

Distinct brain loci in deductive versus probabilistic reasoning

DANIEL OSHERSON,*§ DANIELA PERANI,† STEFANO CAPPÀ,‡ TATIANA SCHNUR,†
FRANCO GRASSI† and FERRUCCIO FAZIO†

*DIPSCO, Scientific Institute H. San Raffaele, Via Olgettina 60, I-20132 Milan, Italy; †INB-CNR, University of Milan, Scientific Institute H. San Raffaele, Via Olgettina 60, I-20132 Milan, Italy; ‡Neurology Department, University of Brescia Medical School, Ple Ospedale 1, I-25125 Brescia, Italy

(Received 15 April 1997; accepted 14 June 1997)

Abstract—Deductive versus probabilistic inferences are distinguished by normative theories, but it is unknown whether these two forms of reasoning engage similar cerebral loci. To clarify the matter, positron emission tomography was applied during deductive versus probabilistic reasoning tasks, using identical stimuli. Compared to a language comprehension task involving the same stimuli, both probabilistic and deductive reasoning increased regional cerebral blood flow (rCBF) bilaterally in the mesial frontal region and in the cerebellum. In the direct comparison, probabilistic reasoning increased rCBF in left dorsolateral frontal regions, whereas deductive reasoning enhanced rCBF in associative occipital and parietal regions, with a right hemispheric prevalence. The results suggest that reasoning about syllogisms engages distinct brain mechanisms, depending on the intention to evaluate them deductively versus probabilistically. © 1998 Elsevier Science Ltd. All rights reserved.

Key Words: deductive; probabilistic reasoning; positron emission tomography; neural correlates.

Introduction

Normative theories of reasoning distinguish two kinds of persuasive arguments depending on the inferential connection between premises and conclusion. If the truth of an argument's premises guarantee that of its conclusion, the argument is called *valid*, whereas if the premises merely enhance the plausibility of the conclusion, the argument is *probabilistically strong*. Human intuition about validity and probability is limited to inferences of moderate size and reveals systematic imperfections even when applied to simple cases. Nonetheless, starting from adolescence both forms of reasoning are recognizable approximations to their normative counterparts [3, 6, 11].

What is the psychological relation between deductive and probabilistic reasoning? One influential theory conceives both kinds of reasoning as involving the manipulation of 'mental models'. In this view, an argument is evaluated by constructing alternative models of its premises, where each model is a representation of potential circumstances that would render the premises true. The

argument is then judged to be probabilistically strong in case a large proportion of the models generated for the premises render the conclusion true as well; the intuition of validity arises from the limiting case in which this proportion reaches one. Within epistemology, such an account of the relation between validity and probability was proposed by Wittgenstein [[41], §5.15], and followed up by de Finetti [9] and others. A psychological version of the same idea has recently been proposed by Johnson-Laird [23], where it receives detailed and persuasive defense.¶ Moreover, the same theory has been claimed to predict right hemispheric predominance in the manipulation of mental models (see [24, 40]). Johnson-Laird writes: "The model theory also makes a critical prediction about the role of the cerebral hemispheres in reasoning. As Whitaker *et al.* [40] first pointed out, the construction of models is likely to depend on the right hemisphere". The mental models perspective on reasoning thus leads to the following theses:

- (1) The brain structures responsible for deductive and probabilistic reasoning are largely the same.

§ Address for correspondence: DIPSCO, Scientific Institute H. San Raffaele, Via Olgettina 60, I-20132 Milan, Italy; e-mail: osherson@sanity.hsr.it.

¶ For algorithmic simulation of the mental steps presumed to underlie the construction of mental models, see [[25], p. 171 ff.].

(2) The brain structures responsible for deductive reasoning are predominantly right hemispheric.

Of course, it follows from identity theses (1) and (2) that:

(3) The brain structures responsible for deductive and probabilistic reasoning are both predominantly right hemispheric.

Opposing theories conceive deduction as based on mental rules specific to logic [5] and thus envision no particular relation between deductive and probabilistic reasoning. Mental rules for deduction are thought to analyze and transform certain kinds of linguistic structures, namely, the logical forms of sentences. Consequently, such theories would place deductive reasoning in areas involved with linguistic processing, thus principally in the left hemisphere.* It can thus be seen that rule theories suggest the following contrast with thesis (2):

(4) The brain structures responsible for deductive reasoning are predominantly left hemispheric.

