MEASURING CHANGES IN THE VALUE OF THE NUMERAIRE

May 2007

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ABSTRACT

This paper estimates a common component in many price series that has an equiproportional effect on all prices. Changes in this component can be interpreted as changes in the value of the numeraire since, by definition, they leave all relative prices unchanged. The first aim of the paper is to measure these changes. The paper provides a framework for identifying this component, suggests an estimator for the component based on a dynamic factor model, and assesses its performance relative to alternative estimators. Using 187 U.S. time-series on prices, we estimate changes in the value of the numeraire from 1960 to 2006, and further decompose these changes into a part that is related to relative price movements and a residual 'exogenous' part. The second aim of the paper is to use these estimates to investigate two economic questions. First, we show that the size of exogenous changes in the value of the numeraire helps distinguish between different theories of pricing, and that the U.S. evidence argues against several strict theories of nominal rigidities. Second, we find that changes in the value of the numeraire are significantly related to changes in real quantities, and discuss interpretations of this apparent non-neutrality.

JEL codes: E31, C43, C32
Keywords: Inflation, Money illusion, Monetary neutrality, Price index

*We are grateful to Marc Giannoni, Alan Kackmeister, David Lopez-Salido, Ulrich Müller, Brad Strum, and Michael Woodford for useful comments. Reis thanks the Hoover Institution at Stanford University for its support and hospitality under a Campbell national fellowship. Support was provided by the National Science Foundation through grant SES-0617811.

1. Introduction

This paper measures the common component in price changes that has an equiproportional effect on all prices. By construction, changes in this component leave all relative prices unchanged. They are changes in the absolute level of prices, separated from relative-price changes, so they are a measure of "pure" inflation. They are also the changes in the value of the special good that serves the role of the unit of account in which all prices in the economy are denominated. This good is the numeraire and changes in pure inflation are changes in the value of the numeraire.[1]

The numeraire plays a central role in most economic models. A basic principle from rational economic behavior states that relative, but not absolute, prices matter for determining real quantities. Regardless of whether consumers have non-standard preferences, of whether they face constraints on borrowing, of how they perceive trade-offs between the present and the future, or of whether they have full information or not, as long as consumers behave optimally and can shift expenditure between two goods, they will equate the marginal rate of substitution between these goods to their relative prices. Likewise, regardless of the features of technology or corporate control, firms that minimize costs equate the relative marginal product of two inputs to their relative prices, and a proportional increase in all input and output prices leaves production unchanged. In choosing how much to consume or produce, agents consider relative benefits and relative costs, regardless of the unit of account in which these are denominated. Therefore, regardless of how markets clear or how equilibrium is defined, changes in units should have no effect on any real quantity, so pure inflation should be neutral with respect to quantities. There is no "money illusion" in almost all economic models.

This key result follows from the thought experiment: "what if the value of the numeraire changed?" But, as an empirical matter, does it ever change and what drives these changes? Changes in social convention or currency reforms are clear cases of changes in the unit of account. When Turkey in 2005 decided to drop 6 zeroes from its currency or when many European countries in 1999 adopted the Euro, the value of the numeraire changed in these countries. These events are rare, so they do not provide a consistent measure of pure inflation over time for a country. Moreover, these events typically come with many other changes in policy and institutions, so it is difficult to separately identify their effects.

This paper aims to answer two questions: first, does the value of the numeraire ever change? That is, are there changes in the value of the numeraire in the post-war U.S. that are unrelated to relative price changes? And second, are these changes neutral? That is, are changes in real variables unrelated to these changes in the value of the numeraire? David Hume was perhaps the first to ask these questions. In the essay "Of Interest" (1752), he first posed the thought experiment of changing the value of the numeraire: "Were all the gold in England annihilated at once, and one and twenty shillings substituted in the place of every guinea, would money be more plentiful or interest lower?" Hume answered by affirming the neutrality of the numeraire: "No surely: We should only use silver instead of gold. Were gold rendered as common as silver, and silver as common as copper; would money be more plentiful or interest lower? We may assuredly give the same answer. Our shillings would then be yellow, and our halfpence white; and we should have no guineas. No other difference would ever be observed; no alteration on commerce, manufactures, navigation, or interest; unless we imagine, that the colour of the metal is of any consequence." Hume moved further and combined the irrelevance of the numeraire with another proposition—that changes in money lead to changes in the value of the numeraire—to conclude that money itself is neutral. Ever since then, this has been of the most hotly debated issues in economics.

The first part of this paper spans sections 2, 3 and 4 and it is dedicated to estimating changes in the value of the numeraire. Section 2 begins with a framework that relates prices to the value of the numeraire, describes estimators for the change in the value of the numeraire, and discusses identification. We use the term NPI, for numeraire price index, for an estimator for the value of the numeraire that is based on goods prices; section 2 discusses several NPIs, and proposes a particular one, a dynamic NPI based on a dynamic factor model for relative prices that captures the salient serial and cross correlation patterns in a large panel of prices.

Section 3 applies this estimator to U.S. data from 1959 to 2006 on the prices of the different components of personal consumption expenditures (PCE) in the national income accounts. This provides an estimate of changes in the value of the numeraire in the United States, which, while broadly similar to other measures of inflation in their decade-to-decade movements, have some interesting differences. The correlation of

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[1] Since prices are denominated in units of the numeraire, the price of the numeraire is always one, but its

changes in NPI inflation with those in the PCE deflator is only 0.64 and it is less volatile and more persistent.

Section 4 extracts exogenous movements in the numeraire, that is movements in the NPI that are unrelated to relative price movements. The dynamic model that motivates the NPI suggests a straightforward decomposition to isolate these exogenous shocks. Applying this decomposition to the U.S. data, we find that exogenous changes in the numeraire are reasonably large: on average, they account for 7% of the variance of individual price changes and 30% of the variance in the NPI. Section 4 also considers the problem of how to compare different estimators of changes in the value of the numeraire. We measure the performance of an estimator by its accuracy measured by mean squared error (mse) and develop an approach to estimate the mse. We compare alternative estimates of the NPI to find that simple unweighted averages are inaccurate, appropriately weighted averages are more accurate, and our dynamically weighted estimator yields additional increases in accuracy.

The second part of this paper, in sections 5 and 6, applies the estimates of the changes in the value of the numeraire to shed light on two issues. The first application, in section 5, is to assess the validity of different models of pricing. We show that different assumptions on pricing imply different predictions on whether there should be exogenous changes in the numeraire. The quantitatively large exogenous movements in the numeraire that we find in the data cast suspicion on strict theories of nominal rigidities in which some sectoral prices are always sticky, and support instead softer versions of these theories where all prices can change with respect to some shocks. We compare our estimates with artificial data generated by different models of pricing and find further support for this conclusion.

Section 6 answers the question of whether exogenous changes in the numeraire are related to changes in measures of real activity. We find that they are, since we typically reject the null hypothesis that changes in several measures of real activity are independent of exogenous changes in the numeraire. To interpret this result, we show that in general-equilibrium models, exogenous movements in the numeraire can be due to one of two possible shocks that we cannot separately identify. The first is the Hume-shock described above that should be neutral (changes in the units); the second is a particular aggregate shock that changes the total amount of goods while leaving all

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value, in terms of all other goods, can change.

relative prices unchanged (changes in the total account). If we assume that there are only Hume-shocks, then our findings suggest that there is money illusion. If we assume that there is no money illusion, then our findings suggest the existence of the non-neutral aggregate shocks described above, although as it turns out, the empirical analysis suggests these shocks cannot explain a large fraction of the variance of both real quantities and the NPI.

1.1. Related literature

The question of whether there is money illusion dates back at least to John Maynard Keynes (1936) and Irving Fisher (1928). Shafir, Diamond and Tversky (1998) and Fehr and Tyran (2001) presented micro-evidence based on surveys and experiments that changes in the value of the numeraire affect real choices. It is an open question how prevalent is this behavior, and how significant it is for economic aggregates. At the macro level, the absence of measures of changes in the value of the numeraire has precluded tests of money illusion. There is a large literature, going back at least to Phillips (1958), that relates movements in output to movements in inflation. This literature measures inflation typically using the consumer price index (CPI), which measures changes in the cost of living, or the GDP or PCE deflators, which were built to obtain real quantities. By construction, each of these price indices reflects relative-price changes so, in principle, they are not good measures of pure inflation.

Our use of large scale dynamic factor models draws on the literature on their estimation by maximum likelihood (e.g., Quah and Sargent, 1993, and Doz, Giannone and Reichlin, 2006) and principal components (e.g., Forni, Hallin, Lippi and Reichlin, 2000, Bai and Ng, 2002, and Stock and Watson, 2005). We provide a new set of questions to apply these methods. Cristadoro, Forni, Reichlin and Veronesi (2005) use these methods to estimate a common factor on a panel with price and quantity series and ask a different question: whether it forecasts inflation well. Amstad and Potter (2007) address yet another issue, using dynamic factor models to build measures of the common component in price changes that can be updated daily. The common factor in both of these papers is not a measure of changes in the value of the numeraire, since it affects different prices differently. Closest to the approach in this paper is Bryan and Cecchetti (1993), who estimate changes in the value of the numeraire as a common component in a panel of 36 price series assuming that relative prices are independent across goods and use it to forecast future inflation. The goal of this paper is not forecasting, but rather to estimate the changes in the value of the numeraire and the part of these that is unrelated to relative price changes, and we impose less restrictive assumptions than Bryan and Cecchetti (1993).[2]

Our approach of testing pricing theories by using a factor model to extract a common shock is shared with Boivin, Giannoni and Mihov (2007). They extract a macroeconomic shock using many series that include prices and real quantities, estimate the impulse response of individual prices to this shock, and then compare their shape to the predictions of different models of nominal rigidities. In our model, the response of any individual price of the numeraire is, by construction, an immediate jump of the same proportional size as the shock. Thus, the mere existence of a shock with this particular common pattern of responses across all prices provides a test of theories of nominal rigidities. Moreover, in contrast to the Boivin, Giannoni and Mihov, we use only price data (and no quantity data) to build a price index so that we can later ask if it is neutral with respect to quantities.

There is a large literature on measuring inflation and building price indices, and the NPI fits into the class of stochastic price indices described in Selvanathan and Rao (1994). However, there have been very few attempts to measure changes in the value of the numeraire. Jevons (1865) considered this problem and proposed calculating the arithmetic mean of the change in price of each good. Edgeworth replied that it would be better to weight each individual price change by the inverse of its sample standard deviation.[3] The relative efficiency of these estimators is discussed in section 4. Finally, the NPI is a member of the recent family of dynamic price indices that use dynamic models to measure inflation. Other members in this family are the measures of the cost of living in Reis (2005), the deflators to obtain measures of quantities and welfare in Weitzman (1976) and Basu and Fernald (2002), the optimal inflation target for central banks with nominal rigidities in Mankiw and Reis (2003), Huang and Liu (2005), and Strum (2006), and the inflation index for bonds to create a safe real asset in Geanakoplos (2005).

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[2]Bryan, Cecchetti and O'Sullivan (2002) use a version of the Bryan-Cecchetti (1993) model to study the importance of asset prices for an inflation index.

[3]Diewert (1995) traces Edgeworth's proposal, which he scattered across several writings. Diewert (1995) and Wynne (1999) criticize these as well as other stochastic price indices for typically imposing strong arbitrary assumptions on the correlation of relative prices and for not being suited to measure the cost of living. The NPI proposed here imposes less structure on the dynamics of relative price shocks, and is not intended to measure the cost of living, but rather to measure the numeraire.

2. Measuring the numeraire

2.1 A framework for measuring the numeraire

Consider an economy with $M + 1$ goods, indexed by $i = 0, 1, \ldots, M$, where $i = 0$ corresponds to the numeraire good and prices (in units of the numeraire good) are given by P_i. A fall in the value of the numeraire good by 1% means that P_i increases by 1% for $i = 1, \ldots, M$. This suggests writing $\pi_{it} = n_t + r_{it}$, where $\pi_{it} = \ln(P_{it}/P_{it-1})$ is the rate of price change for the i'th good, n_t is the rate of change of the value of numeraire good, and r_{it} is the rate of change of the price of the i'th good relative to other non-numeraire goods. Throughout this paper we refer to n_t as the "numeraire" instead of the more long-winded "change in the value of the numeraire good." The numeraire is identified by the condition that relative price changes must add up to zero, so that the model relating the numeraire to prices is:[4]

$$\pi_{it} = n_t + r_{it} \tag{1}$$

$$M^{-1}\sum_{i=1}^{M} r_{it} = 0. \tag{2}$$

These two equations statistically define the numeraire. Two questions naturally arise: how do numeraire and relative-price changes relate to shocks in economic models? And, do (1) and (2) suffice to identify the numeraire?

To answer the first question, consider a simple Walrasian economy with a representative consumer deriving utility u($\boldsymbol{C_t}$) from the consumption of M goods $\boldsymbol{C_t} = \{C_{i,t}\}_{i=1}^{M}$, which trade at prices $\boldsymbol{P_t} = \{P_{i,t}\}_{i=1}^{M}$. Optimal behavior implies equating the marginal rate of substitution between goods i and j to their relative price. The consumer receives exogenous endowments $\boldsymbol{E_t} = \{E_{i,t}\}_{i=1}^{M}$, that are subject to idiosyncratic exogenous shocks. The endowments cannot be stored over time so, for markets to clear, $\boldsymbol{C_t} = \boldsymbol{E_t}$. Finally, there is an exogenous endowment E_{0t} of a good that yields no utility, call it sea shells, but which the consumer uses to denominate its prices. Since E_{it} is the available amount of good i and P_{it} its unit-price in shells, then $P_{it}E_{it}$ is how many shells the endowment of good i is worth. Since consumers can exchange goods for shells, the total

[4] Allowing for the possibility of an infinite number of goods, (2) should be interpreted as a limit with $M \to \infty$.

worth of all goods in shells, $\sum_i P_{it} E_{it}$, must equal the amount of available shells, E_{0t}.

