Money, Prices, Interest Rates and the Business Cycle

Robert G. King[1]
University of Virginia and
Federal Reserve Bank of Richmond

Mark W. Watson
Princeton University and
Federal Reserve Bank of Chicago

October 1993, Revised July 1995

[1]This paper has benefited from constructive comments by Ken West. Support was provided by the National Science Foundation through grants NSF-91-22463 and SBR-94-09629.

Abstract

The mechanisms governing the relationship of money, prices and interest rates to the business cycle are one of the most studied and most disputed topics in macroeconomics. In this paper, we first document key empirical aspects of this relationship. We then ask how well three benchmark rational expectations macroeconomic models — a real business cycle model, a sticky price model and a liquidity effect model — account for these central facts. While the models have diverse successses and failures, none can account for the fact that both real and nominal interest rates are "inverted leading indicators" of real economic activity. That is, none of the models captures the post-war U.S. business cycle fact that a high real or nominal interest rate in the current quarter predicts a low level of real economic activity two to four quarters in the future.

JEL: 130, 310

1 Introduction

The positive correlation of nominal money and real economic activity over the course of many business cycles is a key empirical fact about the U.S. economy. Further, there is a dynamic dimension to this covariation so strong and stable that a monetary variable has long been included in the Commerce Department's Index of Leading Economic Indicators. While this pattern of cyclical comovement is widely agreed upon, its interpretation is not. Some macroeconomists view money as purely passive, with its positive response to varying levels of economic activity producing the positive correlation. Others view changes in the quantity of money as an important, perhaps dominant, source of economic fluctuations. Frequently, the real effects of monetary changes are suggested to arise from frictions in commodity, labor or financial markets. In economic theories that describe the influence of these frictions, the transmission mechanism from monetary changes to real activity is typically viewed as involving interest rates and the price level.

The primary goal of this paper is to evaluate three models that explain the link between money, prices, interest rates and the business cycle. We do this in three steps. First, we document the cyclical behavior of money, prices and interest rates in the U.S. over the postwar period. Second, we construct three quantitative rational expectations models of macroeconomic activity: (i) a real business cycle model with endogenous money; (ii) a model of commodity market frictions that involves non-neutralities of money arising from gradual adjustment of goods prices; and (iii) a model of financial market frictions that involves non-neutralities of money arising from gradual adjustments of portfolios. Finally, we compare that models' prediction for the business cycle behavior of money, prices, and interest rates with the data. In exploring the predictions of these models, we take the stock of money to be one of several exogenous variables in the system. Thus, all of our models are capable of generating a forecasting role for money relative to real economic activity, similar to that found in the U.S. data. In the real business model, monetary changes can forecast real activity because both productivity and money are related to many underlying sources of shocks and because economic agents know the relationship between money and these real shocks. In the models with "sticky prices" and "liquidity effects" (shorthand names for the models with frictions in the commodity and financial markets, respectively), monetary changes have an additional direct positive effect on aggregate output.

The model economies have diverse successes and failures, some of which are surprising such as the sticky price model's prediction of a countercylcical price level. However, we find that all of the models are highly deficient in the predictions that they make about the relationship of real and nominal rates to the business cycle. Notably, none of the models captures the fact that increases in both the real and nominal interest rate preceded every post-war recession, which is highlighted in our empirical description of post-war U.S. business cycles.

The outline of the paper is as follows. First, the remainder of this introductory section is devoted to a summary of our main findings. Section 2 describes the data and documents its' business cycle characteristics. Section 3 outlines the three macroeconomic models, and Section 4 discusses the quantitative versions of the models including practical issues relating to linearization and parameterization. Our main empirical results are presented in Section 5, and Section 6 concludes. Finally, detailed discussion of the models is contained in the appendices.

1.1 Key Features of Post-War U.S. Business Cycles

In Section 2, we document four key features of macroeconomic data that are important for evaluating models of money and the business cycle.

Predictable and Temporary Business Cycles: Our first finding is that most macroeconomic variables display predictable growth over the business cycle frequencies (which we define as 8 to 32 quarters, the typical durations of business cycles identified using the NBER method). We document this feature of U.S. business cycle activity by showing that there is a "typical spectral shape of growth rates," indicating that the variance of most real series is concentrated in these business cycle frequencies. (That is, most of growth is attributable to periodic components with between 6 and 32 quarters of cyclical duration). Since the growth rate's spectrum is much higher at business cycle frequencies than at very low frequencies, this shape further suggests that there are predictable components of macroeconomic activity that are themselves temporary in terms of their influence on output. Importantly, we also show that the growth rates of nominal money and prices display a similar spectral shape, thus suggesting that variation in nominal magnitudes may be a source of the temporary cyclical fluctuations in real activity.

Procyclical and Leading Money: Our second finding is that there is indeed a strong positive correlation of the nominal money stock with the cycle in real activity, once we have transformed the macroeconomic data to eliminate low frequency ("trend") and very high frequency ("irregular") components. Further, and importantly, this description of the U.S. business cycle also makes nominal money a leading indicator for real activity, by which we mean that increases in money this quarter are positively correlated with high output levels in future periods.

Countercyclical and Leading Interest Rates: Our third finding is that interest rates — both real and nominal — are also important leading indicators for real economic activity. However, they are "inverted indicators" in that the level of the nominal (real) rate is low prior to increases in output 4 to 6 months hence. We also find that the nominal rate tends to be positively correlated with output contemporaneously and with a lag, while the real rate is negatively correlated contemporaneously.

Countercyclical and Leading Prices: Our fourth finding is that the price level bears a complex relationship to the cycle, which appears to be somewhat unstable across time. We do find some evidence of the tendency of prices to lag the cycle in real activity, consistent with the conventional view that price increases lag output increases. However, we also find that the price level is negatively related to output contemporaneously over the post-war period and that it is also a negative leading indicator: price increases this year signal output declines in the future.

1.2 Evaluation of the Macroeconomic Models

To evaluate the models, we examine the implications that these models have for selected variances and covariances, both contemporaneously and at various leads and lags. To make this comparison an interesting one, we allow the models to be driven by rich, dynamically interrelated processes for a vector of shocks, which include money and productivity. Then, we discuss how well each model captures the four sets of core summary statistics — our stylized facts — concerning the interaction of nominal indicators and real activity.[1]

Our results are usefully divided into two categories. The first set concerns the ability of the various models to capture measures of *variability*, such as the empirical spectral shapes of growth rates for various real and nominal variables. The second set concerns the ability of the models to capture concerns the *covariation* of money, interest and prices with the business cycle.

In terms of *variability*, we find that the real business cycle model can capture the spectral shapes of many real and nominal variables, but only when it is driven by a highly volatile "Solow residual" whose growth rate itself has the typical spectral shape. The sticky price model also can match the typical spectral shape, but it does so in part because highly persistent monetary shocks have only a temporary impact on output. However, our version of the sticky price model relies on an underlying "monopolistic competition" framework which dictates that productivity is less cyclically volatile than measured Solow residuals. This smaller volatility of shocks, coupled with demand-side determination of output in the short-run, implies that the sticky price model generates less business cycle variability than is present in the data or the real business cycle model. Both the sticky price and real business cycle models produce too little business cycle variation in real interest rates.

We also find that the liquidity effect model can also generate the typical spectral shape of growth rates, but this outcome is subject to three important qualifications. The first qualification stems from the fact that this model works much like a version of the standard real business cycle model, but one that predicts only small cyclical variation in labor input in response to both monetary and productivity shocks: it thus displays much less real volatility than the other models. The second qualification is

[1] In this paper, we concentrate on such cross-correlations rather than impulse responses to nominal shocks: we plan to undertake this alternative evaluation in a companion paper. The current focus reflects our view that both evaluation methods are desirable; while impulse responses from dynamic models are interpreted more readily than moments, the extraction of structural empirical impulse responses is a subtle, difficult, and controversial activity.

that business cycle variability produced by the model is almost entirely due to real rather than monetary shocks. The third qualification is that the financial market frictions model produces too much volatility in real interest rates.

In terms of *covariability*, the three models have very different ability to match the core facts and all have some substantial difficulties in this regard.

The real business cycle model with endogenous money has some modest success in capturing the comovements between nominal indicators and the business cycle. It captures the covariation of output and the money stock well, which occurs because money and productivity shocks are assumed to be highly correlated and because productivity exerts a strong business cycle influence on output. It also successfully captures the contemporaneous negative correlation of the price level with real activity, despite a substantial procyclicality of the money stock. Finally, and surprisingly, it does a good job of capturing the dynamic interaction of the nominal interest rate with real activity. That is, it predicts that the nominal interest rate should be positively correlated with past and current output (although the magnitudes of these correlations are smaller than those present in the U.S. data). Further, it implies that the nominal interest rate is an inverted leading indicator: for example a rise in the nominal rate will precede a decline in real economic activity (again with a smaller magnitude than the data). Within our specification of the real business cycle model, this occurs because there is a productivity process with a substantial temporary component: an increase in output produces a decline in the price level contemporaneously and a rise in expected inflation, since the output increase and price level decreases are expected to be reversed in future periods. Finally, it predicts that the price level will be an (inverted) leading indicator. However, the RBC model displays a dismal performance in terms of real interest rates: it predicts that the real interest rates should be a positive leading indicator for real activity, while the empirical finding is that real interest rate increases lead output decreases.

The sticky price model also has some success in capturing core patterns of real and nominal interactions, but it is also deficient is in a number of important ways.[2] In terms of successes, it surprisingly predicts that the price level should be negatively correlated with real activity contemporaneously and with a year lead. Principally, this negative contemporaneous correlation stems from the simultaneous effect of productivity shocks: there is predicted to be a strong positive correlation if money is the only driving variable. It also captures the modest negative contemporaneous correlation between the real rate and output. But there are many other deficiencies. Most notably, the sticky price model suggests that a high real or nominal interest rate in the current quarter should be a strong signal of high future economic activity, while the data indicate the opposite. This lack of an inverted leading indicator role

[2] In discussing these successes and shortcomings, it is useful to remember that our version of the sticky price model incorporates the forward-looking behavior of consumption, investment, price setting and money demand (via the nominal interest rate). These features mean that it is capable of generating a reasonable pattern of comovement and relative variability among the real variables.

for interest rates is particularly surprising since our sticky price model predicts that nominal and real rates should be strongly negatively correlated contemporaneously with real output.[3]

In terms of matching the cyclical covariation of money, interest rates, and the price level, the financial market frictions model has a core difficulty: it predicts that real activity is not very responsive to nominal money—there are small "multipliers" attached to nominal shocks—and it predicts that real and nominal interest rates are highly volatile in response to these factors. Consequently, the cyclical covariation of money and economic activity comes primarily from the same source as in the real business cycle model, the assumed correlation between the money and productivity processes. In terms of the price level, it captures a countercyclical response of the price level to output (which results from the dominant role of productivity shocks) but cannot capture the inverted leading indicator role of the price level. While the model does capture the modest negative correlation of the real rate with output, it does so only by also predicting that the nominal rate is negatively correlated with output (a counterfactual implication that it shares with the sticky price model). Finally, like the other two models, the liquidity effect model misses the "negative leading indicator" role of the real interest rate.

Overall, we conclude that all prominent macroeconomic models—those which stress a single set of economic mechanisms—have substantial difficulties matching the core features of nominal and real interactions. Most strikingly, all of the models do a poor job at matching the interaction of real and nominal interest rates with real activity. More generally, our paper documents the diverse successes and failures of these models. By doing so, it suggests that new models, that incorporate new mechanisms or combine of existing mechanisms, will be necessary to explain the main empirical linkages between money, prices, interest rates and the business cycle.

2 Features of Post-War U.S. Business Cycles

We begin by providing a summary of some key features of post-war quarterly U.S. data, with particular emphasis on the characteristics of business cycles present in these data.

2.1 Description of the Data

Throughout this paper we use a data set consisting of output, consumption, investment, employment, prices, wages, the money supply and interest rates. Output is private net national product, less housing and farming. We abstract from government, housing and farming because of measurement problems in inputs and output

[3]That is, as will be discussed further below, our introduction of investment adjustment costs means that nominal increases can lower the real interest rate, even with rational expectations effects on investment.

in these sectors. Prices are the implicit price deflator for this measure of output. Consumption is nondurable plus service consumption. Investment is nonresidential fixed investment. Labor input is private nonagricultural employee hours. (Since average weekly hours per employee varies over the business cycle, employee hours is a more accurate measure of labor input than employment.) Real wages are compensation per hour for the nonfarm business sector divided by the output deflator. Money is the M1 aggregate. The nominal interest rate is the rate on three month treasury bills. The data for output, consumption, investment, employment and money are per capita.[4] We will let y, c, i, n, M, and $M - P$ denote the logarithms of per capita values of output, consumption, investment, employment, money, and real balances; P will denote the logarithm of the price level; w will denote the logarithm of the real wage; finally, R and r will denote the level of nominal interest and rates, respectively.

The sample means and standard deviations of the data are shown in Table 2.1. Since the variables have obvious trends, we present the results for growth rates, which are expressed in percent at annual rates (so that the variables measured as logarithms are differenced and multiplied by 400). Similarly, the interest rate is measured in percent at annual rates. From the sample means in Table 2.1, output, consumption and investment have grown at approximately the same average rate, as have real wages and average labor productivity ($\Delta y_t - \Delta n_t$). Labor input and real balances (per capita) declined over the sample period, but the trend growth rates are not statistically different from zero. The most variable series is investment, while the least variable is consumption.

2.2 The Power Spectrum of Growth Rates

The power spectrum of growth rates of macroeconomic variables provides important information about the nature of business cycles; Figure 2.1 presents the estimated spectra of the postwar U.S. data.[5] As Figure 2.1 indicates, we can discuss the implications of the power spectrum very generally, because there is a typical shape of the power spectrum of growth rates for a wide range of both real and nominal macroeconomic variables. In particular, each of the spectra plotted in the panels of Figure 2.1 has the following broad features: the power spectrum is relatively low at low frequencies (a small number of cycles per period), rises to a peak at a cycle length of about twenty quarters, and then declines at very high frequencies. We call this

[4] All variables are from Citibase, unless otherwise noted. The precise definitions, using Citibase labels are: output=(gdpq-gpbfq-gbuhq-ggnpq)/p16; price deflator=(gdp-gpbf-gbuh-ggnp)/(gdpq-gpbfq-gbuhq-ggnpq); consumption=(gcnq+gcsq)/p16; investment=ginq/p16; employment=lpmnu/p16; real wages=lbcpu/price deflator; money=fm1/p16 (1959-1992) and authors' calculation (see King, Plosser, Stock and Watson (1991) for 1947-1958; nominal interest rates=fygm3; real interest rates= fygm3-400 $* E_t(p_{t+1} - p_t)$, where the forecast of inflation is calculated from the VAR described in footnote 5.1 below.

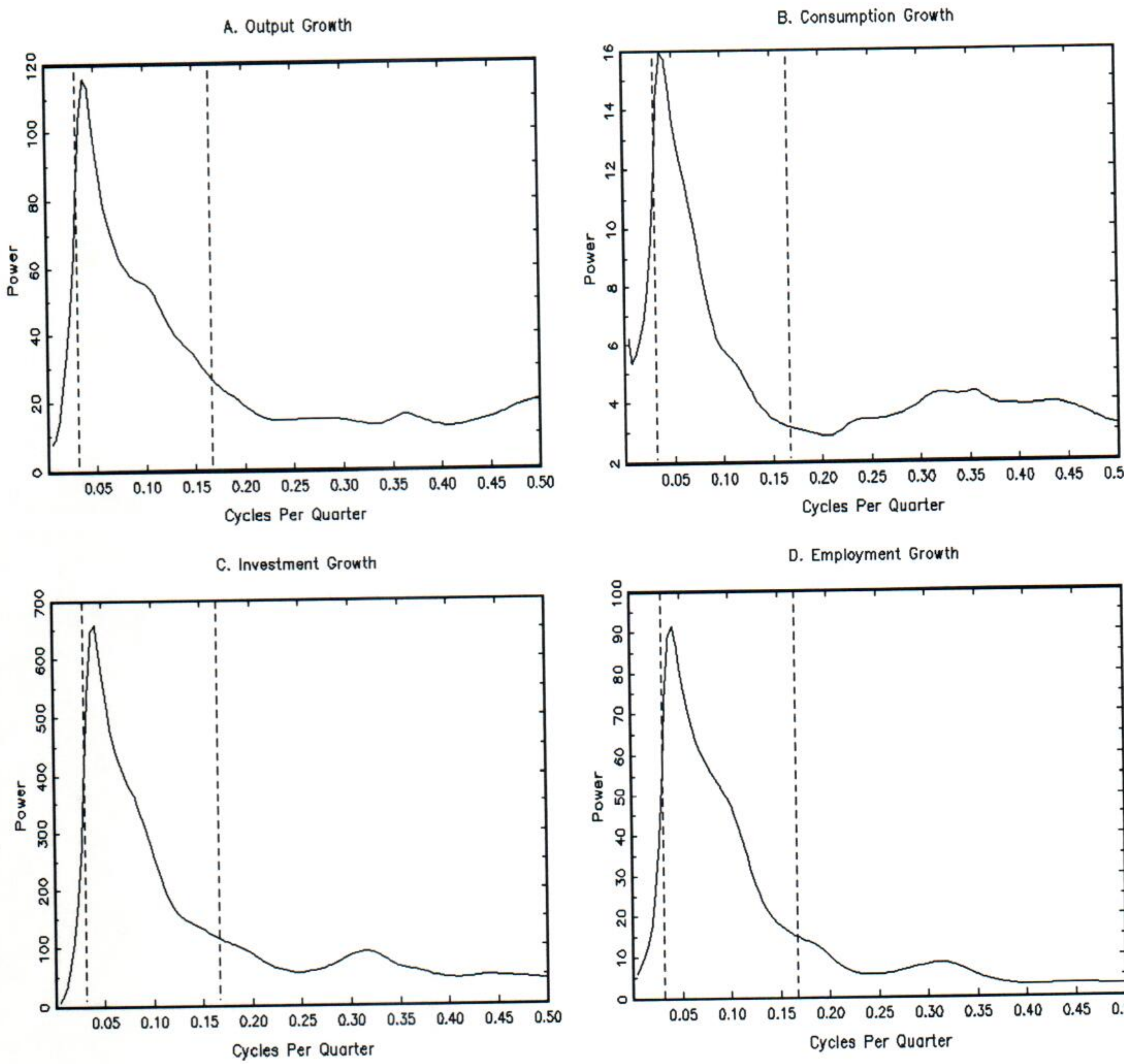
[5] These spectra are calculated from an estimated VAR that will be described in detail in footnote 5.1, below.

Table 2.1
Sample Moments 1949:1 - 1992:4

Variable	Mean	(SE)	Standard Deviation
Δy	1.67	(0.50)	5.15
Δc	1.67	(0.25)	2.21
Δi	1.65	(1.04)	11.15
Δn	-0.11	(0.41)	3.99
Δw	1.63	(0.23)	2.44
$\Delta(M-P)$	-0.36	(1.13)	4.80
ΔM	3.66	(1.23)	4.00
ΔP	4.03	(0.80)	3.24
R	5.19	(1.19)	3.09
r	1.18	(0.76)	2.46

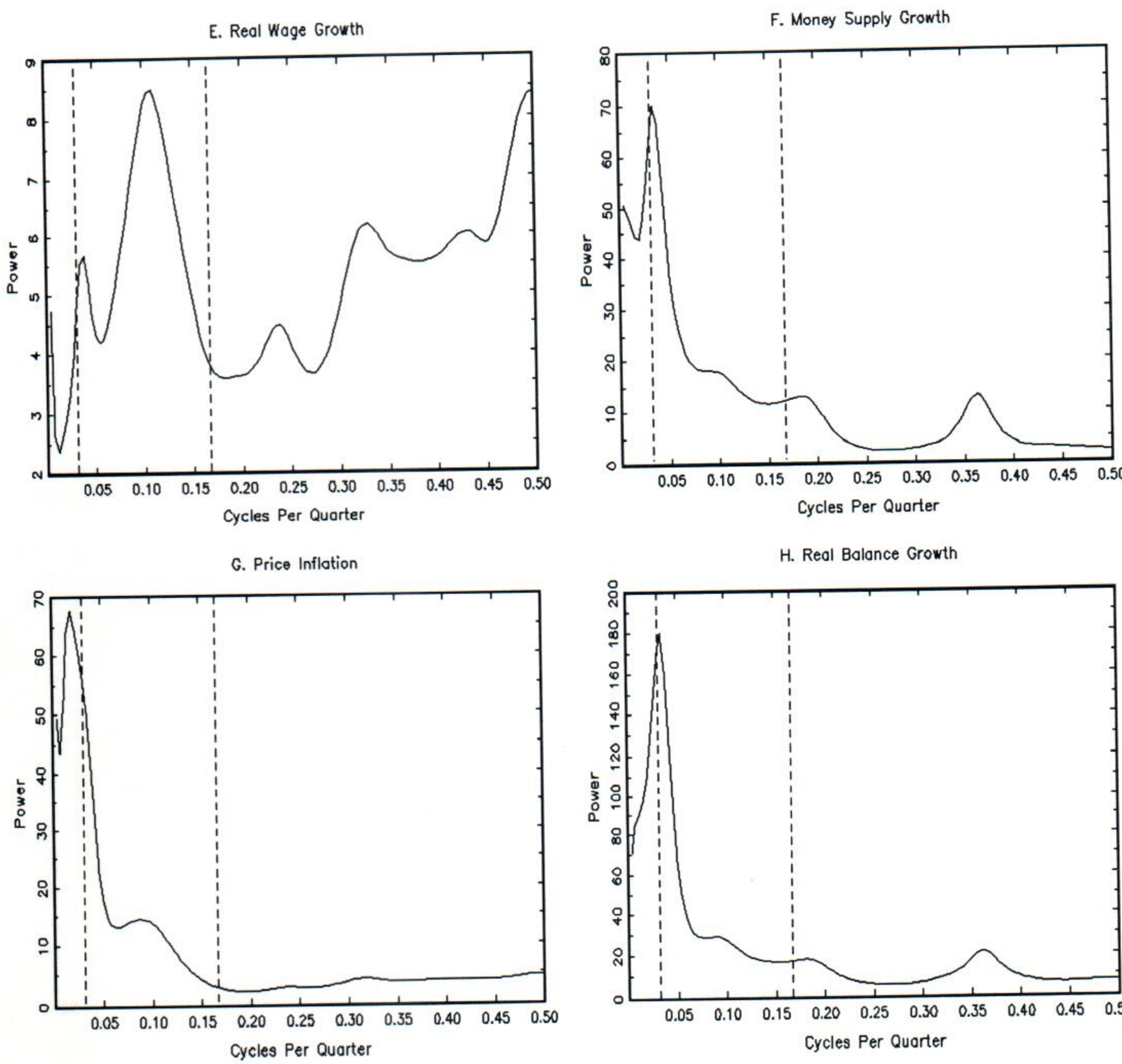
Notes: All values are in percentage points at annual rates. The numbers in parentheses are (autocorrelation robust) estimated standard errors for the sample means, calculated using a AR(6) spectral estimator.

Figure 2.1
Growth Rate Spectra: Postwar Quarterly Data for the U.S.



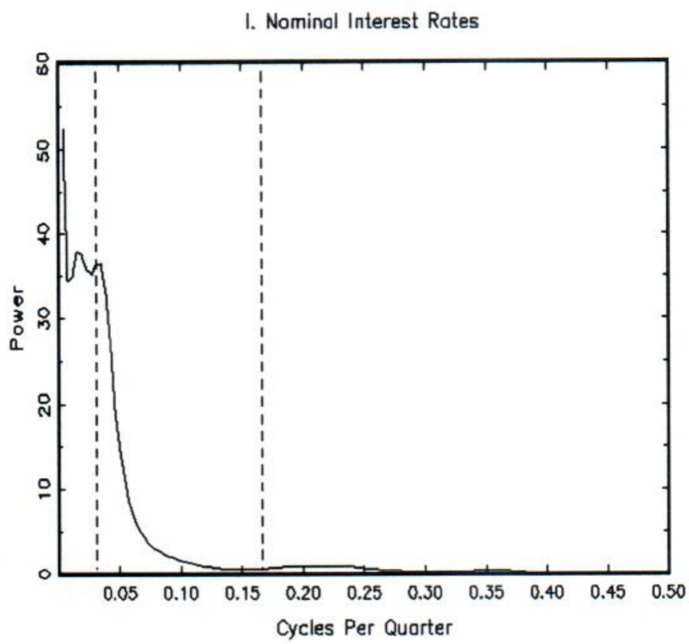
Notes: Vertical lines denote frequencies corresponding to 6 and 32 quarters (frequencies 1/6 and 1/32, respectively).

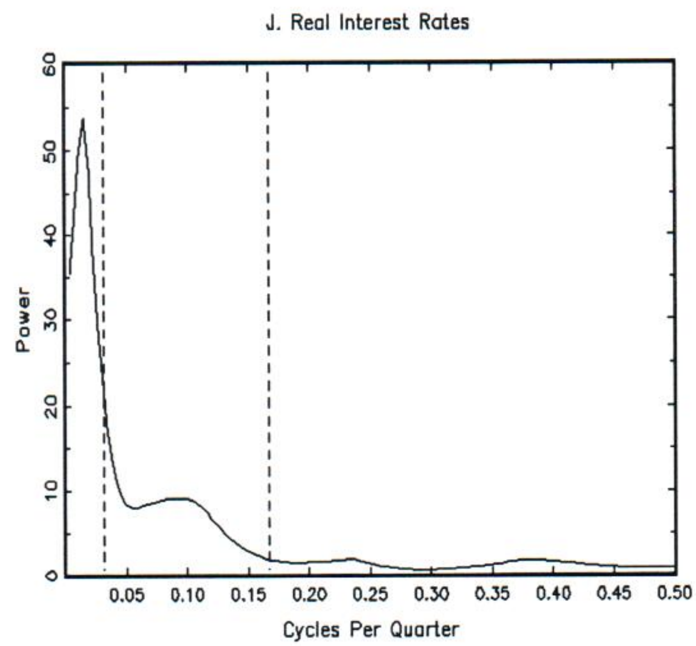
Figure 2.1 (Continued)
Growth Rate Spectra: Postwar Quarterly Data for the U.S.



Notes: Vertical lines denote frequencies corresponding to 6 and 32 quarters (frequencies 1/6 and 1/32, respectively).

Figure 2.1 (Continued)
Growth Rate Spectra: Postwar Quarterly Data for the U.S.





Notes: Vertical lines denote frequencies corresponding to 6 and 32 quarters (frequencies 1/6 and 1/32, respectively).

pattern "the typical spectral shape of growth rates." It is notably different from the typical spectral shape that Granger (1966) identifies for the *levels* of many economic time series, in which much of the power occurs at very low frequencies (evident in the spectra of the levels of real and nominal interest rates in Figure 2.1).

2.2.1 Some Frequency Domain Background

To interpret the typical spectral shape of growth rates, it is useful to briefly review some key elements of time series analysis in the frequency domain. A covariance stationary variable x_t can be decomposed into an integral of periodic components:

$$x_t = \int_0^\pi x_t(\omega)d\omega = \int_0^\pi [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]d\omega \tag{1}$$

where $a(\omega)$ and $b(\omega)$ are uncorrelated random variables with mean zero and common variance $s(\omega)$. Accordingly, the variance can be decomposed as:

$$var(x_t) = \int_0^\pi s(\omega)d\omega$$

where the power spectrum $s(\omega)$ is the contribution to variance at frequency ω. Thus, the height of the spectrum in Figure 2.1 at cycles per period $\frac{\omega}{2\pi}$ indicates the extent of that frequency's contribution to the variance of the growth rate.

2.2.2 The Spectral Shape of Output Growth

A conventional frequency domain definition of business cycles is that these are frequencies between six and thirty-two quarters: this definition derives from the duration of business cycles isolated by NBER researchers using the (non-spectral) methods of Burns and Mitchell (1946). In panel A of Figure 2.1, the power spectrum of output is displayed: the business cycle frequencies lie between the two vertical lines, which correspond to frequencies between $(.03 = \frac{1}{32})$ and $(.16 = \frac{1}{6})$ cycles per period. The crucial point are that (i) the business cycle interval contains the peak in the spectrum; and (ii) the business cycle interval contains the bulk of the variance of output growth. In particular, integrating under the spectrum over that range, we find that 58% of the variance of output growth is at the business cycle frequencies.

This spectral shape of output growth has played an important role in the conclusions of earlier authors about the nature of business cycles. For example, there has been much recent interest in the univariate modeling of the consequences of "stochastic trends" for economic fluctuations (see for example the discussion in Fama (1992) and Cochrane (1994)). This spectral shape has important implications for empirical conclusions of this research. Following Watson (1986), this can be seen by considering the frequency domain interpretation of the trend-cycle decomposition suggested by the work of Beveridge and Nelson (1981). For this purpose, let the stochastic components of output be decomposed as follows:

$$y_t = y_t^\tau + y_t^c$$

where y_t^τ and y_t^c are the trend and cyclical components of output, respectively. Without further assumptions, this decomposition is not operational, but it can be made so by requiring that the trend is a random walk, $y_t^\tau = y_{t-1}^\tau + \varepsilon_t^\tau$, and the cyclical component is stationary. Under these assumptions, the variance at frequency ω of Δy_t can be determined from the decomposition in (1) with $x_t = \Delta y_t$:

$$s_{\Delta y}(\omega) = [var(\Delta y_t^\tau(\omega)) + var(\Delta y_t^c(\omega)) + 2cov(\Delta y_t^\tau(\omega), \Delta y_t^c(\omega))].$$

Since the trend is assumed to be a random walk, it follows that $\Delta y_t^\tau = \varepsilon_t^\tau$, i.e., that $var(\Delta y_t^\tau(\omega))$ is constant across all frequencies. Second, since the cyclical component is stationary, its first difference has no component at frequency zero so that the height of the spectrum at the origin determines the variance of $\Delta y_t^\tau(\omega)$. However, a decomposition at other frequencies cannot be made without additional identifying assumptions.

The restriction employed by Watson (1986) is that $cov(\Delta y_t^\tau(\omega), \Delta y_t^c(\omega)) = 0$ at all frequencies. In this case, the shape of the power spectrum has an immediate and strong implication: there is only a small trend contribution to growth rates (given by the height of the spectrum at frequency zero) and the remainder of the variability arises from highly persistent, but ultimately temporary variations in Δy_t^c. Further, the hump shape of the power spectrum indicates that there is substantial predictability of the cyclical component of output growth, a result that Rotemberg and Woodford (1994) have stressed using time domain methods. This interpretation of the spectral shape for the growth rate of output suggests the need for business cycle models with highly persistent, but ultimately temporary variations in Δy_t^c. Potentially, these models involve the persistent but ultimately neutral effects of nominal variables on real output.

One criticism of this interpretation of the typical growth rate spectral shape is that real business cycle models do not imply that $cov(\Delta y_t^\tau(\omega), \Delta y_t^c(\omega)) = 0$ when they are driven by random walk productivity shocks. Indeed, it is the essence of these models that permanent changes in technology set off "transitional dynamics" in which there is a high amplitude response of investment (an "overshooting" relative to its long-run level) and transitory variation in labor input. However, Watson (1993) documents that the spectrum of output growth in a standard RBC model with a random walk productivity shock does not display the "typical spectral shape of growth rates," suggesting either that the model's real shocks must contain significant transitory components (or mean reversion), or that other transitory shocks must affect output.

2.2.3 Spectra of Real and Nominal Variables

Looking across panels B-D of Figure 2.1 we see that there is a common, hump-shaped spectrum of consumption (panel B), investment (panel C), and employee-hours (panel D). The different height of these spectra reflects the differences in variance documented in Table 2.1. Since the growth rate of consumption is less volatile than the growth rate of output, the average height of the spectrum of consumption must be lower than that of output. Indeed it is, at both the business cycle and higher frequencies. But at the very low frequencies, the height of the spectra are roughly the same because there is stationarity of the ratio of consumption to investment: they display common stochastic trends, as documented in King, Plosser, Stock, and Watson (1991). The spectrum of consumption is also markedly different from that consistent with a logarithmic random walk, which would imply a flat spectrum for consumption growth.[6] Panel C implies that investment is much more volatile than output at business cycle and higher frequencies, but contains roughly the same variability at low frequencies. Finally, from panel D, employment and output have roughly the same variance at business cycle frequencies.

The one exception to our finding of a "typical spectral shape for growth rates" comes in panel E of figure 2.1: the growth rate of real wages contain relatively less business cycle and relatively more high frequency variation than do any of the other series. The spectrum of real wages also displays its peak at about 8 quarters rather than the 20 quarter peak present in other real variables.

The spectra of nominal growth rates are shown in panels F-G of Figure 2.1. There are three characteristics of these spectra that are suggestive about the potential role of nominal variables in the business cycle. First, we see the typical spectral shape in money growth (panel F). The substantial business cycle variability of money growth suggests that variations in money may be an important source of economic fluctuations, leading to the typical spectral shape in the growth rates of other variables. Second, we see the typical spectral shape in price inflation (panel G). However, the peak in the spectrum of price inflation lies at a lower frequency than does the peak in the spectrum of money growth and real variables: it occurs at a periodicity of 51 quarters rather than 20 quarters. This suggests some smoothing of nominal money in prices, i.e., some gradual adjustment of prices. Finally, the spectra of money growth and price inflation have a higher variability at very low frequencies than do real variables: their low frequency components have standard deviations in the 7-8% range, while the standard deviation of output is approximately 3%. This finding suggests the existence of stochastic trend components in nominal variables that are independent from those in real variables.

[6] While consumption growth is predictable, it is not nearly as predictable as output. For example, the regression of Δc_t onto Δc_{t-1}, Δy_{t-1}, and $(y_{t-1} - c_{t-1})$ has an R^2 of .08 and an F-statistic of 5.3. The corresponding regression for Δy_t has an R^2 of .25 and an F-statistic of 19.3. These regression were run over 1947:1-92:4 and included a constant.

2.3 Business Cycle Covariability

We explore the patterns of comovement between real and nominal variables over the business cycle we using two complimentary sets of descriptive statistics. First, Table 2.2 presents the correlations and selected autocorrelations of the variables calculated from the estimated spectral density matrix, but using only the business cycle (6-32 quarter) frequencies.[7] Second, Figure 2.2 plots the business cycle components of the series. These are formed by using an approximate band pass filter to extract the portion of the series associated with cycles of length 6-32 quarters.[8] As a reference, each panel of the figure includes the NBER business cycle reference dates, in addition to the plots of output and the series listed above the plot.

We stress three empirical characteristics of the interaction between money, prices, interest rates and output that are important for our subsequent analysis. First, both nominal and real money are highly correlated with output (from panel A of Table 2.2, the correlation is 0.62 for M and 0.61 for $M-P$). The high degree of business cycle "conformability" of money and output is evident in panels F and G of Figure 2.2 (with the late 1980's being a possible exception). Real money, and to a lesser extent nominal money, appears to lead output over the cycle. From panel B of Table 2.2, the correlation of M_t and y_{t+2} is 0.71, while the contemporaneous correlation is 0.61.

The second important characteristic involves the relation between prices and output. The correlation between P and y over the cycle is -0.35 (panel A of Table 2.2). While this suggests countercyclical movement of prices, it is evident from panel H of Figure 2.2 that prices moved pro-cyclically in some cycles (notably the pre-1970 period) and counter-cyclically in others (notably 1970-1986). The autocorrelations from panel B of Table 2.2 show two important features of the price-output relation. First, there is a tendency for prices to lead output in a countercyclical fashion. (From panel B of Table 2.2, $cor(P_t y_{t+2}) = -0.66$.) Second, at long lags, prices are positively correlated with output. (From panel B of Table 2.2, $cor(P_t y_{t-6}) = +0.20$, and (not shown in the table), this increases to +0.45 when output is lagged 10 quarters.) This positive correlation between prices and lagged values of output is suggestive of price stickiness in response to nominal disturbances, and a model incorporating gradual price adjustment is developed in the next section to investigate this suggestion.

