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The consistency of the subjective concept of randomness

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Abstract

A pervasive bias in the subjective concept of randomness is that people often expect random sequences to exhibit more alternations than produced by genuine random processes. Although this over-alternation bias has received much attention, what is less known is the stability of the bias. Here we examine two important aspects of the over-alternation bias: first, whether this bias is present in stimuli that vary across feature dimensions, sensory modalities, presentation modes, and probing methods; and second, how consistent the bias is across these stimulus variations. In Experiment 1, participants adjusted sequences until they looked maximally random. The sequences were presented as temporal streams of colors, shapes, auditory tones, or tiled as spatial matrices. In Experiment 2, participants produced random matrices by adjusting the color of each cell. We found that participants judged and produced over-alternating stimuli as the most random. Importantly, this bias was consistent across presentation modes (temporal vs. spatial), feature dimensions (color vs. shape), sensory modalities (visual vs. auditory), speed (fast vs. slow), stimulus size (small vs. large matrices), and probing methods (adjusting the generating process vs. individual bits). Overall, the results suggest that the subjective concept of randomness is highly stable across stimulus variations.

Keywords: randomness, modalities, alternation bias, anchoring effect, features

Introduction

In common parlance the term "random" is applied to sequences of events that appear sufficiently disorderly or unstructured. For example, the string *hthhtthttht* of heads and tails from coin tosses might qualify as random, whereas *hhhhhhtttttt* would not. A contrasting usage, adopted here, applies the term to certain mechanisms for generating events, namely, whose successive outputs are independent and unbiased. A standard example is a device *D* that tosses a fair coin repeatedly (ignoring issues about predictability assuming all forces were known). Any sequence of heads and tails produced by such a device is qualified as "randomly generated (by *D*)" regardless of its pattern. Thus, the terminology in this paper follows a "process" rather than "product" conception of randomness (see Eagle, 2014; Earman, 1986 for an extended discussion between the two approaches). Specifically, in our usage a "random" stimulus (or pattern) is an object that has been produced by a random process. Non-random stimuli are defined as productions from a distorted random source.

The randomness of the coin flipper *D* can be compromised in various ways, for example, by making one of its two outcomes more likely than the other. Instead of introducing a bias to the probabilities of the two outcomes, here we consider deviations from stochastic independence by allowing previous flips to influence the next one, while maintaining the equal frequency of the two outcomes. This allows us to identify the lay conception of randomness along a continuum. Specifically, for each number x in the unit interval (from 0 to 1), let $D(x)$ generate a sequence of bits consisting of zeros and ones as follows:

(1) Sequence generation using the device *D*(x): An unbiased coin toss determines the first bit. Suppose that the n^{th} bit has been constructed (for $n \geqslant 1$). Then with probability x the n + 1^{st} bit is set equal to the opposite of the n^{th} bit; with probability $1 - x$ the n + 1^{st} bit is set equal to the n^{th} bit. Repeat this process to generate a sequence of any length.

This procedure was introduced by Zhao, Hahn, and Osherson (2014). It can be seen that *D*(.5) is a genuinely random device. For $x < .5$, *D*(x) tends to repeat itself, resulting in long runs whereas for $x > .5$, *D*(x) tends to alternate. In particular, *D*(0) is uniform, either 0000… or 1111…, while *D*(1) consists of perfectly alternating bits, either 0101… or 1010…. The expected proportion of each bit is 50%, for all $x \in (0, 1)$. For any sequence produced by *D*(x), the expected proportion of alternation — called the "switch rate" — is x. The switch rate of any sequence is calculated by the number of switches between two successive bits divided by the total number of bits in the sequence minus one. For example, the switch rate of the sequence of 11111 or 00000 is 0, and the switch rate of 01010 is 1.

A large body of research on randomness perception has revealed a pervasive bias in people's concept of "random." That is, people often expect random sequences to exhibit shorter runs (greater alternation) than typically produced by random devices, or in our terms, $x > .5$ (Kahneman & Tversky, 1972, Falk & Konold, 1997; Lopes & Oden, 1987; Nickerson & Butler, 2009; Wagenaar, 1972; for reviews on theoretical analysis of major experimental findings see Bar-Hillel & Wagenaar, 1991; Oskarsson et al., 2009).