The M prices in this economy are determined by the system of M equations:

$$\frac{P_{it}}{P_{1t}} = \frac{\partial u(\mathbf{C_t}) / \partial C_{it}}{\partial u(\mathbf{C_t}) / \partial C_{1t}} \quad \text{for } i = 2,\ldots,M \tag{3}$$

$$\sum_{i=1}^{M} P_{it} E_{it} = E_{0t} \tag{4}$$

If the endowment of shells increases, these equations imply that all prices increase by exactly the proportion of this increase. This is an example of a change in the value of the numeraire as described by Hume. If the endowments of all goods except the numeraire change between t-1 and t by the proportions $\{\Delta_i\}_{i=1}^{M}$ that average to zero $\sum_{i=1}^{M} \Delta_i = 0$ and the utility function is Cobb-Douglas, then the changes in log prices add up to zero, giving an example of a pure change in relative prices. Finally, general changes in endowments will lead to both changes in the numeraire and relative prices, with each identified by conditions (1)-(2).[5]

This leads us to the second question, on identification. First note that the formulation of the model in terms of price inflation eliminates the units of the goods. That is, the perennial issue of comparing apples and oranges (here, literally, in their prices) is taken care of. A more subtle problem involves re-bundling of goods. If in the data, we observe fruit salad, we would not want the estimate of changes in the numeraire to depend on whether the salad includes more or less apples relative to oranges. This problem can be formalized as follows: let $\pi_t = (\pi_{1t} \; \ldots \; \pi_{Mt})'$ denote the vector of inflation rates for the M goods, so that $\pi_t = n_t l + r_t$ where l is an M×1 unit vector and r_t is the vector of relative inflation rates r_{it}. Consider rearranging these goods into M bundles. The inflation rates for the new bundles are $\tilde{\pi}_t = \Omega \pi_t$, where Ω_{ij} is the value share of good j in bundle i. Because the rows of Ω sum to one, $\Omega l = l$, so that $\tilde{\pi}_t = n_t l + \tilde{r}_t$, where $\tilde{r}_t = \Omega r_t$. Note that $M^{-1}\sum_i \tilde{r}_{it} = M^{-1}\sum_i \sum_j \Omega_{ij} r_{jt} = M^{-1}\sum_j \Omega_j r_{jt}$, where $\Omega_j = \sum_i \Omega_{ij}$. Therefore, $M^{-1}\sum_i \tilde{r}_{it} = 0$ as long as the arrangement of bundles is uncorrelated with the changes in relative prices, that is $M^{-1}\sum_j \Omega_j r_{jt} = 0$.

It is not clear whether this condition holds in the data. It is possible that when the relative price of apples and oranges changes, makers of fruit salad change the amount of each fruit in the salad in a way that is missed by the statistical agency's price collectors. If they mistakenly record the change in price as if the salad was the same, then the task of separating changes in the numeraire from changes in relative prices is impossible. To make any progress, we must assume (or hope) that this is not the case.

2.2. Estimating the numeraire and the NPI

Were prices on all M goods available, equations (1) and (2) imply that the average change in prices would exactly measure n_t. We suppose instead that data are available for a subset of N goods over T time periods. The sampling of these N goods from the population of M goods introduces statistical uncertainty in the data motivating estimators other than the arithmetic average.[6]

We define a numeraire price index (NPI) as an estimator of n_t constructed from data on individual prices alone and focus on estimators that are linear functions of the inflation rates. A "static" NPI can then be defined by a set of weights $\omega = \{\omega_i\}$, so that the NPI is:

$$\hat{n}_t(\omega) = N^{-1}\sum_{i=1}^{N}\omega_i\pi_{i,t}. \qquad (5)$$

Combining (5) with (1), $\hat{n}_t(\omega) = \bar{\omega}\, n_t + N^{-1}\sum_{i=1}^{N}\omega_i r_{it}$, where $\bar{\omega}$ is the sample mean of ω_i. Evidently, $\hat{n}_t(\omega)$ will be a consistent estimator of n_t (as $N \to \infty$) if $\bar{\omega} = 1$, and if the weights and relative inflation rates are asymptotically uncorrelated, so that $N^{-1}\sum_{i=1}^{N}\omega_i r_{it} \xrightarrow{p} 0$, where the randomness in this quantity is associated with the selection of the N goods from the population of M goods and, potentially, from the choice of weights ω_i.[7]

A dynamic NPI is similarly defined as:

[5] Buiter (2007) discusses the role of the numeraire in new Keynesian models.

[6] We will ignore the introduction of new goods (an increase in M) and assume that the statistical assumptions about the N goods remain satistified throughout the sample period under study.

[7] Bryan and Cecchetti (1993) discuss the correlation between relative price changes and the weights used to construct the CPI and PCE deflator.

$$\hat{n}_t(\omega) = N^{-1}\sum_i\sum_j \omega_{ij}\pi_{it-j}. \qquad (6)$$

In this case, $\hat{n}_t(\omega)$ = $\omega(\mathrm{L})n_t + N^{-1}\sum_i\sum_j \omega_{ij}r_{it-j}$ where $\omega(\mathrm{L}) = \left(N^{-1}\sum_{i=1}^{N}\omega_{ij}\right)\mathrm{L}^j$ and L is the lag operator. This dynamic estimator will be consistent for $\omega(\mathrm{L})n_t$ (which may not equal n_t) if $N^{-1}\sum_i\sum_j \omega_{ij}r_{it-j} \xrightarrow{p} 0$.

These conditions define NPIs that consistently estimate the numeraire or filtered versions of the numeraire ($\omega(\mathrm{L})n_t$), but they still allow for many alternative estimators. One natural way to compare different NPIs and choose among them is by their accuracy, which we will measure by the mean square error (mse), $E[\hat{n}_t(\omega) - n_t]^2$. Section 4 estimates the mse for a variety of static estimators (including the estimators proposed by Jevons and Edgeworth), and compares the efficiency of these estimators to a dynamic estimator that we now describe.

2.3. A dynamic estimator

Different efficient NPIs can be motivated by different statistical assumptions on the behavior of the relative prices used to form the index. For example, suppose that $\{r_{it}\}$ are serially and cross-sectionally uncorrelated and homoskedastic, and there are no restrictions on n_t. In this case, the Gauss-Markov theorem implies that the arithmetic mean of π_{it} is the best linear unbiased estimator of n_t; that is, the Jevons estimator is efficient. If $\{r_{it}\}$ are heteroskedastic, but the other Jevons assumptions are satisfied, then the Edgeworth NPI that weighs each good by the inverse of its variance corresponds to the GLS estimator and dominates the Jevons NPI.

Of course, these assumptions underlying the efficiency of the Jevons and Edgeworth estimators are implausible. Given a large number of price series, it is infeasible to allow for arbitrarily general covariance properties of the relative prices. Yet, it is certainly the case that changes in relative prices are serially and cross-sectionally correlated, and n_t is plausibly serially correlated. These characteristics of the inflation process suggest that dynamic estimators may be more accurate than the Jevons and Edgeworth or other static estimators, raising the challenge of choosing the appropriate weights for the dynamic estimator.

Our approach is to use optimal weights implied by a particular parametric model of inflation. The assumptions underlying this parametric model, while not as strong as the Jevons-Edgeworth assumptions, are also open to challenge. That said, the purpose of the model is not to capture all of the subtleties of the large number of inflation series used in the analysis but rather to provide a set of weights that lead to an accurate NPI. Put differently, the parametric model is meant to capture the key properties of the inflation series as they pertain to estimation of n_t, even though the model is surely misspecified in other respects. As we proceed, we will attempt to guard against faulty inference by using econometric procedures that are robust to this misspecification or, when this is impossible, by investigating the robustness of our conclusions to alternative formulations of the model.

The parametric model is the dynamic factor model:

$$\pi_{it} = n_t + r_{it} \tag{7}$$

$$r_{it} = \lambda_i' f_t + u_{it} \tag{8}$$

$$\Phi(L)\begin{pmatrix} n_t \\ f_t \end{pmatrix} = \varepsilon_t \tag{9}$$

$$\rho_i(\mathrm{L})u_{it} = \alpha_i + e_{it} \tag{10}$$

with restrictions

$$\sum_{i=1}^{N} \lambda_i = 0 \tag{11}$$

$$E(e_{it}) = 0,\ var(e_{it}) = \sigma_i^2,\ E(\varepsilon_t) = 0,\ var(\varepsilon_t) = Q \tag{12}$$

and

$$\{e_{it}\}, \{e_{jt}\}_{j \neq i}, \{\varepsilon_t\} \text{ are mutually and serially uncorrelated sequences} \tag{13}$$

Equation (8) uses a factor model to represent changes in relative prices where f_t is a $k\times 1$ vector of common factors and u_{it} denotes good-specific changes in relative prices. The factors f_t capture aggregate shocks that lead to relative price changes. These could be shocks that affect all sectors, like changes in aggregate productivity, government spending, and monetary policy, or shocks that affect many but not all sectors (so $\lambda_{ij} = 0$ for some i and j) like changes in energy prices, shocks to the weather that affect many goods in the food sector but not manufacturing, or perhaps changes in exchange rates if they affect the prices of tradables but not of non-tradables. Past research focusing on the output of different sectors found that this factor structure for the covariance between shocks can flexibly account for the main features of the economic data (Stock and Watson, 1989, 2005, Forni et al, 2000). Restriction (11) imposes the constraint that the common relative-price shocks f_t do not affect average inflation and identifies n_t as in (2).

Equation (9) allows n_t and f_t to evolve jointly through a parsimonious VAR. If changes in the numeraire may be at least partially related to changes in monetary policy, it is likely that these occur in response to developments in the rest of the aggregate economy.

The good-specific shocks u_{it} are allowed to be serially correlated via (10), but are mutually uncorrelated and uncorrelated with n_t and f_t (see (13)). The restriction that they are uncorrelated with n_t means that changes in the numeraire are not allowed to respond to changes in the price of a single good (say canned tuna) and no other good. The assumption that u_{it} are mutually uncorrelated means that all of the joint correlation between r_{it} is explained by the common factors f_t. This is a strong (and arguably unrealistic) assumption, but a large literature on approximate factor models (Forni et al, 2000, Bai, 2003, Stock and Watson, 2005, Doz et al, 2006) suggests that estimates of n_t and f_t are robust to sufficiently small correlation in these terms. And, as we have already noted, we will use inference methods that are robust to this specification.

Given values of the parameters in (7)-(13), the dynamic NPI is constructed as the minimum mean square error estimator of n_t using the observations on $\{\pi_{i\tau}\}_{i=1,\tau=1}^{i=N,\tau=T}$. Given a set of parameters, this estimator can be computed using standard signal extraction techniques (e.g., the Kalman filter and smoother).

2.4 Estimating the dynamic factor model

Conditional on the number of factors (a problem discussed below), values of the unknown parameters in (7)-(12) can be estimated by Gaussian maximum likelihood, and these are the estimates that we use. There are two concerns with these MLEs. First, as documented in Ball and Mankiw (1995), Bryan and Cecchetti (1994), and Bryan, Cecchetti and Wiggins (1997), disaggregated inflation rates are skewed and fat-tailed. In general, skewness is not a major concern for Gaussian MLEs in models such as this (see Watson, 1989), but excess kurtosis is more problematic. To mitigate the problem we follow Bryan, Cecchetti and Wiggins (1997), by pre-treating the data to eliminate large outliers (see section 3 for more details).

The second concern is the computational complexity of MLEs in models of this size. For example, our benchmark model includes n_t and two additional relative price factors, a VAR(4) for (9), univariate AR(1) models for (10), and N = 187 price series. There are 971 parameters to be estimated.[8] Despite its complexity, the linear latent variable structure of the model makes it amenable to estimation using an EM algorithm with the "E-step" computed by Kalman smoothing and the "M-step" by linear regression. See Watson and Engle (1983) and Shumway and Stoffer (1982) for a general discussion of the EM algorithm in models such as this and the appendix for specific discussion of the implementation used here.

3. The U.S. NPI (1960-2006)

3.1 The data

The price data are monthly chained price indices for personal consumption expenditures by major type of product and expenditure from 1959:1 to 2006:6.[9] Inflation is measured in percentage points at an annual rate using final month of the quarter prices: $\pi_{it} = 400\times\ln(P_{it}/P_{it-1})$, where P_{it} are prices for March, June, September, and December.[10] Prices are for goods at the highest available level of disaggregation that have data for the majority of dates, which gives 214 series. We then excluded series with unavailable observations (9 series), more than 20 quarters of 0 price changes (4 series), and series j if there is another series i such that $Cor(\pi_{it}, \pi_{jt}) > 0.99$ and $Cor(\Delta\pi_{it}, \Delta\pi_{jt}) > .99$ (14 series). This left N = 187 price series. Large outliers were evident in some of the inflation series, and these observations were replaced with local medians. A detailed description of the data and transformations are given in the appendix.[11]

3.2. Choosing the order of the model

[8]The number of unknown parameters is 186 + 185 (λ_i) + 187 (ρ_i) + 187 (α_i) + 187 ($var(e_i)$) + 36 (Φ) + 3 (Q) = 971, where these values reflect the normalizations used for identification.

[9] In principle, any price denominated in dollars could be a part of the sample, including financial prices or the price of labor. We look at these series because they are consistent for a long period of time, but future research could include many more prices. Section 6 will consider financial prices more explicitly.

[10]We considered using monthly, rather than quarterly, price changes, but found that the extra idiosyncratic error in monthly price changes outweighed the benefit of more observations.

[11] The level of aggregation, both over time and across goods, affects the interpretation of the results. In the time dimension, if we looked at century price changes it would be hard to believe that there is money illusion, while if we looked at minute-by-minute price changes, it is hard to believe that prices are not very sticky. In the goods' dimension, we ignore relative-price changes for goods within a sector. It is important to keep in mind that our findings refer to quarterly sectoral prices.

There are three integer parameters that govern the order of the model: the number of lags in the VAR in (9), the number of lags in the univariate autoregressions in (10), and the number of latent factors (n_t and f_t) to include in the model. The benchmark specification uses a VAR(4) model in (9) and AR(1) models in (10); diagnostic tests (not reported) suggested that these lag lengths were adequate, and sections 5 and 6 discusses the robustness of the results to these choices.

Determining the appropriate number of factors is less clear cut. The Bai and Ng (2002) criteria provide consistent (as $\min(N,T) \to \infty$) estimators of the number of the number of factors in models such as this. These estimators are based on the number of dominant eigenvalues of the covariance (or correlation) matrix of the data. Panel (a) of figure 1 shows the largest twenty eigenvalues of the sample correlation matrix. Evidently there is one large eigenvalue, but the figure suggests that it is unclear how many additional factors are necessary. This uncertainty is evident in the Bai-Ng estimates: their ICP_1, ICP_2 and ICP_3 estimates are 2 factors, 1 factor, and 11 factors respectively.

Because of this uncertainty, we estimated models with 1 through 4 factors. These models were unrestricted versions of (7)-(13) that do not impose the restrictions on the factor loadings (unity on the first factor and the restriction (11)). Panel (b) of figure 1 summarizes the fraction of variance explained by the factors for each of these models and for each of the 187 inflation series.[12] To make the figure easier to read, the series have been ordered by the fraction of variance explained by the 1-factor model. The uncertainty in the appropriate number of factors is evident here as well: the second factor improves the fit for several series, but it unclear whether a third, fourth or fifth factor is necessary. In our benchmark model we will use 3 factors (n_t and two relative price factors in f_t). We summarize the key results for other choices in sections 5 and 6.

