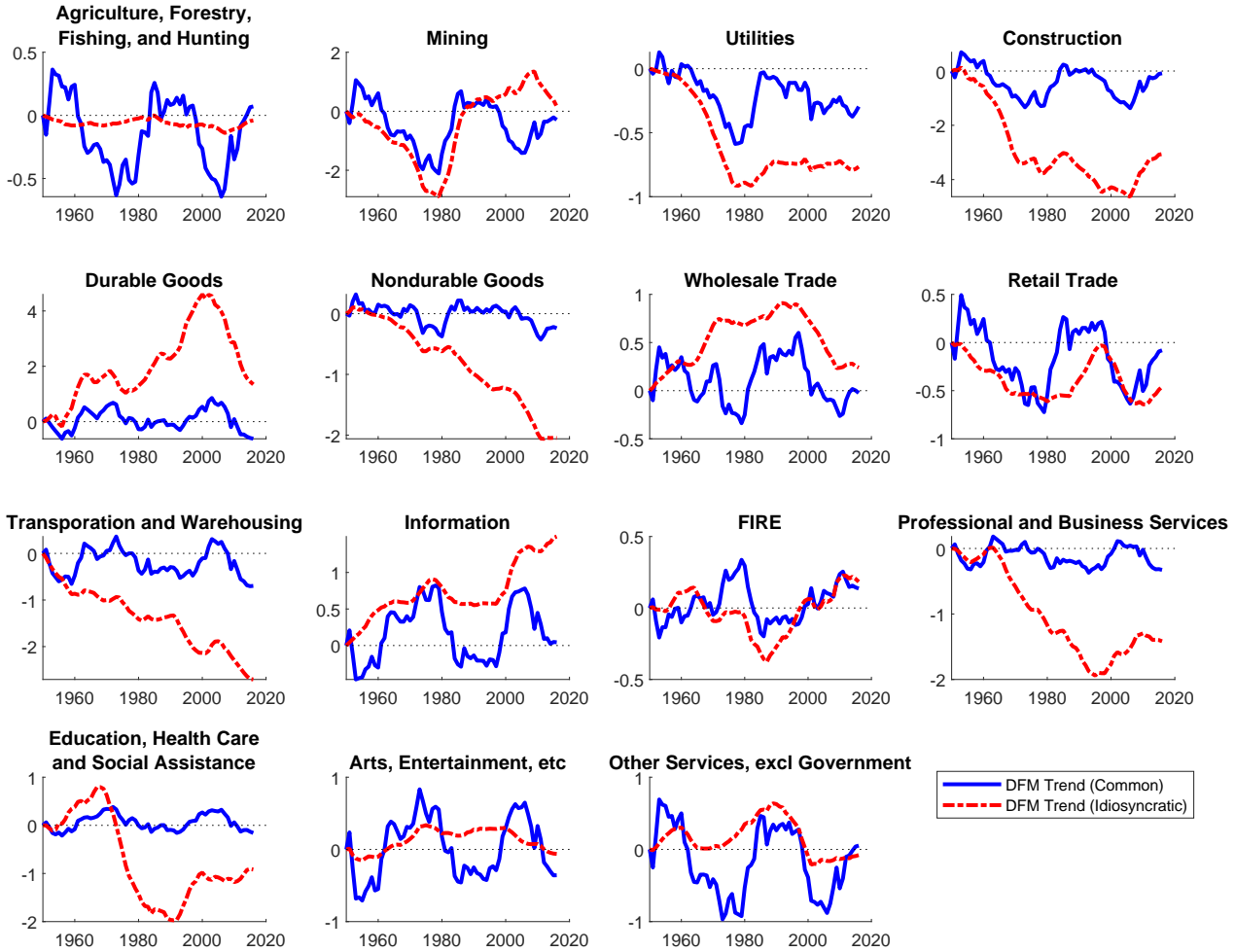
Notes: The figure plots the DFM common and idiosyncratic trend estimates for each sector. The trends are normalized to equal 0 in 1950.

more specific or idiosyncratic purposes.¹⁷

Figures 5 and 6 illustrate the estimated common and sector-specific trends in the growth rates of labor and TFP for each sector. Table 3 highlights selected values of the changes in the trends plotted in the figures. From the beginning of the sample in 1950 until the end of the sample in 2016, the annual trend rate of growth of aggregate labor fell from 1.92 percent to 0.49 percent, a decline of 1.43 percentage points. Much of this decline (1 percentage point) occurred between 1999 and the end of the sample. The dynamic factor model attributes most of the full-sample decline (1.29 percentage points) to sector-specific factors that

¹⁷As shown below, idiosyncratic trends in sectoral TFP growth can still move together in subsets of sectors, even if not across all sectors, over different time periods. The sectoral composition of these subsets changes over time.

Figure 6: Idiosyncratic and Common TFP Trends by Sector from DFM



Notes: See notes to Figure 5.

themselves primarily reflect labor growth declines in the Durable Goods sector (0.97 percentage points). As shown in Figure 5, the secular decline in labor growth in Durable Goods has been large (almost 6 percentage points relative to 1950) and steady throughout the period. Overall, variations in sector-specific trends, $\tau_{j,t}^\ell$, tend to be larger than those in common trends. This result underscores a diversity of sectoral experiences at secular frequencies that stays otherwise hidden in analyses of long-run trends that rely solely on aggregate data.

Similarly, the annual trend growth rate of aggregate TFP fell from 0.83 percent to 0.26 percent over the course of the sample, a decline of 0.57 percentage points. As we saw in Figure 4, and further underscored in

Table 3, this decline was not monotonic: trend annual growth fell by half a percentage point over the period 1950 – 1982, then rebounded over the period 1982 – 1999, before falling again from 1999 to 2016. Over the entire post-war period, roughly a third of the decline is common to all sectors, and the largest sector-specific declines were in Construction (0.15 percentage points, primarily in the first half of the sample) and Nondurable Goods (a near steady decline of 0.19 percentage points over the entire sample period). Remarkably, Figure 6 shows that in the period from 1950 – 1999, sector-specific TFP in Durables led to an increase in aggregate TFP growth (0.39 percentage point in Table 3) that largely offset the decrease in several other sectors including Construction, Nondurable Goods, Transportation and Warehousing, as well as Professional and Business Services. However, since 1999, trend TFP growth in Durable Goods has fallen from 4.2 to 1.1 percent per year (or 3.1 percentage points) and by itself accounts for 0.42 percentage points of the 0.44 decline in aggregate TFP.¹⁸

To assess the implications of the sectoral changes highlighted in this section for the secular behavior of GDP growth, one needs to be explicit about how secular change in one sector potentially impacts other sectors. Put another way, one needs to account for the fact that sectors interact through various input-output and capital flow relationships. We show in the next section that, in the presence of capital accumulation, production linkages between sectors can significantly amplify the effects of structural change in a sector on GDP growth.¹⁹

4 Changing Sectoral Trends and the Aggregate Economy

This section explores how the process of structural change in individual sectors, here captured by the behavior of sectoral TFP and labor growth, shapes the behavior of GDP growth. Consistent with our TFP calculations in Section 3, we consider a canonical multi-sector growth model with competitive product and input markets. Each sector uses materials and capital produced in other sectors, and we allow for less than full depreciation of capital within the period.

The empirical specification in Section 3 leads us to distinguish between persistent and transitory sector-level changes that can arise from either aggregate or idiosyncratic forces. We consider preferences and technologies that are unit elastic in which case the economy evolves along a balanced growth path in the long run. Capital accumulation, however, allows for variations in output growth off that balanced growth path. Given linkages across sectors, structural change in an individual sector affects not only its own value added growth but also that of all other sectors. In particular, capital induces network effects that amplify the effects of sector-specific changes on GDP growth and that we summarize in terms of sectoral multipliers. We show that the approximation in equation (1) holds sector by sector in the special case where both materials and capital are sector-specific. In contrast, the actual structure of the U.S. economy implies a

¹⁸See [Oliner et al. \(2007\)](#) for the role of technological improvements in the IT sector as a driver of TFP growth in the Durable Goods.

¹⁹See [Greenwood et al. \(1997\)](#) for the importance of TFP growth in investment goods producing sectors as a driver of aggregate growth.

balanced growth equation for GDP that is markedly different, and considerably more nuanced, than the simple relationship in equation (1).

4.1 Economic Environment

Consider an economy with n distinct sectors of production indexed by j (or i). A representative household derives utility from these n goods according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=1}^n \left(\frac{c_{j,t}}{\theta_j} \right)^{\theta_j}, \quad \sum_{j=0}^n \theta_j = 1, \quad \theta_j \geq 0,$$

where θ_j is the household's expenditure share on final good j .²⁰

Each sector produces a quantity, $y_{j,t}$, of good j at date t , using a value added aggregate, $v_{j,t}$, and a materials aggregate, $m_{j,t}$, using the technology,

$$y_{j,t} = \left(\frac{v_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{m_{j,t}}{1 - \gamma_j} \right)^{(1-\gamma_j)}, \quad \gamma_j \in [0, 1].$$

The quantity of materials aggregate, $m_{j,t}$, used in sector j is produced with the technology,

$$m_{j,t} = \prod_{i=1}^n \left(\frac{m_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}}, \quad \sum_{i=1}^n \phi_{ij} = 1, \quad \phi_{ij} \geq 0, \quad (4)$$

where $m_{ij,t}$ denotes materials purchased from sector i by sector j . The notion that every sector potentially uses materials from every other sector introduces a first source of interconnectedness in the economy. An input-output (IO) matrix is an $n \times n$ matrix Φ with typical element ϕ_{ij} . The columns of Φ add up to the degree of returns to scale in materials for each sector, in this case unity. The row sums of Φ summarize the importance of each sector as a supplier of materials to all other sectors. Thus, the rows and columns of Φ reflect “sell to” and “buy from” shares, respectively, for each sector.

The value added aggregate, $v_{j,t}$, used in sector j is produced using capital, $k_{j,t}$, and labor, $\ell_{j,t}$, according to

$$v_{j,t} = z_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j} \left(\frac{\ell_{j,t}}{1 - \alpha_j} \right)^{1-\alpha_j}, \quad \alpha_j \in [0, 1].$$

Capital accumulation in each sector follows

²⁰Here, the representative household is assumed to have full information with respect to transitory and permanent changes, as well as between idiosyncratic and common changes, as described in Section 3. In the Technical Appendix, we also consider an imperfect information case in which the representative household cannot distinguish between permanent and transitory components of exogenous changes to the environment. In this alternative scenario, the household faces an additional filtering problem in which it must infer estimates of these components in deciding how much to consume and save in the face of exogenous disturbances.

$$k_{j,t+1} = x_{j,t} + (1 - \delta_j)k_{j,t},$$

where $x_{j,t}$ represents investment in new capital in sector j , and $\delta_j \in (0, 1)$ is the depreciation rate specific to that sector. Investment in each sector j is produced using the quantity, $x_{ij,t}$, of sector i goods by way of the technology,

$$x_{j,t} = \prod_{i=1}^n \left(\frac{x_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}}, \quad \sum_{i=1}^n \omega_{ij} = 1, \quad \omega_{ij} \geq 0. \quad (5)$$

Thus, there exists a second source of interconnectedness in this economy in that new capital goods in every sector are potentially produced using the output of other sectors. Similarly to the IO matrix, a Capital Flow matrix is an $n \times n$ matrix Ω with typical element ω_{ij} . The columns of Ω add up to the degree of returns to scale in investment for each sector or 1 in this case. The row sums of Ω indicate the importance of each sector as a supplier of new capital to all other sectors.

Structural change in each sector j is captured by the composite variable, $A_{j,t}$, which reflects the joint behavior of TFP and labor growth. In particular, under the maintained assumptions, sectoral value added may be expressed alternatively as

$$v_{j,t} = A_{j,t} \left(\frac{k_{j,t}}{\alpha_j} \right)^{\alpha_j},$$

where

$$\Delta \ln A_{j,t} = \Delta \ln z_{j,t} + (1 - \alpha_j) \Delta \ln \ell_{j,t}. \quad (6)$$

As discussed in Section 3, TFP and labor growth each have persistent and transitory characteristics, and each reflects sector-specific and common forces to different degrees. Moreover, observe in Figures 2 and 3 that trend movements in TFP and labor growth often appear unrelated for a given sector. For example, trend TFP growth in Construction has seen a large decline since the 1950s while trend labor growth in that sector has been close to flat and dominated by its common trend or demographics (see Figure 5). Similarly, trend TFP growth in the Durable Goods sector is in 2016 close to where it started in 1950 (Figure 3), with several notable up-and-down swings along the way, while trend labor growth in that sector has steadily declined during the same period. Trend TFP growth in Professional and Business Services has gradually declined over the post-war period while trend labor growth in that sector is essentially flat, and so on. These observations suggest that changes in both labor supply and labor demand potentially play an important role at secular frequencies and that neither force necessarily dominates the other. Furthermore, to the extent that changes in labor demand are tied to technical progress, the relationship is not unambiguous. New technologies can make workers more productive and increase labor demand or, on the contrary, reduce labor demand if these new technologies are primarily labor-saving.²¹

²¹At business cycle frequencies, the behavior of labor and TFP changes across sectors also suggest an important role for labor supply shocks. Specifically, absent supply shocks, idiosyncratic TFP shocks in the standard model imply strong negative comovement between output growth and labor growth across sectors while, in the data, this comovement is unambiguously

In this paper, we take as given the joint behavior of $\{\Delta \ln z_{j,t}, \Delta \ln \ell_{j,t}\}$ in each sector j and interpret these changes as concurrently describing the process of structural change in different sectors. Each component of $\{\Delta \ln z_{j,t}, \Delta \ln \ell_{j,t}\}$ is modeled after the empirical specification in equation (3) with one small change. Specifically, we let

$$\tau_{c,t}^x = (1 - \rho)g_c^x + \rho\tau_{c,t-1}^x + \eta_{c,t}^x, \quad (7)$$

and

$$\tau_{j,t}^x = (1 - \rho)g_j^x + \rho\tau_{j,t-1}^x + \eta_{j,t}^x, \quad (8)$$

where $x = z$ or ℓ , and $\eta_{c,t}^x$ and $\eta_{j,t}^x \forall j$ are mean-zero random variables. We assume that $\rho < 1$ in which case the economy is characterized by a balanced growth path in the long run that we describe below. For values of ρ arbitrarily close to 1, however, the processes for $\tau_{c,t}^x$ and $\tau_{j,t}^x$ become those described in Section 3. Put another way, as in the empirical section, we allow the growth rates of sectoral TFP and labor to have both transitory and persistent - though not quite permanent - components off the balanced growth path. In the quantitative application, we consider values of ρ close to 1 and study transition paths implied by the observed secular behavior of $\Delta \ln z_{j,t}$ and $\Delta \ln \ell_{j,t}$.

