Core Inflation and Trend Inflation

– – Appendix – –

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This appendix contains a detailed description of the estimation methods for the univariate and multivariate UCSVO models, and contains additional empirical results.

1. The univariate model

1.1 The model:

$$\pi_t = \tau_t + \varepsilon_t$$
$$\tau_t = \tau_{t-1} + \sigma_{\Delta\tau,t} \times \eta_{\tau,t}$$
$$\varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t}.$$
$$\Delta\ln(\sigma^2_{\varepsilon,t}) = \gamma_\varepsilon \nu_{\varepsilon,t}$$
$$\Delta\ln(\sigma^2_{\Delta\tau,t}) = \gamma_{\Delta\tau} \nu_{\Delta\tau,t}$$

where (η_ε, η_τ, ν_ε, $\nu_{\Delta\tau}$) are iidN(0, I_4), and s_t ~ i.i.d and independent of (η_ε, η_τ, ν_ε, $\nu_{\Delta\tau}$) with

$$s_t = \begin{cases} 1 \text{ with probability } (1-p) \\ U[2,10] \text{ with probability } p \end{cases}$$

1.2 Priors:

γ_ε, $\gamma_{\Delta\tau}$, p, τ_0, $\ln(\sigma_{\varepsilon,0})$, and $\ln(\sigma_{\Delta\tau,0})$ are independent, with:

$\gamma_\varepsilon \sim U[0, 0.40/\sqrt{np}\ ]$, where np denote the number of periods per year (np = 4 for quarterly data and np = 12 for monthly data);

$\gamma_{\Delta\tau} \sim U[0, 0.40/\sqrt{np}\ ]$;

$p \sim \text{Beta}(\alpha, \beta)$ with $\alpha = (1/(4np))\times(10np)$ and $\beta = [1 - (1/(4np))]\times(10np)$;

and (τ_0, $\ln(\sigma_{\varepsilon,0})$, $\ln(\sigma_{\Delta\tau,0})$) ~ $N(0,\kappa I_3)$, with $\kappa = 10^6$.

The U[2,10] distribution for the values of s_t is approximated with an equally-spaced grid of 9 points. The uniform distributions for γ are approximated by an equally spaced grid of 5 points.

1.3 Approximation of the posterior:

The mean and quantiles of the posterior are approximated by MCMC draws. The details are as follows.

1.3.1 Kim-Shephard-Chib (KSC) (1998) approximate model for stochastic volatility:

Background: A review of KSC approach

Let $x_t = \sigma_t\eta_t$ and $\ln(\sigma_t^2) = \ln(\sigma_{t-1}^2) + \gamma\nu_t$ with $(\eta_t, \nu_t) \sim \text{iidN}(0,I_2)$. Then

$$\ln(x_t^2) = \underline{\ln}(\sigma_t^2) + \ln(\eta_t^2)$$
$$\ln(\sigma_t^2) = \ln(\sigma_{t-1}^2) + \gamma\nu_t$$

which is a linear state-space model with non-Gaussian measurement error $\ln(\eta_t^2) \sim \ln(\chi_1^2)$. KSC suggest approximating the $\ln(\chi_1^2)$ distribution by a mixture of normals so that

$\ln(\eta_t^2) \sim \sum_{i=1}^{n} w_{it}a_{it}$, where w_{it} are iid (0-1) variables with $w_{it} = 1$ for only one value of i at each t, and with $p(w_{it}=1)=p_i$. The a_{it} variables are Gaussian with $a_{it} \sim N(\mu_i, \sigma_i^2)$.

KSC propose an $n = 7$ component Gaussian mixture to approximate the $\ln(\chi_1^2)$ distribution and report the values of (p_i, μ_i, σ_i) for $i = 1, \ldots, 7$. Omori, Chib, Shephard, and Nakajima (2007) propose a more accurate 10-component Gaussian mixture approximation. We use the OCSN approximation.

1.3.2 Details of the MCMC Iterations:

Let Y denote the observed data (π_t, for $t = 1, \ldots, T$). Let θ denote the vector of unknown random variables for which Gibbs draws will be taken.

In this model $\theta = [\{\tau_t\}, \{w_{\varepsilon,i,t}\}, \{w_{\Delta\tau,i,t}\}, \gamma_\varepsilon, \gamma_{\Delta\tau}, \{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}, \{s_t\}, p]$. Partition θ as $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ where

$\theta_1 = \{\tau_t\}, \{w_{\varepsilon,i,t}\}, \{w_{\Delta\tau,i,t}\}$
$\theta_2 = \gamma_\varepsilon, \gamma_{\Delta\tau}, \{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}$
$\theta_3 = \{s_t\}$
$\theta_4 = p$

Gibbs draws

(1) Draw θ_1 from $f(\theta_1 \mid Y, \theta_2, \theta_3, \theta_4)$
(2) Draw θ_2 from $f(\theta_2 \mid Y, \theta_1, \theta_3, \theta_4)$
(3) Draw θ_3 from $f(\theta_3 \mid Y, \theta_1, \theta_2, \theta_4)$
(4) Draw θ_4 from $f(\theta_3 \mid Y, \theta_1, \theta_2, \theta_3)$

Gibbs Draws: Details

(1) Draw θ_1 from $f(\theta_1 \mid Y, \theta_2, \theta_3, \theta_4)$.
That is, draw $\{\tau_t\}$, $\{w_{\varepsilon i,t}\}$, $\{w_{\Delta\tau i,t}\}$ from $f(\{\tau_t\},\{w_{\varepsilon i,t}\},\{w_{\Delta\tau i,t}\}|Y, \theta_2, \theta_3, \theta_4)$.

Write $f(\{\tau_t\},\{w_{\varepsilon i,t}\},\{w_{\Delta\tau i,t}\}|Y, \theta_2, \theta_3, \theta_4) =$
$$f(\{\tau_t\} \mid Y, \theta_2, \theta_3, \theta_4) \times f(\{w_{\varepsilon i,t}\},\{w_{\Delta\tau i,t}\}|Y, \theta_2, \theta_3, \theta_4, \{\tau_t\})$$

(1.a) Draw $\{\tau_t\}$ from $f(\{\tau_t\} \mid Y, \theta_2, \theta_3, \theta_4)$
$$= f(\{\tau_t\} \mid Y, \{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}, \{s_t\})$$

These are draws from a linear Gaussian unobserved components model. Conditional moments can be computed from Kalman smoother, and draws can be conveniently obtained using the insights in Carter and Kohn (1994).

(1.b) Draw $\{w_{\varepsilon i,t}\}$, $\{w_{\Delta\tau i,t}\}$ from $f(\{w_{\varepsilon i,t}\},\{w_{\Delta\tau i,t}\}|Y, \theta_2, \theta_3, \theta_4, \{\tau_t\})$.

(i) With $\{\tau_t\}$ and $\{\sigma_{\Delta\tau,t}\}$ known, $\ln(\eta^2_{\Delta\tau,t})$ can be calculated. Then $\{w_{\Delta\tau i,t}\}$ can be drawn from posterior mixture probability in a straightforward manner.

(ii) With π_t and τ_t known, ε_t can be calculated. With $\{\varepsilon_t\}$ and $\{\sigma_{\varepsilon,t}\}$ known, $\ln(\eta^2_{\varepsilon,t})$ can be calculated. Then $\{w_{\varepsilon i,t}\}$ can be drawn from posterior mixture probability in a straightforward manner.

(2) Draw θ_2 from $f(\theta_1 \mid Y, \theta_1, \theta_3, \theta_4)$
That is, draw γ_ε, $\gamma_{\Delta\tau}$, $\{\sigma_{\varepsilon,t}\}$, $\{\sigma_{\Delta\tau,t}\}$ from $f(\gamma_\varepsilon, \gamma_{\Delta\tau}, \{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}|Y, \theta_1, \theta_3, \theta_4)$.

Write $f(\gamma_\varepsilon, \gamma_{\Delta\tau}, \{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}|Y, \theta_1, \theta_3, \theta_4) =$
$$f(\gamma_\varepsilon, \gamma_{\Delta\tau}|Y, \theta_1, \theta_3, \theta_4) \times f(\{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}|Y, \theta_1, \theta_3, \theta_4, \gamma_\varepsilon, \gamma_{\Delta\tau}).$$

(2a) Draw γ_ε, $\gamma_{\Delta\tau}$ from $f(\gamma_\varepsilon, \gamma_{\Delta\tau}|Y, \theta_1, \theta_3, \theta_4)$

(i) with $\Delta\tau$ and $w_{\Delta\tau i,t}$ known, then

$$\ln\left(\left(\Delta\tau\right)_t^2\right) = \ln(\sigma^2_{\Delta\tau,t}) + \sum_{i=1}^{n} w_{\Delta\tau,i,t} a_{\Delta\tau,i,t}$$

$$\ln(\sigma^2_{\Delta\tau.t}) = \ln(\sigma^2_{\Delta\tau.t-1}) + \gamma_{\Delta\tau} v_{\Delta\tau,t}$$

which is a Gaussian linear state-space model indexed by $\gamma_{\Delta\tau}$. Using a discrete prior for $\gamma_{\Delta\tau}$, the likelihood as a function of $\ln\left(\left(\Delta\tau\right)_t^2\right)$ can be calculated, which then yields the posterior probabilities for the grid of values of $\gamma_{\Delta\tau}$. This enables draws from the posterior.

(ii) draws for γ_ε are computed in a similar fashion.

(2b) Draw $\{\sigma_{\varepsilon,t}\}$, $\{\sigma_{\Delta\tau,t}\}$ from $f(\{\sigma_{\varepsilon,t}\}, \{\sigma_{\Delta\tau,t}\}|Y, \theta_1, \theta_3, \theta_4, \gamma_\varepsilon, \gamma_{\Delta\tau})$.
Draws of $\ln(\sigma^2_{\Delta\tau.t})$ are readily computed from the linear Gaussian state-space model given in 2a. Similarly for $\ln(\sigma^2_{\varepsilon.t})$

(3) Draw θ_3 from $f(\theta_3 \mid Y, \theta_1, \theta_2, \theta_4)$

That is, draw $\{s_t\}$ from $f(\{s_t\}|Y, \theta_1, \theta_2, \theta_4)$.

Write $\ln(\varepsilon_t^2) - \ln(\sigma^2_{\varepsilon,t}) = \ln(s_t^2) + \sum_{i=1}^{n} w_{\varepsilon,i,t} a_{\varepsilon,i,t}$

With ε_t, $\sigma_{\varepsilon,t}$ and $w_{\varepsilon i,t}$ known, then $\ln(\varepsilon_t^2) - \ln(\sigma^2_{\varepsilon,t})$ has a mixture of normal distribution, where the normal distributions have means that depend on the value of s_t. This can be used to form the (discrete) posterior for s_t, from which draws can be made.

(4) Draw θ_4 from $f(\theta_4 \mid Y, \theta_1, \theta_2, \theta_3)$

That is, draw p from $f(p|Y, \theta_1, \theta_2, \theta_3) = f(p|\{s_t\}) = f(p|\{1(s_t > 1)\})$, which is standard given the Beta prior for p.

1.3.3 Number of draws and accuracy

The MCMC iterations described above were initialized with 10,000 iterations. We then carried out 50,000 iterations, saving every 10 draws, yielding 5,000 draws from which the various posterior statistics were estimated. We carried out this process twice using independent starting values for PCE-all; this yielded two estimates of the posterior mean of $\{\tau_t\}$. The mean absolute difference between the two sets of estimates over all t was 0.01 and the largest absolute difference was less that 0.06.