Finally, we must address the issue of unit roots in the model. Several authors (see Pivetta and Reis, 2007, for a detailed discussion) have noted that aggregate inflation is persistent and consistent with a process that contains a unit root in its autoregressive representation. In the models that we have estimated, this persistence is evident by an estimated root in $\Phi(L)$ in (9) that is very close to unity and a few other large roots. In contrast, the estimated roots of $\rho_i(L)$ in (10) are not large. In our benchmark model, we impose two unit roots in $\Phi(L)$; that is, n_t and one of the relative price factors are treated

[12] These measures were computed as $R_i^2 = 1 - [\widehat{\text{var}(u_i)} / s_{\pi_i}^2]$, where $\widehat{\text{var}(u_i)}$ is the estimated variance of u_i implied by the estimated model and $s_{\pi_i}^2$ is the sample variance of π_i.

as I(1) processes. Results in which these unit roots are not imposed turn out to be very similar, and we summarize results for these models in sections 5 and 6.

3.3. Estimates of the U.S. NPI

Figure 2 shows estimates of the NPI constructed from (7)-(13) using the MLEs of the parameter estimates (these are given in the appendix). Also plotted is the more familiar measure of inflation associated with the PCE deflator. The figure shows that the broad movements in the NPI are similar to those of most measures of inflation: after being low in the 1960s, the NPI rose in the 1970s, peaking twice in 1974 and 1980, and then declined throughout the 1980s, remaining low in the 1990s and 2000s. There are a few interesting differences, however. The NPI is smoother and its quarter-to-quarter changes can often go in the opposite direction of those in the PCE deflator. In two episodes, the two differed substantially: in 1986 when the PCE deflator fell sharply while the NPI barely moved, and between 1997 and 1999, when inflation was significantly higher according to the NPI than according to the PCE deflator.

Table 1 displays summary statistics for the NPI and the PCE deflator. The sample correlation between the levels of the two series is high (0.93), but the NPI is less volatile and more persistent than the PCE deflator and the correlation of the changes in the two measures is only 0.64. Table 1 also compares the NPI with the core PCE deflator that excludes food and energy prices. The correlation of its changes with those in the NPI is even lower (0.48), and while the core PCE deflator is less volatile than the PCE deflator, the standard deviation of its changes is still 85% larger than that of the NPI. The last two columns of table 1 compare the NPI with the Jevons and Edgeworth estimates of π_t. Again, the NPI is less volatile and more persistent than these series. As we discuss in section 4, this is consistent with less measurement error in the NPI.

3.4. Comparison with the unrestricted factor model

The NPI model imposes the restrictions that the loading on the first factor must be one for all goods (7), while the loadings on the other factors must each add up to zero across goods (11). We now investigate how restrictive are these conditions.

Figure 3 summarizes the fit of unrestricted factor models without these restrictions. Panel (a) of figure 3 shows the increase in fit, measured as the fraction of (sample) variance of π_i explained by the factors. The median (across the 187 price series) increase in R^2 is less than 1%, and 80% of the series show increases in fit that are less than 3%. The unrestricted model appears to fit better only for a small number of price series: for 10 series the increase in R^2 exceeds 10%.

Panel (b) of figure 3 plots the NPI from the restricted model (previously shown in figure 2) with the estimated first factor from the unrestricted model after the factors were rotated to mimic the factors in the restricted model.[13] The NPI and the first factor from the unrestricted model are essentially identical.

Panels (c) and (d) summarize results from estimating the value of θ_i in the regression:

$$\pi_{it} = \theta_i n_t + \lambda_i' f_t + u_{it}. \tag{14}$$

When $\theta_i = 1$, this corresponds to the model used to construct the NPI. One cannot estimate this regression because n_t and f_t are not observed, but as noted in Watson (1986) and discussed in section 6 below, inference can be carried out by estimating a regression that replaces n_t and f_t with their Kalman smoothed estimates. Panel (c) shows the ordered values of the OLS estimates of θ_i, and panel (d) shows the ordered values of the (4-lag Newey-West) t-statistic testing that θ_i =1. While most of the estimated values of θ_i are close to 1, panel (d) shows far more rejections of the restriction than would be expected by sampling error. For example, over 30% of the *t*-statistics fall outside the standard 5% critical values and over 20% fall outside the 1% critical values. These results suggest that, as a formal matter, the unit factor loading restriction in (7) appears to be rejected by the data. That said, the results in panels (a) and (b) suggest that little is lost by imposing this restriction.

4. Exogenous movements and measures of performance

4.1. Defining the exogenous component of the NPI

Some, perhaps even all, of the movements in the value of the numeraire are associated with current, past, or perhaps future changes in relative prices. Because relative prices movements are inherently non-neutral, it is useful to decompose the NPI

[13] The factors were rotated so that the factor loadings on the first factor average to unity, the loadings on the other factors average to zero, and the factor loadings are orthogonal.

into a component that is correlated with relative price changes and a component that is uncorrelated with these changes. This latter component is associated with what might be called "pure" numeraire shocks, and is given by:

$$v_t = n_t - E\left[n_t \middle| \{r_{i\tau}\}_{i=1,\tau=1}^{N,T}\right] \tag{15}$$

where E is the expectations operator.

In the model (7)-(13), linearity implies that the expectations operator E is the linear projection operator $\hat{E}$ and n_t and f_t are uncorrelated with idiosyncratic changes in sectoral prices (u_{it}), so that $E\left[n_t \middle| \{r_{i\tau}\}_{i=1,\tau=1}^{N,T}\right] = \hat{E}\left[n_t \mid \{f_\tau\}_{\tau=1}^{T}\right]$. Furthermore, because $(n_t \quad f_t')$ follows the VAR in (9), the estimated stochastic process for v_t is readily calculated from the estimated VAR coefficients, as is the estimated realization of v_t.[14]

4.2. Estimates of exogenous movements in the NPI

Figure 4 plots estimates of $n_t - n_{t-8}$ and its decomposition into exogenous shocks ($v_t - v_{t-8}$) and relative-price components. (The figure shows 2-year changes because these are smoother and easier to interpret than quarter-to-quarter changes.[15]) Often n_t and its two components move together. For example, the two spikes in the NPI in the mid and late 1970s came with both large exogenous and relative-price shocks. Likewise, the disinflation of the early 1980s has a large relative-price component, but it is also the largest exogenous contraction in the NPI in the sample. Sometimes though, the two components of the NPI move in different directions. The disinflation of 1991-92 is almost exclusively due to changes in the exogenous component, while the disinflation of the early 2000s comes in spite of the exogenous component being relatively high, and is due to large relative-price contractions.

The estimated VAR implies a standard deviation of Δn_t of 0.75 percent, and a standard deviation of Δv_t of 0.40 percent. This implies that 30% of the variance of n_t is associated with exogenous (non-relative price) factors and the remaining 70% is associated with changes in relative prices. Results analogous to those shown in panel (b) of figure 1, suggests that on average across the 187 series, n_t explains 23% of the sample

[14] The appendix describes how these calculations can be performed.
[15] We show the changes in n_t and its decomposition instead of the levels because n_t is an I(1) variable.

variance of changes in individual prices, so that 7% (0.30×0.23) of the variance of individual inflations is associated with exogenous changes in n_t.

Evidently, there are large exogenous changes in the numeraire in the post-war U.S. that account for a significant share of the overall variance of the numeraire and of the variance of prices.

4.3. Estimating the MSE of estimators of n_t.

So far we have focused on the NPI constructed using the optimal signal extraction weights associated with the estimated dynamic factor model in (7)-(13). But how well does this estimator perform relative to other, perhaps simpler, estimators of n_t?

We assess performance of an estimator by its accuracy, measured by its mean squared error (mse). As discussed in section 2, static estimators of n_t have the form $\hat{n}_t(\omega) = N^{-1}\sum_{i=1}^{N}\omega_i\pi_{i,t}$, where the ω_i are weights that determine the estimator. If the weights average to unity, $\hat{n}_t(\omega) = n_t + N^{-1}\sum_{i=1}^{N}\omega_i r_{it}$, so the mse of the estimator is $E\left(N^{-1}\sum_{i=1}^{N}\omega_i r_{it}\right)^2$. The mse will depend on the weights ω_i (which are known) and the variances and covariances of r_{it} (which are unknown). Because relative price changes are cross-correlated, the covariance terms are likely to be a large component in the mse. Accounting for these covariances presents a challenge.

Our approach uses the decomposition $r_{it} = \lambda_i' f_t + u_{it}$, where $\sum_{i=1}^{N}\lambda_i = 0$ identified the f factors as relative price shocks. Suppose that $N^{-1}\sum_{i=1}^{N}\omega_i\lambda_i$ is sufficiently small so that the mse is dominated by the term $E\left(N^{-1}\sum_{i=1}^{N}\omega_i u_{it}\right)^2$. If the good-specific shocks u_{it} are mutually uncorrelated (as assumed, for example, in the parametric factor model), then it would be reasonably straightforward to estimate the mse using weighted averages of squared residuals. However, while the f_t factors may successfully capture the important aggregate movements in relative prices, they are unlikely to capture all of the variability in relative prices. That is, the elements of u_{it} are likely to be weakly cross-correlated, and this cross-correlation may be quantitatively important for the mse. We use an estimator of the mse that allows for quite general patterns of cross correlation, by exploiting the ability to form uncorrelated groups of prices.

The details for this mse estimator are provided in the appendix, but the basic idea behind it is straightforward, and can be described using the Jevons estimator $\hat{n}_t^J = \bar{\pi}_t$, where $\bar{\pi}_t$ is the cross-sectional average of π_{it}. The sampling error in $\hat{n}_t^J$ is $N^{-1}\sum_{i=1}^{N} u_{it}$, so that the mse of $\hat{n}_t^J$ is given by the usual "long-run" variance of u_{it}, but where "long-run" refers to the cross-section dimension. A variety of cross-sectional long-run variance estimators have been proposed, and we use an estimator based on grouping that is particularly flexible for panel data such as ours.

To see how the estimator works, consider breaking the data into two groups; for simplicity suppose that the groups are of equal size with the first $N/2$ prices in the first group and the final $N/2$ prices in the second group. Let $\hat{n}_{1t}^J$ denote the estimator constructed using data in the first group and $\hat{n}_{2t}^J$ denote the estimator constructed from data in the second group, and suppose for simplicity that there are no relative-price factors f_t. In this case, $\hat{n}_{1t}^J - \hat{n}_{2t}^J = 2(N^{-1}\sum_{i=1}^{N/2} u_{it} - N^{-1}\sum_{i=N/2+1}^{N} u_{it})$. The key grouping assumption is that average error in the first group is uncorrelated with the average error in the second group, which means two terms on the right-hand side of the equality are uncorrelated. In this case the variance of their difference is the same as the variance of their sum: $E(\hat{n}_{1t}^J - \hat{n}_{2t}^J)^2 = 4E(N^{-1}\sum_{i=1}^{N/2} u_{it} + N^{-1}\sum_{i=N/2+1}^{N} u_{it})^2 = 4E(N^{-1}\sum_{i=1}^{N} u_{it})^2 = 4\times\text{mse}(\hat{n}_t^J)$. This suggests that the mse of $\hat{n}_t^J$ can be estimated by the time-series average of the $(\hat{n}_{1t}^J - \hat{n}_{2t}^J)^2$. Importantly, this calculation makes no assumption about the covariance of u_{it} terms within groups. It only assumes that the average value across groups is uncorrelated, and thus is allows potentially rich cross-section correlation patterns. The appendix discusses this grouping mse estimator in detail.

4.4. The accuracy of different estimators of n_t.

Table 2 shows the estimated mse of six estimators of n_t: the Jevons estimator, the Edgeworth estimator, a static estimator that uses weights equal to the PCE expenditure shares in 2005, two principal components estimators, and the dynamic estimator associated with the estimated dynamic factor model. The expenditure-share estimator is a version of the PCE deflator using fixed weights in place of chained weights. The first principal components estimator (labeled PC-Covariance in the table) constructs an NPI as the first principal component of the sample covariance matrix of the inflation data, where the principal component is scaled so that the weights sum to unity. The second principal components estimator (labeled PC-Correlation) is constructed in the same way, but using the sample correlation matrix of the inflation rates.

The simple average used by the Jevons estimator produces an estimator with a large root mean square error (rmse): it is nearly 1 percentage point for the annual change in inflation, and is only slightly smaller for the level of n_t. The Edgeworth estimator is considerably more accurate. The expenditure share estimator performs very poorly even relative to Jevons estimator. The reason is the expenditure share estimator puts considerably weight on a small number of series: over 50% of the weight is placed on only 18 price series, and 1 series (owner-occupied housing) receives over 10% of the weight. The principal components estimator based on the covariance matrix performs very poorly, but the analogous estimator based on the correlation matrix is the most accurate static estimator. The dynamic NPI estimator improves upon all of the static estimators. It is over five times more efficient the Jevons estimator (where efficiency is measured by the ratio the mean squared errors) and over 50% more efficient than the Edgeworth estimator.

There are two additional points to note about the dynamic estimator's mse. First, this estimator would necessarily be more accurate than other estimators if all of the assumption of (7)-(13) were satisfied and the parameters of the model were accurately estimated. However, as we stressed when we introduced the model, this is unlikely to be the case. Importantly, the estimates of the rmse shown in table 2 are based on assumptions much weaker than the assumptions underlying (7)-(13), and the rmse shown in table 2 differs from the estimate computed under (7)-(13). For example, the Kalman smoother associated with (7)-(13) reports a rmse of 0.22 for the level of n_t, which is considerably smaller than the value of 0.32 reported in table 2. The difference between the two estimates suggests, as expected, that the there is positive correlation among the u_{it} terms. The second point is that because the dynamic estimator puts weights on leads and lags of π_{it}, it is biased in the sense that its error includes the component $\omega(L)n_t - n_t$ (see section 2 and the appendix). This component accounts for roughly 15% of the estimator's mse.

Sections 2-4 have considered statistical issues related to the NPI. The next two sections discuss some of the implications of the resulting NPI.

5. Application 1: Assessing pricing theories

Different theories of how firms set prices give different answers to the question: does the numeraire ever exogenously change? Estimates of the exogenous component of the numeraire can therefore provide valuable information on what theories of pricing are consistent with the data. This section shows how to use the information in our estimates of the numeraire.

