Explaining the Increased Variability in Long Term Interest Rates

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1 Introduction

Monetary policy affects the macroeconomy only indirectly. In the standard mechanism, changes in the Federal Funds rate, the Federal Reserve's main policy instrument, lead to changes in longer term interest rates which in turn lead to changes in aggregate demand. Slippage can occur during this process, and this potential slippage is a fundamental problem for monetary policymakers. In particular, long term interest rates sometimes move for reasons unrelated to short term rates, confounding the Federal Reserve's ability to control these long term rates and effect desired changes in aggregate demand. This paper asks whether the link between long rates and short rates has weakened over time, thus making it more difficult for the Federal Reserve to achieve its macroeconomic policy objectives through changes in the Federal Funds rate.

Figure 1 provides the empirical motivation for this inquiry. It plots year-to-year changes in ten year Treasury Bond yields from 1965 through 1998. (The volatile period of the late 1970s and early 1980s has been masked to highlight differences between the early and later periods.) The most striking feature of the plot is the increase in the variability of long term rates in the recent period relative to the earlier period. Indeed, the standard deviation of long rates essentially doubled across the two time periods. What caused this increase in variability? Did a change in the behavior of short term interest rates (caused, for example, by a change in Federal Reserve policy) lead to this dramatic increase in long rate variability? Or rather, is this change in variability caused by changes in factors unrelated to short term rates, often described under the rubric of "term" or "risk" premia?

This paper studies the behavior of short term interest rates over the two sample periods highlighted in Figure 1. It focuses on two key questions. First, has the short term interest process changed? Second, can these changes in the behavior short term interest rates explain the increased volatility in long term interest rates? The answer to both of these questions is yes. Thus, the paper's findings suggest no weakening of the link between short rates and long rates, and thus no weakening of the link between the Federal Reserve's policy instrument and its ultimate objectives.

The variability in long term interest rates is tied to two distinct features of the short rate process: (i) the variability of "shocks" or "innovations" to short term interest rates, and (ii) the persistence (or half-life) of these shocks. In the standard model of the term structure, changes in the variability of short rate innovations lead to one-for-one changes in the variability of the long rate. Thus, holding others things constant, a doubling of the standard deviation of the innovation in short term interest rates would lead to the doubling of the standard deviation of long rates evident in Figure 1.

The relationship between short rate persistence and long rate variability is more complicated. To explain this relationship it is useful to consider an example in which the short term interest rate process can be described by an autoregressive model with one lag (an AR(1)). Let ρ denote the autoregressive coefficient associated with process. When $\rho = 0$, short rates are serially uncorrelated, and shocks have only a one-period effect on the short term interest rate. In contrast, when $\rho = 1$ short rates follow a random walk, so that shocks to the current value of short rates lead to a one-for-one change in all future short rates. When long term interest rates are viewed as discounted sums of expected future short term rates, these different values of ρ imply very different behavior for long term rates. For example, when $\rho = 0$, a change in the current short rate has no implications for future values of short rates, so that long rates move very little. In contrast, when $\rho = 1$, any change in the current short rate is expected to be permanent, all future short rates are expected to change, and this leads to a large change in the long term rate. Values of ρ between 0 and 1 are intermediate between these two extremes, but in a subtle way that will turn out to be important for explaining the increased variability in long term interest rates evident in Figure 1. In particular, for long-lived bonds, a short rate process with $\rho = 0.9$ generates long rates that behave much more like those associated with $\rho = 0$ than with $\rho = 1$. Put another way, changes in the autoregressive parameter ρ have large effects on the behavior long term rates only when ρ is very close to 1. Such a result is familiar from studies of consumption behavior using the present value model, where the variability of changes in consumption increase dramatically as income approaches a unit root process (Deaton (1987, Christiano and Eichenbaum (1990) Goodfriend (1992), Quah (1992)).

As a preview of the empirical results in later sections, we find that the variability of short term interest rate shocks was *smaller* in the later sample period than in the earlier period. If there were no other changes in the short rate process, this should have led to a *fall* in the standard deviation of long term interest rates of approximately 50%, as opposed to the 100% *increase* shown in Figure 1. However, we also find evidence of an increase in persistence: for example, the estimate of the largest autoregressive root in the short rate process (the analogue of ρ from the AR(1) model) increased from 0.96 in the early period to nearly 1.0 in the later period. By itself, this increase in persistence should have led to 3-fold increase in the standard deviation of long rates. Taken together, the decrease in short rate variability and increase in persistence do a remarkably good job explaining the increase in the variability of long rates evident in the data.

The estimated change in the persistence of the Federal Funds process has important implications for the Federal Reserve's leverage on long term rates. For example, the estimated autoregressive process for the early sample period implies that a 25 basis point increase in the Federal Funds rate will lead to only a 3 basis increase in 10-year rates. The autoregressive process for the later period implies that the same increase in the Federal Funds rate will lead to a 15 basis point increase in 10-year rates. Alternatively, the increase in persistence makes it possible to achieve a given change in long-rate with a much smaller change in the Federal Funds rates. The "cost" of this increased leverage is the implicit commitment not to reverse changes in the Federal Funds rate, that is, to maintain the persistence in the short rate process. The benefit of this increased leverage is the reduced variability in the short term rate. These costs and benefits are discussed in detail in Woodford (1999), who argues that it may be beneficial for the monetary authority to commit to making only persistent changes in its policy instrument.

The paper is organized as follows. Section 2 documents changes in the variability of both long term and short term interest rates from the 1960s to the present. This section documents the decrease in variability of short term interest rates (the Federal Funds rate and 3-month Treasury Bill rates) but an increased variability in longer term rates (1-, 5- and 10-year Treasury Bond rates). The relative increase in variability is shown to depend on the horizon of the interest rate – it is much higher for 10-year bonds than for 1-year bonds, for example.

Section 3 studies changes in the persistence of short term interest rates over the two sample periods. It begins by using a hypothetical AR(1) model for short term interest rates to quantify the potential effects of short rate persistence on the variability of long term interest rates. The calculations are carried out using a standard model linking long rates to short rates – the expectations model with a constant term/risk premium. In this model, changes in long term interest rates reflect changes in current and future values of short term interest rates. The persistence of short term interest rates is important because it affects the forecastibility of short term rates and thus the effect of changes of in the short rate on long rates. The results indicate that, when ρ is very near 1, a relatively small change in ρ can lead to a large change in the variability of long term interest rates. The section then presents empirical estimates of the short term interest rate processes for the early and later sample periods using monthly values of the Federal Funds rate. These estimated processes show a fall in the variance of the short rate, but an increase in persistence. Statistical inference about persistence is complicated by the near "unit-root" behavior of the short rate. This leads to bias in the ordinary least squares (OLS) estimates and a non-standard sampling distribution for test statistics for shifts in the process across the two sample periods. The paper corrects the OLS estimates for bias using a procedure developed in Stock (1991) and develops a new statistical test for a change in an autoregression that can applied when data are highly persistent.

In Section 4 the variance of long term interest rates is calculated using the expectations model together with the estimated processes for the short rate. These calculations show that the changes in the estimated short rate process lead to increases in long rate variability quite similar to that found in the long rate data.

Finally, section 5 discusses the robustness of the empirical conclusions to specifics of the econometric specification, and Section 6 concludes. Econometric details concerning tests for changes in the persistence of the short rate process are given in the Appendix.

2 Changes in the Variability of U.S. Interest Rates

Figure 2 plots year-over-year changes in six different interest rates over 1965-1998. As in Figure 1, the data from 1979-1984 are masked to highlight differences between the early sample period (1965:1-1978:9) and the more recent period (1985:1-1998:9). The interest rates range from very short maturity (the Federal Funds Rate) to long maturity (10-Year Treasury Bonds and AAA corporate bonds.).[1] Each series is a monthly average of daily observations of the interest rates measured in percentage points at annual rates. Table 1 presents standard deviations for changes in interest rates over different sample periods. Panel A reports results for the year-over-year changes plotted in figure 2, panel B reports results for monthly changes ($R_t - R_{t-1}$) and the final panel reports standard deviations of residuals from estimated univariate autoregressions.