1.4 Results

The posterior means for τ_t, $\sigma_{\varepsilon,t}$, $\sigma_{\Delta\tau}$, and s_t are plotted in the paper. The posterior distributions for γ_ε, $\gamma_{\Delta\tau}$, and p are summarized in the following tables:

Table A.1: Posterior distribution of γ_ε and $\gamma_{\Delta\tau}$

Value	Prior Prob		Posterior Probability						
			γ_ε				$\gamma_{\Delta\tau}$		
			PCE-all	PCExE	PCExFE		PCE-all	PCExE	PCExFE
0.0001	0.20		0.00	0.02	0.25		0.00	0.00	0.00
0.05	0.20		0.02	0.06	0.20		0.00	0.00	0.00
0.10	0.20		0.09	0.20	0.20		0.06	0.05	0.01
0.15	0.20		0.29	0.35	0.19		0.28	0.27	0.22
0.20	0.20		0.60	0.38	0.17		0.66	0.68	0.77

Table A.2: Posterior distribution of p (selected quantiles)

Quantile	PCE-all	PCExE	PCExFE
0.16	0.02	0.02	0.03
0.50	0.04	0.03	0.05
0.67	0.06	0.05	0.08

2. The 17-component multivariate model

2.1 The model:

$\pi_{i,t} = \alpha_{i,\tau,t}\, \tau_{c,t} + \alpha_{i,\varepsilon,t}\, \varepsilon_{c,t} + \tau_{i,t} + \varepsilon_{i,t}$,

$\tau_{c,t} = \tau_{c,t\text{-}1} + \sigma_{\Delta\tau,c,t} \times \eta_{\tau,c,t}$

$\varepsilon_{c,t} = \sigma_{\varepsilon,c,t} \times s_{c,t} \times \eta_{\varepsilon,c,t}$

$\tau_{i,t} = \tau_{i,t\text{-}1} + \sigma_{\Delta\tau,i,t} \times \eta_{\tau,i,t}$

$\varepsilon_{i,t} = \sigma_{\varepsilon,i,t} \times s_{i,t} \times \eta_{\varepsilon,i,t}$

$\alpha_{i,\tau,t} = \alpha_{i,\tau,t\text{-}1} + \lambda_{i,\tau}\zeta_{i,\tau,t}$ and $\alpha_{i,\varepsilon,t} = \alpha_{i,\varepsilon,t\text{-}1} + \lambda_{i,\varepsilon}\zeta_{i,\varepsilon,t}$

$\Delta\ln(\sigma^2_{\Delta\tau,c,t}) = \gamma_{\Delta\tau,c}\nu_{\Delta\tau,c,t}$, $\Delta\ln(\sigma^2_{\varepsilon,c,t}) = \gamma_{\varepsilon,c}\nu_{\varepsilon,c,t}$, $\Delta\ln(\sigma^2_{\Delta\tau,i,t}) = \gamma_{\Delta\tau,i}\nu_{\Delta\tau,i,t}$, and $\Delta\ln(\sigma^2_{\varepsilon,i,t}) = \gamma_{\varepsilon,i}\nu_{\varepsilon,i,t}$,

2.2 Priors

The priors for $\gamma_{\varepsilon,c}$, $\gamma_{\varepsilon,i}$, $\gamma_{\Delta\tau,c}$, $\gamma_{\Delta\tau,i}$, p_c, p_i, $\tau_{i,0}$, $\ln(\sigma_{\varepsilon,i,0})$, and $\ln(\sigma_{\Delta\tau,i,0})$ are the same priors used in the univariate model and described above.

The values of $\tau_{c,0}$, $\ln(\sigma_{\varepsilon,c,0})$, and $\ln(\sigma_{\Delta\tau,c,0})$ are set to zero. These are normalizations as described in the text.

Let $\alpha_{\Delta\tau,0}$ denote the $n\times1$ vector of factor loadings at $t = 0$. The prior is $\alpha_{\Delta\tau,0} \sim \mathrm{N}(0, \kappa_1^2 ll' + \kappa_2^2 I_n)$, where l is and $n\times1$ vector of 1s, $\kappa_1 = 10$ and $\kappa_2 = 0.4$. The same prior is used for $\alpha_{\varepsilon,0}$ and the two vectors of factor loadings are independent.

Independent inverse Gamma priors are used for the λ parameters. The scale and shape parameters so that the prior corresponds to T_{prior} prior observations with $s^2_{Prior} = \omega^2/T_{Prior}$. We use ω=0.25 and $T_{prior} = T/10$.

2.3 Approximation of the posterior:

The MCMC iterations proceed much as for the univariate model, with the following exception.

θ_1 now contains $\theta_1 = \{\tau_{c,t}\}$, $\{\tau_{i,t}\}$,$\{w_{\varepsilon,i,t}\}$, $\{w_{\Delta\tau,i,t}\}$, $\{\varepsilon_{c,t}\}$, $\{\alpha_{i,\tau,t}\}$, $\{\alpha_{i,\varepsilon,t}\}$, λ_τ, and λ_ε

Partition θ_1 as $(\theta_{1a}, \theta_{1b})$ with $\theta_{1a} = \{\tau_{c,t}\}$, $\{\tau_{i,t}\}$, $\{\varepsilon_{c,t}\}$, $\{\alpha_{i,\tau,t}\}$, $\{\alpha_{i,\varepsilon,t}\}$, λ_τ, λ_ε, and $\theta_{1b} = \{w_{\varepsilon,i,t}\}$, $\{w_{\Delta\tau,i,t}\}$.

As in the univariate model, θ_1 is drawn from $f(\theta_1 \mid Y, \theta_2, \theta_3, \theta_4)$ by drawing

(a) θ_{1a} from $f(\theta_{1a} \mid Y, \theta_2, \theta_3, \theta_4)$

and then

(b) θ_{1b} from $f(\theta_{1a} \mid Y, \theta_2, \theta_3, , \theta_4, \theta_{1a})$.

The draws from (b) are just as in the univariate model. The difference in is (a). Here are the details:

Decompose θ_{1a} as

$\theta_{1a,1} = \{\tau_{c,t}\}, \{\tau_{i,t}\}, \{\varepsilon_{c,t}\}$
$\theta_{1a,2} = \{\alpha_{i,\tau,t}\}, \{\alpha_{i,\varepsilon,t}\}$
$\theta_{1a,3} = \lambda_\tau, \lambda_\varepsilon.$

These are drawn in step (a) using Gibbs sampling (so this is "Gibbs-within-Gibbs").

Steps:
(1a.1) Draw $\{\tau_{c,t}\}, \{\tau_{i,t}\}, \{\varepsilon_{c,t}\}$ from $f(\{\tau_{c,t}\}, \{\tau_{i,t}\}, \{\varepsilon_{c,t}\} \mid Y, \theta_2, \theta_3, \theta_4, \theta_{1a,2}, \theta_{1a,3})$
Which are draws from factors from a linear Gaussian SS model.

(1a.2) Draw $\{\alpha_{i,\tau,t}\}, \{\alpha_{i,\varepsilon,t}\}$ from $f(\{\alpha_{i,\tau,t}\}, \{\alpha_{i,\varepsilon,t}\} \mid Y, \theta_2, \theta_3, \theta_4, \theta_{1a,1}, \theta_{1a,3})$
Which are draws from factors from a linear Gaussian SS model.

(1a.3) Draw $\lambda_\tau, \lambda_\varepsilon$ from $f(\lambda_\tau, \lambda_\varepsilon \mid Y, \theta_2, \theta_3, \theta_4, \theta_{1a,1}, \theta_{1a,2})$
These are posterior draws from $f(\lambda_\tau, \lambda_\varepsilon \mid \{\alpha_{i,\tau,t}\}, \{\alpha_{i,\varepsilon,t}\})$.
When there is no TVP, $\lambda_\tau = \lambda_\varepsilon = 0$ and step (1a.3) is skipped.

2.4 Results

Parameters for common factors:
Figure 3 in the text shows posterior estimates for τ_t, $\sigma_{\Delta\tau,c,t}$, $\sigma_{\varepsilon,c,t}$, and $s_{c,t}$. The remaining parameter "common" parameters are $\gamma_{\Delta\tau,c}$, $\gamma_{\varepsilon,c}$, and p_c, and their posteriors are summarized in the tables below.

Table A.3: Posterior distribution for $\gamma_{\Delta\tau,c}$ and $\gamma_{\varepsilon,c}$

Value	Prior Prob		Posterior Probability	
			$\gamma_{\Delta\tau,c}$	$\gamma_{\varepsilon,c}$
0.0001	0.20		0.00	0.14
0.05	0.20		0.00	0.17
0.10	0.20		0.03	0.19
0.15	0.20		0.20	0.23
0.20	0.20		0.76	0.27

Table A.4: Posterior distribution of p_c (selected quantiles)

16%	**50%**	**67%**
0.08	0.06	0.03

Results for sector-specific parameters:

Table A.5: Posterior distribution for $\gamma_{\Delta\tau,i}$

Value of γ	0.00	0.05	0.10	0.15	0.20
	Prior probability				
	0.20	0.20	0.20	0.20	0.20
Sector	*Posterior probability*				
Motor vehicles and parts	0.20	0.19	0.21	0.21	0.21
Furn. & dur. household equip.	0.29	0.28	0.20	0.15	0.09
Rec. goods & vehicles	0.23	0.24	0.22	0.18	0.13
Other durable goods	0.23	0.25	0.22	0.17	0.12
Food & bev. for off-premises consumption	0.29	0.26	0.21	0.15	0.09
Clothing & footwear	0.31	0.27	0.20	0.14	0.09
Gasoline & other energy goods	0.21	0.21	0.22	0.20	0.17
Other nondurables goods	0.32	0.27	0.20	0.13	0.09
Housing excl. gas & elec. util.	0.07	0.10	0.15	0.27	0.41
Gas & electric untilies	0.22	0.21	0.21	0.20	0.17
Health care	0.24	0.22	0.20	0.17	0.17
Transportation services	0.28	0.27	0.21	0.14	0.10
Recreation services	0.34	0.29	0.19	0.12	0.06
Food serv. & accom.	0.25	0.23	0.21	0.15	0.15
Fin. services & insurance	0.06	0.09	0.19	0.32	0.34
Other services	0.31	0.27	0.20	0.13	0.09
NPISH	0.02	0.03	0.07	0.25	0.63

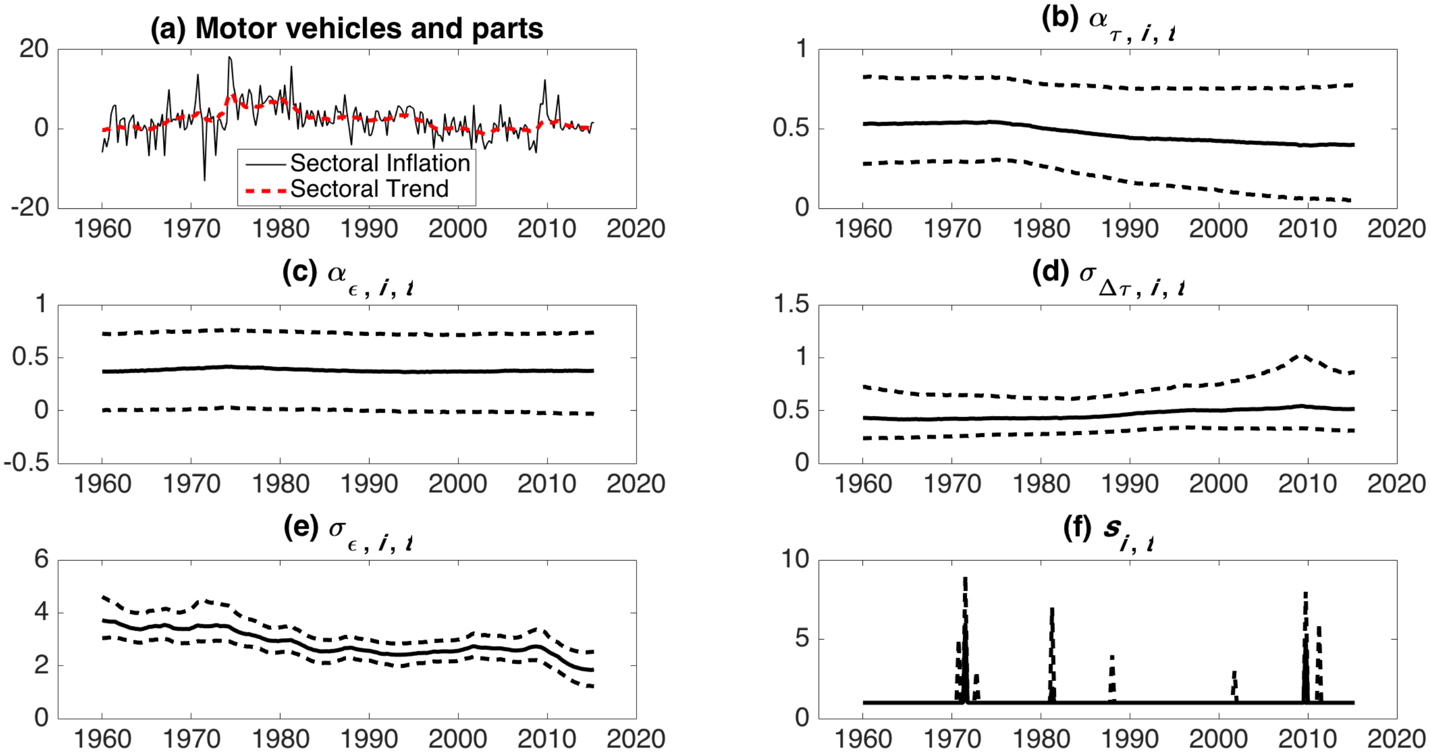
Table A.6: Posterior distribution for $\gamma_{\varepsilon,i}$