In flexible-price models, prices equal whatever the firms wish them to be at any time. Some aggregate shocks affect the desired prices of all firms equally, so all prices move in tandem, and the numeraire can change exogenously in these models. For example, exogenous monetary policy shocks lead to exogenous changes in the numeraire in these models.

In models with nominal rigidities, there is a discrepancy between desired and actual prices. For our purposes, theories of nominal rigidities can be grouped into two sets. *Strict* theories of rigid prices assume that, at any date, there are always some prices in the economy that cannot respond to current conditions. In this group are the time-dependent models of sticky prices of Taylor (1980) and Calvo (1983) as well as the state-dependent model of Sheshinski and Weiss (1977), since in these models there are always some firms that keep their prices fixed. Also in this group are the sticky-contracts model of Fischer (1977) and the sticky-information model of Mankiw and Reis (2002), which assume that there are always some firms that cannot contract on or are not aware of the current news and so cannot respond to them. In these strict models of pricing, there are no exogenous changes to the numeraire. It is never the case that, in response to a shock, all firms change their prices in exactly the same proportion at the same time since there are always some firms that do not adjust at all.

Soft theories of price rigidity instead assume that prices respond imperfectly to some shocks, leading to nominal rigidities and monetary non-neutrality, but are able to respond immediately to some other shocks. The classic example is the Lucas (1972) model where all firms respond immediately to anticipated money changes, but differentially with respect to unanticipated money shocks. This may also be the case in the inattention model of Mackowiak and Wiederholt (2007), if firms choose to monitor closely changes in the numeraire so they can react almost instantly to them, whereas they only slowly become aware of other shocks. Models with sticky but indexed prices like in Yun (1996) also belong in this group if there are shocks to the anchor to which prices are indexed. As long as one of the shocks to which all prices can respond immediately affects all firms equally, then soft theories predict that there will be exogenous changes in the numeraire.

In the U.S. data, we found that there are exogenous changes in the numeraire that are quantitatively large and account for a large share of the variance of the numeraire and individual price changes. We take this as *prima facie* evidence against strict theories of pricing rigidities. We elaborate on this conclusion further by considering in turn: (i) the role of the relative-price common shocks in estimating exogenous changes in the numeraire, (ii) the role of heterogeneity in using our quarterly sectoral results, (iii) alternative specifications of the empirical model, and (iv) the quantitative implications of our results against the models.

5.1. The numeraire in a simple model of staggered prices

Consider a simple economy with one firm producing each good i. There is one I(1) shock m_t that makes all optimal prices rise by the same proportion, but firms adjust their prices only every two periods, so that when a shock hits, some firms react to it instantly while others do so one period later. In this case, inflation for good i is: $\pi_{it} = I_{it} m_t + I_{it-1} m_{t-1} + u_{it}$, where I_{it} is an indicator function that equals one if firm i adjust at t and is zero otherwise. Because firms adjust every two periods, $I_{it} + I_{it-1} = 1$, so rearranging the expression above:

$$\pi_{it} = m_t + I_{it-1}\, \Delta m_{t-1} + u_{it}. \tag{16}$$

Data generated by this model would fit the setup in (7)-(8) and (11) with a relative-price factor that is function of Δm_t. Projecting m_t on leads and lags of the relative price factor Δm_t gives a perfect fit so v_t is zero.

This simple example illustrates a more general point. If firms staggeredly update prices to a common shock that eventually (but not instantly) moves all prices in the same proportion, then all of the changes in the numeraire are associated with relative-price movements. Therefore, there will be no "pure" numeraire shocks as long as we allow for stationary relative-price factors, as we did in our benchmark specification.

5.2. The numeraire in simple general-equilibrium models

Consider a standard general-equilibrium model with a representative consumer that receives utility from consuming goods from each of N sectors indexed by i, where there is a continuum of differentiated goods in each sector. The consumer obtains utility from a constant-elasticity aggregator of the consumption of each sector, which in turn is a Dixit-Stiglitz aggregator. There is a continuum of firms hiring labor from the consumer, each a monopolist for each variety of a good, and all engaging in monopolistic competition. This setup has become the standard workhorse to study models of pricing, inflation, and monetary policy, as surveyed in Woodford (2003). It implies the following log-linear approximate equilibrium relation:

$$p_{it}^{*} = p_t + \alpha y_t + a_{i,t} \tag{17}$$

where p_{it}^{*} is the desired price set by any firm in sector i at date t, p_{it} is the actual price set, $p_t = N^{-1}\sum_{i=1}^{N} p_{it}$ is the static cost-of-living price index, y_t is output, α is the elasticity of marginal costs with respect to output, and a_{it} are idiosyncratic productivity shocks that average to zero across sectors. All variables are in logs. We assume that nominal income, $m_t = p_t + y_t$, follows an exogenous process and the productivity in sector i has two exogenous components: $a_{it} = \theta_i g_t + z_{it}$, one common to all but with different impact across sectors and another that is good-specific. For concreteness, we assume that m_t, g_t, and z_{it} all follow independent random-walks.

Different models of pricing imply different relations between actual and desired prices. In this section, we consider three simple versions of the most popular alternatives to see what they imply for the numeraire. The first model, flexible prices, assumes that desired and actual prices are the same. The second model has sticky prices as in Taylor (1980). A fraction $\omega > 0$ of the N sectors have flexible prices, but in a fraction $1 - \omega$ of the sectors, half of the prices can only change at even dates and half at odd dates. Given this constraint, if a firm in sector i adjusts at date t, it optimally chooses the price $0.5(p_{it}^{*} + E_t p_{it+1}^{*})$, whereas if it doesn't adjust then its price is the same as in the last period. The third model has sticky contracts or information as in Fischer (1977). In a fraction $1 - \omega$ of sectors, half of the firms update their information at even dates and the other half at odd dates. If a firm updates information at date t, then it chooses price p_{it}^{*};

otherwise, it uses its 1-period old information to set $E_{t-1}p^*_{it}$.

Given these assumptions, the appendix shows that:

Proposition 1: *All three models of pricing imply that:*

$$\pi_{it} = n_t + \lambda_i' f_t + u_{it} \quad \text{with} \quad \sum_{i=1}^{N} \lambda_i = 0,$$

as in our dynamic NPI models. With flexible prices, $n_t = v_t = \Delta m_t$. With sticky prices, $n_t = \gamma_1 n_{t-1} + \gamma_2 \Delta m_t + \gamma_3 \Delta m_{t-1}$ where the γ_i depend on α and ω and one of the factors, $f_{3t} = \gamma_1 f_{3t-1} + \alpha (1 - \gamma_2) m_t - \alpha (\gamma_1 + \gamma_3) m_{t-1}$ with $\lambda_{3i} = I_i - (1 - I_i)\, \omega / (1 + \omega)$ where I_i is an indicator equal to 1 if the sector has flexible prices. Therefore, $v_t = 0$. With sticky information, $n_t = \beta_1 \Delta m_t + \beta_2 \Delta m_{t-1}$ where the β_i depend on α and ω, $f_{3t} = \alpha(1 - \beta_1) \Delta m_t - \alpha\beta_2 \Delta m_{t-1}$ and λ_{3i} is the same as in the sticky-price model. Therefore, $v_t = 0$.

In these models, only m_t shocks do not average to zero across sectors and move the numeraire. In flexible-price models, all prices increase immediately in response to them by the same amount so no relative-price changes. In both the sticky-price and sticky-information models, only a share of firms adjust their prices, so the change in the numeraire comes with a change in the relative-price between these firms and the remaining. Therefore, there are no pure numeraire shocks.[16]

If all sectors are equally sticky ($\omega = 0$), then it will still be the case that only some of the firms adjust to the shock on impact, so changes in the numeraire come with changes in the relative-prices across goods. However, we do not observe the prices of goods, but that of sectors, and in each sector exactly half of the firms adjust to the shock and half do not. Therefore, all sectoral prices increase in exactly the same proportion, so there are no sectoral relative-price changes and we identify exogenous changes in the numeraire. Formally, a corollary of the proposition is that with $\omega = 0$ then $\lambda_{3i} = 0$ so that $v_t = n_t$. In other words, strict pricing theories may be consistent with exogenous changes in the numeraire, but only because we observe prices at the sectoral and not good level, and only if all sectors are exactly symmetric and identical in their degree of stickiness.

In summary, strict models of sticky prices imply that there are no exogenous movements in the numeraire except under the implausible homogeneity assumption described in the last paragraph. That said, these models might still be consistent with the empirical analysis showing a large exogenous component of the numeraire if (i) the empirical model is misspecified (containing the wrong number of relative price factors, unit roots, and so forth), or (ii) the sampling uncertainty in the estimated model is sufficiently large to account for a large estimated exogenous component even when the true component is zero. The next two subsections address these possibilities.

5.3 Results for alternate specifications of the empirical model

Table 3 presents results from estimating models using different assumptions on the number of factors, their order of integration, and the order of the VAR. The first row of table 3 presents results from the benchmark model with three factors, an assumption that n_t and one of the relative price factors follow I(1) processes, and a VAR(4) for these factors. The next block of rows show results for models with different treatment of unit roots and with different numbers of factors. The benchmark model is labeled as "(1,1,0)" because it has three factors and the first two factors (n_t and the first f_t) are I(1). The second model in the table, labeled (0,0,0), is also a three-factor model but without unit roots imposed, and so on for the other models. The final two rows of table 3 shows results for models with VAR lag lengths that differ from the benchmark model. The first set of results in the table shows the correlation of $\Delta\hat{n}_t$ from the model with the corresponding estimate for benchmark model and the rmse of the estimates of n_t using the procedure described in the last section. The second set shows the model's implied standard deviation of the change in n_t and the change the exogenous component v_t.

With a few notable exceptions, the results in table 3 are similar to the results for the benchmark model. That is, the models produce estimates of n_t that look like the estimates shown in figure 2, the standard deviation of Δn is roughly 0.75%, roughly 30% of the variance of Δn is unrelated to relative price changes, and the models produce estimators for n_t with an rmse of approximately 0.3. The two notable exceptions are the four factor model with no unit roots (the "0,0,0,0" model), which produces a very volatile Δn_t and an rmse that is roughly twice as large as the benchmark model; and the VAR(6) model, in which Δn behaves like the benchmark model, but a smaller fraction of Δn is associated with the exogenous component.

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[16] As the intuition indicates, the result on v_t in the proposition extends to fixed prices and information for more than two periods and to most linear stochastic processes for the exogenous variables.

5.4. Monte Carlo experiments using standard general-equilibrium models

To evaluate the potential for large sampling error in the estimated exogenous components of the numeraire we conducted a small Monte Carlo experiment using variants of the flexible price, sticky-price, and sticky-information models outlined above. Parameter values in these models were chosen to roughly match the sample data, and 187 sectoral inflation rates were generated for 190 time periods (as in our sample data). Using the notation in section 5.2 the details of the models are as follows: Δm_t followed a random walk, Δg_t followed an AR(1) model with coefficient 0.8, Δz_{it} was white noise, θ_i were i.i.d. uniformly distributed random variables with a mean of zero, and the relative variance of Δm, Δg, and Δz were chosen to approximately match the fraction of the variance explained by the first two factors in the empirical model. The sectoral sticky-price model follows Carvalho (2006), in which each sector contains a large number of firms with prices that adjust as in Calvo (1983) with sector-specific probability ϕ_i. In our experiments, the ϕ_i were i.i.d. and uniformly distributed on [0,1]. The sectoral sticky-information follows Carvalho and Schwartzman (2006) in which each sector contains a large number of firms, each of which sets its price optimally given its information, and the firm is allowed to update its information each period with probability ϕ_i as in Mankiw and Reis (2002). As in the sticky-price model, the probability is the sector-specific parameter and in our experiments $\phi_i \sim$ i.i.d. U[0,1].

The flexible price model contains the numeraire (Δm_t) and one relative price factor (g_t), so in our experiments we used the simulated flexible price data to estimate a 2-factor models, where the factors followed a VAR(1). In the sticky-price and sticky-information model, staggering induces a large number of 'static' factors in the formulation (7)-(8). In our experiments with simulated data from the sticky-price or sticky-information we estimated factor models as in our benchmark empirical model allowing for 3 relative price factors and a VAR(4) for the evolution of the factors.

Because each Monte Carlo replication was computationally demanding – we carried out over 5000 EM iterations for each simulated dataset – the experiment involved only 10 simulated data sets from each model. In spite of the small number of replications, the conclusions are sharp and are summarized Table 4. In the flexible-price model, all of the variation in the numeraire is associated with exogenous variation, and each of the empirical models reached this conclusion. In the sticky-price and sticky-information models, none of the variation in the numeraire is associated with exogenous variation. In the simulations, the average fraction of exogenous variation was small for both of these models (2% and 7%), while the largest amount of variation found in any of the simulations was somewhat larger (15% and 19%). Thus, it appears that sampling error cannot reconcile these formulations of the sticky-price and sticky-information models with the large exogenous variation in the numeraire evident in the U.S. inflation data (30%).

6. Application 2: Are exogenous changes in the value of the numeraire neutral?

We will say that exogenous changes in the value of the numeraire (as defined in (15)) are neutral if they are uncorrelated with real economic quantities. Specifically, let q_t denote a series representing a real quantity such as the rate of growth of GDP, and consider the linear regression

$$q_t = \gamma_0 + \gamma(\mathrm{L})\Delta v_t + \eta_t. \qquad (18)$$

where $\gamma(\mathrm{L})$ is two-sided. We say that Δv_t is neutral for q_t if $\gamma(\mathrm{L}) = 0$, so q_t is uncorrelated with shocks to prices that are unrelated to changes in relative prices. Put differently, we define neutrality as the proposition that any correlation between q_t and nominal inflation rates arises from the correlation of q_t with changes in relative prices.