Because observed labor growth, $\Delta \ln \ell_{j,t}$, is treated as given, we consider two polar cases that bound counterfactual exercises studying the implications of trend labor growth changes in individual sectors. At one extreme, counterfactual changes in trend labor growth within a sector are independent of changes in trend labor growth in other sectors and, therefore, fully reflected as changes in the growth rate of the aggregate labor force. At the other extreme, counterfactual changes in trend labor growth within a sector are reallocated across all other sectors according to their respective labor shares. In the latter case, the growth rate of the aggregate labor force is unaffected.

4.2 Model Parameters

Our choice of model parameters follows mostly Foerster et al. (2011) and is governed by the BEA Input-Output (IO) accounts and Fixed Asset Tables (FATs).

The consumption bundle shares, $\{\theta_j\}$, value-added shares in gross output $\{\gamma_j\}$, capital shares in value added, $\{\alpha_j\}$, and material bundle shares, $\{\phi_{ij}\}$, are obtained from the 2015 BEA Make and Use Tables. The Make Table tracks the value of production of commodities by sector, while the Use Table measures the value of commodities used by each sector. We combine the Make and Use Tables to yield, for each sector, a table whose rows show the value of a sector's production going to other sectors (materials) and households (consumption), and whose columns show payments to other sectors (materials) as well as labor and capital. Thus, a column sum represents total payments from a given sector to all other sectors, while a row sum gives the importance of a sector as a supplier to other sectors. We then calculate material bundle shares, $\{\phi_{ij}\}$,

positive. One way to generate positive comovement between output growth and labor growth across sectors from TFP changes is to assume that these changes are dominated by aggregate or common forces. However, as discussed in Section 3, this does not appear to be the case empirically.

as the fraction of all material payments from sector j that goes to sector i . Similarly, value-added shares in gross output, $\{\gamma_j\}$, are calculated as payments to capital and labor as a fraction of total expenditures by sector j , while capital shares in value added, $\{\alpha_j\}$, are payments to capital as a fraction of total payments to labor and capital. The consumption bundle parameters, $\{\theta_j\}$, are likewise payments for consumption to sector j as a fraction of total consumption expenditures.

The parameters that determine the production of investment goods, $\{\omega_{ij}\}$, are chosen similarly in accordance with the BEA Capital Flow table from 1997, the most recent year in which this flow table is available. The Capital Flow table shows the flow of new investment in equipment, software, and structures towards sectors that purchase or lease it. By matching commodity codes to sectors, we obtain a table that has entries showing the value of investment purchased by each sector from every other sector. A column sum represents total payments from a given sector for investment goods to all other sectors, while a row sum shows the importance of a sector as a supplier of investment goods to other sectors. Hence, the investment bundle shares, $\{\omega_{ij}\}$, are estimated as the fraction of payments for investment goods from sector j to sector i , expressed as a fraction of total investment expenditures made by sector j .

The capital depreciation rates are chosen to be consistent with capital accumulation as described in the BEA's Fixed Asset Tables. We construct Divisia aggregates for our 15-sector aggregation from detailed real net capital stocks and investment, and calculate capital depreciation rates such that net-stocks and investment are consistent with the capital accumulation equation in each sector. Because the implied depreciation rates vary over time, we fix each sector's depreciation rate at its post-2000 average as a benchmark.

For conciseness, we use the following notation throughout the paper: we denote the vector of household expenditure shares by $\Theta = (\theta_1, \dots, \theta_n)$, the matrix summarizing value added shares in gross output by sector, $\Gamma_d = \text{diag}\{\gamma_j\}$, the matrix of input-output linkages by $\Phi = \{\phi_{ij}\}$, the capital flow matrix by $\Omega = \{\omega_{ij}\}$, the matrix summarizing capital shares in value added by sector, $\alpha_d = \text{diag}\{\alpha_j\}$, and the matrix summarizing sector-specific depreciation rates by $\delta_d = \text{diag}\{\delta_j\}$.

4.3 Some Benchmark Results in an Economy Without Growth

A special case of the economic environment presented above is one where $\alpha_j = 0 \forall j$, and $\{\ln z_{j,t}, \ln \ell_{j,t}\}$ are modeled as stationary processes in levels rather than growth rates, in which case $\ln A_{j,t}$ is also stationary in levels around a constant mean in each sector. This special case reduces to the economy studied in [Long and Plosser \(1983\)](#) - though in that paper, materials, $m_{j,t}$, are used with a one-period lag - or [Acemoglu et al. \(2012\)](#). Aggregate value added or GDP, V_t , is then equivalent to the aggregate consumption bundle, $\prod_{j=1}^n \left(\frac{c_{j,t}}{\theta_j}\right)^{\theta_j}$, and

$$\frac{\partial \ln V_t}{\partial \ln A_{j,t}} = s_j^v, \quad (9)$$

where s_j^v is sector j 's value added share in GDP, and where these shares may be summarized in a vector, $s^v = (s_1^v, \dots, s_n^v)$, given by $s^v = \Theta(I - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$. Consistent with [Hulten \(1978\)](#) or more recently [Gabaix \(2011\)](#), a sector's value added share entirely summarizes the effects of structural change in that

sector on the level of GDP. Accordingly, [Acemoglu et al. \(2012\)](#) refer to the object $\Theta(I - (I - \Gamma_d)\Phi')^{-1}\Gamma_d$ as *the influence vector*.²²

When at least some sectors use capital in production, so that $\alpha_j > 0$ for some j , the economy becomes dynamic and, absent shocks, converges to a steady state in levels in the long-run. Getting rid of the t subscripts to denote variables in that steady state, and letting $A = (A_1, \dots, A_n)$ represent the long-run vector of composite exogenous sectoral states, a version of equation (9) holds in the limit as $\beta \rightarrow 1$,

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v, \quad (10)$$

where η is an adjustment factor approximately equal to the inverse of the mean labor share across sectors. In particular, when sectors use capital with the same intensity, $\alpha_j = \alpha \forall j$, then $\eta = \frac{1}{1-\alpha}$. The value added shares in this case, s^v , are given by $\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d\Omega']^{-1}/\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d\Omega']^{-1}\mathbf{1}$, where $\mathbf{1}$ is a unit vector of size n . When $\beta < 1$, the influence vector also depends on sectoral depreciation rates, δ_j , and equation (10) holds as an approximation that depends on $\frac{\beta}{1-\beta(1-\delta_j)} \times \delta_j$ which, for standard calibrations of β , is close to 1.²³ As underscored by [Baqae and Farhi \(2017b\)](#), both equations (9) and (10) may be interpreted in terms of macro-envelope conditions. When preferences and technology are Cobb-Douglas, neither expressions (9) nor (10) has sectoral states, A_j , affecting value added shares, s_j^v .

4.4 Balanced Growth

The effects of structural changes are less straightforward during transitions to the steady state in an economy with capital. Furthermore, because the data described in Section 3 reveals important persistent components in sectoral growth rates, we frame the effects of sectoral structural change on the aggregate economy in terms of growth rate elasticities, $\partial \Delta \ln V_t / \partial \Delta \ln A_{j,t}$.

In an economy with steady state growth, sectoral value added shares in GDP will depend on the entire distribution of growth rates characterizing structural changes, $\Delta \ln A_j$, $j = 1, \dots, n$, in addition to reflecting the sectoral network implicit in the IO and capital flow matrices. As described above, these shares are then used to calculate overall GDP growth, $\Delta \ln V$, by way of a Divisia index. More importantly, because of production linkages, structural change in a sector, $\Delta \ln A_j$, potentially helps determine value added growth in every other sector along the balanced growth path. This mechanism, therefore, amplifies the effects of sector-specific structural change on GDP growth in a way that can be summarized in terms of a multiplier for each sector. In some sectors, these multipliers can scale the impact of structural changes on GDP growth by multiple times their share in the economy.

Consider a non-stochastic steady state path where all disturbances - persistent and transitory as well as idiosyncratic and common - are set to zero. The non-stochastic steady state is defined by a balanced growth

²²Observe that TFP scales value added rather than gross output in this paper. Consequently, the influence vector reflects sectoral value added shares in GDP rather than shares of gross output in GDP or Domar weights.

²³See Appendix B.

path determined by constant growth in each sector,

$$\Delta \ln A_{j,t} = \bar{g}_j = \bar{g}_j^z + (1 - \alpha_j) \bar{g}_j^\ell, \quad (11)$$

where

$$\bar{g}_j^z = \lambda_{j,\tau}^z g_c^z + g_j^z \text{ and } \bar{g}_j^\ell = \lambda_{j,\tau}^\ell g_c^\ell + g_j^\ell. \quad (12)$$

In other words, sectoral structural growth in the steady state, \bar{g}_j , reflects steady state sectoral TFP growth, \bar{g}_j^z , and sectoral labor growth, \bar{g}_j^ℓ . The long-run growth rates of TFP and labor in each sector in turn reflect aggregate (common) components, $(\lambda_{j,\tau}^z g_c^z, \lambda_{j,\tau}^\ell g_c^\ell)$, and idiosyncratic components, (g_j^z, g_j^ℓ) , respectively.

4.4.1 Sectoral Growth in Value Added

Let $\Delta \ln \mu_t^v = (\Delta \ln \mu_{1,t}^v, \dots, \Delta \ln \mu_{n,t}^v)'$ denote the vector value added growth by sector. Then, along the balanced growth path, $\Delta \ln \mu^v$ is constant and given by

$$\Delta \ln \mu^v = \left[I + \alpha_d \Omega' \underbrace{(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d}_{\Xi'} \right] \bar{g}_a, \quad (13)$$

where $\bar{g}_a = (\bar{g}_1, \dots, \bar{g}_n)'$ is the vector of sectoral structural growth rates.²⁴ Equation (13) describes how structural growth in a given sector, \bar{g}_j , affects value added growth in all other sectors, $\Delta \ln \mu_i^v$. This relationship involves the direct effects of sectors' structural growth on their own value added growth, $I \bar{g}_a$, and the indirect effects that sectors have on other sectors through the economy's sectoral network of materials and investment, $\alpha_d \Omega' \Xi' \bar{g}_a$. Specifically,

$$\frac{\partial \Delta \ln \mu_j}{\partial \bar{g}_j} = 1 + \alpha_j \sum_{k=1}^n \omega_{kj} \xi_{jk} \text{ and } \frac{\partial \Delta \ln \mu_i}{\partial \bar{g}_j} = \alpha_i \sum_{k=1}^n \omega_{ki} \xi_{jk}, \quad (14)$$

where $(\xi_{j1}, \dots, \xi_{jn})$ is a column of Ξ' equal to the Leontief inverse, $(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$, diagonally weighted by the matrix of value added shares in gross output, Γ_d , in equation (13). Thus, along the balanced growth path, sectoral linkages make it possible for a structural change in a given sector j , $\partial \bar{g}_j$, to affect value added growth in every other sector, i , so long as that sector uses capital in production, $\alpha_i > 0$. Otherwise, value added growth in a sector with $\alpha_j = 0$ is entirely determined by its own structural growth rate, $\frac{\partial \Delta \ln \mu_j}{\partial \bar{g}_j} = 1$. In this sense, the presence of capital accumulation plays a central role for the sectoral growth implications of production linkages.

To gain intuition into how structural growth in different sectors help determine value added growth in other sectors, observe that the effect of a change in sector j 's structural growth rate on sector i is given by the (i, j) element of $\alpha_d \Omega' \Xi'$. Each of these (i, j) elements in turn contains all of the elements

²⁴See Appendix C.