To clarify the empirical status of the foregoing claims, we used positron emission tomography (PET) to examine brain activation during three kinds of reasoning tasks. The three tasks will be called 'logic', 'probability' and 'meaning'. In the logic task, subjects were asked to distinguish valid from invalid arguments. Arguments in the probability and meaning tasks had the same layout as in the logic task, but none were valid. For probability, subjects were asked whether the conclusion had a greater chance of being true than false, supposing the truth of the premises. The meaning task required subjects to examine premises and conclusion individually and determine whether any had anomalous content; it served as a baseline condition since no more than sentence comprehension was involved. As explained below, stimulus presentation was arranged so that during PET scanning identical arguments were evaluated either for validity, probability or anomaly. Sample arguments are presented in Table 1.

Table 1. Sample arguments: A is valid, B invalid, C has anomalous content

(A)	None of the bakers play chess. Some of the chess players listen to opera. Some of the opera listeners are not bakers.
(B)	Some of the computer programmers play the piano. No one who plays the piano watches soccer matches. Some computer programmers watch soccer matches.
(C)	All the engineers own a computer. None of the engineers has been to school. All the people who own computers are married.

B was used for test, and thus evaluated on separate occasions for validity, probability and anomaly.

*Computer simulation of one version of a mental rule theory is provided in [[34], Ch. 3].

Methods

Subjects

Ten right-handed males aged 21–25 years, and recruited from local universities, served as subjects. All claimed to have little or no training in formal logic. The protocol was approved by the Ethics Committee of the San Raffaele Hospital, where the experiment was performed. Subjects provided written consent to participate.

Experimental tasks

Each of the logic, probability and meaning tasks consisted of four arguments, the first two called 'warm-up', the last two 'test'. The warm-up arguments in a given task varied with respect to possession of the target property (validity, probability or anomaly), and differed across tasks. For example, the warm-up task for logic consisted of one valid and one invalid argument, ordered randomly. Validity is an objective property of arguments, so there was no difficulty distinguishing valid from invalid. Our choice of anomalous statements similarly left little doubt about how to characterize arguments along this dimension. On the other hand, probability is more subjective, so an attempt was made on purely intuitive grounds to select (invalid) arguments that varied in probability.

In contrast to warm-up, the test arguments for all three tasks were identical, namely, a pair of invalid arguments with no anomalous content. Thus, the mental activity required to solve test problems depended only on the kind of reasoning engaged, and practiced during warm-up; the stimuli were identical for the three tasks at the moment of data acquisition. We constructed two other sets of tasks of the same design, using different arguments. The three sets of stimuli yield nine tasks, namely, three each for logic, probability and meaning.

The procedure consisted of a 30-min training session followed by PET scanning within 24 hr. The training session explained the character of the three kinds of reasoning tasks, stated that all arguments concerned a small town in northeastern Italy, and gave examples (no argument figuring in training appeared later among the experimental stimuli). For the PET session, the nine tasks were individually randomized for each subject with the constraint that successive tasks always differ (for example, two logic tasks could not be given successively). The four arguments of a given task were presented sequentially, each for 25 sec with no interstimulus interval. They appeared in the center of a 36-cm video monitor placed 60 cm from the subject's eyes. Relevant instructions from the training session were repeated prior to each task in order to ensure that the subject evaluated the succeeding four arguments in terms of the appropriate distinction (valid/invalid, probable/improbable or anomalous/normal). Subjects studied each argument silently, and made no response during the 100 sec of the task. Warm-up stimuli were evaluated for the 50 sec prior to PET acquisition. Then, without alerting the subject, PET scanning began with the first test argument and continued for the 50 sec required to complete the task. The monitor was then blanked, and the subject closed his eyes for 20 sec (this allowed completion of PET acquisition). Subsequently, the four arguments of the task were re-presented and the subject was asked to indicate the evaluation he had made of each argument during its initial presentation. This part of the procedure lasted around 2 min, and was extended to a 10-min pause between tasks (thereby allowing radioactivity to decay to the baseline level).

