Appendix V-A
Analytical Methods

BACKGROUND

Electric utility load pat ems can be most conveniently summarized in what is known as a "load-duration curve. " The curve for a hypothetical utility is shown in figure V-A-1. It shows the number of hours per year the demand for electricity is greater than or equal to all demands from zero to the annual peak For example, the figure shows that the power company in question had a maximum load of 100 MWe and a minimum load of 25,0 MWe (i. e , the company produced at least 250 MWe for all 8,760 hours of the year). The company met a load which was greater than or equal to 50 MWe for at least 3,504 hours during the year Loads will increase with each new year as a result of population growth and increases in the electricity consumed by each person. The increase in per capita consumption is a result of a shift to electric heating and other electric appliances.

If solar equipment is installed in a significant fraction of the buildings served by a utility, the load pattern which it must meet could be significantly affected. Figure V-A-1 illustrates two extreme possibilities. We assume that curve 1 indicates the load-duration curve which a utility could expect if no solar equipment were installed. If solar equipment requiring supplementary power during a utility's peak demand hours and not during off peak hours was installed, a load-duration curve having roughly the shape of curve 2 would result.

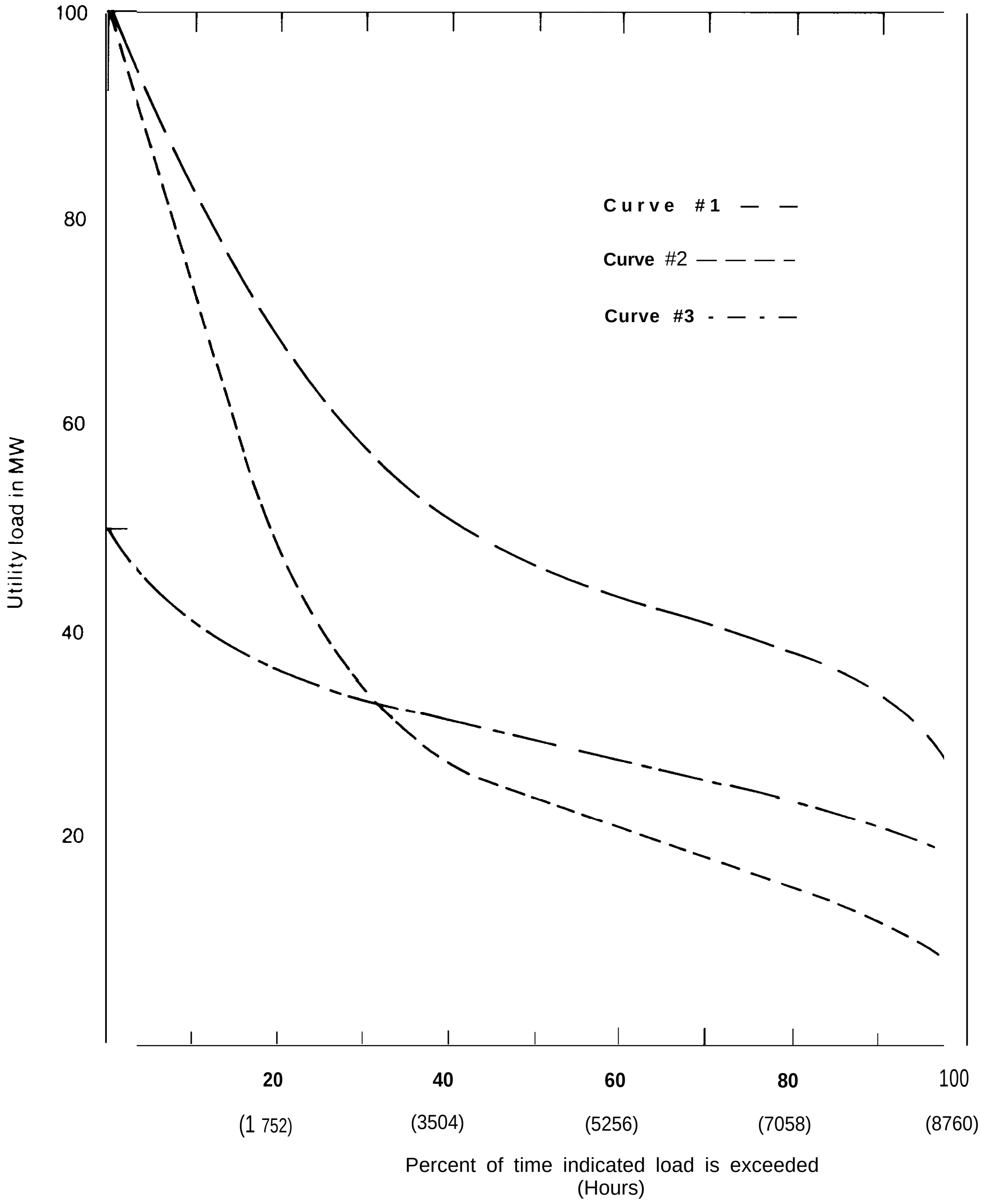
The amount of electricity sold would consequently be reduced, but costs would not be reduced proportionately because a large fraction of utility costs are independent of the amount of electricity generated. In this case, a utility will have proportionately more peaking plants with relatively small capital costs, Unfortunately, such plants are less efficient than large plants in both their fuel consumption and operating and maintenance expenses. Curve 3 indicates a situation where the solar equipment installed does not require supplementary power during the utility's peak demand hours, In this case, more efficient generating facilities (baseload plants and cycling plants) would be used to produce a greater fraction of the total utility load, resulting in a lower cost for each kilowatt-hour generated.