Table 4: Ω , Capital Flow Table

Sector	AFFH	Mining	Util.	Const.	Dur.	Nondur.	Whsl.	Ret.	Trans.	Inf.	FIRE	Prof.	Educ., Arts,	Other
				Goods	Goods	Goods	Trd.	Trd.	and			and	Health	Serv.,
									Ware.			Bus.	Soc.	etc.
												Serv.	Ass.	Excl.
														Gov.
Agriculture, Forestry, Fishing, Hunt.	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mining	0	.5	0	0	0	0	0	0	0	0	0	0	0	0
Utilities	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Construction	.12	.07	.47	.03	.15	.14	.14	.52	.14	.2	.69	.12	.38	.57
Durable Goods	.7	.31	.36	.76	.58	.61	.6	.35	.69	.52	.21	.45	.43	.3
Nondurable Goods	0	0	.04	0	0	0	0	.01	0	0	0	.01	0	.01
Wholesale Trade	.1	.05	.04	.08	.07	.08	.07	.04	.06	.06	.02	.07	.09	.05
Retail Trade	.04	.01	.01	.04	.01	.02	.03	.02	.01	.02	.03	.04	.02	.03
Transportation and Warehousing	.02	.01	.01	.02	.01	.01	.01	.01	.01	.01	0	.01	0	.01
Information	0	.02	.03	.04	.03	.02	.05	.02	.02	.1	.01	.06	.03	.01
FIRE	0	0	0	0	0	0	0	0	0	0	.01	0	0	0
Prof. and Bus. Services	.02	.04	.04	.04	.16	.12	.09	.03	.06	.1	.02	.25	.06	.03
Education and Health Care	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Entertainment and Food Serv.	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Other Services, Except Government	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes: The entries in the table show the share of investment goods used by sector j that originated in sector i . Entries are computed from the 1997 Capital Flow Table.

of the vector $(\xi_{j1}, \dots, \xi_{jn})$ in the j^{th} column of the weighted Leontief inverse, Ξ' , as described in equation (14). The j^{th} column of the weighted Leontief inverse in turn will reflect the j^{th} column of the transposed capital flow matrix, Ω' (i.e. the j^{th} row of Ω or the degree to which sector j produces new capital for other sectors), as well as the j^{th} column of the transposed IO matrix, Φ' , (i.e. the j^{th} row of Φ or the degree to which sector j produces materials for other sectors). To see this, observe that the Leontief inverse, $(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$, can be alternatively expressed as the limit of $(\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi') + (\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^2 + (\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^3 + \dots + (\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^n$. Each column of Ω' and Φ' is weighted by that column's corresponding sector's share of value added and materials in gross output respectively, $\Gamma_d \Omega'$ and $(I - \Gamma_d) \Phi'$. Ultimately, sectors that play a central role in producing capital or materials for other sectors will be associated with a column of the weighted Leontief inverse, Ξ' , whose elements are relatively large. The individual elements of the Neumann series, $(\alpha_d \Gamma_d \Omega' + (I - \Gamma_d) \Phi')^k$, $k = 1, \dots, n$, describe feedback effects in which a change in the structural growth rate of some sector, j , impacts the price and quantities of j 's goods purchased by another sector, i , which in turn impacts the price and quantities of i 's goods purchased by other sectors including j . This process then feeds back into the prices and quantities of goods that sector j sells to sector i in the next round, and so on.

Table 4 shows the Capital Flow matrix of the U.S. economy, Ω , for the 15 sectors considered in this paper. As the table makes clear, the production of investment goods in the U.S. is concentrated in relatively few sectors, with many sectors not producing any capital for other sectors while Construction and Durable Goods produce close to 80 percent of capital in almost every sector. Construction comprises residential

Table 5: Φ , Make-Use Table

Sector	AFFH	Mining	Util.	Const.	Dur.	Nondur.	Whsl.	Ret.	Trans.	Inf.	FIRE	Prof.	Educ., Arts,	Other	
				Goods	Goods	Goods	Trd.	Trd.	and			and	Health,Ent.,	Serv.,	
									Ware.			Bus.	Soc.	etc.	Excl.
												Serv.	Ass.		Gov.
Agriculture, Forestry, Fishing, Hunt.	.39	0	0	0	.01	.13	0	0	0	0	0	0	0	.01	0
Mining	.01	.27	.32	.02	.02	.16	0	0	0	0	0	0	0	0	0
Utilities	.01	.02	.02	0	.01	.01	.01	.01	.01	0	.03	.01	.02	.02	.01
Construction	.01	.04	.04	0	0	0	0	0	.01	0	.07	0	0	0	.01
Durable Goods	.04	.15	.02	.38	.57	.06	.03	.03	.07	.12	.02	.06	.07	.05	.19
Nondurable Goods	.26	.1	.11	.13	.09	.37	.04	.05	.21	.04	.01	.05	.12	.22	.07
Wholesale Trade	.1	.05	.03	.09	.09	.07	.08	.05	.07	.04	.01	.02	.04	.04	.04
Retail Trade	0	0	0	.15	0	0	0	.01	.01	0	0	0	0	.01	.03
Transportation and Warehousing	.05	.08	.17	.04	.04	.05	.13	.14	.27	.03	.02	.04	.03	.03	.02
Information	0	.01	.02	.02	.02	.01	.06	.06	.02	.36	.05	.08	.04	.04	.04
FIRE	.09	.1	.08	.07	.03	.02	.21	.29	.17	.11	.49	.22	.33	.21	.37
Prof. and Bus. Services	.02	.17	.15	.1	.12	.11	.39	.3	.14	.21	.24	.43	.26	.27	.17
Education and Health Care	0	0	0	0	0	0	0	.02	0	0	0	0	.03	0	.01
Entertainment and Food Serv.	0	.01	.02	.01	.01	.01	.02	.01	.01	.06	.03	.05	.03	.06	.02
Other Services, Except Government	0	0	.01	.01	0	0	.04	.02	.01	.02	.02	.03	.04	.02	.03

Notes: The entries in the table show the share of materials used by sector j that originated in sector i . Entries are computed from the 2015 BEA Make and Use Tables.

and non-residential structures, including infrastructure such as power plants or pipelines used to transport crude oil for example, but also the maintenance and repair of highways, bridges, and other surface roads. The bulk of capital produced by the Durable Goods sector resides in Motor Vehicles, Machinery, and Computer and Electronic Products. Other sectors recorded as producing capital goods for the U.S. economy include Wholesale Trade, Retail Trade, and Professional and Business Services. In the Professional and Business Services sector, the notion of capital produced for other sectors is overwhelmingly composed of Computer System Designs and Related Services. As a practical matter, the distinction between materials and investment goods is not always straightforward. The BEA distinguishes between materials and capital goods by estimating the service life of different commodities and, consistent with a time period in this paper, commodities expected to be used in production within the year are defined as materials.²⁵ From the Capital Flow table, we expect that columns of the Leontief inverse, Ξ' , associated with Construction and Durable Goods will have relatively large elements.

Table 5 shows the IO matrix or Make-Use table for the U.S. economy, Φ . Compared to the Capital Flow table, the production of materials is considerably less concentrated in that all sectors produce materials for

²⁵As noted in Foerster et al. (2011), there are a couple of notable measurement issues related to the construction of the Capital Flow table. First, the table accounts for the purchases of new capital goods but not used assets. Thus, for example, a firm's purchase of a used truck in a sector from another sector will not be recorded as investment in the capital flow table even though the truck's remaining service life may be well in excess of a year. Second, McGrattan and Schmitz (1999) note that a non-trivial portion of maintenance and repair takes place using within sector resources, yet many of the diagonal elements of the Capital Flow table are very small or zero. Results presented here are robust up to an adjustment that assumes that an additional 25 percent of capital expenditures takes place within sectors.

Table 6: Ξ' , Weighted Leontief Inverse

Sector	AFFH	Mining	Util.	Const.	Dur.	Nondur.	Whsl.	Ret.	Trans.	Inf.	FIRE	Prof.	Educ., Arts,	Other	
				Goods	Goods	Goods	Trd.	Trd.	and			and	Health,	Ent.,	
									Ware.			Bus.	Soc.	etc.	
												Serv.	Ass.	Excl.	
														Gov.	
Agriculture, Forestry, Fishing, Hunt.	.53	.08	.02	.13	.43	.15	.19	.03	.08	.04	.15	.25	0	.01	.01
Mining	.01	1.06	.01	.12	.38	.07	.12	.02	.06	.04	.11	.26	0	.01	.01
Utilities	.01	.16	.67	.21	.35	.07	.11	.03	.07	.05	.11	.24	0	.02	.01
Construction	.01	.05	.01	.64	.38	.07	.12	.06	.05	.04	.1	.21	0	.01	.01
Durable Goods	.02	.06	.01	.1	.87	.08	.15	.02	.05	.05	.1	.28	0	.01	.01
Nondurable Goods	.07	.18	.02	.12	.4	.5	.16	.02	.07	.05	.11	.3	0	.02	.01
Wholesale Trade	.01	.03	.01	.1	.29	.05	.76	.02	.06	.05	.13	.27	0	.01	.02
Retail Trade	.01	.03	.01	.14	.23	.05	.08	.6	.06	.05	.17	.24	.01	.01	.01
Transportation and Warehousing	.02	.05	.01	.11	.31	.09	.11	.02	.6	.04	.15	.24	0	.01	.01
Information	.01	.04	.01	.13	.38	.06	.12	.02	.05	.69	.14	.31	0	.03	.02
FIRE	.01	.04	.02	.32	.34	.05	.1	.04	.04	.05	.86	.27	0	.02	.02
Prof. and Bus. Services	.01	.02	.01	.08	.19	.04	.06	.02	.03	.05	.12	.9	0	.02	.01
Education and Health Care	.01	.03	.01	.09	.18	.05	.07	.02	.03	.04	.16	.2	.59	.02	.02
Entertainment and Food Serv.	.02	.04	.01	.13	.23	.08	.09	.02	.04	.04	.15	.25	0	.55	.02
Other Services, Except Government	.01	.03	.01	.12	.23	.04	.07	.02	.03	.04	.16	.18	0	.01	.65

Notes: See text for definition of Ξ .

all other sectors, though to varying degrees. From the Make-Use table, Professional and Business Services, Finance, Insurance, and Real Estate (FIRE), and to a degree Nondurable Goods, all play an important role in providing intermediate inputs to the U.S. economy. Observe that while Professional and Business Services figures prominently in Φ , this sector is not nearly as dominant as Durable Goods or Construction are in Ω .

In contrast to the sectors that play a key role in Ω or Φ , output produced in sectors such as Agriculture, Forestry, Fishing, and Hunting, or Entertainment and Food Services, is mostly consumed as a final good. Therefore, columns of the Leontief inverse associated with these sectors will have elements that tend to be small. Observe, however, that a considerable fraction of Agriculture's expenditures on materials is spent within its own sector so that the diagonal element of Ξ' associated with that sector is likely to still be significant.

Putting the information from the Capital Flow and Make-Use tables together, Table 6 shows the resulting Leontief inverse matrix, weighted by the diagonal matrix Γ_d . As expected, elements of the Construction and Durable Goods columns of the Leontief inverse are uniformly large, while those of the Agriculture, Forestry, Fishing, and Hunting or Entertainment and Food Services columns are comparatively unimportant. Because columns of the Leontief inverse determine the extent to which structural change in one sector affects value added growth in every other sector, these columns also ultimately make up a network multiplier for each sector that summarizes the importance of structural change in that sector for aggregate GDP growth.

Before turning our attention to sectoral multipliers, we make one last observation regarding the role of sectoral linkages along the balanced growth path. In particular, consider the special case in which each

sector produces its own capital, $\Omega = I$, as in Horvath (1998) or Dupor (1999), and its own materials, $\Phi = I$. In other words, production requires capital and materials but both are sector specific. In that case, there are effectively no sectoral linkages and we have that

$$\Delta \ln \mu_{j,t}^v = \frac{1}{1 - \alpha_j} \bar{g}_j^z + \bar{g}_j^\ell, \quad (15)$$

which is the sectoral analog to equation (1). In an environment where sectors mostly produce their own capital and material, equation (1) will hold individually sector by sector.

4.4.2 Aggregate Balanced Growth Implications in a Canonical Model with Linkages

Given the vector of value added growth, $\Delta \ln \mu^v$, the Divisia aggregate index of GDP growth is $\Delta \ln V = s^v \Delta \ln \mu^v$ or

$$\Delta \ln V = \sum_{j=1}^n s_j^v \left[\bar{g}_j + \sum_{i=1}^n \alpha_j \omega_{ij} \sum_{k=1}^n \xi_{ki} \bar{g}_k \right], \quad (16)$$

so that, holding shares constant,

$$\frac{\partial \Delta \ln V}{\partial \bar{g}_j} = s_j^v + s_j^v \alpha_j \sum_{k=1}^n \omega_{kj} \xi_{jk} + \sum_{i \neq j} s_i^v \alpha_i \sum_{k=1}^n \omega_{ki} \xi_{jk}, \quad (17)$$

where the second and third terms in this last expression capture the weighted network sectoral effects discussed above in the context of the (diagonally weighted) Leontief inverse, Ξ' .²⁶ When all sectors use little or no capital in production, $\alpha_j = 0 \forall j$, the effects of structural change in a given sector j , $\partial \bar{g}_j$, on GDP growth will be well approximated by its share in the economy, s_j^v . This case recovers a version of of Hulten's theorem (1978) but in growth rates. More generally, equation (17) suggests the presence of a network multiplier effect that varies by sector and that depends not only on the importance of sectors as suppliers of capital and materials to other sectors, ω_{ki} and ξ_{jk} , but also on the extent to which the latter sectors use capital in their own production, α_i . The effects of sectoral change, $\partial \bar{g}_j$, on GDP growth may be summarized as a direct effect, $s^v I$, and an additional indirect effect resulting from sectoral linkages, $s^v \alpha_d \Omega' \Xi'$. Hence, we define the combined direct and indirect effects of structural change on GDP growth in terms of network multipliers, $s^v (I + \alpha_d \Omega' \Xi')$.