[7]In terms of the spectral representation in (1), this table shows the correlation of the series constructed as:

$$x_t = \int_{\omega_1}^{\omega_2} [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]d\omega$$

with ω_1 and ω_2 representing the business cycle frequencies and x_t representing the log-level of the relevant series.

[8]The series are formed by passing the data through symmetric two-sided filter with 12 leads and lags. The filter weights are chosen to produce an optimal (L^2) approximation to the exact 6-32 quarter band pass filter, subject to the constraint the filter has zero gain at frequency 0. See Baxter and King (1995).

Table 2.2
Characteristics of Business Cycle Components of Postwar U.S. Data

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

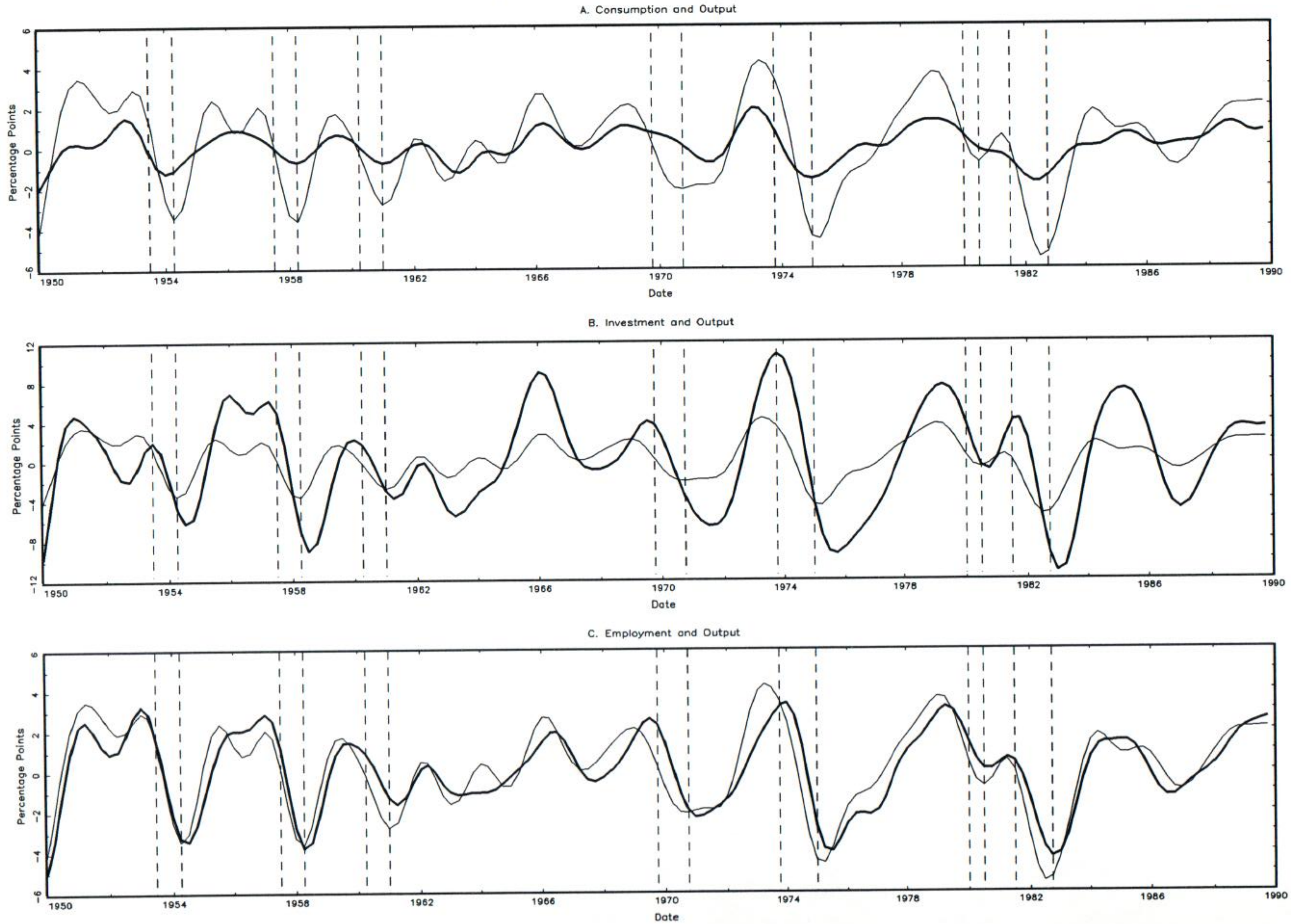
y	c	i	n	w	M	P	M-P	R	r
2.72	0.91	0.84	0.89	0.25	0.62	-0.35	0.61	0.30	-0.27
	1.01	0.77	0.84	0.31	0.61	-0.34	0.60	0.27	-0.19
		6.38	0.91	0.18	0.43	-0.17	0.38	0.57	-0.09
			2.42	0.11	0.52	-0.10	0.40	0.58	-0.08
				0.70	0.38	-0.57	0.57	-0.12	-0.39
					1.92	-0.34	0.86	-0.03	-0.36
						1.57	-0.78	0.33	0.36
							2.87	-0.20	-0.44
								1.34	0.28
									1.46

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.15	0.03	0.25	0.50	0.75	0.93	1.00	0.93	0.75	0.50	0.25	0.03	-0.15
c	-0.06	0.12	0.31	0.52	0.72	0.86	0.91	0.85	0.69	0.47	0.22	-0.01	-0.20
i	0.11	0.32	0.54	0.74	0.88	0.91	0.84	0.66	0.41	0.16	-0.06	-0.23	-0.36
n	0.12	0.31	0.52	0.73	0.89	0.95	0.89	0.72	0.45	0.17	-0.09	-0.29	-0.43
w	0.12	0.18	0.21	0.22	0.22	0.23	0.25	0.28	0.30	0.30	0.26	0.19	0.08
M	-0.05	0.04	0.15	0.28	0.42	0.55	0.62	0.64	0.58	0.47	0.33	0.17	0.01
P	0.20	0.12	0.05	-0.03	-0.12	-0.23	-0.35	-0.48	-0.59	-0.66	-0.66	-0.60	-0.48
M-P	-0.14	-0.04	0.07	0.20	0.35	0.49	0.61	0.69	0.71	0.68	0.58	0.44	0.27
R	0.48	0.54	0.58	0.60	0.57	0.47	0.30	0.07	-0.19	-0.43	-0.61	-0.71	-0.74
r	0.07	0.14	0.19	0.19	0.10	-0.07	-0.27	-0.43	-0.52	-0.50	-0.41	-0.27	-0.14

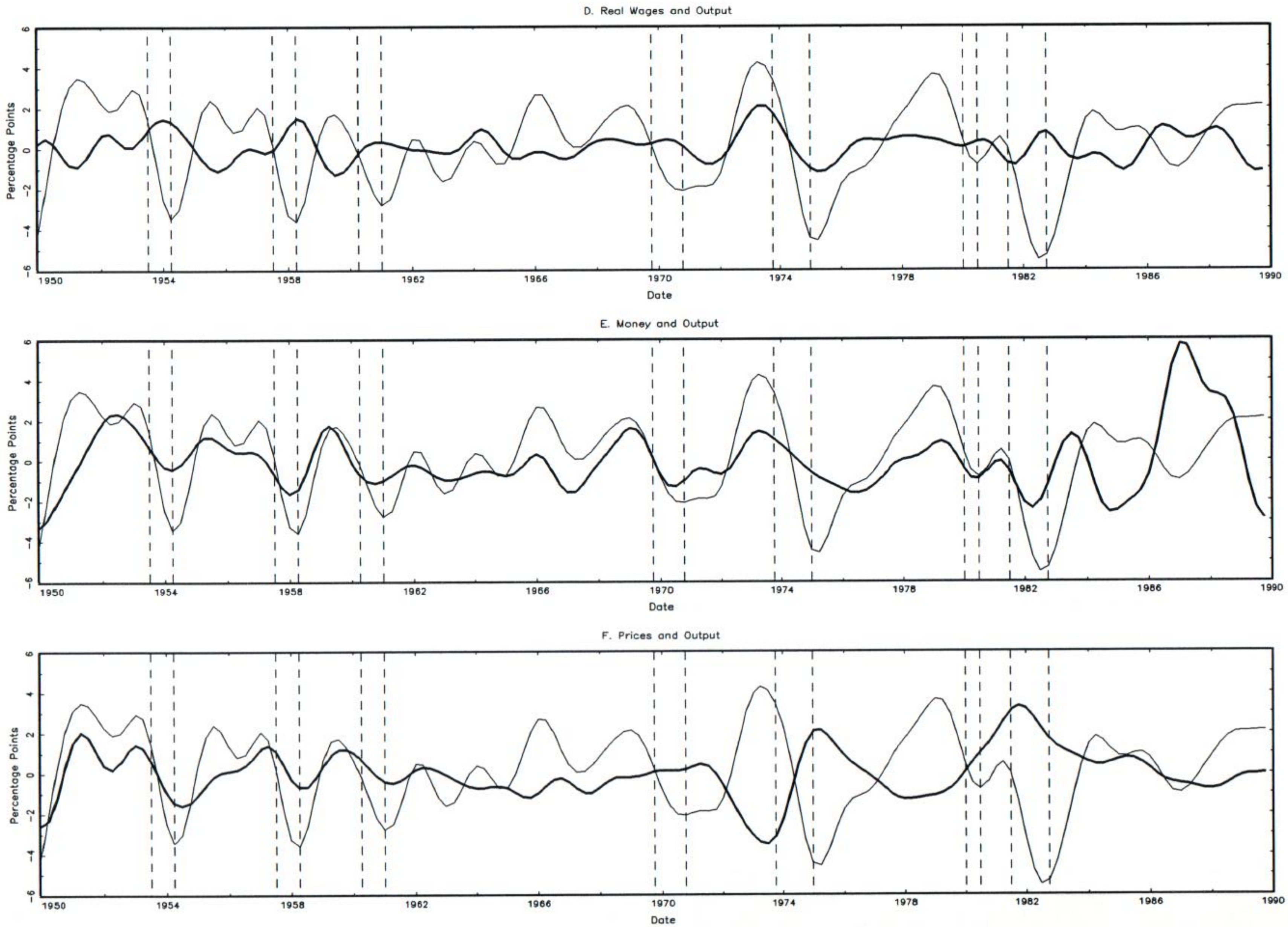
Notes: These results were calculated from the estimated spectral density matrix described in Section 5, footnote 14. Autocovariances were calculated from the relation: $\lambda_{\tau}^{xy}=(1/\pi)\int_{-\pi}^{\pi}e^{i\omega\tau}S^{xy}(\omega)d\omega$, where λ_{τ}^{xy} is the τ'th cross-autocovariance of x_t and y_t, and $S^{xy}(\omega)$ is the cross-spectrum. All series (except the interest rates) are multiplied by 100, so that the standard deviations in panel A represent percentage points. Interest rates are in percentage points at an annual rate.

Figure 2.2
Business Cycle Components of Quarterly U.S. Data



Notes: Each panel shows the 6-32 quarter bandpass filtered series for output (thin line) and other series (thick line) shown in the panel heading. Vertical lines are the NBER peak and trough dates.

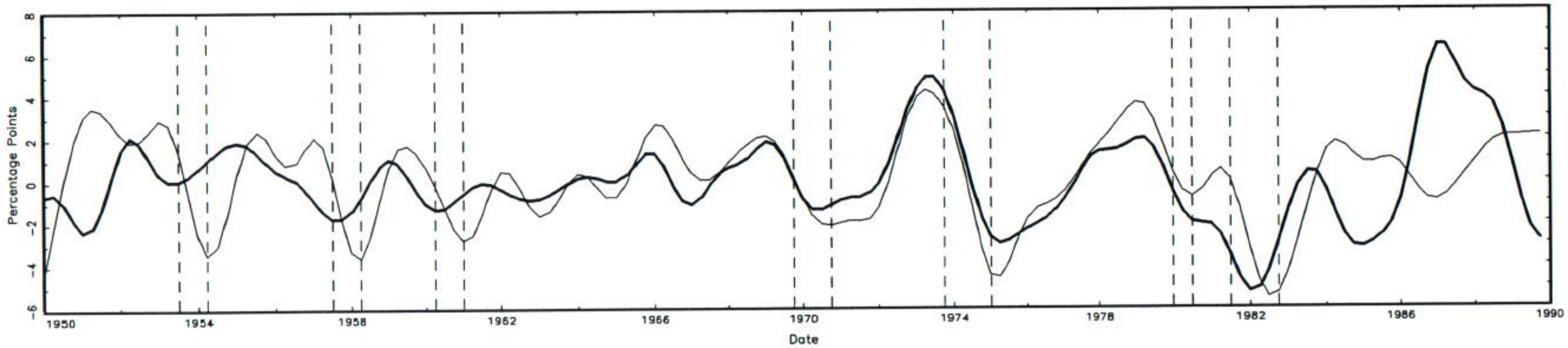
Figure 2.2 (Continued)
Business Cycle Components of Quarterly U.S. Data



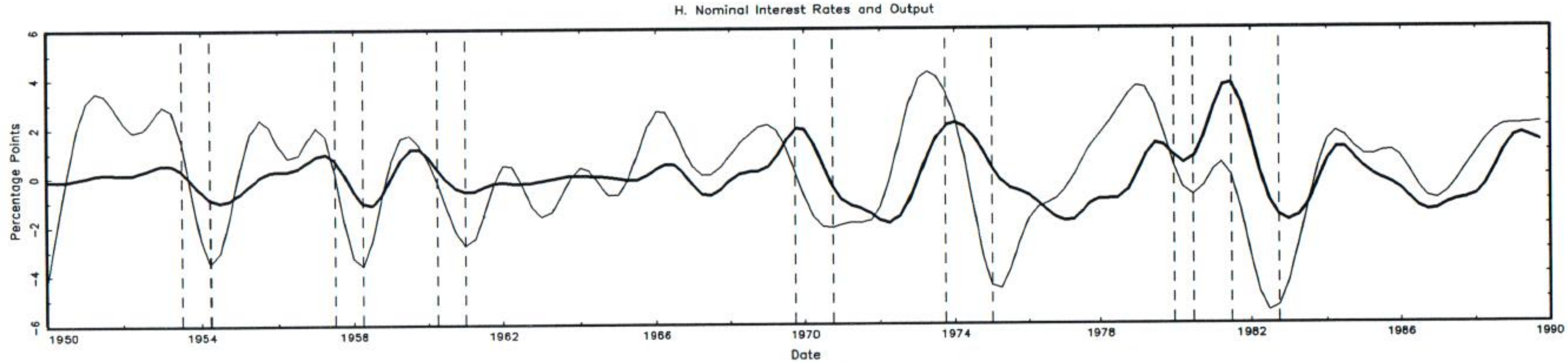
Notes: Each panel shows the 6-32 quarter bandpass filtered series for output (thin line) and other series (thick line) shown in the panel heading. Vertical lines are the NBER peak and trough dates.

Figure 2.2 (Continued)
Business Cycle Components of Quarterly U.S. Data

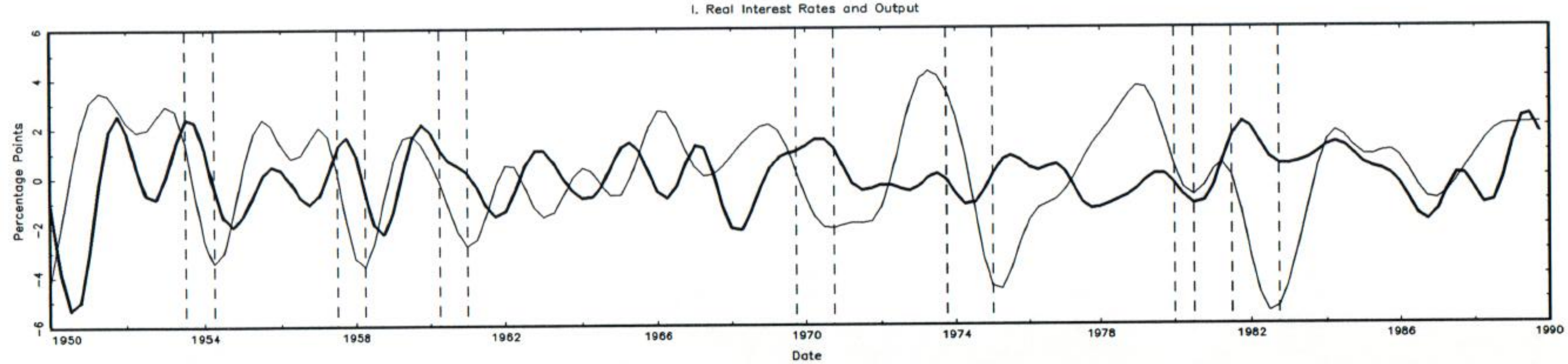
G. Real Balances and Output



H. Nominal Interest Rates and Output



I. Real Interest Rates and Output



Notes: Each panel shows the 6-32 quarter bandpass filtered series for output (thin line) and other series (thick line) shown in the panel heading. Vertical lines are the NBER peak and trough dates.

Finally, the third important characteristic is the systematic cyclical pattern of interest rates: nominal interest rates and output are positively correlated ($cor(R_t y_t) = 0.30$), more highly correlated when nominal rates are lagged ($cor(R_t y_{t-3}) = 0.60$), and strongly negatively correlated with future output $cor(R_t y_{t+6} = -0.74)$. Real interest rates move countercyclically and lead output over the cycle ($cor(r_t y_t) = -0.27$, and $cor(r_t y_{t+2}) = -0.52$). The leading countercyclical nature of real interest rates is suggestive of the types of mechanisms stressed in the models of financial market frictions that we survey in the next section.

These three characteristics of the business cycle have been documented by many empirical researchers using a variety of methods; perhaps most notably by business cycle analysts using methods that descended from the work of Burns and Mitchell (1946). (For a detailed discussion, see Zarnowitz and Boschan (1975).) For example, in the Commerce Department's system of cyclical indicators for the U.S., both nominal and real money are categorized as "leading indicators" with average cyclical leads of 4 and 2 quarters respectively.[9] Real money ($M2$) is one of the 11 series making up the Department's monthly Index of Leading Indicators. Interest rates and general measures of price inflation are categorized as lagging indicators, and both are components of the Department's Index of Lagging Indicators.[10] On the other hand, business cycle analysts indicator researchers have long recognized the negative relation between interest rates and leads of output (see Zarnowitz (1988)).[11]

3 Overview of Models

We consider three classes models with distinctly different mechanisms linking nominal and real variables over the business cycle: real business cycle models; models with prices that are gradually adjusting due to frictions in product markets; and models with gradual adjustment of portfolios due to frictions in financial markets. In this section, we provide an introduction to the specific versions of each type of model that we use in the remainder of our paper. We begin with some discussion of features that are common to all models and then discuss the details of individual setups.

3.1 Common Features

All our models incorporate a representative household and a representative firm, so we begin by discussing aspects of their behavior that are common features in the analysis below.

[9]See, *Handbook of Cyclical Indicators*, Table 8.

[10]Specifically, the Index of Lagging Indicators contains the average prime rate and the change in the CPI for services. Sensitive material prices are included in the Index of Leading Indicators, but this series behaves much differently than the general price level that we consider here.

[11]Indeed, bond prices are used by Moore (1991) in his "Long" leading indicator series.

3.1.1 The Representative Household

The representative household chooses a plan for consumption ($\{c_t\}_{t=0}^{\infty}$) and leisure ($\{l_t\}_{t=0}^{\infty}$) to maximize expected lifetime utility:

$$E_0[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)],$$

where $u(c, l)$ is the momentary utility function and β is the discount factor for future utility flows. With leisure determined (and also another use of time, h_t, in one of the models below), the representative household's labor supply n is then given as a residual from the endowment of time, which is taken to be unity:

$$n_t = 1 - l_t - h_t.$$

In all of the models that we construct below, individuals may freely adjust work effort so as to maximize momentary utility. Thus, we have the requirement that the marginal rate of substitution of leisure for consumption equals the real wage:

$$\frac{\partial u(c_t, l_t)/\partial l_t}{\partial u(c_t, l_t)/\partial c_t} = \frac{W_t}{P_t} = w_t \tag{2}$$

where W_t and P_t are the nominal wage rate and price of consumption, respectively, and w_t is the real wage rate.

3.1.2 The Representative Firm

The representative firm chooses a plan for production, labor demand, and investment so as to maximize the expected present value of its real profits (π),

$$E_0[\sum_{t=0}^{\infty} \beta^t \rho_t \pi_t]$$

where $\beta^t \rho_t$ is the discount factor applied to date t cash flows. Profits are assumed to be the value of output less the wage bill and investment.

The firm's output (y) is related to capital (k) and labor (n) inputs according to a production function,

$$y_t = a_t f(k_t, n_t), \tag{3}$$

where a_t is an exogenous shifter of total factor productivity. The various models considered below will impose different restrictions on the function f. The capital stock evolves as the net result of investment expenditure and depreciation,

$$k_{t+1} - k_t = \phi(i_t/k_t)k_t - \delta k_t, \tag{4}$$

where δ is the rate of depreciation and the strictly concave function ϕ embeds the idea that there are increasing costs of rapidly adjusting the stock of capital.

Optimal capital accumulation generally involves two efficiency conditions on the part of the firm. The first specifies that the marginal value of capital is equated to the marginal cost of investment:

$$\psi_t \phi'(i_t/k_t) = \frac{\partial \pi_t}{\partial i_t}.$$

In this expression, ψ_t is the date t Lagrange multiplier that indicates the value of an additional unit of capital installation (of a small change in k_{t+1} within the constraint (4)) and $\partial \pi_t/\partial i_t$ is the reduction in profits necessitated by the purchase of investment goods. The second efficiency condition specifies that:

$$\psi_t = E\{\beta \psi_{t+1} v(i_{t+1}/k_{t+1}) + \beta \rho_{t+1} \frac{\partial \pi_{t+1}}{\partial k_{t+1}} | I_t\}. \tag{5}$$

where $v(i_t/k_t) = (1-\delta) - (i_t/k_t)\phi'(i_t/k_t) + \phi(i_t/k_t)$ and where I_t is the set of all information realized up through the end of period t. This condition stems from selecting k_{t+1} optimally: it therefore requires that the shadow value of a unit of k_{t+1}, which is a measure of cost, is equated to the relevant expected benefit measure, which includes the effects of k_{t+1} on both future capital accumulation and profits. For each model we will develop this condition in greater detail, essentially by detailing $\rho_{t+1}\partial \pi_{t+1}/\partial k_{t+1}$.

3.1.3 An Economy-Wide Constraint

In each model there is an economy-wide constraint on the uses of output:

$$y_t = c_t + i_t.$$

This constraint highlights the fact the models ignore (i) fiscal interventions, including policies describing taxation and government purchases; and (ii) international trade.

3.2 The Real Business Cycle Model

Our analysis of the real business cycle model presumes that (i) production takes place according to a constant returns-to-scale production function; and (ii) firms and households interact in frictionless, competitive markets for final product, factors, and finance.

Firms thus maximize their profits, $\pi_t = a_t f(k_t, n_t) - w_t n_t - i_t$, by choosing labor input such that the marginal product $a_t \partial f(k_t, n_t)/\partial n_t$ is equated to the real wage w_t. Correspondingly, the effect of capital on profit, $\partial \pi_t/\partial k_t$, is $a_t \partial f(k_t, n_t)/\partial k_t$.

Households maximize lifetime utility subject to an intertemporal budget constraint, $E_0[\sum_{t=0}^{\infty} \beta^t \rho_t (c_t + w_t l_t)] \leq E_0[\sum_{t=0}^{\infty} \beta^t \rho_t (\pi_t + w_t)]$. The condition for optimal intertemporal allocation of consumption is thus that:

$$\partial u(c_t, l_t)/\partial c_t = \lambda \rho_t$$

where λ is the Lagrange multiplier on the wealth constraint and ρ_t is the real discount factor.

Using methods standard in the real business cycle literature, it is possible to generate a dynamic log-linear system that describes the evolution of the real economy by taking approximations to efficiency conditions and market-clearing conditions (see e.g., King, Plosser and Rebelo [1988a]). In addition to describing the behavior of output, consumption, investment, and labor input, this system also details how the real interest rate and real wage rate depend on economic conditions (specifically on the productivity factor a_t and the capital stock k_t).

To consider the behavior of nominal variables, we append a money demand function of the form:

$$\log(M_t) = \log(P_t) + m_y \log(y_t) - m_R \log(1 + R_t) - V_t$$

where M_t is the level of the date t money stock; P_t is the date t price level; R_t is the date t nominal interest rate; and V_t is a date t disturbance to the money demand function. (Using $\log(1 + R) \approx R$, we will also sometimes write this expression in the semi-logarithmic form: $\log(M_t) = \log(P_t) + m_y \log(y_t) - m_R R_t - V_t$.) We also incorporate the Fisherian theory of interest rate determination, written as:

$$R_t = r_t + E_t(\log(P_{t+1})) - (\log(P_t)),$$

where r_t is the real interest rate and $E_t(\log(P_{t+1}))$.

We append a money demand function rather than deriving it from a deeper specification of transactions technology because we want our model to display an exact neutrality with respect to variations in expected inflation of a cyclical and secular form. This strong classical dichotomy makes clear the origins of various results discussed below.

To study how the model's nominal variables evolve with the real equilibrium, we must specify a driving process for the three shift variables in the model (a, M, and V). We permit the money stock to be endogenous in the sense that it depends on the innovations to other shift variables, which should let us mimic the implications of explicitly modeling the monetary sector in a real business cycle model (as in King and Plosser [1984]).

A summary of the log-linearized version of the complete model is provided in Appendix C.

3.3 A Model of Commodity Market Frictions

Recent work in Keynesian macroeconomics has stressed three major departures from the real business cycle framework described above. First, imperfect competition rationalizes price setting behavior by firms. Second, introduction of "overhead" components of labor and capital makes productive activity exhibit increasing returns to scale.[12] Third, various schemes for gradual adjustment of prices have been incorporated.[13] Appendix A provides details for our version of a Keynesian model that incorporates these features. We summarize that key characteristics of that model here.

Households: In our consideration of the sticky-price model, we consider only modifications on the side of firms: the consumers in our economy are free to choose optimal consumption and labor supply plans as in the real business cycle model (although the opportunities that they face will typically differ). We also continue to assume that there is the same money demand function specified above.

Firms, Markups and Price Adjustment: To consider price-setting by firms, it is standard to investigate the operation of a model with imperfect competition. Notably, one can "disaggregate" the preceding real business cycle model, considering consumption as $c_t = [\int c_t(\omega)^\nu d\omega]^{(1/\nu)}$, where ω is an index of an individual firm and $0 < \nu < 1$. This specification implies that demand for product ω has a constant price elasticity, $(\nu - 1)^{-1}$. This leads firms to set price that are a constant markup over marginal cost (MC_t). Denoting the gross markup μ, then $\mu = 1/v$, which means that the gross markup is larger than 1.

In its entirely real form, the monopolistic competition macroeconomic model is closely related to the standard real business cycle model, but there are several important exceptions. First, the business cycle behavior of aggregate output is more strongly linked to fluctuations in labor and capital input:

$$\log(y_t/y) \approx \log(a_t/a) + \mu s_n \log(n_t/n) + \mu s_k \log(k_t/k)$$

In this expression, s_n and s_k are the shares of labor and capital income in value-added and μ is the gross "markup." Second, there are implications for the elasticities of marginal product schedules:

$$\log(\frac{\partial y_t}{\partial n_t}/\frac{\partial y}{\partial n}) \approx \log(a_t/a) - \frac{s_k \mu}{\zeta}\frac{n}{\widetilde{n}}\frac{\widetilde{k}}{k}\log(n_t/n) + \frac{s_k \mu}{\zeta}\log(k_t/k)$$

[12]The introduction of increasing returns to scale technologies and consequent monopolistic competition are surveyed in Rotemberg (1987): NBER Macroeconomics Annual, "The New Keynesian Microfoundations."

[13]Models of gradual adjustment of prices or wages have been developed in an important line of research beginning with Fischer (1977), Gray (1978) and Phelps and Taylor (1977). A key element of these theories is that at least some price or wage setters make their adjustment decisions only infrequently: their prices or wages hence cannot be altered in response to new information at subsequent dates. In the current paper, we explore the implications of a specific model of gradual price adjustment developed by Calvo (1983) and Rotemberg (1982a,1982b): this model has the attractive feature that the aggregate price level evolves as a first-order autoregression that is driven by factors which we discuss in greater detail below. In the earlier version of this paper and in King (1994), we studied models of wage and price adjustment developed more closely along the lines of Fischer, Gray and Phelps-Taylor. Those models were essentially moving average models of wage and price adjustment.

$$\log(\frac{\partial y_t}{\partial k_t}/\frac{\partial y}{\partial k}) \approx \log(a_t/a) + \frac{s_n\mu}{\zeta}\log(n_t/n) - \frac{s_n\mu}{\zeta}\frac{k}{\tilde{k}}\frac{\tilde{n}}{n}\log(k_t/k)$$

In these expressions, ζ is the elasticity of substitution between capital and labor in the production function f, $n/\tilde{n}$ is the ratio of total labor input to variable labor input (non-overhead labor); $k/\tilde{k}$ is the ratio of total capital input to variable capital input (non-overhead capital), and s_n, s_k, and μ are as defined above. The comparable expressions for the real business cycle model involve setting $\mu = 1$ and $n/\tilde{n} = k/\tilde{k} = 1$. That is, in general, the existence of overhead capital and labor changes the responsiveness of marginal products to changes in input quantities. When $\frac{n}{\tilde{n}} = \frac{k}{\tilde{k}}$, the elasticities of marginal products with respect to factor inputs are simply μ times their corresponding values in the real business cycle model.

To incorporate sticky prices into this model, Calvo (1983) and Rotemberg (1982a, 1982b) develop dynamic price-setting rules that are summarized by the following pair of equations:

$$\log(P_t) - \log(P_{t-1}) = \varphi[\log(P_t^*) - \log(P_{t-1})]$$

$$\log(P_t^*) = E_t[\sum_{j=0}^{\infty}(\beta\varphi)^j \log(\mu MC_{t+j})]$$

That is, the change in the price level at date t depends on the gap between a "target" price level and last period's price. In turn, the target price level is a distributed lead of that which would be charged in the static monopolistic competition model, a fixed markup of marginal cost. Calvo rationalizes this pair of specifications with the assumption that only a fraction φ of firms adjust their price each period and that this adjustment opportunity is allocated randomly across firms. It is consequently optimal to choose a price target that is an average of the prices that would otherwise be chosen ($\log(\mu MC_{t+j})$). Rotemberg (1982a, 1982b) rationalizes this specification by assuming that individual firms have quadratic costs of adjusting prices. Each author assumes that firms satisfy demand at the posted price.

Marginal cost for the firm is simply given by the cost of labor and the marginal product schedule:

$$MC_t = \frac{W_t}{\partial y_t/\partial n_t}. \tag{6}$$

The firm minimizes the cost of required production by selecting labor and capital efficiently given the exogenously specified level of demand. In the short-run, with capital predetermined and a given a level of output that must be produced, y_t, the firm simply must hire labor to produce output. Its "effective" demand for labor is thus implicit in the requirement that $y_t = a_t f(k_t, n_t)$, so that labor demand is positively influenced by output and negatively influenced by productivity and capital (as discussed by Barro and Grossman [1976], for example). Thus, locally, it follows that:

$$\log(n_t) \approx \frac{1}{\mu s_n}\log(y_t) - \frac{1}{\mu s_n}\log(a_t) - \frac{s_k}{s_n}\log(k_t)$$

Correspondingly, the value of having an additional unit of the capital stock is the implied cost reduction from reduced labor purchases. Hence

$$\frac{\partial \pi_t}{\partial k_t} = w_t \frac{\partial y_t/\partial k_t}{\partial y_t/\partial n_t}. \tag{7}$$

That is, if there is an additional unit of capital, it produces additional output $\partial y_t/\partial k_t$, with associated real labor cost savings of $w_t(\partial y_t/\partial n_t)^{-1}$.

3.4 A Model of Financial Market Frictions

An important recent strain of macroeconomic literature has stressed the role of financial market frictions in generating "liquidity effects" on nominal and real interest rates. In this section, we briefly present a recent model developed by Christiano and Eichenbaum (1993) that incorporates two main frictions discussed in this literature: (i) the requirement that some portfolio decisions are made without complete information about all shocks within the period, notably prior to the actions of the monetary authority; and (ii) costs of adjusting portfolio positions.

Households: The preferences of the household are as described above, but the opportunity to trade in goods and financial markets is more restricted. Notably, consumption expenditure in the current period must be paid for with "money to spend," S_t, or current labor income, $W_t n_t$:

$$P_t c_t \leq S_t + W_t n_t \tag{8}$$

The form of this constraint has two important implications. First, households can adjust labor supply so that (8) is satisfied, i.e., they are always on a "labor supply schedule" of sorts despite the financial market frictions. Second, this constraint takes the form that it does—rather than the more traditional "cash in advance" constraint of Lucas (1990), which would have the form of $P_t c_t \leq S_t$ in the current setup—because firms are required to pay for labor at the start of each period. This requirement also necessitates some (costly) borrowing on the part of firms, with implications for their labor demand that are considered further below.

There are time costs of adjusting the nominal portfolio holdings, S_t, of the form

$$h_t = h(S_t/S_{t-1}),$$

where $h(S_t/S_{t-1})$ is such that marginal and average time costs are positive and marginal costs are increasing ($h > 0, h' > 0, h'' > 0$). Incorporation of these costs implies that the time constraint is $1 - l_t - n_t - h_t = 0$.

A key friction in the liquidity effect model is that agents must select S_t without knowing the date t values of the money stock or technology shock. After shocks occur, the cash constraint (8) establishes a value of having an additional unit of money to spend,

$$\Delta_t = \frac{\partial u(c_t, l_t)/\partial c_t}{P_t} - \frac{\partial u(c_t, l_t)}{\partial l_t} h_t' \frac{1}{S_{t-1}} + \beta E[\frac{\partial u(c_{t+1}, l_{t+1})}{\partial l_t + 1} h_{t+1}' \frac{S_{t+1}}{(S_t)^2} | I_t], \tag{9}$$

which indicates that entering period t with an additional unit of S allows for the purchase of $(1/P_t)$ units of consumption and also has implications for time costs of adjusting nominal portfolios (higher at t and lower at $t+1$). This value of "money to spend" has the dimension of a utility discount factor for nominal cash flows: amounts of utility per dollar at date t. Thus, it is natural that under an efficient plan for S that is established at the start of period t, Δ_t must grow faster if there is a lower nominal rate of interest:

$$E[\Delta_t - \beta R_t \Delta_{t+1}] | I_{ot} = 0. \tag{10}$$

Notice that in (9) and (10), we have introduced the notation I_{ot} and I_t to indicate, respectively, actions that are taken at the beginning of period t, i.e., without knowledge of the shocks that are impinging on the macroeconomy within t, and at the end of period t.

On the side of the firms, there are also implications of the financial market frictions. First, firms select investment and labor demand decisions taking into account the fact that their owners face a delay in spending the profits flowing from the enterprize. Thus, in particular, it follows that the real discount factor to be applied to date t cash flows from the firm is $\rho_t = \beta^t P_t \Delta_{t+1}$ and the firm maximizes $E_0[\sum_{t=0}^{\infty} \beta^t \rho_t \pi_t]$ with $\pi_t = a_t f(k_t, n_t) - (1 + R_t) w_t n_t - i_t$, and the labor cost term reflecting the requirement that labor payments must be made in advance. Thus, the efficient labor demand decision sets:

$$a_t \frac{\partial f(k_t, n_t)}{\partial n_t} = (1 + R_t) w_t$$

and the efficient investment demand decision sets:

$$\phi'(\frac{i_t}{k_t}) = \frac{P_t \Delta_{t+1}}{\psi_t}$$

with the evolution of the shadow price of capital following (5) with $\rho_{t+1}(\partial \pi_{t+1}/\partial k_{t+1}) = \Delta_{t+1} P_t a_t [\partial f(k_t, n_t)/\partial k_t]$.

