Fig. 1. Two Slocum gliders in summer 2003. Each is about 1.5 meters long. Motion in the vertical plane follows a sawtooth trajectory. A rudder is used to steer in the horizontal plane. Maximum depth is 200 meters and average forward speed relative to the flow is approximately 35 cm/s. During the AOSN 2003 experiment, the gliders were configured to surface and communicate as frequently as every two hours.

Fig. 2. Sensor measurement locations (Spray). Each point represents the location of a profile.

Fig. 3. Sensor measurement locations (Slocum). Each point represents the location of a profile.

Fig. 4. Snapshots in time of glider formation starting at 18:03 UTC on August 6, 2003 and moving approximately northwest. The vectors show the estimate of minus the temperature gradient at the group’s center of mass at 10 meters depth. The gray-scale map corresponds to temperature measured in degrees Celsius. The three smaller black circles correspond to the initial positions of the gliders.

Fig. 5. Error map at different times during the AOSN 2003 experiment. Blue represents small error (good coverage) and red and white represents high error (poor coverage). For each panel, black dots indicates the reported position of the vehicle at the given time. The white dots represent their positions during the last 12 hours. The magenta line encloses all the points where the error has been reduced from its initial state by at least 85%. The sampling metric is shown on Fig. 6. Notice that all the gliders are clustered near the coast on August 10th explaining the drop in coverage performance visible on Fig. 6.

Fig. 6. Sampling metric (solid curve) in units of entropic information (see (10)) and number of profiles (shadowed area) for AOSN 2003. Each cross corresponds to a panel of Fig. 5. On August 10th (day 223), the number of profiles is still high but the metric indicates relatively poor coverage. The second panel of Fig. 5 explains this loss of performance by a poor distribution of the gliders in the bay on that day.

Fig. 7. Cartoons of vehicles moving around closed curves with prescribed relative phases; a) Two vehicles with relative phase equal to zero move around a circle; b) Two vehicles with relative phase equal to π move around a circle; c) Two vehicles with relative phase equal to π and each vehicle moving around a different circle; d) A closed curve with rotational order of symmetry L = 4. Four vehicles move around it with fixed relative phase.

Fig. 8. The six possible different symmetric patterns for N = 12 corresponding to M = 1, 2, 3, 4, 6 and 12. The top left is the synchronized state and the bottom right is the splay state. The number of collocated headings is illustrated by the width of the black annulus denoting each phase cluster.

Fig. 9. A numerical simulation of the splay state formation starting from random initial conditions using the control (22) with N = 12, ω0 = 0.1, κ = ω0 and K = ω0². Each vehicle and its velocity is illustrated by a black circle and an arrow. Note that the center of mass of the group, illustrated by a crossed circle, is fixed at steady-state.

Fig. 10. Simulation results for N = 12 and B = 3 starting from random initial conditions with block all-to-all spacing coupling and three fixed beacons at (R0³, R0⁶, R0⁹) = (−30, 0, 30). Phase coupling is all-to-all and block all-to-all with the potential (26). The simulation parameters are κ = ω0 = 1/10.

Fig. 11. a) The vectors d and d’ used to identify the position of the vehicle (larger white circles) relative to the focii (solid circles) for an ellipse centered at R0 and rotated by μ0. b) Depicts the angles ψ, ψ’, α, and λ used in the control design. Note that λ = 0 for stable elliptical motion with positive rotation.

Fig. 12. Non-dimensional metric Φ for one vehicle on an elliptical trajectory with semi-major and -minor axis lengths a and b. The gray scale is proportional to the value of Φ, from low uncertainty (dark) to high (light). The sampling numbers are S3 = 2, Sh = 1, Sn = 0.1, Sp = 3. The minimum gives the optimal ellipse (a circle) a = b = 0.256.

Fig. 13. Snapshots in time of the error maps associated with the optimal elliptical trajectories for selected values of the parameters. Sn = 0.1 and Sp = 3. Vehicle position is represented by a small circle and velocity by a vector.

Fig. 14. Optimal value of metric Φ as a function of S3 and Sh for Sn = 0.1 and Sp = 3 with a single vehicle on an elliptical trajectory. The elliptical trajectory at each point yields the minimal value of Φ for the corresponding values of S3 and Sh. The gray scale is proportional to the value of Φ, from low uncertainty (dark) to high (light). Within numerical accuracy, Φ is independent of Sh, the shape of the domain. The plot shows that the same performance can be achieved on a rectangle of any aspect ratio (with the appropriately shaped optimal trajectory).

Fig. 15. Top Panel: Value of the metric for the optimal circular trajectory of one vehicle as a function of S3. Bottom Panel: Radius of the optimal circle as a function of S3. Each curve correspond to different values of the sensor noise Sn and the vehicle speed Sp. Notice that Φ does not depend on Sp. Moreover, the optimal radius does not depend on Sn.
Fig. 16. Optimal ellipse trajectories for two vehicles in a square domain with $S_3 = 1$. The left column shows the simulated trajectories using the feedback control from Section VI to stabilize the vehicles to the optimal ellipses with the control gains $\kappa_k = 1/a_k$ and $K = 0.05$, where $a_k$ is the semi-major axis of the $k$th ellipse for $k = 1, 2$. The right column shows the resulting error maps (gray scale) for the steady-state measurement distribution. The rows represent simulations #1, #2, #3, #6 (see Table II). The small circles and heavy vectors show the positions and velocities, respectively, of the vehicles at the time shown. The light arrows represent the direction (and not magnitude) of the flow, if present in the simulation.