Supporting Information

Starling flock networks manage uncertainty in consensus at low cost

George F. Young[1], Luca Scardovi[2], Andrea Cavagna[3], Irene Giardina[3], Naomi E. Leonard[1,*]

1 Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ, USA

2 Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON, Canada

3 Istituto dei Sistemi Complessi, Consiglio Nazionale delle Ricerche and Dipartimento di Fisica, Universitá Sapienza, Rome, Italy

∗ E-mail: naomi@princeton.edu

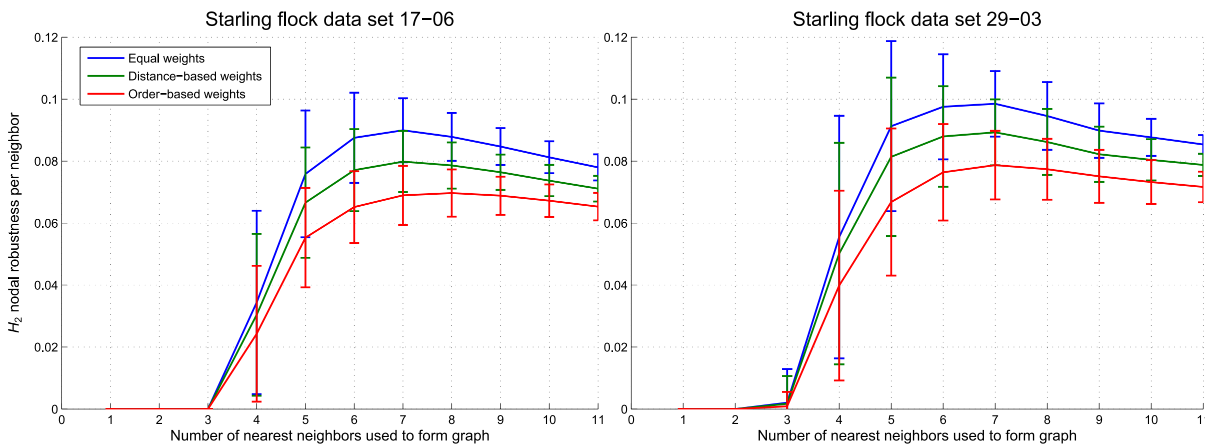

Figure S1. Average H_2 nodal robustness per neighbor as a function of the number of nearest neighbors (m) used to form the graph for two different flocks, with the edge weights computed in three different ways. Although equal edge weights were used throughout this paper, a plausible alternative is that greater weight is given to closer neighbors. In each case, the edge weights in the sensing graph are normalized so that the sum of all weights used by any individual bird is 1. The blue curves show results with equal edge weights (i.e. $a_{i,j} = \frac{1}{m}$ when an edge is present), as used in the rest of this paper. The green curves show results with edge weights inversely proportional to the distance between birds (i.e. $a_{i,j} \propto \frac{1}{d_{i,j}}$ when an edge is present, where $d_{i,j}$ is the distance between birds i and j). The red curves show results with edge weights decreasing linearly according to the ordering of the neighbors from closest to furthest, such that the $(m+1)^{\mathrm{st}}$ neighbor has a weight of 0. Both additional weighting schemes decrease the importance of neighbors that are further away, although the order-based scheme is more "radical" since neighbors tend to be spaced closer than in a geometric progression. In every case, decreasing the weight given to further neighbors tends to decrease the overall robustness, but the location of the peak remains unchanged except for the order-based scheme in flock 17-06. These results suggest that a substantial variation in edge weights is required to move the peak of the robustness per neighbor curve, and furthermore that overall robustness is decreased by doing so.