The over-alternation bias has been typically examined using judgment or production paradigms (e.g., Kahneman & Tversky, 1972; Wagenaar, 1972), where participants either judged how random a sequence appeared, or produced a sequence as if it were generated by a random process. The over-alternation bias explains the gambler's fallacy (Kahneman & Tversky, 1972; Reuter et al., 2005; Wagenaar, 1988), and can be driven by limitations in working memory (Baddeley, 1966; Hahn & Warren, 2009; Kareev, 1992). Despite the overwhelming evidence for the over-alternation bias, what has been given little attention is whether this bias is consistent across sensory modalities, stimulus feature dimensions, presentation modes, and tasks.

The primary goal of the current study is thus to examine the concept of randomness across multiple domains. This will offer insights on the stability or consistency of the over-alternation bias. In Experiment 1, participants observed a sequence of bits produced by $D(x)$ for as long as they wanted. They were asked to adjust the sequence by increasing the alternations or the repetitions of the bits, in order to make the sequence look maximally random. In other words, they manipulated the switch rate x until they were satisfied that the sequence was being generated "randomly." We then measured how close x was to .5 (genuine randomness). Of principal interest was the stability of the switch rate across variations in the stimuli that represent the bits generated by $D(x)$. In Experiment 2, we examined the stability of the switch rate in a randomness production task. The two experiments investigated the consistency of people's randomness concept across stimulus features, modalities, presentations, and tasks. A summary of the stimulus conditions was shown in Table 1 below.

Table 1. Summary of stimulus conditions in Experiments 1 and 2.

Experiment	Probing method	Starting point	Presentation	Features
1	Adjust the switch rate of the stimulus until it looks maximally random	The starting point of the stimulus switch rate was randomly determined from 0 (fully repeating) to 1 (fully alternating)	Temporal	Colored squares: green and blue
			Temporal	Black shapes: squares and circles
			Temporal	Tones: high and low pitches
			Spatial	10×10 matrices
			Spatial	80×80 matrices
2	Change the color of each cell until the matrix looks maximally random	The starting point of the switch rate was 0 (fully repeating)	Spatial	10×10 matrices
		The starting point of the switch rate was 1 (fully alternating)	Spatial	10×10 matrices
		The starting point of the switch rate was 0.5 (fully random)	Spatial	10×10 matrices

Experiment 1

The goal of this experiment was to examine the subjective concept of randomness across stimulus feature dimensions, sensory modalities, and presentation modes.

Participants

Forty-six undergraduate students (29 female, mean age=20.7 years, SD=2.1) from the University of British Columbia (UBC) participated for course credit. Participants in both experiments provided informed consent. Both experiments reported here have been approved by the UBC Behavioral Research Ethics Board.

Apparatus

In all experiments, participants were seated 50cm from a computer monitor (refresh rate = 60Hz) and used stereo headphones for auditory stimuli. Stimuli were presented and responses were collected using JAVA and Python interfaces.

Stimuli

Temporal sequences. There were six temporal trials, each containing a binary sequence. The six trials consisted of two color trials, two shape trials, and two auditory trials (Fig.1a). In each color trial, the two bits were represented by a green square (RGB values: 3, 254, 82) and a blue square (RGB values: 6, 32, 244). In each shape trial, the two bits were a black square and a black circle. The square width and the circle diameter subtended 5.1°. In each auditory trial, the two bits were a high tone (pitch: 392Hz) and a low tone (pitch: 262Hz). For each type of trial, there was one fast trial and one slow trial. In a fast trial, each bit was presented for 400ms and the inter-stimulus interval (ISI) was 200ms. In a slow trial, each bit was again presented for 400ms but the ISI was 1000ms.

Spatial matrices. There were six spatial trials, each containing a matrix. The six trials consisted of three small matrices and three large matrices (Fig.1b). Each matrix was constructed by tiling either vertically or horizontally a sequence of green and blue squares. The large matrix was 80×80, with 6400 bits subtending 14°. The small matrix was 10×10, with 100 bits subtending 11.3°.

Thus, there were 12 trials in total for each participant, allowing the following comparisons in presentation mode temporal versus spatial, color versus shape, visual versus auditory, fast versus slow, and small versus large matrices. These exhaustive comparisons permitted the examination of consistency of randomness judgments across various stimulus domains within the same individual participant.