PET method and data analysis

Subjects were studied in the supine position using a PET tomograph GE-Advance (General Electric Medical System,

Milwaukee, WI, U.S.A.) with collimating septa retracted [10]. The system has 18 rings allowing 35 transaxial images with a slice thickness of 4.25 mm, covering an axial field of view of 15.2 cm. Transmission data were acquired using a pair of rotating pin sources filled with ^{68}Ge (10 mCi/pin). A filtered back-projection algorithm was employed for image reconstruction, on a 128×128 matrix with pixel size 1.9 mm, using a Hanning filter (cut-off 4 mm width) in the transaxial plane, and a ramp filter (cut-off 8.5 mm) in the axial direction. Regional cerebral blood flow (rCBF) was measured by recording the distribution of radioactivity following intravenous bolus injection of 5 mCi of $^{15}\text{O}\text{-H}_2\text{O}$ through a forearm cannula [15, 28]. The integrated counts, collected for 70 sec, starting 20 sec after injection, were used as an index of rCBF. Image manipulation and statistical analysis were performed in MATLAB 4.2 (Math Works, Natick, MA, U.S.A.) using statistical parametric mapping (SPM95, Wellcome Department of Cognitive Neurology, London, U.K.). Individual PET data were oriented along the intercommisural line and transformed into a standard stereotactic space. Global differences in cerebral blood flow were covaried out for all voxels and comparisons across conditions were made using *t* statistics with appropriate linear contrasts, and then converted to *Z*-scores [17, 18]. Only regional activations significant at $P < 0.001$ (thresholding SPM $Z > 3.09$) were considered.

Results

Task performance

We first consider whether subjects remained focussed on the tasks during PET scanning. The brief period allotted for the evaluation of arguments combined with the long interscan interval was expected to promote mental concentration during PET acquisition, and subjects subsequently reported no difficulty in this regard. The subjects' accurate performance on the logic arguments confirms the focussed character of their thought. They averaged 83% correct responses to the logic problems, with range 67–100%. This level of accuracy is equal to that of college students evaluating syllogisms without time stress [[34], pp. 232–234]. For the probability arguments there are no objectively correct answers. That the subjects were able to sustain their attention on the task is nonetheless suggested by their concordant judgment, reflected in a Kuder–Richardson reliability coefficient of 0.87. (Exactly half of the 12 arguments figuring in the probability tasks were judged 'probable' by a majority of subjects.)

PET data analysis

Table 2 reports the activation foci and the *Z*-scores found in the comparison between each experimental condition (logic and probability taken separately) versus the baseline meaning task. The table also shows the foci for the meaning task when compared to the two reasoning conditions pooled. The foci are identified via their stereotactic coordinates, measured in millimeters, relative to

the anterior–posterior commissure, corresponding to the Talairach and Tournoux atlas [38].

Consider first the comparison with meaning when the logic and probability tasks are taken separately. We found common foci of activation in the left frontal mesial cortex (dorsal frontal gyrus, BA 6), right and left cerebellar hemisphere, and vermis. We also found common activations for the two reasoning tasks in subcortical structures, namely, the left thalamus and right caudate. A distinctive focus of activation for the logic task was found in the left cuneus (BA 18), whereas a distinctive focus for probability appeared in the left anterior cingulate gyrus (BA 24/32).

Now let us consider the activation foci found for the meaning task when compared to the pooled data from the two experimental conditions. The comparison reveals bilateral foci of activation in the temporal poles (BA 38), inferior and middle temporal gyri (BA 20, 21), and in the orbital frontal cortex (BA 11). In addition, there were activations in the superior frontal (BA 9) and fusiform (BA 37) gyri of the left hemisphere, and in the inferior frontal (BA 45, 47) and superior temporal (BA 22) gyri of the right hemisphere.

Table 3 shows the stereotactic coordinates and *Z*-scores of the activation foci found in direct comparisons of the two experimental conditions (logic and probabilistic reasoning). The same foci are shown in Fig. 1. In probabilistic reasoning we found activations in the right insula and in the left prefrontal cortex, specifically, in the middle (BA 10) and the superior frontal (BA 8) gyri. In contrast, deductive reasoning activated the right anterior cingulate gyrus (BA 24/32) and a set of posterior brain regions, prevalently in the right hemisphere, namely: right superior parietal lobule (BA 7), right superior and middle occipital gyri (BA 19), right thalamus, left and right precuneus (BA 7), and left cuneus (BA 18).*

Discussion

The principal result of this study is the difference in brain activity associated with probabilistic and deductive reasoning (Table 3). We discuss these differences first, and then turn to the differences between the reasoning tasks and the baseline condition. When we write '*X* compared to *Y*' we refer to the activation observed in the

*Principal component analysis [16] of the PET dataset yielded highly similar profiles between tasks belonging to the same kind of reasoning. In particular, the first component (45% of the variance) cleanly separates the neural sites subserving the six reasoning tasks (three each for logic and probability) from the sites subserving the three anomaly detection tasks. The second component (19% of the variance) separates the neural sites for probability from those for logic. This suggests uniform brain activation across each of the three tasks used to induce deductive reasoning, and similarly for probability and anomaly.