In order to quantify both the extent of the impact and whether it is adverse or beneficial, it is necessary to construct a "typical" utility. From this, several load-duration curves for the utility's operation can be constructed for 1985, involving a variety of scenarios both with and without solar equipment. These hypothetical load-duration curves can then be used to determine the kinds of equipment utilities will have to install to meet the demand of their customers, and the loadfactors for each piece of generating equipment. In turn, electricity costs and the utility's fossil-fuel requirements can be estimated for each scenario,

CHARACTERISTICS OF A "TYPICAL" UTILITY

The model utility examined is as close as possible to a "typical" utility which matches the national average for privately owned electric utilities wherever possible. The following sections briefly outline the physical and financial structure of the utility at the end of 1975.

Figure V-A-1.—A Typical Load-Duration Curve for an Electric Utility



Source: OTA.

CHARACTERISTICS OF THE MODEL UTILITIES

The hourly loads used to evaluate the cost of generation are constructed by combining the hourly electrical loads which apply to individual building types The method for determining the hourly electric demands of individual buildings is explalned in Volume II, The number of customers in a typical private U. S. utility is shown in table V-A-I. The number of buildings of each type used to construct the model utility used here for analysis is shown in table V-A-2.

Table V-A-1.—Average Characteristics of Privately Owned Utilities in 1974

	Number of customers, 1974 average	Demand in millions of kWh, 1974 average
Residential customers	255,000	1,925
Commercial customers	31,703	1,488
Industrial customers	1,491	2,532
Other (railroads, street lighting, etc.)	20	223

SOURCE: *Statistics of Privately Owned Electric Utilities in the United States*, 1974, FPC, page XXXI.

Table V-A-2.—Numbers of Buildings in "Typical" Utility Modeled Which Use Heating and Cooling and Hot Water Equipment*

	Albuquerque	Boston	Fort Worth	Omaha
Single family detached houses				
Baseboard resistance heating	10,470	8,080	11,790	7,719
Central electric air-conditioning	35,450	29,823	44,130	48,200
Window air-conditioning	8,163	5,040	11,790	7,719
Electric hot water	15,840	18,080	16,970	17,970
Total single family houses	55,920	55,920	55,920	55,920
Townhouses				
Baseboard resistance heating	648	1,429	2,010	1,350
Central electric air-conditioning	4,132	3,464	4,950	5,610
Window air-conditioning	424	895	2,010	1,350
Electric hot water	1,043	1,211	1,120	1,208
Total townhouses	6,960	6,960	6,960	6,960
Low rise apartments				
Baseboard resistance heating	201	444	624	419
Central electric air-conditioning	1,282	1,075	1,536	1,741
Window air-conditioning	132	278	624	419
Electric hot water	324	376	348	375
Total low rise apartments	2,160	2,160	2,160	2,160
High rise apartments				
Fancoil resistance heating	28	62	87	58
Baseboard electric heating	28	62	87	58
Central electric chiller	196	188	30	300
Window air-conditioning	196	188	30	300
Electric hot water	90	104	97	104
Total high rise apartments	600	600	600	600
Shopping centers				
Central electric chiller	30	0	30	30
Electric resistance heating	15	0	15	15
Total shopping centers	30	0	30	30

● All figures are number of buildings not units—the townhouses have 8 units each, the low rise apartments 36 units each, and the high rise apartments 196 units each.