Procedure

Temporal trials. For each temporal trial, the starting switch rate of the sequence was randomly determined from 0 to 1. Each bit was presented on the screen for 400ms, and the ISI was 200ms for a fast trial, and 1000ms for a slow trial. As participants viewed each bit, they were asked to adjust the sequence by clicking on two buttons, one labeled "more repeating" and the other "less repeating", in order to make the sequence maximally random. As soon as the participant pressed a button, the switch rate of the sequence changed by a constant amount. Specifically, if the participant pressed on the "more repeating" button, the switch rate decreased by 0.025; if the participant pressed on the "less repeating" button, the switch rate increased by 0.025. Participants were encouraged to view as many bits as possible before they made a decision, and also to adjust the sequence as many times as they wanted.[1] There was no time limit for any trial. When the sequence was rendered maximally random in the judgment of the participant, s/he pressed another button to end the trial.

Spatial trials. For each spatial trial, the starting switch rate of the sequence was randomly determined from 0 to 1. The tiling (vertical or horizontal) of the sequence in the matrix was randomly determined. Participants were asked to adjust the matrix by clicking on either the "more repeating" or the "less repeating" button, to make the matrix look maximally random. As soon as the participant pressed a button, the switch rate of the sequence changed by 0.025 as in the temporal trials, and a new matrix was presented with a randomly determined tiling direction. Participants were encouraged to view each matrix for as long as desired, and to make as many adjustments as they wanted. There was again no time limit for any trial. If the matrix looked maximally random, participants pressed another button to end the trial. The order in which participants completed the temporal or the spatial trials was randomized.

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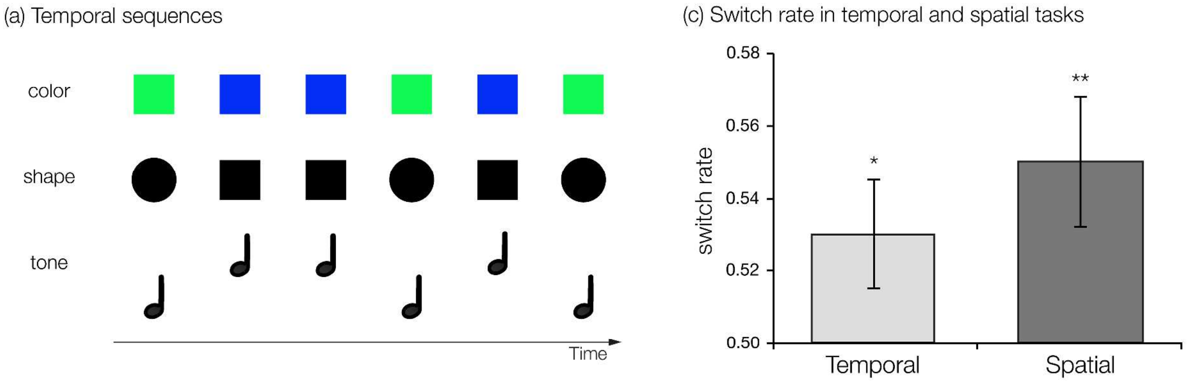
[1] The encouragement to observe long stretches of the sequence was a hedge against unlucky outputs from $D(x)$. It is possible even for $D(.5)$ to relentlessly produce the same bit over and over. (Notice that all sequences of the same length have the same probability of random generation.) There is thus no guarantee that bad luck doesn't infect the results reported below, only low probability that such is the case.