The regression in (18) cannot be computed directly because Δv_t is not observed. However, as discussed in Watson (1986), inference about $\gamma(\mathrm{L})$ can be carried out in a suitably modified version of the regression using estimates of v_t in place of the true regressors. Consider the regression:

$$q_t = \gamma_0 + \gamma(\mathrm{L})\Delta v_t + \theta(\mathrm{L})f_t + b_t. \qquad (19)$$

where $\theta(\mathrm{L})$ is two-sided (and generally non-zero) and b_t represents changes in q_t that are unrelated to Δv_t and f_t. Let $\hat{v}_t$ and $\hat{f}_t$ denote the estimates of v_t and f_t computed using the Kalman smoother, that is $\hat{v}_t$ and $\hat{f}_t$ are projections of v_t and f_t onto $\{\pi_{i\tau}\}_{i=1,\tau=1}^{i=N,\tau=T}$. Then:

$$q_t = \gamma_0 + \gamma(\mathrm{L})\Delta \hat{v}_t + \theta(\mathrm{L})\hat{f}_t + \tilde{b}_t \qquad (20)$$

where $\tilde{b}_t = b_t + \gamma(\mathrm{L})(\Delta v_t - \Delta \hat{v}_t) + \theta(\mathrm{L})(f_t - \hat{f}_t)$. Assuming that the signal extraction model is well specified, $\Delta\hat{v}_t$ and $\hat{f}_t$ are uncorrelated with the signal extraction errors ($\Delta v_\tau - \Delta\hat{v}_\tau$) and ($f_\tau - \hat{f}_\tau$) for all t and τ. Thus, if the regression error b_t in (19) is uncorrelated with $\{\pi_{i\tau}\}_{i=1,\tau=1}^{i=N,\tau=T}$ (the series used to construct $\hat{v}_t$ and $\hat{f}_t$), then the regressors in (20) are uncorrelated with the error term $\tilde{b}_t$, and inference about γ(L) can be carried out using OLS regression methods.[17]

Table 5 summarizes results from regressions of the form (20) using several measures of q_t: the rates of growth of real GDP, real PCE, aggregate (non-agricultural) employment, the index of industrial production, and changes in the civilian unemployment rate. Panel (a) shows detailed results for the regression that includes one lead/lag of $\Delta\hat{v}_t$ and five leads/lags of $\hat{f}_t$, and panel (b) summarizes results for different lead/lag lengths.[18] Looking at panel (a), the partial R^2 for $\Delta\hat{v}_t$ is small in all of the regressions, but the estimated regression coefficients are reasonably large and statistically significant. For example, in the real GDP regression, the partial R^2 is only 5%, but the point estimates suggest that a one standard deviation change in Δv_t is associated with a 0.5% cumulative increase in real GDP ($\gamma(1)\times 0.40/4$), and the estimated coefficients and sum of coefficients are statistically significant at the 1% level. Panel (b) indicates that these results are generally robust to the number of leads and lags included in the regression, although the standard error for the estimated value of $\gamma(1)$ is large when $p = 4$. Thus, the neutrality hypothesis is rejected.

There are at least four important specification errors that might explain the rejection of neutrality. First, perhaps the determinants of quantities include other relative-price shocks not captured by the f_t shocks, but which are correlated with the numeraire. Natural candidates are intertemporal relative prices, since the NPI estimates used only static goods' prices.[19] To investigate this, Panel (c) adds leads and lags of three financial variables (stock returns, short-term interest rates, the spread between long and short rates) as additional controls in the regression. These controls reduce the estimated sum of coefficients by roughly 40% and the partial R^2 falls to less than .03. Yet, while

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[17] To see why b_t would be uncorrelated with $\pi_{i\tau}$, note that equation (5.2) already includes the common-factor determinants of prices, and it is unlikely that the idiosyncratic shocks u_{it} affect the aggregate real quantity q_t.

[18] The number of leads and lags of $\hat{f}_t$ was set to equal the number of leads and lags of $\hat{v}_t$ plus four to guard against truncation bias in θ(L) that might induce correlation between $\hat{v}_t$ and the error term.

[19] However, our data include the prices of several durable goods that depend on intertemporal prices.

the estimated effects are smaller, they remain statistically significant.

A second potential misspecification is associated with errors in measuring real GDP and real consumption caused by inappropriate price deflators. There may be a correlation between Δv_t and the deflated values of nominal GDP and PCE despite no relation between their true quantity counterparts and Δv_t. To investigate this potential problem, we repeated that analysis using the NPI as the deflator for GDP and PCE. These results are shown in panels (a)-(c) for the series labeled $GDP^{\dagger}$ and $PCE^{\dagger}$. This change leads to a reduction in the estimated coefficients and partial R^2 values, and now neutrality cannot be rejected for GDP for many of the specifications.

A third explanation follows from potential misspecification of the factor model used to estimate the numeraire and its exogenous component. To investigate this, we repeated the analysis using the alternative factor model specifications discussed in Table 3. Panel (d) of table 5 summarizes the results from these alternative specifications, and the results indicate the non-neutrality results are robust to these changes in the factor model specification.

A fourth explanation for the neutrality rejection is that n_t is affected by real shocks that are not reflected in relative prices, and we now turn to this explanation.

6.1. Interpreting the non-neutrality of the numeraire

To see what determines exogenous movements in the numeraire, go back to the simple Walrasian economy of section 2 where equilibrium prices solve equations (3)-(4). As we showed there, an increase in the endowment of the numeraire good changes the value of the numeraire with no changes in relative prices. We also noted that most changes in the relative endowment of all other goods come with relative-price changes so, even if they affect the numeraire, they lead to no "pure" numeraire shocks. But now consider a very specific aggregate endowment shock, A_t. If this shock raises the endowment of all goods but the numeraire proportionally, and if the utility function is homothetic, then all prices fall by exactly the proportion A_t. Thus, special shocks that induce changes in consumption without changing the marginal rate of substitution (as in the case of proportional shocks to the endowment of all goods, no storage, and homothetic utility) are also "pure" numeraire shocks.

Given data on a subset of the prices in this Walrasian economy, our statistical-measurement model recovers $n_t = e_{0t} + a_t$, where e_{0t} and a_t are the rates of growth of E_{0t}

and A_t. Changes in e_{0t} are neutral with respect to quantities, while changes in a_t are not. This conclusion—that the numeraire can change due to changes in units or in response to a very particular type of aggregate shock—extends to many general-equilibrium models.

Thus, the non-neutrality of the numeraire can be interpreted in two ways. One interpretation assumes that there are no a_t shocks, so all exogenous movements in the n_t correspond to Hume's thought experiment. In this case, the non-neutrality of the n_t implies that there is money illusion in the world. The second interpretation assumes that Hume was right and there is no money illusion. In this case, non-neutrality indicates the presence of a_t shocks.

Unfortunately, the relative variances of e_{0t} and a_t cannot be separately identified from the price and quantity data used in the neutrality regressions. However, the small values of the partial R^2's from these regressions implies that a_t cannot be a quantitatively important determinant of both q_t (one of the real aggregates used in the neutrality regressions) and n_t. That is, if a_t is an important determinant of q_t, the small partial R^2 implies that most of the variance of n_t is associated with e_{0t}. Alternatively, if a_t is an important determinant of n_t, then a_t cannot explain much of the variance of q_t.

7. Conclusion

There are many measures of inflation, at least as many as there are uses for it. Whether to measure the cost of living, to deflate nominal quantities into real counterparts, or to guide monetary policy, there are several available measures of the overall increase in prices. From the perspective of economic theory, one particularly interesting use of a measure of inflation is to understand changes in the value of the numeraire. Economists sometimes refer to these as pure inflation, changes in the unit of account, in the absolute level of prices, or in the overall price level, and economic theories predict whether and when the numeraire's value should change and what the effects of these changes are.

Measuring the value of the numeraire is naturally difficult, since the concept itself is more a fruit of thought experiments and theoretical exercises than something easily observed. As a result, there have been few systematic attempts to measure it in the data. The goal of this paper was to make some progress on measuring changes in the value of the numeraire and understanding their effects.

Our approach was akin to some of the work on Solow (1957) residuals. Solow started from a simple theory, the aggregate production function, linking raw inputs to capital to measure shifts in this function as residuals; we started from a simple dynamic factor model of relative prices, to measure changes in the value of the numeraire as statistical estimates. The size of Solow residuals put in question the role of factor accumulation on economic growth; the size of our estimated changes in the value of the numeraire put in question strict theories of nominal rigidities. Assuming that Solow residuals measure only technical progress, there are large enough technological changes to generate sizeable business cycles; assuming that all changes in the value of the numeraire are due to Hume-shocks, there is money illusion. Without this assumption, models and data on capital utilization and the quality of inputs provide some information on the quantitative properties of shocks to technology; without the assumption that all numeraire changes are Hume-shocks, models and information from regressions of quantities on the numeraire provide some information on the quantitative properties of shocks to the numeraire.

We specified the problems of measuring the numeraire, extracting its exogenous components, assessing their neutrality, and measuring the performance of different estimators. We proposed a dynamic numeraire price index based on a factor model for relative price changes and estimated it in the U.S. data from 1959 to 2006. This produced time-series for the numeraire and its exogenous shocks and we described some of their properties. We hope that future research will look deeper into these time-series in the same way that past research has looked at shocks to technology, monetary policy, and fiscal policy, among others.

Our estimates shed some light on two issues. First, the large exogenous changes in the value of the numeraire that we found are hard to square with strict models of price rigidities but are more favorable to their soft alternatives. Second, we found that exogenous changes in the value of the numeraire were not neutral with respect to real quantities, and noted that this could be interpreted as a sign of either money illusion or of the presence of a particular type of aggregate shocks.

Our estimates of the changes in the value of the numeraire are certainly not perfect. We hope, however, that by stating the challenges and putting forward a benchmark, we can motivate future research to come up with better estimators. Likewise, we are sure that our findings will not settle the debates on what is the best theory of pricing, whether there is money illusion or monetary neutrality, or what aggregate shocks are more important. Our more modest hope is that they offer a new perspective on how to bring data to bear on these long-standing questions.

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Appendix

All price series are from NIPA Table 2.4.4U available from http://www.bea.gov/national/nipaweb/nipa_underlying/SelectTable.asp. Quarterly inflation rates were computed using the first difference of logarithms of the price indices for the last month of the quarter. Inflation observations that differed from the series median by more that six times the interquartile range were replaced by the local median computed using the six adjacent observations. The table below shows the price index from the NIPA table, the series description, the standard deviation of the (outlier-adjusted) series over 1959:2-2006:2 and the 2005 PCE expenditure share. To save space, the final four columns of this table are used to show the estimated parameters from the benchmark 3-factor model.

Table A1: Series Descriptions, Summary Statistics,
and Parameter Estimates from the Benchmark 3-factor Model

Num.	Label	Description	s_π	2005 Share	Benchmark Model Parameters			
					λ_1	λ_2	ρ	σ_e
001	P1NFCG D	New foreign autos	4.5	0.5	1.14	0.00	-0.13	0.88
002	P1NETG D	Net transactions in used autos	1.8	0.4	2.35	0.42	0.15	2.71
003	P1MARG D	Used auto margin	6.9	0.3	1.09	0.18	0.02	4.22
004	P1REEG D	Employee reimbursement	7.5	0.0	1.11	0.15	-0.19	1.68
005	P1TRUG D	Trucks, new and net used	4.8	2.4	1.25	-0.09	-0.12	0.96
006	P1TATG D	Tires and tubes	5.8	0.3	0.15	0.57	0.12	1.27
007	P1PAAG D	Accessories and parts	5.5	0.4	-0.21	-0.04	0.26	1.15
008	P1FNRG C	Furniture, incl. matt. and bedsprings	4.1	0.9	0.53	0.30	-0.29	0.77
009	P1MHAG D	Major household appliances	4.0	0.4	0.84	0.13	0.09	0.73
010	P1SEAG D	Small electric appliances	5.0	0.1	1.06	0.35	0.12	0.93
011	P1CHNG C	China, glassware, tableware, and utensil	6.7	0.4	1.32	0.93	-0.28	1.25
012	P1TVSG D	Television receivers	5.4	0.2	1.16	0.47	0.42	0.99
013	P1AUDG D	Audio equipment	5.2	0.3	0.57	0.06	-0.17	1.17
014	P1RTDG D	Records, tapes, and disks	4.9	0.2	-0.21	0.07	-0.06	1.17
015	P1MSCG D	Musical instruments	4.0	0.1	0.41	0.22	-0.13	0.85
016	P1FLRG D	Floor coverings	5.8	0.2	0.60	0.09	-0.24	1.27
017	P1CLFG D	Clocks, lamps, and furnishings	6.0	0.4	1.22	0.45	-0.04	1.29
018	P1TEXG D	Blinds, rods, and other	8.6	0.1	1.54	1.07	-0.28	1.81
019	P1WTRG D	Writing equipment	5.1	0.0	0.18	-1.01	-0.28	1.06
020	P1HDWG D	Tools, hardware, and supplies	4.7	0.1	0.56	0.14	-0.04	1.05
021	P1LWNG D	Outdoor equipment and supplies	5.1	0.0	0.73	0.13	-0.16	1.11
022	P1OPTG C	Ophth. prd, and orthopedic appliances	2.8	0.3	0.29	-0.05	-0.07	0.55
023	P1CAMG D	Photographic equipment	6.0	0.1	1.26	0.04	0.34	1.25
024	P1BCYG D	Bicycles	4.3	0.1	-0.09	0.30	-0.15	0.90
025	P1MCYG D	Motorcycles	4.7	0.2	1.18	-0.11	0.01	1.00
026	P1AIRG D	Pleasure aircraft	7.2	0.0	0.05	0.57	0.06	1.64
027	P1JRYG C	Jewelry and watches (18)	7.3	0.7	0.15	0.33	-0.21	1.67
028	P1BKSG C	Books and maps (87)	5.8	0.5	1.00	-0.37	-0.25	1.23
029	P1GRAG D	Cereals	6.3	0.4	-1.34	-0.19	0.45	1.34
030	P1BAKG D	Bakery products	4.6	0.6	-0.22	0.25	0.14	1.01
031	P1BEEG D	Beef and veal	13.0	0.4	-4.16	-0.28	-0.16	2.88
032	P1PORG D	Pork	6.9	0.3	-3.52	-0.91	0.19	3.96
033	P1MEAG D	Other meats	8.3	0.2	-2.72	-0.74	0.17	1.84
034	P1POUG D	Poultry	7.0	0.5	-2.23	0.03	-0.20	4.06
035	P1FISG D	Fish and seafood	5.7	0.2	-0.69	0.01	0.18	1.22
036	P1GGSG D	Eggs	7.4	0.1	-5.34	-0.42	-0.03	6.63
037	P1MILG D	Fresh milk and cream	6.9	0.2	-1.10	0.04	-0.03	1.63
038	P1DAIG D	Processed dairy products	6.2	0.5	-1.19	0.08	0.28	1.32
039	P1FRUG D	Fresh fruits	4.5	0.3	-0.89	0.21	-0.07	3.55
040	P1VEGG D	Fresh vegetables	9.3	0.4	-2.70	-0.21	-0.41	6.59
041	P1PFVG D	Processed fruits and vegetables	5.7	0.2	0.40	0.15	0.38	1.21
042	P1JNBG D	Juices and nonalcoholic drinks	6.4	0.8	0.16	0.64	0.32	1.22
043	P1CTMG D	Coffee, tea and beverage materials	1.8	0.2	1.49	0.89	0.58	2.31
044	P1FATG D	Fats and oils	9.3	0.1	-0.60	1.33	0.52	1.71
045	P1SWEG D	Sugar and sweets	6.3	0.5	-0.97	0.36	0.27	1.37
046	P1OFDG D	Other foods	4.1	1.3	0.11	0.05	0.11	0.76
047	P1PEFG D	Pet food	3.9	0.3	-0.19	0.04	-0.04	0.79