An additional equilibrium condition arises as a result of the joint actions of households, firms, and financial intermediaries. That is, at the start of each period, the household splits its monetary wealth into an amount that is deposited with financial intermediaries and an amount that is retained as "spending money": $M_{t-1} = Q_t + S_t$, where Q_t is the volume of deposits. The total volume of loans that financial intermediaries can make for the purpose of financing purchases of labor by the firm is thus $W_t n_t = Q_t + (M_t - M_{t-1})$, where the latter component is newly printed money injected via open market operations. But since, (8) is satisfied as an equality in equilibrium it follows that

$$P_t c_t = M_t.$$

4 Quantitative Models

Our objectives in this section are two-fold. First, we describe our quantitative implementation of the theoretical models sketched in the previous section. Second, we explore the internal propagation of the models by specifying simple processes for the exogenous shifts in technology shocks, money supply and demand. In the following section we investigate the properties of the model with driving processes that closely match postwar U.S. data.

4.1 Approximate Model Solutions

Our analysis will be carried out using approximate model solutions as in Kydland and Prescott (1982), King, Plosser, and Rebelo (1982a, 1982b), and elsewhere. We begin by log-linearizing the n equations that describe each of the economies around the applicable steady state point; we then reduce the dynamic system to a state space form; and finally we solve the resulting rational expectations linear difference system using techniques like those developed in Blanchard and Kahn (1980).

Specifically, after log-linearization each model can be written as a dynamic linear rational expectations model of the form:

$$AE_tY_{t+1} = BY_t + C_0X_t + C_1E_tX_{t+1} + ...C_pE_tX_{t+p} \qquad (11)$$

where the vector Y is a vector of n endogenous variables, the vector X is a vector of m exogenous variables, E_t denotes conditional expectation, and $A, B, C_0, ...C_p$ are coefficient matrices. Some of the endogenous variables ($K = \Phi Y$) are predetermined (Φ is a selection matrix which identifies the locations of these variables in Y). Other variables respond to some or all of the information that arrives at date t. A notable feature of our solution methodology is that the A and B may be singular matrices and the solution algorithm begins by undertaking state reduction numerically.

The exogenous variables are related to an underlying set of driving variables, δ_t, according to:

$$X_t = Q\delta_t \qquad (12)$$

$$\delta_t = \rho\delta_{t-1} + \xi_t \qquad (13)$$

Equations (11)-(13) can then be solved to yield:

$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} \Pi_{YK} & \Pi_{Y\delta} \\ 0 & Q \end{bmatrix} \begin{bmatrix} K_t \\ \delta_t \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} K_{t+1} \\ \delta_{t+1} \end{bmatrix} = \begin{bmatrix} M_{KK} & M_{K\delta} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} K_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \xi_{t+1}, \tag{15}$$

which specifies that the state of the economy is given by the driving variables (δ) and the predetermined components of the endogenous variables.[14]

The equations of the dynamic log-linear rational expectations models that we construct are reported in Appendix C. These specifications show that the coefficient matrices depend on three types of information. First, they depend on attributes of the steady state. For example, in the real business cycle model, the production function (3) implies that output is related to its sources and uses according to:

$$\log(y_t/y) = \log(a_t/a) + s_n \log(n_t/n) + s_k \log(k_t/k) = s_c \log(c_t/c) + s_i \log(i_t/i)$$

where s_c (s_i) is the share of consumption (investment) in national expenditure and s_n (s_k) is the share of labor (capital) in national income. Second, the coefficients depend on aspects of the near-steady-state behavior of the economy. For example, the efficiency condition for investment (5) implies that:

$$\log(i_t/i) - \log(k_t/k) = \eta[\log(\psi_t/\psi) - \log(\rho_t/\rho)]$$

where $\eta = [\frac{i}{k}\phi''/\phi']^{-1}$ governs the response of i/k to variations in the Tobin's q measure, ψ/ρ. Finally, the coefficients depend of the parameters of the driving processes.

Since the economic models considered in the previous section have many common elements in terms of the parameters of A, B, and C_i, we begin by discussing the choice of parameters that are common across our models. We then later add some additional, model-specific parameter information in subsections below.

4.2 Common Aspects of Model Parameterization

Parameter values for each of the models are chosen so that steady states values match estimates of average growth rates and specific "great ratios" calculated from the postwar data. Specifically, using estimates constructed in King, Plosser and Rebelo (1988a), parameter values are chosen so that in steady-state each model implies $s_n =$ .58, a per capita annual growth of 1.6%, an annual depreciation rate of 10%, and a real annual interest rate of 6.5%. We also assume that the investment adjustment cost function, ϕ, is such that there are no average or marginal adjustment costs local

[14] King and Watson (1995a, 1995b) discuss the specifics for finding a solution of the form (14) and (15) for linear rational expectations models that can be written as (11)-(13). The methods described there are important in the present context because they apply to singular (in the sense that A and B are less than full rank, as will be the case in all of our models) and that have multistage timing structures (as in the liquidity effect model).

to the steady-state; i.e., that $\phi(i/k) = i/k$ and that $\phi' = 1$. As discussed in the appendices, these specifications are sufficient to determine many of the "great ratios" of this economy, including the shares s_i and s_c as well as the capital-output ratio (k/y).

A notable feature of the three models that we are studying is that these steady-state attributes are not affected by our introduction of monopolistic competition or financial market frictions. In particular, as discussed in detail in the appendix, the "great ratios" are invariant to monopolistic competition because of particular assumptions about the nature of long-run equilibrium and about the relative importance of labor and capital in the specification of "fixed costs." Moreover, and also discussed in the appendix, the key steady-state ratios of real variables are invariant to the level of sustained inflation. This similarity of steady-states in all of the models is convenient because it allows us to focus on the implications of a small set of structural factors that are important for economic fluctuations.

Our models also have a common determination of the long-run level of labor input. We assume that the momentary utility function takes the form:

$$u(c,l) = \frac{1}{1-\sigma}[c^{\theta} l^{1-\theta}]^{1-\sigma}$$

The marginal rate of substitution of leisure for consumption (2) together with $c/y, n/l$, and s_n can then be used to determine the value of the preference parameter θ.[15] By contrast, the preference parameter σ, which governs the intertemporal substitutability of consumption and leisure, cannot be determined from steady-state information, and we simply assume $\sigma = 1$ so that utility is logarithmic. We also assume throughout that there is a unitary elasticity of substitution between capital and labor in production, i.e., that the production function takes on a Cobb-Douglas form.

4.3 The Real Business Cycle Model

Real business cycle models are typically built with two additional assumptions about parameters. First, there are assumed to be only small adjustment costs for investment, so that ϕ'' is very small (or zero) and $\eta = [\frac{i}{k}\phi''/\phi']^{-1}$ is correspondingly very larger (or infinite). We accordingly adopt this assumption, setting $\phi'' = 0$. Second, the technology driving process is assumed to be a low order autoregression that generates

[15] The equality of the real wage and the marginal rate of substitution (2) implies:

$$\frac{\partial u(c,l)/\partial l}{\partial u(c,l)/\partial c} = \frac{1-\theta}{\theta}\frac{c}{l} = w,$$

so that

$$\frac{1-\theta}{\theta}\frac{c}{y}\frac{n}{l} = \frac{wn}{y},$$

which determines θ from the ratios $c/y, n/l$, and s_n. See appendix B for some additional discussion of the interpretation in the presence of inflation-tax effects on labor input.

a great deal of persistence. For the purpose of the expository discussion in this section, we assume that $\log(a_t)$ follows a random walk. (Again, this assumption is dropped in the next section, where we use a process that is fitted to the data.)

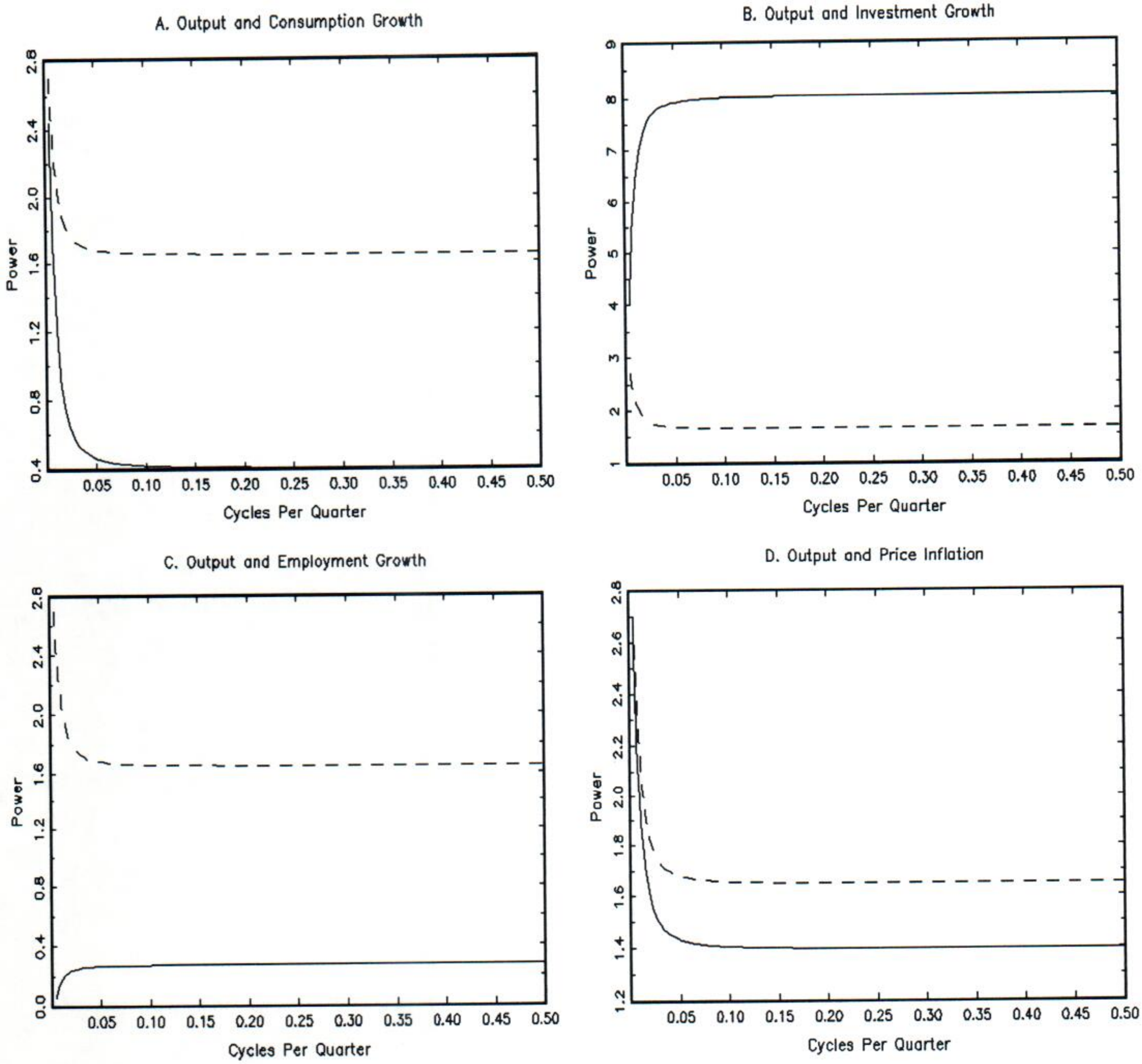
To complete the model, we need to specify the money demand and money supply processes. We use a log-linear money demand function, $\log(M_t) = \log(P_t) + m_y \log(y_t) - m_R R_t + V_t$, with $m_y = 1$ and $m_R = -.01$. The former is essentially the long-run income elasticity estimate found in Lucas (1988) and Stock and Watson (1993). Those analyses estimate the long-run interest sensitivity to be $m_R = -.10$. We use a value that is much smaller because we think there is a smaller degree of substitution in money demand over business cycles than in the long-run; at the same time, it will turn out that $m_R = -.01$ is a sufficiently large value that a number of surprising results will stem from it. Since the V_t and M_t have no effect on real variables in this model, their driving processes are not specified in this section.

Impulse Responses: Since the impulse responses to a permanent 1% shock to a_t have been much discussed in the real business cycle literature, our consideration of them will be brief. The long run effect of this shock on output, consumption and investment is 1.7%, since the direct productivity effects are amplified by the accumulation of capital (the impact effect of 1% is translated into a $(1 - s_k)^{-1}$ % long run effect). In order to produce the requisite capital accumulation, investment displays an "overshooting" of its long-run response and consumption is correspondingly less responsive than output in the long-run. There is also a moderate labor response, which is substantially smaller than in the case of stationary productivity shocks that has also been extensively discussed in the literature (such as $\log(a_t/a) = \rho \log(a_{t-1}/a) + \varepsilon_{at}$ with $\rho = .95$). The labor response is smaller because permanent productivity shocks raise wealth and expectations of future real wages—two factors leading to reductions in labor input—leaving only the intertemporal substitution effect of high real interest rates to stimulate labor supply.

There is a large, immediate negative effect of productivity shocks on the price level, of the form suggested by Mankiw (1989) and others. The central reason for this is that there is an increase in the real demand for money arising from a larger volume of real income. However, the price level decline is smaller than the increase in output because nominal interest rates rise with a positive shock to technology. (This rise in nominal interest occurs because the increase in the real rate of interest is only partly offset by a decline in the expected rate of inflation.) Consequently, the price level is less variable than real activity in the short-run, although the long-run variability of the price level is entire determined by the response of output to the permanent technology shocks.

Spectra of Real and Nominal Variables: The spectra of the growth rates of output, consumption, investment and labor have previously been studied by Watson (1993) and Soderlind (1994). Figure 4.1 reports the spectra of each of these real variables (output is the common, dashed line in each of the subplots). These show that the RBC model produces a greater amount of very low frequency variation in growth

Figure 4.1
Growth Rate Spectra: Real Business Cycle Model



Notes: Each panel shows the spectra for output growth (dashed line) and other series (solid line) shown in the panel heading. The spectra were calculated with an innovation variance equal to unity.

rates in output and consumption (variations in "stochastic trend") than it does business cycle variation. However, for investment, there is a greater extent of higher frequency—at both business cycle and irregular frequencies—than there is stochastic trend variation. This feature of the spectrum reflects the "investment overshooting" displayed in the impulse responses considered earlier. However, since the trends in consumption, investment and output are common, the height of the growth rate spectrum at the zero frequency is the same in all of the graphs.[16] Finally, as suggested by the impulse responses, the price level displays smaller high frequency variability than does output, but with common low frequency variability.

The outstanding feature of these figures is that the "typical spectral shape of growth rates" is not captured by the basic real business cycle model. This departure provides a major motivation for models that provide a short-run influence of nominal variables on real economic activity. Such models have the potential to capture large business cycle variations without large trend variations.

4.4 The Implications of Commodity Market Frictions

The next model that we consider involves two major departures from the basic RBC model. First, there is monopolistic competition and an increasing returns to scale production function. Second, there is gradual price adjustment toward a target price level. Each of these features is quantitatively important in determining the dynamic behavior of real and nominal variables.

There are three new parameters that we must specify in this model. First, motivated by empirical studies like those of Hall , we set the value of the markup of price over marginal cost, μ, to be 1.5. Second, we assume that the steady-state ratios of variable to total values of the labor and capital are equal, i.e., $(\tilde{n}/n) = (\tilde{k}/k)$. As we show in the appendix, their common value can be determined from μ, under the assumption that entry eliminates any steady-state profits. Specifically, when $\mu = 1.5$, $(\tilde{n}/n) = (\tilde{k}/k) = .67$. Third, we must specify a price adjustment parameter in the adjustment equation $P_t - P_{t-1} = \varphi[P_t^* - P_{t-1}]$. We choose $\varphi = .10$, which implies that 10% adjustment of discrepancies are eliminated per quarter. Finally, we set the investment cost parameter, η, to be equal to 1 in line with the Chirinko's (1993) overview of empirical investment functions: while not strictly required for the study of commodity market frictions, the substantial investment adjustment costs implied by this elasticity are consistent with much conventional macroeconometric work.

The most interesting features of this model can be seen by solving this model a simple driving process for money: we adopt the specification used in King's (1994) earlier work, $M_t - M_{t-1} = \frac{1}{2}[M_{t-1} - M_{t-2}] + \varepsilon_{Mt}$. This specification implies that there is a long-run effect of 2% on the level of the money stock if there is a 1% monetary injection any date.

[16] Since we are discussing shape of spectra in this section, we have simply normalized the innovation variance of technology shocks to unity for the purpose of computing these figures.

Impulse Responses: Figure 4.2 displays the impulse responses of real and nominal macroeconomic variables to a one percent monetary shock. The key features are as follows. First, despite the fact that the level of the money stock increases for several quarters before reaching its long run level of 2%, output increases most in the initial period and then dies away.[17] Investment and consumption inherit this overall shape, but there is greater short-run responsiveness of investment than consumption due to the "permanent income" specification of preferences even though there are substantial investment adjustment costs. Given that output is demand-determined, labor input must rise to produce requisite output in the short-run; given that μs_n is .87 for our specification, it follows that labor input must rise by about 1.1 times the increase in output in the first two periods. The real interest rate declines in response to this monetary expansion, in contrast to King (1995), where strong "investment accelerator" effects caused it to rise. The presence of major investment adjustment costs implies mitigates the accelerator effects in our model.

Real money balances rise substantially in response to the monetary shock, although by only a fraction of the output increase because the nominal interest rate also rises substantially. This nominal interest rate rise reflects a general tendency in sticky price models driven by persistent nominal shocks: there must be substantial expected inflation due to the gradual adjustment of prices.

Interestingly, there are important real propagation mechanisms that are built into this model, which appear stronger than in their RBC counterpart. Notably, after about twenty quarters, labor is back to its long-run level, but output displays much greater persistence. This reflects the fact that the elasticity of output with respect to capital is $s_k = .42$ in the RBC model and that it is μs_n=.73 in this monopolistic competition model.

Spectra: The spectra of growth rates of real and nominal variables are reported in Figure 4.3: in each of these panels, the dashed line is output growth as in the prior figure. The results are strikingly different from the RBC model. Output, consumption and investment display "humps" in the business cycle frequencies, although they are not as pronounced as the ones that we saw in the U.S. data above. There is also a natural ordering in terms of variability: the volatility of investment is higher than that of output, while the volatility of consumption is lower. The spectrum of price level growth, by contrast, displays no peak at the business cycle frequencies but rather a substantial amount of low frequency variability.

These features are naturally linked to the economic mechanisms present in the model with commodity market frictions: monetary disturbances produce highly persistent, but ultimately, temporary movements in output. The permanent income model of consumption embedded into the model implies that consumption will be less volatile and investment more volatile than output.

[17]This contrasts with the "hump shaped" response of output to a monetary injection that King (1995) generates in a model without forward-looking money demand and without investment adjustment costs.

Figure 4.2
Dynamic Responses to Innovation in Money Growth
Sticky-Price Model with Money Process: $\Delta m_t = \frac{1}{2}\Delta m_{t-1} + \epsilon_t$

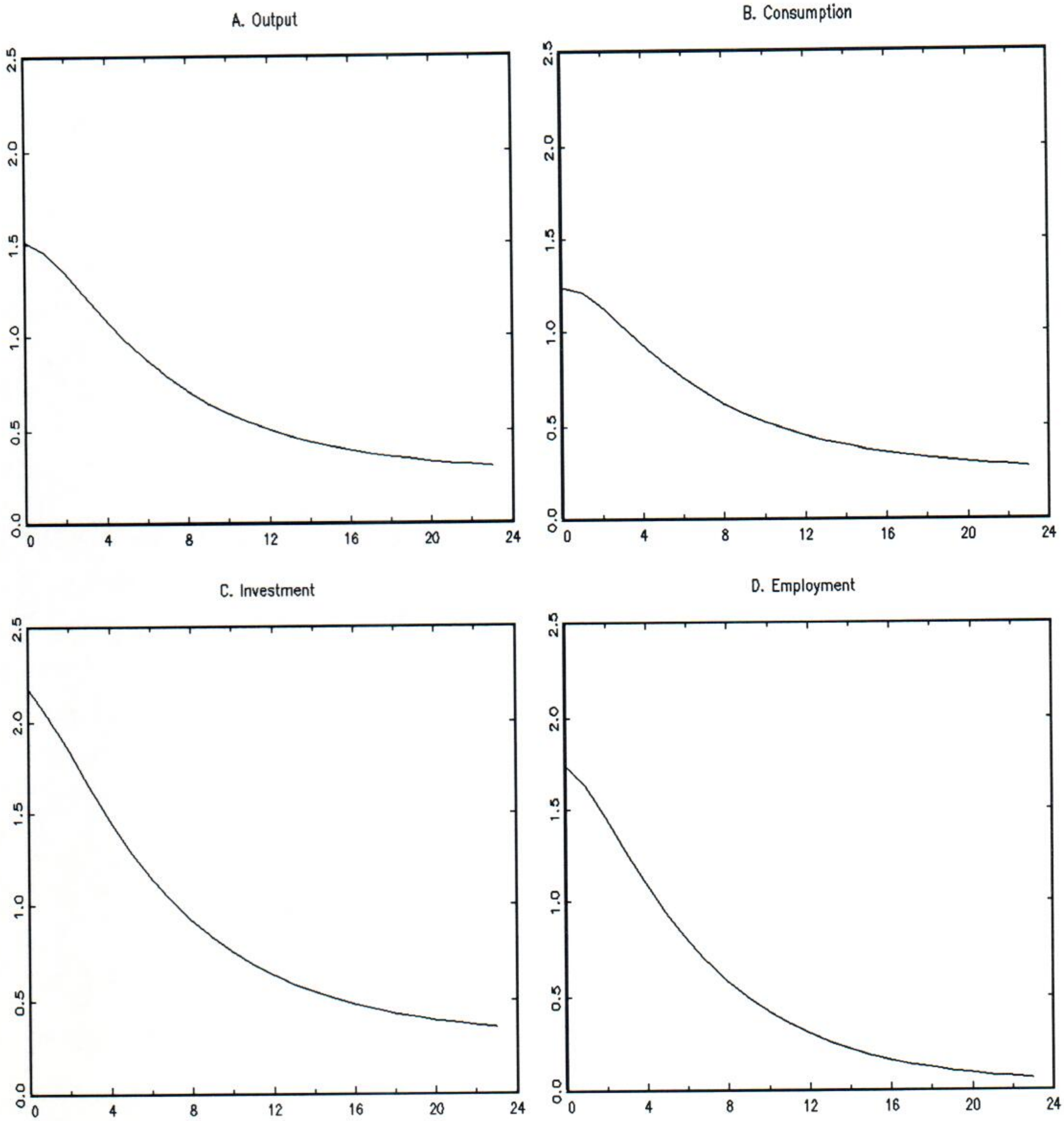


Figure 4.2 (Continued)
Dynamic Responses to Innovation in Money Growth
Sticky-Price Model with Money Process: $\Delta m_t = \frac{1}{2}\Delta m_{t-1} + \epsilon_t$

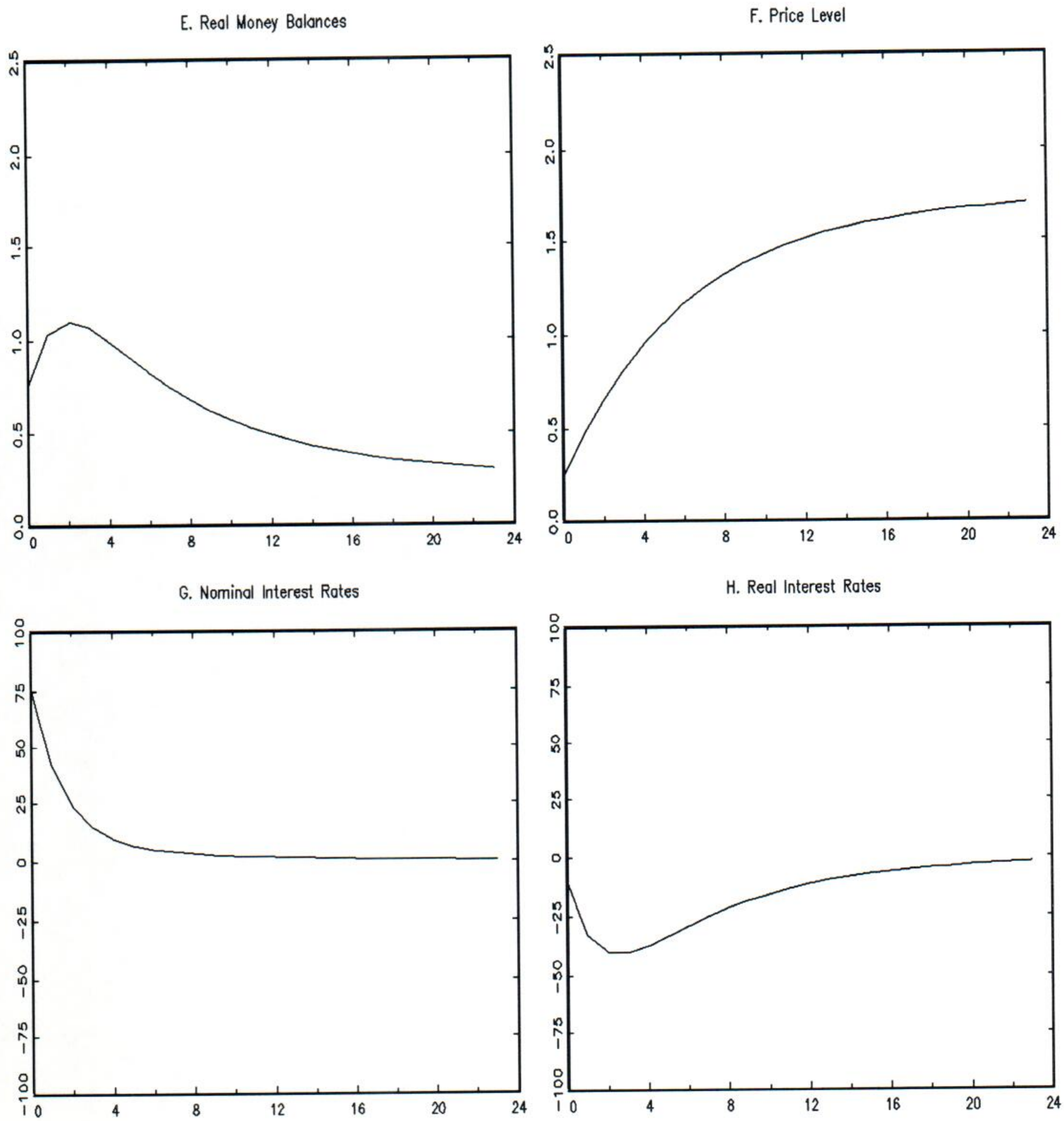
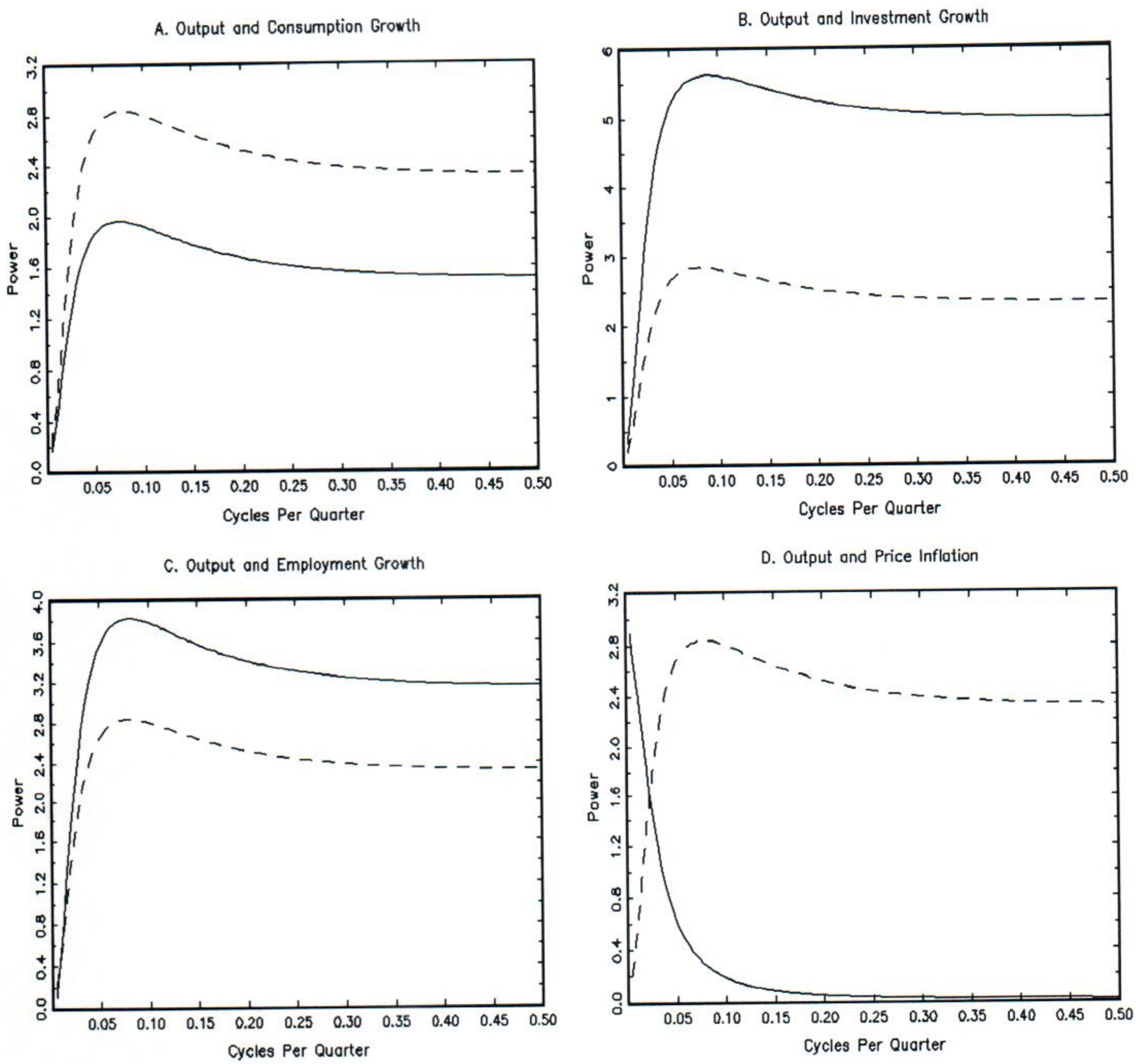


Figure 4.3
Growth Rate Spectra
Sticky-Price Model with Money Process: $\Delta m_t = \frac{1}{2}\Delta m_{t-1} + \epsilon_t$
(Productivity and money demand innovations set to zero)



Notes: Each panel shows the spectra for output growth (dashed line) and other series (solid line) shown in the panel heading. The spectra were calculated with a innovation variance equal to unity.

4.5 Financial Market Frictions

To develop the quantitative version of the financial market frictions model, we need to specify the time costs of adjusting portfolios. As in Christiano and Eichenbaum (1993), the natural procedure is to specify that there are small average and marginal time costs near the steady state position. We assume that in steady-state, individuals spend one percent of their working time in portfolio rearrangement, so that $\underline{h} = .01 * \underline{n} = .002$ and that the initial steady state position involves an annual inflation rate of 4%. Then, we assume that a rise in the inflation rate by 4% would increase h to $(1.06)\underline{h}$ and a similar decline would move h to $(.95)\underline{h}$. These assumptions are sufficient to determine the derivatives of portfolio adjustment cost function: $h' = 5.5\underline{h}$ and $h'' = 100\underline{h}$.

4.5.1 One-Time Changes in the Money Stock

Under our assumptions on the h function, there is some important propagation of the effects of one-time monetary shocks, as suggested by the work of Christiano and Eichenbaum (1992), because there is a gradual adjustment of "spending money" to the actual money stock shown in Figure 4.4 and a corresponding temporary increase in bank deposits and loans. The economy displays a sustained increase in output, investment, consumption and labor supply and a sustained decrease in real and nominal interest rates. However, the price level moves immediately to just below its long-run level and then gradually approaches it through time.

The economic mechanisms are fairly direct. First, because the nominal rate is low in a sustained manner, there is a sustained increase in the demand for labor at a given real wage rate. Firms accordingly offer higher real wages (not shown) and successfully attract additional labor input, which produces additional real output. Second, because the real interest rate is low in a sustained manner and because the increased demand for labor raises the marginal product of capital, there is a substantial increase in investment, even despite substantial investment adjustment costs ($\eta = 1$).