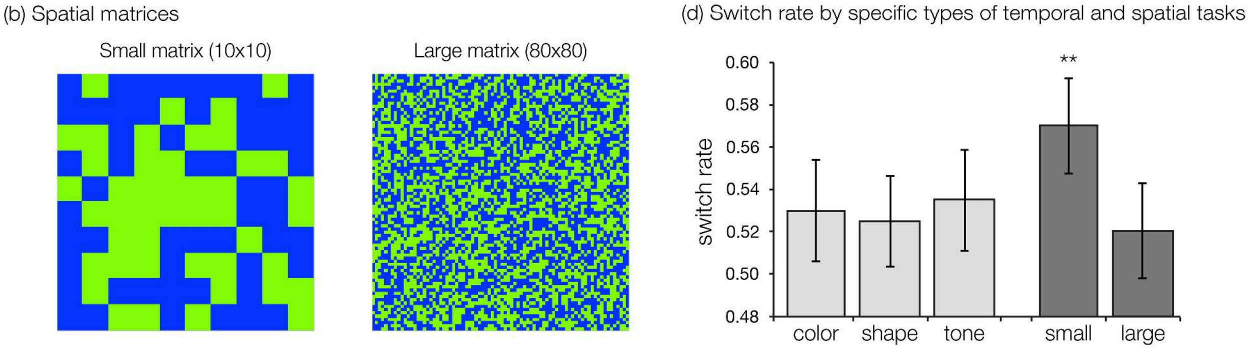


Figure 1. Stimuli and results for Experiment 1. (a) Three types of temporal sequences were presented. In the color sequence, the two bits were green and blue squares. In the shape sequence, the two bits were circles and squares. In the auditory sequence, the two bits were high and low tones. Each sequence started with a random switch rate, and participants adjusted the switch rate until the sequence looked maximally random. (b) Two types of matrices were presented, a small10×10 matrix, and a large 80×80 matrix. A fully alternating sequence tiled horizontally or vertically would result in a striped pattern, rather than a checkerboard pattern. Each matrix was generated from a sequence of blue and green squares, tiled vertically or horizontally. Participants adjusted the switch rate of the matrix until it looked maximally random. (c) The switch rate of temporal and spatial trials. (d) The switch rate of each type of temporal and spatial trials. Error bars indicate ±1 between-subjects SEM. (*p<.05, **p<.01)

Results and discussion

For each trial, the switch rate of the sequence that the participant judged to be maximally random was recorded. Across participants, the average switch rate of the temporal trials was 0.53 (SD=0.10), reliably above 0.5 (fully random) [$t(45)=2.03$, $p<.05$, $d=0.30$]. The average switch rate of the spatial trials was 0.55 (SD=0.12), again reliably above 0.5 [$t(45)=2.71$, $p<.01$, $d=0.40$], but not different from that of the temporal trials [$t(45)=0.80$, $p=.43$, $d=0.14$]. This reveals an over-alternation bias for both temporal and spatial trials (Fig.1c). To confirm that the initial switch rate of each sequence was not biased, we found that the starting switch rate was not different from 0.5 for the temporal trials [M=0.49, SD=0.12, *t*(45)=0.42, *p*=.68, *d*=0.06], or for the spatial trials [M=0.50, SD=0.12, *t*(45)=0.14, *p*=.89, *d*=0.02]. These results suggest that over-alternating sequences were judged to be maximally random, regardless of the temporal or the spatial presentation mode.

The specific types of temporal trials were compared. The average switch rate was 0.533 (SD=0.16) for the color trials and 0.525 (SD=0.15) for the shape trials, and the two were not different [*t*(45)=0.30, *p*=.77, *d*=0.05]. The average switch rate for the auditory trials was 0.535 (SD=0.16). Importantly, there was no difference in the switch rate among the color, shape, and auditory trials [*F*(2,132)=0.03, *p*=.97, η_p^2<0.001]. Moreover, the average switch rate was 0.528 (SD=0.13) for the fast trials and 0.534 (SD=0.12) for the slow trials, and the two were not different [*t*(45)=0.28, *p*=.78, *d*=0.05]. No specific trial type produced a switch rate reliably above 0.5 [*t*'s(45)<1.90, *p*>.06, *d*<0.29]. Nonetheless, the switch rates of all trial types were remarkably similar (between 0.525 and 0.535, Fig.1d). This suggests that the randomness judgment was consistent across feature dimensions, sensory modalities, and presentation speed.

For spatial trials, the average switch rate was 0.57 (SD=0.15) for small matrices, reliably above 0.5 [*t*(45)=3.28, *p*<.01, *d*=0.48]. The average switch rate was 0.52 (SD=0.15) for large matrices, not different from 0.5 [*t*(45)=0.89, *p*=.38, *d*=0.13], or from that of the small matrices [*t*(45)=1.84, *p*=.07, *d*=0.35]. There was a significant correlation in the switch rates between small matrices and temporal trials [*r*(44)=0.39, *t*=2.79, *p*<.01], but not between large matrices and temporal trials [*r*(44)=0.13, *t*=0.86, *p*=.40]. This suggests that the over-alternation bias was more prominent when the sample that the participants experienced at a given moment in time was small (100 bits) than when the sample was large (6400 bits).