The figures and table show that the volatility of long term rates is much higher in the recent period than in the 1965-78 sample period, but that this not true for short-term rates. For example, from panel A of Table 1, the standard deviation of year-over-year changes in 10-Year Treasury Bond rates increased from 0.69 (69 basis points) in the 1965-78 period to 1.29 (129 basis points) in the 1985-98 period. A similar large increase is evident for AAA Corporate bond rates and for 5-Year Treasury bonds. At the shorter end of term structure volatility did not increase. Indeed there is substantial reduction in the variability of the Federal Funds rate from 2.44 (244 basis points) to 1.50 (150 basis points).

The remainder of the table investigates the robustness of this conclusion about volatility to both sample period and data transformation. The table shows that this conclusion does not depend on of the precise dates used to define the "early" and "recent" periods. These 1965-1978 and 1985-1998 dates were chosen somewhat arbitrarily, and the same volatility results hold for a wide range of cut-off dates used to define the sample periods. Thus for example, defining the early period as 1955-78 and the recent period as 1992-1998 leads to the same conclusions. However, results do change if the volatile period of the late 1970s and early 1980s is included: from Table 1 interest rates were much more volatile in this period

[1] All of the data are from the DRI database. The series are FYFF, FYGM3, FYGT1, FYGT5, FYGT10 and FYAAAC.

than they were either before 1979 or after 1984. Finally, the results from different panels show that same qualitative conclusion follows when year-over-year differences are replaced with monthly differences or with residuals from univariate autoregressions. For example, from panel C, the standard deviation of the residuals in a univariate autoregression for 10-Year Treasury Bond rates increased from 19 basis points in 1965-78 to 25 basis points during 1985-78. The corresponding standard deviation for the Federal Funds rate fell from 38 to 21 basis points.

Since the variability of short term was smaller in the later sample period than in the early period, it is clear that changes in the variability of short rates cannot explain the increased variability of long rates. We will have to look elsewhere, and with in mind the next section investigates changes in persistence in the short rate process.

3 Changes in the Persistence of U.S. Interest Rates

Before examining the empirical results on the persistence of short-term interest rates, it is useful to review the mechanism that links changes in short-rate persistence with changes in long-rate variability. This mechanism can be described using a simple expectations model of the term structure. Thus, let R_t^h denote the yield to maturity on an h-period pure discount bond, and assume that these yields are related to short term rates by

$$R_t^h = \frac{1}{h}\sum_{i=0}^{h-1} E_t R_{t+i}^1$$

where R_t^1 is the corresponding rate on a 1-period bond. This relation can be interpreted as a risk-neutral arbitrage relation. Now, suppose that short term rates follow an AR(1) process

$$R_t^1 = \rho R_{t-1}^1 + \varepsilon_t$$

so that $E_t R_{t+i}^1 = \rho^i R_{t+i}^1$ for $i \geq 1$. Then

$$R_t^h = R_t^1 [\frac{1}{h}\sum_{i=0}^{h-1} \rho^i]$$

so that long rates are proportional to short rates, with a factor of proportionality that is an increasing function of the persistence parameter ρ. Complications to the model (incorporation of term/risk premia, allowance for coupon payments, etc.) change details of the link between long rates and short rates, but do not change the key feature of the model that long rates depend on a sequence of expected future short rates, and that the variance of this sequence depends critically on the persistence of shock to short term rates.

Figure 3 gives a sense of the quantitative impact of short-rate persistence on long-rate variability. Using the expectations relation given above, it plots the standard deviation of year-over-year changes in R_t^h (that is, $R_t^h - R_{t-12}^h$) as a function of ρ. Results are shown for different maturities h, and the scale of the plot is fixed by setting the innovation variance of short rates (σ_ε^2) equal to 1. The plot shows the functions for values of ρ between 0.95 and 1.00, which is the relevant range for the monthly data studied in this paper. For short maturities (small values of h) ρ does not have much of an effect on the standard deviation interest rate. For example, as ρ increases from 0.95 to 1.00 the standard deviation of 1-period rates increases by a factor of 1.1 (from 3.1 to 3.5). However, ρ has a large effect of the variability of long-term interest rates. When $h = 120$ (a 10-year bond when the period is a month), then as ρ increases from 0.95 to 1.00, the resulting standard deviation of long rates increases by a factor of 7 (from 0.5 to 3.5). Moreover, the rate of increase in the standard deviation increases with the value of ρ. Thus, the implied changes in the volatility of long rates across sample periods will depend both on the level of ρ and on its change. We now move to the empirical evidence on persistence.

Table 2 contains estimates of the persistence in short term rates for the two sample periods. Results are presented for both the Federal Funds and the 3-month Treasury Bill rate. Univariate autoregressions are fitted to the series, and persistence is measured by the largest root of the implied autoregressive process. The effect of shocks on long horizon forecasts of short rates is determined by this largest autoregressive root, and hence this parameter summarizes most of the information about the link between short rates and long-term interest rate variability. This parameter is denoted by ρ as in the AR(1) model discussed above.

The first entry for each sample period is the OLS estimate of ρ (denoted ρ_{ols}) computed from an $AR(6)$ model. (The next section will discuss the robustness of results to the lag length in the autoregression.) The values of ρ_{ols} are very large for both interest rates and over both sample periods. Thus, short rates were apparently highly persistent in both sample periods. There is some evidence of a small increase in ρ in the latter sample period: the value of ρ_{ols} increases from 0.97 to 0.98 for Federal Funds and from 0.96 to 0.98 for 3-month Treasury bills. However, interpreting these changes is difficult because of statistical sampling problems associated with highly persistent autoregressions. These problems are well known in autoregressions with "unit-roots," but similar problems also arise when roots are close to unity. Thus, a short statistical digression is necessary before discussing the other entries in Table 2.

When values of ρ are close to 1 and the sample size is moderate (as it is here), then the sampling distributions of OLS estimators and test statistics differ markedly from the distributions that arise in the classical linear regression model. In particular, ρ_{ols} is biased, and the usual t-statistics have non-normal distributions Confidence intervals for ρ cannot be constructed in the usual way. Of course, so long as ρ is strictly less than 1, the usual asymptotic statistical arguments imply that these problems disappear for a "suitably" large sample size. Unfortunately, the sample size used in this paper (and the sample size commonly used in empirical macroeconomic research) is not large enough for the conventional asymptotic normal distributions (based on stationarity assumptions,) to provide an accurate approximation to the sampling distribution of the usual OLS statistics. We must use alternative and more accurate approximations.

In empirical problems when ρ is close to 1 (say in the range $0.90 - 1.01$) and the sample size is moderate (say less than 200 observations), econometricians have found that "local-to-unity" approximations provide very good approximations to the sampling distribution of OLS statistics[2]. These approximations will be used hereIn the present context, these approximations will be used to construct unbiased estimators of ρ, confidence intervals for ρ, and Prob-values for tests for changes for ρ over the two sample periods. Specifically, "median-unbiased" estimates and confidence intervals for ρ are constructed from the Dickey-Fuller τ^{μ} statistic using the procedures developed in Stock (1991).[3] Tests for changes in ρ

[2] Important early references in econometrics include Cavanagh (1985), Phillips (1987) and Stock (1991).

[3] The median unbiased estimator,which will be denoted ρ_{mub}, has the property that $\Pr ob(\ \rho_{mub} \leq \rho) = \Pr ob(\rho_{mub} \geq \rho) = 0.5$.

across the two sample periods are carried out using the usual Chow-F statistic. This statistic is computed as the Wald statistic from changes in the values of ρ_{ols} over the two sample periods. The regressions are estimated separately in each sample period, so that all of the coefficients are allowed to change, but the Wald statistic tests for a change in the largest root only. (Changes in the other autoregressive parameters will have little effect on the variance of long rates, and so we focus the test on the largest root.). The statistical significance of the Chow statistic can be determined using Prob-values computed from the local-to-unity probability distributions. These alternative Prob-values are described in detail in the Appendix. As the appendix shows, the Prob-value depends on the true, and unknown, value of ρ. Thus, rather than reporting a single Prob-value, an upper and lower bound is reported.