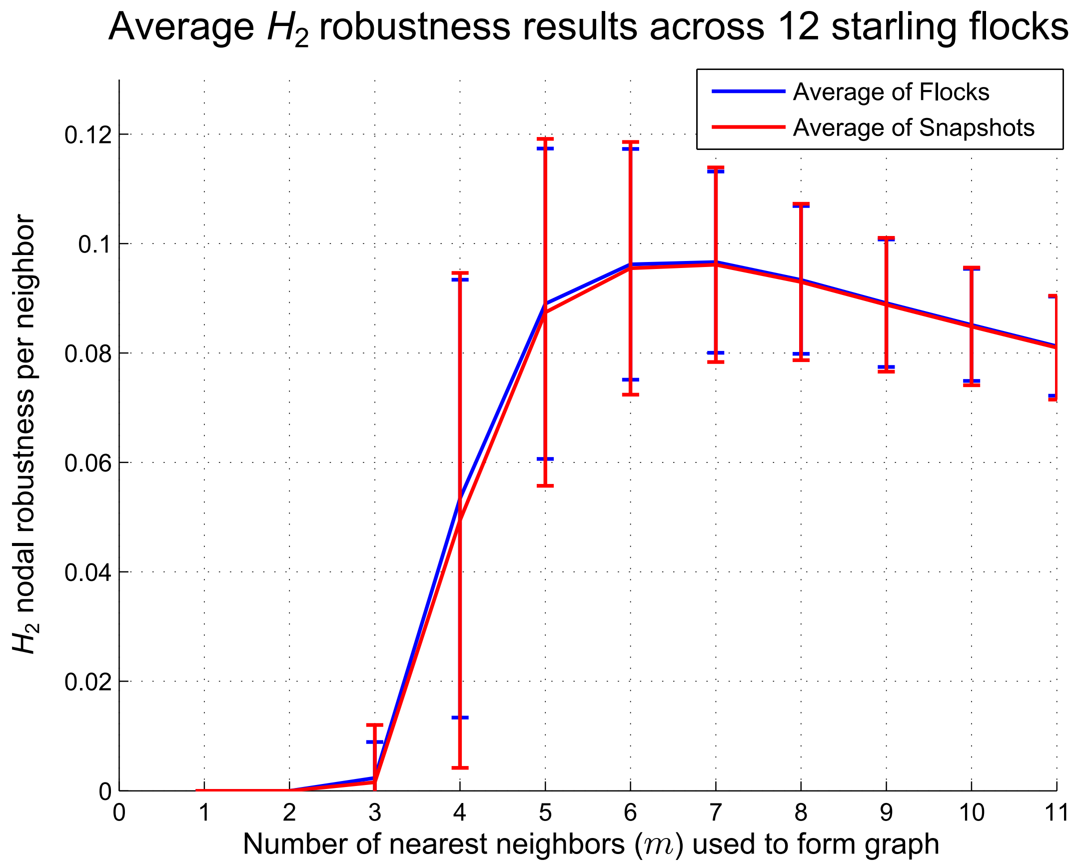

Figure S2. Average H_2 nodal robustness per neighbor as a function of the number of nearest neighbors (m) used to form the graph, with the average taken in two different ways. The blue curve shows the average of the twelve flock averages (as in Fig. 1 from the main text), while the red curve shows the average of the 394 snapshots taken across all flocks. In each case, the error bars show standard deviation. Since the results of the two averages match so closely while the number of snapshots taken of each flock varied between 16 and 80, this suggests that each snapshot may be taken to be an independent observation.