It is possible to produce additional propagation of the effects of monetary shocks by further increasing the scope of portfolio adjustment costs. However, such additional increases also have consequences that are unattractive unless they are accompanied by changes in other model parameters. Notably, with greater portfolio sluggishness, it follows that there will be longer intervals of low real interest rates, low nominal interest rates, and high real labor input. These factors all work to further increase the responsiveness of the initial period investment demand to a one-time increase in the quantity of money, while having relatively little effect on output in the initial period output. Accordingly, with higher portfolio adjustment costs, it follows that the price level "over shoots" its long-run level and real consumption correspondingly declines (recall that consumption, the price level, and money are linked by $M = Pc$). Hence, there are important limits on the extent to which portfolio adjustment costs can be

Figure 4.4
Dynamic Responses to Innovation in Money Growth
Financial Market Friction Model with Money Process: $\Delta m_t = \epsilon_t$

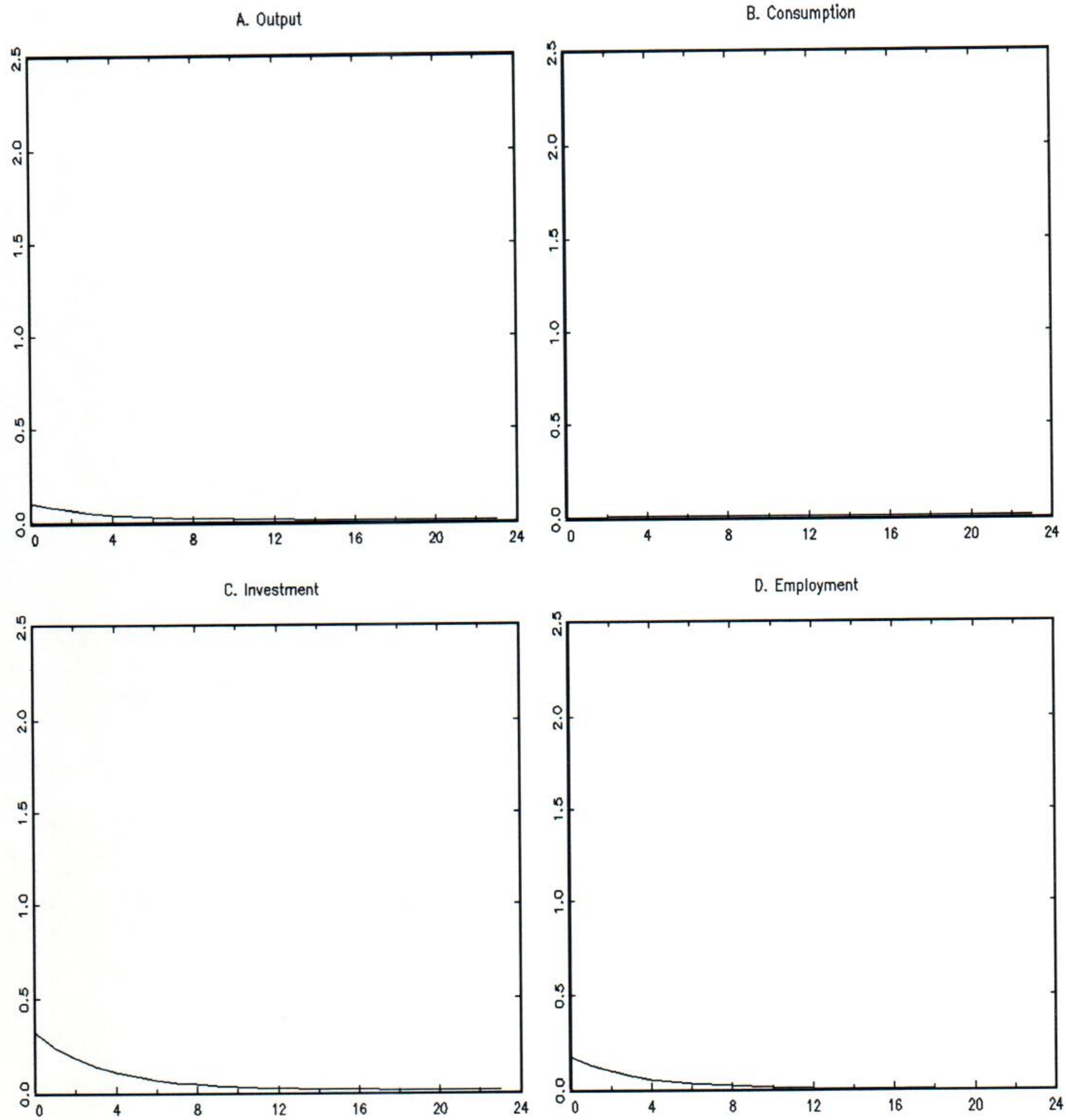
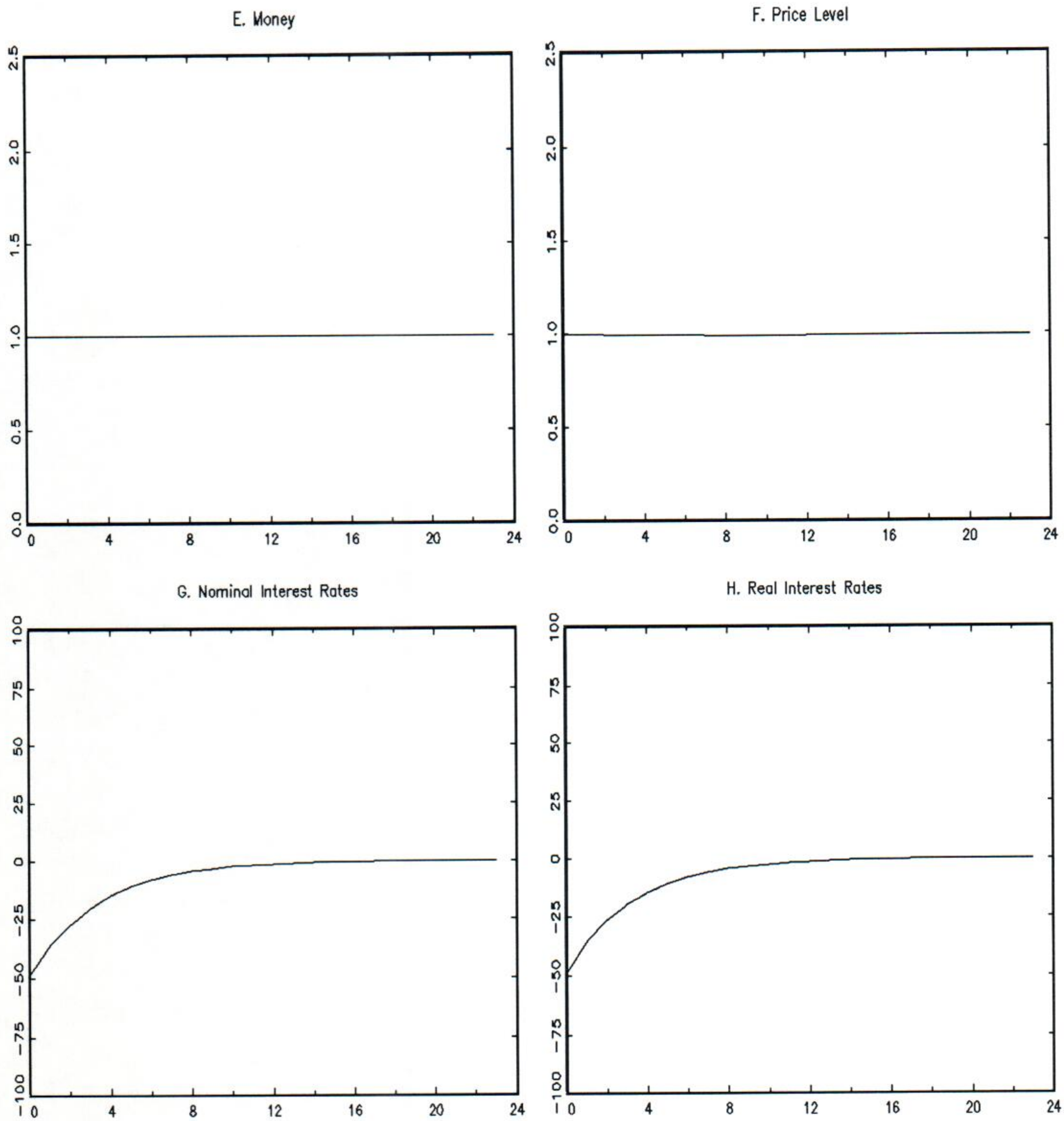


Figure 4.4 (Continued)
Dynamic Responses to Innovation in Money Growth
Financial Market Friction Model with Money Process: $\Delta m_t = \epsilon_t$



used to generate persistence, unless one is also willing to raise investment adjustment costs so as to curb the expectations-induced shifts in investment demand. However, altering investment adjustment costs plays havoc with the response of the financial market frictions model to productivity and other real shocks.

4.5.2 Effects of Positive Serial Correlation in Money Growth

If there is positive serial correlation in money growth, there are also some unusual features of the response of the financial market frictions model to a shock to money. Figure 4.5 displays these responses under the assumption that there is a monetary rule of the form used in the prior section: $M_t - M_{t-1} = \frac{1}{2}[M_{t-1} - M_{t-2}] + \varepsilon_{Mt}$. As seen above, this specification implies that there is a long-run effect of 2 % on the level of the money stock if there is a 1% monetary injection currently.

There are two key implications of this modification. First, the increased persistence of the exogenous shock process leads to an increased responsiveness of investment to monetary shocks in line with the discussion above. Thus, in Figure 4.5, we see a high amplitude investment response, coupled with a decline in consumption at the time of the initial monetary injection. The second effect, stressed in prior work by Christiano (1992), is that expected inflation results from this shock. In the model considered here, it follows that this tendency is sufficiently strong that there is actually a rise in the nominal interest rate in response to the initial monetary injection. Surprisingly, however, this decline in the nominal interest rate does not produce a decline in the quantity of labor input because of the decrease in consumption discussed earlier. Instead, the nominal rate induced decline in labor demand is overwhelmed by an increase in labor supply associated with the decline in consumption.

Overall, our consideration of the financial market frictions model suggests that its dynamic response paths to monetary shocks are fragile, in ways that are associated with the role of expectations about future real and nominal variables.

5 Empirical Evaluation of the Models

In this section, we evaluate how well our three basic macroeconomic models capture two sets of stylized facts about post-war U.S. business cycles: (i) the patterns of business cycle variability, as revealed by the spectra of growth rates and the standard deviations of business cycle components of economic activity; and (ii) the comovements of real and nominal variables.

5.1 Specification of realistic driving processes

In the last section, we showed selected impulse response function and spectra from versions of the models that incorporated very simple driving process. This was done so that we could easily focus on the internal dynamic mechanisms of the models.

Figure 4.5

Dynamic Responses to Innovation in Money Growth

Financial Market Friction Model with Money Process: $\Delta m_t = \frac{1}{2} m_{t-1} + \epsilon_t$

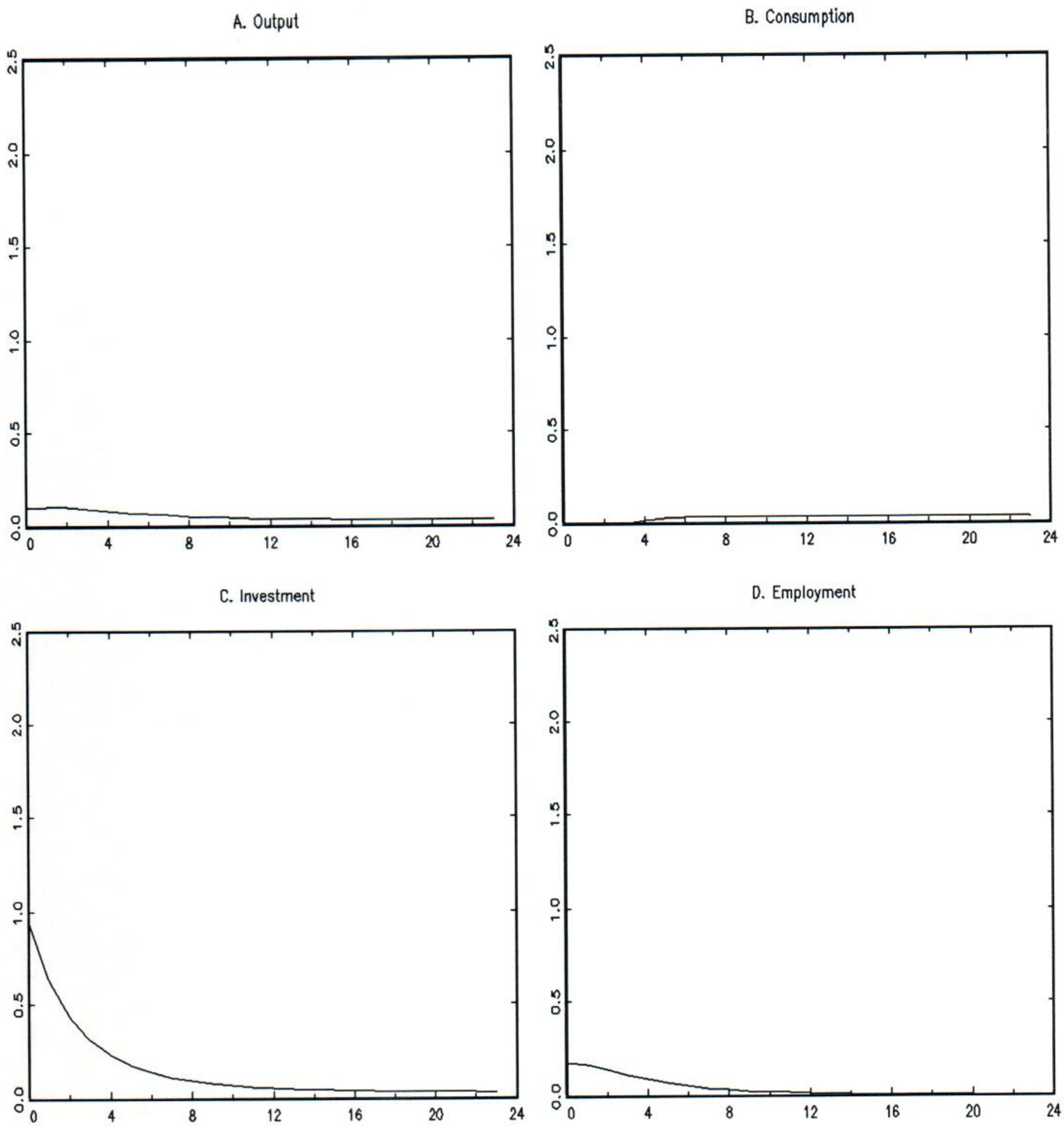
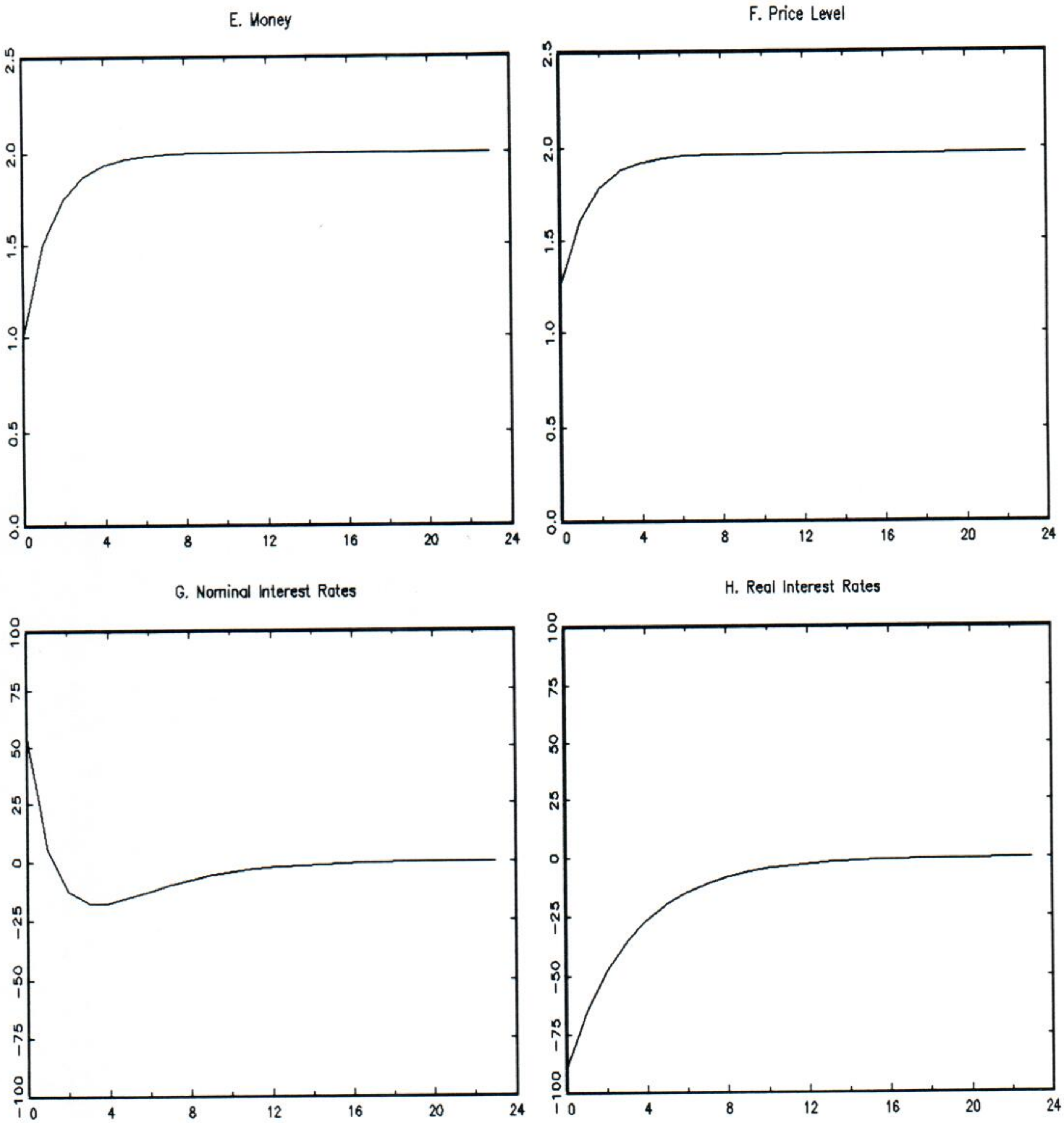


Figure 4.5 (Continued)
Dynamic Responses to Innovation in Money Growth
Financial Market Friction Model with Money Process: $\Delta m_t = \frac{1}{2} m_{t-1} + \epsilon_t$



Here, our purpose is different: we want to see whether the models produce outcomes that are broadly consistent with the business cycle characteristics of the post-war U.S. data. For this purpose, we estimate more "realistic" processes for the driving variables. Two goals underlie the specification of the driving processes used in this section. First, and most obviously, we want the autocovariance properties of the model's driving process to mimic those of the data. Second, we want the driving processes to be general enough, so that, at least in principle, the autocovariances of the models' variables can match those of the data. This latter requirement means that the driving processes must be specified in terms of a large number of underlying shocks so that the models can potentially produce variables that, like the data, have a nonsingular spectral density matrix.[18]

To achieve these goals, we require that each of the driving processes to match the fitted moving average representation of a particular linear combination of variables obtained from VAR fitted to the postwar data. Specifically, let y_t^d, c_t^d, i_t^d, n_t^d, w_t^d, M_t^d, P_t^d, and R_t^d denote the postwar values of the logarithms of output, consumption, etc., that were described in detail in Section 2 above. (The superscript "d" is used as reminder that these are data and may differ from their counterparts generated by the models.) Let $\delta_t = \rho\delta_{t-1} + h\xi_t$ denote the companion form of the VAR representing these data, and let x_t denote the exogenous driving variables in one of the models. The process for x_t is then specified as:

$$x_t = C(L)\xi_t \tag{16}$$

where $C(L)$ is the lag polynomial generated by the state-space model:

$$x_t = Q\delta_t \tag{17}$$

$$\delta_t = \rho\delta_{t-1} + h\xi_t. \tag{18}$$

In (16), the vector ξ_t has the same characteristics as the VAR residuals estimated from the data: ξ_t is an 8×1, zero mean, white noise vector. Thus, in the RBC model, with $x_t = (a_t, M_t, v_t)$, the three driving variables depend on eight shocks. If only three shocks were used, then any subset of four or more variables in the models would be dynamically singular. This is avoided by allowing ξ_t to include eight distinct shocks.

The matrix Q in (17) is model specific and is chosen so that the driving variables in the models have autocovariances that match their empirical counterparts in the data. Specifically, in the RBC model, $x_t = (a_t, M_t, v_t)$ and Q is chosen so that the autocovariances of Δa_t, Δm_t, and v_t match those of $\Delta a_t^d = \Delta y_t^d - \theta_n \Delta n_t^d$, M_t^d, and $v_t^d = -M_t^d + P_t^d + m_y y_t^d + m_R R_t^d$ respectively, with $\theta_n = 0.58$, estimated as labor's

[18]Models with multiple shocks can't eliminate singularities when the endogenous variables are functions only of current and lagged values of the driving variables. However, in models with forward looking expectations and multiple driving shocks the endogenous variables will in general, depend on all of the shocks in the system.

average share of national income, $m_y = 1$ and $m_R = -0.01$, as discussed above. The variables (Δa_t^d, M_t^d,v_t^d) are the data's natural analogues of model's exogenous variables (Δa_t, ΔM_t, v_t), except that Δa_t^d excludes the term $-\theta_k \Delta k_t^d$ on the grounds that this term has a very small variance, and is poorly measured in the data. In the sticky-price model, the definition of x_t is the same, except that $\theta_n = 0.87$, which is labor's average share multiplied by a markup parameter of 1.5. Mechanically, this leads to a less volatile series for productivity shocks, with implications that we trace out below. In the financial market frictions model, the variables are the same as the RBC model, except that v_t is excluded from x_t.[19]

The spectra of these driving processes is shown in Figure 5.1. The first two panels show the spectra of annual rates of growths of productivity and money ($400\Delta a_t$ and $400\Delta M_t$, respectively), and the last panel shows the spectrum of the level of v_t. Each of these processes differs markedly from the simple processes analyzed in the last section. The growth rates for both productivity and money are positively serially correlated with significant mass at the business cycle frequencies. The productivity process in the sticky price model is less variable than in the other models because of the larger labor elasticity in the production function. The process for velocity, while stationary, is highly persistent, with a spectral shape similar to nominal interest rates (see Figure 2.1, panel I).

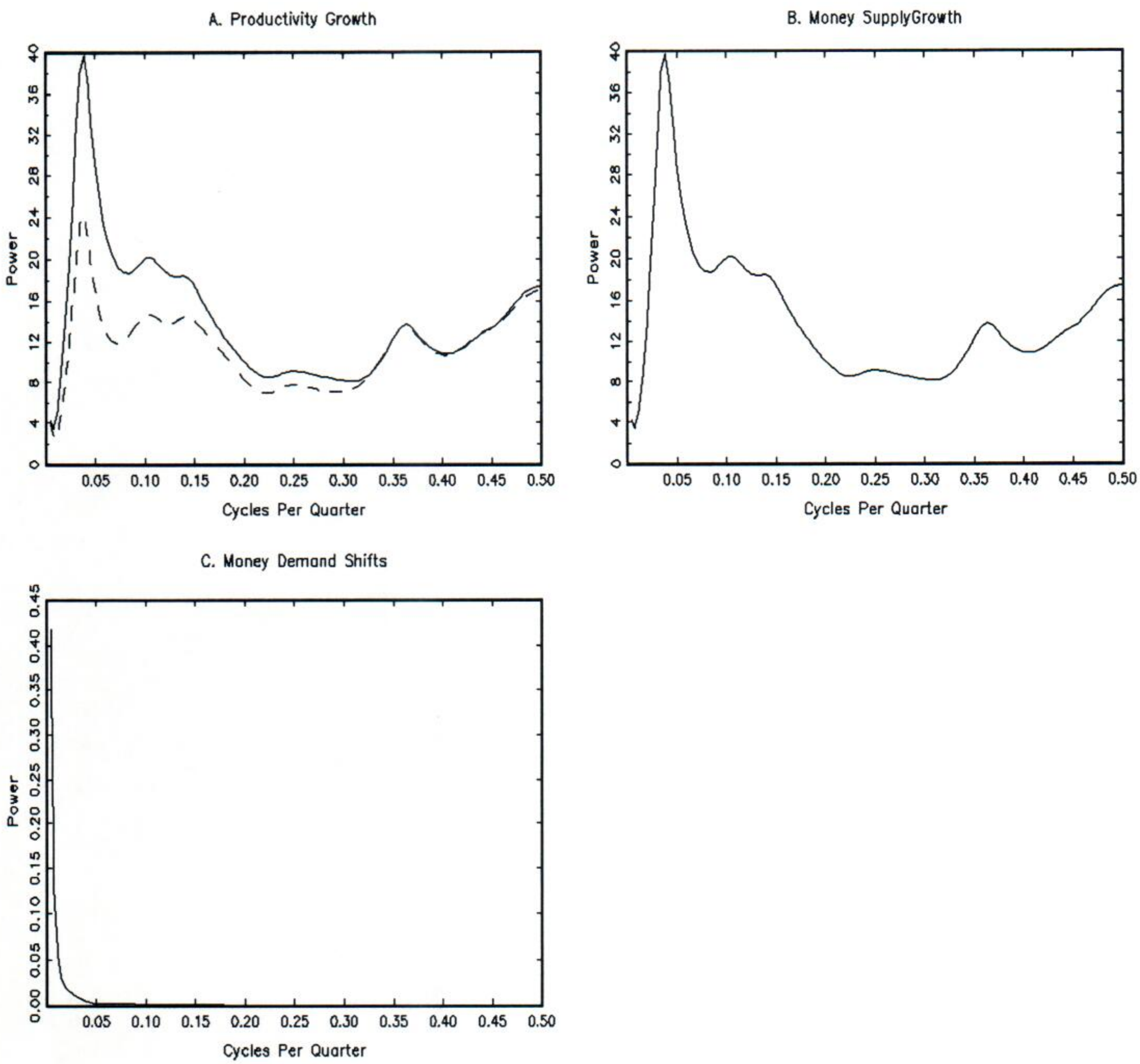
5.2 Results for the Three Macro Models

The second moment properties of the macroeconomic models, when they are driven by these realistic processes, are summarized in Figure 5.2 and Tables 5.1-5.2. The figures and tables highlight different aspects of the operation of these models.

Figure 5.2 shows the spectra of the growth rates of macroeconomic variables in the models along with the estimated spectra of the growth rates of the counterpart variable in the postwar U.S. data. It thus displays the extent to which the models capture the variability of the growth rates of output, consumption, money, etc., at different

[19]The estimated VAR underlying (18) was specified using $\Delta n_t^d, \Delta w_t^d, \Delta M_t^d, R_t^d, y_t^d - c_t^d, y_t^d - i_t^d, (M_t^d - P_t^d - y_t^d)$ and $(w_t^d - y_t^d + n_t^d)$. This mixture of levels and differences uses integration characteristics of the data familiar from a large body of empirical research. In particular, the specification imposes three unit roots or stochastic trends in the system. These trends are shared by the variables in a way that is consistent with (i) balanced growth in y, c and i, (ii) stable long-run money demand with unit income elasticity (see Lucas (1988) and Stock and Watson (1993)), and (iii) balanced real wage and labor productivity growth. Of course, during estimation, the VAR is free to ignore these relations by differencing the level variables. That is, while this specification imposes a minimum of three unit roots, it also accommodates higher order integration. Thus, for example, it nests specifications with integrated interest rates, money growth, price inflation, and money demand. In addition to forming the basis for (18), the estimated VAR was also used to calculate estimated spectra of the data shown in Figure 2.1. The VAR included a constant term and was estimated over the period 1949:1-1992:4. (Data before 1949:1 was used to initialize the VAR.) The estimated data spectra were computed using a VAR with six lags. (Although similar results can be obtained using standard nonparametric estimators.)

Figure 5.1
Spectra of Driving Processes



Notes: The solid lines show the spectra for the Real Business Cycle Model (panels A-C), the Sticky-Price Model (panels B-C), and the Financial Market Friction Model (panels A-B). The dashed line denotes the spectra for the growth rate of productivity in the Sticky-Price Model.

Table 5.1
Correlation Matrix of Business-Cycle Components of Variables
(Standard Deviation Shown on Diagonal)

A. Real Business Cycle Model

y	c	i	n	w	M	P	M-P	R	r	a	v
2.74	0.50	0.99	0.98	0.91	0.64	-0.32	0.65	0.58	0.82	0.98	0.25
	0.51	0.40	0.35	0.82	0.40	-0.27	0.44	0.36	0.13	0.46	-0.04
		8.74	1.00	0.85	0.62	-0.31	0.63	0.57	0.85	0.98	0.27
			2.02	0.82	0.61	-0.30	0.62	0.56	0.87	0.98	0.28
				0.83	0.62	-0.34	0.64	0.56	0.60	0.87	0.14
					1.92	-0.22	0.88	0.26	0.51	0.61	-0.35
						1.25	-0.67	-0.17	-0.38	-0.38	0.47
							2.51	0.28	0.58	0.65	-0.50
								1.05	0.32	0.51	-0.08
									0.43	0.89	0.23
										1.60	0.26
											2.03

B. Sticky-Price Model

y	c	i	n	w	M	P	M-P	R	r	a	v
1.56	0.99	0.98	0.60	0.98	0.42	-0.16	0.42	-0.31	-0.45	0.74	0.52
	1.48	0.94	0.60	0.99	0.44	-0.14	0.43	-0.37	-0.51	0.72	0.54
		1.82	0.57	0.93	0.37	-0.19	0.38	-0.18	-0.33	0.74	0.46
			1.20	0.71	-0.01	0.18	-0.05	-0.05	-0.35	-0.09	0.55
				1.67	0.39	-0.09	0.37	-0.34	-0.51	0.62	0.57
					1.92	-0.25	0.96	-0.56	-0.49	0.48	-0.35
						0.58	-0.49	0.12	0.04	-0.37	0.33
							2.14	-0.54	-0.45	0.53	-0.41
								1.24	0.91	-0.30	-0.29
									1.45	-0.22	-0.43
										1.26	0.19
											2.03

C. Financial Market Friction Model

y	c	i	n	w	M	P	M-P	R	r	a
1.64	0.98	0.91	0.51	0.65	0.65	-0.26	0.98	-0.28	-0.41	1.00
	1.68	0.81	0.55	0.63	0.63	-0.30	1.00	-0.44	-0.47	0.97
		1.79	0.37	0.60	0.60	-0.13	0.81	0.11	-0.21	0.92
			0.17	0.65	0.64	0.20	0.55	-0.70	-0.58	0.44
				1.95	1.00	0.55	0.63	-0.32	-0.23	0.61
					1.92	0.55	0.63	-0.31	-0.22	0.61
						1.56	-0.30	0.09	0.23	-0.29
							1.68	-0.44	-0.47	0.97
								1.68	0.59	-0.22
									1.91	-0.35
										1.60

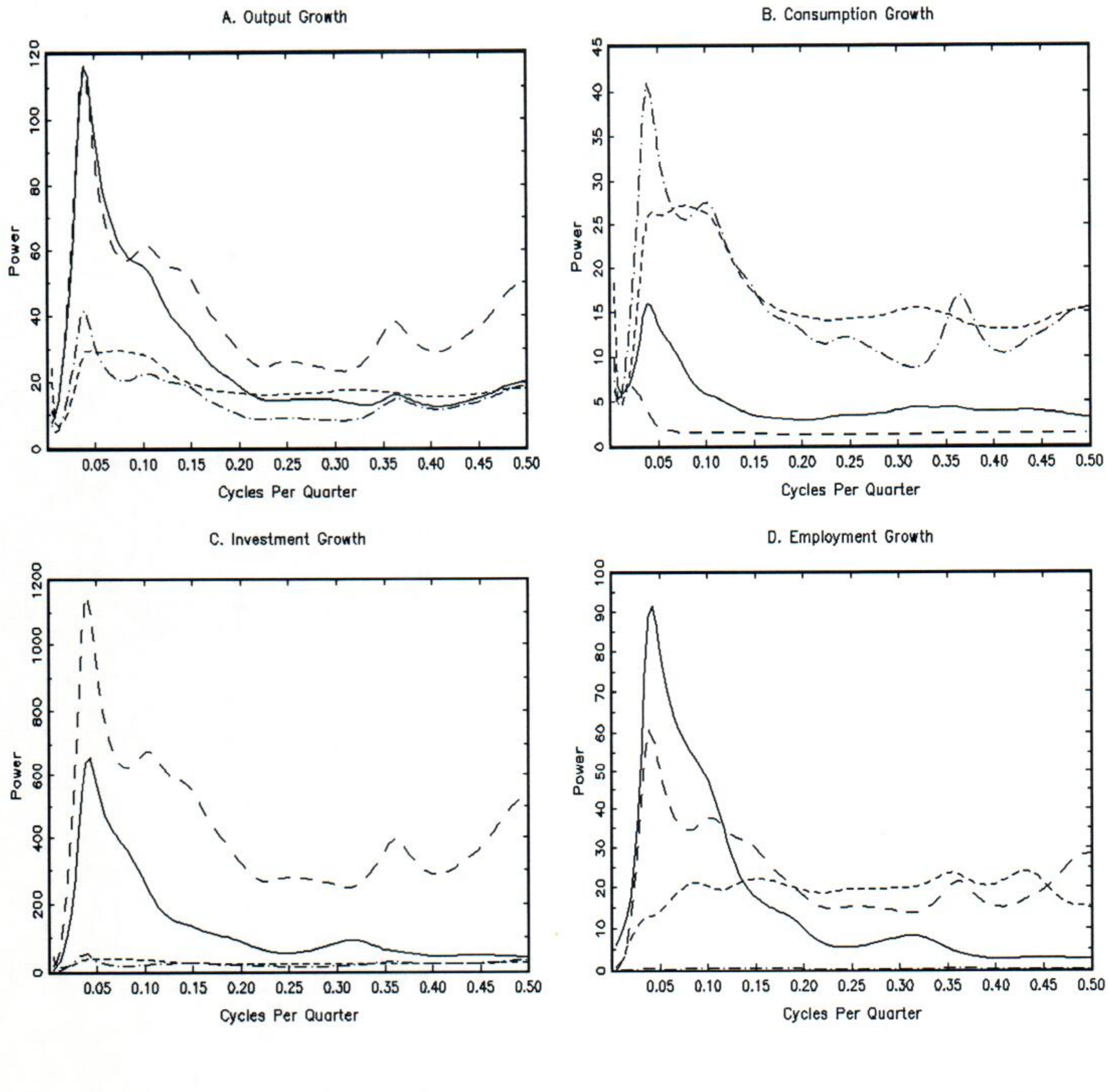
Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table 5.2 Summary of Cross-Correlations

	$\sigma_{y,bc}$	Correlations of y_{t+k} with: y_t	M_t			P_t			r_t			R_t		
		±4	-4	0	4	-4	0	4	-4	0	4	-4	0	4
Data	2.72	0.25	0.15	0.62	0.33	0.05	-0.35	-0.66	0.19	-0.27	-0.41	0.58	0.30	-0.61
A. Baseline Models														
RBC	2.74	0.23	0.27	0.64	0.18	-0.08	-0.32	-0.46	-0.23	0.82	0.61	0.25	0.58	-0.13
SP	1.56	0.10	0.38	0.42	-0.21	0.18	-0.16	-0.40	-0.53	-0.45	0.46	-0.57	-0.31	0.39
LE	1.64	0.23	0.31	0.65	0.13	0.07	-0.26	0.02	-0.51	-0.41	0.50	-0.39	-0.28	0.48
B. Other Models														
RBC-I	2.74	0.23	0.00	0.00	0.00	0.04	-0.45	-0.32	-0.23	0.82	0.61	0.34	0.64	-0.12
RBC, η=1	1.59	0.24	0.32	0.62	0.10	-0.37	-0.48	-0.06	-0.56	-0.31	0.55	-0.45	-0.62	0.18
RBC, m_R=-.10	2.74	0.23	0.27	0.64	0.18	-0.52	-0.56	0.21	-0.23	0.82	0.61	0.56	0.15	-0.68
SP-I	1.98	0.13	0.50	0.42	-0.20	0.60	0.30	-0.41	-0.41	-0.34	0.36	-0.40	0.23	0.43
SP, η^{-1}=0	3.11	0.02	0.22	0.52	0.04	0.25	-0.07	-0.55	-0.34	0.89	0.22	-0.22	0.85	0.04
SP-M	1.31	0.24	0.76	0.64	-0.30	0.78	0.55	-0.34	-0.59	-0.13	0.46	-0.51	0.38	0.56
SP, m_R=0	2.39	0.05	-0.00	0.51	0.28	0.56	0.17	-0.71	-0.59	-0.20	0.56	-0.57	0.00	0.56
SP-M, m_R=0	1.31	0.27	0.44	0.97	0.19	0.65	0.73	0.01	-0.57	-0.11	0.65	-0.57	0.05	0.72
SP, m_R=-.10	1.44	0.10	0.32	-0.21	-0.39	0.03	0.30	0.13	-0.43	-0.64	0.23	-0.53	-0.53	0.21
LE-I	1.58	0.24	-0.00	0.06	0.03	-0.16	-0.60	-0.12	-0.53	-0.31	0.53	-0.03	-0.06	0.02
LE-M	0.12	0.18	-0.02	0.76	0.45	-0.11	0.67	0.52	0.32	-0.42	-0.08	-0.42	-0.71	0.22

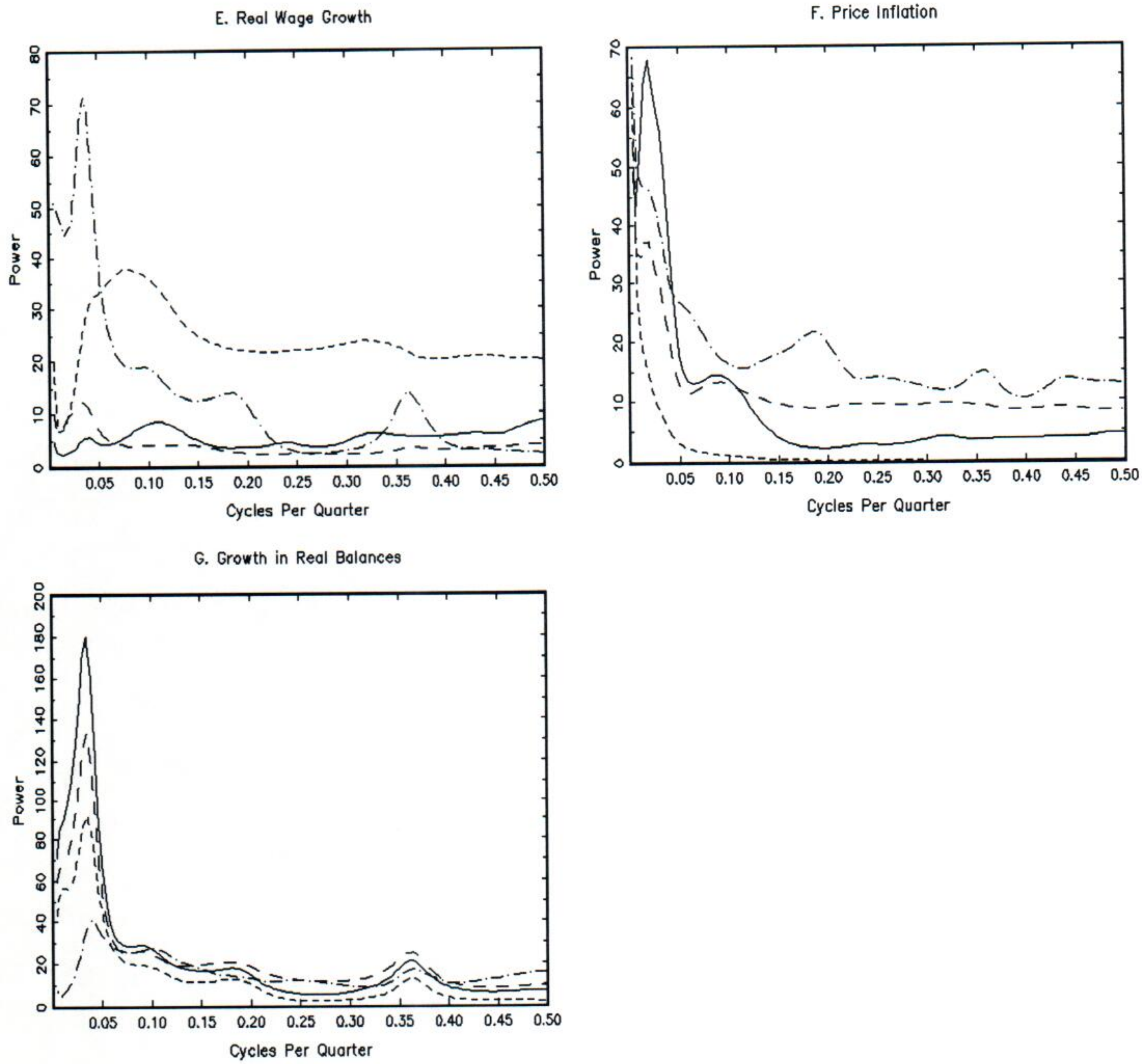
Notes: The column labeled $\sigma_{y,bc}$ shows the standard deviation of the business cycle component of output (in perentage points). RBC, SP and LE refer to the real business cycle model, sticky-price, and liquidity effect models, respectively. The suffix "-I" indicates the model solved under the assumption that the driving processes are independent. The suffix "-M" indicates the model solved using money as the only driving process. The paramter η^{-1} is the investment adjustment cost parameter, which is equal to 0 in the baseline RBC model and 1 in the other baseline models. The paramter m_R is the interest rate semi-elasticity of money demand; it is equal to -.01 in the baseline RBC and SP models.