With this background, the other entries in Table 2 can now be discussed. The unbiased estimates are reported in the column labeled ρ_{mub} (the *mub* subscript stands for "Median UnBiased), and these are followed by the 90% confidence interval for ρ. The point estimates suggest that persistence was higher in the second period; for example, using the Federal Funds rates the value of ρ_{mub} increased from 0.96 to 1.00. However, the confidence intervals show that there is a rather wide range of values of ρ that are consistent with the data – the first period confidence interval for Federal Funds is 0.91-1.01 and this shifts up to 0.94-1.02 in the second period. The overlap in these confidence intervals suggests that the apparent shift in ρ is not highly statistically significant, and this is verified by the Chow-statistic which has a Prob-value that falls between 0.30 and 0.64. Thus, there is some evidence of a shift in the largest root, in a direction consistent with the behavior of long-term rates, but the shift is small and the exact magnitude is difficult to determine because of sampling error. However, when ρ is near 1, small changes in its value can have large changes in the variability of long term interest rates.

4 Implications of the Changes in the Short rate process on Long rate variability

The changes in long rate volatility associated with the changes in the short rate process depend on the specifics of model linking short rates to long rates. Before computing the variability of long rates associated with the estimated short rate processes from the last section, there are three issues that need to be addressed in the present context.

First, the data used here, while standard, are not ideal. The data are not point sampled, but rather are monthly observations of daily averages. The bonds contain coupon payments, which were absent from in the simple theory presented above. The calculations presented below are based on two approximations. First, the process for 1-month rates is estimated using the Federal Funds data. This is a rough approximation that uses a monthly average of daily rates as a monthly rate. As it turns out, similar results obtain if the Federal Funds process was replaced with the estimated process for the 3-month Treasury Bills, and so the precise choice of short rate does not seem to matter much. The second approximation adjusts the present value expectations model for coupon payments using the approximation in Shiller, Campbell and Schoenholitz (1983). Specifically, the expectational equation for long rates becomes

$$R_t^h = \frac{1-\beta}{1-\beta^h} \sum_{i=0}^{h-1} \beta^i E_t R_{t+i}^1$$

where $\beta = .997$.

The second issue involves the expectations theory described above. That model used an AR(1) driving process for short rates, and constructed expectations using this process. The univariate process for short rates in more complicated than an AR(1); moreover expectations can be formed using an information set richer than one containing only lags of short rates. Extending the calculations to account for higher order univariate AR process is straightforward, as this merely involves computing the terms $E_t R_{t+i}^1$ from a higher order AR model. However, accounting for a wider information set is more problematic. A standard and powerful approach to this problem is to construct bounds on the implied variance of long rates from the short process, using for example, the approach in Shiller (1981). Unfortunately, this approach requires stationarity of the underlying data, and thus the bounds are likely to be inaccurate for the highly persistent data studied here. West (1988) proposes bounds for the expectational present value model based on the innovations in the univariate processes and shows that these bounds hold for integrated as well as stationary processes. Unfortunately, West's results hold only for the infinite horizon model, and the model here is finite horizon.[4] Another approach is simply to specify a more general information set and carry out the analysis using, say, a vector autoregression instead of a univariate autoregression. However, the statistical analysis becomes increasingly complicated in a VAR with highly persistent variables. For all of these reasons, the analysis here will be carried out using a univariate AR.

Finally, the calculations reported here ignore all term/risk premia and other deviations from the simple expectations theory. As mentioned above, even in more complicated versions of the models, the first order impact of short rate persistence on long rate variability occurs through the expected present value expression from the version of the model used here. With these limitations in mind, we now discuss the implied variability in long term rates.

The results are summarized in figure 4 and table 3 which shows the implied variability of interest rates computed from the expectations model, using the estimated short-rate process over the different sample periods and for different values of ρ. Results are shown for four maturities. Each panel of figure 4 shows the variability of year-over-year changes in the interest rate implied by the estimated AR(6) model for the Federal Funds rate estimated imposing the value of ρ shown on the $x-$axis. Results are shown for both sample periods. Highlighted on the graphs are the results that impose the OLS and the median unbiased estimates of ρ from Table 2. (The value of ρ_{ols} is shown by a circle and ρ_{mub} is shown by a square.). In each panel, the variance function for the second period lies below the

[4] In West's present value model $y_t = E_t \sum_{i=0}^{h} \beta^i x_{t+i}$, the key restriction is that $E_t \beta^h x_{t+h}$ converges to zero in mean square as $h \to \infty$. This suggests that West's bounds will provide a good approximation in the finite horizon model so long as $E_t \beta^h x_{t+h}$ is small. Thus, the quality of the approximation will depend on the size of $(\beta\rho)^h$, where ρ is the largest autoregressive root. In the term structure model $\beta = 0.997$ and the x_t process is highly persistant, with a largest autoregessive root of, say $\rho = 0.99$. Thus, for $h = 120$, $(\beta\rho)^h = 0.16$, which implies that $E_t \beta^h x_{t+h}$ will often be substantially different from 0.

function for the first period. This shift is caused by the decrease in variance of the AR errors estimated for the second period. The vertical distance between the curves shows the change in variance for a given value of ρ. To compute the variance across periods, the value of in ρ in each sample period must be specified. Thus, the change in variability across the two periods using the OLS estimates of ρ (ρ_{ols}) is given by vertical displacement of the two circles plotted in the graph. The displacement of the squares gives the change using the median unbiased estimator (ρ_{mub}). The implied standard deviation for the four maturities, for both sample periods, and for ρ_{ols} and ρ_{mub} are given in table 3. For comparison, the table also gives the period-specific sample standard deviations for the Federal Funds rate and the rates on 1-, 5- and 10-year Treasury Bonds.

There are substantial differences in the results across the four panels in the graph. For one month rates (panel A), variability is essentially independent of ρ and thus the model predicts a substantial decrease in variability during the second period. Since the Federal Funds rate data were use to estimate the short rate process, this decrease in variability is essentially equal to the sample values – see the first row of Table 3. For 1-year rates (panel b of figure 4 and the second row of table 3) variability is also predicted to decrease in the second period, but the decrease is far less than for 1-month rates, and depends on the estimator of ρ used. (The implied decrease in the standard deviation is 49 basis points using ρ_{ols} and 30 basis points using ρ_{mub}.). In the sample, there was a small increase (14 basis points) in the standard deviation of 1 year interest rates. At longer maturities (panels C and D of figure 4 and the last two rows of table 3), variability is predicted to increase in the second period, and again, the amount of the increase depends on the estimator of ρ that is used. The increase is not particularly large using ρ_{ols} (less than 20 basis points); however it is much larger using ρ_{mub} (70 basis points). The small bias correction incorporated in ρ_{mub} results in this large change because it pushes the second period estimate of ρ very close to 1 and because the variance function is rapidly increasing in this region.

While the estimated change in persistence, as measured by ρ_{mub}, explains much of the increase in variability in long term interest rates, there is still a large amount of the variability in long rates that is left unexplained. For example, in the first sample period the model's implied standard deviation for 5-year rates is 52 basis points, while the sample standard deviation of actual 5-year rates is 89 basis points. This leaves a "residual" component, orthogonal to short term rates, with a standard deviation of 72 basis points ($72 = \sqrt{89^2 - 52^2}$) representing the difference between the actual 5-year rates and the value implied by the expectations model. Interestingly, a residual component of similar size (69 basis points) is necessary in the second sample period. (A somewhat larger residual is required for 10-year rates.) Thus, although the simple expectations model with constant term/risk premium and simple information structure leaves much of the variability in long-rate unexplained in both sample periods, it does explain the lion's share of the increase in variability across the two sample periods.

