Has the Business Cycle Changed? Evidence and Explanations

Appendix

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1. Model-Based Calculations

This section of the appendix provides additional details for the model-based calculations summarized in Tables 6 and 8 in the text. It begins with a description of the models, and then presents expanded versions of text Tables 6 and 8.

The Rudebusch-Svensson Model

The model consists of three equations. The Phillips curve is specified as:

$$\Delta\pi_{t+1} = \alpha_0 + \alpha_{\pi 1}\Delta\pi_t + \alpha_{\pi 2}\Delta\pi_{t-1} + \alpha_{\pi 3}\Delta\pi_{t-2} + \alpha_y y_t^{gap} + \varepsilon_{t+1}, \tag{1.1}$$

where π_t is the inflation rate and y_t is the output gap. The IS equation is specified as:

$$y_{t+1}^{gap} = \beta_0 + \beta_{y1} y_t^{gap} + \beta_{y2} y_{t-1}^{gap} + \beta_r(\bar{R}_t - \bar{\pi}_t) + \eta_{t+1}. \tag{1.2}$$

where $\bar{R}_t$ and $\bar{\pi}_t$ are the four quarter averages of R_t (the Federal Funds rate) and π_t computed over time periods t–3 to t. The model is closed using the Taylor rule specification from Judd and Rudebusch (1998), written here as:

$$R_{t+1} = \phi_0 + \phi_{R1}R_t + \phi_{R2}R_{t-1} + \phi_\pi\bar{\pi}_{t+1} + \phi_{y1} y_{t+1}^{gap} + \phi_{y2} y_t^{gap} + \xi_{t+1} \tag{1.3}$$

For our calculations $\pi_t = 400\times\ln(P_t/P_{t-1})$, where P_t is the quarterly value of the U.S. GDP deflator; $y_t^{gap} = 100\times(y_t - y_t^{trend})$ where y_t is the quarterly value of real GDP and y_t^{trend} is the fitted value from a regression of y_t onto $(1, t, t^2)$ over the sample period 1959:1-2002:4.

The parameters in equations (1.1) and (1.2) were estimated over the sample period 1960:1-2002:4; the parameters of (1.3) were estimated over 1960:1-1978:4 and 1984:1-2002:4. OLS estimates and heteroskedastic robust standard errors are given in the attached table.

Table A.1
Parameter Estimates for the Rudebusch-Svensson Model

Parameter	1960:1-2002:4	1960:1-1978:4	1984:1-2002:4
α_0	0.000 (0.078)		
$\alpha_{\pi 1}$	–0.425 (0.084)		
$\alpha_{\pi 2}$	–0.350 (0.073)		
$\alpha_{\pi 3}$	–0.194 (0.092)		
α_y	0.113 (0.025)		
β_0	0.239 (0.111)		
β_{y1}	1.209 (0.081)		
β_{y2}	–0.280 (0.079)		
β_r	–0.081 (0.033)		
ϕ_0		0.981 (0.321)	–0.023 (0.156)
ϕ_{R1}		1.124 (0.139)	1.386 (0.150)
ϕ_{R2}		–0.469 (0.136)	–0.506 (0.107)
ϕ_π		0.187 (0.050)	0.270 (0.120)
ϕ_{y1}		0.046 (0.086)	0.404 (0.098)
ϕ_{y2}		0.079 (0.099)	–0.362 (0.085)
σ_ε	1.035		
σ_η	0.816		
σ_ξ		0.711	0.437