048	P1MLTG D	Beer and ale, at home	3.6	0.7	0.42	0.18	0.13	0.66
049	P1WING D	Wine and brandy, at home	3.9	0.2	-0.51	0.14	-0.02	0.79
050	P1LIQG D	Distilled spirits, at home	2.1	0.2	-0.17	-0.40	0.25	0.54
051	P1OPMG D	Other purchased meals	2.8	4.5	-0.15	0.09	0.30	0.32
052	P1APMG C	Alcohol in purchased meals	3.7	0.6	0.45	-0.06	-0.16	0.79
053	P1MFDG D	Food supplied military	3.0	0.0	-0.20	0.10	0.25	0.40
054	P1FFDG C	Food produced and consumed on farms	0.9	0.0	-4.86	-1.37	-0.09	4.98
055	P1SHUG C	Shoes (12)	3.8	0.6	-0.01	0.41	0.01	0.78
056	P1WGCG D	Clothing for females	4.5	1.8	-0.14	0.30	0.02	1.10
057	P1WICG D	Clothing for infants	8.9	0.1	1.40	0.58	-0.33	1.88
058	P1MBCG D	Clothing for males	3.5	1.2	0.30	0.34	0.11	0.74
059	P1MSGG D	Sewing goods for males	6.4	0.0	0.28	0.25	-0.29	1.46
060	P1MUGG D	Luggage for males	2.6	0.0	1.29	1.25	-0.21	2.82
061	P1MICG C	Std. clothing issued to military personnel	2.8	0.0	0.28	0.16	0.15	0.43
062	P1GASG D	Gasoline and other motor fuel	4.2	3.2	-6.30	1.54	-0.13	5.37
063	P1LUBG D	Lubricants	5.5	0.0	-0.37	0.47	0.37	1.09
064	P1OILG D	Fuel oil	3.7	0.1	-7.75	2.55	0.21	4.84
065	P1FFWG D	Farm fuel	6.0	0.0	-3.91	1.84	0.14	3.38
066	P1TOBG C	Tobacco products	7.5	1.0	0.36	-0.70	0.06	1.83
067	P1SOAG D	Soap	4.9	0.1	1.21	0.25	-0.13	0.92
068	P1CSMG D	Cosmetics and perfumes	4.3	0.2	1.07	0.17	-0.24	0.78
069	P1SDHG C	Semidurable house furnishings	7.4	0.5	1.76	0.64	-0.44	1.40
070	P1CLEG D	Cleaning preparations	4.2	0.4	0.66	0.13	0.09	0.75
071	P1LIGG D	Lighting supplies	7.2	0.1	0.87	0.53	-0.13	1.59
072	P1PAPG D	Paper products	5.6	0.3	0.36	0.40	0.04	1.17
073	P1RXDG D	Prescription drugs	4.0	2.6	0.33	-0.62	0.67	0.55
074	P1NRXG D	Nonprescription drugs	4.0	0.3	0.91	-0.45	0.10	0.64
075	P1MDSG D	Medical supplies	3.7	0.1	0.77	-0.58	-0.13	0.64
076	P1GYNG D	Gynecological goods	4.2	0.0	1.02	0.24	-0.08	0.68
077	P1DOLG D	Toys, dolls, and games	5.4	0.6	1.04	0.47	0.10	1.08
078	P1AMMG D	Sport supplies, including ammunition	4.7	0.2	0.35	0.15	-0.16	1.06
079	P1FLMG D	Film and photo supplies	4.6	0.0	0.62	-0.25	0.10	1.06
080	P1STSG D	Stationery and school supplies	4.7	0.1	0.91	0.50	-0.04	0.95
081	P1GREG D	Greeting cards	4.8	0.1	0.92	0.50	-0.04	0.97
082	P1ABDG C	Expenditures abroad by U.S. residents	16.8	0.1	0.28	0.54	0.18	4.02
083	P1MGZG D	Magazines and sheet music	5.5	0.3	0.66	-0.44	-0.31	1.17
084	P1NWPG D	Newspapers	3.8	0.2	0.87	0.24	0.14	0.78
085	P1FLOG C	Flowers, seeds, and potted plants	6.7	0.2	0.57	0.29	-0.12	1.54
086	P1OMHG D	Owner occupied mobile homes	2.5	0.4	0.03	-0.74	-0.30	0.24
087	P1OSTG D	Owner occupied stationary homes	2.4	10.7	0.00	-0.75	-0.17	0.19
088	P1TMHG D	Tenant occupied mobile homes	3.8	0.1	0.07	-0.75	-0.26	0.77
089	P1TSPG D	Tenant occupied stationary homes	2.4	2.8	-0.04	-0.77	-0.31	0.17
090	P1TLDG D	Tenant landlord durables	3.8	0.1	0.45	-0.51	0.25	0.66
091	P1FARG C	Rental value of farm dwellings (26)	4.3	0.2	-0.27	-0.15	0.70	0.84
092	P1HOTG D	Hotels and motels	6.3	0.6	0.19	-0.01	-0.10	1.38
093	P1HFRG D	Clubs and fraternity housing	2.9	0.0	0.03	-0.65	-0.33	0.43
094	P1HHEG D	Higher education housing	3.0	0.2	-0.15	-0.78	0.04	0.54
095	P1HESG D	El. and secondary education housing	8.9	0.0	0.16	-0.84	-0.36	2.01
096	P1TGRG D	Tenant group room and board	3.4	0.0	-0.12	-0.70	-0.38	0.60
097	P1ELCG C	Electricity (37)	5.7	1.5	0.43	-0.16	0.23	1.15
098	P1NGSG C	Gas (38)	2.6	0.8	0.35	0.19	0.44	2.71
099	P1WSMG D	Water and sewerage maintenance	3.9	0.6	0.88	-0.50	0.20	0.75
100	P1REFG D	Refuse collection	4.1	0.2	1.02	-0.56	0.29	0.75
101	P1LOCG D	Local and cellular telephone	4.5	1.3	0.41	-0.84	0.05	0.98
102	P1OLCG D	Local telephone	4.4	0.6	0.05	-1.00	0.00	1.00
103	P1LDTG D	Long distance telephone	5.3	0.3	0.15	-0.31	0.33	1.24
104	P1INCG D	Intrastate toll calls	5.1	0.1	-0.08	-0.66	0.36	1.17
105	P1ITCG D	Interstate toll calls	6.3	0.2	0.38	0.09	0.23	1.52
106	P1DMCG D	Domestic service, cash	4.3	0.2	0.27	0.10	0.24	0.98
107	P1DMIG D	Domestic service, in kind	6.0	0.0	-1.76	-0.21	-0.03	1.24
108	P1MSEG D	Moving and storage	3.7	0.2	0.15	0.09	-0.03	0.69
109	P1FIPG D	Household insurance premiums	3.7	0.2	0.13	-0.49	0.32	0.84
110	P1FIBG D	Less: Household insurance benefits paid	3.3	0.1	0.86	0.38	-0.28	0.40
111	P1RCLG D	Rug and furniture cleaning	4.4	0.0	0.33	0.06	-0.36	0.79
112	P1EREG D	Electrical repair	3.8	0.1	0.06	0.12	0.17	0.79
113	P1FREG D	Reupholstery and furniture repair	3.2	0.0	-0.11	-0.20	0.13	0.74
114	P1MHOG D	Household operation services, n.e.c.	3.7	0.2	0.03	0.09	-0.02	0.73
115	P1ARPG D	Motor vehicle repair	2.9	1.7	0.17	0.06	0.30	0.34
116	P1RLOG D	Motor vehicle rental, leasing, and other	4.9	0.6	0.82	0.15	-0.16	0.96
117	P1TOLG C	Bridge, tunnel, ferry, and road tolls	6.2	0.1	0.00	-0.75	-0.19	1.42
118	P1AING C	Insurance	4.2	0.7	0.84	-0.73	0.13	3.61

119	P1IMTG C	Mass transit systems	5.4	0.1	0.09	-0.45	0.09	1.35
120	P1TAXG C	Taxicab	5.7	0.0	0.05	0.22	0.02	1.27
121	P1IBUG C	Bus	9.2	0.0	-0.10	-0.37	-0.20	2.13
122	P1IAIG C	Airline	15.0	0.4	-0.64	0.75	-0.04	3.60
123	P1TROG C	Other	9.1	0.1	-0.23	-0.04	-0.05	2.11
124	P1PHYG C	Physicians	3.3	4.0	0.63	-0.09	0.50	0.42
125	P1DENG C	Dentists	2.7	1.0	0.39	-0.22	0.17	0.48
126	P1OPSG C	Other professional services	3.2	2.7	0.61	0.04	0.25	0.50
127	P1NPHG C	Nonprofit	3.1	4.4	0.05	-0.02	0.03	0.48
128	P1GVHG C	Government	4.3	1.4	-0.10	-0.06	0.51	0.76
129	P1NRSG C	Nursing homes	3.3	1.3	0.05	0.11	-0.30	0.62
130	P1MING C	Medical care and hospitalization	0.3	1.4	-0.90	-0.95	0.29	4.89
131	P1IING C	Income loss	5.7	0.0	0.70	-1.74	0.64	4.86
132	P1PWCG C	Workers' compensation	8.1	0.2	-0.55	0.26	0.80	1.16
133	P1MOVG C	Motion picture theaters	4.1	0.1	0.05	0.08	0.15	1.07
134	P1LEGG C	Leg. theaters and opera,	4.2	0.1	0.13	0.11	0.16	1.10
135	P1SPEG C	Spectator sports	4.1	0.2	-0.15	-0.34	-0.08	1.03
136	P1RTVG C	Radio and television repair	3.1	0.1	0.28	-0.52	0.33	0.62
137	P1CLUG C	Clubs and fraternal organizations	4.2	0.3	-0.13	0.42	-0.27	0.77
138	P1SIGG D	Sightseeing	5.3	0.1	0.04	0.00	-0.07	1.21
139	P1FLYG D	Private flying	9.8	0.0	0.48	0.19	-0.28	2.27
140	P1BILG D	Bowling and billiards	4.1	0.0	0.46	-0.31	0.05	0.96
141	P1CASG D	Casino gambling	2.9	0.9	-0.28	0.10	-0.22	0.32
142	P1OPAG D	Other com. participant amusements	2.8	0.3	0.27	0.06	0.16	0.59
143	P1PARG C	Pari-mutuel net receipts	4.8	0.1	-0.66	-0.09	0.51	0.99
144	P1PETG D	Pets and pets services excl. vet.	3.6	0.1	-0.12	-0.07	0.00	0.76
145	P1VETG D	Veterinarians	3.0	0.2	-0.18	-0.23	0.13	0.67
146	P1CTVG D	Cable television	7.0	0.7	0.18	-0.21	0.08	1.76
147	P1FDVG D	Film developing	3.8	0.1	0.76	-0.08	0.39	0.85
148	P1PICG D	Photo studios	3.8	0.1	0.12	-0.12	0.09	0.89
149	P1CMPG D	Sporting and recreational camps	3.4	0.0	0.09	-0.04	-0.07	0.81
150	P1HREG D	High school recreation	4.7	0.0	0.05	-0.14	-0.22	1.12
151	P1NECG D	Commercial amusements n.e.c.	3.4	0.6	0.25	0.00	-0.05	0.80
152	P1NISG D	Com. amusements n.e.c. except ISPs	3.3	0.4	0.12	-0.05	-0.04	0.80
153	P1SCLG D	Shoe repair	3.3	0.0	0.04	-0.27	0.12	0.64
154	P1DRYG D	Drycleaning	3.6	0.1	0.30	0.18	0.24	0.52
155	P1LGRG D	Laundry and garment repair	3.6	0.1	-0.03	0.07	0.12	0.57
156	P1BEAG D	Beauty shops, including combination	3.9	0.5	0.08	-0.09	0.17	0.76
157	P1BARG D	Barber shops	2.8	0.0	0.01	0.08	0.11	0.56
158	P1WCRG D	Watch, clock, and jewelry repair	3.3	0.0	-0.01	-0.30	-0.03	0.66
159	P1CRPG D	Miscellaneous personal services	3.8	0.5	0.17	0.11	-0.02	0.62
160	P1BROG C	Brokerage charges and inv. couns.	1.2	1.0	0.30	0.50	0.01	5.18
161	P1BNKG C	Bnk srv. chges, trust serv., s-d box rental	5.7	1.2	1.81	-0.70	0.39	1.02
162	P1IMCG D	Commercial banks	2.4	1.0	-0.18	0.76	0.18	2.93
163	P1IMNG D	Other financial institutions	15.0	1.4	0.19	-0.32	0.58	3.05
164	P1LIFG C	Exp. of handl. life ins. and pension plans	2.3	1.2	-0.37	-0.24	0.49	0.45
165	P1GALG C	Legal services (65)	4.4	1.0	0.60	-0.41	0.14	0.91
166	P1FUNG C	Funeral and burial expenses	3.2	0.2	0.47	-0.61	0.35	0.57
167	P1UNSG D	Labor union expenses	4.1	0.2	-0.32	0.29	0.07	0.74
168	P1ASSG D	Profession association expenses	6.5	0.1	-0.23	0.03	-0.37	1.33
169	P1GENG D	Employment agency fees	5.5	0.0	1.40	-0.11	-0.04	1.03
170	P1AMOG D	Money orders	5.3	0.0	1.12	-0.24	-0.21	1.09
171	P1CLAG D	Classified ads	5.4	0.0	1.15	-0.23	-0.16	1.09
172	P1ACCG D	Tax return preparation services	5.2	0.1	0.97	-0.31	-0.11	1.12
173	P1THEG D	Personal business services, n.e.c.	7.1	0.1	0.61	-0.55	-0.03	1.66
174	P1PEDG D	Private higher education	4.4	0.7	-0.25	-0.13	0.02	0.89
175	P1GEDG D	Public higher education	4.1	0.7	0.52	-0.27	0.07	0.89
176	P1ESCG D	Elementary and secondary schools	4.3	0.4	-0.47	0.20	-0.02	0.84
177	P1NSCG D	Nursery schools	4.8	0.1	-0.63	0.01	0.02	1.05
178	P1VEDG D	Commercial and vocational schools	4.1	0.4	-0.96	-0.38	0.20	0.88
179	P1REDG D	Foundations and nonprofit research	4.5	0.2	-0.37	-0.27	-0.03	1.05
180	P1POLG D	Political organizations	8.2	0.0	0.04	0.39	-0.32	1.83
181	P1MUSG D	Museums and libraries	5.7	0.1	-0.70	0.08	-0.13	1.18
182	P1FOUG D	Foundations to religion and welfare	5.4	0.2	-0.54	0.09	0.01	1.11
183	P1WELG D	Social welfare	3.3	1.7	-0.39	0.12	-0.01	0.54
184	P1RELG D	Religion	5.0	0.7	0.17	0.19	-0.09	1.11
185	P1AFTG D	Passenger fares for foreign travel	9.8	0.5	-0.95	0.39	-0.08	2.32
186	P1USTG D	U.S. travel outside the U.S.	9.6	0.6	-2.04	0.50	0.15	2.16
187	P1FTUG D	Foreign travel in U.S.	3.6	1.0	-0.20	0.00	0.04	0.62

A.2 State-space representation of the dynamic factor model, the log-likelihood function, and the EM algorithm.