Table 2. Coordinates and Z-scores for regions showing differential activation during logical and probabilistic reasoning compared to the meaning task

Region	Coordinates			Z-score
	x	y	z	
Logic vs Meaning				
L dorsal frontal gyrus (BA 6)	-16	2	52	4.29
L cuneus(BA 18)	-10	-86	24	3.57
L thalamus	-10	-16	12	3.37
R caudate	16	2	20	3.91
R thalamus	10	-26	4	3.50
R cerebellum	4	-80	-32	4.54
L cerebellum	4	-74	-36	3.52
Vermis	-2	-64	-16	3.71
Probability vs Meaning				
L dorsal frontal gyrus (BA 6)	-14	-2	48	3.71
L ant. cingulate gyrus (BA 24/32)	-2	14	32	3.41
L thalamus	-16	-16	20	4.02
R mid frontal gyrus (BA 10)	26	40	16	3.64
R caudate nucleus	8	14	8	3.15
R cerebellum	6	-84	-28	4.65
L cerebellum	-6	-64	-36	3.39
Vermis	0	-68	-16	3.41
Meaning vs Logic + Probability				
L temporal pole (BA 38)	-32	8	-28	4.76
L fusiform gyrus (BA 37)	-40	-36	-8	3.73
L sup. frontal gyrus (BA 9)	-14	46	32	3.61
L mid temporal gyrus (BA 21/22)	-46	-30	4	3.25
L inf. temporal gyrus (BA 20)	-40	-8	-24	3.22
R inf. frontal gyrus (BA 45)	48	28	12	4.18
R inf. frontal gyrus (BA 47)	48	18	-4	4.12
R temporal pole (BA38)	48	8	8	3.80
R inf. temporal gyrus (BA 20)	50	-14	-24	3.80
R mid temporal gyrus (BA 21)	50	-8	-16	3.75
R sup. temporal gyrus (BA 22)	54	-2	0	3.49
R/L orbitofrontal cortex (BA 11)	0	40	-20	3.99

Coordinates are in mm relative to the anterior-posterior commissure, corresponding to the Talairach and Tournoux atlas [38]. SPM $P < 0.001$ throughout.

Table 3. Coordinates and Z-scores for regions showing differential activation during logical versus probabilistic reasoning

Region	Coordinates			Z-score
	x	y	z	
Probability vs Logic				
R insula	26	28	16	3.34
L mid frontal gyrus (BA 10)	-40	48	16	3.88
L sup. frontal gyrus (BA 8)	-12	24	44	3.33
Logic vs Probability				
R cingulate gyrus (BA 24/32)	8	38	0	4.18
R sup. parietal lobule (BA 7)	30	-76	40	4.06
R precuneus (BA 7)	8	-76	48	3.45
R mid occipital gyrus (BA 19)	46	-76	-4	3.38
R thalamus	14	-28	12	3.36
R sup. occipital gyrus (BA 19)	22	-80	24	3.31
L cuneus (BA 18)	-2	-94	24	3.72
L precuneus (BA 7)	-10	-80	44	3.53

Coordinates are in mm relative to the anterior-posterior commissure, corresponding to the Talairach and Tournoux atlas [38]. SPM $P < 0.001$ throughout.

