Appendix A.–The Mathematics Workshop Project: Intervention With a Difference at the University of California, Berkeley

The Mathematics Workshop Project was created by Uri Treisman as a component of the University of California, Berkeley's Professional Development Program (PDP).[1] Since 1978, Black and Hispanic undergraduates in this honors program have earned higher mean grades in calculus at Berkeley than nonworkshop minority students and have graduated from the university at rates roughly comparable to those of white and Asian students. The program was created in response to concerns about the low achievement among Black undergraduates at Berkeley in mathematics, but its unique format grew out of research undertaken by Treisman to explain why Blacks were having such intense problems in their adjustment to university life.

Treisman sought to answer not "Why do Blacks do so badly in mathematics?" but rather, "Why are Chinese students so successful in a subject that non-Chinese minority students find so daunting?" He assumed, quite plausibly, that Blacks might enjoy the same levels of success of the Chinese if a means to promote successful study habits and a productive approach to mathematics could be determined.

For many years, these two groups have been at very different points in the academic pecking order at Berkeley: Chinese students have traditionally been the most accomplished mathematics students at the university, while Blacks have been the least accomplished. For example, in 1975 only 2 of the 21 Black students who enrolled in the first course of the three-term calculus sequence managed to complete the last term in the sequence with a grade higher than "C." Since calculus is required for most of the academic majors that minority students at Berkeley pursue (e.g., architecture/ environmental design, business, engineering, natural sciences, and premedicine), this pattern of failure for Blacks has had devastating consequences for their academic persistence and graduation.

In 1975 Treisman first interviewed 20 Black and 20 Chinese students about their study habits and their methods of preparing for examinations. He subsequently observed them around the clock — in their homes, on dates, as they interacted with family and

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1. This appendix is based on Robert Fullilove, "Images of science: Factors Affecting the Choice of Science as a Career," OTA contractor report, 1987, and especially on an interview of Treisman who elaborated on the origins of his work. Treisman's approach to the creation of programs for minority students in mathematics is featured here.

friends — for almost 18 months to obtain some sense of how their adjustment to campus life and to the study of mathematics differed.

Among Black students he found a pattern of cultural, social, and academic isolation. Many of these students had come from high schools where few students attended college. They had achieved success in secondary mathematics by becoming extremely self-reliant and by becoming relatively isolated from the social life pursued by other students. At the university these habits of isolation — studying long hours alone, resisting the temptation to hang out with friends — persisted. Unfortunately, such patterns proved harmful to their adjustment to the university: most of these students became lost and confused by the blistering pace of first-semester calculus. They were unwilling to seek help from other students or from many of the remedial programs created to assist minority students. They saw these programs as having been created for poorly prepared, weak students, and their pride simply would not permit them to admit to others that they were struggling. Thus:

> The freshman year at Berkeley was a time of rude awakening and disorienting surprises, even for many black students who had attended academically reputable, predominantly white high schools. Even though these students were relatively well-prepared academically, the pace and intensity of competitive first-year mathematics and science courses coupled with the unexpected social isolation they encountered prevented many of them from getting their bearings or developing adequate study habits; thus, few did well in their courses. [2]

The 20 Chinese students in his study, by contrast, almost immediately upon their matriculation at the university, found friends and classmates with whom they studied regularly. Twelve of the 20 formed informal study groups that became a vehicle for mastering mathematics and for becoming acquainted with life in the university.

> Composed of students with shared purpose, the informal study groups of Chinese freshmen enabled their members not only to share mathematical knowledge but also to "check out" their understanding of what was being required of them by their professors and, more generally, by the University. These students learned quickly, for example, that the often-quoted rule of thumb for estimating the number of hours that one should devote to study — two hours for each class hour — was seriously

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2. Philip Uri Treisman, University of California, Berkeley, "A Study of the Mathematics Achievement of Black Students at the University of California, Berkeley," doctoral dissertation, 1985, p. 22.

> misleading. The blacks whom I had interviewed devoted approximately eight hours per week to homework and study for their four-unit math course; the Chinese devoted roughly fourteen hours per week to these same tasks.[3]

Treisman was particularly struck by the efficiency with which Chinese students within these groups mastered critical concepts in the course, concepts that, by contrast, left many of the Black students in his study bewildered. Black students, Treisman observed, were frequently stumped by a problem whose solution consumed hours of their time — often without success.

The Chinese students, when confronted with a similar problem, were quickly able to consult others in their study group. Typically, if no one in the group had come up with a solution, group members concluded that the problem was difficult enough and significant enough to warrant consulting the teaching assistant (TA) for help. Black students, by contrast, almost never sought out such assistance, particularly from the TA, because they were fearful that they would be exposing a weakness.