NOTE: Detailed assumptions about the buildings modeled can be found in volume 11, chapter I *Analytical Methods.*

Commercial demands are approximated by simply using 30 shopping centers. A more detailed model would require many more load types–schools, hospitals, etc. industrial demand was approximated as a weekly load which is not weather dependent. Hourly industrial loads used in the utility model are shown in table V-A-3. They were chosen after examining a number of actual utility industrial loads. There are great variations in these loads around the country, and the data used can only show one "typical" pattern. Weekend loads were assumed to be 40 percent of the weekday loads. The total yearly industrial load served by the utility is 2.532×10^9 kWh.

Table V-A"3.—Industrial Load Profile Used

Hour	Weekday	Weekend
1	.2813E + 06	.1125E + 06
2	.2665E + 06	.1066E + 06
3	.2517E + 06	.1007E + 06
4	.2576E + 06	.1031E + 06
5	.2606E + 06	.1042E + 06
6	.2724E + 06	.1090E + 06
7	.2872E + 06	.1149E + 06
8	.3761E + 06	.1504E + 06
9	.4087E + 06	.1635E + 06
10	.4323E + 06	.1729E + 06
11	.4531E + 06	.1812E + 06
12	.4649E + 06	.1860E + 06
13	.4412E + 06	.1765E + 06
14	.4501E + 06	.1800E + 06
15	.4679E + 06	.1872E + 06
16	.4383E + 06	.1753E + 06
17	.3405E + 06	.1362E + 06
18	.3228E + 06	.1291E + 06
19	.3405E + 06	.1362E + 06
20	.3317E + 06	.1327E + 06
21	.3169E + 06	.1267E + 06
22	.3169E + 06	.1267E + 06
23	.3139E + 06	.1256E + 06
24	.3021E + 06	.1208E + 06

LOAD DIVERSITY

The load diversity factor used in forming an aggregated utility load was determined as follows:

1. It was assumed that heating and air-conditioning loads had no diversity, since all buildings in the area would be affected by approximately the same weather at approximately the same time. (This is a conservative assumption. Most utilities cover areas sufficiently large enough to expect some weather diversity. Tielines can be used between widely spaced utilities to even out weather loads.)
2. It was assumed that the hourly variation of "miscellaneous electric loads" and domestic hot water loads was the same for each building of a similar type. When they were added, however, it was assumed that each sequence started at a different time. The spread in "start times" was assumed to be the spread during which people wake up in the morning, eat, and go to work which, in turn, was assumed to be the same as the spread of traffic during rush-hour peaks in major cities (i. e., a normal distribution with a standard deviation of approximately 1 hour).

This technique is expressed quantitatively in the expression below:

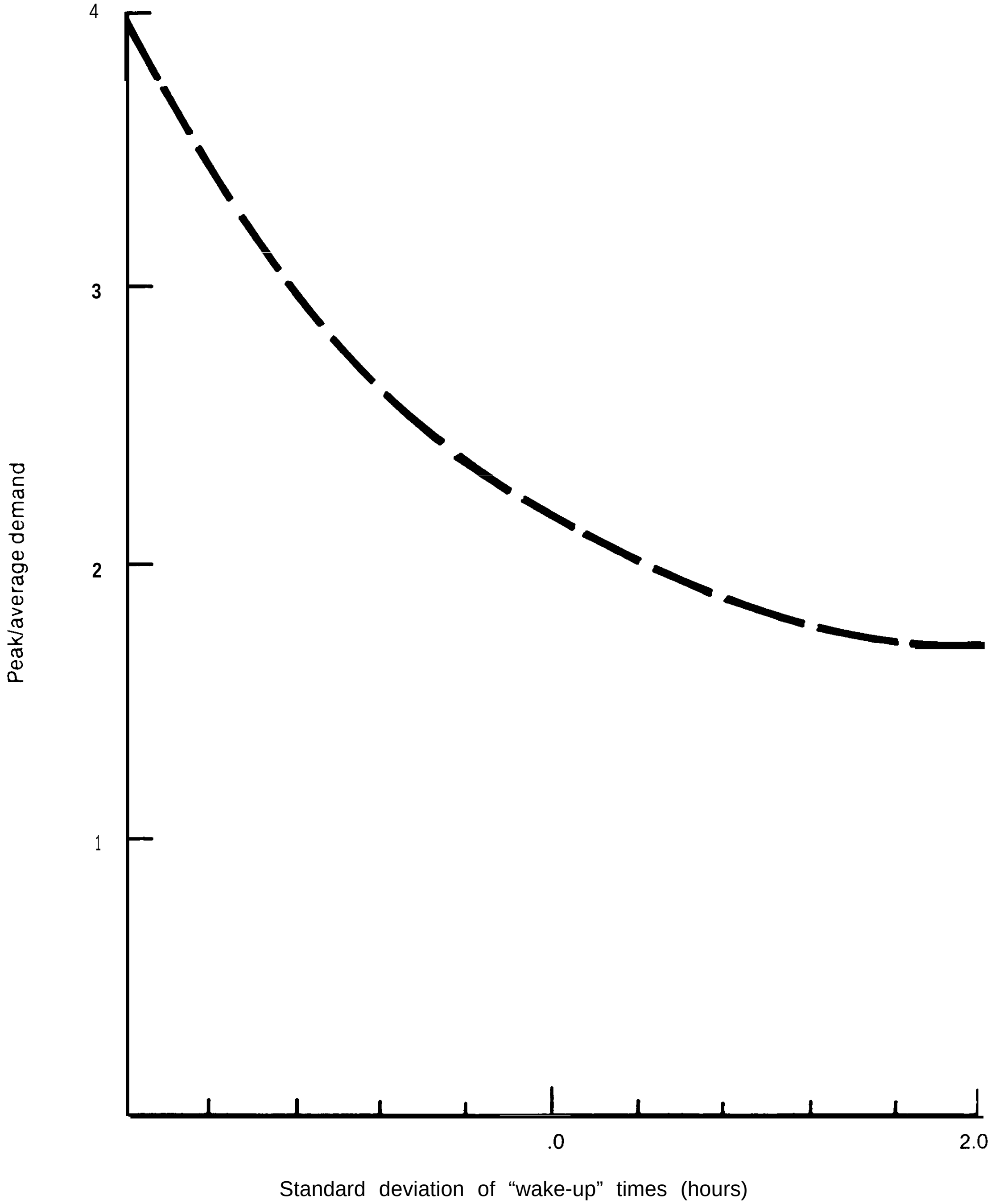
$$f_N(t) = \frac{N}{\sigma\sqrt{2\pi}} \int_{\infty}^{00} f(t+x)e^{-x^2/2\sigma^2}dx \quad (1)$$