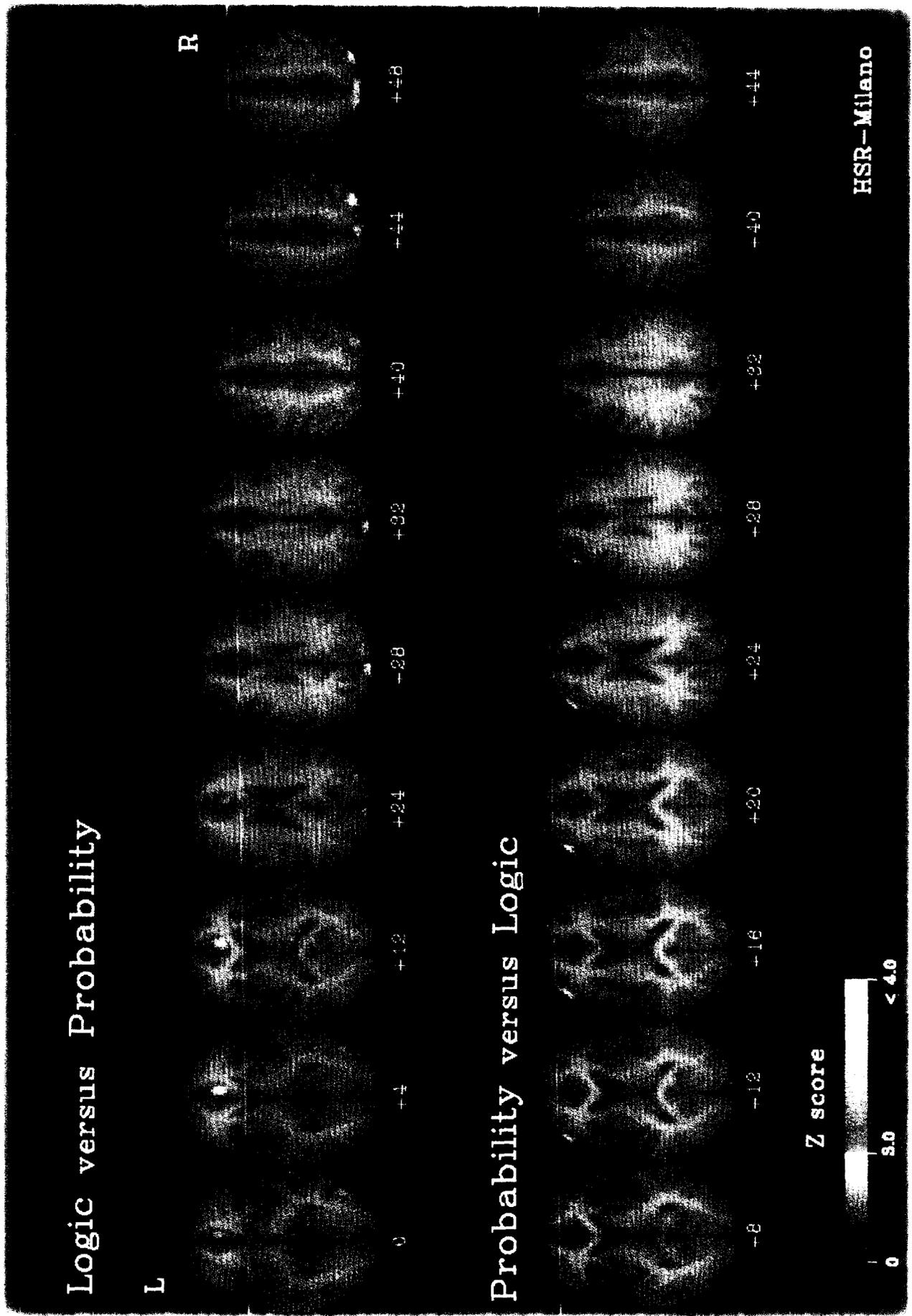


Fig. 1. Activation foci in the direct comparison of logic and probability tasks, superimposed over a magnetic resonance imaging atlas.

performance of task *X* that is observed at a significantly lower level in the performance of task *Y*.

Probability compared to deduction

Compared to deductive reasoning, the probability task produced activation in the left dorsolateral frontal cortex (BA 8 and 10), as well as in the right insular cortex. Studies with brain-damaged patients help to explain these findings, even though none appears to have considered probabilistic reasoning *per se*. In a study by Smith and Milner [37], 24 abstract figures were presented various numbers of times to patients with focal cortical ablations for treatment of epilepsy. The subjects were asked to estimate the frequency of appearance of each figure. Only patients with left or right mediolateral frontal corticectomy were impaired, while subjects with temporal and temporo-hippocampal lesions performed comparably to controls. Given that the frontal ablation patients performed normally in recognizing the figures, their disorder was considered to reflect an inability to estimate relative frequency. Since relative frequency is a principal source of information about probability [19], the frontal ablation patients can thus be considered impaired in at least one aspect of probabilistic thought. Another task that may be related to probabilistic reasoning is 'cognitive estimation'. It has been shown that, compared to retro-rolandic damage, frontal lesions on either side of the cortex disturb answers to questions such as 'how fast do race horses gallop?' [35]. In a related study of focal cortical ablations it was found that patients with dorsolateral frontal lesions made more errors in pricing objects (such as a car, a TV set, etc.) than patients with temporal and temporo-hippocampal lesions [36]. Both cognitive estimation and pricing require the selection of a value along a continuous dimension. Chance is likewise a continuous dimension, so determining the probability of a statement might require similar skills. Overall, the foregoing studies provide reason to believe that the dorsolateral frontal cortex is involved in the kind of judgment requested in our probability task.