It became apparent to Treisman that group study offered many options that would be particularly useful to Black students at the university. First, study groups would provide an efficient vehicle for mastering the challenges of calculus. The interaction of students as they struggled with difficult, challenging problems appeared to have clear benefit for students who were prone to getting stuck. Second, study groups would provide students with an opportunity to combine their social and academic lives, and in so doing, combat much of the social isolation that Treisman had observed among the Black students in his study.

In order to avoid the appearance of being "just another remedial program," Treisman and the staff of the Professional Development Program billed their Mathematics Workshop program as an honors program. The "honors" label was not difficult to sell. PDP is sponsored by the University's Academic Senate under the auspices of a standing committee of the Senate, the Special Scholarships Committee. Created in 1964 this committee has counted some of the university's finest scholars (including two Nobel prize winners) among its members. Having such a committee sponsor an honors program, therefore, was consistent with student expectations of how the university functions.

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3. Ibid., p. 13.

The workshop's honors focus was not meant to suggest that its participants were selected because of their superior academic credentials; rather, the workshops would require that each student strive to earn honors-level grades as a condition of his or her participation. One clearcut benefit has been derived from this emphasis. The workshops attract highly-motivated students who see a direct relationship between working for high grades and achieving their career or graduate school objectives. Thus since the creation of the Mathematics Workshops, PDP students typically put twice as much time into studying each night as is suggested by conventional campus wisdom. This increased "time-on-task" is believed to explain, in part at least, why workshop students do so well.

The work that students are asked to complete in each workshop is intended to be of a more formal nature than the work Treisman observed among the study groups of the Chinese students in his study. However, the basic principles that made these informal groups so successful — the intense discussion and debate between students around difficult problems in mathematics — were retained and elaborated on. These features remain a distinct component of the program today.

The current version of the program centers around workshops that enroll approximately 20 to 25 students each. Each workshop meets for 2 hours twice a week. Each workshop session consists of both individual and group work that is centered on the problems contained in a "worksheet." Worksheet problems typically include:

> (1) old chestnuts that appear frequently on examinations but rarely on homework assignments; (2) monkey wrenches — problems designed to reveal deficiencies either in students' mathematical backgrounds or in their understanding of basic course concept; (3) problems that introduce examples or counterexamples to shed light on, or deli m it, major course concepts or theorems; (4) problems designed to deepen the student's understanding of and facility with mathematical language; and (5) problems designed to help students master what, in workshop parlance, is known as "street mathematics" — the computational tricks and shortcuts known to many of the best students, but which are neither mentioned in the textbook nor taught explicitly by the instructor.[4]

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4. Ibid., pp. 42-43.

Students work on these problems alone at first, then together in a group of four or five other students, all of whom have been working with the same problems. The major objective of the group work is to have students communicate with others about their efforts to develop solutions. This communication may be facilitated in a number of ways: 1) students may be asked to present their problem solutions to others in their group (or if the situation warrants it, to the entire workshop); 2) two or three students may be asked to edit another student's work, paying particular attention to issues of mathematical accuracy (e.g., was the correct form followed?), and to the elegance and clarity of the student's conclusions; and 3) students who appear to be well advanced in their work may be asked to tutor slower students until everyone in the group has arrived at the same level of expertise.

The advantage of these approaches is that the art of communicating complex ideas and concepts is an important means through which students organize and clarify their thoughts. As Treisman observes: "By continually explaining their ideas to others, students acquire the same benefits of increased understanding that teachers themselves regularly experience." If students find it difficult to express themselves, they become immediately aware of the inconsistencies in their understanding. Moreover, their efforts to make themselves understood — particularly in the face of pointed, thoughtful probing by the listeners — may also lead them to explore facets of a particular concept that might not otherwise have occurred to them. Discussing the solutions to worksheet problems also provides students with an opportunity to practice the skills and to exhibit the mastery of course concepts that they are expected to demonstrate on quizzes and exam i nations.

Students are not alone in the workshops, however. **A** workshop leader— typically a graduate student in mathematics or physics or some other similarly quantitative field – will be responsible for the preparation of the worksheets and for directing the activities of workshop students. Leaders are taught to be unobtrusive. Their major task is to ensure that students are communicating effectively about the work at hand.