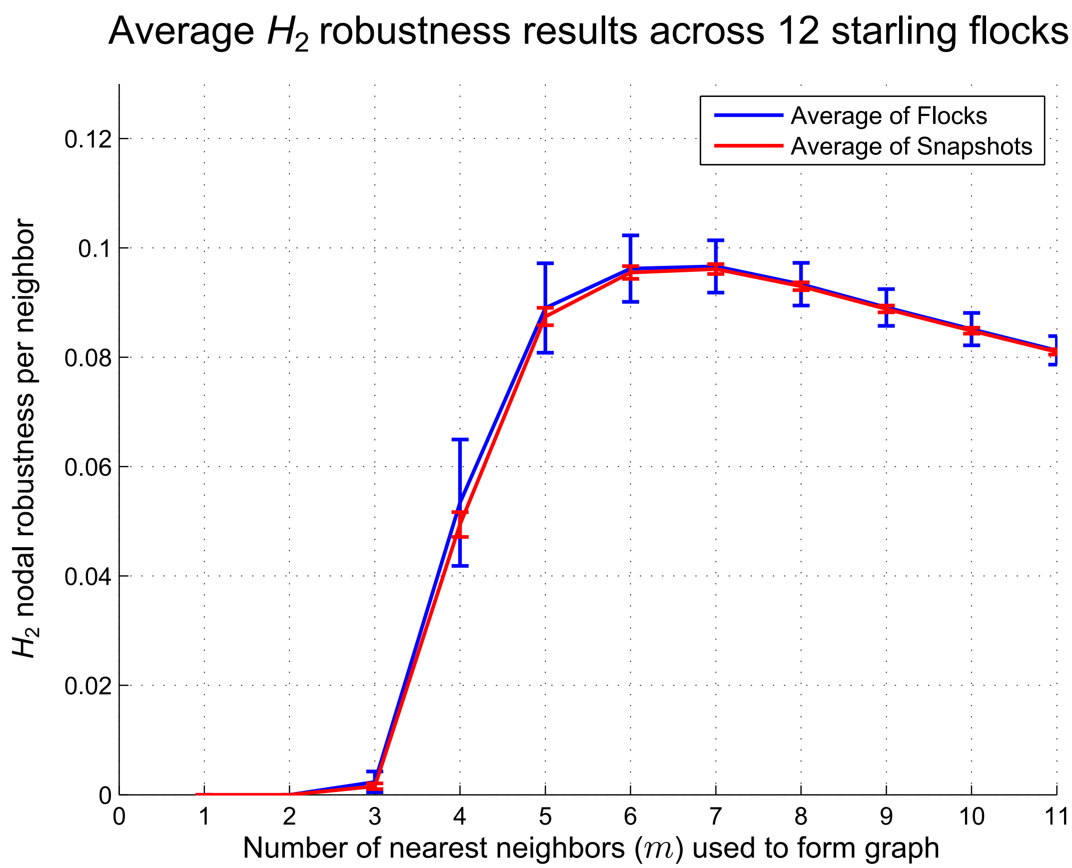

Figure S3. Average H_2 nodal robustness per neighbor with standard error as a function of the number of nearest neighbors (m) used to form the graph, with the average taken in two different ways. The blue curve shows the average of the twelve flock averages (as in Fig. 1 from the main text), while the red curve shows the average of the 394 snapshots taken across all flocks. In each case, the error bars show standard error. By treating each snapshot as independent (see Fig. S2), the standard error is reduced and we can be more certain that the peak robustness per neighbor occurs at $m = 6$ or $m = 7$.