Finally, we observed a strong anchoring effect for all types of trials in the experiment. Specifically, the switch rate was highly correlated with the starting switch rate for both temporal trials [$r(44)=0.50$, $t=3.85$, $p<.001$] and spatial trials [$r(44)=0.44$, $t=3.23$, $p<.01$], and also for color [$r(44)=0.58$, $t=4.67$, $p<.001$], shape [$r(44)=0.46$, $t=3.44$, $p<.01$], auditory trials [$r(44)=0.59$, $t=4.85$, $p<.001$], as well as for small matrices [$r(44)=0.55$, $t=4.33$, $p<.001$] and large matrices [$r(44)=0.49$, $t=3.72$, $p<.001$].

Taken together, these results suggest that over-alternating sequences were judged as maximally random. Importantly, this bias was consistent across presentation modes (temporal vs. spatial), feature dimensions (color vs. shape), sensory modalities (visual vs. auditory), speed (fast vs. slow), and stimulus size (small vs. large matrices).

Experiment 2

In Experiment 1, any adjustment made by the participants altered the underlying process that generated the stimuli, and thus resulted in an entirely new and different sequence. A different method to probe people's concept of randomness is to have participants produce each bit in the sequence as if the bits are generated by a random process. Thus, Experiment 2 employed this paradigm to see if the over-alternation bias was consistent with that observed in Experiment 1.

Participants

Sixty-five undergraduate students (47 female, mean age = 21.5 years, SD = 2.9) from UBC participated for course credit.

Stimuli and procedure

Participants completed 12 trials in total. In each trial, they were first presented with a matrix and then asked to adjust the cells in the matrix to make it maximally random. The initial matrix was fully uniform, alternating, or random. Participants were encouraged to change as many cells in the matrix as they wanted, in order to make the matrix maximally random. This method was comparable to that in Experiment 1 where a strong anchoring effect was observed.[2]

Each matrix was 10×10 with 100 cells in total, subtending 15°. Each cell could be either black or white, representing the two possible bits. Of the 12 trials, there were three types of matrices which were initially presented to participants: uniform with all black or white cells (switch rate of 0), fully alternating with a sequence (switch rate of 1) tiled horizontally or vertically, and fully random with a random sequence (switch rate of 0.5) tiled horizontally or vertically. The trials were presented in a random order.

In each trial, participants first viewed the initial matrix, and then clicked on any cell in the matrix to reverse its color. They were told to produce a maximally random matrix by changing as many cells as they like. As in Experiment 1, there was no time limit.

Results and discussion

To compute the switch rate of a produced matrix, the matrix was transformed into two binary sequences, one by extracting the bits across columns horizontally through the matrix, and another by traversing across rows vertically through the matrix. The switch rates of the two sequences were computed and then averaged. The average switch rate was 0.53 (SD=0.07), reliably above 0.5 [$t(64)=3.48$, $p<.001$, $d=0.43$]. This again shows an over-alternation bias in the production of random matrices. The current switch rate was not reliably different from the switch rate of the small matrices (0.57) in Experiment 1 [$t(109)=1.78$, $p=.08$, $d=0.36$], or from the

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[2] It is difficult to compare our paradigms with a temporal production task where participants generate a random sequence. This is because participants are not exposed to any initial sequence in the temporal production task, and thus their produced sequence is free from any anchoring effect. Therefore, we did not include a temporal production task in this experiment. Nonetheless, to provide evidence for our concern, we did run a temporal production task where each participant produced a 100-bit sequence of Ts and Hs. The average switch rate of the produced sequence was 0.68 (SD=0.12), reliably above 0.5 [$t=11.54$, $p<0.001$, $d=1.43$]. This switch rate is consistent with previous studies on randomness production (Bar-Hillel & Wagenaar, 1991), but much higher than the switch rate in our tasks.

switch rate of temporal trials (0.53) in Experiment 1 [*t*(109)=0.06, *p*=.95, *d*=0.01]. This reveals a consistent concept of randomness between the two experiments using different probing methods.