where f(t) is the hourly load profile of an individual house, $f_N(t)$ is the hourly load profile of an aggregate of N such individual houses if N is large and if "wake-up times" are distributed in a Gaussoan distribution with a standard deviation of ó hours. The results of this smoothing for a variety of different values of the standard deviation are shown in figures V-A-2 and V-A-3.

SIZING GENERATING EQUIPMENT

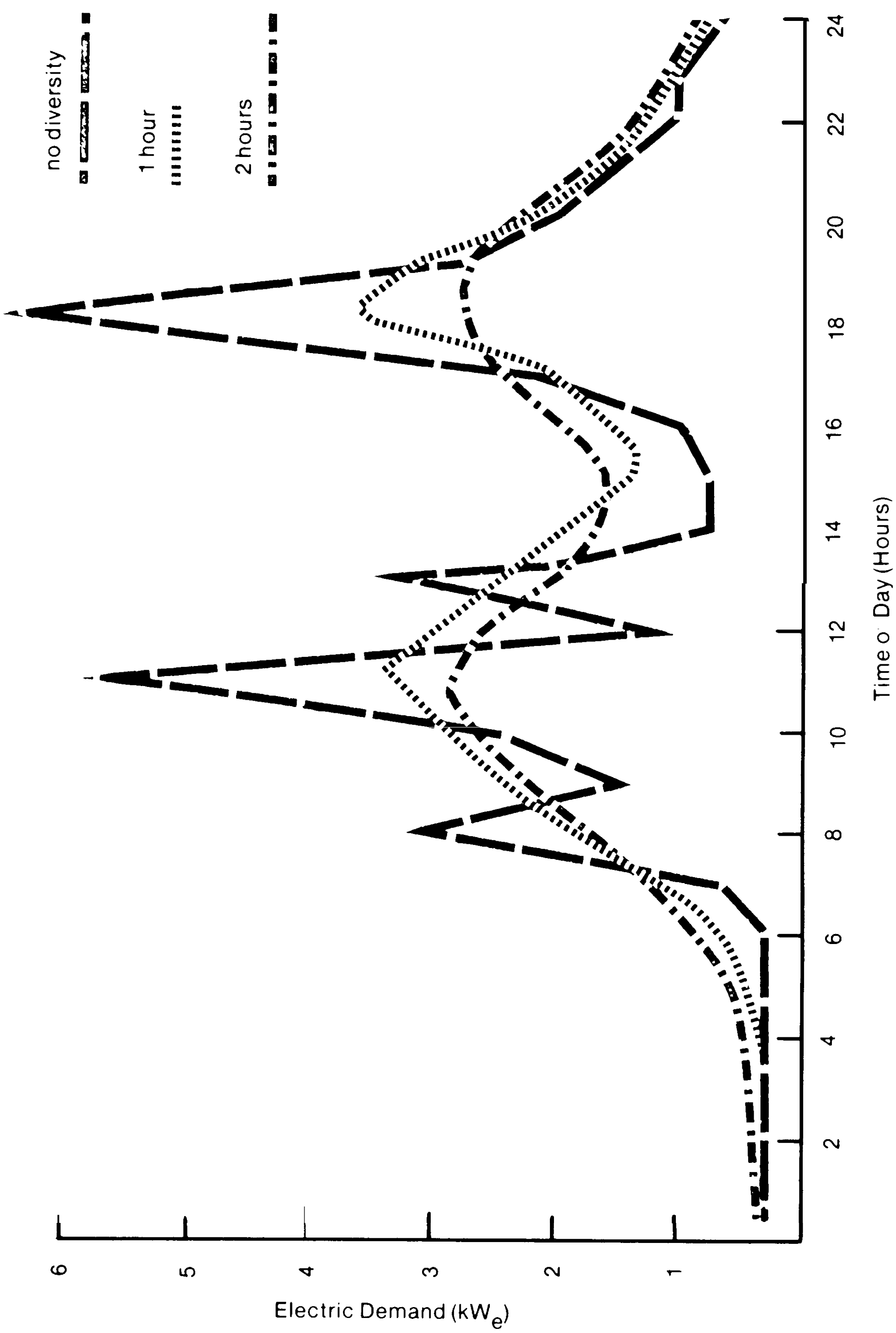
The generating equipment installed by the utility depends on the load-duration curve (which characterizes customer demands), costs of purchasing and operating alternative types of generating equipment, kinds of financing available to the utility,