The Stock-Watson Structural VAR

The structural VAR is based on a 4-variable, 4-lag VAR that includes the logarith of output (y), inflation (π), interest rates (R), and the logarithm of commodity prices (Z). The structural model includes an IS equation, a forward-looking New Keynesian Phillips curve, a forward looking Taylor-type monetary policy rule, and an exogenous process for commodity prices:

$$y_t = \theta r_t + \text{lags} + \varepsilon_{y,t}$$

$$\pi_t = \gamma Y(\delta)_t + \text{lags} + \varepsilon_{\pi,t}$$

$$R_t = \beta_\pi \overline{\pi}_{t+h/t} + \beta_y \overline{y}^{gap}_{t+h/t} + \text{lags} + \varepsilon_{r,t}$$

$$Z_t = \text{lags} + \alpha_y \varepsilon_{y,t} + \alpha_\pi \varepsilon_{\pi,t} + \alpha_r \varepsilon_{r,t} + \varepsilon_{z,t}$$

where $r_t = R_t - \overline{\pi}_{t+k/t}$ is the real interest rate, $\overline{\pi}_{t+k/t}$ is the expected average inflation rate over the next k periods, where k is the term of the interest rate R; $Y(\delta)_t = \sum_{i=0}^{\infty} \delta^i y^{gap}_{t+1/t}$ is the discounted expected future output gap, and $\overline{y}^{gap}_{t+h/t}$ is the expected future average output gap over the next h periods measured in percentage points. We have used generic notation "lags" to denote four unrestricted lags of variables in each of these equations.

The VAR is estimated using $\Delta y_t = \ln(\text{GDP}_t/\text{GDP}_{t-1})$ where GDP is the real value of GDP; $\Delta\pi_t = 400 \times \{\ln(P_t/P_{t-1}) - \ln(P_{t-1}/P_{t-2})\}$ where P is the GDP deflator; R_t is the 1-year Treasury Bond rate; $Z_t = \ln(Pcom_t/Pcom_{t-1})$ is the spot market commodity price index.

We assume that the structural parameters θ, γ, and δ are constant throughout the sample period, and allow β_π, β_y, and the coefficients on lags to change across the two sample periods. The values of θ, γ, and δ are specified *a priori*, with $\theta = -.002$ (note that y is the logarithm of quarterly real *GDP* and r is measured in percentage points at an annual rate), $\gamma = 0.30$ and $\delta = 0.50$. (See Stock and Watson (2002) for a discussion of these values and for results using other values of these parameters.) Under the assumption that ε_R is uncorrelated with ε_y and ε_π, the parameters β_π and β_y can be estimated by IV methods using the reduced form VAR residuals. Parameter values estimated over the two subsamples and heteroskesdastic robust standard errors are given in the following table.

Table A.2
Estimated Taylor Rule Coefficients for the SW Structural VAR Model

Parameter	1960:1-1978:4	1984:1-2002:4
β_π	–0.744 (0.194)	1.432 (0.227)
β_y	0.161 (0.181)	0.451 (0.146)

The Smets-Wouter Models

The Smets-Wouter US model (SWUS) and EuroArea model have a common structure. They differ from another in two ways. First, the parameter values differ: the SWUS parameter values we fit to U.S. data over 1957-2002 and the SWEA parameter values were fit to European data over 1980-1999. Second, they have different low-frequency characteristics: the SWEA model using uses linearly detrended values of real variables, and this results in stationary dynamics for all of the variables in the model; the SWUS model includes common real I(1) stochastic trends shared by the models real variables and an independent I(1) stochastic trend in inflation.

The models share a common specification of the Taylor rule

$$R_t = \bar{\pi}_{t-1} + \rho(R_{t-1} - \bar{\pi}_{t-1}) + (1-\rho)\{r_\pi(\pi_{t-1} - \bar{\pi}_{t-1}) + r_y\, y_{t-1}^{gap}\} +$$

$$r_{\Delta\pi}\Delta(\pi_t - \bar{\pi}_t) + r_{\Delta y}\Delta\, y_t^{gap} + \eta_t \qquad (1.4)$$

with $\bar{\pi}_t = \rho_\pi \bar{\pi}_{t-1} + \varepsilon_t$. The specification of the other equations in the models can be found in Smets and Wouters (2003a, 2003b).

Our simulations of the models used the posterior modes reported in Smets and Wouters (2003a, 2003b). The values of the Taylor rule coefficients are given in the following table.

