Imperfect Information and Wage Inertia in the Business Cycle: A Comment

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In a recent article, Gertler (1982) presents a model that explains certain stylized facts concerning the relative behavior of the money supply and nominal wages. One additional feature of the model is the behavior of real output, which is found to have a serially correlated response to a money supply shock. This serial correlation appears to arise from an information structure that precludes agents from directly disentangling permanent versus transitory movements in relevant state variables. This information structure complicates the forecasting problem that must be solved to analyze the model and affects the behavior of wages and the variance of output. It will not, however, lead to serial correlation in real output. An algebraic error is responsible for the serial correlation found by Gertler.

This note corrects the error and presents an alternative solution to the forecasting problem. Gertler constructs forecasts of the relevant state variable through the use of a Kalman filter. While this technique is appropriate for the problem, a simpler solution is available. This solution is based on an equivalent representation of the stochastic process generating the disturbances of the model. This equivalent representation can then be used, in a straightforward way, to describe wage dynamics and the variance of real output.

It is sufficient to analyze Gertler's model in the case where there are only monetary disturbances. The relevant equations are

$$w_t = m_{t|t-1}, \tag{1}$$

$$y_t - y_t^* = \pi_2(m_t - m_{t|t-1}), \tag{2}$$

$$m_t = \overline{m}_t + \Gamma_t, \tag{3}$$

$$\overline{m}_t = \rho_2\overline{m}_{t-1} + \eta_t, \quad 0 < \rho_2 < 1, \tag{4}$$

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where (defined as logarithms of deviations from trend) y_t is real output, y_t^* is the market-clearing value of y_t, w_t is the nominal wage, m_t is the money supply, $\overline{m}_t$ is the "permanent" component of m_t, π_2 is a positive constant, $\{\Gamma_t\}$ and $\{\eta_t\}$ are independent white-noise sequences, and $x_{t-k|t-j} = E_{t-j}(x_{t-k})$ for any variable x. My equations (1) and (2) describe the reduced forms for w_t and y_t, given a structural setup where nominal wages are fixed at the beginning of the period, before the realization of the shocks Γ and η. My equations (3) and (4) describe the evolution of the money supply. Agents observe m_t, but not its components, the "permanent" component, $\overline{m}_t$, and the "transitory" component, Γ_t. The forecast $m_{t|t-1}$ and forecast error $m_t - m_{t-1}$ can be expressed as

$$m_{t|t-1} = \rho_2 \overline{m}_{t-1|t-1}, \tag{5}$$

$$m_t - m_{t|t-1} = m_t - \rho_2 \overline{m}_{t-1|t-1}. \tag{6}$$

The logic behind this solution to the forecasting problem is straightforward. Suppose that we are at time $t - 1$ and seek to forecast m_t, then clearly only $\overline{m}_{t-1}$ matters. Since $\overline{m}_{t-1}$ is unobserved, we must use $\overline{m}_{t-1|t-1}$, the estimate of $\overline{m}_{t-1}$ constructed from $m_{t-1}, m_{t-2}, \ldots$. The Kalman filter is an appropriate device for forming this estimate, and one finds

$$\overline{m}_{t-1|t-1} = (1 - d_m)\overline{m}_{t-1|t-2} + d_m m_{t-1}, \tag{7}$$

where d_m is derived from the steady-state Kalman filter equations (see Gertler's eqq. [14]–[16]).

Gertler's algebraic mistake arises in the computation of d_m. The coefficient δ_2 that enters d_m in his equation (16) should be divided by ρ_2^2.[1] The correct formulation yields a filter that converts the right-hand side of my equation (6) into white noise.[2] Though the forecast errors of the unobserved variable $\overline{m}_t$ are serially correlated, the same cannot be true for the prediction errors of the observed component m_t. The latter result from optimal forecasts, using proper information sets. Equation (2) therefore implies that $y_t - y_t^*$ must be serially uncorrelated.

To solve the forecasting problem without the algebraic solution of

[1] Also, the expression for the constant π_1 in Gertler's eq. (12) should be amended to $\alpha\beta/[1 + \beta(1 - \alpha)]$. This correction does not affect the analysis.