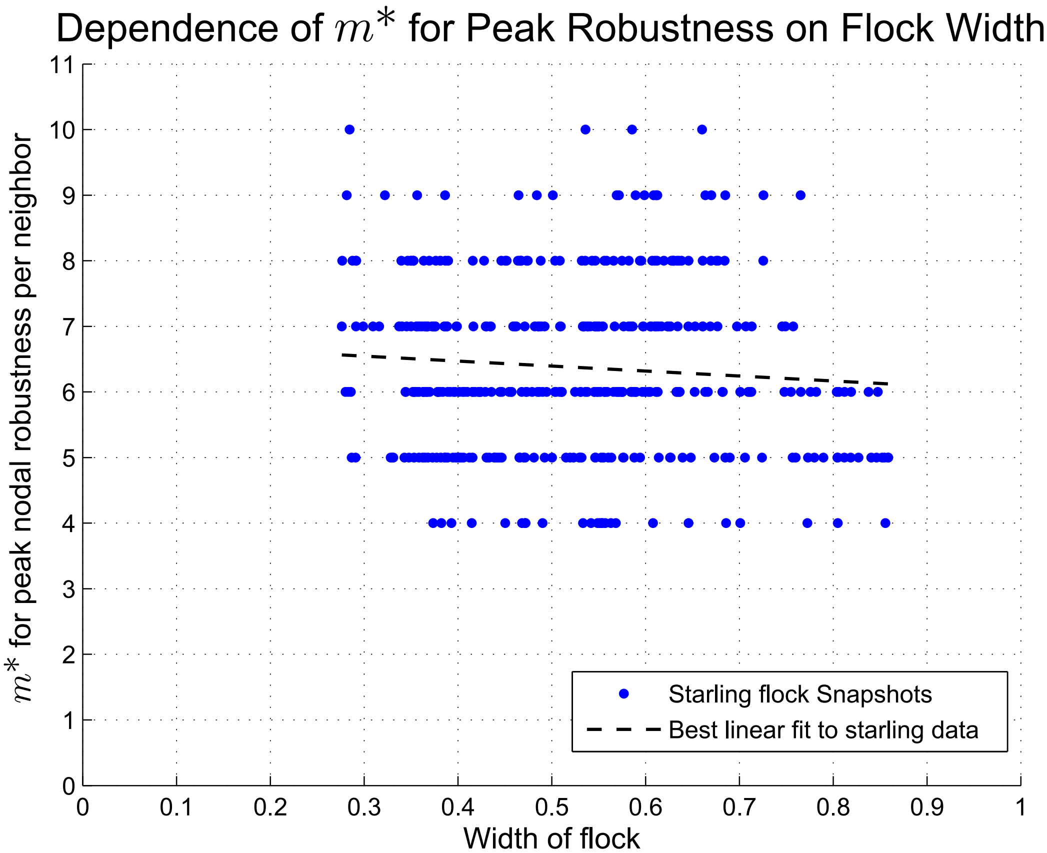

Figure S4. Dependence of the optimum number of neighbors (m^*) on the width of the flock. Flock width is defined as the ratio of intermediate to largest dimension of an ellipsoid having the same principal moments of inertia as the flock. No significant dependence on width is observed, with the best linear fit having negligible slope and an R^2 value of 0.0064.