Figure 5.2
Spectra of Data and Models



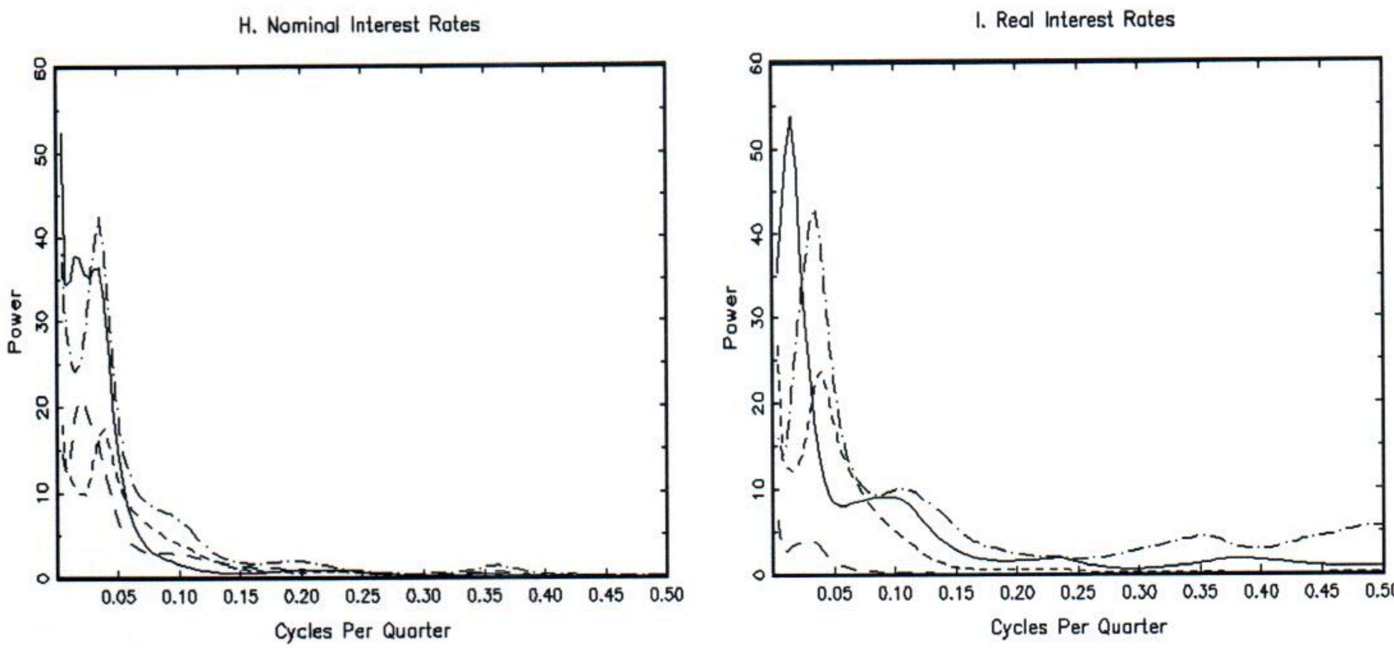
Notes: Each panel shows the spectra of postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

Figure 5.2 (Continued)
Spectra of Data and Models



Notes: Each panel shows the spectra of postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

Figure 5.2 (Continued)
Spectra of Data and Models



Notes: Each panel shows the spectra of postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

frequencies; we focus our discussion on the business cycle frequencies, periodicities between 6 and 32 quarters.

Tables 5.1 and 5.2 present the information on the levels of the economic variables, once these have been passed through the business cycle filter that eliminates trend and irregular components, as described in Section 2. Table 5.1 shows the correlation matrix of the business cycle filtered versions of the variables; Table 5.2 shows selected cross-correlations of the filtered series with the filtered values of output (contemporaneously and at a one year lead and lag). We view these correlations as describing the comovement of individual series with the business cycle, since we take output as our reference measure of the business cycle.

Before describing the results for the specific models, we highlight five low-frequency features of the model spectra. First, since each of these models exhibits long-run balanced growth, the height of the spectra at frequency zero for Δy, Δc, and Δi are equal. Second, since long-run growth arises from movements in productivity, the low frequency behavior of Δy, Δc, and Δi is closely related to the low frequency behavior of the driving process for Δa_t (the long run effect of a permanent technology shock on output is $1/\alpha$, where α is the elasticity of output with respect to labor input; hence the height of the spectrum for output is about 3 times that of technology given our assumption that $\alpha = .6$ and the fact that the spectrum is a variance). Third, per-capita employee hours, n_t, is stationary in each of the models, and hence has no spectral mass at frequency zero. Fourth, the velocity of money $(y - M + P)$ is stationary in each of the models, so that long-run movements in real balances match those of y, which in turn implies that over low frequencies the spectra of $\Delta(M - P)$ and Δy coincide. Finally, stationary velocity together with the long-run neutrality of money in each of the models, implies that at frequency zero, the spectrum of ΔP is equal to the sum of the spectrum of ΔM and the spectrum of Δy. We will now discuss each of the models in turn.

5.2.1 Variability in the RBC Cycle Model with Endogenous Money

The RBC model — using the standard parameterization employed here and the driving processes described above — produces output, consumption, and investment that behave much like the data in terms of their spectra. This finding is strikingly different from that of Watson (1993) and is traceable to a simple difference from the assumptions of that paper: we assume that the driving process for technology displays substantial mean reversion, rather than being a random walk. As discussed in the last section, the implied dynamics of output variables closely matches the assumed process for productivity, and thus the shape of the spectra for these models reflects the assumed spectra of the input process for a_t.

Thus, the spectra for Δy, Δc, and Δi are quite similar to the spectra estimated from the data as shown in panels A,B, and C of Table 5.2. The only notable difference between the model and data spectra for y, c, and i is that consumption is less and investment is more cyclically volatile in the model than in the data, a finding which is reflected in both the height of the spectra and in the standard deviations reported in Table 5.1. This outcome reflects the fact that the "permanent income" determination of consumption implies substantial smoothing in the face of mean reversion in productivity.[20] Interestingly, panel E shows that employment is somewhat less volatile in the model than in the data, but less markedly so than in Watson's (1993) study. (From panel A of Table 5.1, the cyclical standard deviation of employment is 2.02% in the model, and from panel A of Table 2.2, the standard deviation of the data is 2.42%.). Again this result is traceable to a key feature of the real business cycle model: there is substantial intertemporal substitution in labor input when there is substantial mean reversion in productivity. Overall, this neoclassical model of consumption, investment and income determination works well, at least in terms of the characteristics shown here. It does somewhat less well for real wages, where there are larger differences between the data and model spectra; however, the differences for the other models are much larger. The RBC model seriously underpredicts the variance of the real interest rate: the standard deviation for the real rate in the model is 0.43 and the corresponding value for the data is 1.46. Finally, panel G shows that the spectrum of real balances in the RBC model is close to estimates from the data and shares the "typical spectral shape" of the growth rates of other real variables.

The RBC model also has volatility implications for nominal variables, and some of these are at substantial variance with the empirical estimates. Panel F shows that the real business cycle model displays too little price volatility, although there is a peak in the spectrum at the business cycle frequencies. (In terms of the standard deviations in Table 2.2 and 5.1, the standard deviation of the business cycle component of the price level is 1.57% in the data, but it is only 1.25% in the model). Further, the real business cycle model implies too little volatility in the nominal interest rate, as indicated by panel H of Figure 2.

5.2.2 Comovement in the RBC Model with Endogenous Money

The cyclical covariability of key real and nominal variables is summarized in Table 5.2. This table shows the cyclical cross correlation between output and money, prices, and nominal and real interest rates both contemporaneously and at a lead and lag of 4 quarters. Panel A of the table summarizes the results for the data and then for our baseline parameterization of the RBC, sticky-price and financial market friction models. Panel B shows results for various modifications of the baseline models that we have produced to help understand how the results depend on our assumptions about driving processes and model parameters.[21]

[20]However, this "defect" could be easily remedied by allowing small adjustment costs in investment. The relative variability of investment and consumption are quite close to the data when the model is solved with $\eta^{-1} = .05$.

[21]More detailed results are presented in Appendix D for all of the models in Table 5.2.

We will use Table 5.2 repeatedly in the following manner. First, we compare the first row of the table (the data) with results from each of the models. Thus comparing the first and second rows of the table shows that the RBC model closely captures the cyclical behavior of *money* evident in the data. Of course, since money is neutral in this model, all of the covariability between money and output arises from the assumed correlation of the input processes for a_t and M_t. Here, the close match between the data and model arises from two related features. First, as we stressed above, y_t is highly correlated with a_t in the RBC model. Second, y_t^d is highly correlated with a_t^d in the data. Thus, since the model's correlation between a_t and M_t matches the data, the same is expected for y_t and M_t. The cross correlations for the other variables are less prone to match those in the data by construction. Money is also a leading indicator for output: $cor(M_t, y_{t+4}) = 0.18$, but somewhat less so than in the data, where $cor(M_t, y_{t+4}) = 0.33$. The first row of panel B shows the results from solving the model with independent driving processes. That is, in the model, each of the driving processes has the same autocovariances/spectrum as in the benchmark model, but all cross-autocovariance/cross spectra are set to zero. Here, since money and productivity are uncorrelated, so are money and output.

The *price level* in the model is countercyclical ($cor(P_t, y_t) = -.35$ in the data and $cor(P_t, y_t) = -.32$ in the model). The RBC model also predicts that prices should be an inverted leading indicator for output ($cor(P_t, y_{t+4}) = -.46$) but not as strongly as in the data ($cor(P_t, y_{t+4}) = -.66$). Interestingly, the countercyclical nature of prices obtains in this model in spite of the strong positive feedback from output to money (more precisely, from a_t to M_t). Indeed, when the model is solved is solved using the same univariate processes for the driving variables, but assuming no feedback (the first row of panel B of Table 5.1), the correlation between prices and output is negative: monetary changes are partly accommodating productivity changes in the model, so that the price level is less strongly countercyclical when output changes.

Nominal interest rates in the model show much the same cyclical lead-lag relation as the data, albeit with smaller correlations. However, this isn't true of the *real rate of interest.* In the data, the real interest rate is negatively correlated with contemporaneous values of output and even more highly negatively correlated with output four quarters hence. In the model, r_t is highly positively correlated with y_t and y_{t+4}. This result obtains in the model because output is driven by persistent changes in productivity. Positive productivity disturbances lead to expected growth in consumption, associated increases in real interest rates, and higher current and future output. As shown in row 2 of panel B of the table, this procyclicality of real rates depends on the assumed investment adjustment cost parameter: with large investment adjustment costs, real rates become negatively correlated with output.[22] However, this large value of the adjustment cost parameter also eliminates the cyclical variability of labor input in the real business cycle model: large investment adjustment costs make

[22]By large investment adjustment costs, we mean that we use the same parameter values that are employed in the sticky price model. This involves changing η^{-1} from 0 to 1.

it less desirable for agents to intertemporally substitute labor input.

5.2.3 Variability in the Sticky Price Model

There are several noteworthy aspects of the sticky-price model in terms of its implications for business cycle variability. To begin, from panel A of Figure 5.2 and panel C of Table 5.1, output in the model is less variable than in the data or in the RBC model. There are two reasons for this. First, the assumed input for a_t is less variable in the sticky-price model (see Panel A of Figure 5.1) than in the RBC model: this is an essential feature of the underlying monopolistic competition model.[23]Second, the assumed level of investment adjustment costs is higher in this model than in the RBC model ($\eta = 1$ in the sticky price model and $\eta^{-1} = 0$ in the RBC model). In line 5 of panel B of Table 5.2, results are reported for the sticky price model under the zero investment adjustment costs assumption: it produces more volatility in output than is present in the data.

The fact that there are high investment adjustment costs in the sticky price model has implications for the variability of investment and consumption: the spectrum of consumption growth shown in panel B of figure 5.2 indicates that there is much more consumption variability than in the RBC model and a spectral shape that broadly resembles that found in the data. However, there is much less volatility of investment than is found in the data.[24]

The price level in this model is very smooth (Table 5.1 shows that the standard deviation in the model is .58 and it is 1.57 in the data). In terms of the power spectrum, the model does not deliver a hump at the business cycle frequencies: there is simply great power at very low frequencies. Potentially, these two features may indicate that there is "too much price stickiness" present in this economy, but we have not experimented with the sensitivity of the shape of the spectra to the chosen value of the price adjustment parameter.

Finally, the sticky price model predicts much more volatility in real interest rates than does the RBC model and, in fact, virtually exactly the amount that is present in the data (the standard deviation of the real interest rate in Table 2.2 is 1.46 and it is 1.45 in the data). This is due to a combination of two features: the effects of nominal shocks on the real rate and the presence of investment adjustment costs. We

[23]We have seen above that monopolistic competition implies that $a_t = y_t - \mu s_n n_t - \mu s_k k_t$ rather than $a_t = y_t - s_n n_t - s_k k_t$ as in the RBC model. Since n moves roughly one-for-one with output (as seen in section 2 above), this makes a much less volatile.

[24]The RBC model has similar spectra for y, c, and i when solved using $\eta = 1$. However, in the RBC model with high investment adjustment costs, employment variability is very low; yet in the sticky-price model, it remains high (see panel D of figure 5.2). Roughly, this occurs because in the sticky-price model employment must be used to produce the changes in output if a demand shock occurs and must move inversely with productivity shocks so that a given demand-determined level of output is produced. These required changes in employment also lead to changes in real wages as is evident in panel E of figure 5.2.

will return later to discussing some aspects of the effect of investment adjustment costs.

5.2.4 Comovement in the Sticky Price Model

Turning to the cyclical covariance properties of the model, several additional and surprising results stand out. First, there is important contemporaneous correlation between *money* and output, although not as much as is present in the data ($cor(M_t, y_t)$ = .42 in the model and $cor(M_t, y_{t+4})$ = .62 in the data). However, money is negatively related to future values of output. This result is not the result of feedback in the driving process; it continues to obtain when independent driving processes are used (row 4 of panel B) or when the model is solved using money as the only driving process (row 5). Instead, this negative correlation arises from two aspects of the model: (i) mean reversion in the money process; and (ii) the positive relation of nominal interest rates and aggregate demand associated with interest elastic money demand. To see why these aspects of the model are important, note that when M_t is high, then mean reversion implies that it is expected to decline. This, together with sticky prices, leads to declines in expected future prices, interest rates and output. When $m_R = 0$ so that money demand is not interest elastic, the link between nominal interest rates and aggregate demand is broken and the negative correlation between M_t and y_{t+4} disappears (rows 7 and 8).

The cyclical behavior of *price level* in this model also differs from what one might expect: the price level is negatively correlated with current output and it is even more strongly negatively correlated with future output (the magnitude of these correlations is somewhat smaller than those in the data). Finally, there is a small positive correlation with lagged output, rather than the large one that one might guess would describe a model with sticky prices. These surprising results are traceable to two features of the model that we construct. First, if we make money independent of productivity or if we make it the only driving process, then a positive correlation emerges (see line 4 of panel B of Table 5.2 for the independent process assumption and lines 6 and 8 for money as the only shock).[25] These modifications also typically introduce a very large, positive correlation between lagged output and the price level. Second, the forward-looking nature of money demand plays a crucial role in governing whether the model predicts that the price level will be an inverted leading indicator.

[25]These experiments thus shed light on one sticky price model's answer to a conjecture raised by Ball and Mankiw [1994]. These authors argued that high-pass filtering (of the specific sort undertaken with the Hodrick-Prescott filter) gives rise to a tendency for output and the price level to be negatively related in sticky price models that are driven entirely by demand shocks. The band-pass filters that we employ might well be subject to the same criticism, since these are closely related to the HP filter. However, the price level is positively related to output in all of the "money shock only" models that we study in panel B of table 5.2. For our models, the correct interpretation of the negative correlation is that there are indeed productivity shocks that are a major source of business cycles.

If we assume that $m_R = 0$ and money shocks only, as in line 8 of panel B of Table 5.2, then there is no correlation.

The *nominal interest rate* in this sticky price model is a positive leading indicator of output and a negative lagging indicator; in the data the opposite occurs. Further, in the entire battery of modifications of the sticky price model that we study in panel B of Table 5.2, there is no modification that makes the nominal interest rate an inverted leading indicator.

The *real interest rate* in this sticky price model is negatively related to output. This reflects a feature of the sticky price model discussed above: investment adjustment costs induce diminished ability to substitute over time: with high adjustment costs, even the RBC model implies that the real interest rate should be negatively correlated with output (see line 2 of panel B of Table 5.2). However, it is again the case that neither the basic model (or any of the modifications that we study) makes the real interest rate an inverted leading indicator for output. In the data, we find that $cor(r_t, y_{t+4}) = -.41$ and in the basic sticky price model $cor(r_t, y_{t+4}) = .46$.

5.2.5 Variability in the Liquidity Effect Model

The baseline results for the financial frictions model are most notable for what they *don't* say about the relation between financial market frictions and the business cycle. That is, when looking at the real variables, y, c, i, n, and r, the results for the baseline liquidity effect (LE) model are very close to what one obtains from the RBC model with the same investment cost parameter ($\eta = 1$). The reason is that there are very small "multipliers" attached to the effect of nominal money on real economic activity, despite the presence of liquidity effects, so that the spectra are essentially those of the RBC model (i.e., are produced by productivity shocks). The only real variable with different behavior in the RBC ($\eta = 1$) and LE models is that the wage rate w, which is more variable in the LE model. Its' increased variability can be traced to the variability in nominal interest rates, which affects labor demand in this model, as discussed in Section 3.

5.2.6 Comovement in the Liquidity Effect Model

The comovement of real activity and nominal variables stems from a surprising source in the liquidity effect model. Because the causal role of monetary shifts on output is small (money is close to neutral in our parameterization of the model), essentially all of the correlation between money and output arises from the assumed correlation of money and productivity. To see this, note that the baseline version of the model does capture the cyclical correlation of *money* and output: $cor(M_t y_t) = .65$ in Panel A of Table 5.2. However, this correlation falls to .06 when the money and productivity processes are assumed to be independent (row 10 of panel B of Table 5.2). Moreover, when the model is solved using money as the only driving process, the cyclical standard deviation of output falls from 1.6 to 0.1 (in row 11).

The cyclical behavior of the *price level* is also very reminiscent of that found in the real business cycle model. P is negatively correlated with the y contemporaneously, as in the data. However, there is not a quantitatively important negative leading indicator relationship predicted by the LE model, in contrast to the RBC model.

However, the financial market frictions model does not inherit the problems that the RBC model has in capturing the contemporaneous relationship between *real interest rates* and output, for two reasons. First and most important, like the sticky price model, our LE model builds in high investment adjustment costs. Secondly, the real interest rate and output are negatively associated for the small part of output that is attributable to the non-neutral effects of monetary shocks (see row 11 of panel B). However, the LE model does not produce a real interest rate that is an inverted leading indicator: it implies that $cor(r_t, y_{t+4}) = .50$, while in the data $cor(r_t, y_{t+4}) = -.41$.

Moreover, the LE model produces an altered pattern of correlations of *nominal interest rates* with output, that eliminates the success that the RBC model had in matching these correlations. It implies that the nominal rate should be negatively associated with output contemporaneously and should be a positive leading indicator; the data display a positive contemporaneous association and an inverted leading indicator role for the nominal rate.[26]

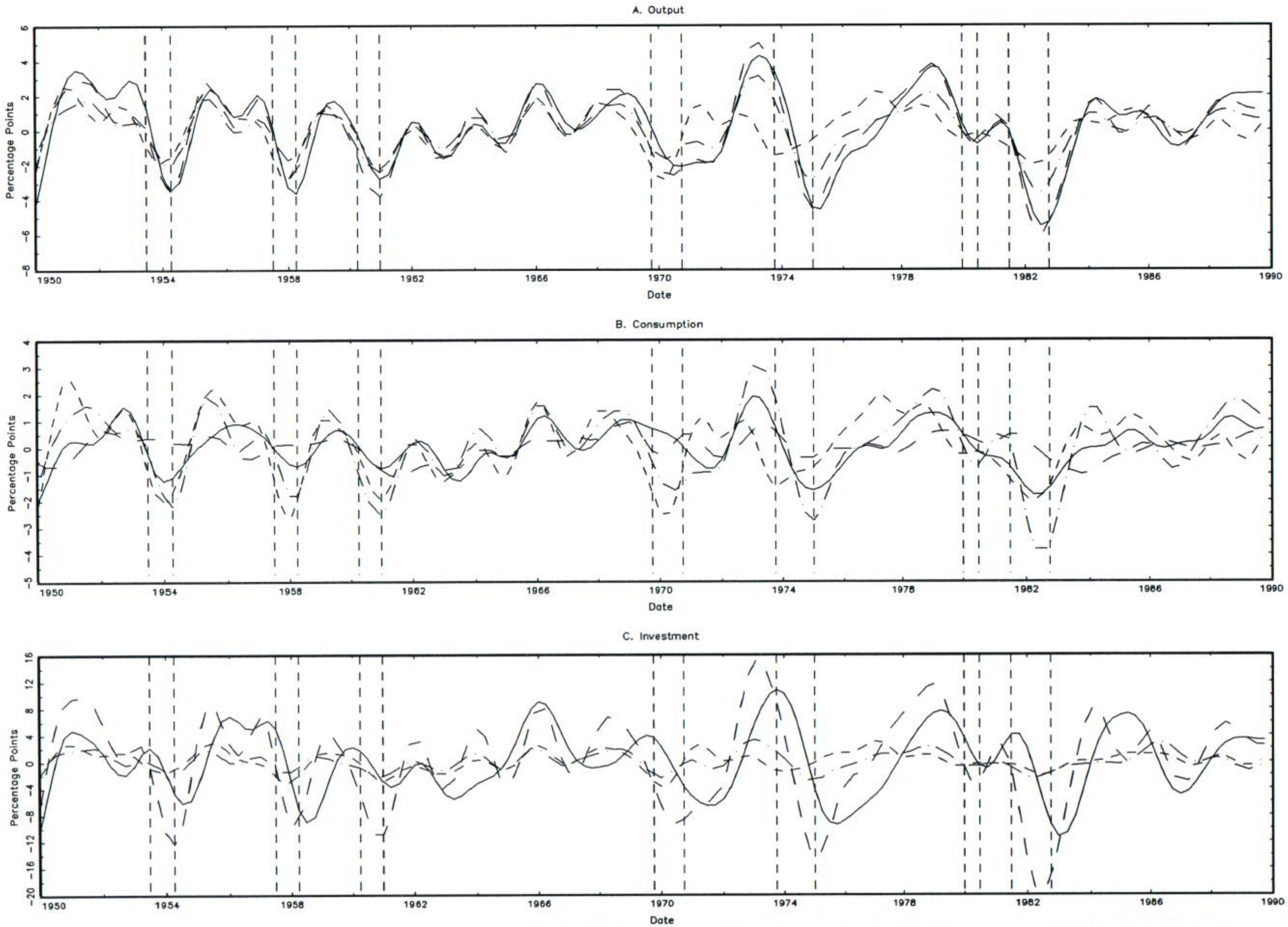
5.3 Explaining Postwar Business Cycles

Figure 5.3 shows fitted values from each of the three models for the postwar U.S. data. These were obtained by solving the models using the data's VAR residuals for ξ_t in equation (16). The resulting fittted values were then filtered to highlight their business cycle components using the same bandpass filter used to produce Figure 2.2. Figure 5.4 shows the implied values of the fitted driving processes for the models.

Figure 5.3 reinforces many of the conclusions reached above. First, the fitted values for the RBC model closely match the data; this is less true for the other two models, where the fitted values are less variable than the data. The same result was evident from the spectrum. Similarly evident from the spectrum, investment is too variable in the RBC model and too smooth in the other models. Interestingly, the fitted values for prices also match the data more closely in the RBC model than in the other two models. Prices from the sticky price model are too smooth over the business cycle; prices in the liquidity effect model have the right overall variability, but

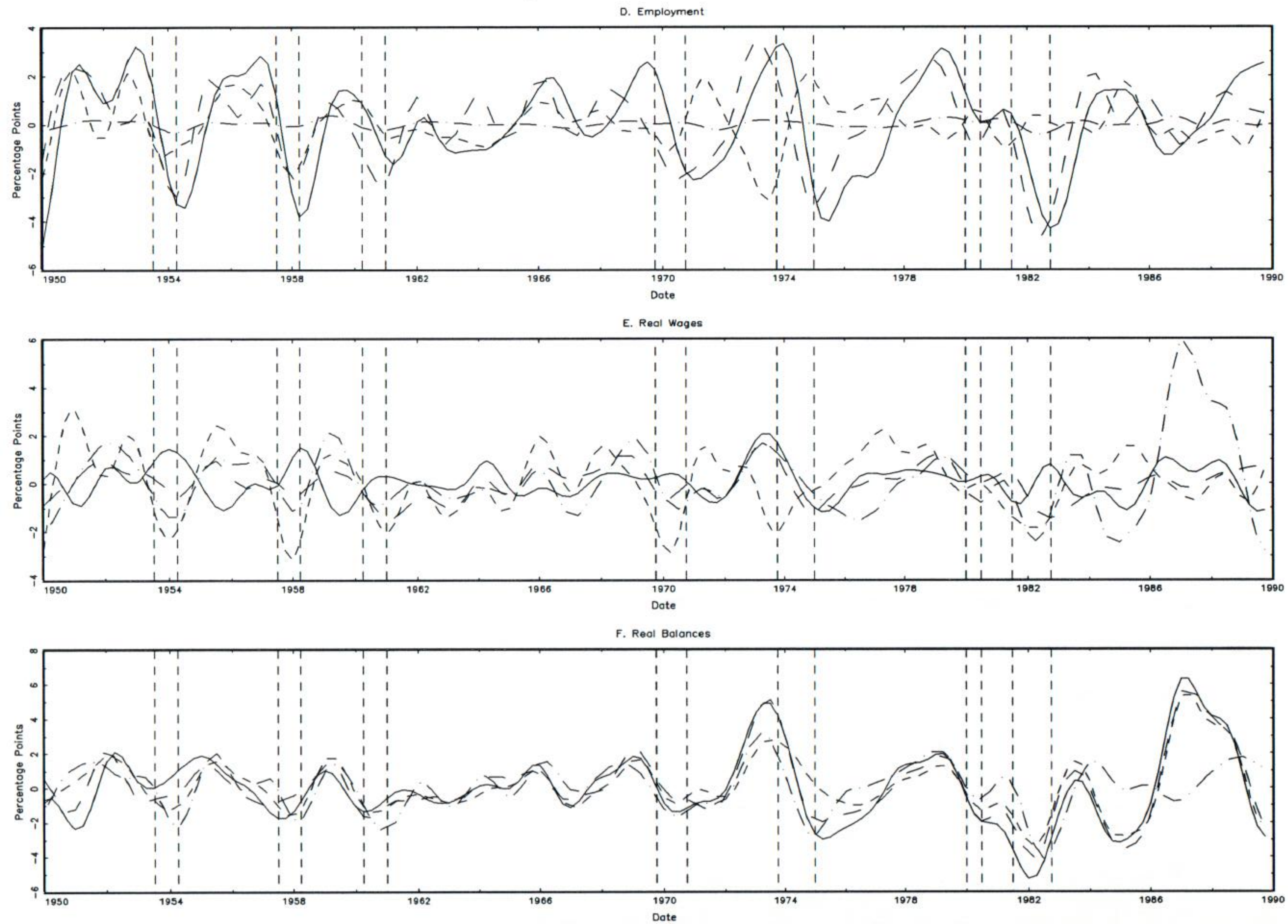
[26]It is perhaps useful to note that the model with just monetary shocks (line 11 of panel B of Table 5.2) does capture some of the lead-lag relations evident in the data, even though the size of the real multipliers on money in the model are very small. For example, when the model is solved using money as the only driving process, *real interest rates* are countercyclical and do lead output somewhat ($cor(r_t y_t) = -.42$ and $cor(r_t y_{t+1}) = -.48$). Similarly, money is procyclical and slightly leading ($cor(M_t y_t) = .76$ and $cor(M_t y_{t+1}) = .78$). However, this version of the model predicts that there should be a pattern of correlations between *nominal interest rates* and output that is very different from that found in the data.

Figure 5.3
Business Cycle Components of Data and Models



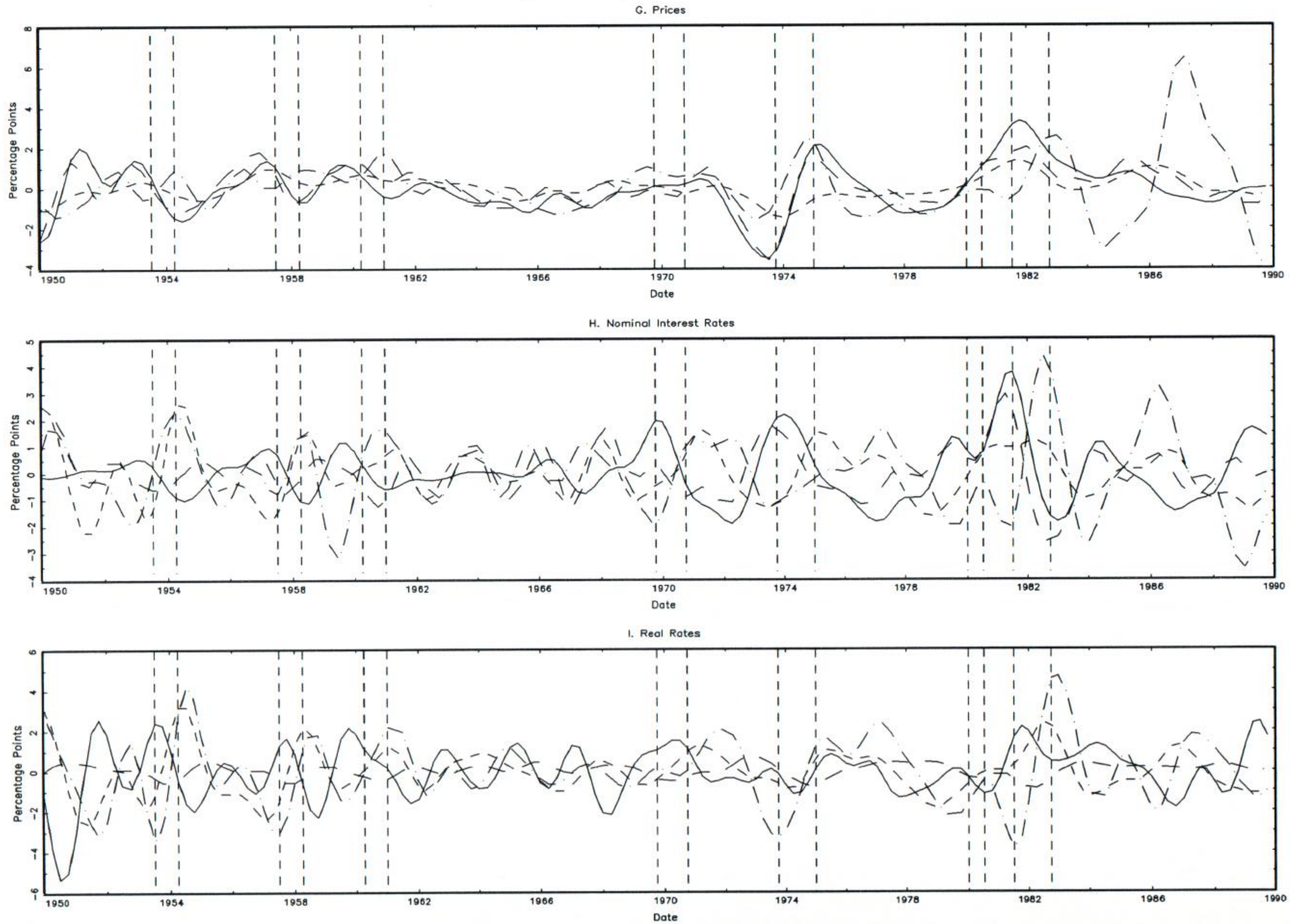
Notes: Each panel shows the 6-32 quarter bandpass filtered series for postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

Figure 5.3 (Continued)
Business Cycle Components of Data and Models



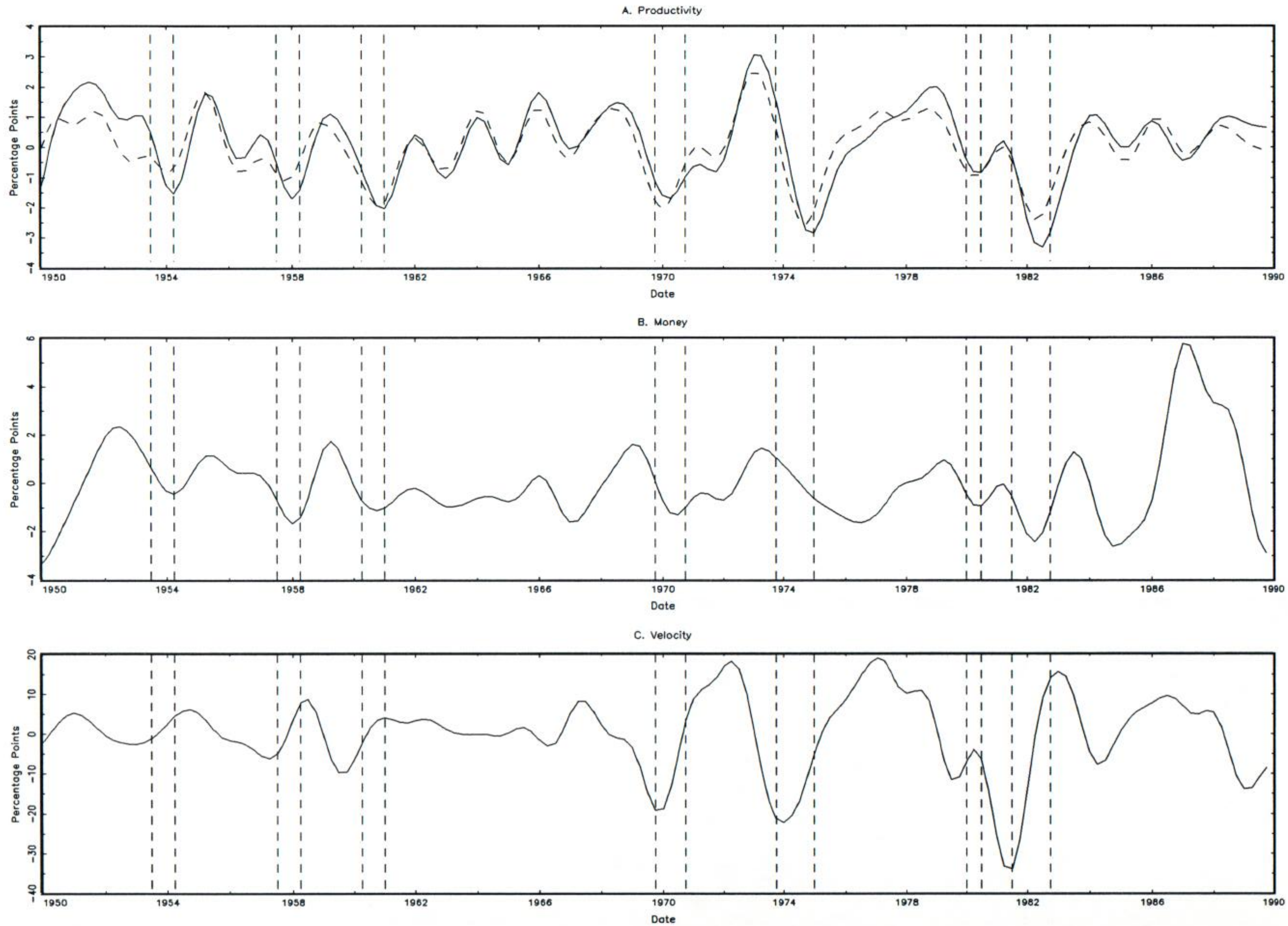
Notes: Each panel shows the 6-32 quarter bandpass filtered series for postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

Figure 5.3 (Continued)
Business Cycle Components of Data and Models



Notes: Each panel shows the 6-32 quarter bandpass filtered series for postwar quarterly U.S. data (solid line), the Real Business Cycle Model (long dashes), the Sticky-Price Model (short dashes), and the Financial Market Frictions Model (dash-dots). Driving processes for the models are described in the text.