[2] The algebraic mistake has only a minuscule effect on the numerical computations in Gertler's examples. Therefore, the correction does not significantly affect his fig. 1, which portrays the variation of wages relative to the money supply for various cases. His fig. 3, the spectrum of $y_t - y_t^*$, is incorrect. It suggests slight serial correlation, when in fact it should portray white noise. However, Gertler does mention that the model could be modified to produce significant persistent real effects of monetary shocks by including adjustment costs, e.g.

the steady-state Kalman filter, note that m_t has an ARMA (1, 1) representation. To see this, rewrite my equations (3) and (4) as

$$(1 - \rho_2 B)m_t = \eta_t + (1 - \rho_2 B)\Gamma_t, \tag{8}$$

where B is the backshift operator. Since η_t and Γ_t are white noise, the autocorrelation of the right-hand side of my (8) is nonzero for lag one and zero for all lags greater than one. We can therefore apply Granger's lemma (see Anderson 1975) and rewrite (8) as

$$(1 - \rho_2 B)m_t = (1 - \theta B)\mu_t, \quad 0 < \theta < 1, \tag{9}$$

where μ_t is white noise. The one-step-ahead forecast and forecast errors are therefore

$$m_{t|t-1} = \rho_2 m_{t-1} - \theta\mu_{t-1}, \tag{10}$$

$$m_t - m_{t|t-1} = \mu_t. \tag{11}$$

To see the relationship between this formulation and the Kalman filter formulation, note that the properties of the Kalman filter imply that $\rho_2\overline{m}_{t-1|t-1}$ is the linear minimum mean square error predictor of m_t. But so is $m_{t|t-1}$, given in (10), so that $\overline{m}_{t-1|t-1} = m_{t-1} - (\theta_2/\rho_2)\mu_{t-1}$.

The salient features of wage dynamics presented in Gertler's figure 1 can now be derived. My equations (1) and (10) imply that $(1 - \rho_2 B)w_t = (\rho_2 - \theta)\mu_{t-1}$, so that w_t follows an AR (1) process with innovation $(\rho_2 - \theta)\mu_{t-1}$. From my equation (9), m_t follows an ARMA (1, 1) process with innovation μ_t, and the ratio of the spectrum of w_t to the spectrum of m_t is

$$\frac{s_w(\omega)}{s_m(\omega)} = (\rho_2 - \theta)^2(1 + \theta^2 - 2\theta \cos \omega)^{-1}. \tag{12}$$

This, of course, is just the spectrum of an AR (1) process with autoregressive parameter θ and innovation variance $(\rho_2 - \theta)^2$.

Figure 1 of Gertler shows my (12) for various values of $\lambda_2 = \sigma_\Gamma^2/\sigma_\eta^2$. To see the relationship between λ_2 and θ, equate the first autocorrelation of the right-hand side of my equation (8) with that of the right-hand side of (9), that is, $\theta/(1 + \theta^2) = \rho_2/(\lambda_2^{-1} + 1 + \rho_2^2)$. Therefore, θ is an increasing function of λ_2 with $\lim_{\lambda_2 \to 0} \theta = 0$ and $\lim_{\lambda_2 \to \infty} \theta = \rho_2$. As λ_2 increases $s_w(\omega)/s_m(\omega)$ falls because of the first term on the right-hand side of (12) and becomes more concentrated in the low frequencies because of the second term.

In a private communication, Gertler has shown that my equations (8) and (9) can also be used to demonstrate that an increase in the variance of either of the shocks to the money supply will have a magnified effect on the variance of output.[3] From (2) and (9), $y_t - y_t^*$

[3] This type of result is suggested by Gertler (p. 977) but was not shown explicitly.

$= \pi_2(m_t - m_{t|t-1}) = \rho_2\mu_t$, so that var $(y_t - y_t^*) \equiv \sigma_y^2 = \pi_2^2\sigma_\mu^2$. From (8) and (9),

$$\sigma_\mu^2 = \frac{(1 + \rho_2^2)\sigma_\Gamma^2 + \sigma_\eta^2}{1 + \theta^2}$$

and

$$\frac{\partial\sigma_\mu^2}{\partial\sigma_\Gamma^2} = \frac{1 + \rho_2^2 - 2\rho_2\theta}{1 - \theta^2} \geq 1, \quad \frac{\partial\sigma_\mu^2}{\partial\sigma_\eta^2} = \frac{1}{1 - \theta^2} \geq 1.$$

Furthermore,

$$\lim_{\lambda_2 \to 0} \frac{\partial\sigma_\mu^2}{\partial\sigma_\Gamma^2} = 1 + \rho_2^2$$

and

$$\lim_{\lambda_2 \to \infty} \frac{\partial\sigma_\mu^2}{\partial\sigma_\Gamma^2} = 1.$$

The intuition behind this result is reasonably straightforward. Suppose, for example, that λ_2 is very small so that most of the shock to money is permanent. An increase in the variance of the transitory shock has two effects. First, it adds a direct unforecastable component to the money supply. Second, it decreases the information in the money supply concerning permanent movements. This leads to less precise estimates of the permanent component and less accurate forecasts of future values of the money supply.

The main purpose of Gertler's model was to explain the observed dynamics of nominal wages. His discussion of serial correlation in output was a side issue. The analysis of wage dynamics that culminated in his equation (17) is correct. The algebraic error merely produces slightly different lag weights in this equation. In many models, however, a primary concern is to include rigidities or information structures that lead to serial correlation in output. The main purpose of this note has been to point out that the information structure used by Gertler is not sufficient for this purpose.

References

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