The results derived here, based on a simple version the expectations theory of the term structure, are consistent with results derived by other researchers using reduced form time series methods. For example, the expectations theory together with a process for the short term interest can be used to calculate the change in the long-rate associated with a given change in the short rate. Using the first period estimates (and the values of ρ_{mub} shown in Table 2) the model predicts that a 25 basis point change in the Federal Funds rate would lead to a 3 basis point change in the 10-year Bond rate. The second period estimates imply that the same 25 basis point change in the Federal Funds Rate would lead to a 15 basis point in the long rate. Mehra (1996) estimates a reduced form time series model (a vector error correction model) of long rates, inflation and inflation over the 1957-1978 and 1979-1995 sample periods. His estimated models predict that a 25 basis point change in the Federal Funds rate led to $3-7$ basis point change long rates in the early period and a $7-12$ basis change in the later period.

5 Robustness of Results

The results presented in the last section were based on AR(6) models estimated over the sample periods 1965:1-1978:9 and 1985:1-1998:9. This section discusses the robustness of the paper's main findings to specification of lag length in the autoregression and to choice of sample period. The empirical results are summarized in Table 3. The first row in each panel of the table shows the results from the specification used in the last section. Thus these results are the same as reported in Table 2. Each of the following rows summarizes results from a different specification of either lag-length or sample period. Panel A of the table shows results for the Federal Funds rate and the panel B shows results for the 3-month Treasury Bill rate.

The AR lag length of 6 used in the baseline specification was suggested by the AIC and by t-tests on the autoregressive coefficients. Much shorter lag lengths were suggested by the BIC. Table 2 shows results from specifications using 2, 4 and 8 lags. Each of these alternative specifications yield first period estimates of ρ that are lower than the estimates from the AR(6) model; second period estimates are essentially unchanged. The first period differences in ρ_{ols} are small, but the differences are more substantial for the ρ_{mub}. However, even ignoring the large amount of sampling error associated with these estimates, the new point estimates have little effect on the variance of long term rates. From figure 3, the long-rate variance function is relatively flat over the range of first-period ρ estimates given in Table 3. Thus, (from figure 3), the implied first-period standard deviation of long term interest rates changes is 0.11 when $\rho = .93$ (the value of ρ_{mub} from the AR(4) first-period model for the Federal Funds rate) and increases to only 0.18 as ρ increases to 0.96 (the corresponding value of ρ_{mub} in the AR(6) model). Both of these specifications imply a much larger second period standard deviation (1.48 and 0.975 for the AR(4) and AR(6) models, respectively) since the second period values of ρ_{mub} are very close to 1.0 in both specifications. Thus lag length choice appears to have little effect on the qualitative conclusions.

The choice of sample period has a more important effect. The baseline sample periods 1965:1-1978:9 and 1985:1-1998:9 were chosen to eliminate the large variability in interest rates during the late 1970s and early 1980s. With this volatile period eliminated, two samples of equal size were chosen (with 1998:9 being the last sample period available when this research was started). Aside from equating statistical power in each sample, there is no compelling reason why the early and recent samples should be of equal size, and the last two rows of the table show results from increasing the early sample period (by changing the beginning date to 1955:1) and decreasing the recent sample period (by changing the beginning date to 1992:1). Since the 1992-1998 sample period is very short, an AR(2) was model was used for this specification. Evidently, the choice of the second period has little effect on the estimates of ρ, but the choice of first sample period does. Estimates of ρ are larger for both interest rates in the extended sample period 1955-1978 than in the 1965-1978. This increase should not be surprising given the behavior of interest rates over 1955-1978, where the dominant feature of the data is an increase in the "trend" level of interest rates. However, since this paper's analysis focuses on the behavior of long rates as they are affected by expected future short rates, the question is whether investors in the late 1950s anticipated this trend rise in interests, as would be suggested by ex-poste fitted values from the univariate autoregression. This seems unlikely.

6 Summary and Discussion

This paper has documented the increase in the variability of long term interest rate changes during 1985-1998 relative to 1965-1978. In contrast, the variability of short term interest rates decreased in the latter period. A possible explanation for this differential behavior is a change in the persistence of changes in short term rates: expectations theories of the term structure imply that such shifts in persistence will have a large effect on the variability of changes in long term rates but have little effect on the variability of changes in short rates. Point estimates of the largest autoregressive root for short rates show an increase in persistence that is large enough to explain the increase variability in long rates. However, the short rate persistence parameter is imprecisely estimated, so that it is impossible to reach definitive conclusions based on this analysis. This lack of precision raises two issues: one related to statistical technique and one related to learning about changes in central bank policy.

The statistical issue follows from the observation that persistence in short term interest rates have a large effect on long term rates, and yet the persistence parameter cannot be precisely estimated using the short rates. (Confidence intervals for ρ reported in tables 2 and 3 are very wide.) This suggests that information in long rates could potentially sharpen the estimates of ρ, perhaps quite significantly. This is indeed the case if one is willing to assume that a version of the present value model holds. This observation is used in Valkanov (1998) who uses the model's implied cointegration between long and short rates to construct improved estimators for ρ. He then uses this improved estimator to overcome inference problems identified by Elliott (1998) in his critique of cointegration methods. Indeed, in a comment on a preliminary draft of this paper, Valkanov (1999) used his method to construct estimates of ρ together with 90% confidence intervals for the time periods 1962:1-1978:8 and 1983:1-1991:2 using data on the Federal Funds rate and 10 year Treasury Bonds. He finds an estimate of ρ of .96 (with a 90% confidence interval of $0.93 - 0.98$) in the early period and an estimate of 0.99 (with a 90% confidence interval of $0.99 - 1.00$) in the later period (Valkanov (1999), table 2c) . His point estimates are essentially identical to the values of ρ_{mub} reported in Table 2, but as expected from the use of a more efficient procedure, his confidence intervals are considerably narrower than the results presented in Table 2.

The large sampling uncertainty associated with estimates of the short rate persistence suggests that the market will learn about changes in persistence very slowly from observing realization of short term interest rates. A central bank interested in increasing the persistence of short term interest rates (for the reason suggested in Woodford (1999), for example) would have to follow this policy for a considerable time to convince a market participant who relied only on econometric evidence that such a change had indeed taken place. For example, suppose that the Federal Funds process changed from one with a largest root of 0.96 to one with a largest root 0.99, and after 10 years in the new regime an econometrician tested the null hypothesis that $\rho = 0.96$ versus the alternative that $\rho > .96$ using a standard t-test at the 5% significance level. The econometrician would (correctly) reject this null only about 50% of the time. (That is, the power of the test using 10 years of data is roughly 0.50). Thus, it is likely that econometrician would have to observe the new Federal Funds process for quite some time before he concluded that the process had changed. This highlights the role of other devices (institutional constraints, public statements, etc.) to more quickly convince a wary public.

This paper has presented econometrician evidence suggesting that changes in the Federal Funds rate are more persistent now than they were in the 1960s and 1970s. Why did this change occur? This is an important question, on which I can offer but a few remarks. One possible explanation follows from decomposing the Funds rate into a real rate and an inflation component. Iif movements in the real rate are transitory, then the persistence in the Funds rate will be driven by the inflation component. Thus, an increase in the persistence of inflation is a possible explanation of the increased persistence in the Funds rate. This explanation does not seem promising. For example, the values of ρ_{mub} computed for using CPI inflation *fell* from 0.98 in the earlier sample period to 0.92 in the latter period. Thus, inflation seems to have become *less* persistence, and this implies that some of the explanation must lie in the persistence of the real component of the Funds rate. There is growing econometric evidence that the Federal Reserve's "reaction function" linking the Federal Funds rate to expected future inflation and real activity has been quite different in under Chairman Volker and Greenspan that under the previous three Chairs. For example, Clarida, Gali and Gertler (1999) present evidence suggesting that the Federal Reserve responded more aggressively to expected future inflation post-1979 than in the previous two decades. Their evidence also suggests that Federal Reserve more aggressively smoothed the funds rate in this latter period, consistent with the increased persistence found here. Changes in this reaction function undoubtedly contain the key to explaining the increased persistance in the Federal Funds rate process.