Deduction compared to probability

The prevalent frontal pattern of activation found with probabilistic reasoning can be contrasted with the foci associated with deductive reasoning on identical arguments. For deductive reasoning, the activation was predominantly posterior and bilateral, with a right-side prevalence. There was involvement of the associative visual areas (cuneus, precuneus, middle and superior occipital gyri), as well as the right superior parietal lobule and thalamus. A considerable literature—involving both neuropsychological investigations [39] and PET procedures [8, 21, 22, 27]—implicates these areas in visuo-spatial processing, including form discrimination and

imaginative operations. A relationship between visuo-spatial operations and syllogistic reasoning is suggested by the fact that syllogistic reasoning protocols from naive subjects often involve spatial diagrams resembling Euler circles; the remaining subjects typically connect selected terms of the premises, often using arrows [14]. Both strategies have a geometrical character that would be expected to require visuo-spatial processing. Further evidence comes from clinical and PET studies of deductive reasoning involving spatial-relational terms, like 'taller than'. A study by Caramazza *et al.* [7] reported right brain-damaged, non-aphasic subjects to have difficulty with verbal reasoning tasks involving antonymic contrast ('taller than A but shorter than C'). The findings suggest reliance for such reasoning on visuo-imaginative strategies, involving the right hemisphere. A recent PET study using similar tasks showed significant activation of the right lateral and mesial parietal cortex during reasoning, compared to control conditions in which reasoning could not be initiated [2].

The only significant anterior activation found for logic compared to probability was in the right anterior cingulate cortex. The activation of this area has frequently been observed in PET experiments, and related to attentional processes, in particular to selective or divided attention [8, 29]. In the work of Posner and Petersen [33], the anterior cingulate is conceived as a component of the 'anterior attentional system', devoted to executive control. One aspect of this control function is thought to be the integration of linguistic and spatial processing [32]. Its relative increase in activation during deductive reasoning could thus be due to the greater involvement of spatial processing in logic compared to probability (both tasks obviously presuppose a linguistic component).

The comparisons discussed to this point allow tentative evaluation of the theses formulated in the introductory section. Our data cast doubt on thesis (1), and favor thesis (2) over thesis (4). In other words, the brain structures responsible for deductive and probabilistic reasoning appear to be substantially distinct; moreover, deductive reasoning activates sites predominantly in the right rather than left hemisphere.

Reasoning compared to anomaly detection

Comparison of each reasoning task with the baseline condition (requiring only reading for meaning) revealed common activation in the left medial frontal cortex (area 6), the cerebellum and in several subcortical structures. Cerebellar activation was particularly extensive. The foregoing structures have been shown to be activated in verbal working memory tasks [12, 30], and are considered to reflect articulatory rehearsal [1]. Cerebellar activation has also been implicated in the manipulation of transient, non-verbal information [26].

The activation of the foregoing structures is consistent

with the character of our two reasoning tasks, which require the integration of information in separate premises, and thus probably entail continuous rehearsal of sentences. Indeed, the role of working memory in syllogistic reasoning has been documented via an interference paradigm [20]. In contrast, the meaning task can be performed without integrating information across sentences, since each sentence may be examined separately for anomaly. It is thus likely that the meaning task requires no verbal rehearsal. The activation found in comparing the reasoning tasks with the baseline condition may therefore be explained in terms of the differential requirements for verbal rehearsal in reasoning compared to anomaly detection.

Anomaly detection compared to reasoning

The reverse subtraction (reading for meaning vs reasoning) was associated with activation in a widespread network of cortical areas in both hemispheres. The areas of activation in the left hemisphere included most of the classical perisylvian language areas, as well as the temporal pole. These foci have been shown in PET studies to be involved in processing prose passages [13, 31]. Similar patterns of activation in homologous contralateral hemispheric areas have been reported for comprehending metaphors [4] and 'theory of mind' stories [13].