> Towards these ends, the leader circulates among the students and listens carefully to their discussions. When he suspects that students are not listening carefully to one another, he intercedes, perhaps asking a student to restate something he has said more precisely or to explain in ore detail the steps by which he arrived at the solution to a certain problem.[5]

5. Ibid., pp. 44-45.

One clearcut advantage of the group study format used in the workshop is that anyone listening to the conversations students are having about their work has a unique glimpse of the mathematical thought processes of each of the speakers. As students discuss their struggles with the material, they are literally making their problem solving algorithms public. At the same time, these conversations provide the workshop leader with numerous opportunities to determine the degree to which students have mastered important material and key ideas. If students are unclear about the work, their problems will quickly become obvious.

The workshop, therefore, is an ideal instructional setting: it offers students an opportunity to practice the skills they will be expected to demonstrate in quizzes and examinations; it forces students to communicate with each other in a fashion that promotes greater mastery of difficult concepts as well as familiarity with the language and syntax of mathematics; and finally, it provides instructors with a vehicle for monitoring the progress of students as they master course materials.

Data on student achievement suggest that the program has been extremely successful:

- Black students at Berkeley are at greater risk of academic failure and are more prone to leave college before graduation than any comparable group of students. Significantly, 55 percent of the 231 Black students who were enrolled in the workshop program between 1978 and 1985 earned grades of “B-" or better in calculus; only 21 percent of the 284 nonworkshop Black students who took calculus during this period earned comparable grades. The mean final calculus grade for workshop students was 2.6 (N=231); the comparable mean for nonworkshop Blacks was 1.9 (N=284).

- The workshops also had a dramatic impact on student failure in mathematics: during the period between 1978-85, only 8 Black workshop students in 231 (3 percent) failed calculus; by comparison, 105 of 284 (37 percent) nonworkshop Black students failed the course.

- Perhaps the most significant impact of the workshops was on the mathematics achievement of poorly prepared students — those who entered the university with SAT mathematics scores in the lowest tercile (200 to 460) of the score distribution. The mean final grade in calculus for Black workshop students with poor mathematics preparation (2.2; n=56) was four-tenths of a grade point higher than that of nonworkshop Black students (1.9; n=42) with ‘strong preparation" in mathematics (defined as students with an SAT math score above 550).

- Participation in the workshops was also associated with high retention and graduation rates. Approximately 65 percent of all Black workshop students who entered the university in 1978 and 1979 (47/72) had graduated or were still enrolled in the spring semester of 1985. The comparable rate for nonworkshop Black students entering the university in those same years was 47 percent (132/281). The proportion of workshop students earning degrees in science and/or mathematics-related fields was 44 percent — the comparable rate among nonworkshop students was 10 percent. Comparable rates of achievement and persistence have been reported for Hispanic workshop students as well.

Treisman's success with this approach extends beyond the boundaries of the Berkeley campus. Successful adaptations of the workshop program—defined as programs whose Black and Hispanic students have earned final mean grades in calculus of 2.7 or better — have been created at the University of California-Los Angeles, University of California-San Diego, University of California-Santa Cruz, and California State Polytechnic University-Pomona. Similar secondary-level adaptations have been created for high school students in Albany, Richmond, Stockton, and Orange County, California.

These adaptations are by no means exact clones of the Mathematics Workshop program at Berkeley, but they all share key features. Treisman stresses that at the college level there is much to be learned by studying successful students. Successful students typically have a "bag of tricks" for dealing with institutional bureaucracies (e.g., how to navigate the financial aid mess, how to locate helpful TAs, how to approach faculty members if you have a "dumb" question), as well as useful strategies for succeeding academically (e.g., which campus classrooms are open all night, which questions always appear on so-and-so’s examinations). Where possible, these pieces of received wisdom need to be incorporated into the design of programs that serve students at all educational levels and should be integrated into the academic and personal advising that students are given by program staff. In Treisman's words:

> What should not be overlooked here is the fact that every minority student who gets an "A" in a mathematics course that other equally (or better) prepared students failed may have learned something that we should pass on to others. Too much educational research concentrates on explaining the variance in performance when, in reality, it is the unexplained variance — typified by the kids whose success can't be explained by race, SES, prior levels of preparation for mathematics, or time-spent-on-task — that may hold the answer to some of our knotty questions about how to design programs that promote student success.

Treisman goes on to note that the most successful teaching techniques are those that attempt to have students approach mathematics the way mathematicians do, by looking for and examining patterns. In too many instances, mathematics instruction fails to provide students with an opportunity to explore mathematics or to play with the patterns that fascinate and entrance mathematicians.[6] Instead, the curriculum concentrates on rote procedures and on getting the right answer. 'Students are taught, in other words, to focus on one of the end products of mathematics —the answer— and not on the potentially fascinating process we engage in to generate that answer."