A Computing Prob-values for the Chow Test Statistic for the Largest Autoregressive Root

This appendix describes the method used to compute the Prob-values for tests of changes in the largest autoregressive root of a univariate autoregression. The specification is the AR(p) autoregression

$$x_t = \mu + u_t$$

with

$$u_t = \sum_{i=1}^{p} \phi_i u_{t-1} + \varepsilon_t$$

where x_t denotes the level of the interest rate, μ is a constant denoting the average level of the process in the stationary model and u_t is a stochastic term. The u_t process can be rewritten as

$$u_t = \rho u_{t-1} + \sum_{i=1}^{p-1} \pi_i (u_{t-i} - u_{t-i-1}) + \varepsilon_t$$

were $\rho = \sum_{i=1}^{p} \phi_i$ and $\pi_i = -\sum_{j=i+1}^{p} \phi_j$. The parameter ρ is thus the sum of the AR coefficients. When one root of the AR polynomial $1 - \sum_{i=1}^{p} \phi_i z^i$ is close to 1 and all of the other roots are larger than 1, then ρ is also an approximately equal to the inverse of the root closest to unity. In this case ρ is usually called the "largest" root because its inverse is the largest eigenvalue of the companion matrix of the model VAR(1) representation.

In this appendix we study the behavior of statistics in a setting where ρ is modeled as close to 1.0, written as

$$\rho_T = 1 + \frac{c}{T}$$

This artificial dependence of ρ on the sample size T facilitates the analysis of continuous asymptotic limits as $T \to \infty$.[5] To simplify notation, the presentation will be done in the AR(1) model, so that $\pi_i = 0$, for $i = 1, ..., p-1$. For the test statistics used in this

[5] To see this, contrast the discontinous results

$$\lim_{T\to\infty} \rho^T = \left\{ \begin{array}{c} 0 \text{ when } |\rho| < 1 \\ 1 \text{ when } \rho = 1 \\ \infty \text{ when } \rho > 1 \end{array} \right\}$$

paper, the inclusion of extra lags has no affect on the limiting distribution, and in this sense the presentation here is without loss of generality. Following the discussion of the limiting distribution of the Chow test statistic, the appendix discusses the numerical procedure used to compute the Prob-values shown in tables 2 and 3.

A.1 Asymptotic Distribution in the AR(1) Model

Assume

$$u_t = \rho_T u_{t-1} + \varepsilon_t$$

where u_0 is a finite fixed constant, $t = 1, ..., T$ and ε_t is a martingale difference sequence with $E(\varepsilon_t^2 \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) = 1$ and with $\sup_t E\varepsilon_t^4 < \infty$, and where $\rho_T = 1 + \frac{c}{T}$.

Let $\widehat{\rho}_1$ denote the OLS estimator of ρ constructed from the regression of x_t onto $(1, x_{t-1})$ using the early sample period $t = 1, ..., T_1$ and let $\widehat{\rho}_2$ denote the corresponding estimator constructed using the later sample period $t = T_2, ..., T$. Assume

$$\lim_{T\to\infty} \frac{T_1}{T} = \tau_1$$

and

$$\lim_{T\to\infty} \frac{T_2}{T} = \tau_2$$

with $0 < \tau_1 < \tau_2 < 1$. Denote the sample means by

$$\begin{aligned}
\overline{x}_{1,T} &= \frac{1}{T_1} \sum_{t=1}^{T_1} x_t \\
\overline{x}_{2,T} &= \frac{1}{T - T_2 + 1} \sum_{t=T_2}^{T} x_t
\end{aligned}$$

and the demeaned series by

$$\begin{aligned}
x_{1,t}^{\mu} &= x_t - \overline{x}_{1,T} \\
x_{2,t}^{\mu} &= x_t - \overline{x}_{2,T}
\end{aligned}$$

---

with the continuous result

$$\lim_{T\to\infty} (\rho_T)^T = e^c \text{ when } \rho_T = 1 + c/T.$$

The limiting behavior of these series is related to the behavior of the diffusion process $J_c(s)$, generated by

$$dJ_c(s) = cJ_c(s)ds + dW(s)$$

for $0 \leq s \leq 1$, where $W(s)$ is a standard Wiener process. In particular

$$\frac{1}{\sqrt{T}} x^{\mu}_{1,[sT]} \Rightarrow J_c(s) - \tau_1^{-1} \int_0^{\tau_1} J_c(r)dr \equiv J^{\mu}_{1,c}(s) \text{ for } 0 < s \leq \tau_1$$

$$\frac{1}{\sqrt{T}} x^{\mu}_{2,[sT]} \Rightarrow J_c(s) - (1-\tau_2)^{-1} \int_{\tau_2}^{1} J_c(r)dr \equiv J^{\mu}_{2,c}(s) \text{ for } \tau_2 \leq s < 1$$

The Chow F-statistic for testing $H_o : \rho_1 = \rho_2$ is

$$F = \frac{(\widehat{\rho}_1 - \widehat{\rho}_2)^2}{[\sum_{t=1}^{T_1} (x^{\mu}_{1,t-1})^2]^{-1} + [\sum_{t=T_2}^{T} (x^{\mu}_{2,t-1})^2]^{-1}}.$$

The limiting behavior follows from considering the terms

$$U_{1,T} \equiv \frac{1}{T} \sum_{t\ \equiv 1}^{T_1} \varepsilon_t x^{\mu}_{1,t-1} \Rightarrow \int_0^{\tau_1} J^{\mu}_{1,c}(s)dW(s) \equiv U_1$$

$$U_{2,T} \equiv \frac{1}{T} \sum_{t\ \equiv T_2}^{T} \varepsilon_t x^{\mu}_{2,t-1} \Rightarrow \int_{\tau_2}^{1} J^{\mu}_{2,c}(s)dW(s) \equiv U_2$$

$$V_{1,T} \equiv \frac{1}{T^2} \sum_{t\ \equiv 1}^{T_1} (x^{\mu}_{1,t-1})^2 \Rightarrow \int_0^{\tau_1} (J^{\mu}_{1,c}(s))^2 ds \equiv V_1$$

$$V_{2,T} \equiv \frac{1}{T^2} \sum_{t\ \equiv T_2}^{T} (x^{\mu}_{2,t-1})^2 \Rightarrow \int_{\tau_2}^{1} (J^{\mu}_{2,c}(s))^2 ds \equiv V_2$$

Defining

$$\gamma_{1,T} = T(\widehat{\rho}_1 - \rho) \text{ and } \gamma_{2,T} = T(\widehat{\rho}_2 - \rho),$$

the F can be written as

$$F = \frac{(\gamma_{1,T} - \gamma_{2,T})^2}{V_{1,T}^{-1} + V_{2,T}^{-1}}.$$

Since

$$\gamma_{1,T} = \frac{\frac{1}{T}\sum_{t=1}^{T_1} \varepsilon_t x^{\mu}_{1,t-1}}{V_{1,T}} = \frac{U_{1,T}}{V_{1,T}} \Rightarrow \frac{U_1}{V_1} = \gamma_1$$

and

$$\gamma_{2,T} = \frac{\frac{1}{T}\sum_{t=T_2}^{T} \varepsilon_t x^{\mu}_{2,t-1}}{V_{2,T}} = \frac{U_{2,T}}{V_{2,T}} \Rightarrow \frac{U_2}{V_2} = \gamma_2$$

by the continuous mapping theorem, then

$$F \Rightarrow \frac{(\gamma_1 - \gamma_2)^2}{V_1^{-1} + V_2^{-1}}$$

which provides a representation for the limiting distribution of F in terms of functionals of the diffusions $J_c(s)$.