Value of γ	0.00	0.05	0.10	0.15	0.20
	Prior probability				
	0.20	0.20	0.20	0.20	0.20
Sector	*Posterior probability*				
Motor vehicles and parts	0.04	0.27	0.26	0.22	0.20
Furn. & dur. household equip.	0.00	0.00	0.24	0.45	0.30
Rec. goods & vehicles	0.37	0.29	0.18	0.10	0.06
Other durable goods	0.00	0.00	0.04	0.23	0.73
Food & bev. for off-premises consumption	0.00	0.00	0.01	0.16	0.84
Clothing & footwear	0.00	0.04	0.30	0.36	0.29
Gasoline & other energy goods	0.00	0.00	0.05	0.29	0.66
Other nondurables goods	0.15	0.25	0.26	0.20	0.14
Housing excl. gas & elec. util.	0.00	0.01	0.07	0.30	0.62
Gas & electric untilies	0.00	0.00	0.02	0.23	0.74
Health care	0.00	0.00	0.05	0.28	0.67
Transportation services	0.00	0.00	0.09	0.47	0.45
Recreation services	0.05	0.13	0.22	0.30	0.30
Food serv. & accom.	0.07	0.21	0.29	0.26	0.17
Fin. services & insurance	0.00	0.00	0.00	0.03	0.97
Other services	0.01	0.04	0.18	0.35	0.42
NPISH	0.00	0.00	0.00	0.11	0.89

Table A.7: Posterior distribution of p_i (selected quantiles)

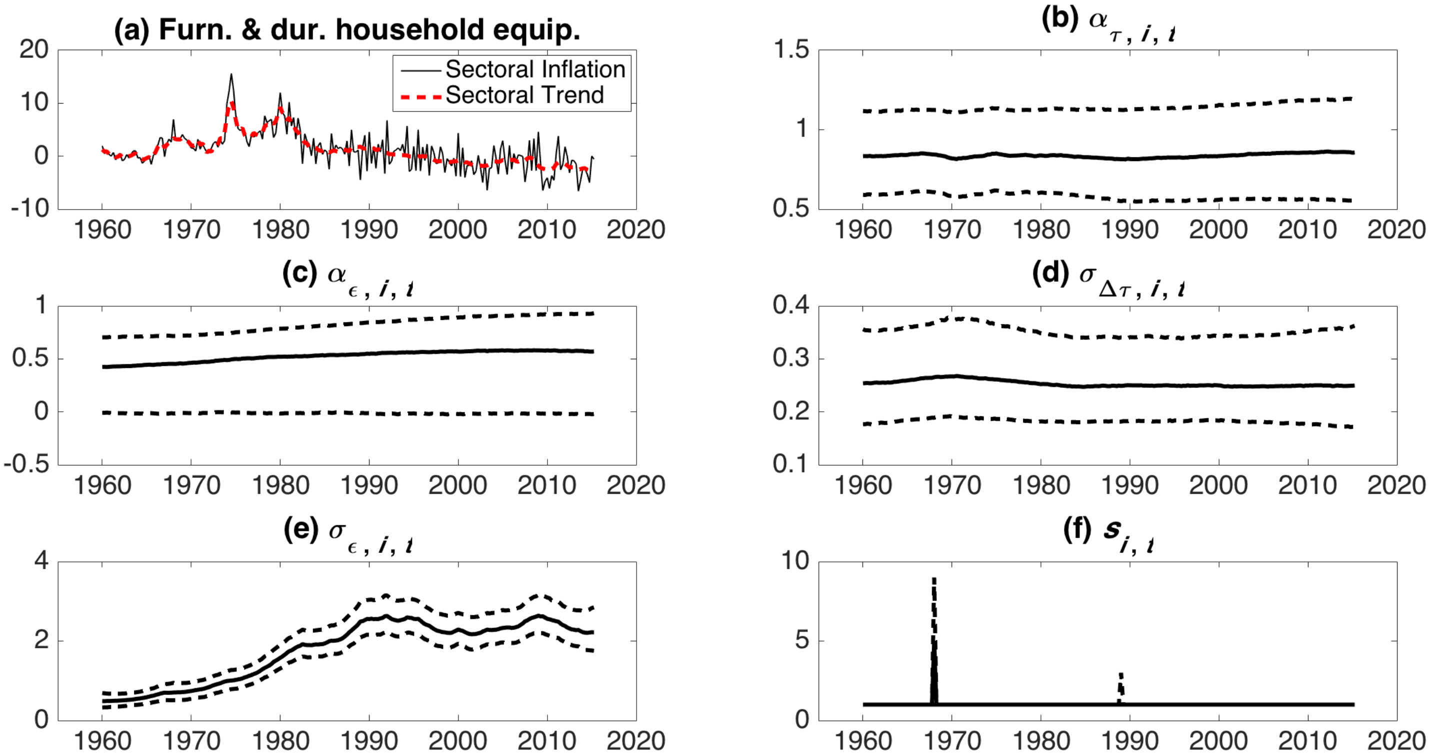
Sector	**16%**	**50%**	**67%**
Motor vehicles and parts	0.02	0.03	0.05
Furn. & dur. household equip.	0.01	0.02	0.04
Rec. goods & vehicles	0.02	0.04	0.06
Other durable goods	0.02	0.04	0.06
Food & bev. for off-premises consumption	0.01	0.02	0.04
Clothing & footwear	0.01	0.02	0.04
Gasoline & other energy goods	0.06	0.09	0.13
Other nondurables goods	0.03	0.05	0.08
Housing excl. gas & elec. util.	0.02	0.03	0.04
Gas & electric untilies	0.04	0.07	0.11
Health care	0.01	0.02	0.04
Transportation services	0.01	0.02	0.03
Recreation services	0.02	0.03	0.06
Food serv. & accom.	0.01	0.02	0.04
Fin. services & insurance	0.05	0.08	0.11
Other services	0.02	0.04	0.06
NPISH	0.01	0.02	0.03

Figure A.1



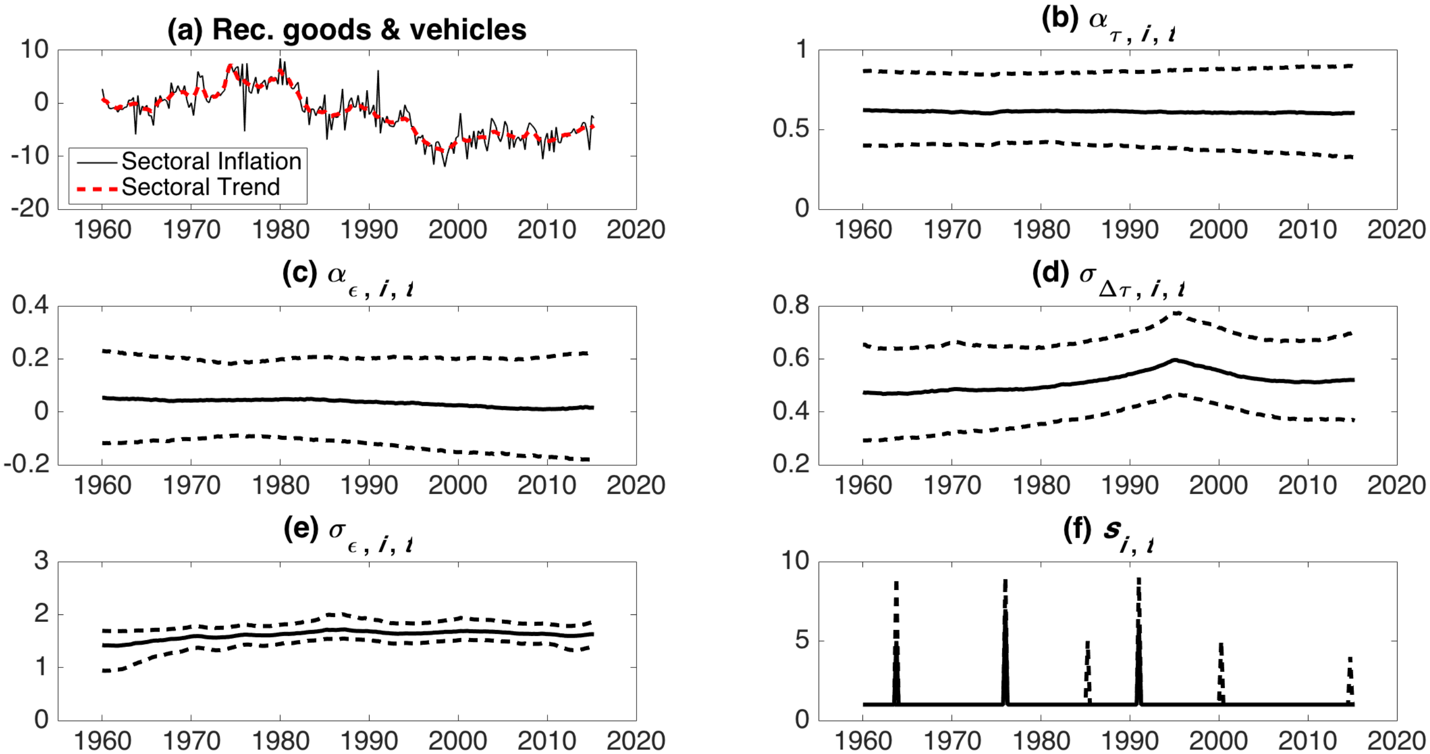
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.2



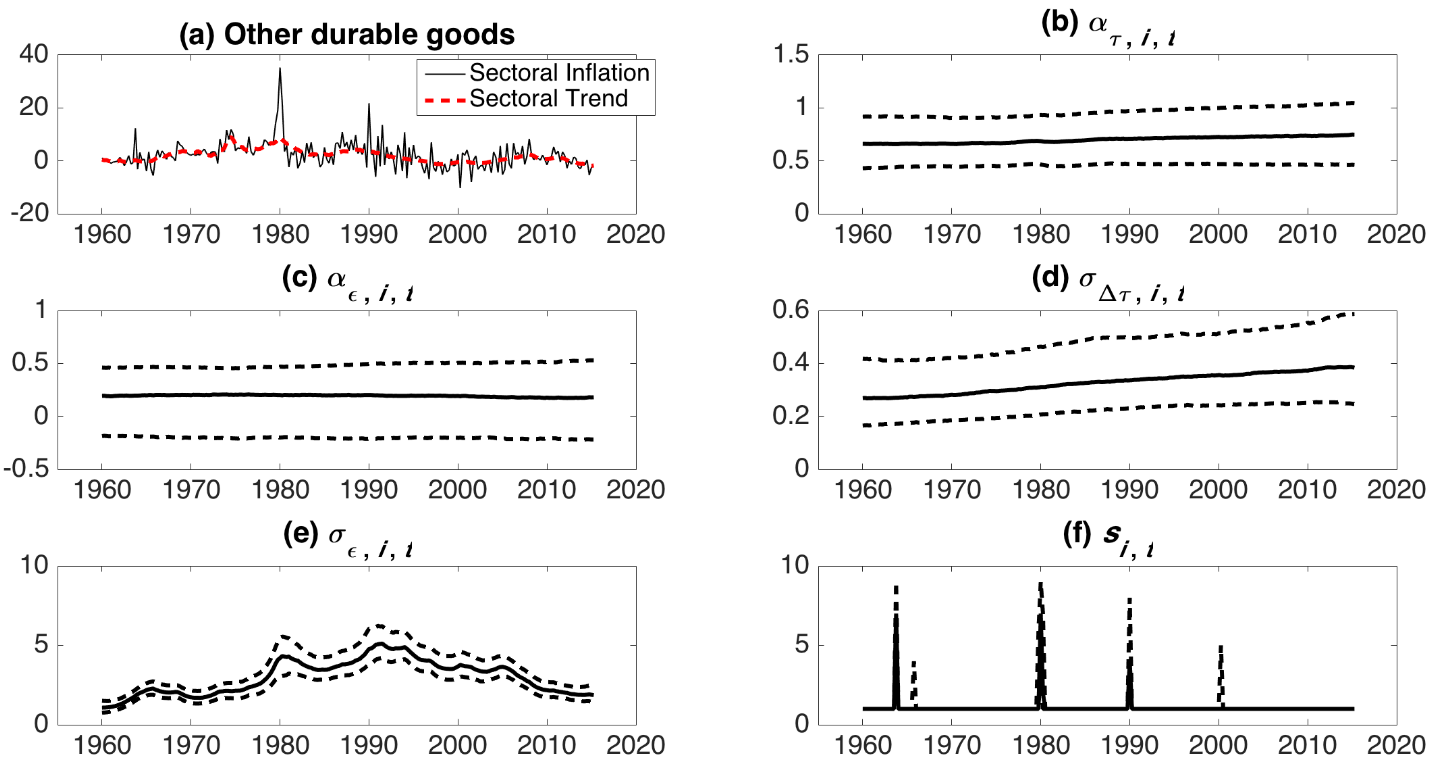
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.3