Figure 5.4
Business Cycle Components of Driving Processes



Notes: The solid lines show the 6-32 quarter bandpass filtered driving variables for the Real Business Cycle Model (panels A-C), the Sticky-Price Model (panels B-C), and the Financial Market Friction Model (panels A-B). The dashed line denotes the filtered productivity series in the Sticky-Price Model.

the correlation with the data is not as high as in the RBC model. In particular, the liquidity effect model predicts large movements in prices in the late 1980's associated with the large increases in money. This doesn't occur in the RBC model: real balances increase in the late 1980's associated with lower than average nominal interest rates. Finally, all of the models do very poorly matching both real and nominal interest rates. There is little relation between the data and the fitted values of interest rates from any of the models.

6 Conclusions

We have explored the implications of three protype macroeconomic models of the relationship between money, price, interest rates and the business cycle: a real business cycle model with endogenous money, a model with real effects of money arising from sticky prices and a model with real effects of money arising from financial market frictions. While these models have some disparate success at matching aspects of the correlation of nominal variables with real output, they also have some common failings: all of the models simply do a poor job at matching the interaction of real and nominal interest rates with real activity. By documenting the diverse successes and failures of these models, our work indicates that new models–which incorporate new mechanisms or combine aspects of existing mechanisms—will be necessary to explain the main empirical linkages between money, prices, interest rates and the business cycle.

References

[1] Barro, R.J. and H.I. Grossman (1976), *Money, Employment and Inflation*, Cambrige: Cambridge University Press.

[2] Baxter, M. and M.J. Crucini (1993), "Explaining Saving-Investment Correlations," *American Economic Review*, Vol. 83, No. 3, pp. 416-36.

[3] Baxter, M. and R. King (1995), "Approximate Bandpass Filters for Economic Time Series," manuscript, the University of Virginia.

[4] Beveridge, S. and C. Nelson (1980), "A New Approach to Decomposition of Time Series into Permanent and Transitory Components with Particular Attention to the Measurement of the 'Business Cycle'," *Journal of Monetary Economics*, 7, 151-77.

[5] Blanchard, O.J. and C. Kahn (1980), "The Solution of Linear Difference Models Under Rational Expectations," *Econometrica*, vol. 48, pp. 1305-12.

[6] Burns A. and W. Mitchell (1947), *Measuring Business Cycles*, National Bureau of Economic Research: New York.

[7] Calvo, G.A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12: 383-98.

[8] Chirinko, Robert S. (1993), "Business Fixed Investment Spending: A Critical Survey of Modeling Strategies, Empirical Results, and Policy Implications," *Journal of Economic Literature*, Vol. XXXI.

[9] Christiano, L.J. (1988), "Why Does Inventory Investment Fluctuate So Much," *Journal of Monetary Economics*, 21, 247-280.

[10] Christiano, L.J. (1991), "Modeling the Liquidity Effect of a Monetary Shock," *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter, 3-34.

[11] Christiano, Lawrence.J. and M. Eichenbaum (1992), "Liquidity Effects and the Monetary Transmission Mechanism," *American Economic Review*, Vol. 82, 346-53.

[12] Christiano, Lawrence.J. and M. Eichenbaum (1993), "Liquidity Effects, Monetary Policy and the Business Cycle, " *Journal of Money, Banking and Credit*, forthcoming.

[13] Cochrane, John H. (1994), "Permanent and Transitory Components of GNP and Stock Prices," *Quarterly Journal of Economics*, Vol. CIX, February, pp. 241-66.

[14] Fischer, S., "Long-Term Contracts, Rational Expectations and the Optimal Money Supply Rule," *Journal of Political Economy*, vol. 85, pp. 191-205.

[15] Fama, E.F. (1992), "Transitory Variation in Investment and Output," *Journal of Monetary Economics*, Vol. 30, pp. 467-80.

[16] Fuerst, Timothy (1992), "Liquidity, Loanable Funds, and Real Activity," *Journal of Monetary Economics*, Vol. 29, pp. 3-24.

[17] Granger, C.W.J. (1966), "The Typical Spectral Shape of an Economic Variable," *Econometrica*, Vol. 34, pp. 150-61.

[18] Gray, J. (1978), "On Indexation and Contract Length," *Journal of Political Economy*, vol. 86, pp. 1-18.

[19] *Handbook of Cyclical Indicators*, U.S. Department of Commerce, 1984, U.S. Government Printing Office.

[20] King, R.G. (1995), "Money and Business Cycles," *Journal of Monetary Economics*, forthcoming.

[21] King, R.G. and C.I. Plosser (1984), "Money, Credit and Prices in a Real Business Cycle," *American Economic Review*.

[22] King, R.G., C.I. Plosser and S.T. Rebelo (1988a), "Production, Growth, and Business Cycles: I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21, pp. 195-232.

[23] King, R.G., C.I. Plosser and S.T. Rebelo (1988b), "Production, Growth, and Business Cycles: II. New Directions," *Journal of Monetary Economics*, 21, pp. 309-342.

[24] King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review*, LXXXI, pp. 819-40.

[25] King, R.G. and M.W. Watson (1995a), "The Solution of Singular Linear Difference Equations Under Rational Expectations," manuscript, the University of Virginia.

[26] King, R.G. and M.W. Watson (1995b), "System Reduction and Solution Algorithms for Singular Linear Difference Systems Under Rational Expectations," manuscript, the University of Virginia.

[27] King, R.G. and M.W. Watson (1995c), "Timing Models," manuscript, the University of Virginia.

[28] Kydland, F.E. and E.C. Prescott (1982), "Time To Build and Aggregate Fluctuations," *Econometrica*, 50, pp. 1345-1370.

[29] Kydland, F.E. and E.C. Prescott (1990), "Business Cycles: Real Facts and a Monetary Myth," *Quarterly Review*, Federal Reserve Bank of Minneapolis, Spring, pp. 3-18.

[30] Long, John and Charles Plooser, (1983), "Real Business Cycles," *Journal of Political Economy,* 91, 39-69.

[31] Lucas, R.E. (1988), "Money Demand in the United States: A Quantitative Review," *Carnegie-Rochester Conference Series on Public Policy*, 29, 137-168.

[32] Lucas, R.E. (1990), "Liquidity and Interest Rates," *Journal of Economic Theory*, 50, 237-64.

[33] Mankiw, N.G. (1989), "Real Business Cycles: A New Keynesian Perspective," *Journal of Economic Perspectives*, 3, 79-90.

[34] Moore, G.H. (1991), "Long Leading Indicators," Center for International Business Cycle Research, Columbia Business School.

[35] Phelps, E. and J.B. Taylor, "Stabilizing Powers of Monetary Policy Under Rational Expectations," *Journal of Political Economy*, 85, no. 1 (February 1977), pp. 163-190.

[36] Prescott, E. C. (1986), "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25 (Autumn), 11-44.

[37] Rotemberg, J. (1982a), "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49: 517-31.

[38] Rotemberg, J. (1982b), "Sticky Prices in the United States," *Journal of Political Economy*, 90: 1187-1211.

[39] Rotemberg, J. (1987), "The New Keynsian Microfoundations," *NBER Macro Annual*, MIT Press: Cambridge, Ma.

[40] Rotemberg, J. and M. Woodford (1994), "Is the Business Cycle a Necessary Consequence of Stochastic Growth?" MIT working paper 3653-94-EFA, February.

[41] Soderlind, P. (1994), "Cyclical Properties of a Real Business Cycle Model," *Journal of Applied Econometrics*, Vol. 9: S113-144.

[42] Stock, J. and M. Watson (1993), "A Simple Estimator of Cointegrating Vectors in Higher Order Cointegrated Systems," *Econometrica*,61, 783-820.

[43] Taylor, J.B., "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, vol. 88, pp. 1-23.

[44] Tobin, James (1970), "Money and Income: Post hoc ergo propter hoc?" *Quarterly Journal of Economics*, 84, pp. 301-329.

[45] Watson, Mark W. (1986), "Univariate Detrending with Stochastic Trends," *Journal of Monetary Economics*, Vol. 18, pp. 49-75.

[46] Watson, Mark W. (1993), "Measures of Fit for Calibrated Models," *Journal of Political Economy*, 101, pp 1011-41..

[47] Zarnowitz, Victor (1988), "Corporate Bond Prices as a Leading Indicator," in G. Moore (eds), *A Review of the Leading, Coincident and Lagging Indicators*, Center for International Business Cycle Research, Columbia Business School.

[48] Zarnowtiz, V. and C. Boschan (1975), "Cyclical Indicators: An Evaluation and New Leading Indexes," *Business Conditions Digest*, May.

A Appendix: The Differentiated Products Framework

Our analysis in the main text concerns the behavior of a sticky price economy that takes as a precondition an economy with a large number of differentiated products. In this appendix, we discuss various aspects of the real differentiated products economy in greater detail. [27] Our specific objective is to derive conditions describing behavior of supply and demand in a representative industry.

We assume that there is a continuum of products on the interval $0 \leq \omega \leq \Omega$. Throughout, we assume that each of these products has the same weight in aggregate economy. Further, in some of the discussion below, we assume that Ω is endogenously determined in the long run.

A.1 Aggregation of Products

Aggregate consumption and investment, which enter into utility and production, are represented by:

$$c_t = [\int_0^{\Omega} c_t(\omega)^{\nu} d\omega]^{(1/\nu)} \tag{19}$$

$$i_t = [\int_0^{\Omega} i_t(\omega)^{\nu} d\omega]^{(1/\nu)} \tag{20}$$

where ω is the index of an individual product. We consider the behavior of these quantities relative to a real steady-state growth path in which $i_t(\omega) = i_0\gamma^t$ and $c_t(\omega) = c_0\gamma^t$. The approximate dynamics of these aggregates are then given by

$$\frac{dc_t}{c_t} = \int_0^{\Omega} \frac{dc_t(\omega)}{c_t} d\omega \tag{21}$$

$$\frac{di_t}{i_t} = \int_0^{\Omega} \frac{di_t(\omega)}{i_t} d\omega \tag{22}$$

Notice that this set of approximate dynamics is independent of ν, which governs only substitution across the differentiated products.

We will also consider a nominal steady-state growth path with an aggregate price index P growing at the (gross) growth rate Γ, i.e., $P_t = P_0\Gamma^t$. This fixed-weight price index is defined as

$$P_t = \int_0^{\Omega} P_t(\omega) d\omega \tag{23}$$

[27]The material in the appendix has benefited from presentation in Economics 835 at the University of Virginia. Particular thanks go to Tim Adam and Dennis Starleaf

where $P_t(\omega)$ is the nominal price of the consumption-investment good ω. Evaluating the local behavior near $P_t(\omega) = P_t$, we get that:

$$\frac{dP_t}{P_t} = \int_0^{\Omega} \frac{dP_t(\omega)}{P_t} d\omega \tag{24}$$

Finally, there are expenditures on consumption and investment goods given by:

$$X_{ct} = \int_0^{\Omega} P_t(\omega) c_t(\omega) d\omega \tag{25}$$

$$X_{it} = \int_0^{\Omega} P_t(\omega) i_t(\omega) d\omega \tag{26}$$

and the local behavior of these expenditures near $P_t(\omega)c_t(\omega) = P_t c_t$ and $P_t(\omega)i_t(\omega) = P_t i_t$ is:

$$\frac{dX_{ct}}{X_{ct}} = \frac{dP_t}{P_t} + \frac{dc_t}{c_t}$$

$$\frac{dX_{it}}{X_{it}} = \frac{dP_t}{P_t} + \frac{di_t}{i_t}$$

so that proportional changes in aggregate nominal expenditure are the sum of proportional changes in the price level and aggregate real expenditure.

A.2 Demand Behavior in the Differentiated Products Setup

We obtain demands for $c_t(\omega)$ and $i_t(\omega)$ by maximizing (19) subject to (25) and (20) subject to (26). Since these are formally equivalent problems, we only discuss the determination of consumption in detail. Let the Lagrange multiplier attached to (25) be Λ: then, the first order conditions are

$$\Lambda P_t(\omega) = \frac{\partial c_t}{\partial c_t(\omega)} = [c_t(\omega)]^{\nu-1}[c_t]^{1-\nu}$$

for all products ω. Requiring that (25) is satisfied at a given level of expenditure, we get that:

$$X_{ct} = \int_0^{\Omega} P_t(\omega) c_t(\omega) d\omega = \int_0^{\Omega} \frac{1}{\Lambda}[c_t(\omega)]^{\nu}[c_t]^{1-\nu} d\omega = \frac{1}{\Lambda} c_t$$

Hence, the demand behavior of consumers in industry ω is:

$$P_t(\omega) = [c_t(\omega)]^{-\varepsilon}[c_t]^{\varepsilon-1} X_{ct} \tag{27}$$

where $\varepsilon = 1 - \nu$ is the (constant) elasticity of the inverse demand function.

Using this expression (and the comparable one for investment), we need to establish two results. First, what is the effect of an increase in the number of products on the demand for a particular product ω? Second, what is are the determinants of the demand for industry ω's product?

A.3 The Effect of An Increase in the Number of Products

In some our the analysis below, we are concerned with how an increase in the number of products, Ω, will affect the level of demand for a representative product: if there is a common price P charged for each product and a given level of consumer expenditure X_c, then (26), tells us directly that the level of common level of demand for each product must be:

$$c = \frac{1}{\Omega}\frac{X_c}{P}$$

Using the analogous expression for investment, we get that:

$$y = c + i = \frac{1}{\Omega}\frac{X_c + X_i}{P} \tag{28}$$

That is, a larger number of products lowers the demand for each individual product. We will employ this result later in our discussion of long-run scale of industry.

A.4 The Local Demand Function for Industry ω

We also need to spell out the determinants of demand for industry ω, taking into account the fact that there is demand for both consumption and investment purposes. Total consumption and investment demand in the industry, $y_t(\omega) = c_t(\omega) + i_t(\omega)$, also has a constant elasticity form, $y_t(\omega) = [P_t(\omega)]^{(-1/\varepsilon)}\ \Phi_t$, with $\Phi_t = [(c_t)^{\varepsilon-1}X_{ct}]^{\frac{1}{\varepsilon}} + [(i_t)^{\varepsilon-1}X_{it}]^{\frac{1}{\varepsilon}}$, so that

$$\frac{dy_t(\omega)}{y_t(\omega)} = -\frac{1}{\epsilon}\frac{dP_t(\omega)}{P_t(\omega)} + \frac{d\Phi_t(\omega)}{\Phi_t(\omega)}.$$

Letting real expenditure be denoted as $x_t = c_t + i_t$, then near the steady state $P_t(\omega)c_t(\omega) = P_t c_t$ and $P_t(\omega)i_t(\omega) = P_t i_t$,

$$\frac{d\Phi_t}{\Phi_t} = [\frac{1}{\varepsilon}\frac{dP_t}{P_t} + \frac{dx_t}{x_t}].$$

Thus, demand for product ω satisfies:

$$\frac{dy_t(\omega)}{y_t(\omega)} = \frac{dx_t}{x_t} - \frac{1}{\varepsilon}[\frac{dP_t(\omega)}{P_t(\omega)} - \frac{dP_t}{P_t}]$$

i.e., changes in demand arise from changes in total expenditure and from changes in industry ω's relative price. The price elasticity of demand is $(1/\varepsilon)$.

A.5 Imperfect Competition and the Markup

We are now in a position to describe the pricing practices of monopoly suppliers, who understand that the price of the product is determined by:

$$P_t(\omega) = [y_t(\omega)]^{-\varepsilon}[\Phi_t]^{\varepsilon}$$

where $\Delta_t(\omega)$ is the quantity demanded and Φ_t is the aggregate demand shift factor described above. Given this specification, the marginal revenue for a firm is:

$$\frac{\partial[P_t(\omega)y_t(\omega)]}{\partial y_t(\omega)} = (1-\varepsilon)\ [y_t(\omega)]^{-\varepsilon}[\Phi_t]^{\varepsilon} = (1-\varepsilon)\ P_t(\omega)$$

Equating marginal revenue to marginal cost, there is a fixed markup ratio implied by this specification:

$$\mu = \frac{1}{1-\varepsilon}$$

where μ is the "gross markup" of price above marginal cost, i.e., $\mu = 1$ under perfect competition.

A.6 Short-Run Marginal Cost

We will assume that there is a competitive economy-wide labor market, which determines a nominal wage rate, W_t. Then, with a predetermined capital stock, marginal cost is simply:

$$W_t / \frac{\partial y_t(\omega)}{\partial n_t(\omega)} \tag{29}$$

where $n_t(\omega)$ is the quantity of labor input and $y_t(\omega)$ is the quantity of output in industry ω. That is, the marginal cost of a unit of output depends positively on the wage rate (W_t) and the amount of labor necessary to produce a unit of output $\partial n_t(\omega)/\partial y_t(\omega)$.

Comparably, the value of having an additional unit of capital at date t is given by the value of the labor input that it allows one to replace. That is, letting the short-run value of nominal profits be $\Pi_t^s(\omega) = P_t(\omega)y_t(\omega) - W_t n_t(\omega)$

$$\frac{\partial \Pi_t^s}{\partial k_t} = W_t \frac{\partial y_t(\omega)}{\partial k_t(\omega)} / \frac{\partial y_t(\omega)}{\partial n_t(\omega)} \tag{30}$$

We discuss the calculation of the elasticities of marginal products below, but (29) and (30) provide basic definitions of marginal cost and the shadow rental value of capital used in construction of our log-linear macroeconomic model.

A.7 Long-Run Marginal Cost

We assume that the production function is such that there is constant marginal cost independent of the scale of output, although we permit declining average cost in one of the cases explored below. That is, given a rental price of capital Z and a wage W, there will be a unit marginal cost $\Xi(W, Z, a)$ that will be invariant to the scale of output. The familiar microeconomic problem of determining this minimum cost implies that $\Xi(a\partial f/\partial k) = Z$ and $\Xi(a\partial f/\partial n) = W$. Since all industries are the same in the long-run, there is the same price in every industry, $P = \mu\Xi$.

A.8 Real Value Added in a Typical Industry

Under imperfect competition, nominal value added in sector ω will be $V_t(\omega) = P_t(\omega)y_t(\omega)$, which is comprised of profits, $\Pi_t(\omega)$, wage payments, $W_t n_t(\omega)$, and the implicit rental value of capital, $Q_t k_t(\omega)$. Hence,

$$\frac{dV_t(\omega)}{V_t(\omega)} = \frac{dP_t(\omega)}{P_t(\omega)} + \frac{dy_t(\omega)}{y_t(\omega)}.$$

Using an equal weight index for real gross national product, we have $y_t = \int_0^\Omega y_t(\omega)d\omega$ and nominal gnp is $Y_t = \int_0^\Omega p_t(\omega)y_t(\omega)d\omega$. Hence, it follows that:

$$\frac{dY_t}{Y_t} = \frac{dP_t}{P_t} + \frac{dy_t}{y_t}$$

Thus, when we deflate changes in sector ω by the general price index, it follows that

$$\frac{dV_t(\omega)}{V_t(\omega)} - \frac{dP_t}{P_t} = [\frac{dP_t(\omega)}{P_t(\omega)} - \frac{dP_t}{P_t}] + \frac{dy_t(\omega)}{y_t(\omega)} = \frac{dy_t(\omega)}{y_t(\omega)} = \frac{dy_t}{y_t}$$

with the last two equalities following for a typical industry. Accordingly, we concentrate on a single representative industry and drop the ω notation.

A.9 The Relationship Between Inputs and Outputs

Differentiating the production function, we get:

$$\frac{dy_t(\omega)}{y_t(\omega)} = \frac{da_t(\omega)}{a_t(\omega)} + \frac{[(a\partial f/\partial n)n]}{y}\frac{dn_t(\omega)}{n_t(\omega)} + \frac{[(a\partial f/\partial k)k]}{y}\frac{dk_t(\omega)}{k_t(\omega)}$$

and then using the efficiency conditions for labor and capital, we get:

$$\frac{dy_t(\omega)}{y_t(\omega)} = \frac{da_t(\omega)}{a_t(\omega)} + \frac{Wn}{\Xi y}\frac{dn_t(\omega)}{n_t(\omega)} + \frac{Zk}{\Xi y}\frac{dk_t(\omega)}{k_t(\omega)}$$

and then taking $P = \mu\Xi$ and $V = Py$, we get:

$$\frac{dy_t(\omega)}{y_t(\omega)} = \frac{da_t(\omega)}{a_t(\omega)} + \mu s_n\frac{dn_t(\omega)}{n_t(\omega)} + \mu s_k\frac{dk_t(\omega)}{k_t(\omega)} \qquad (31)$$

where $s_n = Wn/Py$ and $s_k = Zk/Py$. We now explore two different specifications that are consistent with the key assumption of constant marginal cost used in the preceding derivations.

A.10 Implications of Overhead Capital and Labor

Suppose that there is a production function that includes overhead capital and labor requirements:

$$y_t = a_t f(n_t - n_t^*, k_t - k_t^*),$$

where n_t^* and k_t^* are the quantities of overhead labor and capital. Defined in terms of variable labor and capital, $\tilde{n}_t = n_t - n_t^*$ and $\tilde{k}_t = k_t - k_t^*$, the production function is neoclassical. Hence, there are fixed costs of productive activity, $W_t n_t^* + Z_t k_t^*$, and variable costs, $W_t \tilde{n}_t + Z_t \tilde{k}_t$: total costs are simply $W_t n_t + Z_t k_t$. With cost minimization and f homogenous of degree one in variable inputs, the variable costs are proportional to production: $\Xi(W_t, Z_t, a_t) y_t = W_t \tilde{n}_t + Z_t \tilde{k}_t$, where $\Xi(W_t, Z_t, a_t)$ is the unit cost stemming from variable inputs. In this setup, we thus have that:

$$\frac{dy_t}{y_t} = \frac{da_t}{a_t} + \theta_n \frac{d\tilde{n}_t}{\tilde{n}_t} + \theta_n \frac{d\tilde{k}_t}{\tilde{k}_t}$$

where $\theta_n = [(a\partial f/\partial n)\tilde{n}]/y$ and $\theta_k = [(a\partial f/\partial k)\tilde{k}]/y$ with $\theta_k + \theta_n = 1$. From cost minimization, $\theta_n = [W\tilde{n}]/[\Xi y]$ and $\theta_k = [Z\tilde{k}]/[\Xi y]$. Since $V = Py = \mu \Xi y$, $d\tilde{n}/\tilde{n} = (n/\tilde{n})/(dn/n)$, and $d\tilde{k}/\tilde{k} = (k/\tilde{k})/(dk/k)$, it then follows that

$$\frac{dy_t}{y_t} = \frac{da_t}{a_t} + s_n \; \mu \; \frac{dn_t}{n_t} + s_k \; \mu \; \frac{dk_t}{k_t} \tag{32}$$

where s_k and s_n are the value-added shares and μ is the gross markup. In this expression, the shares and the proportionate changes in factor inputs reflect total use of labor and capital, not just the variable components as in the preceding expression. Notice that this implies that the existence of overhead capital and labor or imperfect competition has implications only through the level of the markup and the levels of the factor shares. In our constant elasticity case, the markup is pinned down from the preference side.

A.11 Economic Profits and Factor Shares

We explore two cases, which highlight the range of alternative implications that may arise.

Case 1: If there are no overhead uses of factor inputs, pure economic profits arise in equilibrium. Generally, the shares of value added may be written as:

$$s_\pi + s_n + s_k = 1$$

and the fact that $\Pi = Py - Wn - Zk$ with no overhead costs means $\Pi = \mu \Xi y - \Xi y$ or that $s_\pi = (\mu - 1)/\mu$. Hence, using the conventional procedure of measuring labor income and computing capital's share as a residual, we have that:

$$s_k = 1 - s_n - s_\pi = \frac{1}{\mu} - s_n$$

With $s_n = .6$ and $\mu = 1.5$, for example, then $s_k = .07$. Hence, there will be a much larger effect of labor on output than in a neoclassical model with the same labor's share ($s_n = .6$ and $\mu = 1.5$ imply that $s_n\mu = .9$) and a much smaller effect of capital on output ($s_k = .1$ and $\mu = 1.5$ imply that $s_k \ \mu = .10$).

Case 2: If there are no profits in equilibrium, then there must be an adjustment of the size of the market so that revenue from the markup is just offset by fixed cost. With overhead labor and capital:

$$\Pi = V - Wn - Zk = \mu\Xi y - \Xi y - Wn^* - Zk^*$$

If profit is to be zero, then it follows that

$$y = \frac{Wn^* + Zk^*}{\Xi}\frac{1}{\mu - 1}$$

Hence, market size is positively influenced by the scale of fixed costs and negatively influenced by the magnitude of markups. Using the expression developed in (28) above, this alternatively determines the range of products that are compatible with long-run equilibrium.

Notice that there is no need to adjust the shares in this case because there are no economic profits. Notice also that in this case, which we use in the main text, the combination of imperfect competition and overhead costs leads to a general increase in the effect of factors on output. Using the same numbers as in the previous case, it follows that $s_n\mu = .78$ and $s_k\mu = .52$ when $s_\pi = 0$.

A.12 Implications for Elasticities of Marginal Product

The constant returns-to-scale component of the production function, $a_t f(\tilde{n}_t, \tilde{k}_t)$, has the conventional elasticities of its marginal products. Letting ξ_{xy} denote the elasticity of the marginal product of x with respect to y, it follows that

$$\xi_{\tilde{n}\tilde{n}} = -\frac{\theta_k}{\zeta}$$

$$\xi_{\tilde{n}\tilde{k}} = \frac{\theta_k}{\zeta}$$

$$\xi_{\tilde{k}\tilde{k}} = -\frac{\theta_n}{\zeta}$$

$$\xi_{\tilde{k}\tilde{n}} = \frac{\theta_n}{\zeta}$$

where ζ is the local elasticity of substitution—defined with respect to variable inputs—and the θ's are the shares of variable cost described above. (There are also elasticities with respect to a but these are unity).

We want to convert these to elasticities in terms of observable factor inputs, n and k. There are two steps. First, we must undertake a rescaling of factor input, comparable to that undertaken in deriving (32). That is, while the marginal products of total and variable inputs are the same (so that $\xi_{\widetilde{x}\widetilde{y}} = \xi_{x\widetilde{y}}$ for $x = k, n$ and $y = k, n$), the new elasticities require scaling by the ratio of total to variable input (so that $\xi_{xy} = \xi_{x\widetilde{y}}\,(y/\widetilde{y})$ for $x = k, n$ and $y = k, n$). Second, we must replace the unobserved factor shares of variable inputs with observable factor share information. We utilize the facts that $s_n\mu = \theta_n(n/\widetilde{n})$ and $s_k\mu = \theta_k(k/\widetilde{k})$. Hence, the relevant elasticities with respect to total labor ($n = \widetilde{n} + n^*$) and ($k = \widetilde{k} + k^*$) can be written as:

$$\xi_{nn} = -\frac{s_k\mu}{\zeta}\frac{n}{\widetilde{n}}\frac{\widetilde{k}}{k}$$

$$\xi_{nk} = \frac{s_k\mu}{\zeta}$$

$$\xi_{kk} = -\frac{s_n\mu}{\zeta}\frac{k}{\widetilde{k}}\frac{\widetilde{n}}{n}$$

$$\xi_{kn} = \frac{s_n\mu}{\zeta}$$

Notice that these specification make marginal products much more responsive to changes in factor inputs than in the perfect competition model. For example, with $\mu = 1.5$, it follows that there is a 1.5 times larger effect of a change in labor input on the marginal products: this strengthens the "accelerator effect" present in the basic neoclassical model, by which an expansion of future labor input raises the marginal product of future capital and consequently current investment. However, there is also a tendency for the marginal product of labor to become correspondingly more responsive to a change in labor input, although this depends on the extent of overhead labor and capital.

A.13 Symmetric Overhead Costs

If the size of the market adjusts so as to eliminate pure economic profits and there are symmetric overhead costs, then some simplification of the foregoing results. First, if there are no pure profits, then it follows that: $Py = \mu\Xi y = W(n^* + \widetilde{n}) + Z(k^* + \widetilde{k})$. Hence, if $n^*/\widetilde{n} = k^*/\widetilde{k}$, it follows that $W(n^* + \widetilde{n}) + Z(k^* + \widetilde{k}) = \phi[W\widetilde{n} + Z\widetilde{k}] = \phi\Xi y$, where $\phi = n/\widetilde{n} = k/\widetilde{k}$. Hence, it also follows that $\phi = \mu$, i.e., the gross markup is just high enough to meet the ratio of total to variable inputs. Accordingly, there is a simple link between the elasticities of marginal products under perfect and imperfect competition: they are simply scaled by μ, with the extent of overhead labor and capital not entering as a separate parameter. This can be of some quantitative significance for the behavior of marginal products. For example, since $\xi_{nn} = -\frac{s_k \mu}{\zeta} \frac{\tilde{n}}{n} \widetilde{\frac{k}{k}} = -\frac{s_k}{\zeta}\mu$, it follows that with $s_k = .4$ and $\zeta = 1$, then $\xi_{nn} = -.4$: labor's marginal product (the real wage) will be move $-.4$ for every one percent variation in labor input. But in the imperfect competition model with $\mu = 1.5$, the same change in labor input will require a $-.6\%$ change in the real wage.

B Appendix: The Financial Market Frictions Model

This appendix reports the derivation of the equilibrium conditions for the financial market frictions model of Christiano and Eichenbaum (1993). For this purpose, it is convenient to use a dynamic programming because of the multistage information structure that is present in the setup.

We will work with an economy that is assumed to be transformed to eliminate the effects of deterministic trend growth in nominal and real variables. The gross growth rate of the money stock is γ_M. (That is, under certainty, the behavior of the money stock is given by $\underline{M}_t = \gamma_M \underline{M}_{t-1}$ where the underbar denotes the level of the money stock prior to transformation). We will also assume that there is trend growth in real activity, at gross rate γ. arising from labor augmenting technical progress. Given these two features, the model implies that the trend in the price level will be $\gamma_P = \gamma_M/\gamma$. Our specifications of preferences and opportunities for the household will assume that these trends have been removed in a model-consistent manner, which will sometimes involve the presence of the growth parameters in various specifications. This transformation is accomplished by dividing all real variables by γ^t and all nominal variables by γ_P^t.

B.1 Household Choices

The household's dynamic programming problem will be two stage as a result of the informational structure.

After Realization of Aggregate Information: With all shocks revealed at date t, the household will maximize momentary utility, $u(c_t, l_t)$, treating asset accumulation in interest-bearing bank deposits (B_t) and "spending money" decisions (S_t) as predetermined. The maximization is subject to the cash for goods constraint:

$$P_t c_t \leq S_t + W_t n_t \tag{33}$$

and the time constraint:

$$n_t + l_t + h_t = 1 \tag{34}$$

with the choice variables being c_t, l_t and n_t. Since cash balances are predetermined relative to this subperiod, so too is the time devoted to portfolio adjustment, h_t.

$$h_t = h(\frac{S_t}{S_{t-1}}\gamma_P) \tag{35}$$

Efficient selection of consumption, leisure, and work effort implies that:

$$\frac{\partial u(c_t, l_t)/\partial l_t}{\partial u(c_t, l_t)/\partial c_t} = \frac{W_t}{P_t} = w_t \tag{36}$$

Another result of this efficient consumer choice is an indirect utility function which specifies $v(S_t, S_{t-1}; W_t, P_t)$. Following conventional "envelope theorem" arguments, the partial derivatives of v are:

$$\frac{\partial v(S_t, S_{t-1}; W_t, P_t)}{\partial S_t} = \frac{\partial u(c_t, l_t)/\partial c_t}{P_t} - \frac{\partial u(c_t, l_t)}{\partial l_t} h_t' \frac{1}{S_{t-1}} \gamma_P$$

and

$$\frac{\partial v(S_t, S_{t-1}; W_t, P_t)}{\partial S_{t-1}} = \frac{\partial u(c_t, l_t)}{\partial l_t} h_t' \frac{S_t}{(S_{t-1})^2} \gamma_M$$

where h_t' is the first derivative of the h function evaluated at date t. That is, the primary value of having an additional unit of "spending money" is that it yields consumption according to its real purchasing power, but there are also some time costs of obtaining this purchasing power which must be paid. Further, since adjustment of the level of "spending money" depends on the prior level, there are also costs savings attached to having higher S_{t-1}.