Table 7 shows the direct and combined effects of structural change in the different sectors we consider

²⁶Along the balanced growth path, sectoral value added shares in GDP, s^v , are functions of the model's underlying parameters including, in the general case, the distribution of sectoral structural growth rates, \bar{g}_a . As shown in the Technical Appendix, $s^v = \frac{\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d G_d \Omega']^{-1}}{\Theta[\Gamma_d^{-1}(I - (I - \Gamma_d)\Phi') - \alpha_d G_d \Omega']^{-1} \mathbf{1}}$ where $G_d = \text{diag} \{ [1 - (1 - \delta_i) \prod_{k=1}^n \prod_{\ell=1}^n (1 + \bar{g}_\ell)^{-\omega_{ki} \xi_{\ell k}}] \Delta_i \}$ with $\Delta_i = \frac{\beta \prod_{j=1}^n \prod_{k=1}^n (1 + \bar{g}_k)^{\xi_{kj} \theta_j}}{1 - \beta(1 - \delta_i) [\prod_{j=1}^n \prod_{k=1}^n (1 + \bar{g}_k)^{\xi_{kj} (\theta_j - \omega_{ji})}]}$. Furthermore, changes in sectoral shares induced by a change in structural growth in a sector k , $\frac{\partial s_j^v}{\partial \bar{g}_k}$ tend to be small for overall GDP growth, consistent with Figure 1, and the notion that since shares must sum to 1, $\sum_j \frac{\partial s_j^v}{\partial \bar{g}_k} = 0$.

Table 7: Sectoral Network Multipliers

Sector	s^v	$s^v \alpha_d \Omega' \Xi'$	$s^v (I + \alpha_d \Omega' \Xi')$
Agriculture, Forestry, Fishing, Hunt.	.03	.01	.03
Mining	.02	.03	.05
Utilities	.02	.01	.03
Construction	.05	.14	.19
Durable Goods	.13	.28	.41
Nondurable Goods	.09	.03	.13
Wholesale Trade	.07	.08	.15
Retail Trade	.08	.02	.11
Transportation and Warehousing	.04	.03	.07
Information	.05	.03	.08
FIRE	.19	.05	.24
Prof. and Bus. Services	.09	.15	.23
Education and Health Care	.06	0	.06
Entertainment and Food Serv.	.04	.01	.04
Other Services, Except Government	.03	.01	.04

Notes: This table decomposes each sector's total multiplier (column 3) into a direct effect (column 1) and an indirect effect (column 2).

on GDP growth given Tables 4 and 5. The importance of Construction and Durable Goods as suppliers of investment goods results in their having not only a large value added share in GDP, 5 and 13 percent respectively, but also in their having large spillover effects on other sectors. In particular, the network multipliers for the Construction and Durable Goods sectors come out to more than 3 times their share, 0.19 and 0.41 respectively. Considering that trend TFP growth in Construction fell by nearly 4 percentage points between 1950 and 2014 in Figure 3, this gives us, all else equal, a roughly 0.7 percentage point effect on trend GDP growth from TFP changes in Construction alone. In practice, the aggregate growth effects from combined changes in TFP and labor in Construction are likely smaller since labor growth has been moderately positive in that sector throughout the post-war. Nevertheless, the effect from Construction is large not only because of its central role as a producer of capital, including commercial and residential structures for all sectors, but also because capital depreciates only partially in any given year. It is apparent from Table 7 that the effects of structural change on GDP growth are always at least as large as sectoral shares. The sectoral network multipliers roughly double the share of Professional and Business Services, from 0.09 to 0.23, and raise the already large share of FIRE from 0.19 to 0.24. In other sectors, such as Agriculture, Forestry, Fishing and Hunting, or Entertainment and Food Services, the network multipliers are small or negligible as suggested by the corresponding columns of the weighted Leontief inverse. Because the same network relationships embodied in the Capital Flow table, Ω , and Make-Use table, Φ , determine

the importance that sectors have in the economy both as a share of value added and through their spillover effects, sectors with relatively larger shares in GDP will also tend to be associated with large network multipliers.

A key implication of Table 7 is that the effects of sectoral change on GDP growth arise in part through a composition effect. Therefore, secular changes in GDP growth can take place without observable changes in aggregate TFP growth. For example, consider purely idiosyncratic changes in TFP growth, $\partial \bar{g}_j^z$, that leave aggregate TFP growth unchanged, $\sum_{j=1}^n s_j^v \partial \bar{g}_j^z = 0$. Despite aggregate TFP growth not changing, these idiosyncratic changes may nevertheless have an effect on GDP growth since the sum of sectoral multipliers is larger than 1.

Finally, we return to Table 2 and the calculation in Section 3.2, but this time using the expression derived for the economy with production linkages across multiple sectors, equation (16). In this expression, we continue to use the numbers in Table 2, specifically the second and third columns for \bar{g}_j^z and \bar{g}_j^l , the fourth column for s_j^v , together with the BEA estimates of α_d , Γ_d , Ω' and Φ' . Using these estimates, equation (16) now gives a growth rate for GDP that jumps to 3.02 percent compared to 2.49 percent in the aggregate one-sector model and 3.25 percent in the data.

Evidently, the balanced growth equations (1), (13), and (16) hold only in the steady state and ignore important endogenous dynamics driven by capital accumulation. In other words, these equations all miss dynamic terms that capture deviations from steady state growth. Thus, we now turn our attention to the model solution and associated Markov decision and policy rules.

5 Quantitative Findings

Having described the balanced growth path implications of sectoral structural change, we now interpret the secular trend in aggregate GDP growth estimated in Section (2) as part of a slow-moving transition towards a balanced growth path. Therefore, to assess how sectoral trends in different sectors play out in other sectors over time, as well as in terms of overall GDP growth, we first work out the transition dynamics of the model. Given the economy's balanced growth path, we normalize the model's variables by appropriate factors (these are also the factors used in Appendix C) so as to make the model's optimality conditions and resource constraints stationary in the detrended variables. The scaling factors used in detrending differ with the variables under consideration but always involve the entire distribution of growth rates summarized in \bar{g}_a . Detrended variables are constant along the balanced growth path and while consumption, investment, materials, and gross output all grow at the same rate within each sector, these variables grow at different rates across sectors. Prices in different sectors, as well as their implied price indices, also all grow at different rates along the balanced growth path but in such a way as to generate constant shares along that path. After linearizing the model's dynamic equations around the steady state in detrended variables, Markov decision and policy rules may be readily obtained using standard linear rational expectations solution toolkits,

including in this case [King and Watson \(2002\)](#).²⁷

5.1 Historical Decompositions

In exploring the aggregate implications of structural change in individual sectors, the previous section highlighted the importance of accounting for production linkages and their network effects. In other words, in any counterfactual exercise, individual sectors cannot be modeled in isolation. In this section, we wish to recover the dynamic implications of the model for sectoral value added growth, $\Delta \ln v_{j,t}$, and aggregate GDP growth, $\Delta \ln V_t = \sum_{j=1}^n s_j^v \Delta \ln v_{j,t}$, when driven by the different sectoral processes estimated in Section 3.

In Appendix D, we show that during transitions to the balanced growth path, the vector of value added growth rates, $\Delta \ln v_t$, evolves according to

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a + \alpha_d \Delta \hat{k}_t + \hat{A}_t + \alpha_d \Omega' \Xi' \hat{A}_{t-1}, \quad (18)$$

where the vector $\hat{k}_t = (\hat{k}_{1,t}, \dots, \hat{k}_{n,t})'$ denotes the log deviations of capital from its appropriate level and $\hat{A}_t = (\hat{A}_{1,t}, \dots, \hat{A}_{n,t})'$ with $\hat{A}_{j,t} = \Delta \ln A_{j,t} - \bar{g}_j$. The vector \hat{k}_t follows from the model's equilibrium Markov decision rules and is thus a function of previous endogenous states, \hat{k}_{t-1} , and the various shocks affecting the economy (i.e. persistent and transitory, as well as idiosyncratic and common). The first term on the right-hand side of (18) captures the sectoral steady state growth paths described in equation (13). The additional terms represent the dynamics of value added growth induced by the driving processes, \hat{A}_t , and by endogenous adjustments to capital in different sectors, $\Delta \hat{k}_t$. In the absence of shocks, $\Delta \hat{k}_t$ and \hat{A}_t are zero in the long run and $\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a$ as in equation (13).

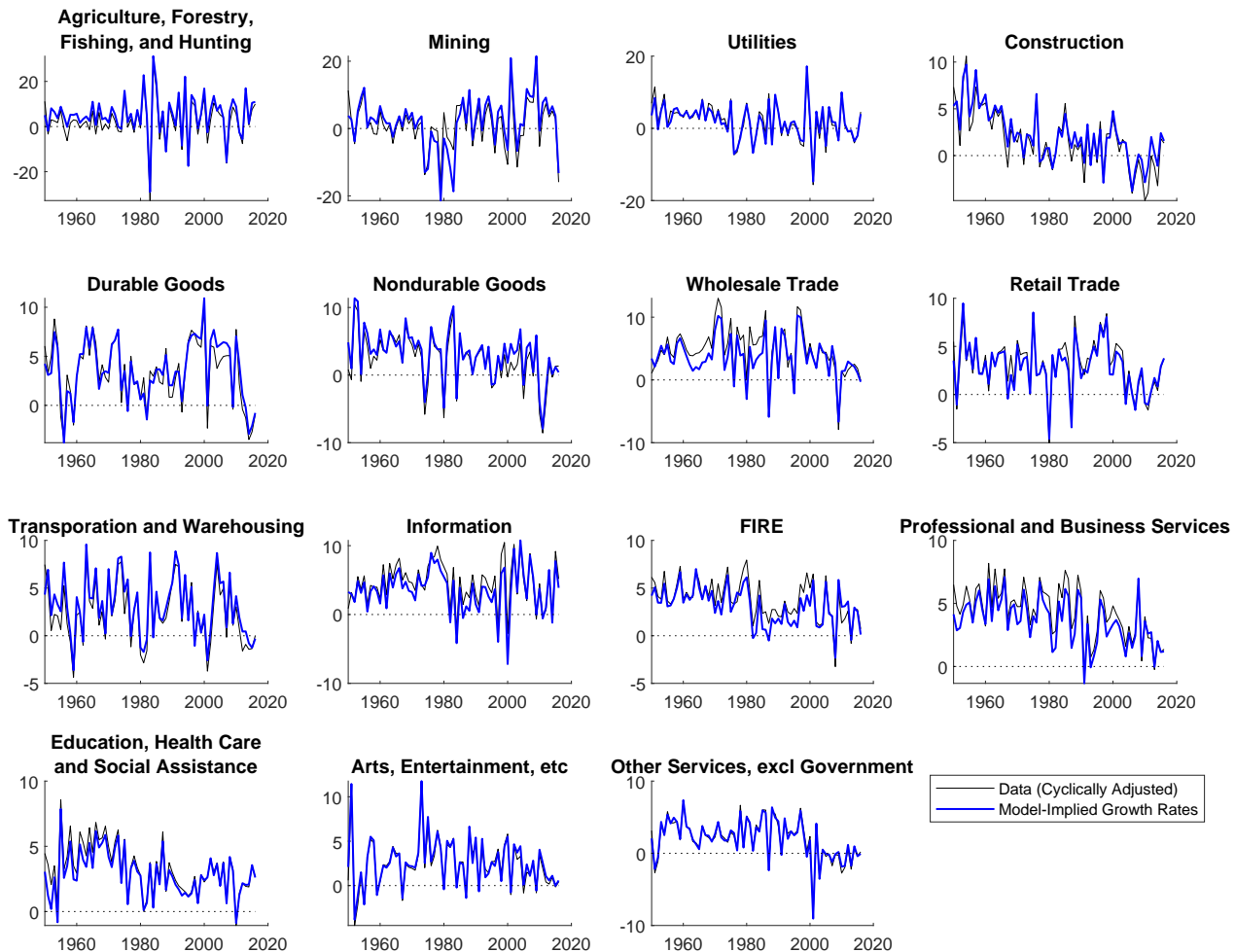
Equation (18) implies both contemporaneous and lagged effects of structural changes, \hat{A}_t , on sectoral value added growth, $\Delta \ln v_t$, towards balanced growth. The contemporaneous effect of a structural change in sector j , $\partial \Delta \ln A_{j,t}$, on its own value added, $\Delta \ln v_{j,t}$, is one-for-one whereas this effect is zero on value added growth in other sectors. It follows that in the period in which a sector experiences a structural change, the effect of that change on GDP growth is entirely captured by that sector's share in GDP, $\frac{\partial \Delta \ln V_t}{\partial \Delta \ln A_{j,t}} = s_j^v$. Thus, a version of Hulten's theorem (1978) holds in growth rates on impact.