A.2 Approximating Prob-values

The limiting distribution of F is seen to depend on three parameters τ_1, τ_2 (through the limits in the integrals) and the value of c (through the mean reversion in the diffusion process J_c). Quantiles of the limiting distribution (an hence Prob-values for the test statistic) can be approximated by repeated simulations of F using a large sample size and for fixed values of τ_1, τ_2, and c, and ε_t chosen as $Niid(0,1)$ random variables. The Prob-values reported in the paper resulted from $10,000$ replications from a sample of size 500. The parameters τ_1 and τ_2 were chosen as T_1/T and T_2/T where T_1 denotes the first break point and T_2 denotes the second break point. The distribution also depends on c, which governs how close ρ is to unity. Unfortunately, this parameter cannot be consistently estimated. (Equivalently, in finite samples the distribution of F depends on ρ, and small changes in ρ – like those associated with sampling error – lead to large changes in the quantiles of this distribution.) Thus, selecting the correct distribution of F requires knowledge of c (equivalently, ρ). Since c is unknown, the distribution is computed for an range of values in $-25 \leq c \leq 10$ and the resulting minimum and maximum Prob-value over all of the these values of c is reported in the table. Viewing c as unknown, classical approaches (which must hold for all values of the "nuisance parameter" c) would use the upper Prob-value. The lower bound gives the smallest Prob-value that would be obtained in c were known.

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Figure 1
10 Year Treasury Bond Yields
Annual Differences

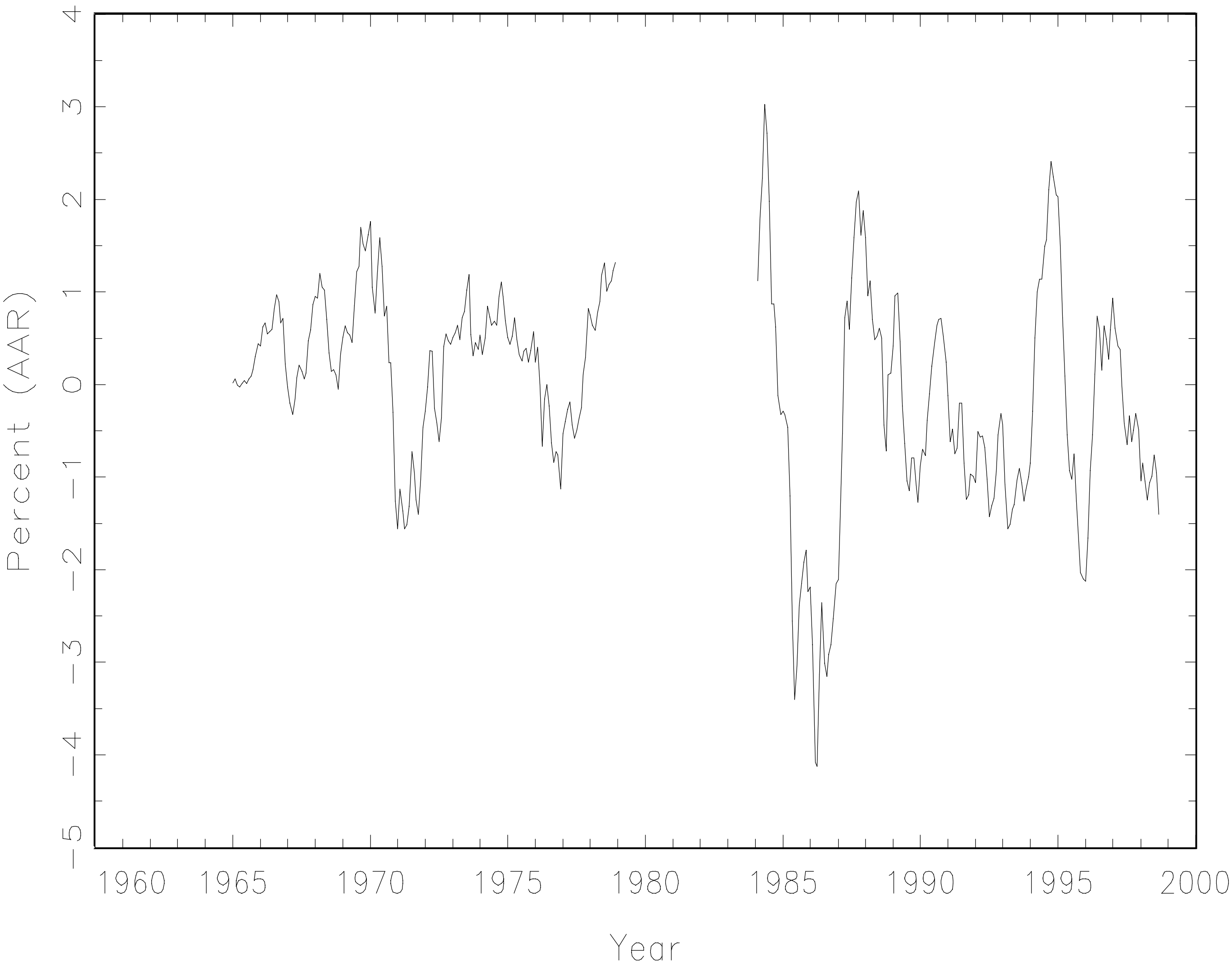

Figure 2
Annual Differences of Interest Rates

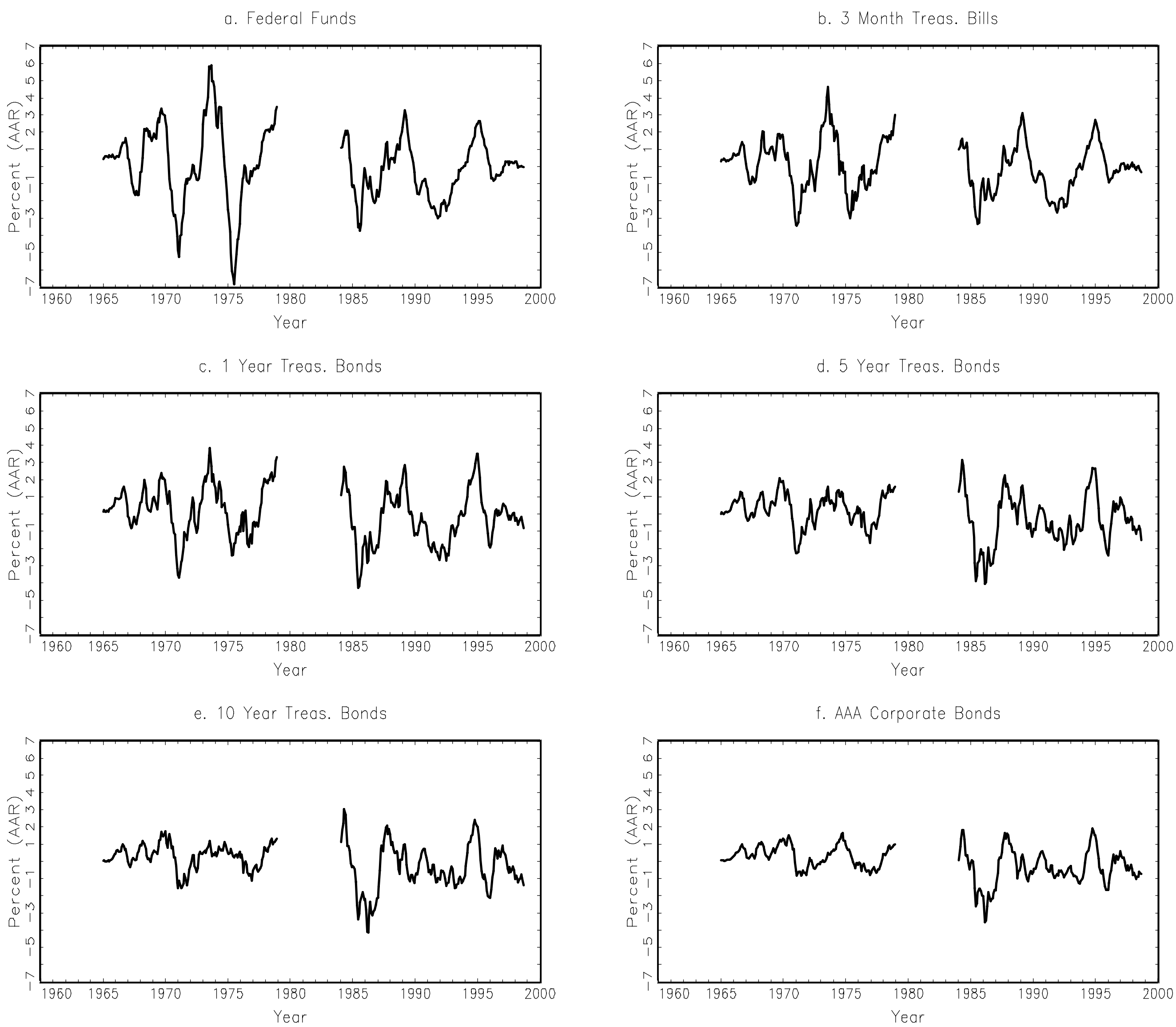

Figure 3: Annual Differences in Interest Rates
Implied Standard Deviation from AR(1) Model