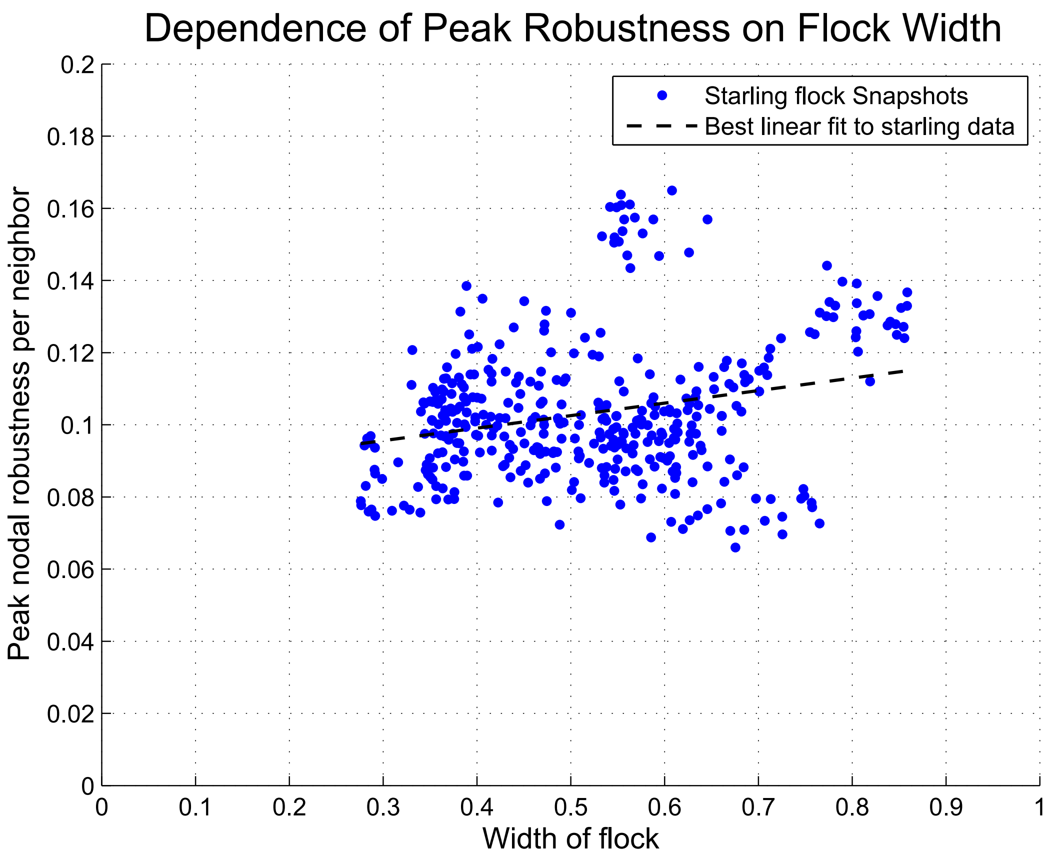

Figure S5. Dependence of the peak value of robustness per neighbor on the width of the flock. Flock width is defined as the ratio of intermediate to largest dimension of an ellipsoid having the same principal moments of inertia as the flock. No significant dependence on width is observed, with the best linear fit having a slight positive slope and an R^2 value of 0.0635.