In subsequent periods, the change in GDP growth stemming from sectoral structural changes will reflect the network effects of production linkages, $s^v \alpha_d \Omega' \Xi' \Delta \ln A_t$. Along transitions, structural changes will also affect GDP growth indirectly in-so-far as they are reflected by endogenous changes in the capital stock of different sectors, $s^v \alpha_d \Delta \hat{k}_t$, through optimal investment decisions.

Before assessing the effects of structural changes in individual sectors, we discuss the dynamic model's behavior when driven by the processes estimated in Section 3. Relative to the empirical model in that section, the structural model adds two key features. Aside from allowing for a rich network of production linkages in both materials and capital, it introduces endogenous dynamics by way of capital accumulation.

²⁷For a detailed description of this sequence of steps and all their subsequent implications for this section, see the Technical Appendix.

Figure 7: Sectoral Value Added Growth Rates - Data and Model (Percentage points at an annual rate)

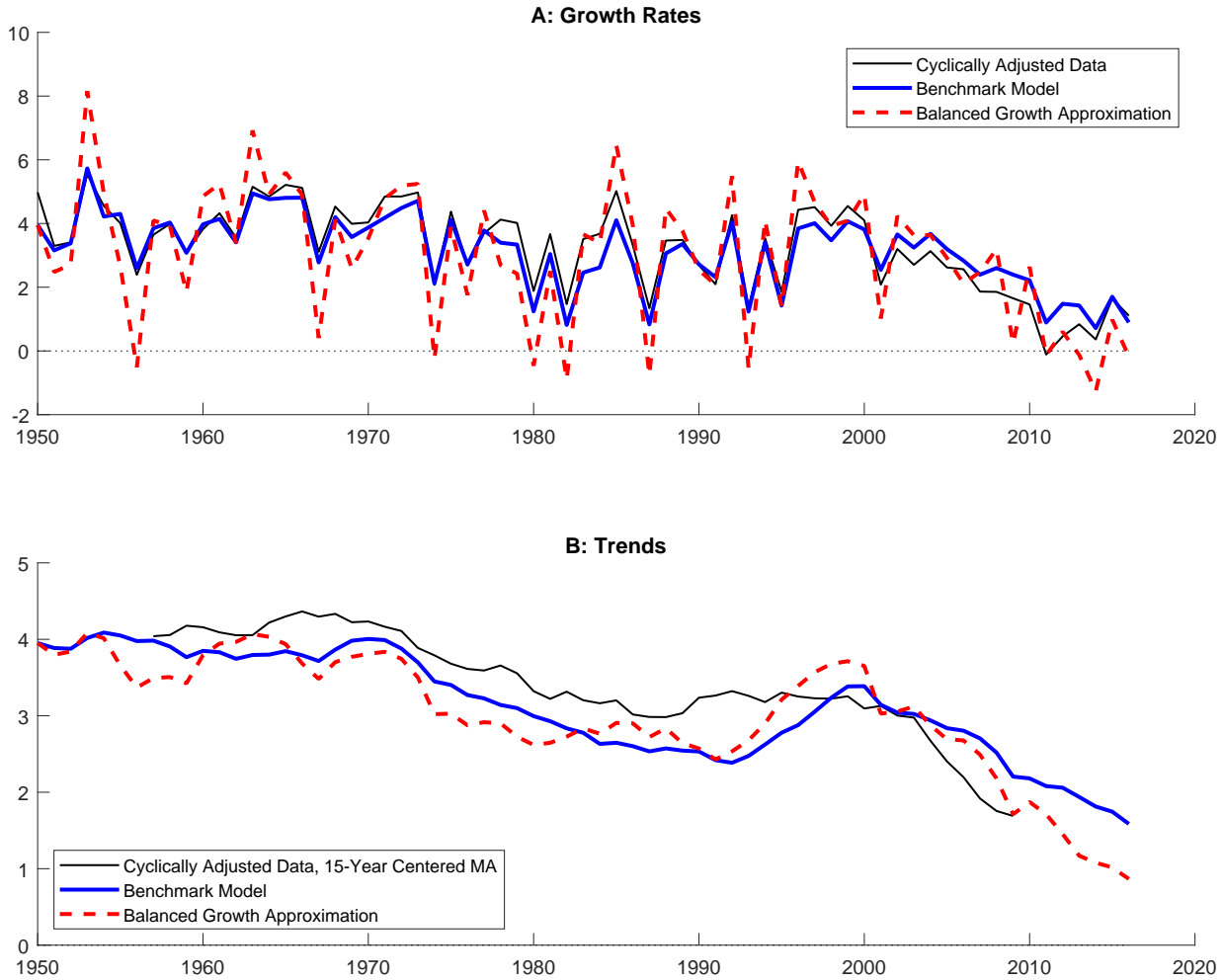


Notes: The figure shows the growth rate in value added for each sector. The model-implied value added is computed from equation (18) with values of $\hat{A}_{j,t}$ from the DFM model.

Figure 7 illustrates the behavior of value added growth in each sector, $\Delta \ln v_{j,t}$, in the data and in the model when driven by the processes estimated in Section 3. As the figure makes clear, the model performs remarkably well in capturing sectoral value added growth simultaneously across virtually every sector. The fit is particularly close in some key sectors highlighted above, such as Construction and Durable Goods, as well as most other sectors. The model misses somewhat on the dynamics of Wholesale Trade in the early part of the sample.

In Figure 7, a downward trend is clearly noticeable in Construction more than in any other sector. This feature of the data is significant in that the Construction sector was also associated with a large multiplier in the previous section. This large multiplier in turn reflected the importance of that sector in

Figure 8: GDP Growth - Data and Model



Notes: Panel (A) shows share-weighted value added growth rates from Figure 7. Panel (B) shows 15-year centered moving averages from panel (A) (thin black line) and the model-implied values of growth rates using equation (18) and the τ -components of $\hat{A}_{j,t}$. The dashed lines are balanced growth approximations using equation (16) but with the disturbances to TFP and labor growth in place of \bar{g}_j .

producing capital (commercial and residential structures as well as equipment) for almost every sector in the economy. A downward trend is also noticeable in Professional and Business Services though considerably less pronounced.

Figure 8A illustrates the behavior of GDP growth in the model and in the data. The red line depicts the growth rate of GDP as approximated by the balanced growth formula in equation (16) but driven by the full set of disturbances to TFP and labor growth in place of \bar{g}_j in that equation. In other words, the red line shows the model's predicted GDP growth rate implied by the dynamics embodied in the exogenous

driving processes but absent any internal (endogenous) dynamics from the model. The figure shows that this predicted growth rate is able to move ‘locally’ with the data. However, it is also considerably more volatile than actual GDP growth. In contrast, the blue line depicts the behavior of GDP growth in the full model as implied by the law of motion for sectoral value added growth in equation (18) together with the Markov decision rules for \hat{k}_t . This line is considerably smoother and closely matches the actual behavior of GDP growth (in black). The model underpredicts GDP growth slightly in the late 1990s and overstates the level of GDP growth somewhat in the 2000s but it matches its overall rate of decline almost exactly during the latter period.

Figure 8B illustrates the implications of the dynamic multi-sector model for trend GDP growth. Specifically, it illustrates the model’s solution when driven only by the disturbances associated with the persistent components of labor and TFP growth, $\Delta\tau_{c,t}^x$ and $\Delta\tau_{j,t}^x$, $x = \ell, z$. The red line represents the trend growth rate implied by the balanced growth approximation (16) when driven only by these shocks. The blue line in Figure 8B is the model-implied trend growth rate of GDP taking into account the model’s internal dynamics. The black line depicts a 15–year centered moving average of actual (cyclically adjusted) GDP growth. The full model indicates that trend GDP growth rate has steadily declined by over 2.3 percent over the post-war period. In the next sections, therefore, we unpack this slow decline in GDP growth in terms of the history experienced by individual sectors through counterfactual exercises.

5.2 Implications of Changing Sectoral Trends for GDP Growth

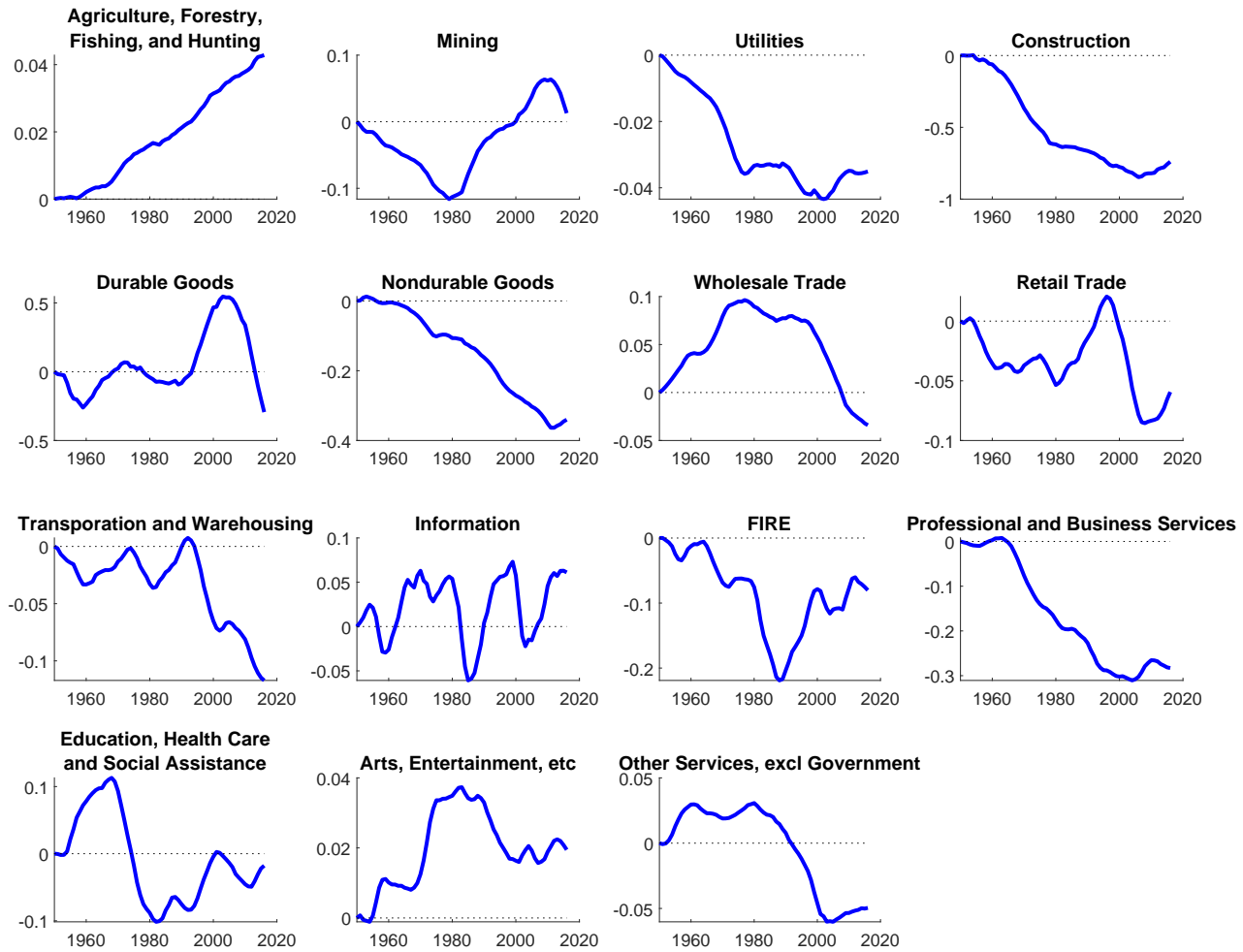
The multi-sector balanced growth model of Section 4 provides an accounting framework for decomposing the changes in the trend growth rate of GDP shown in Figure 8, panel B, into sources associated with common and sector-specific shocks. More precisely, using equation (18) and the definition of A_t , the model-based trend in GDP growth rates can be written as its balanced growth value, say $\mu_{\Delta V}$, and a distributed-lag of past trend shocks:

$$\Delta \ln V_t = \mu_{\Delta V} + \left[\beta_{c,z}(L)\Delta\tau_{c,t}^z + \beta_{c,\ell}(L)\Delta\tau_{c,t}^\ell \right] + \sum_{j=1}^n \left[\beta_{j,z}(L)\Delta\tau_{j,t}^z + \beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell \right], \quad (19)$$

where the distributed lag coefficients, $\beta(L)$, are functions of the model parameters and the factor loadings that link the common shocks to the sectors. Figure 9 plots the historical contribution of each sector’s contribution to $\Delta \ln V_t$, that is $\zeta_{j,t} = \beta_{j,z}(L)\Delta\tau_{j,t}^z + \beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell$; Figure 13 plots the contribution of the common shocks, $\zeta_{c,t} = \beta_{c,z}(L)\Delta\tau_{c,t}^z + \beta_{c,\ell}(L)\Delta\tau_{c,t}^\ell$ (in each figure, pre-sample values of $\Delta\tau$ are set to zero, so initial values of ζ are equal to zero.)