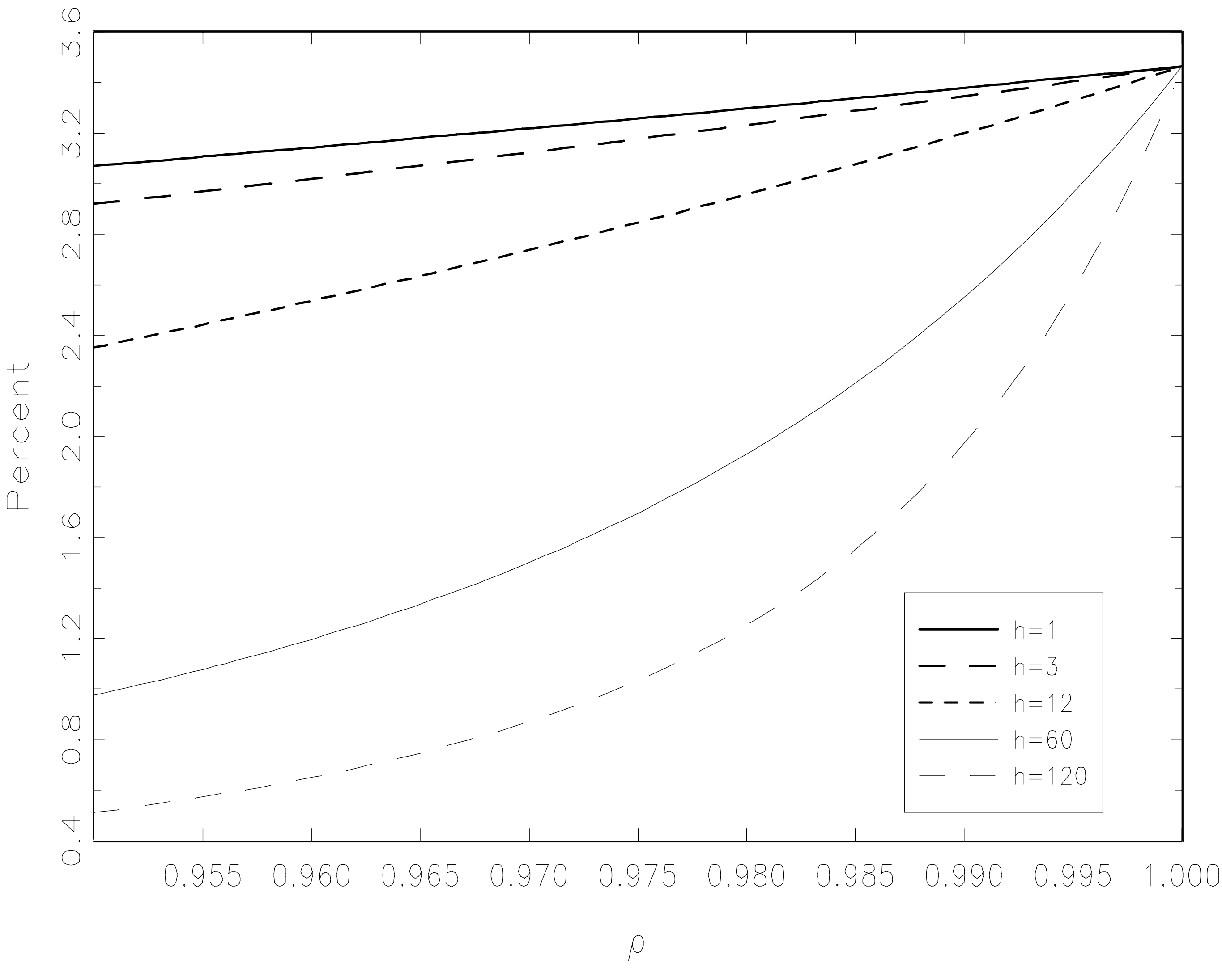

Figure 4: Annual Differences in Interest Rates
Implied Standard Deviation from Fitted AR Models
(Circle=ρ_{ols}, Square=ρ_{mub})

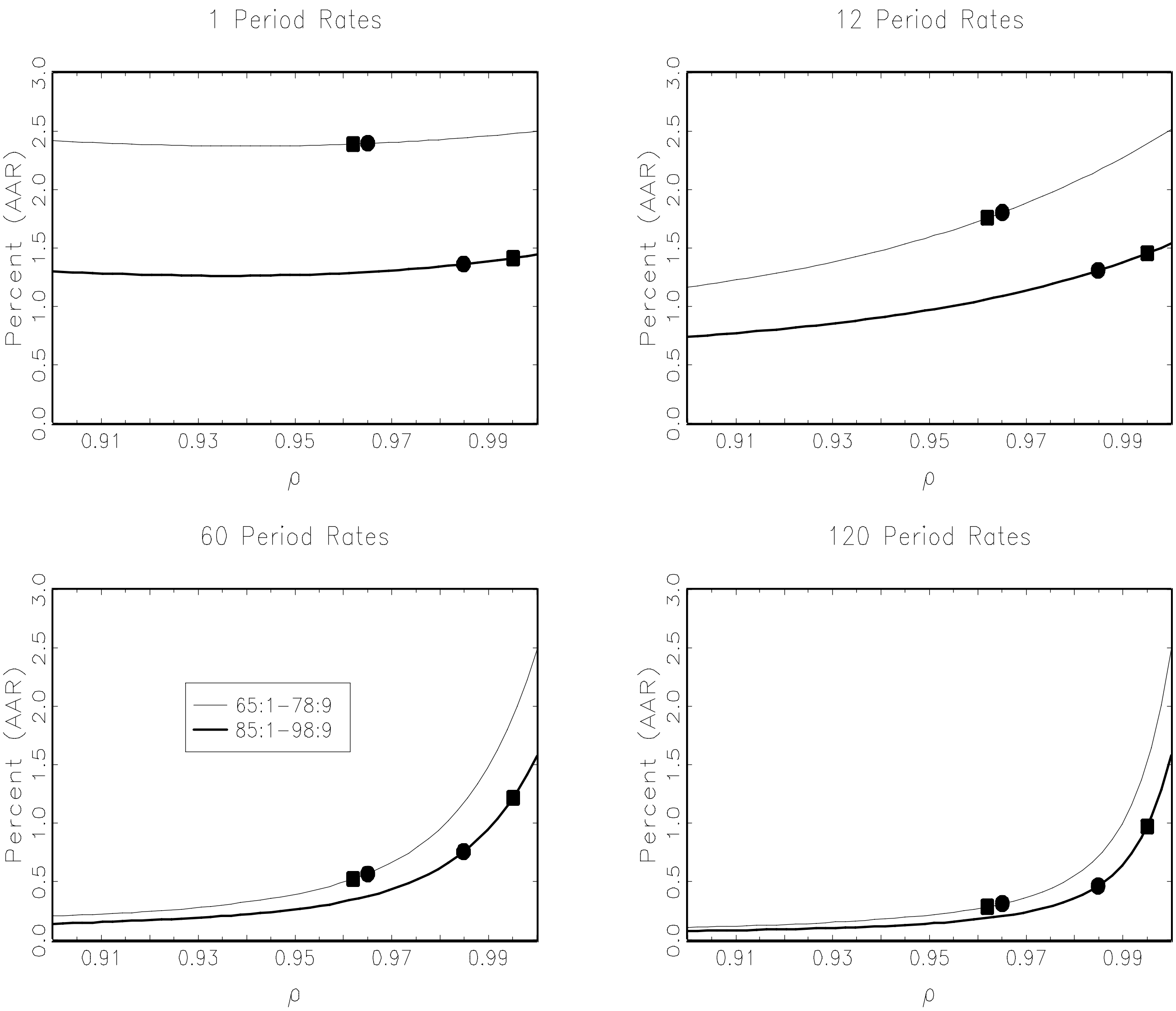

Table 1

Standard Deviations of Interest Rate Changes
(Percent at Annual Rates)