In the Stockton Summer Math Institute, a project that Treisman directed in 1987, with support from the Hitachi Foundation, the search for pattern was placed at the core of the curriculum of a summer program for ninth graders. In one of the courses offered, students were introduced to variables as 'pattern generalizers" and were given an opportunity to use variables to understand arithmetic progressions.

Not surprisingly, the program involves working with interesting problems and working in small groups of the type used in PDP's Mathematics Workshops. Preliminary reports of the achievement of Stockton Summer students, the majority of whom were minority students, strongly suggests that there is considerable merit to this approach. At the beginning of the program, the mean percentile score of participating students on a test of mathematical problem solving skills was 27; at the end of the program the mean was 78. Student attendance and morale were described as "excellent," and observers feel that the model has tremendous potential as a tool for assisting students to make a

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6. In this vein, see Lynn Arthur Steen, "Out From Underachievement," *Issues in Science & Technology*, vol. 5, fall 1988, pp. 88-93.

successful transition from middle school mathematics to the college-track algebra course in high school.

Treisman's final observation is related to teachers and their role in mathematics instruction. He points out that in all too many schools with predominantly minority enrollments, mathematics is taught by teachers who are not trained for it and who may have been assigned teaching duties in the subject against their will. Efforts to reform teaching techniques and the content of the curriculum pass these teachers by because they have neither the time, the opportunity, nor the interest in learning how to teach the subject well. If change is to occur, a number of important alterations must be made in the way we try to affect how teachers teach mathematics.

The Stockton Summer Math Institute provides key insights to the nature of these changes. Teachers who participated in the institute were actively involved in the development of the institute's curriculum and in the preparation of teaching materials. In many teacher training programs, teachers are treated as "students": they become passive learners who have little or no opportunity to bring their own classroom expertise to bear as they learn new techniques and ideas.

Stockton Institute teachers, however, adapted the materials they would use in the classroom from a variety of texts and teaching materials, guided in large part by their own sense of what they knew to be effective methods for presenting topics to their students. If curriculum materials are developed on the assumption that they should be "teacher proof" — that is, able to be used without any direct involvement of the teacher — is it any wonder that teachers ignore them?

From these experiences, Treisman concludes: 1) mathematics is something that students must do, not as a set of rules that they must memorize, but rather as an activity in which they must be actively engaged; and 2) instruction works best when students are given an opportunity to communicate with each other about their work and when teachers are in a position to observe, and, where necessary, intervene, in that communication.

Treisman is a mathematician first and a teacher second. His philosophy of teaching reflects a desire to provide students with opportunities to do the things that mathematicians do, to search for and examine patterns. One sees in the PDP programs a greater concern with the process of doing mathematics — designing classroom opportunities to observe what students are doing with mathematics problems — and less focus on the products of students' labors— i.e., the answers. There is a concern with maximizing opportunities for teachers to observe students at work, doing the kinds of problems and exercises that elicit the skills and abilities we are most interested in having students acquire. Finally, group work has the potential to influence the dynamics of peer groups within the schools. At present, in all too many schools, academic achievement, particularly in mathematics and science, is not valued by students.

Finally, and most significantly perhaps, Treisman's work helps to place much of the research in mathematics education in some much needed perspective. Much research has focused on the structural barriers (both personal and systemic) that inhibit (or promote) student success in science and mathematics courses. What has been suggested here is: 1) that students succeed when the proper conditions for success are provided; and 2) that we do, in fact, know something about what those conditions must be. The key to students' success, Treisman has noted, is not the student's "native ability" for mathematics, but rather the institutional ability to design instructional settings that promote excellence. Workshops students, Treisman is fond of saying, provide an "existence proof" — they demonstrate how much can be achieved if the proper conditions are created and maintained. They also demonstrate that mathematics excellence can be achieved by minority students if it is demanded of them.[7]

7. A fitting postscript is that in June 1988 the Charles A. Dana Foundation awarded $737,000 to the University of California, Berkeley to establish a center to assist other colleges in educating minority students in mathematics and science. In Treisman's words: "The Dana Award [for Pioneering Achievement in Higher Education] legitimized our work." See Liz McMillen, "Dana Awards for Undergraduate Education and Health Prompt Debate on the Proper Role of Foundations," *The Chronical of Higher Education*, Nov. 2, 1988, p. A-28-30. While honoring individuals, such awards dramatically raise the visibility of issues and allow for the replication of a successful program like the Mathematics Workshop Project on many other campuses.