The sectoral results shown in Figure 9 suggest that, over the entire post-war period, the Construction sector is the largest single source of decline in trend GDP growth at approximately 0.75 percentage points. That is, more than 30 percent of the trend decline in GDP growth in the last 60 years is associated with this sector alone. Other sectors that contributed materially to the fall in trend GDP growth in the last 6 decades include Professional and Business Services and Nondurable Goods with both sectors contributing

Figure 9: Model-Based Historical Decomposition of Trend GDP Growth Rates: Contributions from Sectoral Trends

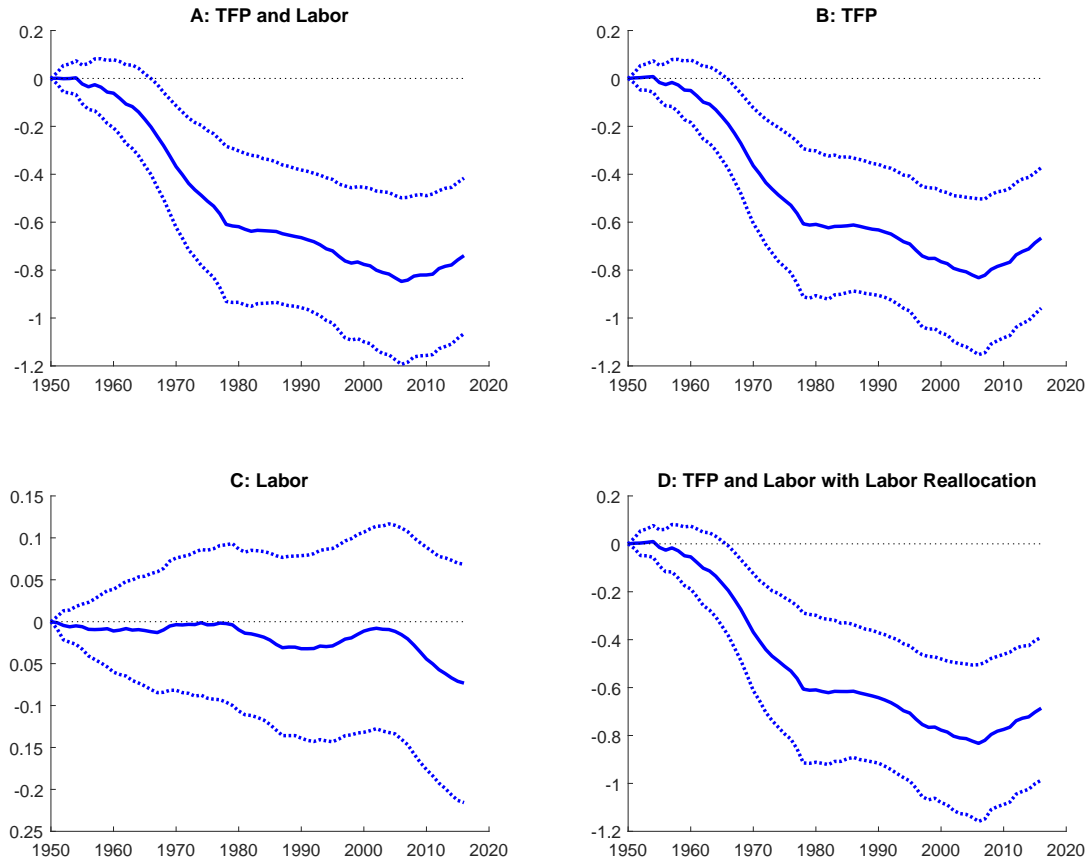


Notes: These figures show the effect of the sector-specific trend shocks ($\Delta\tau_{j,t}^z$ and $\Delta\tau_{j,t}^\ell$) on the growth rate of GDP using the model-based dynamic multipliers.

around a 0.3 percentage point loss. The Durable Goods sector contributed large decade-long up-and-down swings in trend GDP growth between 1950 and the end of the sample. The most-recent upswing in Durable Goods, contributing by itself almost 0.5 percentage points to trend GDP growth between the late 1980s and 2000, partly reflects technical progress in the production of semiconductors that was much publicized during that period. The subsequent decrease in Durable Goods accounts for more than a third (0.7 percentage points) of the 1.8 percentage point post-1999 decline in the growth rate of GDP. Finally, some sectors, such as Information and Wholesale Trade, have somewhat offset the longer run decrease in GDP growth.

Figure 10 looks more closely at the aggregate effects of structural change in the construction sector. Panel

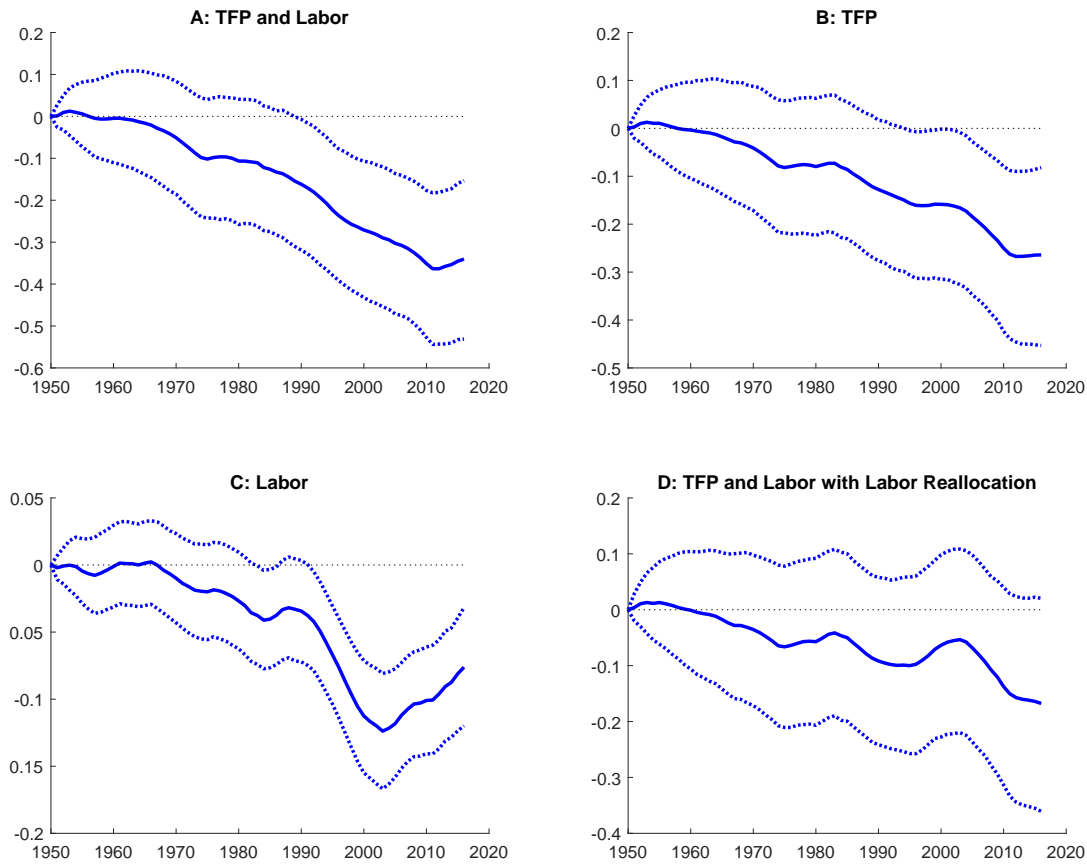
Figure 10: Model-Based Historical Decomposition of Trend GDP Growth Rates: Contributions from Construction



Notes: Panels A-C show the effect of the sector-specific trend shocks ($\Delta\tau_{j,t}^z$ and $\Delta\tau_{j,t}^\ell$) on the growth rate of GDP using the model-based dynamic multipliers. Panel D shows these effects after reallocating $\Delta\tau_{j,t}^\ell$ to the other sectors. Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters.

A shows the composite effect of construction-specific changes in trend values of labor and TFP that was shown previously in Figure 9. Panels B and C show the separate effects from labor ($\beta_{j,\ell}(L)\Delta\tau_{j,t}^\ell$) and TFP ($\beta_{j,z}(L)\Delta\tau_{j,t}^z$). Evidently, decreases in the rate of growth of TFP in the construction sector explain most of its deleterious effect on trend GDP growth. Of course, one interpretation of these historical decompositions is as counterfactual paths of GDP growth setting all other shocks to zero. In this interpretation, Panel A of Figure 10 shows the path of the trend growth rate of GDP under the counterfactual absence of changes in common and sector-specific shocks other than construction ($\Delta\tau_{c,t}^z = \Delta\tau_{c,t}^\ell = \tau_{j,t}^z = \Delta\tau_{j,t}^\ell = 0$ for $j \neq$ Construction). This interpretation of Figure 10 raises the question of how to account for the reduction in labor associated with $\Delta\tau_{j,t}^\ell$ in the construction sector. Did it disappear, as implicitly assumed in Panels A and C, or did it move to other sectors? Panel D shows the counterfactual historical path of trend growth rates

Figure 11: Model-Based Historical Decomposition of Trend GDP Growth Rates: Contributions from Non-durable Goods



Notes: See notes to Figure 10.

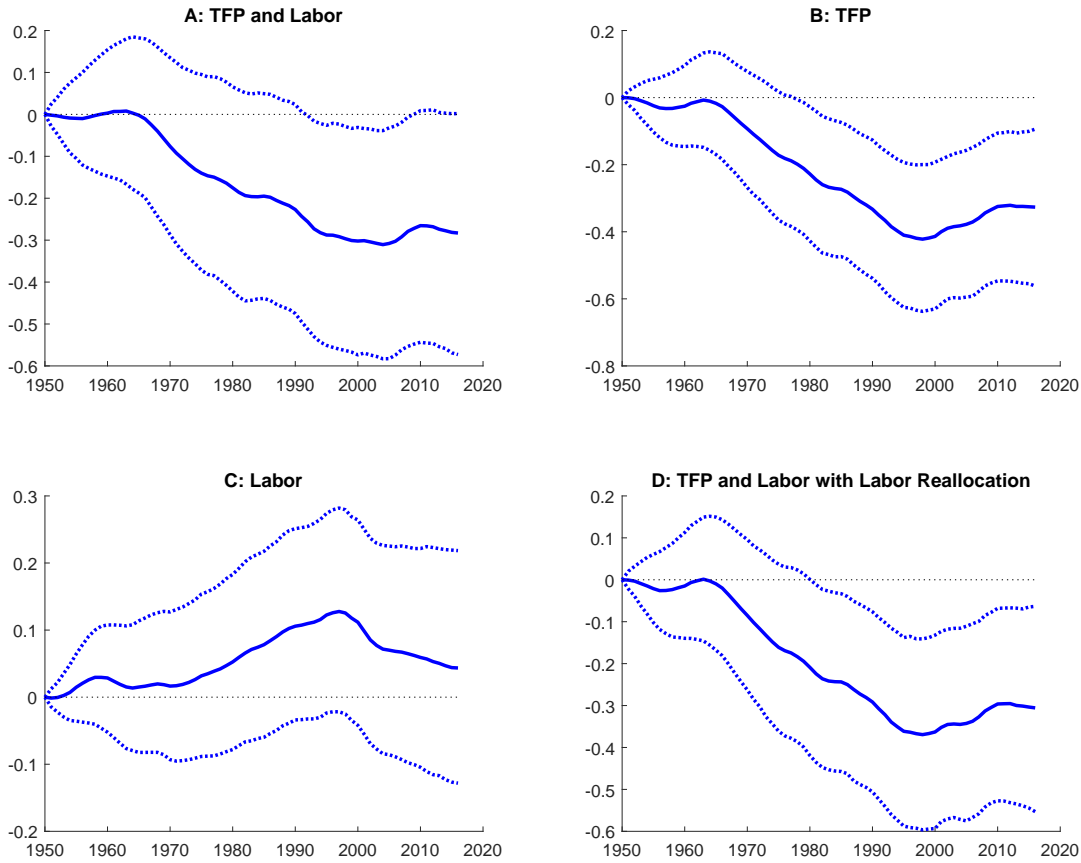
in GDP after allocating all of the sectoral losses of labor in Construction to the other sectors in proportion to their labor shares, so there is no change in the aggregate supply of labor.²⁸ Because the behavior of labor growth is not the dominant force driving secular changes in the Construction sector, panels A and D show similar historical paths.

Figures 11 and 12 repeat this exercise for two other important sectors: Nondurable Goods and Professional and Business Services. (Results for the other sectors are shown in the Technical Appendix.) Taken together, Figures 11 and 12 underscore the importance of sector-specific structural change for trend GDP growth. Remarkably, the Construction, Nondurable Goods, and Professional and Business Services sectors account for around almost 60 percent of the 2.3 percentage point decrease in trend GDP growth since 1950.

In contrast, Figure 13 illustrates the implications of the common or aggregate trends in labor and

²⁸Recall also our discussion of the limits on cross-sectoral substitution of KLEMS sectoral labor measures in footnote 16.