Figure V-A-2.— Ratio of Peak Demands to Average Demands as a Function of the Standard Deviation of "Wake-Up" Times



Source: OTA

Figure V-A-3.—Hourly "Miscellaneous" Electric Load Profile for Single Family House With Different Types of Diversity



Source OTA.

and current and projected fuel costs. As selecting the best mix of generating equipment is a complex task, a greatly simplified approximation of the techniques actually employed by utilltles is used, The selection of the appropriate mIX of such plants for meeting any load-duration curve is dominated by the fact that: (1) larger plants have relatively high efficiencies and high initial costs and, as a result, are profitable only if operated for a large fraction of the year; and (2) smaller gas turbines and internal combustion systems are relatively inexpensive to purchase, but have relatively high-cost fuel consumption and are thus best used in situations where they operate only a few hours each day Larger plants, therefore, are used to meet the "baseload" requirements of the utility (the loads which will be constant throughout the year), with the smaller plants used for intermediate and "peaking" purposes when the demand is greater than the baseload plant capacity

Storage can be used to meet some demands during peak periods If the utility has facilities for storing energy. Peaking plants would then be used only when storage output capabilities were exhausted

The heuristic arguments given above can be quantified quite easily if a few simplifying approximations are made The basic parameter used to evaluate a utility system is the total "levelized" annual cost of producing electricity (which is called C_T in the following discussion). This cost is the sum of the cost of capital invested in equipment and the average annual fuel, operating, and maintenance costs, Following the notation developed in the discussion of economic and financial analysis found in volume 11, chapter 1, the levelized annual cost of a piece of equipment is given as follows:

$$\begin{aligned} \text{Cost} &= k_1 \times \text{(Initial equipment cost)} \\ &+ k_2 \times \text{(amount of fuel used annually)} \\ &+ k_3 \times \text{(annual operating cost)} \end{aligned} \qquad (2)$$

Here k, is the effective cost of capital, k_2 is a "levelized" fuel cost (which may differ from current fuel costs because of projected fuel price increases), and k_3 is a multiplier leveling the presumed inflation of operating costs.