Let π_t, u_t, and e_t be $N\times1$ vectors containing $\{\pi_{it}\}$, $\{u_{it}\}$, and $\{e_{it}\}$, Λ be an $N\times k$ matrix with λ_j as its j'th column, α an $N\times1$ vector, and let R and ρ denote $N\times N$ diagonal matrices. Then, then the dynamic factor model (7)-(13) can be written as[20]

$$\pi_t = n_t l + \Lambda f_t + u_t \tag{A.1}$$

$$u_t = \alpha + \rho u_{t-1} + e_t \tag{A.2}$$

$$\Phi(L)\begin{pmatrix} n_t \\ f_t \end{pmatrix} = \varepsilon_t \tag{A.3}$$

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim WN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} R & 0 \\ 0 & Q \end{pmatrix}\right) \tag{A.4}$$

where ρ is a diagonal matrix with diagonal elements ρ_i and we have assumed an AR(1) model for u_{it}. It is convenient to write the model in state-space form as:

$$y_t = Hs_t + e_t \tag{A.5}$$

$$s_t = Fs_{t-1} + G\varepsilon_t \tag{A.6}$$

where $y_t = \pi_t - \rho\pi_{t-1} - \alpha$, $s_t = (x_t^{'}\ x_{t-1}^{'}\ \dots\ x_{t-p+1}^{'})'$ with $x_t = (n_t\ f_t')'$,

$H = \begin{bmatrix} l & \Lambda & -\rho l & -\rho\Lambda & 0_{(N,(p-2)*(k+1))} \end{bmatrix}$, $F = \begin{pmatrix} \Phi_1,\dots,\Phi_{p-1} & \Phi_p \\ I_{(p-1)(k+1)} & 0_{(p-1)(k+1),k+1} \end{pmatrix}$, and

$C = \begin{pmatrix} I_{k+1} \\ 0_{(p-1)(k+1)} \end{pmatrix}$.

The Gaussian log-likelihood for the unknown parameters conditional on $\{y_t\}_{t=2}^{T}$ can be computed using the Kalman filter innovations and their variances as described in Hamilton (1993, Chapter 13).

The EM algorithm is a well-known approach (Watson and Engle, 1983) to maximize the Gaussian log-likelihood function for state-space problems. The method is

[20] We use l to denote the $N\times1$ vector of 1s, I_j to denote an identity matrix of size, and $0_{i,j}$ to denote an $i\times j$ matrix of zeros. For any matrix X, $X_{(i:j,k:l)}$ is the block with its i^{th} to j^{th} row and k^{th} to l^{th} column.

convenient here because it straightforward to compute the expected value of the "complete data" ($\{y_t, s_t\}$) sufficient statistics conditional on the observed data ($\{y_t\}$), and because maximization of the complete data Gaussian likelihood follows from familiar regression formulae. The standard linear regression formulae are modified in two ways to estimate the parameters in (A.5)-(A.6). First, Gauss-Seidel/Cochrane-Orcutt iterations are used to estimate ρ conditional on α and Λ, and α and Λ conditional on ρ. Second, Λ is estimated subject to the constraint $l'\Lambda = 0$ in (11) using the standard restricted least squares formula.

While the model contains a large number of unknown parameters (971 in the benchmark model), there are two features of the model that make estimation feasible. First, while N is large, because R is diagonal, the sufficient statistics for the complete data likelihood can be computed in $O(Tm)$ calculations, where m is the dimension of the state vector s. Second, because N and T are large, the principal component estimators of (n_t f_t) are reasonably accurate and regression based estimators of the model parameters can be constructed using these estimates of the factors. These principal component based estimates serve as useful initial values for the MLE algorithm. (See Doz, Giannone and Reichlin, 2006, for further discussion.) Results reported in the text are based on 40,000 EM iterations, although results using 5,000 iterations are essentially identical.

A.3 MLEs for the benchmark model

Table A1 includes the estimates of Λ, ρ, and σ_e for the benchmark 3-factor model. The estimated parameters in the VAR(4) state transition equation are

$$\Phi_1 = \begin{bmatrix} 0.40 & -0.10 & 0.35 \\ 0.44 & 0.63 & -0.01 \\ -0.72 & -0.25 & 1.33 \end{bmatrix}, \; \Phi_2 = \begin{bmatrix} 0.73 & 0.06 & -0.28 \\ -0.19 & 0.06 & 0.06 \\ 1.14 & 0.21 & -0.71 \end{bmatrix}, \; \Phi_2 = \begin{bmatrix} 0.00 & -0.13 & -0.05 \\ -0.45 & 0.16 & -0.10 \\ -0.30 & -0.36 & 0.36 \end{bmatrix}$$

$$\Phi_4 = \begin{bmatrix} -0.13 & 0.17 & -0.01 \\ 0.20 & 0.15 & 0.12 \\ -0.11 & 0.39 & -0.11 \end{bmatrix}, \; Var(\varepsilon) = \begin{bmatrix} 0.40 & -0.16 & 0.45 \\ -0.16 & 1.0 & 0 \\ 0.45 & 0 & 1.0 \end{bmatrix}$$

A.4 Estimating v_t

Recall that $v_t = n_t - \hat{E}(n_t \mid \{f_\tau\}_{\tau=1}^T)$, where $\hat{E}$ denotes the linear projection operator. The projection $\hat{E}(n_t \mid \{f_\tau\}_{\tau=1}^T)$ can be computed from the Kalman smoother from a state space system with state equation given by (A.6) and observation equation given by f_t = [0 I_k $0_{(k,\,(k+1)p)}$]s_t. The covariance matrix from the Kalman smoother yields the variance of v_t. Finally, letting $q_{t/T} = \hat{E}(q_t \mid \{\pi_{i\tau}\}_{i=1,\tau=1}^{N,T})$ for any variable q_t, the law of iterated expectations implies that $v_{t/T}$ can be computed from the formula $v_t = n_t - \hat{E}(n_t \mid \{f_\tau\}_{\tau=1}^{T})$ by replacing n_t by $n_{t/T}$ and f_τ by $f_{\tau/T}$.

A.5. Grouping MSE estimator for the NPI

First consider the static estimator $\hat{n}_t(\omega) = N^{-1}\sum_{i=1}^{N}\omega_i\pi_{it}$, so that

$$\hat{n}_t(\omega) - n_t = (\bar{\omega}-1)n_t + \left[N^{-1}\sum_{i=1}^{N}\omega_i\lambda_i'\right]f_t + N^{-1}\sum_{i=1}^{N}\omega_i u_{it} \tag{A.7}$$

We will assume that the last term dominates this expression. Specifically, we assume that $E\left[\sqrt{N}(\hat{n}_t(\omega)-n_t)\right]^2 = E\left[N^{-1/2}\sum_{i=1}^{N}\omega_i u_{it}\right]^2 + o(1)$. (This will be true, for example, when $\bar{\omega} = 1$, $|f_t|$ is bounded and $\sum_{i=1}^{N}\omega_i\lambda_i' \sim o(N^{1/2})$. Note that $\sum_{i=1}^{N}\omega_i\lambda_i' = 0$ when ω_i = 1 for all i.) The goal then is to estimate $E\left[N^{-1/2}\sum_{i=1}^{N}\omega_i u_{it}\right]^2$.

To do this, organize the prices into two groups, where for notational convenience the first group consists of the first N_1 prices and the second group consists of the remaining $N-N_1$ prices. Let $\hat{n}_{1t} = N_1^{-1}\sum_{i=1}^{N_1}\omega_i\pi_{it}$, $\hat{n}_{2t} = (N-N_1)^{-1}\sum_{i=N_1+1}^{N}\omega_i\pi_{it}$, and $\bar{\omega}_1$ and $\bar{\omega}_2$ denote the sample means of the weights in the two groups. Thus

$$\begin{aligned}\sqrt{N}\left(\hat{n}_{1t} - (\bar{\omega}_1/\bar{\omega}_2)\hat{n}_{2t}\right) = &\left[\frac{1}{\pi}N^{-1/2}\sum_{i=1}^{N_1}\omega_i\lambda_i' - \frac{\bar{\omega}_1}{\bar{\omega}_2(1-\pi)}N^{-1/2}\sum_{i=N_1+1}^{N}\omega_i\lambda_i'\right]f_t \\ &+ \frac{1}{\pi}N^{-1/2}\sum_{i=1}^{N_1}\omega_i u_{it} - \frac{\bar{\omega}_1}{\bar{\omega}_2(1-\pi)}N^{-1/2}\sum_{i=N_1+1}^{N}\omega_i u_{it}\end{aligned} \tag{A.8}$$

where $\pi = N_1/N$. Again, assume the term involving f_t is negligible (that is, the weighted average of the λ's are sufficiently close to zero in both groups), let $a_N = \pi^{-1}$, $b_N = \bar{\omega}_1/[\bar{\omega}_2(1-\pi)]$, $c_{1t} = N^{-1/2}\sum_{i=1}^{N_1}\omega_i u_{it}$, $c_{2t} = N^{-1/2}\sum_{i=N_1+1}^{N}\omega_i u_{it}$, and note that the goal is to estimate $E\left[N^{-1/2}\sum_{i=1}^{N}\omega_i u_{it}\right]^2 = E[c_{1t} + c_{2t}]^2$. The key assumption for the mse estimator that we use is that $E(c_{1t}c_{2t}) \to 0$ as N grows large, that is, that the weighted average of the u_{it}'s in the first group is asymptotically uncorrelated with the weighted average u_{it} in the second group. In this case $E\left[N^{-1/2}\sum_{i=1}^{N}\omega_i u_{it}\right]^2 = E(c_{1t}^2) + E(c_{2t}^2) + o(1)$. To see why this assumption is useful write

$$
\begin{aligned}
E\left[\sqrt{N}\left(\hat{n}_{1t} - (\bar{\omega}_1/\bar{\omega}_2)\hat{n}_{2t}\right)\right]^2 &= E\left[\frac{1}{\pi}N^{-1/2}\sum_{i=1}^{N_1}\omega_i u_{it} - \frac{\bar{\omega}_1}{\bar{\omega}_2(1-\pi)}N^{-1/2}\sum_{i=N_1+1}^{N}\omega_i u_{it}\right]^2 + o(1) \\
&= E\left[a_N c_{1t} - b_N c_{2t}\right]^2 + o(1) \\
&= a_N^2 E(c_{1t}^2) + b_N^2 E(c_{2t}^2) + o(1) \\
&= \left[E(c_{1t}^2) + E(c_{2t}^2)\right]\left[\frac{a_N^2 + b_N^2}{2}\right] + \left[E(c_{1t}^2) - E(c_{2t}^2)\right]\left[\frac{a_N^2 - b_N^2}{2}\right] + o(1)
\end{aligned}
$$

where the first equality follows from (A.8) and the assumption that the term involving f_t is negligible, the second line uses the definition of a_N, b_N, c_{1t} , and c_{2t}, the third line uses the assumption that c_{1t} and c_{2t} are asymptotically uncorrelated, and the final line follows by rearranging terms.

To complete the argument, assume either that (i) $E(c_{1t}^2) = E(c_{2t}^2)$ (so that the weighted average u_{it}'s in the two groups have the same variance), or that (ii) $a_N = b_N$ (so that the groups have comparable weights). Under either assumption

$$
E\left[c_{1t} + c_{2t}\right]^2 = \frac{2}{a_N^2 + b_N^2} E\left[\sqrt{N}\left(\hat{n}_{1t} - (\bar{\omega}_1/\bar{\omega}_2)\hat{n}_{2t}\right)\right]^2 + o(1) \qquad \text{(A.9)}
$$

which suggests the estimator

$$
\widehat{mse[\hat{n}_t(\omega)]} = \frac{2}{a_N^2 + b_N^2} T^{-1} \sum_{t=1}^{T}\left(\hat{n}_{1t} - (\bar{\omega}_1/\bar{\omega}_2)\hat{n}_{2t}\right)^2 \qquad \text{(A.10)}
$$

which is the estimator used in Table 2 for the static estimators.

The mse estimator for the dynamic estimator is formed similarly, but with two key differences. First, ω_i, $\bar{\omega}$, $\bar{\omega}_1$, $\bar{\omega}_2$ and so forth are now lag polynomials. The calculations above are then interpreted as yielding spectra or autocovariance generating functions. These yield the mse of $E\left[N^{-1/2}\sum_{i=1}^{N}\omega_i(L)u_{it}\right]^2$. The second difference is that the first term on the right hand side of (A.7) is now ($\bar{\omega}$(L) − 1)n_t; in general $\bar{\omega}$(L) ≠ 1, so that the mse of $\hat{n}_t(\omega)$ must include the variance of ($\bar{\omega}$(L) − 1)n_t, and this variance depends on the assumed process for n_t. We have computed this variance using the estimated VAR in (9).

The results shown in table 2 are computed using the estimators described above with the first 94 prices used to form the first group and the remaining 93 prices in the second group. This grouping was suggested by the ordering of the prices listed in table A1. Table A2 below shows results in which prices were randomly assigned to the two groups. These estimates are somewhat lower than the estimates shown in table 2. One interpretation of these results is that with random assignment, c_{1t} and c_{2t} are positively correlated and this is positive correlation results in a downward biased estimate of the rmse.