Figure 12: Model-Based Historical Decomposition of Trend GDP Growth Rates: Contributions from Professional and Business Services



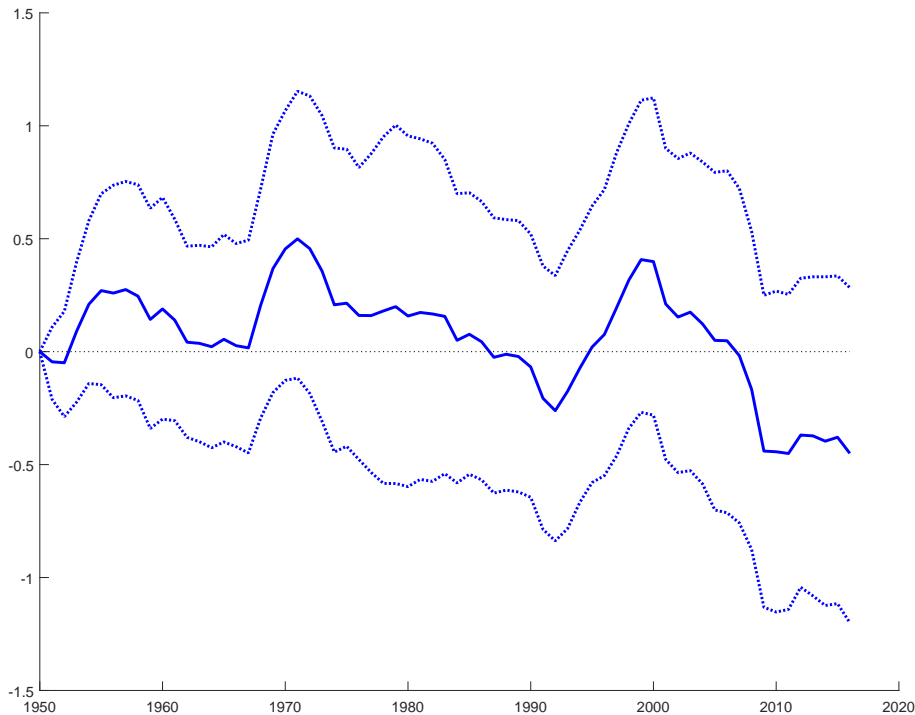
Notes: See notes to Figure 10.

TFP growth estimated in Section 4. Figure 13 suggests that these common trends are responsible for 0.4 percentage points or 17 percent of the aggregate trend decline in GDP growth since 1950. Thus, sector-specific rather than aggregate factors shaped the major part of the secular behavior of GDP growth over the period 1950 – 2016.

5.3 Implications for Future Trend Growth

Given the trends in labor growth and TFP growth estimated in this paper, and their implications for the aggregate U.S. economy through sectoral multipliers, a natural question is: what do those trends imply going forward? As indicated by equation (18), disturbances to trend labor growth and TFP growth are endogenously propagated over time through investment decisions and thus have persistent effects. The random walk dynamics of the trend components of labor growth and TFP growth imply that forecasts of

Figure 13: Model-Based Historical Decomposition of Trend GDP Growth Rates: Contributions from Common Trends



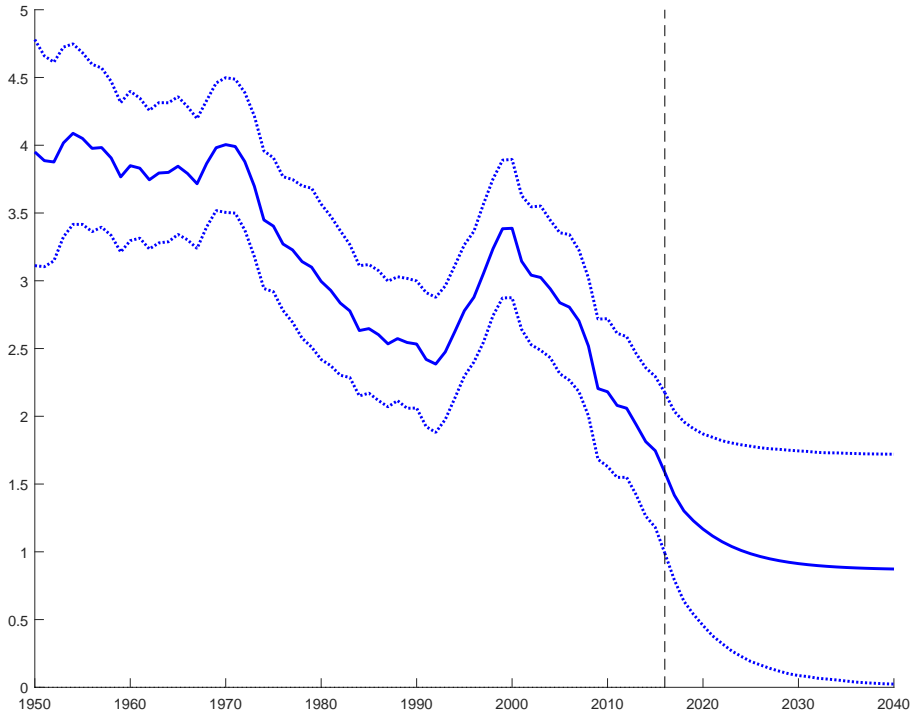
Notes: This figure show the effect of the common trend shocks ($\Delta\tau_{c,t}^z$ and $\Delta\tau_{c,t}^\ell$) on the growth rate of GDP using the model-based dynamic multipliers. Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters.

these components are equal to their value in 2016, the last year of the data.

Figure 14 shows predicted trend growth rates for GDP from the dynamic multi-sector model as past disturbances play out through the model’s internal dynamics. The figure indicates that absent the realization of positive and persistent disturbances to TFP and labor growth, trend GDP growth will continue to fall by around 0.60 percentage points over the next 10 years. In this scenario, both trend labor and TFP growth remain constant, but the internal dynamics of capital accumulation imply continued slowing of trend GDP growth. The 68 percent credible bands indicate there is substantial uncertainty about the exact level of trend GDP going forward but, absent positive shocks, the model predicts a slowdown ahead nonetheless.

Table 8 shows the contributions of the common and sector-specific trend components in generating the decline in trend GDP growth of around 0.60 percentage points going forward. While there is a role for common trends, they play a smaller role than the sector-specific trends, particularly in TFP growth. The decomposition into each individual sector shows that a decline in Durable Goods is the key driver of lower trend GDP growth in the future. The fall in large part reflects the propagation of the collapse associated

Figure 14: In-sample and predicted out-of-sample trend rate of growth in GDP



Notes: Blue dots indicate 68 percent credible intervals conditional on balanced growth model parameters.

with the Durable Goods sector that began in 2000 (see Figure 9). This effect is mitigated somewhat by a turn-around in Construction. However, the marked slowdown in Durable Goods affects other sectors through production linkages, and through capital accumulation implies a pervasive impact that is sustained 10 years into the future.

6 Concluding Remarks

In this paper, we estimate trends in TFP and labor growth across major U.S. production sectors and explore the role they have played in shaping the secular behavior of GDP growth. We find that trends in TFP and labor growth have generally decreased across a majority of sectors since 1950. More than 2/3 of the secular decline in aggregate TFP growth results from the combination of sector-specific rather than aggregate disturbances. Similarly, trend labor growth has also been dominated by sector-specific factors, especially after 1980 and the latter part of the post-war period.

We embed these findings into a dynamic multi-sector framework in which materials and capital used by different sectors are produced by other sectors. The presence of capital, in particular, allows changes

Table 8: Model-based Forecasts of Changes in Trend GDP Growth

	2016 – 2021			2021 – 2026		
	VA	Labor	TFP	VA	Labor	TFP
Aggregate	-0.47	-0.16	-0.31	-0.15	-0.03	-0.12
Common	-0.13	-0.06	-0.07	-0.02	0.00	-0.02
Sector specific (<i>total</i>)	-0.34	-0.10	-0.24	-0.13	-0.04	-0.10
Sector specific (<i>by sector</i>)						
Agriculture	-0.00	-0.00	0.00	-0.00	-0.00	-0.00
Mining	-0.02	-0.01	-0.01	-0.01	-0.00	-0.00
Utilities	-0.00	-0.00	0.00	-0.00	-0.00	0.00
Construction	0.04	-0.01	0.05	0.02	-0.00	0.02
Durable Goods	-0.34	-0.09	-0.26	-0.14	-0.03	-0.11
Nondurable Goods	0.00	0.00	0.00	0.00	-0.00	0.00
Wholesale Trade	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00
Retail Trade	0.00	0.00	0.00	-0.00	0.00	-0.00
Trans. & Ware.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Information	-0.00	-0.00	0.00	0.00	-0.00	0.00
FIRE	-0.01	0.00	-0.01	-0.00	0.00	-0.00
PBS	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
Educ. & Health	-0.00	0.01	-0.01	-0.00	0.00	-0.00
Ent. & FS	-0.00	-0.00	0.00	0.00	-0.00	0.00
Oth. serv.	0.00	0.00	0.00	0.00	0.00	-0.00

in TFP or labor growth in a given sector to affect value added growth in every other sector. This feature leads to quantitatively important sectoral multiplier effects on GDP growth that reflect the importance of different sectors as suppliers of capital or materials to other sectors. The strength of these linkages result in GDP growth multipliers that for some sectors can be as large 3 times their value added share.

Ultimately, sector-specific rather than aggregate factors in TFP and labor growth explain the major part of low frequency variations in U.S. GDP growth. Changing sectoral trends in the last 6 decades, translated through the economy's production network, have on net lowered trend GDP growth by around 2.3 percentage points. The Construction sector, more than any other sector, stands out for its contribution to the trend decline in GDP growth over the post-war period, accounting for 30 percent of this decline. Moreover, the process of capital accumulation means that these structural changes have endogenously persistent effects.

Thus, absent the realization of predominantly positive and persistent disturbances to TFP and labor growth, we estimate that trend GDP growth will continue to fall over the next 10 years.

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Appendix A: Estimation of the Multivariate Unobserved Components Model

The DFM has the form: $\Delta \ln x_{j,t} = \lambda_{j,\tau}^x \tau_{c,t}^x + \lambda_{j,\varepsilon}^x \varepsilon_{c,t}^x + \tau_{j,t}^x + \varepsilon_{j,t}^x$, where $x = z$ or ℓ and $j = 1, \dots, n$. The τ components evolve as random walks and the ε components as white noise. More specifically, we assume that the $2(n+1) \times 1$ vector $\zeta_t = \left(\Delta \tau_{c,t}^x, \varepsilon_{c,t}^x, \{\Delta \tau_{j,t}^x, \varepsilon_{j,t}^x\}_{j=1}^n \right)$ is *i.i.d.* with $\zeta \sim N(0, \Sigma_\zeta)$, where Σ_ζ is a diagonal matrix. The factor loadings and scale of the common factors are not separately identified; thus we normalize $var(\Delta \tau_{c,t}^x) = var(\varepsilon_{c,t}^x) = 1$, $\sum_{j=1}^n \lambda_{j,\tau}^x \geq 0$, and $\sum_{j=1}^n \lambda_{j,\varepsilon}^x \geq 0$ to identify the scale and sign of the factors.

We estimate the model using Bayesian methods. We use independent priors for all the parameters. The priors for the factor loadings are $\lambda_{j,\tau}^x \sim N(0, 1)$ and $\lambda_{j,\varepsilon}^x \sim N(0, 16)$. The priors for $var(\Delta \tau_{j,t}^x)$ and $var(\varepsilon_{j,t}^x)$ are inverse Gamma priors with shape and scale parameters that correspond to T_{prior} observations with $s_{prior}^2 = \omega_{prior}^2 / T_{prior}$. We use $T_{prior} = 2$ and $s_{prior}^2 = 0.25^2$ for $var(\Delta \tau_{j,t}^x)$ and $T_{prior} = 2$ and $s_{prior}^2 = 3.00^2$ for $var(\varepsilon_{j,t}^x)$.

The posterior is approximated using Gibbs-MCMC methods. Let X denote the data, ζ denote the values for ζ_t for $t = 1, \dots, T$, and θ denote the parameter values $(\lambda_{j,\tau}^x, \lambda_{j,\varepsilon}^x, var(\Delta \tau_{j,t}^x), var(\varepsilon_{j,t}^x))$ for $j = 1, \dots, n$. The first Gibbs step draws $\zeta | (X, \theta)$ using standard formulae related to the Kalman smoother. The second Gibbs step draws θ from the posterior of $\theta | (X, \zeta)$ that is also known in closed form from familiar linear regression formulae. The estimates reported in the paper rely on one million MCMC draws after 5k initial draws.

The Technical Appendix contains additional details.

Appendix B: The Influence Vector in a Model with Capital

The competitive equilibrium of the economy described in the main text is defined as a set of gross output prices, $p_{j,t}^y$, value added prices, $p_{j,t}^v$, materials prices, $p_{j,t}^m$, and investment goods prices, $p_{j,t}^x$, for all sectors j and dates t , and allocations, $(c_{j,t}, C_t, x_{ij,t}, m_{ij,t}, x_{j,t}, m_{j,t}, v_{j,t}, y_{j,t}, k_{j,t+1}, \ell_{j,t})$, such that

- i) the representative household maximizes its utility,
- ii) the representative firm in each sector maximizes profits
- iii) all markets clear.

Absent frictions, equilibrium allocations may be found by solving a planner's problem whose corresponding decentralization and definitions of prices are straightforward.

Suppose that sectoral structural changes embodied in $\{z_{j,t}, \ell_{j,t}\}$ are stationary in levels, with non-stochastic steady state denoted by $\{z_j, \ell_j\}$. Then, in the absence of shocks, the economy converges to a steady state in the long-run indexed by $A = (A_1, \dots, A_n)$, where $A_j = z_j + (1 - \alpha_j)\ell_j$, $j = 1, \dots, n$.

