Appendix B: THE PRODUCT-LIMIT ESTIMATOR

The product-limit estimator for the distribution of lifetime cost, \( S(c) \) is given by:

\[
\hat{S}(c) = \prod_{j \leq c} P_j,
\]

where \( c \) is a level of lifetime cost attained by a case who died. 

\( j \) is a ranking from 1 to \( J \) for the \( J \) levels of lifetime cost attained by cases who died. 

\( P_j \) is the proportion of cases surviving to attain higher cost levels among all cases observed to attain the \( j \)th cost level. 

\[
\prod_{j \leq c} \]

\( N \) is the product calculated over all \( j \) less than \( c \). 

\( \hat{\lambda}(c) \) is the estimated proportion of all cases whose lifetime cost will be more than \( c \). 

The product-limit estimate of median lifetime cost is the cost level \( c \) for which \( \hat{S}(c) = 0.5 \); the estimate of mean lifetime cost is the area beneath a plot of \( \hat{S}(c) \); namely,

\[
\hat{\mu} = \sum_j \hat{S}(c_j) (c_j - c_{j-1})
\]

Ninety-five percent confidence limits are presented for this mean lifetime cost, using the variance estimator

\[
\text{Var}(\hat{\mu}) = \prod_c (\hat{A}^2/\%)/[n \cdot (n-\%)]
\]

where \( d_c \) is the number of cases who die at cost level \( c \), and \( n_c \) is the number of cases who attain a cost of \( c \) or more, and

\[
\hat{A} = \prod_{j > c} \hat{S}(c_j) (c_j - c_{j-1})
\]