The annual cost of operating a given piece of generating equipment can then be written as follows:*

$$C_n = k_1 K_n C_n + T K_n (k_{2n}/\eta_n + \alpha_n k_3) \qquad (3)$$

where:

C_n = annual cost ($/year)
k_1 = cost of capital (1/year)
k_{2n} = levelized fuel cost ($/kWh)
k_3 = multiplier for O&M (dimensionless)
K_n = size of the plant (kW)
C_n = cost of the equipment ($/kW)
T = number of hours per year equipment is used (hours/year)
η_n = the efficiency of the equipment (dimensionless)
α_n = O&M cost ($/kWh)

The subscript *n* refers to the type of plant where:

n = 1 for a baseload plant
n = 2 for an intermediate or cycling plant
n = 3 for a peaking plant

The cost per kW_e of this equipment can then be written as follows:

$$C_n = C_n/K_n = a_n + b_n T \qquad (4)$$

where

$$\begin{aligned} a_n &= k_1 C_n \text{ (fixed cost)} \\ b_n &= k_{2_n}/\eta_n + \alpha_n k_3 \text{ (variable cost)} \end{aligned} \qquad (5)$$

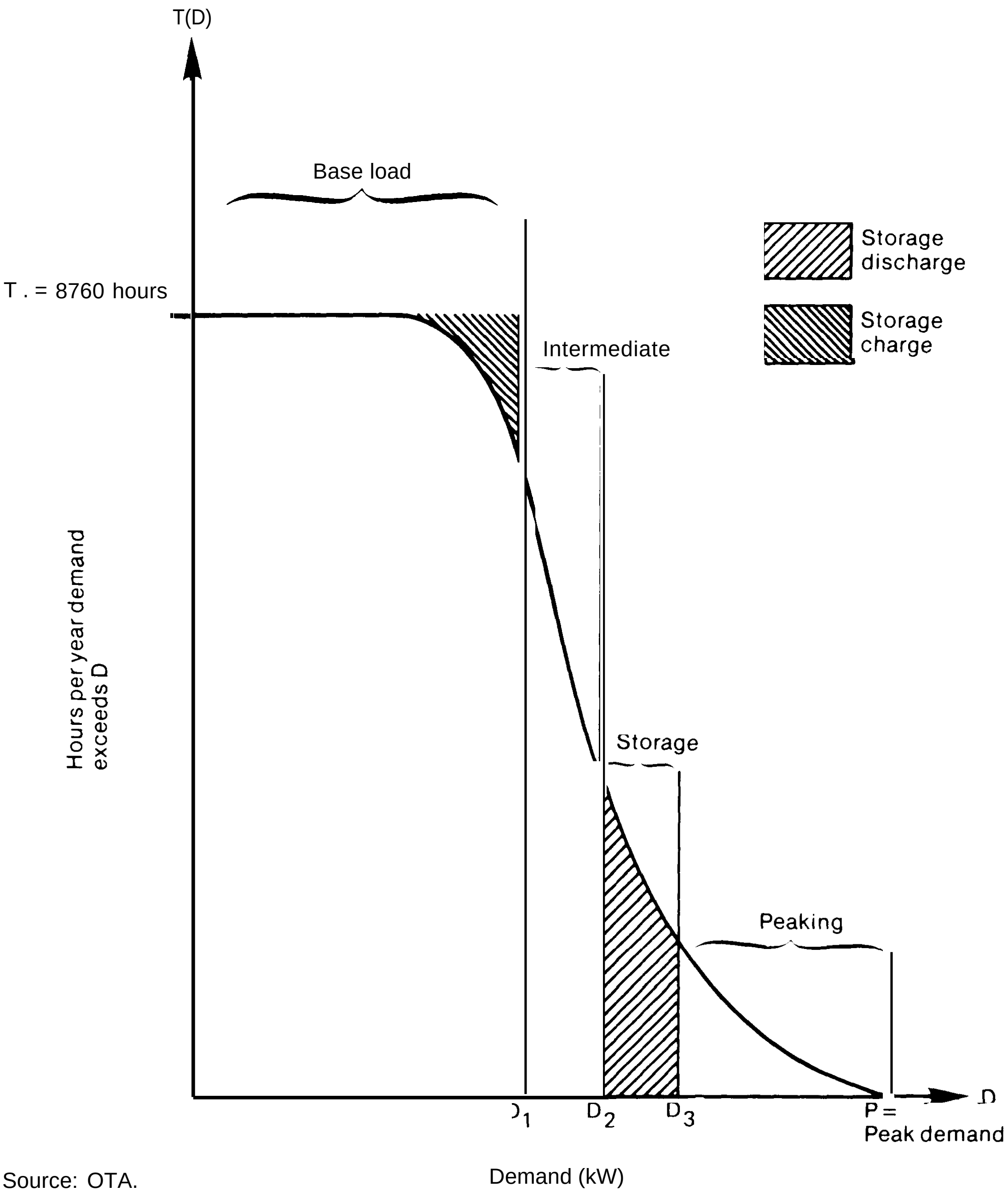
This analysis is somewhat artificial in that it has been assumed that operating costs are directly proportional to the amount of electricity generated. In fact, of course, these costs are a more complex function of operating time and will not be zero even when no energy is being generated. A straightfor-

* Index

ward improvement to the current model would be to assume that the operating costs were of a form X + YT, but the current method was chosen for simplicity. Another approximation which has been made is that the efficiency is independent of the operating strategy.