Table A.3
Full-Sample Estimated Taylor Rule Coefficients from the Smets-Wouter Models

Parameter	Europe 1980-1999	US 1957-2002
ρ	.96	.88
r_π	1.68	1.49
r_y	.40	.24
$r_{\Delta\pi}$	0.14	.18
$r_{\Delta y}$	0.16	.24
ρ_π	.92	1.00 (constrained)
σ_η	.32	.84
σ_ε	.07	.30

These values were used for the baseline versions of the models. For the “pre-1979” calculations, the Taylor rule coefficients were modified so that the central bank was more accommodative to inflation, subject to the constraint that the model still had a unique rational expectations equilibrium. For both models this was accomplished by setting $r_\pi = 0.97$.

Detailed results of the counterfactual model simulations

Table 6 in the text reports results from model simulations for each of the four models discussed above. The "Base Model" results for the Rudebusch-Svensson model were computed using equations (1.1)-(1.2) with parameter values shown in the column labeled "1960:1-2002:4" of Table A.1, and equation (1.3) with parameter values from the column labeled "1984:1-2002:4." The model was simulated using the residuals from these equations over the 1984:1-2002:4 sample period. The results in Table 6's column labeled "Base + Pre-79 Monetary Model" was constructed in the same way, except that the parameter values for (1.3) came from the "1960:1-1978:4" column of Table A.1. The "Base Model" results in Table 8 are the same as those Table 6. Table 8's "Base + Pre-79 shocks" results are computed from 1960:1-1978:4 residuals computed from (1.1)-(1.2) and the residuals from (1.3) came from the "1960:1-1978:4" column of Table A.1. The model is simulated using the coefficient values for (1.3) from the "1984:1-2002:4" column of Table A.1.

The Base Model results for the SVAR model were computed from the structural VAR estimated over the 1984:1-2002:4 sample period along with the residuals from the period. The "Base + Pre-79 Monetary Model" results were computed using the 1984:1-2002:4 residuals along with the VAR estimated over the 1960:1-1978:4 sample period. (Note that the structural parameters θ, γ, and δ are the same in both sample periods.) The "Base Model" results in Table 8 are the same as those Table 6. Table 8's "Base + Pre-79 shocks" results are computed from 1960:1-1978:4 "structural" residuals together with VAR coefficient values estimated over the 1984:1-2002:4 sample.

The Base Model results for the Smets-Wouters models were computed using posterior model parameter estimates from Smets and Wouters (2003a, 2003b). The standard deviation are the implied population standard deviation of $y_t - y_{t-4}$ using these parameter values. As discussed above, the results for "Base + Pre-79 Monetary Model" used the same parameters except that r_π was reduced to $r_\pi = 0.97$.

The following table provides additional results for these experiments.

Table A.4
a. Additional Results for the Rudebusch-Svensson and Structural VAR Models

	Rudebusch-Svensson			SW Structual VAR		
	Base Model	Base + Pre-79 Monetary Policy	Base + Pre-79 shocks	Base Model	Base + Pre-79 Monetary Policy	Base + Pre-79 shocks
$\sigma(y_t - y_{t-4})$	1.67	1.74	2.60	1.67	1.63	2.36
$\sigma(\pi_t - \pi_{t-4})$	1.00	1.04	1.85	1.00	1.42	1.25
$\sigma(\bar{\pi}_t)$	0.85	3.73	1.34	0.85	1.12	0.80
$\mu(\bar{\pi}_t)$	2.50	8.36	2.76	2.50	6.87	2.18
$\bar{\pi}_{2002:4}$	1.33	15.60	0.56	1.33	7.38	2.29

b. Additional Results for the Smets-Wouter Models

	Smets-Wouter US		Smets-Wouter EuroArea	
	Base Model	Base Model with $r_\pi = 0.97$	Base Model	Base Model with $r_\pi = 0.97$
$\sigma(y_t - y_{t-4})$	2.40	2.33	1.63	1.88
$\sigma(\pi_t - \pi_{t-4})$	1.83	2.19	1.59	1.85
$\sigma(\bar{\pi}_t)$	NA	NA	1.29	2.11