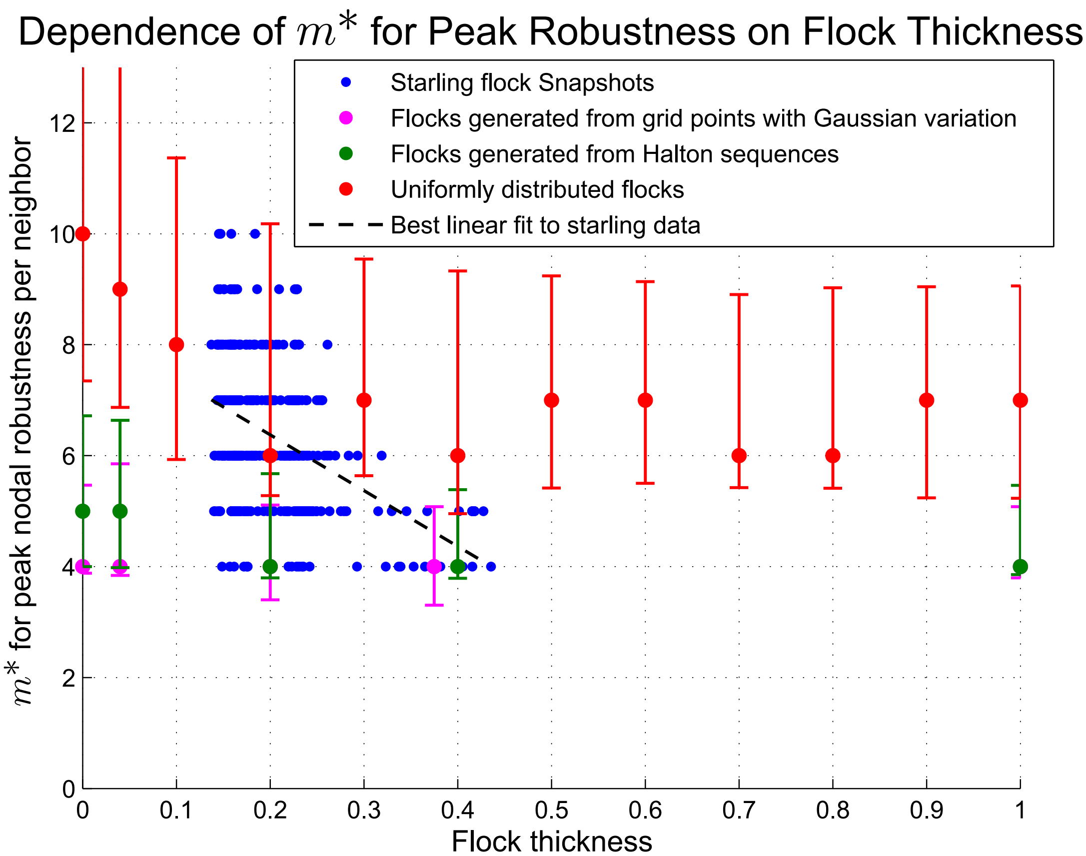

Figure S6. Dependence of the optimum number of neighbors (m^*) on the thickness of the flock. In addition to starling data plotted in blue, results are shown from flocks randomly generated from three different distributions within a rectangular prism. These distributions are as follows: points arranged in a grid and then perturbed with Gaussian noise (in magenta), points generated from Halton sequences (in green) and points taken from a uniform distribution (in red). Each data point shown from the random flocks is the average result from generating 100 separate flocks, each containing approximately 1200 individuals. The error bars show the range of values for which the robustness per neighbor is within 90% of the peak. As the random flocks become more ordered, the m^* values decrease and there is less of a dependence on thickness, with the most ordered flocks showing no thickness dependence. Compared to these three distributions, the starling flocks appear closest to uniform, with slightly more "order."

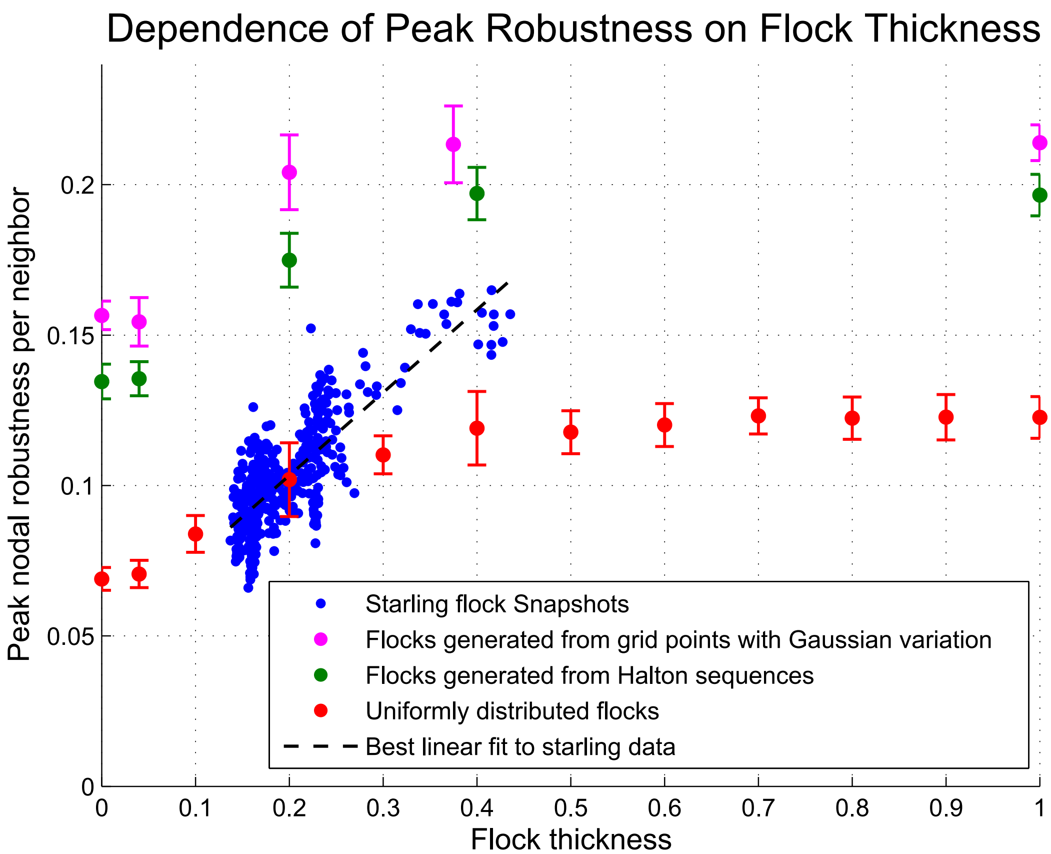

Figure S7. Dependence of the peak value of robustness per neighbor on the thickness of the flock. In addition to starling data plotted in blue, results are shown from flocks randomly generated from three different distributions within a rectangular prism. These distributions are as follows: points arranged in a grid and then perturbed with Gaussian noise (in magenta), points generated from Halton sequences (in green) and points taken from a uniform distribution (in red). Each data point shown from the random flocks is the average result from generating 100 separate flocks, each containing approximately 1200 individuals. The error bars show standard deviation. As the random flocks become more ordered, the peak values increase but the same thickness trends are apparent, i.e., the curves all show a sigmoidal shape in the increase in peak value with increasing thickness. The starling flocks are close to the uniform flocks in most cases, with higher robustness values in other cases.