In the steady state, first-order conditions imply a set of relationships between the prices of materials, investment, and value added,

$$\Theta \ln p^y = 0,$$

$$\begin{aligned}\ln p^v &= \Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] \ln p^y, \\ \ln p^x &= \Omega' \ln p^y,\end{aligned}$$

where $p^v = (p_1^v, \dots, p_n^v)'$, $p^x = (p_1^x, \dots, p_n^x)'$, and $p^y = (p_1^y, \dots, p_n^y)'$.

From the definition of value added, we have that

$$\ln v_j = \ln A_j + \alpha_j \ln \left(\frac{k_j}{\alpha_j} \right),$$

and from the Euler equation governing the optimal choice of capital in each sector,

$$\frac{k_j}{\alpha_j} = \left(\frac{p_j^v v_j}{p_j^x} \right) \left(\frac{\beta}{1 - \beta(1 - \delta_j)} \right).$$

Combining these expressions gives

$$\ln v_j = \ln A_j + \alpha_j \ln (p_j^v v_j) - \alpha_j \ln p_j^x + \alpha_j \ln \left(\frac{\beta}{1 - \beta(1 - \delta_j)} \right),$$

or in matrix form,

$$(I - \alpha_d) \ln (p^v \cdot \times v) = \ln A + \ln p^v - \alpha_d \ln p^x + \alpha_d \ln \Delta_d,$$

where $(p^v \cdot \times v)$ represents the vector of nominal value added, $\{p_j^v v_j\}$, $\alpha_d = \text{diag}(\alpha_j)$, and $\Delta_d = \text{diag} \left(\frac{\beta}{1 - \beta(1 - \delta_j)} \right)$. Substituting for investment and value added prices on the right-hand-side of this last expression, we obtain

$$\ln p^y = (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d \Omega')^{-1} [(I - \alpha_d) \ln (p^v \cdot \times v) - \ln A - \alpha_d \ln \Delta_d]. \quad (\text{B.1})$$

From the resource constraints in each sector j , and the optimal allocation of materials and investment in the economy, it follows that

$$\frac{p_j^v v_j}{\gamma_j} = \theta_j C + \sum_{i=1}^n \phi_{ji} (1 - \gamma_j) \frac{p_i^v v_i}{\gamma_i} + \sum_{i=1}^n \omega_{ji} \frac{\beta \delta_j}{1 - \beta(1 - \delta_j)} \alpha_i p_i^v v_i,$$

or in matrix form,

$$\Gamma_d^{-1} (p^v \cdot \times v) = \Theta' C + \Phi (I - \Gamma_d) \Gamma_d^{-1} (p^v \cdot \times v) + \Omega \Delta_d \delta_d \alpha_d (p^v \cdot \times v), \quad (\text{B.2})$$

so that

$$\frac{(p^v \cdot \times v)}{C} = ([I - \Phi(I - \Gamma_d)] \Gamma_d^{-1} - \Omega \Delta_d \delta_d \alpha_d)^{-1} \Theta'.$$

Aggregate GDP (in units of the final consumption bundle) is then given by

$$V = \mathbf{1}'(p^v \cdot v) = \mathbf{1}'\psi C,$$

where $\psi = ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1}\Theta'$.

Substituting the expression for value added in (B.2) into the equation for gross output prices, (B.1), and using the fact that the ideal price index for the final consumption bundle implies $\Theta \ln p^y = 0$, we obtain

$$\ln C = \frac{\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} [\ln A + \alpha_d \ln \Delta_d - (I - \alpha_d) \ln \psi]}{\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} (I - \alpha_d)\mathbf{1}},$$

where note that $\Theta (\Gamma_d^{-1} [I - (I - \Gamma_d)\Phi'] - \alpha_d\Omega')^{-1} (I - \alpha_d)\mathbf{1} = 1$.

It follows that

$$\frac{\partial \ln V}{\partial \ln A_j} = \frac{\partial \ln C}{\partial \ln A_j} = \left\{ ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta' \right\}_j.$$

At the same time, the vector of value added shares or influence vector, s^v , is given by

$$\frac{(p^v \cdot v)}{V} = \frac{([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1} \Theta'}{\mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\Delta_d\delta_d\alpha_d)^{-1} \Theta'}.$$

Therefore, in the limit case where $\beta \rightarrow 1$, $\Delta_d\delta_d \rightarrow I$, and

$$\frac{\partial \ln V}{\partial \ln A_j} = \eta s_j^v,$$

where $\eta = \mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'$ is approximately the inverse of the mean labor share across sectors. To see this, observe that η can also be expressed as $\eta = \frac{\mathbf{1}' ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'}{\mathbf{1}' (I - \alpha_d) ([I - \Phi(I - \Gamma_d)]\Gamma_d^{-1} - \Omega\alpha_d)^{-1} \Theta'}$. Thus, when $\alpha_j = \alpha \forall j$, $\eta = \frac{1}{1-\alpha}$.

Appendix C: Balanced Growth with Sectoral Linkages in Materials and Investment

We begin by normalizing some of the model's key variables with respect to sector-specific factors, $\mu_{j,t}$, to be determined below, so as to yield a system of equations that is stationary in the normalized variables. If all growth rates are constant, the resource constraint in any individual sector implies that all variables in that constraint must grow at the same rate along a balanced growth path. Thus, define $\tilde{y}_{j,t} = y_{j,t}/\mu_{j,t}$,

$\tilde{c}_{j,t} = c_{j,t}/\mu_{j,t}$, $\tilde{m}_{ji,t} = m_{ji,t}/\mu_{j,t}$, and $\tilde{x}_{ji,t} = x_{ji,t}/\mu_{j,t}$. Then, the economy's resource constraint becomes

$$\tilde{c}_{j,t} + \sum_{i=1}^n \tilde{m}_{ji,t} + \sum_{i=1}^n \tilde{x}_{ji,t} = \tilde{y}_{j,t}.$$

Similarly, the production of investment goods may be re-written as

$$\tilde{x}_{j,t} = \prod_{j=1}^n \left(\frac{\tilde{x}_{ij,t}}{\omega_{ij}} \right)^{\omega_{ij}},$$

where $\tilde{x}_{j,t} = x_{j,t}/\prod_{j=1}^n \mu_{i,t}^{\omega_{ij}}$, while the capital accumulation equation becomes

$$\tilde{k}_{j,t+1} = \tilde{x}_{j,t} + (1 - \delta_j) \tilde{k}_{j,t} \prod_{j=1}^n \left(\frac{\mu_{i,t-1}}{\mu_{i,t}} \right)^{\omega_{ij}},$$

where $\tilde{k}_{j,t+1} = k_{j,t+1}/\prod_{j=1}^n \mu_{i,t}^{\omega_{ij}}$.

The expression for value added may be written as

$$v_{j,t} = A_{j,t} \left(\frac{\tilde{k}_{j,t} \prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}}}{\alpha_j} \right)^{\alpha_j},$$

so that, defining $\tilde{v}_{j,t} = v_{j,t}/A_{j,t} \left(\prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}$, it becomes

$$\tilde{v}_{j,t} = \left(\frac{\tilde{k}_{j,t}}{\alpha_j} \right)^{\alpha_j}.$$

The composite bundle of materials used in sector j may be expressed as

$$\tilde{m}_{j,t} = \prod_{j=1}^n \left(\frac{\tilde{m}_{ij,t}}{\phi_{ij}} \right)^{\phi_{ij}},$$

with $\tilde{m}_{j,t} = m_{j,t}/\prod_{j=1}^n \mu_{i,t}^{\phi_{ij}}$. Therefore, gross output in terms of detrended variables is given by

$$\tilde{y}_{j,t} \mu_{j,t} = \left(\frac{\tilde{v}_{j,t} A_{j,t} \prod_{j=1}^n \mu_{i,t-1}^{\alpha_j \omega_{ij}}}{\gamma_j} \right)^{\gamma_j} \left(\frac{\tilde{m}_{j,t} \prod_{j=1}^n \mu_{i,t}^{\phi_{ij}}}{1 - \gamma_j} \right)^{1 - \gamma_j},$$

or alternatively,

$$\tilde{y}_{j,t} = \left(\frac{\tilde{v}_{j,t}}{\gamma_j} \right)^{\gamma_j} \left(\frac{\tilde{m}_{j,t}}{1 - \gamma_j} \right)^{1 - \gamma_j} \left[\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}} \mu_{i,t}^{(1 - \gamma_j) \phi_{ij}} \right].$$

We can now use the expression in square brackets to solve for the normalizing factors, $\mu_{j,t}$, as a function of the model's underlying parameters.

First, re-write the term in square brackets as

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \left(\prod_{j=1}^n \frac{\mu_{i,t-1}^{\gamma_j \alpha_j \omega_{ij}}}{\mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}}} \right) \left(\prod_{j=1}^n \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}} \right),$$

where this last expression involves the growth rate of $\mu_{i,t}$. Then, along a balanced growth path where all variables grow at constant rates, the term $\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t}^{(1-\gamma_j)\phi_{ij}} \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij}}$ must be constant. Thus, we choose $\mu_{j,t}$ such that

$$\frac{A_{j,t}^{\gamma_j}}{\mu_{j,t}} \prod_{j=1}^n \mu_{i,t}^{\gamma_j \alpha_j \omega_{ij} + (1-\gamma_j)\phi_{ij}} = 1.$$

This choice is without loss of generality since any other constant in place of 1 will be differenced out along the balanced growth path.

Taking logs of both sides of the above expression, we have

$$\gamma_j \ln A_{j,t} - \ln \mu_{j,t} + \sum_{i=1}^n (\gamma_j \alpha_j \omega_{ij} + (1-\gamma_j)\phi_{ij}) \ln \mu_{i,t} = 0,$$

or in vector form,

$$\Gamma_d \ln A_t - \ln \mu_t + \Gamma_d \alpha_d \Omega' \ln \mu_t + (I - \Gamma_d) \Phi' \ln \mu_t = 0,$$

which gives us

$$\ln \mu_t = \Xi' \ln A_t \tag{C.1}$$

where

$$\Xi' = (I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d$$

is the Leontief inverse, $(I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1}$, diagonally weighted by value added shares in gross output, Γ_d .

Going back to equation (11) in the main text, and writing the vector of productivity growth rates as $\Delta \ln A_t = \bar{g}_a$, it follows that

$$\Delta \ln \mu_t = \Xi' \bar{g}_a = \Xi' (\bar{g}_z + (I - \alpha_d) \bar{g}_\ell).$$

Thus, let $\mu_{j,t}^v$ denote the normalizing factor for value added in sector j defined above,

$$\mu_{j,t}^v = A_{j,t} \left(\prod_{j=1}^n \mu_{i,t-1}^{\omega_{ij}} \right)^{\alpha_j}.$$

Then, using equation (C.1), we have that

$$\ln \mu_t^v = \ln A_t + \alpha_d \Omega' \Xi' \ln A_{t-1},$$

or, along a steady state growth path,

$$\Delta \ln \mu_t^v = \left[I + \alpha_d \Omega' \underbrace{(I - \alpha_d \Gamma_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d}_{\Xi'} \right] \bar{g}_a. \quad (\text{C.2})$$

Appendix D: The Dynamics of Value Added Growth

We wish to recover the implications of the model for sectoral value added growth, $\Delta \ln v_{j,t}$, and aggregate GDP growth, $\Delta \ln V_t$.

Observe that

$$\Delta \ln \tilde{v}_{j,t} = \alpha_j \Delta \ln \tilde{k}_{j,t},$$

and that $\Delta \ln \tilde{k}_{j,t} = \Delta \hat{k}_{j,t}$, where $\Delta \hat{k}_{j,t}$ follows directly from the Markov decision rules. Then, by definition of $\tilde{v}_{j,t}$,

$$\begin{aligned} \Delta \ln v_{j,t} &= \Delta \ln \tilde{v}_{j,t} + \Delta \ln A_{j,t} + \sum_{i=1}^n \alpha_j \omega_{ij} \Delta \ln \mu_{i,t-1} \\ &= \alpha_j \Delta \hat{k}_{j,t} + \hat{A}_{j,t} + \bar{g}_j + \sum_{i=1}^n \alpha_j \omega_{ij} \Delta \ln \mu_{i,t-1}, \end{aligned}$$

or, in vector form,

$$\Delta \ln v_t = \alpha_d \Delta \hat{k}_t + \hat{A}_t + \bar{g}_a + \alpha_d \Omega' \Delta \ln \mu_{t-1}.$$

The sectoral detrending factors, $\mu_{j,t}$, solve

$$\Delta \ln \mu_t = \Xi' \Delta \ln A_t,$$

where

$$\Xi' = (I - \Gamma_d \alpha_d \Omega' - (I - \Gamma_d) \Phi')^{-1} \Gamma_d.$$

Then,

$$\Delta \ln v_t = \alpha_d \Delta \hat{k}_t + \hat{A}_t + \bar{g}_a + \alpha_d \Omega' \Xi' (\hat{A}_{t-1} + \bar{g}_a)$$

or

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a + \alpha_d \Delta \hat{k}_t + \hat{A}_t + \alpha_d \Omega' \Xi' \hat{A}_{t-1},$$

with $\Delta \ln V_t = s^v \Delta \ln v_t$. When all shocks are set to zero, so that $\Delta \hat{k}_t = \hat{A}_t = \hat{A}_{t-1} = 0$, we recover the sectoral balanced growth paths for sectoral value added,

$$\Delta \ln v_t = (I + \alpha_d \Omega' \Xi') \bar{g}_a.$$