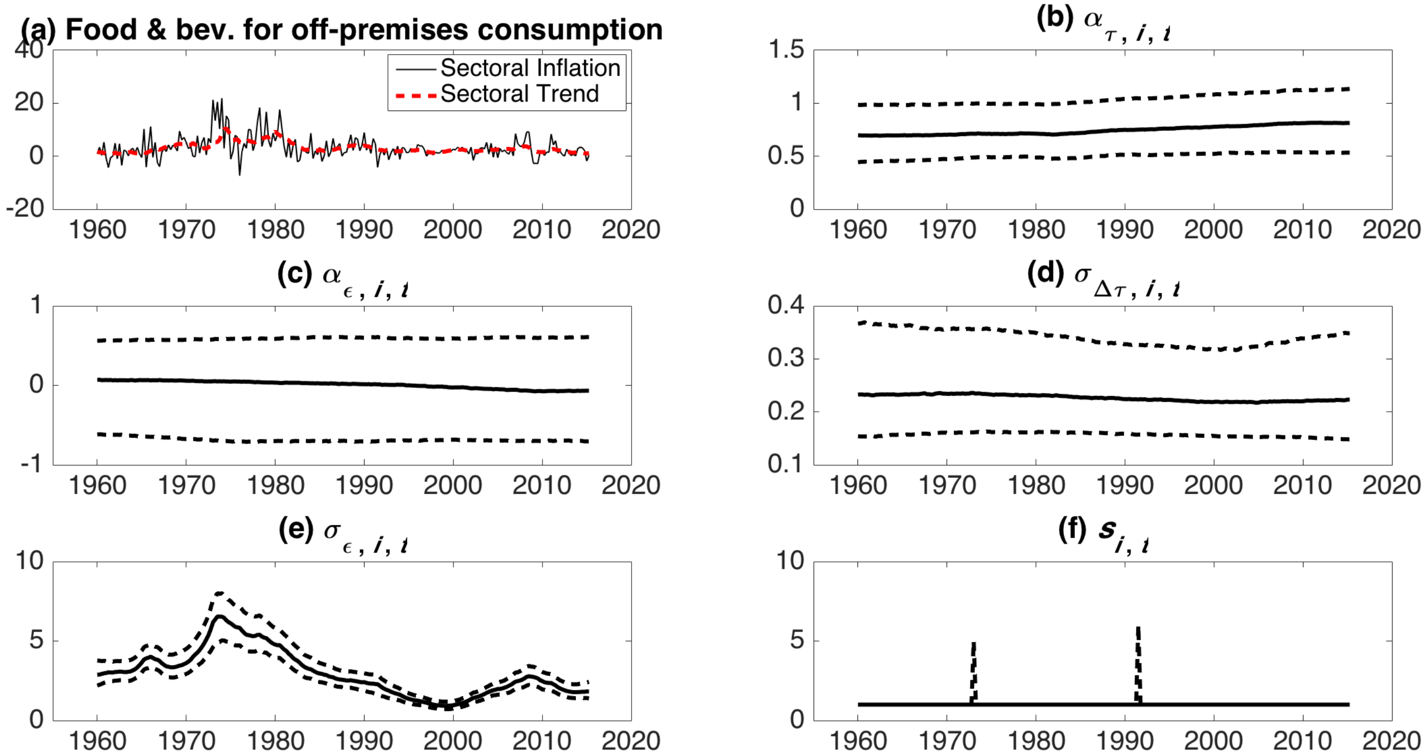
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.4



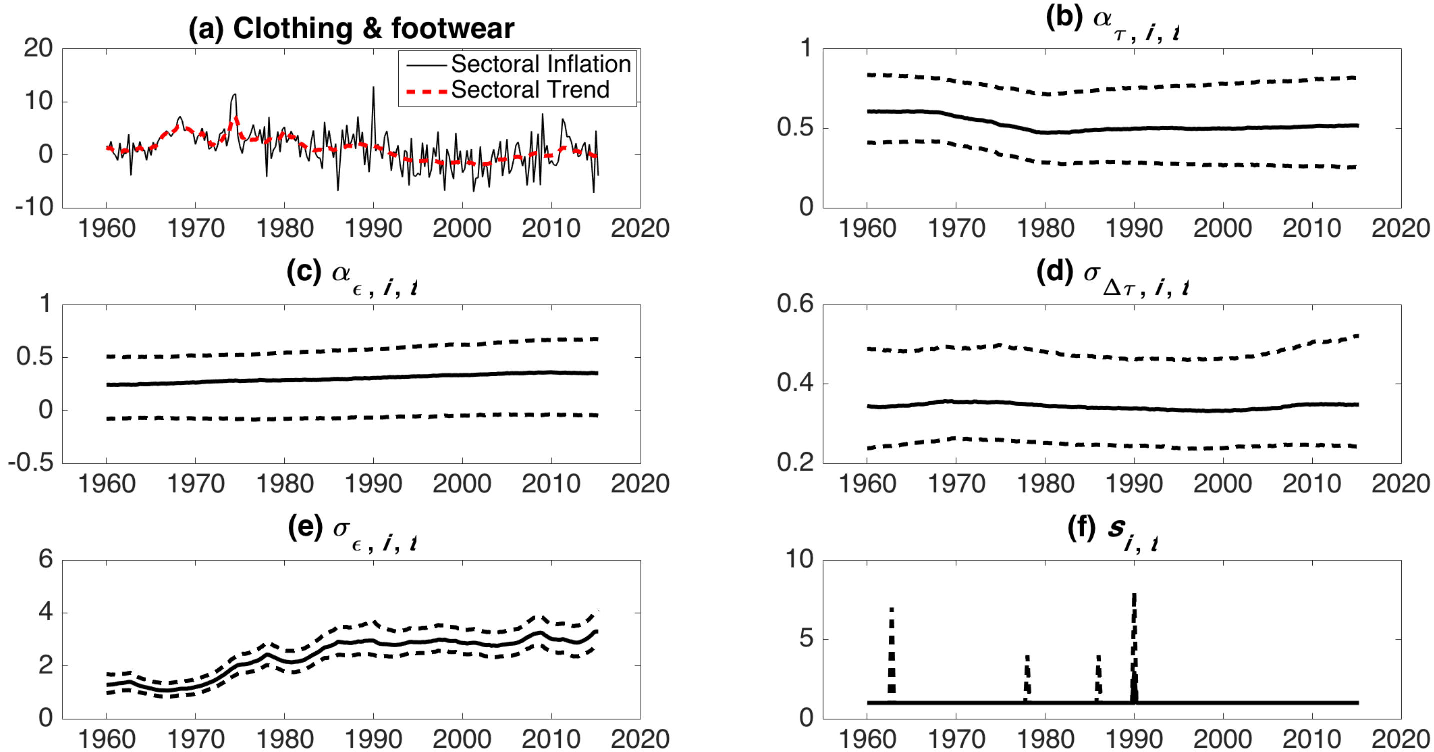
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.5



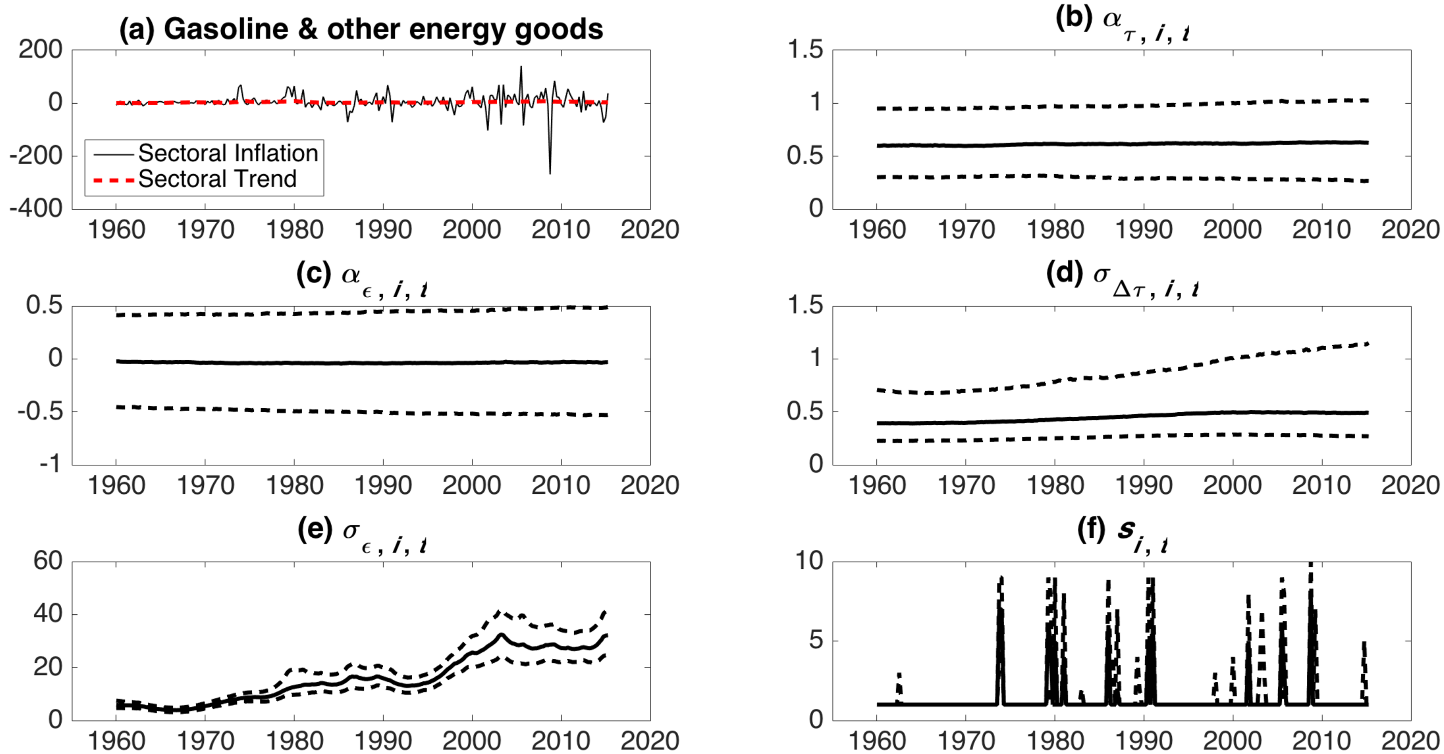
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.6