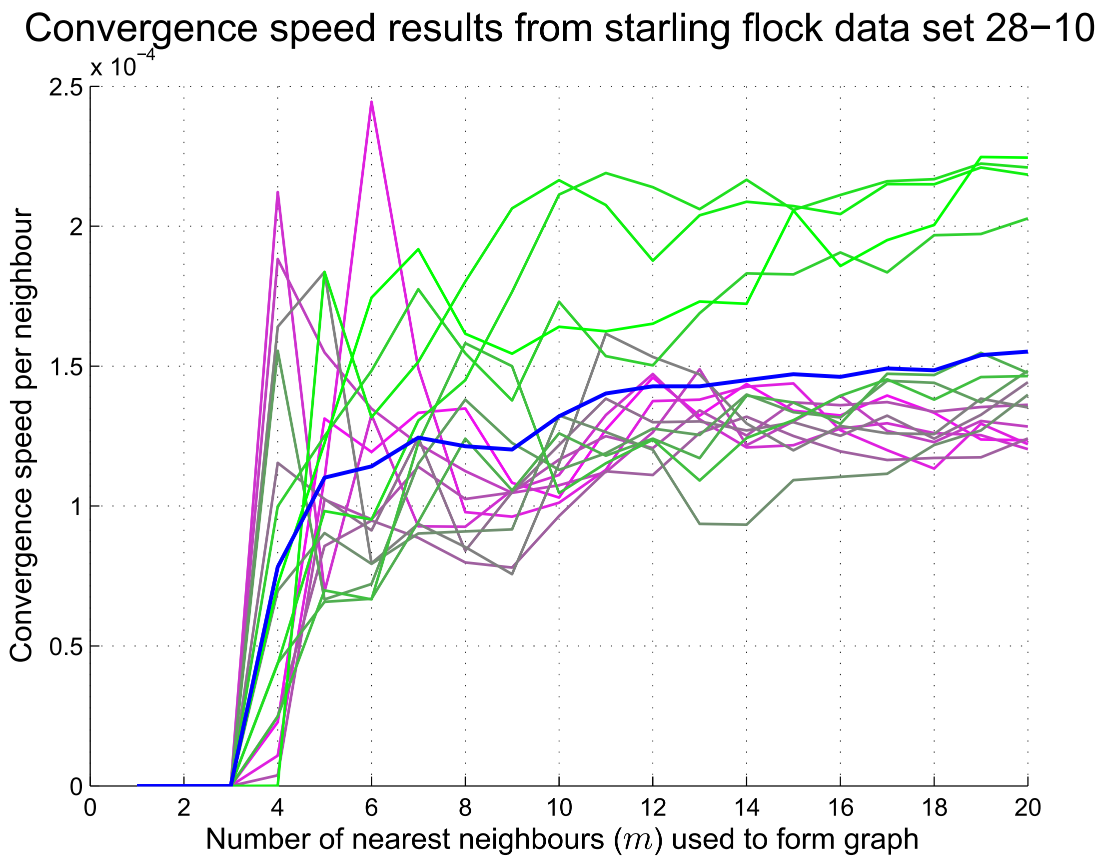

Figure S8. Speed of convergence to consensus (in the absence of noise) per neighbor as a function of the number of nearest neighbors (m) used to form the graph, for one starling flock containing approximately 1300 birds. Speed is computed as the real part of the second-smallest eigenvalue of the Laplacian matrix - this is the exponential rate of convergence for the system in Eq. (2) of the main text. The units of speed are normalized with the fastest possible speed (in the case of every individual sensing every other individual) being 1. The thin lines show results for each snapshot, while the thick blue line shows the average over all snapshots. On average, speed per neighbor increases with m, with no maximum observed for $m < 20$. In fact, when m is equal to the number of birds in the flock, the speed per neighbor will be approximately 7.7×10^{-4}, significantly larger than the values for small m.