The utility's total operating costs can then be approximated by examining the load-duration curve. Figure V-A-4 (which is an inverted load-duration curve) illustrates the sequence in which loads are met by generating plants. T(D) is the number of hours per

Figure V-A-4.— Inverted Load-Duration Curve of a Typical Electric Load



Source: OTA.

year when the utility's load exceeds D kilowatts. When there is no storage, the approximate cost is given by:

$$C_T = b_1 \int_{0} T(D)dD + b_2 \int_{D_1}^{D_2} T(D)dD + b_3 \int_{D_2}^{P} T(D)dD \qquad (6)$$

The optimum set of equipment to meet the loads is then determined by minimizing this function with respect to D, and D,. This minimum occurs when:

$$T(D_1) = \frac{a_2 - a_1}{b_1 - b_2}$$
$$T(D_2) = \frac{a_3 - a_2}{b_2 - b_3} \qquad (7)$$

The optimum size of the plants is then given by:

(capacity of baseload plants) = D_1
(capacity of intermediate plants) = $D_2 - D_1$
(capacity of peaking plants) = $P - D_2$

The approximate cost of generation for the year is then given by using these quantities in equation [1).

Provision for Reserve Margin

Two major approximations have been made in obtaining costs in this way: (1) no provision is made for maintenance cycles, the need to maintain reserve capacity for unanticipated failures, and the need to maintain some capacity as spinning reserve, and (2) no provision is made for the costs associated with starting or shutting off a plant and the inefficiencies of running at partial loads.

The first difficulty is handled by simply increasing the assumed capacity of each type of plant by *20* percent. If the analysis shows that an optimum size for baseload plants is D_1, it will be assumed that the utility actually installs baseload capacity equal to $(1.2)D_1$. **In actual utilities, these reserve margins are computed by carefully analyzing the reliability and maintenance schedules of each plant in the system,**

The second problem can only be eliminated with a detailed examination of the actual sequence with which plants are turned on and off. However, this is beyond the scope of this study. Choosing appropriate average values for operating costs and efficiencies should produce results which are sufficiently close to those of a detailed model, and serve the purpose of looking for major impacts of different load patterns.

Transmission and Distribution Costs

In addition to generating costs, the utility will have expenses associated with transmission and distribution. These will vary greatly, since they are a function of the spacing of the utility's generating facilities and the location of its customers. In this simple model, it is assumed that the transmission and distribution costs are in direct proportion to the utility's peak generating capacity. While generating plants represent approximately 44 percent of the total value of electric utility plants and equipment, approximately 60 percent of the new capital invested by the electric utilities in recent years has been invested in generating equipment and this trend is expected to continue (see table V-2 in the main text).[1,2] It is therefore assumed that for each dollar invested in generating capacity the utility invests $0.67 in transmission, distribution, and other equipment.

In addition, the cost of maintaining transmission and distribution facilities is assumed to be proportional to the total investment in such facilities. Maintenance of transmission and distribution equipment in 1974 cost privately owned electric utilities in the **United States approximately 3.3 percent for every dollar invested in such equipment.[3] It**

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[1] *Statistics of* Private/y *Owned Electric Utilities in* the *United* States, 1974, Federal Power Commission, p xx I

[2] 26th Annual Electrical Industry Forecast, *Electrical World,* September 15, 1975, p 49

[3] *Statistics of Privately Owned Electric Ut///tles In the United States, 1974, Federal Power Commission, pp xxx VI, xxx vii, arrd xx I*

is therefore assumed that the annual transmission and distribution operating expenses are 0.033 x (the investment in transmission and distribution equipment) = 0.022 x investment in generating equipment).

The costs of generating facilities fuel costs and operating costs actually used in the analysis are summarized in table V-A-4. Some of the characteristics of the solar energy equipment examined are summarized in table V-A-5.