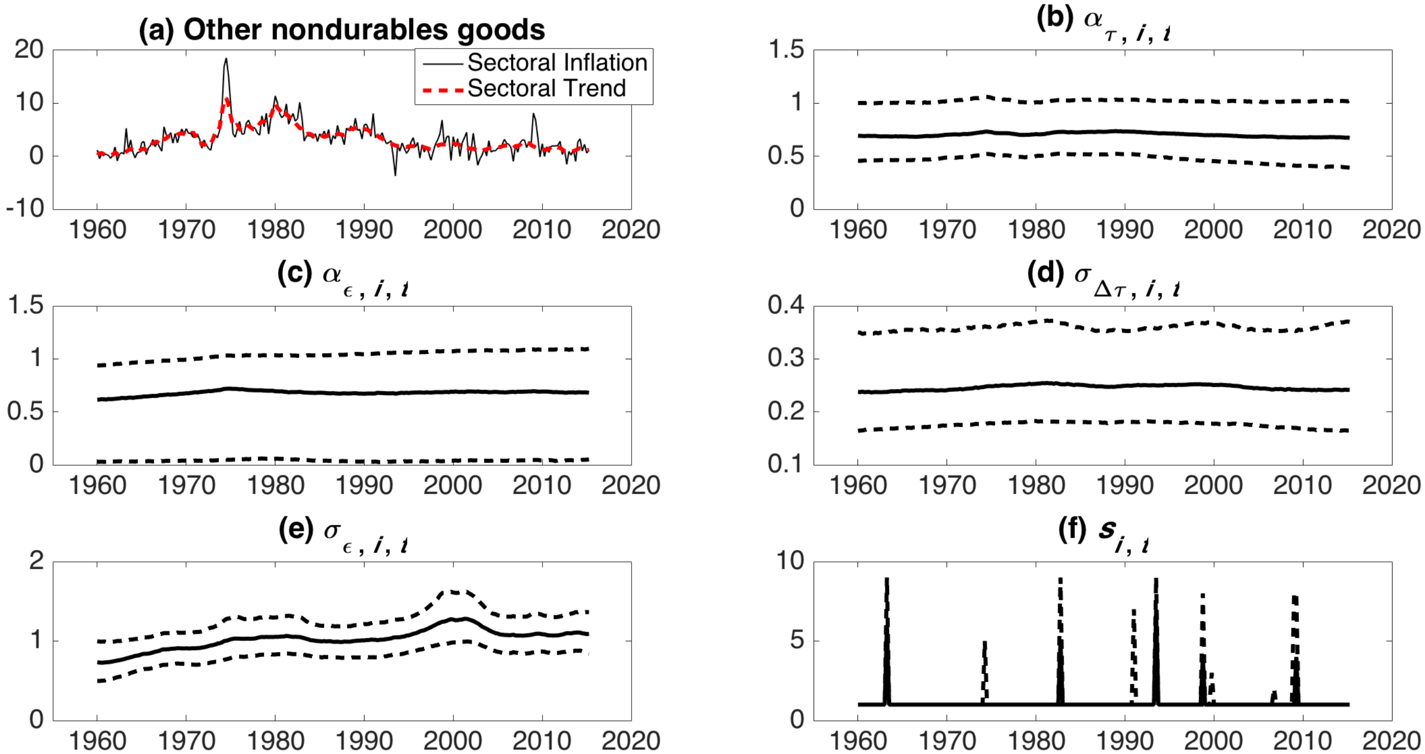
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.7



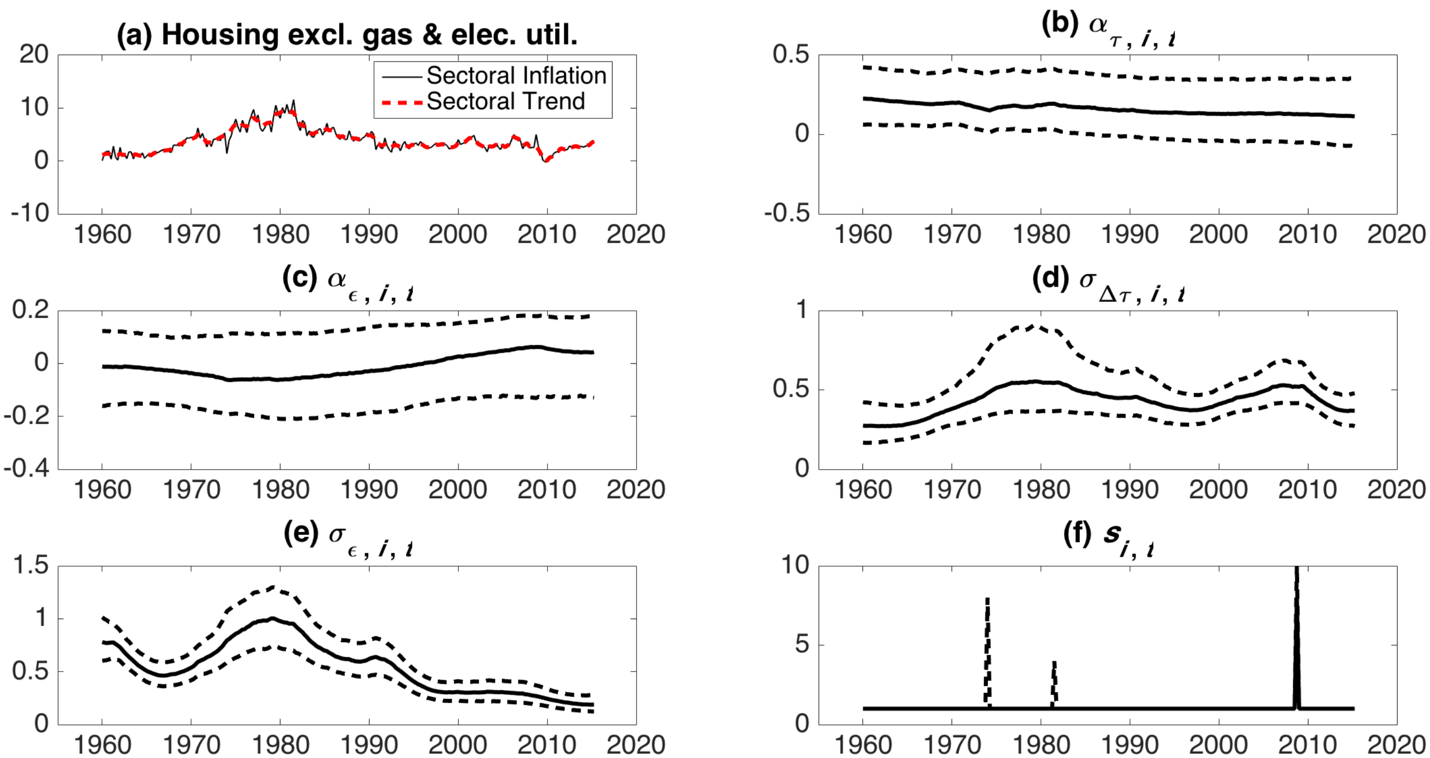
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.8



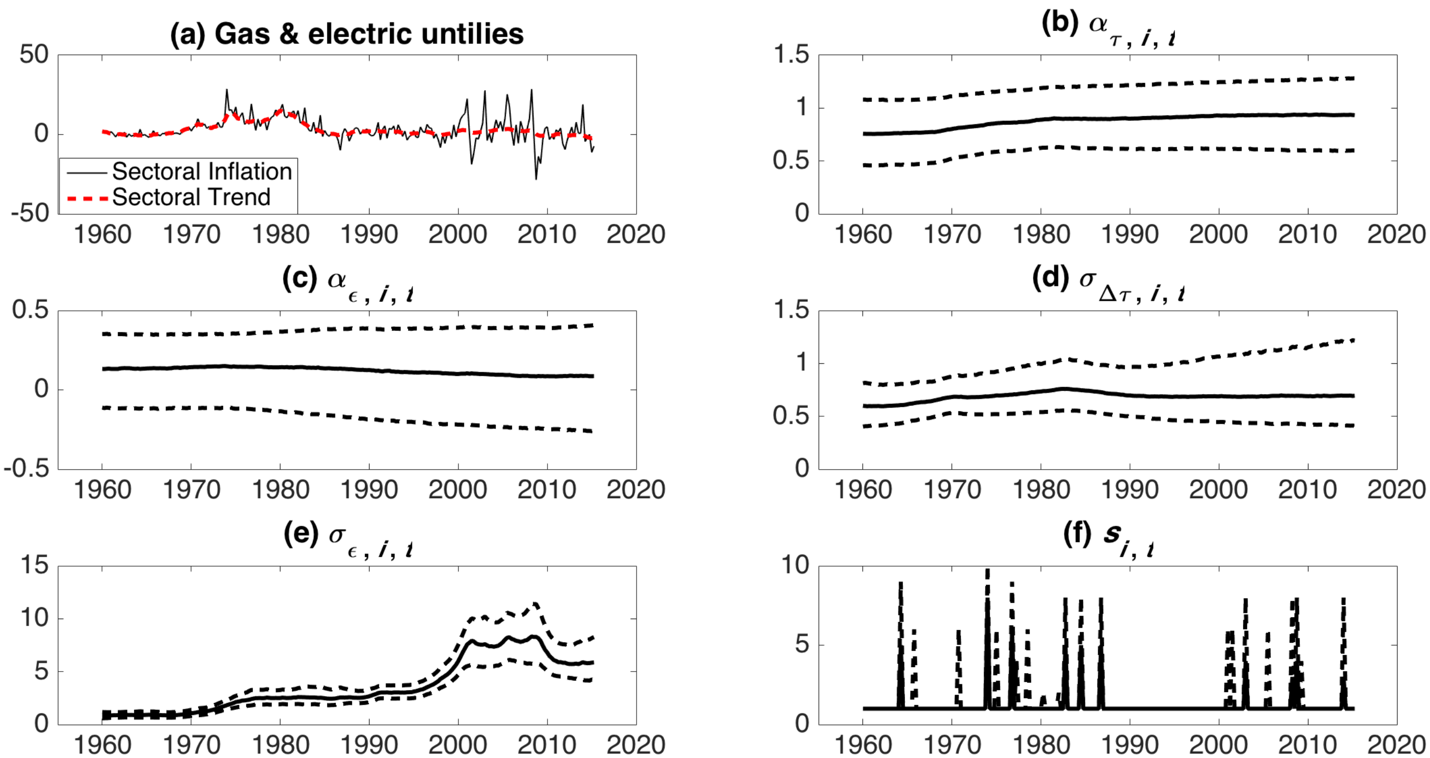
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.9



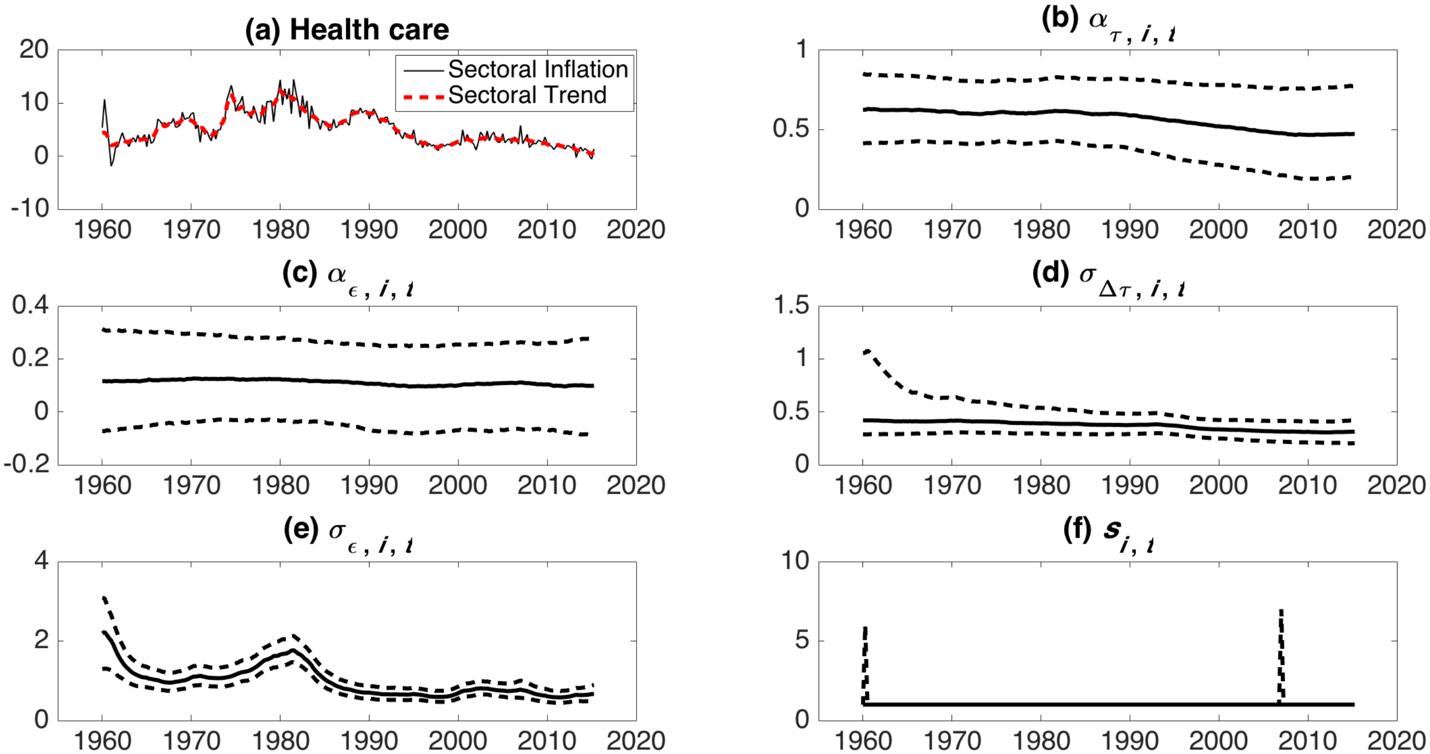
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.10