A. Year-over-Year Differences

	Interest Rate					
Sample Period	FedFunds	3M-TB	1Y-TB	5Y-TB	10Y-TB	Corp
1965:1- 1978:9	2.44	1.50	1.40	0.89	0.69	0.60
1985:1- 1998:9	1.50	1.37	1.54	1.40	1.29	1.00
1978:10-1984:12	4.12	3.29	3.12	2.35	2.06	1.81
1955:1- 1978:9	2.02	1.33	1.30	0.83	0.62	0.51
1992:1- 1998:9	1.28	1.17	1.38	1.24	1.07	0.81

B. First Differences

	Interest Rate					
Sample Period	FedFunds	3M-TB	1Y-TB	5Y-TB	10Y-TB	Corp
1965:1- 1978:9	0.44	0.37	0.37	0.26	0.20	0.13
1985:1- 1998:9	0.23	0.21	0.28	0.30	0.27	0.21
1978:10-1984:12	1.36	1.10	1.03	0.68	0.57	0.47
1955:1- 1978:9	0.38	0.32	0.32	0.22	0.17	0.11
1992:1- 1998:9	0.15	0.16	0.23	0.26	0.23	0.17

C. AR Innovations

	Interest Rate					
Sample Period	FedFunds	3M-TB	1Y-TB	5Y-TB	10Y-TB	Corp
1965:1- 1978:9	0.38	0.35	0.34	0.25	0.19	0.12
1985:1- 1998:9	0.21	0.18	0.25	0.27	0.25	0.19
1978:10-1984:12	1.23	0.94	0.87	0.56	0.50	0.40
1955:1- 1978:9	0.35	0.30	0.29	0.21	0.16	0.10
1992:1- 1998:9	0.14	0.15	0.21	0.23	0.20	0.16

Notes: Entries are the sample standard deviations of the series over the sample period given in the table's first row. Year-over-year differences are $R_t - R_{t-12}$, first differences are $R_t - R_{t-1}$ and AR innovations are residuals from AR(6) models that incorporate a constant term.

Table 2

Largest Autoregressive Roots for Short Term Interest Rates

Interest Rate	Sample Period: 1965:1-1978:9 ρ_{ols}	ρ_{mub}	90% CI	Sample Period: 1985:1-1998:9 ρ_{ols}	ρ_{mub}	90% CI	Chow Test F_ρ	P-Value
Federal Funds	0.97	0.96	0.91-1.01	0.98	1.00	0.94-1.02	1.19	0.30-0.64
3-Month TBill	0.96	0.98	0.93-1.02	0.98	0.99	0.94-1.02	1.17	0.31-0.64

Notes: ρ_{ols} is the OLS estimate of ρ constructed from an AR(6) model that included a constant term. ρ_{mub} is the median unbiased estimator of ρ constructed from the Dickey-Fuller τ^μ statistic as described in Stock (1991). The 90% confidence interval is also computed from τ^μ using Stock's procedure. F_ρ is the Chow F-statistic for testing for change in ρ across the two sample periods. The column labeled P-value shows that upper and lower bound for the F-statistic P-value using the procedure described in the appendix.

Table 3

Standard Deviation of Annual Changes in Interest Rates

Maturity	Sample Period: 1965:1-1978:9 Actual	Implied by ρ_{ols}	Implied by ρ_{mub}	Sample Period: 1985:1-1998:9 Actual	Implied by ρ_{ols}	Implied by ρ_{mub}
1 Month	2.44	2.39	2.39	1.50	1.36	1.41
12 Month	1.40	1.80	1.76	1.54	1.31	1.46
60 Month	0.89	0.57	0.52	1.40	0.76	1.22
120 Month	0.69	0.31	0.29	1.29	0.46	0.98

Notes: Entries show the actual (sample value) and implied standard deviations of year-to-year changes in interest rates ($R^h_t - R^h_{t-12}$) for different horizons. Entries labeled Actual are taken from Table 1 are the sample values for the Federal Funds rate and the rates for 1-,5-, and 10-Year Treasury Bonds. The columns labeled ρ_{ols} and ρ_{mub} were computed using the expectations model and the estimated AR(6) processes using the Federal Funds rate over the sample periods shown, and imposing the values of ρ_{ols} and ρ_{mub} listed in Table 2. These values correspond to the circles and squares shown in figure 4.

Table 4

Largest Autoregressive Roots for Different Specifications

A. Federal Funds Rate

Specification Change from Baseline	First Sample Period ρ_{ols}	ρ_{mub}	90% CI	Second Sample Period ρ_{ols}	ρ_{mub}	90% CI	-- Chow Test -- F_{ρ}	P-Value
None	0.97	0.96	0.91-1.01	0.98	1.00	0.94-1.02	1.19	0.30-0.64
AR(2)	0.96	0.92	0.86-1.00	0.99	1.00	0.95-1.02	2.46	0.13-0.43
AR(4)	0.96	0.93	0.87-1.01	0.99	1.00	0.95-1.02	2.34	0.14-0.45
AR(8)	0.96	0.95	0.90-1.01	0.99	1.00	0.95-1.02	1.65	0.22-0.55
SD 1955	0.98	0.98	0.96-1.01	0.98	1.00	0.94-1.02	0.06	0.81-0.93
SD 1992, AR(2)	0.96	0.92	0.86-1.00	0.98	1.01	0.95-1.04	1.55	0.24-0.57

B. 3-Month Treasury Bill Rate

Specification Change from Baseline	First Sample Period ρ_{ols}	ρ_{mub}	90% CI	Second Sample Period ρ_{ols}	ρ_{mub}	90% CI	-- Chow Test-- F_{ρ}	P-Value
None	0.96	0.98	0.93-1.02	0.98	0.99	0.94-1.02	1.17	0.31-0.64
AR(2)	0.95	0.97	0.92-1.01	0.99	1.00	0.95-1.02	1.73	0.21-0.54
AR(4)	0.95	0.95	0.90-1.01	0.99	1.00	0.95-1.02	2.66	0.12-0.40
AR(8)	0.95	0.97	0.91-1.01	0.98	0.99	0.94-1.02	1.85	0.20-0.52
SD 1955	0.98	1.00	0.97-1.01	0.98	0.99	0.94-1.02	0.01	0.93-0.98
SD 1992, AR(2)	0.98	1.00	0.97-1.01	0.98	0.99	0.94-1.02	0.00	0.96-0.99

Notes: The first column shows the change in the specification from the baseline AR(6) model incorporating a constant (from Table 2). The baseline specification was estimated over the sample periods 1965:1-1978:9 and 1985:1-1998:9. AR(p) denotes an AR(p) model with a constant was used. "SD 1955" denotes a specification with the first sample period from 1955:1-1978:9. "SD 1992, AR(2)" denotes an AR(2) with second sample period from 1992:1-1998:9.