Prior to Realization of Aggregate Information: At the start of the period, agents solve the dynamic programming problem:

$$J(S_{t-1}, B_{t-1}, X_t) = \max E\{v(S_t, S_{t-1}; W_t, P_t) + \beta J(S_t, B_t, X_{t+1})\} | I_{0t}$$

where X_t represents the state of the dynamic macroeconomic system, which agents view as beyond their control, and I_{0t} is information available at the start of the period. The maximization is with respect to S_t and the household's financial asset position evolves according to $\gamma_P B_t = (1 + R_t)G_t$ and $B_{t-1} = G_t + S_t$. (The date t interest rate unknown as of the start of the period.) Accordingly, the efficiency condition for selection of the level of "spending money" is:

$$\Omega_t = E\{\frac{\partial v(S_t, S_{t-1}; W_t, P_t)}{\partial S_t} + \beta \frac{\partial J(S_t, B_t, X_{t+1})}{\partial S_t}\} | I_{0t}$$

and that for holdings of interest-bearing deposits (choice of G) is:

$$\Omega_t = E\{\beta [\frac{(1 + R_t)}{\gamma_P}] \frac{\partial J(S_t, B_t, X_{t+1})}{\partial B_t}\} | I_{0t}$$

where Ω_t is the multiplier on the constraint $B_{t-1} = G_t + S_t$. Further, standard envelope theorem arguments imply that

$$\frac{\partial J(S_{t-1}, B_{t-1}, X_t)}{\partial B_{t-1}} = \Omega_t$$

and that

$$\frac{\partial J(S_{t-1}, B_{t-1}, X_t)}{\partial S_{t-1}} = E\{\frac{\partial v(S_t, S_{t-1}; W_t, P_t)}{\partial S_{t-1}}\} | I_{ot} = E\{\frac{\partial u(c_t, l_t)}{\partial l_t} h_t' \frac{S_t}{(S_{t-1})^2} \gamma_P\} | I_{ot}.$$

It is useful to combine these efficiency conditions into two specifications that we will use in our model economy. First, the *ex poste* (conditional on I_t rather than I_{ot}) value of a unit of "spending money" is:

$$\Delta_t = \frac{\partial u_t/\partial c_t}{P_t} - \frac{\partial u_t}{\partial l_t}h'_t\frac{1}{S_{t-1}}\gamma_P + \beta E\{\frac{\partial u_{t+1}}{\partial l_{t+1}}h'_{t+1}\frac{S_{t+1}}{(S_t)^2}\gamma_P\}|I_t \tag{37}$$

where by $\partial u_t/\partial c_t$ we mean $\partial u(c_t, l_t)/\partial l_t$, etc.

Using the definition of Δ_t, the preceding efficiency conditions also imply that the marginal expected benefits from holding bonds and "spending money" must be equated, $\Omega_t = E\{\Delta_t\}|I_{ot}$, and that

$$E\{\Delta_t - (1+R_t)\frac{\beta}{\gamma_M}\Delta_{t+1}\}|I_{ot} = 0, \tag{38}$$

which is the requirement that the return to holding "spending money" is equated to the nominal interest rate (suitably modified for discounting as well as trend growth in prices and output, i.e., the factors which determine the certainty level of the nominal interest rate).

To cast our model into first-order form, we also need to include the specification:

$$S_{(t+1)-1} = S_t \tag{39}$$

Taken together, (33), (34), (36), (37), (38), and ((39) provide a description of the household's efficient choices and constraints in this financial market frictions economy.

B.2 Firms and Investment

In the financial market frictions model, firms borrow from banks to finance purchases of labor input and finance investment out of retained earnings. The following conditions governing efficient investment and capital choices on the part of the firms, presuming that they act on the basis of I_t to maximize the utility of their owners.

The discussion is facilitated by viewing firms as containing two parts. First, an essentially static component simply rents capital and labor from households. In a typical "cash in advance" model, this part of the firm collects revenue of $P_t y_t$ at the end of period t and distributes these cash flows as profits and wages at that time, which is too late for individuals to earn date t interest so that they are subject to some real consequences of expected inflation. However, in the current setup, the firms borrow on behalf of the worker, providing them with a cash transfer in the amount $W_t n_t$. Thus, the competitively determined wage rate is:

$$W_t = \frac{1}{1+R_t}[P_t a_t \frac{\partial f(k_t, n_t)}{\partial n_t}]. \tag{40}$$

The firm pays its rentals at the end of the period, so that:

$$Z_t = [P_t a_t \frac{\partial f(k_t, n_t)}{\partial k_t}].$$

is the competitively determined flow rental price of capital.

Second, to assure that the firm maximizes the expected utility of its owners, we view the household as operating the second part of the firm, which is responsible for its investment activity. This requires some small modifications of the preceding dynamic programming framework, which we sketch here. One is that there are now additional inflows to and outflows from the households "bank balance," B, so that the date t constraint is modified to:

$$B_{t-1} + D_{t-1} = B_{t-1} + \frac{1}{\gamma_P}(Z_{t-1}k_{t-1} - P_{t-1}i_{t-1}),$$

That is, dividend income D (capital rental income less investment expenditure) augments the asset holdings of the household. As above, the household evaluates payments into its bond account according to a multiplier Ω_t so the efficiency condition for investment is:

$$\psi_t \phi'(i_t/k_t) = \frac{1}{\gamma_P} E\{P_t \Omega_{t+1}\} | I_t \tag{41}$$

and the capital accumulation condition is:

$$\gamma \psi_t = \beta E\{\psi_{t+1} \upsilon(i_{t+1}/k_{t+1}) + \beta \frac{1}{\gamma_P} \Omega_{t+2} Z_{t+1}\} | I_t,$$

Equivalently, the firm may be viewed as selecting investment and capital formation to maximize discounting profits, using the multiplier as its discount factor.[28]

For the purpose of programming this model, it is convenient to have an expression for a discounted real marginal product of capital:

$$z_t^* = \beta E[\Omega_{t+1} P_t a_t \frac{\partial f(k_t, n_t)}{\partial k_t}] | I_t \tag{42}$$

[28] These investment and capital efficiency conditions look very different from the ones in the main text. Alternatively, if we define:

$$\rho_t = \frac{1}{\gamma_P} E\{P_t \Omega_{t+1}\} | I_t$$

these conditions may be written in a more familiar form as:

$$\psi_t \phi'(i_t/k_t) = \rho_t$$

and as:

$$\gamma \psi_t = \beta E\{\psi_{t+1} \upsilon(i_{t+1}/k_{t+1}) + \rho_{t+1} z_{t+1}\} | I_t,$$

which are the familiar expressions.

which makes the capital efficiency condition very simple:

$$\gamma\psi_t = \beta E\{\psi_{t+1}v(i_{t+1}/k_{t+1}) + \beta z^*_{t+1}\}|I_t. \tag{43}$$

The production function and accumulation technologies are as in the real business cycle model:

$$y_t = a_t f(k_t, n_t) \tag{44}$$

$$\gamma k_{t+1} = (1-\delta)k_t + \phi(\frac{i_t}{k_t})k_t \tag{45}$$

There are market-clearing conditions in the markets for goods and money:

$$c_t + i_t = y_t \tag{46}$$

$$P_t c_t = M_t \tag{47}$$

The latter is standard. The former arises from the lending activities of banks, which loan out $M_t - S_t$ as to finance firms' expenditure on labor, so that $M_t - S_t = W_t N_t$

Finally, there is a condition determining the expected real interest rate:

$$1 + r_t = E[(1 + R_t)\frac{P_t}{P_{t+1}}]|I_t \tag{48}$$

C Appendix: Construction of Log-Linear Models

This section provides a more detailed discussion of the quantitative models discussed in the main text. Our procedure is to outline the model equations for each framework, to discuss their implications for the steady-state, and then to discuss the linearization of the model equations we use.

In this appendix, we thus provide sufficient information about the operation of our economies that a researcher should be able to replicate our model simulation results using the model solution methods of King and Watson [1994]. Since these three model economies are also used as examples of modern dynamic models in our model solution work, this appendix therefore also explains the models in greater detail for that purpose.

C.1 The Real Business Cycle model

The equations describing the real business cycle model are grouped into the following five blocks.

Utility Function and Efficiency Conditions for Consumption and Leisure: The two conditions describing efficient selection of labor supply and consumption:

$$\partial u(c_t, l_t)/\partial c_t = \rho_t \tag{49}$$

$$\partial u(c_t, l_t)/\partial l_t = \rho_t w_t \tag{50}$$

where we further assume that the momentary utility function takes the form:

$$u(c, l) = \frac{1}{1-\sigma}(c^{\theta} l^{1-\theta})^{1-\sigma}$$

so that $\partial u(c_t, l_t)/\partial c_t = \theta \ c^{\theta(1-\sigma)-1} \ l^{(1-\theta)(1-\sigma)}$ and $\partial u(c_t, l_t)/\partial l_t = (1-\theta) \ c^{\theta(1-\sigma)} \ l^{(1-\theta)(1-\sigma)-1}$.

This utility specification implies that if the original, growing economy has a discount factor of b then the discount factor for the stationary economy that we study is $\beta = b\gamma^{\theta(1-\sigma)}$, where γ is the growth factor for the steady state. Further, the real interest rate in the original growing economy is determined by the requirement that:

$$1 = bE_t[\frac{\partial u(\tilde{c}_{t+1}, l_{t+1})/\partial \tilde{c}_{t+1}}{\partial u(\tilde{c}_t, l_t)/\partial \tilde{c}_t}] = \frac{\beta}{\gamma} \ E_t[\frac{\partial u(c_{t+1}, l_{t+1})/\partial c_{t+1}}{\partial u(c_t, l_t)/\partial c_t}]$$

where $\tilde{c}$ denotes the level of consumption in the original growing economy.

Resource Constraints for Time and Goods: There are constraints on uses of time and goods;

$$n_t + l_t = 1. \tag{51}$$

$$y_t = c_t + i_t \tag{52}$$

Production Function and Production Efficiency Conditions: There is a production function;

$$y_t = a_t f(k_t, n_t) \tag{53}$$

and there are equations describing the determination of real wages and real rentals;

$$w_t = a_t \frac{\partial f(k_t, n_t)}{\partial n_t} \tag{54}$$

$$z_t = a_t \frac{\partial f(k_t, n_t)}{\partial k_t} \tag{55}$$

Monetary Equilibrium: There is a monetary equilibrium condition,

$$\log(M_t) = \log(P_t) + m_y \log(y_t) - m_R \log(1 + R_t) - V_t \tag{56}$$

which we write directly in loglinear form.

Efficient Investment and Capital Accumulation: There are three equations describing efficient investment and capital accumulation

$$\psi_t \phi'(i_t/k_t) = \rho_t \tag{57}$$

$$\gamma\psi_t = E\{\beta\psi_{t+1}\nu(i_{t+1}/k_{t+1}) + \beta\rho_{t+1}z_{t+1}\}|I_t. \tag{58}$$

where $\nu(i_{t+1}/k_{t+1}) = (1-\delta) + \phi(i_{t+1}/k_{t+1}) - (i_{t+1}/k_{t+1})\phi'(i_{t+1}/k_{t+1})$. The evolution of capital is governed by:

$$\gamma k_{t+1} = (1-\delta)k_t + \phi(\frac{i_t}{k_t})k_t \tag{59}$$

Asset Return Specifications: There are equations describing determination of real and nominal interest rates;

$$1 = \frac{\beta}{\gamma} E_t\{(1 + r_t)\frac{\rho_{t+1}}{\rho_t}\} \tag{60}$$

$$R_t = r_t + E_t(\log(P_{t+1})) - (\log(P_t)), \tag{61}$$

To solve the model quantitatively, we must specify the utility $(u(c, l))$, production $(AF(k, n))$ and accumulation $(\phi(i/k))$ specifications. We use a combination of the existing methods, choosing an explicit functional form (as in Kydland and Prescott [1982] or Christiano [1988) and choosing an implicit functional form (as in King, Plosser and Rebelo [1988]).

C.1.1 The Steady State

We now describe aspects of the steady state of the real business cycle model, concentrating on the extent to which a procedure of "matching" aspects of the steady-state restricts the parameters of the model.

The momentary utility function, u, determines the steady-state combination of consumption and leisure that our representative agent selects, given the real wage. Hence, it follows that empirical estimates of three ratios can be used to determine the parameter θ in the specification above. Specifically, the ratio of (50) to (49) implies that:

$$\frac{1-\theta}{\theta}\frac{c}{l} = w \text{ or } \frac{1-\theta}{\theta}\frac{c}{y}\frac{n}{l} = \frac{wn}{y}$$

That is, specification of information on c/y, n/l, and wn/y is sufficient to determine a value of the parameter θ. By contrast, the parameter σ is not determined from steady state conditions since it governs the near-steady state behavior of consumption. It will be set equal to the conventional value of unity in our work: this implies that the utility function is $\theta \log(c)+(1-\theta)\log(l)$.

The steady-state capital-output ratio and investment-output ratios are determined as follows. Combining (57), (58), and (59), we arrive at the following condition:

$$\frac{r+\delta-[\phi-(\frac{i}{k})\phi']}{\phi'} = z \tag{62}$$

That is, the steady state real rental on capital, z, depends on the conventional $r+\delta$ plus terms reflecting investment adjustment costs.

We will assume that there are no average or marginal adjustment costs, locally to the steady state, so that $\phi = i/k$ and that $\phi' = 1$. These conditions thus imply that steady-state z is simply $r+\delta$. (They also imply that the ϕ function takes the form displayed in figure appbl). Further, since $\phi = i/k$ near the steady-state, (59) dictates that:

$$\frac{i}{k} = (\gamma - 1) + \delta \tag{63}$$

which is the familiar requirement that the "investment rate" i/k depends positively on the growth and depreciation rates. (This condition also holds more generally, since (59) specifies that $\phi(\frac{i}{k}) = (\gamma-1)+\delta$ in steady-state). Finally, these conditions also imply that the steady-state value of $\nu = 1-\delta$ and that the steady-state value of $\psi = \rho$.

Our procedure here is thus to "implicitly specify" the ϕ function. It is equivalent to specifying an explicit functional form, such as $\phi(i/k) = \kappa_0[(i/k)-\kappa_1]^{\kappa_2}$ and then requiring that the κ_i parameters are such that our level and derivative requirements are met. With an explicit functional form, we are left with the value of κ_2 as free; this corresponds to ϕ'' being free when one determines the function ϕ implicitly.

Taking (62) and (63) together with the definition of $s_k = (zk/y)$, we have that:

$$\frac{k}{y} = \frac{s_k}{r+\delta}$$

$$s_i = (\frac{i}{k})(\frac{k}{y}) = \frac{\gamma - 1 + \delta}{r+\delta}\ s_k$$

Thus, specification of empirical information on the growth rate $(\gamma - 1)$, the depreciation rate (δ), the real interest rate (r), and the share of capital (s_k) is sufficient to determine these ratios. Similarly, the ratios s_c and s_n are determined as $s_c = 1 - s_i$ and $s_n = 1 - s_k$.

C.1.2 Approximation of Model Equations

We now discuss the approximation of the model equations laid out above.

Marginal Utilities: The first two equation, (49) and (50), do not require approximation with the specified utility function, since they are directly log-linear.

$$[\theta(1-\sigma) - 1]\log(c_t/c) + [(1-\theta)(1-\sigma)]\log(l_t/l) = \log(\rho_t/\rho) \tag{64}$$

$$\theta(1-\sigma)\log(c_t/c) + [(1-\theta)(1-\sigma) - 1]\log(l_t/l) = \log(\rho_t/\rho) + \log(w_t/w) \tag{65}$$

In these expressions, as in others below, we use the symbol without a time subscript to indicate a steady state value.

Resource Constraints: Log-linear approximation of the resource constraints may be accomplished in one of two ways. As in Christiano [1988], one may write the resource constraint $n_t + l_t = 1$ as $l\exp(\log(l_t/l) + n\log(n_t/n)) = 1$ and then take a Taylor series approximation in $\log(l_t/l)$ and $\log(n_t/n)$. The result of this approximation, around the point $\log(l_t/l) = 0$ and $\log(n_t/n) = 0$ is that $l\log(l_t/l) + n\log(n_t/n) = 0$ since $\frac{d}{dx}\exp(x) = \exp(x)$ and $\exp(0) = 1$. *Alternatively, one may simply totally differentiate the expression, rewrite it as* $n\frac{dn_t}{n} + l\frac{dl_t}{l} = 0$ and then replace dn_t/n with $\log(n_t/n)$. The results are the same:

$$n\log(n_t/n) + l\log(l_t/l) = 0 \tag{66}$$

$$s_c\log(c_t/c) + s_i\log(i_t/i) = \log(y_t/y) \tag{67}$$

Production Function and Marginal Product Schedules: The conventional approximation of the constant returns-to-scale production function is:

$$\log(y_t/y) \approx \log(a_t/a) + s_n\log(n_t/n) + s_k\log(k_t/k) \tag{68}$$

$$\log(\frac{\partial y_t}{\partial n_t}/\frac{\partial y}{\partial n}) \approx \log(a_t/a) - \frac{s_k}{\zeta}\log(n_t/n) + \frac{s_k}{\zeta}\log(k_t/k) \tag{69}$$

$$\log(\frac{\partial y_t}{\partial k_t}/\frac{\partial y}{\partial k}) \approx \log(a_t/a) + \frac{s_n}{\zeta}\log(n_t/n) - \frac{s_n}{\zeta}\log(k_t/k) \tag{70}$$

where $\zeta > 0$ is the elasticity of substitution of capital for labor labor in the production function. The derivation of these approximations is based on standard microeconomic arguments and is reviewed in King, Plosser and Rebelo [1988, supplementary note #2]. Since it governs the near-steady state behavior of the response of relative factor inputs to relative factor prices, ζ cannot be determined from steady state conditions. Thus, we are free to choose this parameter and, as conventional, we set it equal to unity so that the production function is "Cobb Douglas". Our procedure of implicitly specifying the production function is equivalent to choosing the parameters of the CES production specification, $y = a[(1-\alpha)k^\kappa + \alpha n^\kappa]^{(1/\kappa)}$, with $\kappa < 1$, so as to have a specified level of the marginal product of capital in steady state ($z = r + \delta$) and a specified value for capital's share. These two requirements would determine a and α but not κ, leaving us free to specify it; this is analogous to the freedom in the implicit specification since $\zeta = 1/(1-\kappa)$.

Monetary Equilibrium: The money demand function is directly written in logarithmic form.

$$\log(M_t) = \log(P_t) + v_y \log(y_t) - v_R \log(1+R_t) - V_t \tag{71}$$

and requires specification of the parameter v_y and v_R.

Investment and Capital: The three conditions governing the efficient selection of investment and capital are approximated as follows. The approximation of (57) is given by:

$$\log(\psi_t/\psi) + [\frac{(\frac{i}{k})\phi''}{\phi'}][\log(i_t/i) - \log(k_t/k)] = \log(\rho_t/\rho)$$

so that it contains a "slope of the investment demand function," $\eta = [(\frac{i}{k})\phi''/\phi']^{-1}$, that cannot be determined from steady-state information. The approximation of (58) is derived as follows. First, the total differential is:

$$\gamma \ d\psi_t = \beta E\{\nu \ d\psi_{t+1} + \psi \ d\nu_{t+1} + z \ d\rho_{t+1} + \rho \ dz_{t+1}\}|I_t.$$

where $d\nu_{t+1} = d[(1-\delta) + \phi(i_{t+1}/k_{t+1}) - (i_{t+1}/k_{t+1})\phi'(i_{t+1}/k_{t+1})]$. Thus, $d\nu_{t+1} = -(i/k)[\phi'' d(i_{t+1}/k_{t+1})]$, which simplifies to $-[(i/k)\phi'/\eta][d(i_{t+1}/i) - d(k_{t+1}/k)$ using the definition of η. Hence, using $\psi = \rho$, $\beta\nu/\gamma = (1-\delta)/(1+r)$ and $\beta z/\gamma = (r+\delta)/(1+r)$, we have:

$$\begin{aligned}\log(\psi_t/\psi) = \tfrac{1-\delta}{1+r}E\{\log(\psi_{t+1}/\psi) - \tfrac{i/k}{\eta(1-\delta)}[\log(i_{t+1}/i) - \log(k_{t+1}/k)]\}|I_t \\ + \tfrac{r+\delta}{1+r}E\{\log(\rho_{t+1}/\rho) + \log(z_{t+1}/z)\}|I_t\end{aligned} \tag{72}$$

Comparably, the approximation of the accumulation equation yields:

$$\gamma \log(k_{t+1}/k) = [1-\delta + \frac{i}{k}\phi' - \phi]\log(k_t/k) + [\frac{i}{k}\phi']\log(i_t/i)$$

where under our steady state assumptions $[1-\delta+\frac{i}{k}\phi'-\phi] = (1-\delta)$ and $[\frac{i}{k}\phi'] = \frac{i}{k}$.

Real and Nominal Interest Rates: Finally, we approximate condition (60), describing the behavior of the real interest rate, in a slightly different manner.

$$r_t - r = E_t\{\log(\rho_{t+1}/\rho) - \log(\rho_t/\rho)\} \tag{73}$$

where $r = \frac{\gamma}{\beta} - 1$ is the level of the steady-state real interest rate. The "semilogarithmic" treatment of the interest rate reflects the fact that we want to consider questions like "if consumption is expected to grow at one percent per quarter, then how much of an increase in the level of the real interest rate will occur?"

Finally, the nominal interest equation is directly loglinear, although it may be viewed as an approximation to $1 = \frac{\beta}{\gamma}E_t\{(1+R_t)\frac{P_t}{P_{t+1}}\frac{\rho_{t+1}}{\rho_t}\}$:

$$R_t = r_t + E_t(\log(P_{t+1})) - (\log(P_t)), \tag{74}$$

C.2 The Sticky Price Model

If we were simply building an imperfect competition version of the real business cycle model, then the modifications of the preceding setup would be relatively straightforward. We would need to incorporate only the effects of imperfect competition and the effects of an altered production technology. Hence, we begin by discussing that straightforward modification

C.2.1 A Real Business Cycle Model with Imperfect Competition

The specifications of (49), (50),(51) and (52) are unaffected by the introduction of imperfect competition, since these are just the consumer's first-order conditions and the resource constraints on time and goods. However, as discussed in appendix A, we alter the production technology to include overhead factor requirements:

$$y_t = a_t f(k_t - k_t^*, n_t - n_t^*) \tag{75}$$

and imperfect competition alters the equations describing the relationship of wages and rentals to marginal products. Using the three specifications described in appendix A, $P_t = \mu\Xi_t$, $W_t = \Xi_t a_t \partial f(k_t - k_t^*, n_t - n_t^*)/\partial n_t$ and $Z_t = \Xi_t a_t \partial f(k_t - k_t^*, n_t - n_t^*)/\partial k_t$ together with the definitions $w = W/P$ and $z = Z/P$, we get that

$$w_t = \mu a_t \frac{\partial f(k_t - k_t^*, n_t - n_t^*)}{\partial n_t} \tag{76}$$

$$z_t = \mu a_t \frac{\partial f(k_t - k_t^*, n_t - n_t^*)}{\partial k_t} \tag{77}$$

The monetary equilibrium condition (56) is also unaltered by the introduction of imperfect competition and the modification of the technology to include overhead labor and capital requirements.

There three equations describing efficient investment and capital accumulation are simply taken to describe the selection of total capital (fixed plus variable) and the related investment. These can be thought of as arising from the actions of a competitive sector supplying capital or, equivalently, the actions of a cost-minimizing firm undertaking internal investment (in which case the rental price z is a within-firm transfer price). Hence, we assume that there is no alteration of (57), (58), or (59).

Finally, there are no changes in the asset-pricing specifications (60) or (61).

C.2.2 The Steady-State

The steady-state of the imperfect competition-overhead factor model differs only a little from that of the competitive real business cycle model.

In terms of the preference parameter θ, it is influenced by imperfect competition only if the expression the shares $s_c, n/l$ or s_n are altered: if these are being determined empirically, then there is no modification of θ.

The steady-state requirement that the variable component of capital stock has the required long-run rental price may be written as:

$$Z = P(r+\delta) = \Xi\, a\frac{\partial f(k-k^*, n-n^*)}{\partial k} \tag{78}$$

i.e., that the rental is the investment good price multiplied by interest and depreciation cost factors. Multiplying both sides of the first equality in this expression by the *total* capital stock as a ratio to national income, k/Py, we get that:

$$s_k = \frac{k}{y}(r+\delta)$$

which is the expression that we previously used to determine the capital-output ratio. This expression is invariant to the extent of competition and overhead factors. The determination of s_i then follows immediately from this specification as above.

This invariance does not mean that there may not be effects of imperfect competition or overhead factor requirements on the steady state capital-output ratio. For example, suppose that the steady state capital stock is determined from the second pair of equalities in (78) and that the underlying production function is Cobb-Douglas in variable inputs: $y = a[k-k^*]^{1-\alpha}[n-n^*]^{\alpha}$. Then, using $P = \mu\Xi$, we find that:

$$\frac{k-k^*}{y} = \frac{(1-\alpha)}{(r+\delta)\mu}$$

which implies that less perfect competition (higher μ) lowers the capital output ratio and that additional overhead capital requirements raise it. The point is simply that we are implicitly allowing $(1-\alpha)$ to adjust in determining the model's implication for the capital-output ratio. That is, we are asking "what capital output ratio is consistent with observed factor income share and return data" rather than "what are the implications of monopoly power for the capital-output ratio?

C.2.3 Near Steady-State Dynamics

In view of the preceding discussion, the only alteration of the approximation of model equations from the RBC model is that which affects the production function and its marginal products. These topics have been extensively discussed in appendix A above. Thus, the modifications are:

$$\log(y_t) \approx \log(a_t) + \mu s_n \log(n_t) + \mu s_k \log(k_t) \tag{79}$$

$$\log(\frac{\partial y_t}{\partial n_t}/\frac{\partial y}{\partial n}) \approx \log(a_t/a) - \frac{s_k\mu}{\zeta}\frac{n}{\widetilde{n}}\frac{\widetilde{k}}{k}\log(n_t/n) + \frac{s_k\mu}{\zeta}\log(k_t/k)$$
$$\log(\frac{\partial y_t}{\partial k_t}/\frac{\partial y}{\partial k}) \approx \log(a_t/a) + \frac{s_n\mu}{\zeta}\log(n_t/n) - \frac{s_n\mu}{\zeta}\frac{k}{\widetilde{k}}\frac{\widetilde{n}}{n}\log(k_t/k)$$

C.2.4 The Sticky Price Variant

The model with sticky prices is designed to describe near steady state dynamics that incorporate altered real effects of nominal and real disturbances. The price adjustment dynamics are summarized by three equations: a price adjustment rule that links price changes to discrepancies from target:

$$\log(P_t) - \log(P_{t-1}) = \varphi[\log(P_t^*) - \log(P_{t-1})] \tag{80}$$

a specification of the target as related to discounted marginal cost:

$$\log(P_t^*) = E_t[\sum_{j=0}^{\infty}(\beta\varphi)^j \log(\mu MC_{t+j})] \tag{81}$$

and the specification of short-run discussed in appendix A above.

$$MC_t = \frac{W_t}{\partial y_t/\partial n_t}. \tag{82}$$

This last expression can be cast directly in log-linear form using the preceding sections results on marginal product elasticities.

Labor quantities in this model are demand-determined, so that the production function is inverted for this purpose: (79 implies:

$$\log(n_t) \approx \frac{1}{\mu s_n}\log(y_t) - \frac{1}{\mu s_n}\log(a_t) - \frac{s_k}{s_n}\log(k_t)$$

Finally, as discussed in appendix A, the short-run benefit from an additional unit of capital is that it permits the firm to save labor payments, i.e.,

$$\frac{\partial \pi_t}{\partial k_t} = w_t \frac{\partial y_t / \partial k_t}{\partial y_t / \partial n_t}. \tag{83}$$

This expression can also be cast in log-linear from using the previous derivations of marginal product elasticities.

C.3 The Financial Market Frictions Model.

The equations of the financial market frictions model as derived in Appendix B are as follows. There is a cash for goods constraint,

$$P_t c_t \leq S_t + W_t n_t \tag{84}$$

a constraint on the allocation of time,

$$n_t + l_t + h_t = 1 \tag{85}$$

and a function specifying the time spent in portfolio adjustment,

$$h_t = h(\frac{S_t}{S_{t-1}} \gamma_M). \tag{86}$$

There is an efficiency condition governing choice of consumption and labor:

$$\frac{\partial u(c_t, l_t)/\partial l_t}{\partial u(c_t, l_t)/\partial c_t} = \frac{W_t}{P_t} = w_t. \tag{87}$$

There is the shadow value of a unit of spending money,

$$\Delta_t = \frac{\partial u_t / \partial c_t}{P_t} - \frac{\partial u_t}{\partial l_t} h_t' \frac{1}{S_{t-1}} \gamma_P + \beta E\{\frac{\partial u_{t+1}}{\partial l_{t+1}} h_{t+1}' \frac{S_{t+1}}{(S_t)^2} \gamma_P\} | I_t, \tag{88}$$

and an expression governing its efficient evolution

$$E\{\Delta_t - (1 + R_t) \frac{\beta}{\gamma_M} \Delta_{t+1}\} | I_{ot} = 0, \tag{89}$$

together with an expression describing the evolution of that stock,

$$S_{(t+1)-1} = S_t. \tag{90}$$

The firms' efficiency conditions include those for labor demand,

$$W_t = \frac{1}{1 + R_t} [P_t a_t \frac{\partial F(k_t, n_t)}{\partial n_t}]. \tag{91}$$

investment demand,

$$\psi_t \phi'(i_t/k_t) = \frac{1}{\gamma_P} E\{P_t \Omega_{t+1}\} | I_t \tag{92}$$

the shadow rental value of a unit of capital,

$$z_t^* = \beta E[\Omega_{t+1} P_t a_t \frac{\partial f(k_t, n_t)}{\partial k_t}] | I_t, \tag{93}$$

and the evolution of the shadow price of installed capital

$$\gamma \psi_t = \beta E\{\psi_{t+1} v(i_{t+1}/k_{t+1}) + \beta z_{t+1}^*\} | I_t. \tag{94}$$

There is a production function governing output,

$$y_t = a_t f(k_t, n_t) \tag{95}$$

and an accumulation equation governing the evolution of capital,

$$\gamma k_{t+1} = (1 - \delta) k_t + \phi(\frac{i_t}{k_t}) k_t. \tag{96}$$

Finally, there are equilibrium conditions in the markets for goods and money

$$c_t + i_t = y_t \tag{97}$$

$$P_t c_t = M_t \tag{98}$$

and a Fisher equation,

$$1 + r_t = E[(1 + R_t) \frac{P_t}{P_{t+1}}] | I_t. \tag{99}$$

C.4 Steady State Analysis

There is an invariance of the steady-state capital output ratio in this model, which is perhaps surprising given that firms are required to hold cash when they sell output, thus taxing capital income. The reason for this type of superneutrality is that an increase in investment reduces next period's cash dividend to the household (rather than the current period one) so that there is also a sense in which increased inflation lowers the cost of making investments to households. That is, combining the investment and capital efficiency conditions yields:

$$\gamma \psi_t = \beta\{\psi_{t+1} v(i_{t+1}/k_{t+1}) + \psi_{t+1} \phi'(i_{t+1}/k_{t+1}) z_{t+1}\},$$

where $z_t = Z_t/P_t = a_t \partial f(k_t, n_t)/\partial k_t$. Thus, in a steady state where $\psi_t = \psi_{t+1}$, it follows that i/k and z are linked together by:

$$\gamma = \beta\{v(i/k) + \phi'(i/k)z\}.$$

However, the accumulation equation for capital specifies that $\gamma = \phi(i/k) + (1-\delta)$ in a steady state, so that the marginal product $z = a\partial f/\partial k$ is exactly the same as in the RBC model. Hence, there is an equivalent real steady state in terms of the "great ratios."

However, on the labor side, there is a minor modification associated with the effect of inflation on nominal interest rates and, hence, the firm's cost of hiring labor. If we assume that we measure real labor compensation at the level of the firm (so that it includes expenditure for making "spending money loans" to workers), then it follows that the firm's labor cost fraction is:

$$s_n = \frac{na\frac{\partial f}{\partial n}}{y} = (1+R)\frac{Wn}{Py}$$

i.e., it exceeds labor income received by workers by the extent of the interest cost. This requires that we adjust the value of the preference parameter θ as follows:

$$\frac{1-\theta}{\theta}\frac{c/y}{l/n} = \frac{wn}{y} = s_n\frac{1}{1+R}$$

This reflects the adverse effect of the inflation tax on steady-state labor supply: to match an observed ratio of l/n we must therefore have a higher utility weight on consumption when the average inflation rate is higher. However, the effect of an empirically relevant value of R (which is a quarterly interest rate) indicates that this adjustment is minor. Essentially, the financial market frictions model shares the same steady state as the other models, in terms of great ratios, etc.

C.5 Log-linearization of Model Equations

Given the discussion above, it is straightforward to loglinearize most of the equations; we confine our discussion to those that have not been previously discussed in the context of the RBC model or are not direct.

The requirement that households pay cash for goods is simply:

$$\begin{aligned} &s_c[\log(P_t/P) + \log(c_t/c)] \\ &= (s_c - s_n^h)\log(S_t/S) + s_n^h[\log(W_t/W) + \log(n_t/n)] \end{aligned} \tag{100}$$

where by s_n^h we mean the adjusted household value discussed above. Next, the time allocation constraint is approximated as:

$$n\log(n_t/n) + l\log(l_t/l) + h\log(h_t/h) = 1 \tag{101}$$

and the extent of time spent in portfolio adjustment activity is:

$$\log(h_t/h) = (\gamma_P \frac{h'}{h})[\log(S_t/S) - \log(S_{t-1}/S)] \tag{102}$$

Given the specification of utility, the marginal rate of substitution condition is exactly:

$$[\log(l_t/l) - \log(c_t/c) = \log(w_t/w) \tag{103}$$

The log-linearization of the definition of Δ is facilitated by writing it as $\Delta_t = \frac{\partial u_t/\partial c_t}{P_t} - \frac{\partial u_t}{\partial l_t} h'_t \frac{1}{S_{t-1}} \gamma_P + \beta E\{\frac{\partial u_{t+1}}{\partial l_{t+1}} h'_{t+1} \frac{S_{t+1}}{(S_t)^2} \gamma_P\}|I_t = \Delta_{1t} - \Delta_{2t} + \Delta_{3t}$, so that it takes the form:

$$\log(\Delta_t/\Delta) = \frac{\Delta_1}{\Delta}\log(\Delta_{1t}/\Delta) - \frac{\Delta_2}{\Delta}\log(\Delta_{2t}/\Delta) + \frac{\Delta_3}{\Delta}\log(\Delta_{3t}/\Delta)$$

Further, by manipulating the steady-state version of this equation, it is possible to show that $\frac{\Delta_1}{\Delta} = \frac{1}{1-(1-\beta)d}$; $\frac{\Delta_2}{\Delta} = \frac{d}{1-(1-\beta)d}$; and $\frac{\Delta_3}{\Delta} = \frac{\beta d}{1-(1-\beta)d}$ with $d = \frac{h}{n}\frac{h'}{h}\ \gamma_P\ s_n(s_c - s_n)^{-1}$. The individual components are then easy to log-linearize. For example,

$$\begin{aligned}\log(\Delta_{2t}/\Delta) &= \theta(1-\sigma)\log(c_t/c) + [(1-\theta)(1-\sigma)-1]\log(l_t/l) \\ &+ (\tfrac{h''}{h'}\gamma_P)[\log(S_t/S) - \log(S_{t-1}/S)] - [\log(S_{t-1}/S)]\end{aligned}$$

The efficient evolution of this shadow price is described by:

$$E\{\log(\Delta_t/\Delta) - (R_t - R) - \log(\Delta_{t+1}/\Delta)\}|I_{ot} = 0, \tag{104}$$

and the evolution of the stock of spending money is:

$$\log(S_{(t+1)-1}/S) = \log(S_t/S) \tag{105}$$

The firm's efficiency condition is approximated as:

$$\log(W_t/W) = \log(P_t/P) + \log(a_t) + \xi_{nk}\log(k_t/k) + \xi_{nn}\log(n_t/n) - (R_t - R). \tag{106}$$

The remainder of the conditions, those governing investment and capital as well as the sources and uses of output are approximated in essentially the same manner as is in the RBC model. The monetary equilibrium condition is directly in loglinear form:

$$\log(P_t/P) + \log(c_t/C) = \log(M_t/M) \tag{107}$$

D Appendix: Detailed Results Underlying Table 5.2

Tables D.1-D.14 provide detail for the results summarized in Table 5.2.