Among the three types of matrices, there was a reliable difference in the switch rate via a one-way repeated measures ANOVA [*F*(2,189)=3.28, *p*=.04, η_p^2=0.03]. Specifically, when the initial matrix was uniform, the switch rate of the produced matrix was 0.48 (SD=0.13), not different from 0.5 [*t*(64)=1.14, *p*=.26, *d*=0.14], but reliably different from that when the initial matrix was fully alternating [M=0.55, SD=0.05, *t*(64)=5.45, *p*<.001, *d*=0.70], or when the initial matrix was fully random [M=0.56, SD=0.05, *t*(64)=5.19, *p*<.001, *d*=0.74]. The latter two switch rates were not different from each other [*t*(64)=0.62, *p*=.53, *d*=0.06], but were both reliably above 0.5 [alternating: *t*(64)=7.74, *p*<.001, *d*=0.96; random: *t*(64)=8.88, *p*<.001, *d*=1.10].

These results suggest that the produced matrix was over-alternating, and biased toward the initial matrix, showing the same anchoring effect as in Experiment 1. Importantly, the over-alternation bias was consistent with that in Experiment 1, despite the differences in the tasks.

General Discussion

The goal of the present study was to examine consistency in the subjective concept of randomness across different domains. In Experiments 1 and 2, we found a highly stable over-alternation bias across presentation modes (temporal vs. spatial), feature dimensions (color vs. shape), sensory modalities (visual vs. auditory), speed (fast vs. slow), stimulus size (small vs. large matrices), and probing methods (adjusting the generating process vs. individual bits). These results suggest that the subjective concept of randomness is consistent in the face of vast stimulus variations.

In addition, we found a strong anchoring effect in both experiments. Specifically, the switch rate of the sequences that were deemed as most random was correlated with the starting switch rate of the sequence (Experiment 1). Moreover, the switch rate of the produced matrix was lower when the starting matrix was fully uniform than when the starting matrix was fully alternating or random (Experiment 2). Despite the anchoring effect, the over-alternation bias was consistent across the two experiments.

The over-alternation bias observed in the current study was less pronounced than that in previous studies on randomness judgments (Bar-Hillel & Wagenaar, 1991; Falk & Konold, 1997; Lopes & Oden, 1987; Nickerson & Butler, 2009; Wagenaar, 1972; Zhao et al., 2014). The switch rate of the stimuli that were deemed maximally random in our tasks ranged from 0.53 to 0.57, whereas in most previous studies the switch rate was above 0.6. The relatively low switch rate might be driven by the anchoring effect. Since the starting switch rate of the stimuli was around 0.5 for both Experiments 1 and 2, this initial anchor may have weakened the over-alternation bias toward true randomness, lowering the final switch rate of the stimuli that were judged as maximally random.

The most noteworthy finding of the current study was that the over-alternation bias was consistent across various stimulus domains. This consistency suggests that people's concept of randomness is immune to differences in the stimuli used to embody randomness. People's conception of randomness must therefore have a stable abstract character, applying similarly to distinct physical domains.

What explains the consistent over-alternation bias? One explanation focuses on the limitations of working memory (Baddeley, 1966; Hahn & Warren, 2009; Kareev, 1992). People can only hold a limited number of items in working memory, which means that the amount of bits processed at a given moment in time is constrained. This is especially true when people process temporal sequences. Due to local representativeness, people may assume equal frequency of outcomes within a local sequence (Tversky & Kahneman, 1971). Such emphasis on local equality for a limited number of bits held in working memory can cause over-alternation when producing a random sequence. For a spatial matrix, the number of bits that can be processed at a given moment is also constrained depending on the size of the matrix. Within a fixation, there are more bits that can be processed for a large matrix than for a small matrix (because in large matrices each element subtends a smaller visual angle). This suggests that the smaller number of bits to be sampled at any given moment in a small matrix can lead to greater over-alternation, compared to a large matrix. This account is supported by the finding in Experiment 2 that the switch rate of the large matrices (0.52) was not reliably different from 0.5 (true randomness), but the switch rate of the small matrices (0.57) was. This could be due to the possibility that people were able to sample more information from the large matrix, thus their over-alternation bias was reduced.

In conclusion, the current study demonstrates a highly consistent over-alternation bias in people's randomness concept across presentation modes, feature dimensions, sensory modalities, stimulus speed, stimulus size, and probing methods.

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