Table V-A-4.—Characteristics of Equipment Used in the Utility Model

1. Generation costs

	Capital costs* $/kW	Cycle efficiency	Levelized fuel cost ($/kWh)	Variable O&M ($/kWh)
Albuquerque				
Nuclear	796	0.328	0.0056	0.00072
Coal	691	0.338	0.00682	0.00147
Combined cycle	292	0.42	0.020	0.0012
Boston				
Nuclear	845	0.328	0.0056	0.00072
Coal	713	0.347	0.00760	0.00189
Combined cycle	292	0.42	0.020	0.0012
Fort Worth				
Nuclear	748	0.328	0.0056	0.00072
Coal	725	0.327	0.00456	0.00214
Combined cycle	292	0.42	0.020	0.0012
Omaha				
Nuclear	769	0.328	0.0056	0.00072
Coal	668	0.338	0.00626	0.00148
Combined Cycle	292	0.42	0.020	0.012

II. Transmission and distribution costs

O&M—O.022 of investment in generating capacity
Capital charges— 0.67 of generating capacity costs
Efficiency of T&D—O.91

III. Other costs

Cost of capital —O.15
Overhead–0.021 $/kWhr
20% excess generating capacity installed

"Capital costs include an allowance for "fixed operating costs" computed by dividing the fixed operating costs per year by the levelized fixed charge rate.

SOURCE: Technical Assessment Guide, EPRI, August 1977. *Statistics of Privately Owned Electric Utilities*, 1974, Federal Power Commission.

Table V-A-5.—Assumptions About the Nonconventional Systems Used in the Utility Impact Analysis

1. **Albuquerque**

Single family house

–SF-2
Flat-plate collector—30m^2
Thermal output
Low-temperature storage—200 kWh

–SF-2
East-west axis tracking collector—92m^2
Photovoltaic and thermal output
Low-temperature storage—200 kWh
Battery storage—23 kWh

—IF-2
Flat-plate collector—59m^2
Photovoltaic output
Battery storage—20 kWh

–SF-3
Flat-plate collector—30m^2
Thermal output
Low-temperature storage—200 kWh

Table V-A-5.—Continued

–SF-3
Off peak purchase of electricity y
Flat-plate Collector—30m2
Thermal output
Low-temperature storage—232 kWh

High rise apartment buildings

—HR-2
East-west axis tracking Collector—4,263m²
Photovoltaic and thermal output
Low-temperature storage— 17,000 kWh
Battery storage– 170 kWh

II. Boston

Single family houses

—SF-2
Flat-plate collector—45m²
Thermal output
Low-temperature storage—200 kWh

–SF-2
East-west axis tracking collector—92rn²
Photovoltaic and thermal output
Low-temperature storage—200 kWh
Battery storage— 12 kWh

—IF-2
Flat-plate Collector—59m2
Photovoltaic output
Battery storage—20 kWh

High rise apartment buildings

—HR-2
East-west axis tracking collector—4,263m²
Photovoltaic and thermal output
Low-temperature storage— 17,000 kWh
Battery storage— 170 kWh

III. Fort Worth

Single family houses

—SF-2
Flat-plate collector —40m²
Thermal output
Low-temperature storage—200 kWh

—SF-2
East-west axis tracking collector—92m²
Photovoltaic and thermal output
Low-temperature storage—200 kWh
Battery storage— 12 kWh

–IF-2
Flat-plate collector—59m²
Photovoltaic output
Battery storage—20 kWh

High rise apartment buildings

–HR-2
East-west axis tracking collector—4,263m²
Photovoltaic and thermal output
Low-temperature storage— 1,700 kWh
Battery storage— 170 kWh

IV. Omaha

Single family houses

–SF-2
Flat-plate collector—40m²
Thermal output
Low-temperature storage—200 kWh

–SF-2
East-west axis tracking collector—92m²
Photovoltaic and thermal output
Low-temperature storage– 12 kWh

–IF-2
Flat-Plate collector—59m²
Thermal Output
Low-temperature storage—200 kWh

–SF-3
Off peak purchase of electricity y
Low-temperature storage—31 3 kWh

–SF-3
Off peak purchase of electricity
Flat-plate collector—40m²
Low-temperature storage—293 kWh

High rise apartment buildings

—HR-2
East-west axis tracking collector—4,263m²
Photovoltaic and thermal output
Low-temperature storage— 1,700 kWh
Batteries— 170 kWh

—HR-2
(seasonal storage)
Flat-plate collector—4,100m²
Thermal output
Low-temperature storage—1,200,000 kWh

NOTE: SF-2 single family house with heat pump and electric hot water.
IF-2 well-insulated single family house with heat pump and electric hot water.
HR-2 high rise apartment buildings with central electric chiller, for coil resistance heating, central electric hot water.
SF-3 single fmaily houses with window air-conditioners, baseboard resistance heating, and electric hot water.
SF-3 off peak heating and hot water (with or without solar), window air-conditioners, electric resistance furnace, and electric hot water.
SF-3 off peak heating, cooling, and hot water (with or without solar), central electric chiller, electric resistance furnace, and electric hot water.