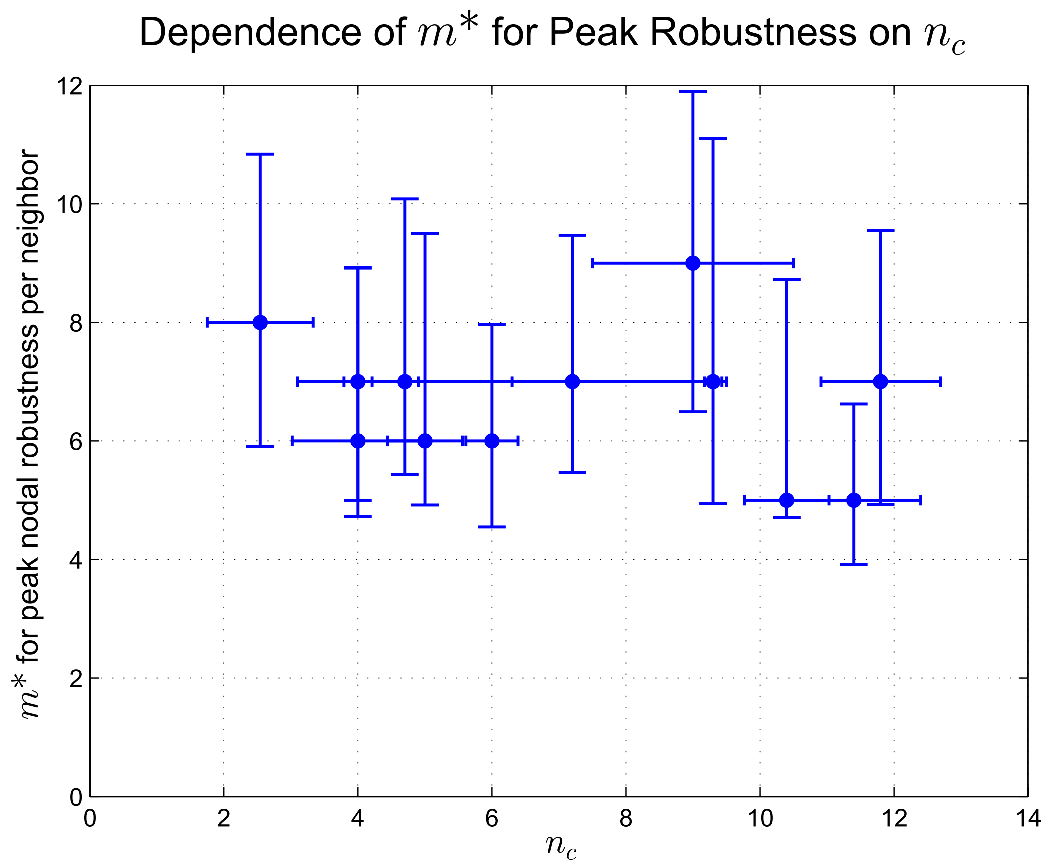

Figure S9. A comparison between the optimal number of neighbors (m^*) and the observed topological range (n_c) for each flock studied in this paper. The vertical error bars show the range of values for which the robustness per neighbor is within 90% of the peak and the horizontal error bars show the error in the estimates of n_c. No significant correlation is observed between these two measures, with a correlation coefficient of approximately -0.24 and a p-value of approximately 0.46. This is not surprising since it seems unlikely that individual starlings would interact with more or fewer neighbors based on the thickness of the flock (or any other bulk flock parameter). In addition, this suggests that the two analyses are independent and there is no underlying mathematical reason why m^* should be so close to n_c.