In summary, the present results suggest that reasoning about syllogisms engages distinct brain mechanisms, depending on the intention to evaluate them deductively versus probabilistically. Likewise, reading the same syllogisms merely for meaning involves a pattern of cortical activation distinct from that obtained during reasoning.

Acknowledgements—We thank two anonymous reviewers for helpful comments. This study was supported by CNR and HFSP grants.

References

1. Baddeley, A. D., *Working Memory*. Oxford University Press, Oxford, 1992.
2. Baker, S. C., Dolan, R. J. and Frith, C. D., The functional anatomy of logic: a PET study of inferential reasoning. *NeuroImage*, 1996, **3**, S218.
3. Baron, J., *Thinking and Deciding*, 2nd edn. Cambridge University Press, Cambridge, 1994.
4. Bottini, G., Corcoran, R., Sterzi, R., Paulesu, E., Schenone, P., Scarpa, P., Frackowiak, R. S. J. and Frith, C. D., The role of the right hemisphere in the interpretation of figurative aspects of language. *Brain*, 1994, **117**, 1241–1253.
5. Braine, M. D. S. On the relation between the natural logic of reasoning and standard logic. *Psychological Review*, 1978, **85**, 1–21.
6. Braine, M. D. S., The 'natural logic' approach to reasoning. In *Reasoning, Necessity, and Logic: Developmental Perspectives*, ed. W. F. Overton. Erlbaum, Hillsdale, NJ, 1990.
7. Caramazza, A., Gordon, J., Zurif, E. B. and De Luca, D. Right hemispheric damage and verbal problem solving behavior. *Brain and Language*, 1976, **3**, 41–46.
8. Corbetta, M., Miezin, F. M., Dobmeyer, S., Shulman, G. L. and Petersen, S. E., Selective and divided attention during visual discriminations of shape, color, and speed: functional anatomy by positron emission tomography. *Journal of Neuroscience*, 1991, **11**(8), 2383–2402.
9. De Finetti B., Foresight: its logical laws and its subjective sources. In *Studies in Subjective Probability*, ed. H. Kyburg and H. Smokler. Wiley, New York, 1964.
10. DeGrado, T. R., Turkington, T. G., Williams, J. J., Stearns, C. W., Hoffman, J. M. and Coleman, R. E., Performance characteristics of a whole-body PET scanner. *Journal of Nuclear Medicine*, 1994, **35**, 1398–1406.
11. Evans, J. St. B. T., *Bias in Human Reasoning*. Erlbaum, Hillsdale, NJ, 1989.
12. Fiez, J. A., Raife, E. A., Balota, D. A., Schwarz, J. P., Raichle, M. E. and Petersen, S. E., A positron emission tomography study of the short-term maintenance of verbal information. *Journal of Neuroscience*, 1996, **16**, 808–822.
13. Fletcher, P. C., Happé, F., Frith, U., Baker, S. C., Dolan, R. J., Frackowiak, R. S. J. and Frith, C. D., Other minds in the brain: a functional imaging study of "theory of mind" in story comprehension. *Cognition*, 1995, **57**, 109–128.
14. Ford, M. Two modes of mental representation and problem solution in syllogistic reasoning. *Cognition*, 1995, **54**, 1–71.
15. Fox, P. and Mintun, M. Noninvasive functional brain mapping by change-distribution analysis of averaged PET images of H₂¹⁵O tissue activity. *Journal of Nuclear Medicine*, 1989, **30**, 141–149.
16. Friston, K. J., Frith, C. D., Liddle, P. F., Frackowiak, R. S. J. The principal component analysis of large (PET) data sets. *Journal of Cerebral Blood Flow and Metabolism*, 1993, **13**, 5–14.
17. Friston, K. J., Ashburner, J., Poline, J. B., Frith, C. D., Heather, J. D., Frackowiak, R. S. J. Spatial registration and normalization of images. *Human Brain Mapping*, 1995, **2**(3), 165–168.
18. Friston, K. J., Holmes, A. P., Worsley, K. J., Poline, J. B., Frith, C., Frackowiak, R. S. J. Statistical parameter maps in functional imaging: a general linear approach. *Human Brain Mapping*, 1995, **2**(4), 189–210.
19. Gigerenzer, G. and Murray, D. J., *Cognition as Intuitive Statistics*. Erlbaum Associates, Hillsdale, NJ, 1987.
20. Gilhooly, K. J., Logie, R. H., Wetherick, N. E. and Wynn, V. Working memory and strategies in syllogistic-reasoning tasks. *Memory and Cognition*, 1993, **21**, 115–124.
21. Haxby, J. V., Grady, C. L., Horwitz, B., Ungerleider, L. G., Mishkin, M., Carson, R. E., Herscovitch, P.,