Table A2. Root Mean Square Error of NPI Estimators (Random Grouping)

Estimator	n_t	$n_t - n_{t-1}$	$n_t - n_{t-4}$
Jevons	0.61	0.80	0.90
Edgeworth	0.32	0.38	0.42
Expenditure Share	0.93	1.34	1.35
Dynamic	0.27	0.35	0.40

A.6. Proof of Proposition 1

With flexible prices, $p_{it} = \alpha m_t + (1 - \alpha)p_t + \theta_i g_t + z_{it}$. Adding over i shows that $p_t = m_t$. Replacing back and taking first differences gives: $\pi_{it} = \Delta m_t + \theta_i \Delta g_t + \Delta z_{it}$. This maps into a dynamic factor NPI model with $n_t = \Delta m_t$, $\lambda_i = \theta_i$, $f_t = \Delta g_t$ and $u_{it} = \Delta z_{it}$.

With sticky prices, first note that prices in a sector with stickiness are: $4p_{it} = p_{it}^* + p_{it-1}^* + E_{t-1}p_{it}^* + E_t p_{it+1}^*$. In sectors with flexible prices, $p_{it} = p_{it}^*$. Summing over all firms over all sectors, gives an expectational difference equation in p_t. The solution to this equation is $p_t = \gamma_1 p_{t-1} + \gamma_2 m_t + \gamma_3 m_{t-1}$, where γ_1, γ_2, and γ_3 are messy functions of α and ω. Then, note that $p_{it} = p_t + I_i(p_{it}^* - p_t) + (1 - I_i)0.25(p_{it}^* + p_{it-1}^* + E_{t-1}p_{it}^* + E_t p_{it+1}^*)$, where I_i is an indicator function equal to 1 if sector i has flexible prices and 0 if it has sticky prices. Substituting the expressions for p_{it}^* and rearranging shows that this maps into a dynamic factor NPI model with numeraire $n_t = \gamma_1 n_{t-1} + \gamma_2 \Delta m_t + \gamma_3 \Delta m_{t-1}$. The first two factors are $\lambda_{1i} = \theta_i$, $f_{1t} = 0.5(\Delta g_t + \Delta g_{t-1})$ and $\lambda_{2i} = I_i \theta_i$, $f_{2t} = 0.5(\Delta g_t - \Delta g_{t-1})$. The third factor is $\lambda_{3i} = I_i - (1 - I_i)\, \omega / (1 + \omega)$, $f_{3t} = \gamma_1 f_{3t-1} + \alpha\, (1 - \gamma_2)\Delta m_t - \alpha\, (\gamma_1 + \gamma_3)\Delta m_{t-1}$. The idiosyncratic shocks are: $u_{it} = I_i \Delta z_{it} + 0.5(1 - I_i)(\Delta z_{it} + \Delta z_{it-1})$.

With sticky information, prices in a sector with stickiness are $2p_{it} = p_{it}^* + E_{t-1} p_{it}^*$, while in a flexible sector $p_{it} = p_{it}^*$. Aggregating shows that $p_t = \beta_1 m_t + \beta_2 m_{t-1}$, where β_1 and β_2 are functions of α and ω. Prices in a sector are: $p_{it} = p_t + I_i(p_{it}^* - p_t) + (1 - I_i)0.5(p_{it}^* + E_{t-1} p_{it}^*)$. Substituting the expressions for p_{it}^* and rearranging gives a model as in (7), (8) and (11) with numeraire $n_t = \beta_1 \Delta m_t + \beta_2 \Delta m_{t-1}$. The other factors and idiosyncratic shocks are exactly as in the sticky price model with only one exception: now $f_{3t} = \alpha(1 - \beta_1)\Delta m_t - \alpha\beta_2 \Delta m_{t-1}$.

Table 1. Sample Moments of the NPI and of alternative measures of inflation

	NPI	*PCE deflator*	*Core PCE deflator*	*Jevons*	*Edgeworth*
Correlation with level of NPI	–	0.93	0.92	0.96	0.98
Correlation with change in NPI	–	0.64	0.48	0.76	0.89
Standard deviation of levels	2.20	2.63	2.25	2.27	2.14
Standard deviation of changes	0.68	1.90	1.26	1.35	0.93
First Autocorrelation of levels	0.95	0.74	0.84	0.82	0.91

Notes: Sample Period 1960:1 – 2006:2.

Table 2. Root Mean Square Error of NPI Estimators

Estimator	n_t	$n_t - n_{t-1}$	$n_t - n_{t-4}$
Jevons	0.74	0.93	0.99
Edgeworth	0.40	0.49	0.50
Expenditure Share	1.01	1.39	1.45
PC-Covariance	2.11	2.85	3.13
PC-Correlation	0.38	0.45	0.49
Dynamic	0.32	0.38	0.41

Notes: Root MSE is in percentage points at annual rate of the level of the NPI, its quarterly change and its annual change. See the text for the definition of the estimators.

Table 3. The numeraire's variance with alternative dynamic specifications

Specification	*Cor(*$\Delta\hat{n}$ *)*	*RMSE*	*Var(Δn)*	*Var(Δv)*
		Baseline		
(1,1,0)	1.00	0.32	0.74	0.41
		Alternative number of factors and I(1) or I(0) specifications		
(0,0,0)	1.00	0.33	0.75	0.41
(1,0,0)	1.00	0.32	0.75	0.42
(1,1,0,0)	0.99	0.35	0.74	0.41
(0,0,0,0)	0.99	0.59	3.05	0.50
(1,0,0,0)	0.91	0.40	0.81	0.30
(1,1)	0.96	0.42	1.62	0.47
(1,0)	0.96	0.31	0.75	0.45
(0,0)	0.96	0.42	1.58	0.46
		Alternative VAR lag length		
VAR(2)	0.99	0.32	0.74	0.47
VAR(6)	0.99	0.33	0.74	0.21

Notes: Column 1 shows the model specification where the numbers in parentheses refer to I(1) or I(0) specifications for the factors. For example, the (1,1,0) specification is the benchmark specification which includes three factors, where the first two factors (the NPI and the first relative price factor) are I(1) and the third factor is I(0); the (0,0,0,0) model has four factors (the NPI and three relative price factors), all of which are I(0). The VAR(2) and VAR(6) models correspond the (1,1,0) model where the factors follow a VAR(2) and VAR(6). The column labeled *Cor(*$\Delta\hat{n}$ *)* shows the correlation of the model's $\Delta\hat{n}_t$ with the corresponding value from the benchmark (1,1,0) model. The column labeled *RMSE* shows the rmse for n_t computed as in table 2. The columns labeled *Var(Δn)* and *Var(Δv)* show the model's implied standard deviation of Δn and Δv.

Table 4. Artificial data from models of pricing and exogenous changes in the numeraire

$Var(\Delta v)/Var(\Delta n)$	*Flexible Price*	*Sticky Price*	*Sticky Information*
Population Value	1.00	0.00	0.0
Smallest Value	0.97	0.00	0.00
Average Value	0.99	0.02	0.07
Largest Value	1.00	0.15	0.19

Notes: This table shows the fraction of variance of Δn attributed to the exogenous component Δv. The first row shows the population value of this ratio for each of the three models. The remaining rows of the table show summary results from estimates constructed from 10 simulated samples for each model. Values shown are the smallest, average, and largest values of the ratios found in the 10 simulations.

Table 5. Neutrality Regressions

$$q_t = \alpha_0 + \sum_{j=-p}^{p} \gamma_j \Delta \hat{v}_{t-j} + \sum_{j=-(p+4)}^{p+4} \theta_j \hat{f}_{t-j} + \tilde{b}_t$$

Panel (a). Results with p = 1 (one lead and lag of Δv_t)

	Dependent variable (q_t)						
	GDP	*PCE*	*EMP*	*UNMP*	*IP*	*GDP*†	*PCE*†
Parameter estimates and standard errors							
γ_{-1}	1.65 (0.68)	1.99 (0.46)	1.11 (0.34)	-0.18 (0.05)	1.74 (1.34)	1.20 (0.73)	1.59 (0.47)
γ_0	1.07 (0.85)	0.95 (0.62)	0.84 (0.42)	-0.23 (0.06)	4.40 (1.77)	1.16 (0.91)	1.01 (0.61)
γ_1	1.99 (0.68)	-0.42 (0.61)	0.74 (0.45)	-0.09 (0.07)	0.87 (1.97)	1.53 (0.78)	-0.41 (0.62)
$\gamma(1)$	4.71 (1.38)	2.53 (0.92)	2.68 (0.85)	-0.50 (0.13)	7.01 (2.79)	3.89 (1.43)	2.19 (0.88)
Statistical significance tests							
F_v	4.29**	8.01**	4.07**	6.42**	3.08*	2.71*	5.27**
F_f	5.94**	8.35**	13.46**	9.65**	5.26**	7.14**	7.92**
Share of variability explained							
R^2_v	0.05	0.06	0.05	0.08	0.03	0.03	0.04
R^2	0.26	0.30	0.34	0.44	0.20	0.22	0.25

Notes: The first block of results shows the estimated values of γ_j with (4-lag Newey-West) standard errors in parentheses, and the sum of coefficients $\gamma(1)$. The second block shows the *F*-statistic testing the null hypotheses that coefficients on Δv are zero (F_v) and that the coefficients on f are zero (F_f), where * and ** indicate significance at the 5% and 1% levels respectively. The final block shows the estimated partial R^2 for v (R^2_v) and the overall R^2. The regressands *GDP*† and *PCE*† are nominal *GDP* and *PCE* deflated by the NPI. The regressand *GDP*, *PCE*, *EMP*, *IP*, *GDP*† and *PCE*† are growth rates shown in percentage at an annual rates. The regressand *UNMP* is the unemployment rate in percentage points.

Table 5. (continued)

Panel (b). Results with different values of *p*

	GDP	*PCE*	*EMP*	*UNMP*	*IP*	*GDP*†	*PCE*†
	Leads and lags: p = 0						
$\gamma(1)$	1.35 (0.90)	1.03 (0.59)	0.97 (0.42)	-0.23 (0.07)	3.91 (1.72)	1.36 (0.93)	1.09 (0.58)
F_v	2.25	3.04	5.35*	12.36**	5.15*	2.15	3.51
R^2_v	0.01	0.01	0.02	0.05	0.02	0.01	0.01
	Leads and lags: p = 2						
$\gamma(1)$	3.29 (1.71)	3.33 (1.46)	3.57 (1.28)	-0.52 (0.17)	7.29 (4.51)	1.92 (1.83)	3.05 (1.41)
F_v	2.95*	4.64**	2.69*	3.98**	1.91	2.85*	3.34**
R^2_v	0.04	0.05	0.05	0.07	0.01	0.03	0.03
	Leads and lags: p = 3						
$\gamma(1)$	3.95 (1.91)	3.02 (1.81)	3.89 (1.34)	-0.54 (0.18)	0.48 (3.84)	2.07 (2.01)	2.27 (1.71)
F_v	2.07*	2.58*	2.59*	2.51*	1.59	1.84	2.14*
R^2_v	0.05	0.03	0.05	0.05	0.01	0.04	0.02
	Leads and lags: p = 4						
$\gamma(1)$	2.61 (2.39)	2.52 (2.05)	6.03 (1.34)	-0.71 (0.25)	4.30 (5.15)	1.11 (2.46)	2.13 (1.94)
F_v	2.52**	2.43**	4.14**	3.08**	1.65	2.65**	2.40*
R^2_v	0.04	0.02	0.08	0.05	0.01	0.04	0.01

Panel (c). Results with p=1 and additional controls *x* added to regression

	GDP	*PCE*	*EMP*	*UNMP*	*IP*	*GDP*†	*PCE*†
	x = SP Returns						
$\gamma(1)$	3.98 (1.29)	2.31 (0.89)	2.56 (0.80)	-0.44 (0.11)	5.25 (2.45)	3.26 (1.38)	1.96 (0.84)
F_v	3.88**	6.44**	4.67**	6.25**	2.42	2.20	4.28**
R^2_v	0.03	0.04	0.05	0.06	0.01	0.01	0.03
	x = *Δ3Month Tbill Rate*						
$\gamma(1)$	3.49 (1.17)	2.18 (0.90)	1.86 (0.78)	-0.34 (0.10)	3.80 (2.65)	2.54 (1.24)	1.79 (0.87)
F_v	3.39*	5.80**	2.85*	5.52**	1.87	1.80	3.65*
R^2_v	0.02	0.04	0.02	0.03	0.01	0.00	0.02
	x = Term Spread						
$\gamma(1)$	4.21 (1.33)	2.09 (0.86)	2.27 (0.79)	-0.41 (0.12)	5.31 (2.72)	3.42 (1.40)	1.75 (0.82)
F_v	3.85**	6.82**	3.33*	5.86**	2.46	2.46	4.78**
R^2_v	0.03	0.04	0.03	0.05	0.02	0.02	0.03
	x = SP Returns, Δ3Month Tbill Rate, Term Spread						
$\gamma(1)$	3.27 (1.21)	2.09 (0.87)	1.89 (0.77)	-0.32 (0.10)	2.88 (2.44)	2.43 (1.29)	1.77 (0.85)
F_v	3.49*	5.18**	3.42*	4.78**	1.23	1.69	3.38*
R^2_v	0.02	0.03	0.02	0.03	0.00	0.00	0.02

Notes: Panel (b) presents values of F_v, F_f, R^2_v and R^2 for different values of p. Panel (c) shows results for regressions augmented with additional control variables and p = 1.

Panel (d). Results for alternative specifications

Specification	*GDP*		*GDP*†	
	$\gamma(1)$	R^2_v	$\gamma(1)$	R^2_v
Baseline				
(1,1,0)	3.27 (1.21)	0.02	2.43 (1.29)	0.00
Alternative number of factors and I(1) or I(0) specifications				
(0,0,0)	4.06 (1.10)	0.04	3.07 (1.16)	0.01
(1,0,0)	3.46 (1.19)	0.03	2.62 (1.25)	0.01
(1,1,0,0)	3.36 (0.99)	0.02	2.48 (1.06)	0.00
(0,0,0,0)	0.16 (0.39)	0.01	0.29 (0.39)	0.01
(1,0,0,0)	4.44 (1.13)	0.03	3.13 (1.15)	0.01
(1,1)	2.88 (1.10)	0.02	2.59 (1.13)	0.01
(1,0)	2.98 (1.10)	0.02	2.41 (1.14)	0.01
(0,0)	3.99 (0.84)	0.06	3.54 (0.83)	0.04
Alternative VAR lag length				
VAR(2)	3.09 (1.14)	0.02	2.48 (1.19)	0.01
VAR(6)	4.48 (1.80)	0.01	3.06 (1.91)	0.00

The four columns show results from neutrality regressions for *GDP* and *GDP*† for the final specification in panel (c) (p = 1 and x =*SP Returns, ∆3Month Tbill Rate, Term Spread*).