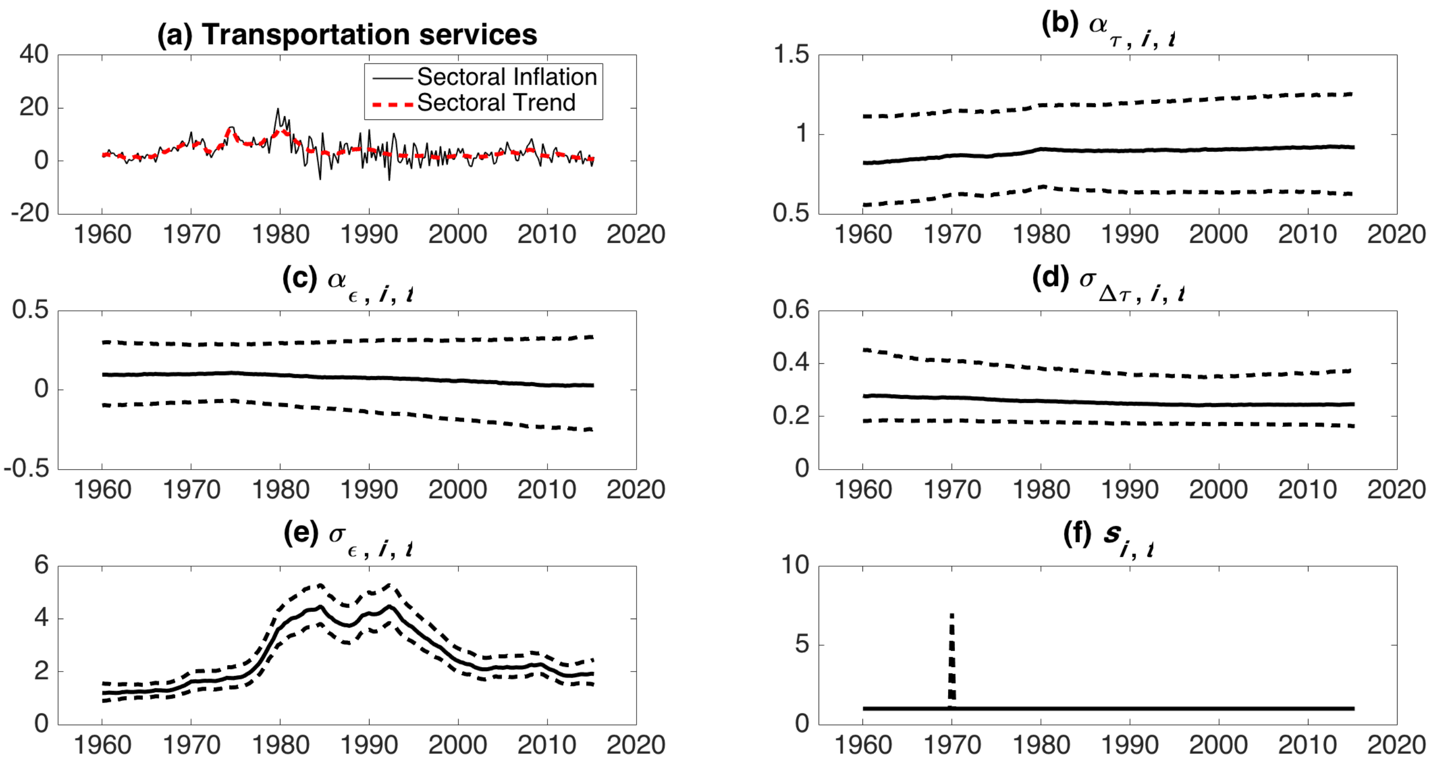
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.11



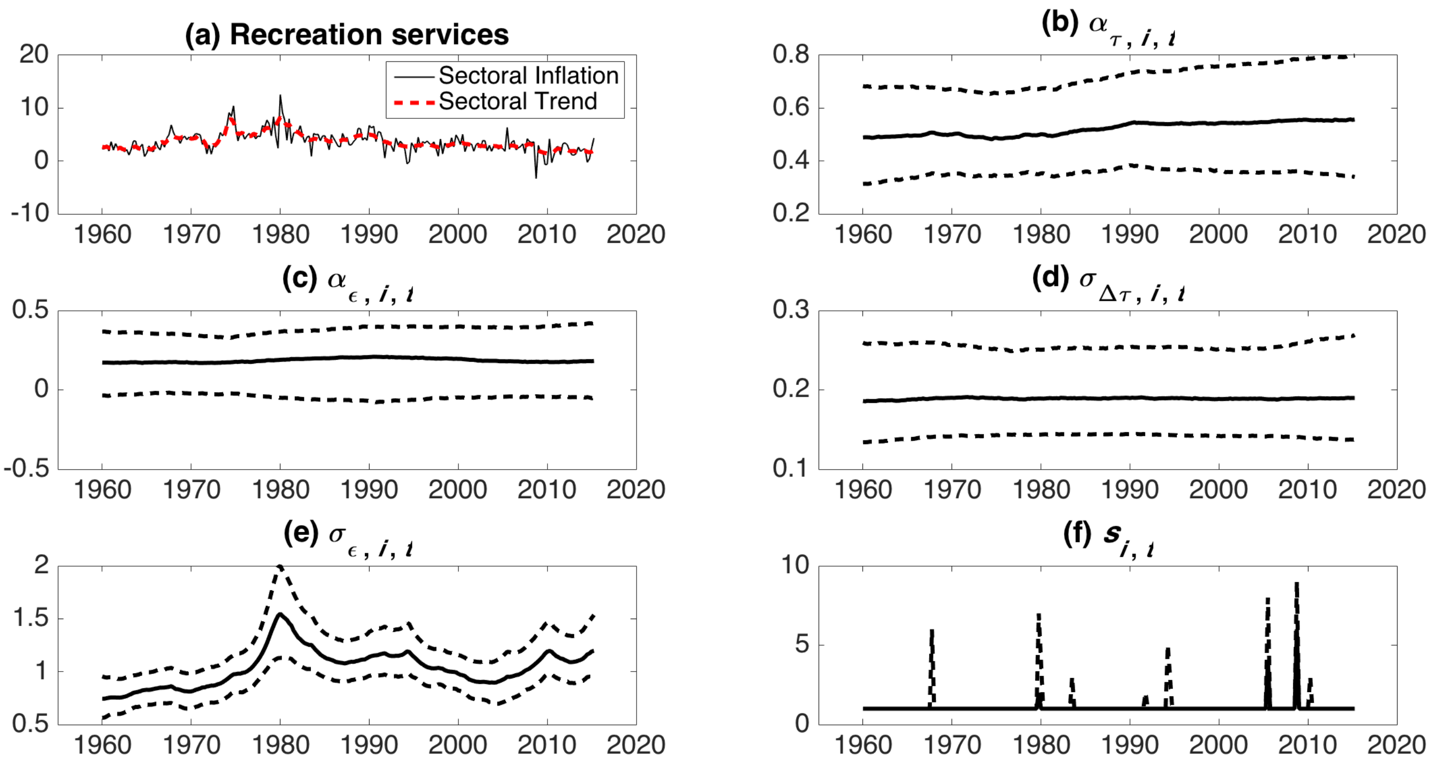
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.12



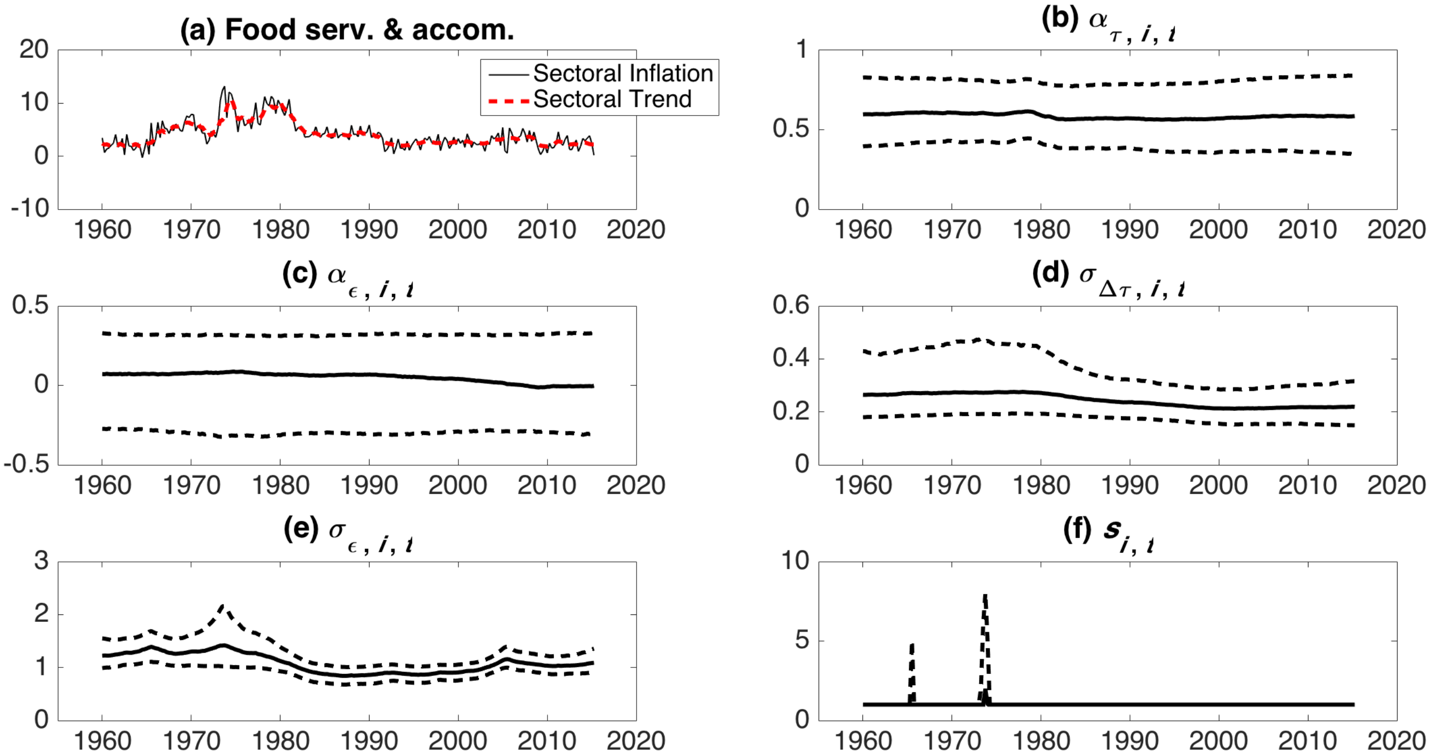
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.13