- Schapiro, M. B. and Rapoport, S. I., Dissociation of object and spatial visual processing pathways in human extrastriate cortex. *Proceedings of the National Academy of Science, U.S.A.*, 1991, **88**, 1621–1625.
22. Hirsch, J., DeLaPaz, R. L., Relkin, R. N., Victor, J., Kim, K., Li, T., Borden, P., Rubin, N. and Shapley, R. Illusory contours activate specific regions in human visual cortex: evidence from functional magnetic resonance imaging. *Proceedings of the National Academy of Science, U.S.A.*, 1995, **92**, 6469–6473.
 23. Johnson-Laird, P. N. Mental models and probabilistic thinking. *Cognition*, 1995, **50**, 171–191.
 24. Johnson-Laird, P. N., Mental models, deductive reasoning, and the brain. In *The Cognitive Neurosciences*, ed. M. S. Gazzaniga. MIT Press, Cambridge, MA, 1995, pp. 999–1008.
 25. Johnson-Laird, P. N. and Byrne, R. M. J., *Deduction*. Erlbaum, Hillsdale, NJ, 1991.
 26. Kim, S.-G., Ugurbil, K. and Strick, P. L., Activation of cerebellar output nucleus during cognitive processing. *Science*, 1994, **265**, 949–951.
 27. Kosslyn, S. M., Thompson, W. L., Kim, I. J. and Alpert, N. M., Topographical representations of mental images in primary visual cortex. *Nature*, 1995, **378**, 496–498.
 28. Mazziotta, J. C., Huang, S. C., Phelps, M. E., Carson, R. E., Donald, N. S. and Mahoney, K. A non-invasive positron computed tomography technique using oxygen-15 labelled water for the evaluation of neurobehavioral task batteries. *Journal of Cerebral Blood Flow Metabolism*, 1985, **5**, 70–78.
 29. Pardo, J. V., Pardo, J. P., Janer, K. W. and Raichle, M. E., The anterior cingulate cortex mediates processing selection in the Stroop attentional conflict paradigm. *Proceedings of the National Academy of Science, U.S.A.*, 1990, **87**, 256–259.
 30. Paulesu, E., Frith, C. D., Frackowiak, R. S. J. The neural correlates of the verbal component of working memory. *Nature*, 1993, **362**, 342–345.
 31. Perani, D., Dehaene, S., Grassi, F., Cappa, S. F., Dupoux, E., Fazio, F. and Mehler, J. Brain processing of native and foreign languages. *NeuroReport*, 1996, **55**, 99–101.
 32. Posner, M. I., Attention in Cognitive neuroscience: an overview. In *The Cognitive Neurosciences*, ed. M. S. Gazzaniga. MIT Press, Cambridge, MA, 1995, pp. 615–624.
 33. Posner, M. I. and Petersen, S. E., The attention system of the human brain. *Annual Review of Neuroscience*, 1990, **13**, 25–42.
 34. Rips, L., *The Psychology of Proof*. MIT Press, Cambridge, MA, 1994.
 35. Shallice, T. and Evans, M. C., The involvement of the frontal lobe in cognitive estimation. *Cortex*, 1978, **14**, 294–303.
 36. Smith, M. A. and Milner, B. Estimation of frequency of occurrence of abstract designs after frontal or temporal lobectomy. *Neuropsychologia*, 1988, **26**, 297–306.
 37. Smith, M. A. and Milner, B. Differential effects of frontal lobe lesions on cognitive estimation and spatial memory. *Neuropsychologia*, 1984, **22**, 697–705.
 38. Talairach, J. and Tournoux, P., *Co-planar Stereotaxic Atlas of the Human Brain*. Thieme, New York, 1988.
 39. Warrington, E. K. and McCarthy, R. A., *Cognitive Neuropsychology*. Academic Press, New York, 1992.
 40. Whitaker, H., Savary, F., Markovits, H. and Grou, C., Inference deficits after brain damage. INS Meeting, San Antonio, 1991.
 41. Wittgenstein, L., *Tractatus Logico-Philosophicus*. Routledge & Kegan Paul, London, 1961. German edition published in 1921.