Notes: $\sigma(y_t - y_{t-4})$ denotes the standard deviation of $y_t - y_{t-4}$, and similarly for $\sigma(\pi_t - \pi_{t-4})$ and $\sigma(\bar{\pi}_t)$. $\mu(\bar{\pi}_t)$ denotes the mean of $\bar{\pi}_t$. $\bar{\pi}_{2002:4}$ is the value of $\bar{\pi}$ in 2002:4.

2. Nonlinearities in the 1960-1978 Taylor rule

Tests for nonlinearities were carried using (1.3) estimated over 1960:1-1978:4. The tests were conducted by adding several "threshold" variables to the base specification. To define these threshold variables, let $F_{x,0.75}$ denote the 75th percentile of the empirical distribution of x over the 1960-1978 sample period, and let $F_{x,\,0.25}$ be similarly defined. Let $\bar{r}_t = \bar{R}_t - \bar{\pi}_t$. The table below shows results with additional variables, the estimated coefficients, standard errors and F-statistics for joint significance.

Table A.5
Tests for Nonlinearity

Regressor	Base Model			
Baseline Regressors				
constant	0.98 (0.32)	1.04 (0.35)	0.77 (0.34)	0.89 (0.25)
R_{t-1}	1.12 (0.14)	1.11 (0.14)	1.15 (0.14)	1.07 (0.14)
R_{t-2}	–0.47 (0.14)	–0.54 (0.13)	–0.49 (0.13)	–0.43 (0.13)
$\bar{\pi}_t$	0.19 (0.05)	0.28 (0.09)	0.19 (0.05)	0.22 (0.05)
y_t^{gap}	0.05 (0.09)	0.05 (0.09)	0.13 (0.11)	0.04 (0,09)
y_{t-1}^{gap}	0.08 (0.10)	0.07 (0.11)	0.06 (0.10)	0.06 (0.10)
Additional Regressors				
$\mathbf{1}(\bar{r}_{t-1} > F_{\bar{r},0.75})\times\bar{r}_{t-1}$		0.07 (0.40)		
$\mathbf{1}(\bar{r}_{t-1} < F_{\bar{r},0.25})\times\bar{r}_{t-1}$		0.27 (0.28)		
$\mathbf{1}(\bar{r}_{t-1} > F_{\bar{r},0.75})$		0.05 (1.00)		
$\mathbf{1}(\bar{r}_{t-1} < F_{\bar{r},0.25})$		0.00 (0.34)		
$\mathbf{1}(y_t^{gap} > F_{y^{gap},0.75})\times y_t^{gap}$			–0.04 (0.26)	
$\mathbf{1}(y_t^{gap} < F_{y^{gap},0.25})\times y_t^{gap}$			–0.13 (0.09)	
$\mathbf{1}(y_t^{gap} > F_{y^{gap},0.75})$			0.02 (1.26)	
$\mathbf{1}(y_t^{gap} < F_{y^{gap},0.25})$			0.05 (0.34)	
$\mathbf{1}(\bar{\pi}_t - \bar{\pi}_{t-8} > F_{\bar{\pi}-\bar{\pi}_{-8},0.75})\times(\bar{\pi}_t - \bar{\pi}_{t-8})$				1.17 (1.58)
$\mathbf{1}(\bar{\pi}_t - \bar{\pi}_{t-8} > F_{\bar{\pi}-\bar{\pi}_{-8},0.75})\times\bar{\pi}_t$				-0.11 (0.13)
$\mathbf{1}(\bar{\pi}_t - \bar{\pi}_{t-8} > F_{\bar{\pi}-\bar{\pi}_{-8},0.75})$				–0.71 (1.65)
F-statistic (p-value) for exclusion of additional regressors		0.51 (0.73)	1.09 (0.36)	0.86 (0.46)

References

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