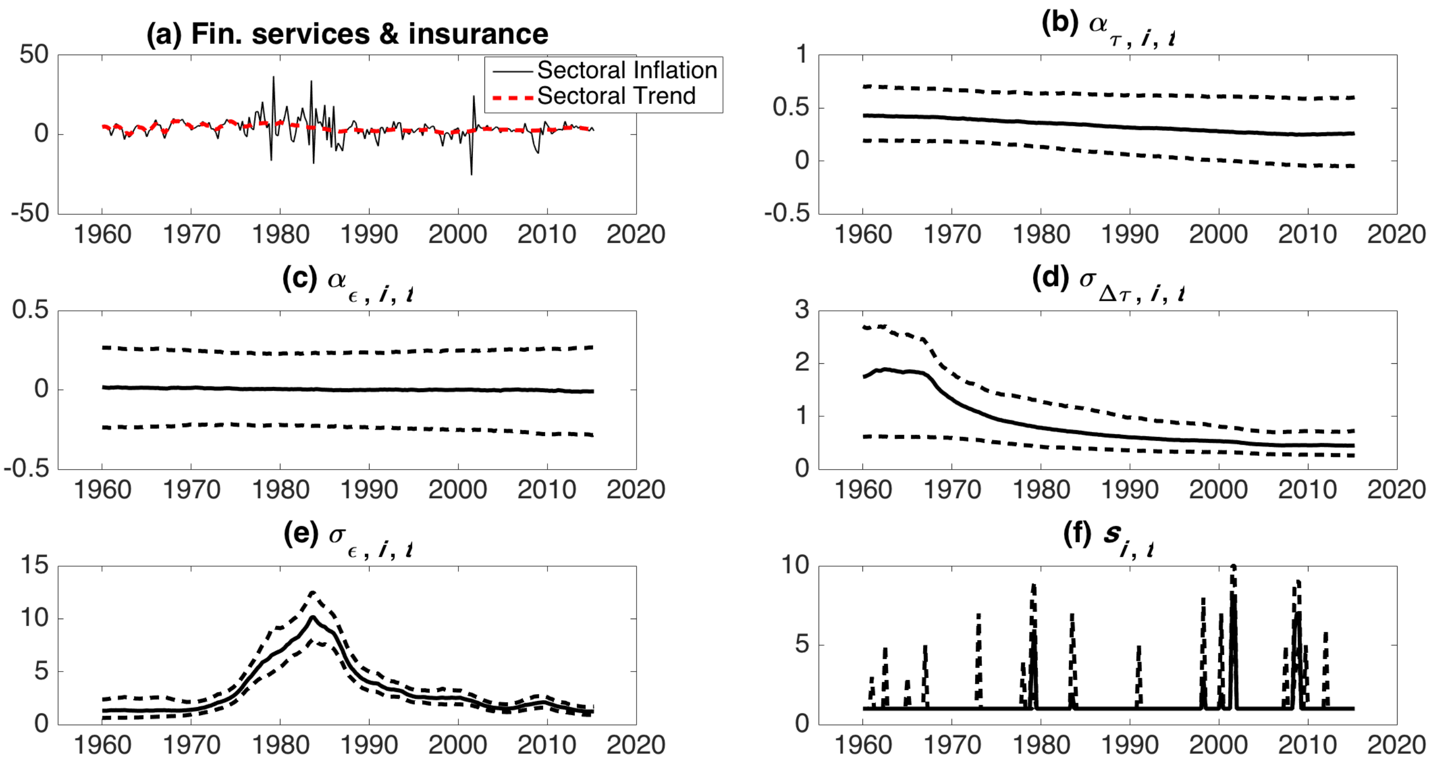
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.14



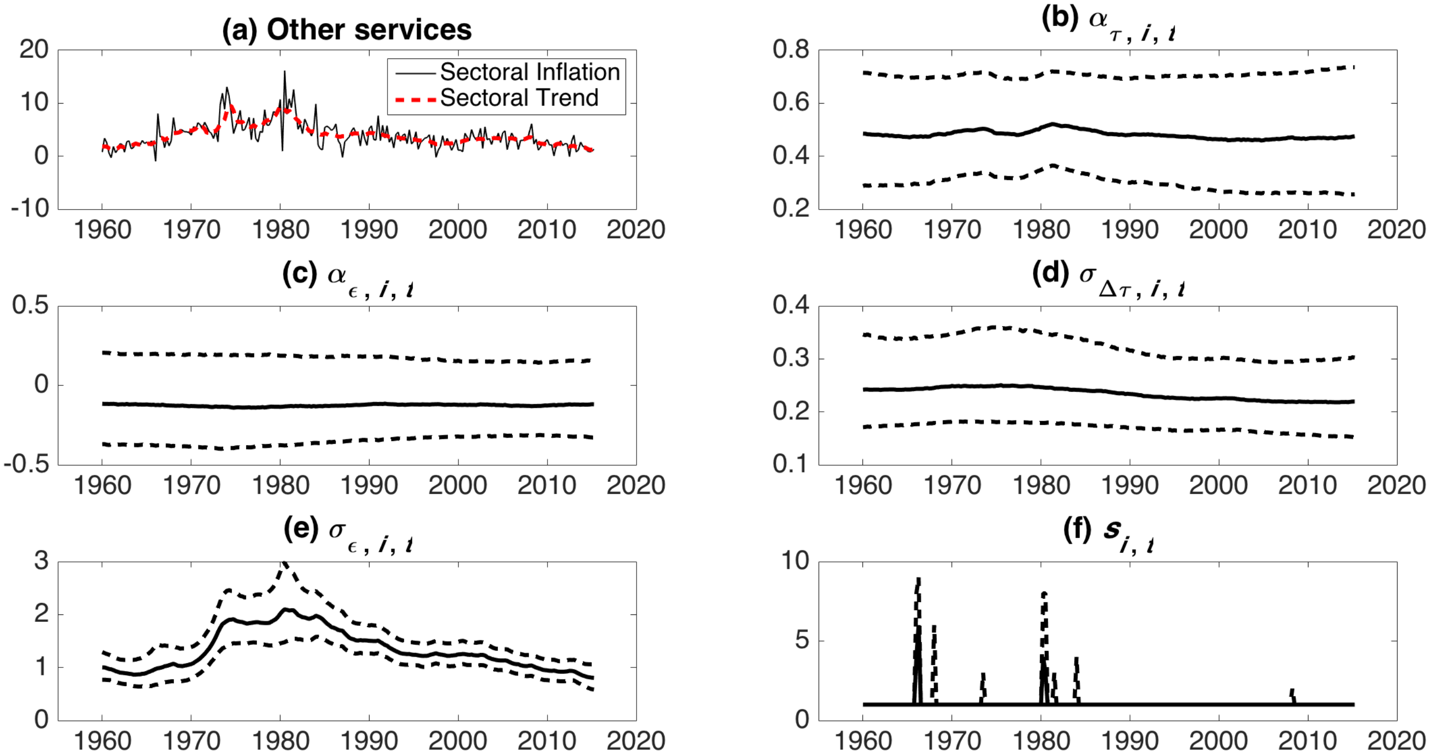
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.15



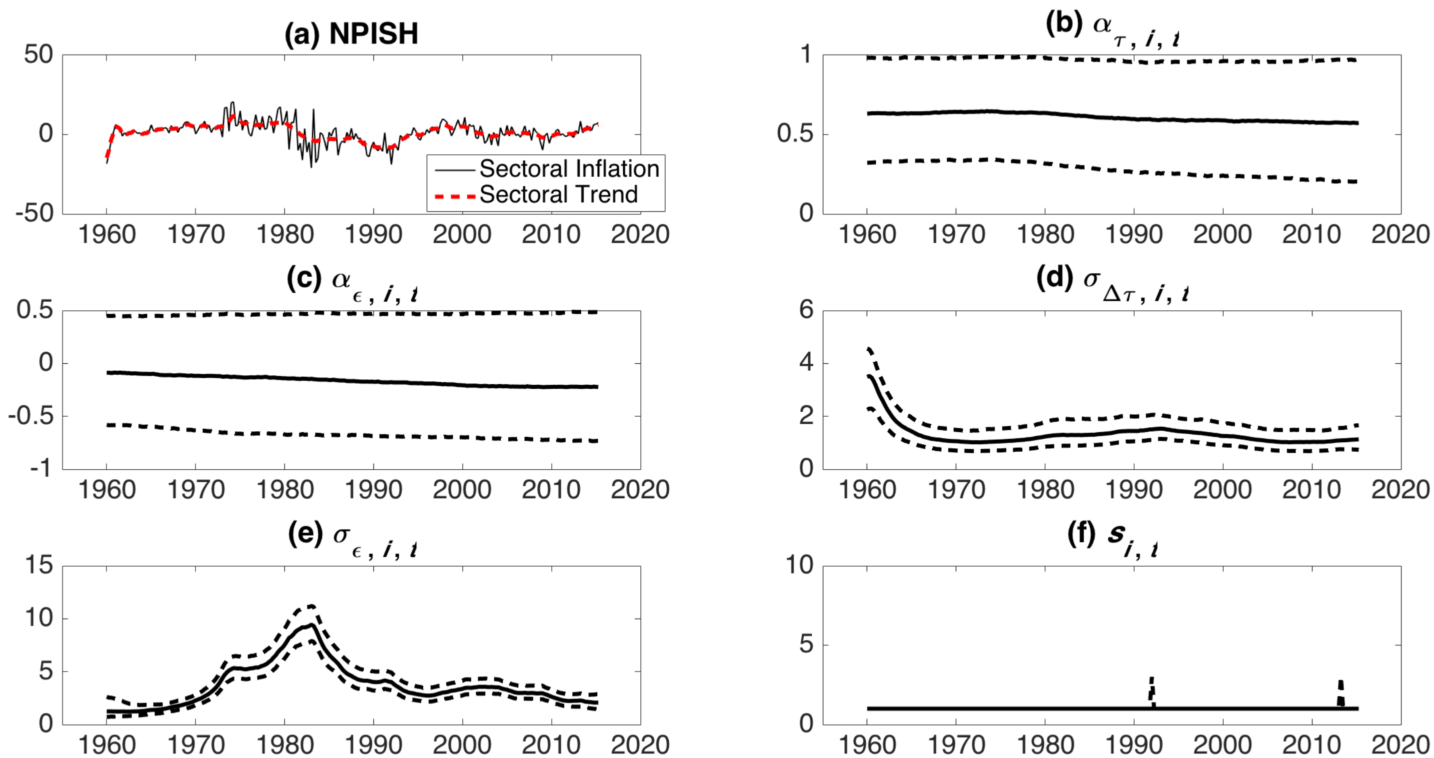
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.16



Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.17



Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

3. The 3-component multivariate model

This model has the same structure and uses the same priors as the 17-component model but uses only 3 components. The results for the model are given below.

Table A.8: Posterior distribution for $\gamma_{\Delta\tau,c}$ and $\gamma_{\varepsilon,c}$

Value	Prior Prob		Posterior Probability	
			$\gamma_{\Delta\tau,c}$	$\gamma_{\varepsilon,c}$
0.0001	0.20		0.03	0.21
0.05	0.20		0.05	0.22
0.10	0.20		0.10	0.22
0.15	0.20		0.29	0.18
0.20	0.20		0.53	0.16

Table A.9: Posterior distribution of p_c (selected quantiles)

16%	**50%**	**67%**
0.02	0.04	0.07

Results for sector-specific parameters:

Table A.10: Posterior distribution for $\gamma_{\Delta\tau,i}$

Value of γ	0.00	0.05	0.10	0.15	0.20
	Prior probability				
	0.20	0.20	0.20	0.20	0.20
Sector	*Posterior probability*				
Core	0.19	0.20	0.20	0.21	0.21
Food	0.30	0.26	0.21	0.14	0.10
Energy	0.24	0.23	0.21	0.18	0.14

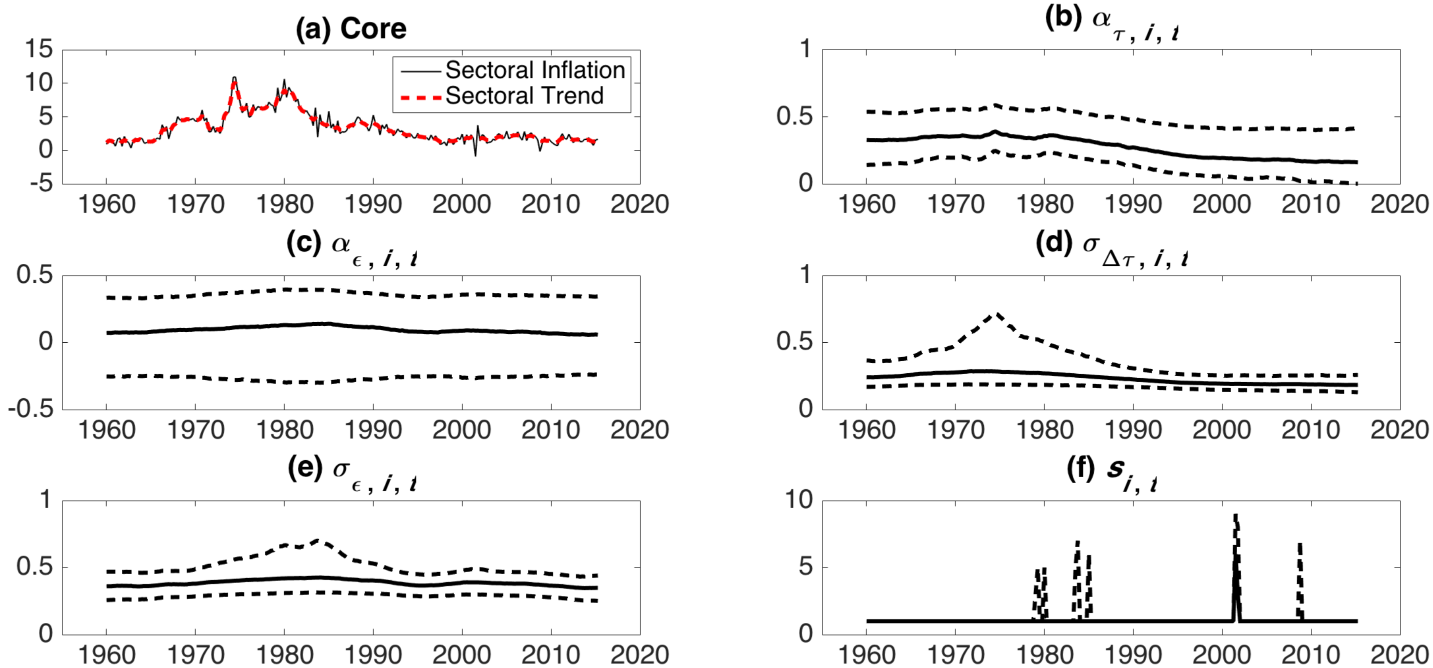
Table A.11: Posterior distribution for $\gamma_{\varepsilon,i}$