Table D.1
Characteristics of Business Cycle Components
Baseline Real Business Cycle Model

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
2.74	0.50	0.99	0.98	0.91	0.64	-0.32	0.65	0.58	0.82	0.98	0.25
	0.51	0.40	0.35	0.82	0.40	-0.27	0.44	0.36	0.13	0.46	-0.04
		8.74	1.00	0.85	0.62	-0.31	0.63	0.57	0.85	0.98	0.27
			2.02	0.82	0.61	-0.30	0.62	0.56	0.87	0.98	0.28
				0.83	0.62	-0.34	0.64	0.56	0.60	0.87	0.14
					1.92	-0.22	0.88	0.26	0.51	0.61	-0.35
						1.25	-0.67	-0.17	-0.38	-0.38	0.47
							2.51	0.28	0.58	0.65	-0.50
								1.05	0.32	0.51	-0.08
									0.43	0.89	0.23
										1.60	0.26
											2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.12	0.03	0.23	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.23	0.03	-0.12
c	0.46	0.57	0.65	0.70	0.70	0.64	0.50	0.32	0.12	-0.06	-0.21	-0.32	-0.40
i	-0.19	-0.05	0.15	0.41	0.68	0.90	0.99	0.94	0.76	0.52	0.27	0.08	-0.07
n	-0.22	-0.08	0.12	0.38	0.65	0.88	0.98	0.94	0.77	0.53	0.29	0.10	-0.05
w	0.15	0.30	0.47	0.66	0.83	0.92	0.91	0.77	0.54	0.28	0.05	-0.14	-0.28
M	0.05	0.15	0.27	0.40	0.52	0.61	0.64	0.60	0.49	0.34	0.18	0.02	-0.12
P	0.07	-0.01	-0.08	-0.14	-0.20	-0.26	-0.32	-0.39	-0.44	-0.47	-0.46	-0.40	-0.31
M-P	0.01	0.12	0.24	0.37	0.50	0.60	0.65	0.65	0.59	0.49	0.36	0.22	0.06
R	0.22	0.23	0.25	0.33	0.44	0.55	0.58	0.50	0.32	0.08	-0.13	-0.29	-0.38
r	-0.48	-0.38	-0.23	0.00	0.29	0.59	0.82	0.93	0.91	0.79	0.61	0.44	0.29
a	-0.23	-0.09	0.11	0.37	0.64	0.87	0.98	0.95	0.80	0.57	0.35	0.16	0.01
v	-0.28	-0.22	-0.12	0.01	0.14	0.23	0.25	0.19	0.09	-0.01	-0.07	-0.08	-0.04

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.2
Characteristics of Business Cycle Components
Baseline Sticky Price Model

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.56	0.99	0.98	0.60	0.98	0.42	-0.16	0.42	-0.31	-0.45	0.74	0.52
	1.48	0.94	0.60	0.99	0.44	-0.14	0.43	-0.37	-0.51	0.72	0.54
		1.82	0.57	0.93	0.37	-0.19	0.38	-0.18	-0.33	0.74	0.46
			1.20	0.71	-0.01	0.18	-0.05	-0.05	-0.35	-0.09	0.55
				1.67	0.39	-0.09	0.37	-0.34	-0.51	0.62	0.57
					1.92	-0.25	0.96	-0.56	-0.49	0.48	-0.35
						0.58	-0.49	0.12	0.04	-0.37	0.33
							2.14	-0.54	-0.45	0.53	-0.41
								1.24	0.91	-0.30	-0.29
									1.45	-0.22	-0.43
										1.26	0.19
											2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.24	-0.11	0.10	0.37	0.67	0.91	1.00	0.91	0.67	0.37	0.10	-0.11	-0.24
c	-0.21	-0.08	0.13	0.41	0.70	0.92	0.99	0.89	0.64	0.33	0.05	-0.15	-0.27
i	-0.28	-0.16	0.03	0.30	0.60	0.86	0.98	0.92	0.72	0.45	0.18	-0.02	-0.16
n	-0.20	-0.12	0.00	0.18	0.38	0.54	0.60	0.52	0.33	0.11	-0.08	-0.19	-0.23
w	-0.22	-0.09	0.12	0.39	0.68	0.91	0.98	0.87	0.62	0.31	0.03	-0.17	-0.28
M	0.23	0.30	0.38	0.46	0.51	0.50	0.42	0.28	0.11	-0.06	-0.21	-0.33	-0.41
P	0.18	0.18	0.18	0.15	0.09	-0.02	-0.16	-0.30	-0.40	-0.44	-0.40	-0.32	-0.20
M-P	0.16	0.22	0.30	0.38	0.43	0.45	0.42	0.33	0.21	0.06	-0.08	-0.21	-0.32
R	-0.21	-0.39	-0.57	-0.69	-0.68	-0.55	-0.31	-0.05	0.17	0.32	0.39	0.40	0.39
r	-0.19	-0.35	-0.53	-0.67	-0.73	-0.65	-0.45	-0.18	0.11	0.33	0.46	0.51	0.49
a	-0.19	-0.10	0.05	0.25	0.47	0.65	0.74	0.72	0.60	0.43	0.25	0.09	-0.04
v	-0.22	-0.08	0.11	0.31	0.48	0.56	0.52	0.38	0.19	0.03	-0.08	-0.10	-0.08

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.3
Characteristics of Business Cycle Components
Baseline Liquidity Effect Model

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a
1.64	0.98	0.91	0.51	0.65	0.65	-0.26	0.98	-0.28	-0.41	1.00
	1.68	0.81	0.55	0.63	0.63	-0.30	1.00	-0.44	-0.47	0.97
		1.79	0.37	0.60	0.60	-0.13	0.81	0.11	-0.21	0.92
			0.17	0.65	0.64	0.20	0.55	-0.70	-0.58	0.44
				1.95	1.00	0.55	0.63	-0.32	-0.23	0.61
					1.92	0.55	0.63	-0.31	-0.22	0.61
						1.56	-0.30	0.09	0.23	-0.29
							1.68	-0.44	-0.47	0.97
								1.68	0.59	-0.22
									1.91	-0.35
										1.60

B. Cross Autocorrelations with Output
[$Cor(x_t y_{t+k})$, where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.12	0.03	0.23	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.23	0.03	-0.12
c	-0.07	0.09	0.29	0.53	0.77	0.94	0.98	0.87	0.65	0.38	0.13	-0.06	-0.20
i	-0.21	-0.09	0.08	0.29	0.54	0.77	0.91	0.92	0.81	0.63	0.43	0.25	0.10
n	0.15	0.20	0.28	0.39	0.50	0.55	0.51	0.38	0.19	-0.03	-0.21	-0.33	-0.39
w	0.09	0.19	0.31	0.44	0.56	0.64	0.65	0.59	0.46	0.30	0.12	-0.04	-0.18
M	0.09	0.19	0.31	0.44	0.56	0.64	0.65	0.59	0.46	0.30	0.13	-0.03	-0.17
P	0.18	0.14	0.07	-0.03	-0.14	-0.23	-0.26	-0.22	-0.13	-0.04	0.02	0.03	0.01
M-P	-0.07	0.09	0.29	0.53	0.77	0.94	0.98	0.87	0.65	0.38	0.13	-0.06	-0.20
R	-0.25	-0.31	-0.39	-0.46	-0.48	-0.43	-0.28	-0.08	0.15	0.35	0.48	0.55	0.56
r	-0.31	-0.39	-0.51	-0.63	-0.69	-0.62	-0.41	-0.11	0.19	0.40	0.50	0.51	0.48
a	-0.17	-0.02	0.18	0.43	0.69	0.90	1.00	0.94	0.76	0.52	0.29	0.09	-0.05

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.4
Characteristics of Business Cycle Components
Real Business Cycle Model, Independent Driving Processes

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
2.74	0.50	0.99	0.98	0.91	0.00	-0.45	0.51	0.64	0.82	0.98	0.00
	0.51	0.40	0.35	0.82	0.00	-0.19	0.22	0.37	0.13	0.46	0.00
		8.74	1.00	0.85	0.00	-0.45	0.51	0.63	0.85	0.98	0.00
			2.02	0.82	0.00	-0.45	0.51	0.62	0.87	0.98	0.00
				0.83	0.00	-0.39	0.44	0.61	0.60	0.87	0.00
					1.92	0.49	0.20	-0.23	0.00	0.00	0.00
						2.88	-0.76	-0.16	-0.52	-0.49	0.52
							2.57	0.00	0.58	0.55	-0.58
								2.24	0.34	0.57	-0.24
									0.43	0.89	0.00
										1.60	0.00
											2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

Series	k: -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.12	0.03	0.23	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.23	0.03	-0.12
c	0.46	0.57	0.65	0.70	0.70	0.64	0.50	0.32	0.12	-0.06	-0.21	-0.32	-0.40
i	-0.19	-0.05	0.15	0.41	0.68	0.90	0.99	0.94	0.76	0.52	0.27	0.08	-0.07
n	-0.22	-0.08	0.12	0.38	0.65	0.88	0.98	0.94	0.77	0.53	0.29	0.10	-0.05
w	0.15	0.30	0.47	0.66	0.83	0.92	0.91	0.77	0.54	0.28	0.05	-0.14	-0.28
M	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P	0.21	0.14	0.04	-0.08	-0.22	-0.35	-0.45	-0.49	-0.47	-0.40	-0.32	-0.23	-0.14
M-P	-0.23	-0.15	-0.05	0.09	0.25	0.40	0.51	0.55	0.53	0.45	0.35	0.26	0.16
R	0.12	0.21	0.34	0.48	0.61	0.68	0.64	0.50	0.29	0.06	-0.12	-0.25	-0.33
r	-0.48	-0.38	-0.23	0.00	0.29	0.59	0.82	0.93	0.91	0.79	0.61	0.44	0.29
a	-0.23	-0.09	0.11	0.37	0.64	0.87	0.98	0.95	0.80	0.57	0.35	0.16	0.01
v	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.5
Characteristics of Business Cycle Components
Real Business Cycle Model, η=1

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.59	1.00	1.00	-0.99	1.00	0.62	-0.48	0.68	-0.62	-0.31	1.00	0.25
	1.59	1.00	-0.99	1.00	0.62	-0.48	0.68	-0.62	-0.31	1.00	0.25
		1.58	-0.99	1.00	0.62	-0.48	0.68	-0.62	-0.31	1.00	0.25
			0.00	-0.99	-0.61	0.45	-0.66	0.64	0.32	-0.99	-0.29
				1.59	0.62	-0.48	0.68	-0.62	-0.31	1.00	0.25
					1.92	-0.34	0.88	-0.56	-0.17	0.61	-0.35
						1.38	-0.75	0.36	0.41	-0.47	0.41
							2.72	-0.58	-0.33	0.67	-0.45
								1.24	0.63	-0.60	-0.32
									1.86	-0.28	-0.19
										1.60	0.26
											2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
c	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
i	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
n	0.13	-0.02	-0.22	-0.46	-0.72	-0.91	-0.99	-0.92	-0.73	-0.48	-0.24	-0.04	0.11
w	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
M	0.12	0.21	0.32	0.44	0.55	0.62	0.62	0.55	0.43	0.27	0.10	-0.06	-0.20
P	-0.20	-0.28	-0.37	-0.45	-0.50	-0.52	-0.48	-0.39	-0.27	-0.16	-0.06	0.03	0.11
M-P	0.18	0.29	0.41	0.54	0.65	0.70	0.68	0.59	0.44	0.27	0.10	-0.05	-0.20
R	-0.11	-0.27	-0.45	-0.62	-0.72	-0.73	-0.62	-0.42	-0.19	0.02	0.18	0.27	0.32
r	-0.37	-0.45	-0.56	-0.65	-0.67	-0.56	-0.31	-0.00	0.28	0.47	0.55	0.55	0.52
a	-0.14	0.01	0.21	0.45	0.71	0.91	1.00	0.94	0.75	0.51	0.27	0.08	-0.07
v	-0.26	-0.19	-0.09	0.03	0.15	0.23	0.25	0.20	0.10	0.00	-0.06	-0.06	-0.02

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.6
Characteristics of Business Cycle Components
Real Business Cycle Model, m_r=-.10

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
2.74	0.50	0.99	0.98	0.91	0.64	-0.56	0.71	0.15	0.82	0.98	-0.09
	0.51	0.40	0.35	0.82	0.40	-0.51	0.54	0.47	0.13	0.46	-0.46
		8.74	1.00	0.85	0.62	-0.53	0.68	0.09	0.85	0.98	-0.04
			2.02	0.82	0.61	-0.51	0.66	0.06	0.87	0.98	-0.01
				0.83	0.62	-0.62	0.73	0.32	0.60	0.87	-0.29
					1.92	-0.44	0.86	-0.06	0.51	0.61	-0.03
						1.80	-0.84	-0.63	-0.19	-0.47	0.68
							3.15	0.32	0.42	0.64	-0.40
								1.13	-0.35	0.02	-0.98
									0.43	0.89	0.40
										1.60	0.04
											12.15

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.12	0.03	0.23	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.23	0.03	-0.12
c	0.46	0.57	0.65	0.70	0.70	0.64	0.50	0.32	0.12	-0.06	-0.21	-0.32	-0.40
i	-0.19	-0.05	0.15	0.41	0.68	0.90	0.99	0.94	0.76	0.52	0.27	0.08	-0.07
n	-0.22	-0.08	0.12	0.38	0.65	0.88	0.98	0.94	0.77	0.53	0.29	0.10	-0.05
w	0.15	0.30	0.47	0.66	0.83	0.92	0.91	0.77	0.54	0.28	0.05	-0.14	-0.28
M	0.05	0.15	0.27	0.40	0.52	0.61	0.64	0.60	0.49	0.34	0.18	0.02	-0.12
P	-0.32	-0.42	-0.52	-0.60	-0.65	-0.64	-0.56	-0.41	-0.21	0.01	0.21	0.36	0.47
M-P	0.21	0.33	0.46	0.59	0.69	0.74	0.71	0.60	0.42	0.20	-0.01	-0.19	-0.34
R	0.52	0.55	0.56	0.54	0.47	0.34	0.15	-0.10	-0.35	-0.56	-0.68	-0.73	-0.70
r	-0.48	-0.38	-0.23	0.00	0.29	0.59	0.82	0.93	0.91	0.79	0.61	0.44	0.29
a	-0.23	-0.09	0.11	0.37	0.64	0.87	0.98	0.95	0.80	0.57	0.35	0.16	0.01
v	-0.57	-0.59	-0.59	-0.55	-0.46	-0.30	-0.09	0.15	0.38	0.57	0.69	0.73	0.71

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.7
Characteristics of Business Cycle Components
Sticky Price Model, Independent Driving Processes

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.98	0.99	0.98	0.82	0.98	0.42	0.30	0.31	0.23	-0.34	0.16	0.56
	1.71	0.94	0.81	0.99	0.48	0.34	0.35	0.14	-0.41	0.15	0.59
		2.72	0.80	0.94	0.32	0.22	0.23	0.37	-0.22	0.17	0.49
			2.48	0.89	0.35	0.40	0.15	0.26	-0.32	-0.43	0.48
				2.25	0.46	0.37	0.30	0.18	-0.40	-0.00	0.58
					1.92	0.54	0.82	-0.31	-0.41	0.00	0.00
						1.09	-0.03	-0.13	-0.24	-0.32	0.41
							1.61	-0.29	-0.33	0.21	-0.27
								1.56	0.71	-0.02	-0.31
									1.24	0.08	-0.61
										1.26	0.00
											2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.17	-0.05	0.13	0.39	0.68	0.91	1.00	0.91	0.68	0.39	0.13	-0.05	-0.17
c	-0.12	-0.00	0.18	0.44	0.71	0.92	0.99	0.88	0.64	0.35	0.09	-0.10	-0.21
i	-0.23	-0.13	0.05	0.31	0.61	0.86	0.98	0.92	0.71	0.44	0.19	0.01	-0.10
n	-0.17	-0.09	0.05	0.27	0.53	0.73	0.82	0.75	0.55	0.31	0.09	-0.06	-0.15
w	-0.14	-0.03	0.15	0.41	0.69	0.91	0.98	0.88	0.64	0.35	0.09	-0.09	-0.20
M	0.33	0.42	0.50	0.56	0.57	0.53	0.42	0.27	0.09	-0.07	-0.20	-0.29	-0.35
P	0.46	0.54	0.60	0.62	0.59	0.47	0.30	0.08	-0.13	-0.30	-0.41	-0.46	-0.48
M-P	0.08	0.14	0.19	0.24	0.29	0.31	0.31	0.27	0.20	0.12	0.04	-0.03	-0.09
R	-0.32	-0.37	-0.40	-0.35	-0.21	0.00	0.23	0.41	0.50	0.49	0.43	0.35	0.29
r	-0.15	-0.27	-0.41	-0.53	-0.58	-0.51	-0.34	-0.11	0.11	0.28	0.36	0.38	0.37
a	-0.07	-0.03	0.00	0.05	0.09	0.13	0.16	0.18	0.18	0.17	0.14	0.11	0.07
v	0.01	0.12	0.28	0.46	0.60	0.64	0.56	0.36	0.12	-0.09	-0.23	-0.30	-0.32

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.8
Characteristics of Business Cycle Components
Sticky Price Model, $\eta^{-1}=0$

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

	y	c	i	n	w	M	P	M-P	R	r	a	v
y	3.11	0.19	0.99	0.93	0.86	0.52	-0.07	0.50	0.85	0.89	0.60	0.55
c		0.52	0.07	0.01	0.60	0.33	0.34	0.22	-0.20	-0.19	0.05	0.17
i			10.36	0.95	0.80	0.49	-0.11	0.48	0.88	0.93	0.60	0.54
n				2.87	0.81	0.35	0.01	0.32	0.92	0.92	0.33	0.59
w					0.89	0.47	0.20	0.39	0.62	0.63	0.30	0.57
M						1.92	-0.23	0.97	0.26	0.34	0.48	-0.35
P							0.49	-0.44	-0.03	-0.15	-0.42	0.36
M-P								2.09	0.25	0.34	0.54	-0.41
R									1.12	0.92	0.32	0.49
r										0.73	0.53	0.51
a											1.26	0.19
v												2.03

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.33	-0.20	0.02	0.32	0.65	0.90	1.00	0.90	0.65	0.32	0.02	-0.20	-0.33
c	0.48	0.65	0.77	0.79	0.69	0.47	0.19	-0.09	-0.31	-0.43	-0.46	-0.45	-0.41
i	-0.39	-0.29	-0.08	0.23	0.58	0.86	0.99	0.93	0.70	0.38	0.07	-0.15	-0.29
n	-0.39	-0.28	-0.07	0.23	0.56	0.83	0.93	0.85	0.60	0.27	-0.02	-0.23	-0.34
w	-0.03	0.15	0.39	0.65	0.85	0.94	0.86	0.63	0.30	-0.03	-0.29	-0.45	-0.51
M	0.05	0.12	0.22	0.34	0.44	0.51	0.52	0.46	0.35	0.20	0.04	-0.11	-0.23
P	0.20	0.23	0.25	0.25	0.20	0.09	-0.07	-0.26	-0.43	-0.54	-0.55	-0.48	-0.36
M-P	-0.00	0.06	0.15	0.25	0.36	0.45	0.50	0.49	0.42	0.31	0.16	0.01	-0.13
R	-0.34	-0.33	-0.22	0.01	0.34	0.65	0.85	0.85	0.66	0.36	0.04	-0.20	-0.32
r	-0.51	-0.48	-0.34	-0.06	0.30	0.66	0.89	0.94	0.79	0.52	0.22	-0.02	-0.16
a	-0.36	-0.31	-0.20	-0.02	0.20	0.43	0.60	0.68	0.66	0.56	0.42	0.27	0.13
v	-0.31	-0.19	-0.01	0.22	0.44	0.56	0.55	0.41	0.20	-0.02	-0.17	-0.22	-0.19

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.9

Characteristics of Business Cycle Components

Sticky Price Model, Money Driving Process Only

A. Correlation Matrix

(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.31	0.99	0.98	1.00	1.00	0.64	0.55	0.62	0.38	-0.13	0.00	0.00
	1.13	0.94	0.98	1.00	0.73	0.60	0.71	0.26	-0.23	0.00	0.00
		1.80	0.99	0.96	0.49	0.44	0.46	0.55	0.03	0.00	0.00
			1.49	0.99	0.58	0.47	0.57	0.44	-0.07	0.00	0.00
				1.49	0.70	0.57	0.68	0.31	-0.19	0.00	0.00
					1.92	0.85	0.96	-0.42	-0.73	0.00	0.00
						0.70	0.67	-0.19	-0.39	0.00	0.00
							1.38	-0.49	-0.82	0.00	0.00
								1.18	0.82	0.00	0.00
									0.70	0.00	0.00
										0.00	0.00
											0.00

B. Cross Autocorrelations with Output

[Cor($x_t y_{t+k}$), where y_t is output

and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
c	-0.02	0.13	0.33	0.55	0.78	0.95	0.99	0.89	0.68	0.43	0.18	-0.02	-0.17
i	-0.23	-0.09	0.10	0.36	0.63	0.86	0.98	0.94	0.78	0.55	0.32	0.13	-0.01
n	-0.18	-0.03	0.17	0.42	0.68	0.90	1.00	0.95	0.77	0.53	0.30	0.11	-0.04
w	-0.06	0.09	0.29	0.52	0.76	0.94	1.00	0.91	0.71	0.45	0.21	0.01	-0.14
M	0.50	0.64	0.76	0.84	0.87	0.80	0.64	0.41	0.14	-0.10	-0.30	-0.44	-0.53
P	0.54	0.67	0.78	0.83	0.82	0.72	0.55	0.31	0.07	-0.16	-0.34	-0.47	-0.56
M-P	0.42	0.55	0.66	0.75	0.79	0.75	0.62	0.41	0.16	-0.07	-0.25	-0.37	-0.46
R	-0.62	-0.60	-0.51	-0.35	-0.12	0.15	0.38	0.55	0.62	0.61	0.56	0.49	0.42
r	-0.54	-0.58	-0.59	-0.56	-0.47	-0.32	-0.13	0.08	0.27	0.39	0.46	0.47	0.46
a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
v	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.10
Characteristics of Business Cycle Components
Sticky Price Model, $m_{R=0}$

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
2.39	0.99	0.98	0.87	0.99	0.51	0.17	0.40	0.00	-0.20	0.52	0.63
	2.27	0.95	0.90	1.00	0.48	0.22	0.36	-0.07	-0.27	0.45	0.66
		2.75	0.79	0.94	0.54	0.05	0.47	0.14	-0.06	0.63	0.55
			2.27	0.94	0.31	0.44	0.12	-0.24	-0.44	0.04	0.74
				2.80	0.45	0.27	0.32	-0.10	-0.31	0.37	0.68
					1.92	-0.06	0.94	0.21	0.13	0.48	-0.31
						0.74	-0.41	-0.49	-0.55	-0.48	0.51
							2.10	0.36	0.31	0.61	-0.46
								3.32	0.97	0.46	-0.31
									3.07	0.39	-0.46
										1.26	-0.02
											2.46

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

							k						
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.32	-0.18	0.05	0.35	0.66	0.91	1.00	0.91	0.66	0.35	0.05	-0.18	-0.32
c	-0.28	-0.14	0.09	0.40	0.70	0.93	0.99	0.88	0.61	0.28	-0.02	-0.24	-0.36
i	-0.37	-0.25	-0.05	0.25	0.57	0.84	0.98	0.94	0.75	0.47	0.18	-0.06	-0.22
n	-0.11	0.05	0.27	0.53	0.77	0.90	0.87	0.69	0.38	0.04	-0.25	-0.44	-0.53
w	-0.25	-0.10	0.13	0.43	0.73	0.94	0.99	0.85	0.57	0.24	-0.07	-0.28	-0.40
M	-0.14	-0.09	-0.00	0.12	0.27	0.41	0.51	0.54	0.50	0.41	0.28	0.14	0.01
P	0.45	0.51	0.56	0.57	0.51	0.38	0.17	-0.10	-0.37	-0.59	-0.71	-0.74	-0.68
M-P	-0.29	-0.26	-0.20	-0.09	0.07	0.24	0.40	0.53	0.59	0.58	0.50	0.39	0.25
R	-0.19	-0.38	-0.57	-0.68	-0.62	-0.37	0.00	0.37	0.61	0.67	0.56	0.37	0.20
r	-0.15	-0.36	-0.59	-0.75	-0.75	-0.55	-0.20	0.18	0.47	0.60	0.56	0.43	0.29
a	-0.52	-0.52	-0.44	-0.27	-0.02	0.27	0.52	0.68	0.72	0.67	0.57	0.44	0.31
v	-0.06	0.05	0.22	0.41	0.59	0.68	0.63	0.43	0.14	-0.15	-0.38	-0.50	-0.52

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.11
Characteristics of Business Cycle Components
Sticky Price Model, Money Process Only, $m_R=0$

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.31	0.98	0.94	1.00	0.99	0.97	0.73	1.00	0.05	-0.11	0.00	0.00
	1.23	0.86	0.97	1.00	0.96	0.77	0.98	-0.12	-0.25	0.00	0.00
		1.67	0.96	0.89	0.88	0.61	0.94	0.34	0.15	0.00	0.00
			1.48	0.98	0.94	0.68	1.00	0.12	-0.05	0.00	0.00
				1.59	0.97	0.75	0.99	-0.06	-0.21	0.00	0.00
					1.92	0.89	0.97	-0.04	-0.18	0.00	0.00
						0.74	0.73	-0.19	-0.27	0.00	0.00
							1.31	0.05	-0.11	0.00	0.00
								2.14	0.96	0.00	0.00
									1.58	0.00	0.00
										0.00	0.00
											0.00

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

Series	k: -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.09	0.07	0.27	0.51	0.75	0.93	1.00	0.93	0.75	0.51	0.27	0.07	-0.09
c	0.02	0.18	0.38	0.60	0.81	0.96	0.98	0.87	0.65	0.39	0.14	-0.06	-0.21
i	-0.27	-0.13	0.06	0.30	0.57	0.80	0.94	0.96	0.85	0.68	0.48	0.29	0.13
n	-0.16	-0.00	0.20	0.45	0.70	0.91	1.00	0.95	0.78	0.56	0.33	0.13	-0.02
w	-0.02	0.14	0.34	0.57	0.79	0.95	0.99	0.89	0.69	0.43	0.19	-0.01	-0.16
M	0.09	0.25	0.44	0.64	0.82	0.95	0.97	0.87	0.67	0.43	0.19	-0.02	-0.19
P	0.37	0.52	0.65	0.75	0.81	0.81	0.73	0.60	0.42	0.21	0.01	-0.18	-0.34
M-P	-0.09	0.07	0.27	0.51	0.75	0.93	1.00	0.93	0.75	0.51	0.27	0.07	-0.09
R	-0.50	-0.54	-0.57	-0.57	-0.47	-0.25	0.05	0.37	0.61	0.72	0.72	0.65	0.55
r	-0.40	-0.47	-0.57	-0.63	-0.59	-0.41	-0.11	0.23	0.50	0.64	0.65	0.57	0.48
a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
v	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.12
Characteristics of Business Cycle Components
Sticky Price Model, m_R=-.10

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a	v
1.44	0.99	0.97	0.71	0.97	-0.21	0.30	-0.25	-0.53	-0.64	0.10	0.66
	1.38	0.93	0.70	0.98	-0.22	0.34	-0.28	-0.59	-0.70	0.10	0.73
		1.67	0.69	0.92	-0.17	0.21	-0.20	-0.38	-0.50	0.11	0.51
			2.11	0.84	-0.52	0.58	-0.59	-0.07	-0.26	-0.62	0.27
				1.79	-0.32	0.44	-0.38	-0.48	-0.62	-0.11	0.64
					1.92	-0.54	0.97	-0.20	-0.28	0.48	-0.03
						0.64	-0.72	-0.04	-0.04	-0.50	0.21
							2.34	-0.15	-0.22	0.53	-0.08
								1.14	0.81	-0.47	-0.97
									0.86	-0.32	-0.80
										1.26	0.34
											12.15

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

Series	k: -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.21	-0.09	0.10	0.37	0.67	0.91	1.00	0.91	0.67	0.37	0.10	-0.09	-0.21
c	-0.16	-0.03	0.16	0.42	0.70	0.92	0.99	0.88	0.63	0.33	0.07	-0.12	-0.23
i	-0.29	-0.20	-0.03	0.24	0.56	0.83	0.97	0.91	0.70	0.41	0.15	-0.03	-0.15
n	-0.48	-0.43	-0.29	-0.06	0.24	0.53	0.71	0.74	0.63	0.45	0.28	0.15	0.07
w	-0.27	-0.15	0.04	0.31	0.61	0.87	0.97	0.90	0.67	0.39	0.14	-0.05	-0.16
M	0.39	0.37	0.32	0.22	0.09	-0.07	-0.21	-0.31	-0.38	-0.40	-0.39	-0.36	-0.32
P	-0.17	-0.08	0.03	0.14	0.24	0.29	0.30	0.27	0.23	0.17	0.13	0.10	0.07
M-P	0.36	0.33	0.25	0.14	0.01	-0.13	-0.25	-0.33	-0.37	-0.38	-0.36	-0.32	-0.28
R	-0.29	-0.42	-0.53	-0.62	-0.66	-0.64	-0.53	-0.36	-0.15	0.05	0.21	0.32	0.38
r	-0.21	-0.31	-0.43	-0.55	-0.66	-0.70	-0.64	-0.48	-0.23	0.03	0.23	0.36	0.42
a	0.40	0.45	0.47	0.44	0.36	0.24	0.10	-0.02	-0.12	-0.19	-0.24	-0.28	-0.29
v	0.18	0.32	0.46	0.60	0.70	0.73	0.66	0.51	0.29	0.07	-0.12	-0.25	-0.33

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.13
Characteristics of Business Cycle Components
Liquidity Effect Model, Independent Driving Processes

A. Correlation Matrix
(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a
1.58	0.98	0.90	0.05	0.06	0.06	-0.60	0.98	-0.06	-0.31	1.00
	1.63	0.80	0.09	0.05	0.05	-0.62	1.00	-0.23	-0.28	0.97
		1.71	-0.05	0.07	0.07	-0.47	0.80	0.32	-0.34	0.90
			0.17	0.67	0.65	0.45	0.09	-0.69	-0.12	-0.03
				1.95	1.00	0.75	0.05	-0.32	-0.06	-0.00
					1.92	0.75	0.05	-0.31	-0.06	0.00
						2.46	-0.62	-0.09	0.14	-0.64
							1.63	-0.23	-0.28	0.97
								1.67	-0.02	-0.01
									1.94	-0.28
										1.60

B. Cross Autocorrelations with Output
[Cor($x_t y_{t+k}$), where y_t is output
and x_t is the series in column 1]

Series	k: -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.11	0.04	0.24	0.48	0.73	0.93	1.00	0.93	0.73	0.48	0.24	0.04	-0.11
c	-0.10	0.05	0.24	0.48	0.72	0.91	0.98	0.91	0.71	0.47	0.23	0.03	-0.11
i	-0.11	0.02	0.20	0.42	0.65	0.83	0.90	0.84	0.67	0.45	0.23	0.05	-0.08
n	-0.01	-0.01	-0.01	0.01	0.02	0.04	0.05	0.05	0.03	0.02	0.00	-0.01	-0.01
w	-0.02	-0.01	-0.00	0.02	0.03	0.05	0.06	0.06	0.05	0.04	0.03	0.03	0.02
M	-0.02	-0.01	-0.00	0.02	0.03	0.05	0.06	0.06	0.05	0.04	0.03	0.03	0.02
P	0.05	-0.04	-0.16	-0.31	-0.45	-0.57	-0.60	-0.55	-0.43	-0.27	-0.12	-0.00	0.09
M-P	-0.10	0.05	0.24	0.48	0.72	0.91	0.98	0.91	0.71	0.47	0.23	0.03	-0.11
R	-0.01	-0.02	-0.03	-0.05	-0.06	-0.07	-0.06	-0.05	-0.02	0.00	0.02	0.03	0.03
r	-0.35	-0.42	-0.53	-0.63	-0.65	-0.54	-0.31	-0.02	0.26	0.45	0.53	0.53	0.50
a	-0.14	0.01	0.20	0.45	0.71	0.91	1.00	0.93	0.75	0.51	0.27	0.08	-0.07

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.

Table D.14

Characteristics of Business Cycle Components

Liquidity Effect Model, Money Process Only

A. Correlation Matrix

(Standard Deviation Shown on Diagonal)

y	c	i	n	w	M	P	M-P	R	r	a
0.12	0.56	-0.11	0.98	0.77	0.76	0.67	0.56	-0.71	-0.42	0.00
	0.37	-0.89	0.52	0.22	0.22	0.03	1.00	-0.97	0.33	0.00
		0.73	-0.07	0.16	0.16	0.34	-0.89	0.77	-0.62	0.00
			0.17	0.67	0.65	0.57	0.52	-0.69	-0.52	0.00
				1.95	1.00	0.98	0.22	-0.32	-0.24	0.00
					1.92	0.98	0.22	-0.31	-0.23	0.00
						1.88	0.03	-0.13	-0.30	0.00
							0.37	-0.97	0.33	0.00
								1.67	-0.13	0.00
									0.48	0.00
										0.00

B. Cross Autocorrelations with Output

[Cor($x_t y_{t+k}$), where y_t is output and x_t is the series in column 1]

	k												
Series	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-0.11	0.00	0.18	0.42	0.70	0.92	1.00	0.92	0.70	0.42	0.18	0.00	-0.11
c	0.25	0.35	0.48	0.61	0.69	0.68	0.56	0.35	0.10	-0.12	-0.28	-0.37	-0.43
i	-0.36	-0.42	-0.48	-0.49	-0.44	-0.31	-0.11	0.10	0.27	0.37	0.43	0.45	0.45
n	-0.14	-0.05	0.10	0.35	0.63	0.87	0.98	0.90	0.68	0.39	0.14	-0.03	-0.12
w	-0.30	-0.18	-0.02	0.21	0.45	0.65	0.77	0.79	0.71	0.58	0.45	0.33	0.23
M	-0.30	-0.19	-0.02	0.20	0.44	0.64	0.76	0.78	0.70	0.58	0.45	0.34	0.23
P	-0.36	-0.26	-0.11	0.09	0.31	0.53	0.67	0.73	0.70	0.62	0.52	0.42	0.32
M-P	0.25	0.35	0.48	0.61	0.69	0.68	0.56	0.35	0.10	-0.12	-0.28	-0.37	-0.43
R	-0.18	-0.28	-0.42	-0.59	-0.73	-0.79	-0.71	-0.51	-0.24	0.02	0.22	0.34	0.40
r	0.23	0.29	0.32	0.23	0.04	-0.21	-0.42	-0.48	-0.40	-0.23	-0.08	-0.01	-0.02
a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: These results were calculated from the spectra density matrix of the models using parameter values discussed in the text. See notes to table 2.2 for additional details.