Value of γ	0.00	0.05	0.10	0.15	0.20
	Prior probability				
	0.20	0.20	0.20	0.20	0.20
Sector	*Posterior probability*				
Core	0.29	0.24	0.19	0.16	0.12
Food	0.00	0.00	0.00	0.11	0.89
Energy	0.00	0.00	0.07	0.33	0.59

Table A.12: Posterior distribution of p_i (selected quantiles)

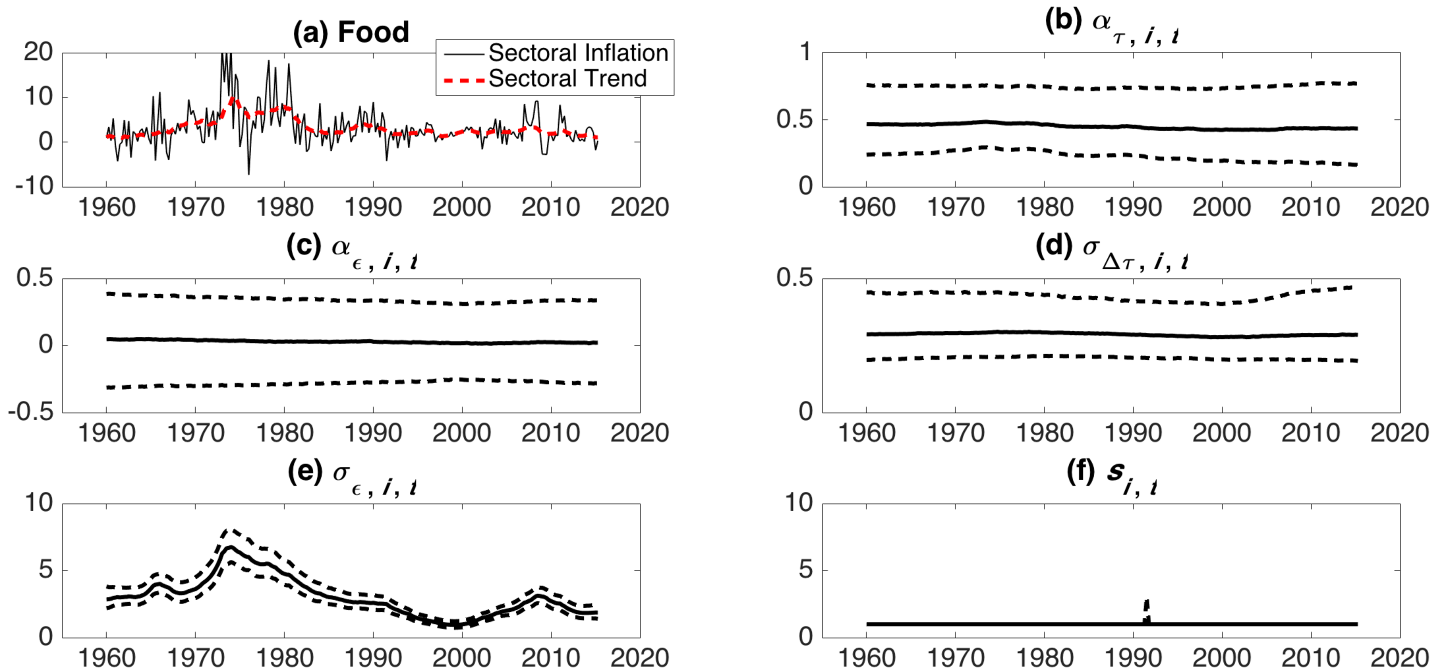
Sector	**16%**	**50%**	**67%**
Core	0.02	0.04	0.06
Food	0.01	0.02	0.03
Energy	0.07	0.10	0.14

Figure A.18



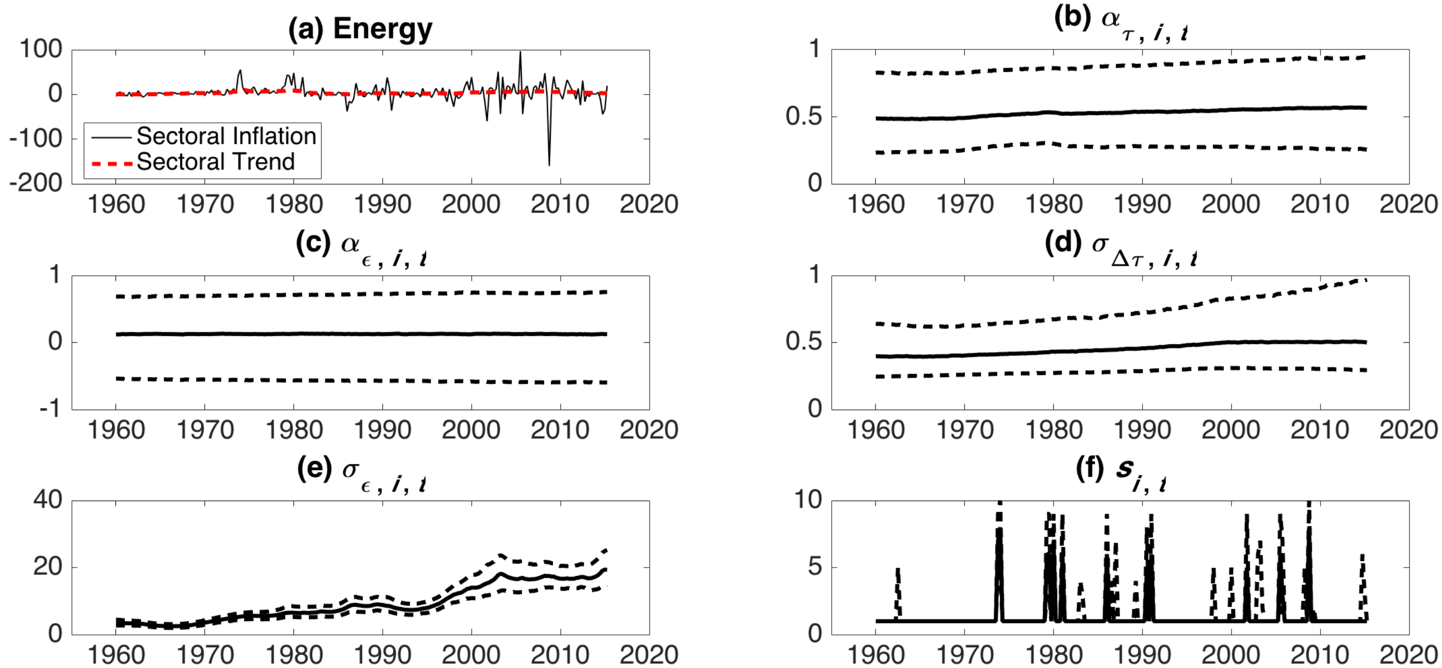
Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.19



Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

Figure A.20



Notes: Panel (a) inflation is the sector shown in the figure heading and the full-sample posterior mean of the sectoral trend. The other panels plot the full-sample posterior median and (point-wise) 67% intervals for the sector-specific parameters.

3. Monthly Models

We also estimated the same univariate and multivariate models using monthly inflation rates. We do not report detailed results for these models, but do report the forecasting performance of these models using recursively computed posterior mean trend estimates.

Table A13: Monthly and quarterly MSFE (1990 – end-of-sample), excluding 2008:Q4

1990-End of Sample, excluding 2008Q4

	4 quarter/12 month-ahead forecasts			**8 quarter/24 month - ahead forecasts**			**12 quarter/36 month - ahead forecasts**	
	Quarterly Model	**Monthly Model**		**Quarterly Model**	**Monthly Model**		**Quarterly Model**	**Monthly Model**
Multivariate UCSVO Forecasts								
17comp	0.62 (0.10)	0.78 (0.14)		0.49 (0.07)	0.68 (0.13)		0.42 (0.08)	0.62 (0.14)
3comp	0.59 (0.09)	0.75 (0.12)		0.49 (0.08)	0.67 (0.13)		0.43 (0.10)	0.62 (0.14)
Univariate UCSVO Forecasts								
PCE-all	0.66 (0.10)	0.70 (0.12)		0.63 (0.13)	0.64 (0.13)		0.57 (0.14)	0.60 (0.14)
PCExE	0.59 (0.10)	0.67 (0.11)		0.50 (0.08)	0.55 (0.09)		0.45 (0.10)	0.52 (0.10)
PCExFE	0.61 (0.10)	0.66 (0.10)		0.49 (0.08)	0.51 (0.08)		0.45 (0.11)	0.49 (0.10)

4. Calculating the approximated weights plotted in Figure 5 of the paper

Ignoring outliers, and conditional on the parameters $\{\alpha_{i,\tau,t}\}$, $\{\alpha_{i,\varepsilon,t}\}$, $\{\sigma_{\Delta\tau,c,t}\}$, $\{\sigma_{\varepsilon,c,t}\}$, $\{\sigma_{\Delta\tau,i,t}\}$, and $\{\sigma_{\varepsilon,i,t}\}$, the model is

$$\pi_{i,t} = \alpha_{i,\tau,t}\,\tau_{c,t} + \alpha_{i,\varepsilon,t}\,\varepsilon_{c,t} + \tau_{i,t} + \varepsilon_{i,t},$$

$$\tau_{c,t} = \tau_{c,t\text{-}1} + \sigma_{\Delta\tau,c,t} \times \eta_{\tau,c,t}$$

$$\varepsilon_{c,t} = \sigma_{\varepsilon,c,t} \times \eta_{\varepsilon,c,t}$$

$$\tau_{i,t} = \tau_{i,t\text{-}1} + \sigma_{\Delta\tau,i,t} \times \eta_{\tau,i,t}$$

$$\varepsilon_{i,t} = \sigma_{\varepsilon,i,t} \times \eta_{\varepsilon,i,t}$$

which is a linear Gaussian state-space model. The Kalman filter provides estimates $\tau_{c,t|t}$ and $\tau_{i,t|t}$, so that the filtered estimate of the aggregate trend is $\tau_{t|t} = \sum_{i=1}^{17} w_{it}\left(\alpha_{i,\tau,t}\tau_{c,t|t} + \tau_{i,t|t}\right)$. Because the model is linear and Gaussian, the filtered estimates are linear functions of current and lagged values of π_{it}, where the time varying weights depend on $\{\alpha_{i,\tau,t}\}$, $\{\alpha_{i,\varepsilon,t}\}$, $\{\sigma_{\Delta\tau,c,t}\}$, $\{\sigma_{\varepsilon,c,t}\}$, $\{\sigma_{\Delta\tau,i,t}\}$, $\{\sigma_{\varepsilon,i,t}\}$, and $\{w_{it}\}$. That is $\tau_{t|t} = \sum_{i=1}^{17}\sum_{j=0}^{t-1}\omega_{ij,t}\pi_{it-j}$, where $\omega_{ij,t}$ are the weights. The values plotted in Figure 5 are $\bar{\omega}_{i,t} = \sum_{j=0}^{3}\omega_{ij,t} / \sum_{k=1}^{17}\sum_{j=0}^{3}\omega_{ij,t}$, where the weights $\omega_{ij,t}$ are computed using the full-sample posterior means of $\{\alpha_{i,\tau,t}\}$, $\{\alpha_{i,\varepsilon,t}\}$, $\{\sigma_{\Delta\tau,c,t}\}$, $\{\sigma_{\varepsilon,c,t}\}$, $\{\sigma_{\Delta\tau,i,t}\}$, and $\{\sigma_{\varepsilon,i,t}\}$.

Additonal References

Carter, C. K., and R. Kohn. 1994. "On Gibbs Sampling for State Space Models".*Biometrika* 81 (3): 541–53.