Appendixes
The DELTA model is a linear programming mathematical model used to calculate the costs of satisfying electric, heating, and cooling demands of a particular electric utility, given the capital equipment in operation in 1980. The model computes a capacity expansion plan that minimizes the time-discounted sum of capital, operating, and fuel costs.

This appendix is divided into three sections: first, a description of the major assumptions used in the formulation of the model; second, a list of all the variables and notations used in the model equation set; and third, the equation-by-equation description.

**Major Assumptions and Model Formulation**

All of the major assumptions used in the DELTA model are described in chapter 5. They are summarized below:

First, the model uses the linear programming type of mathematical programing in its representation of the utility system. Second, the model only simulates cogeneration that is connected to the grid. Third, the model divides the entire commercial sector into three subsectors corresponding to three types of demand patterns: multifamily, hospitals/hotels, and 9-to-5 office buildings. Fourth, the model uses the eight different types of daily load cycles in its representation of electrical, heating, and cooling demands. Fifth, the model has two different types of fuel price paths. Sixth, three different sample utility systems were used. Seventh, different assumptions were made to represent the various technologies, the way they operate, and the financial structure of the utility region.

As mentioned in chapter 5, the model uses three types of energy demands—electrical, thermal heating, and cooling—that must be met by a combination of electrical and thermal generation. A schematic for the general structure of the model (for a particular subsector) is given in figure A-1.

**Notation And Variables Used**

Each variable (representing particular activities of a utility) may have up to five subscripts for its mathematical shorthand. The subscripts are:

- _n_— for each centralized technology type [e.g., _n_ = 1 (baseload), _n_ = 2 (intermediate), _n_ = 3 (peaking)];
- _s_— for subsector (multifamily, hospitals/hotels, 9-to-5 office buildings);
- _y_— for time period (0 is 1980, 1 is 1990, 2 is 2000);
- _d_— for the day type (eight different ones, e.g., peak summer weekend day); and
- _h_— for the hour of the day represented.

For each individual dispersed type of technology represented and each subsector and year, day, and hour _y_, _d_, _h_, we represent the various activities with the following mathematical shorthand. Power output is measured in megawatt hours, while capacity is measured in megawatts. One thermal megawatt is equal to 3.412 million Btu/hr.

- _COG_ is the cogeneration electrical power output in subsector _s_ at time _y_, _d_, _h_.
- _COC_ is the cogeneration power capacity in subsector _s_ at year _y_.
- _T H G_ is the electrical heating power generation in subsector _s_ at time _y_, _d_, _h_.
- _T H C_ is the electrical heating power capacity in subsector _s_ at year _y_.
- _E C D_ is the electric air conditioning output in subsector _s_ at time _y_, _d_, _h_.

Figure A-1.—DELTA Model Structure
ECC\textsubscript{sy} is the electric air conditioning capacity in subsector \( s \) at year \( y \).

TCD\textsubscript{sydh} is the thermal (absorptive) air conditioning output in subsector \( s \) at time \( y, d, h \).

TCC\textsubscript{sy} is the thermal (absorptive) air conditioning capacity in subsector \( s \) at year \( y \).

SPH\textsubscript{sydh} is the thermal heating output in subsector \( s \) in time \( y, d, h \).

There are also the electrical generation variables, that are represented for the different types of centralized technologies \( n \) and time \( y, d, h \):

ELG\textsubscript{nydh} is the central electric power generation for technology \( n \) and time \( y, d, h \) (measured in MWh).

ELC\textsubscript{nY} is the central electric power capacity for technology \( n \) and year \( y \) (measured in MW).

Finally, there are three variables that represent the thermal storage activities for each subsectors and time \( y, d, h \):

TOT\textsubscript{sydh} is the dispersed thermal storage output for subsectors and time \( y, d, h \) (measured in MWh).

TIN\textsubscript{sydh} is the dispersed thermal storage input for subsectors and time \( y, d, h \) (measured in MWh).

TEC\textsubscript{sy} is the dispersed thermal storage energy capacity for subsectors and year \( y \) (measured in MW).

The input and output variables are measured in megawatts, while the capacity is measured in megawatt-hours.

The various technological characteristics are abbreviated mathematically with the following shorthand:

\( A_n \) is the availability of equipment of technology type \( n \) to provide power.

\( MAN_{ny} \) is the amount of time that technology type \( n \) is out of service for maintenance in year \( y \) and day \( d \).

\( C(.) \) is the annual capital cost or operating cost for each capacity activity variable (\( . \)).

\( DISC \) is the real discount factor for both capital and operating costs.

The three types of energy demands are abbreviated:

ELD\textsubscript{sy} is electrical demand in time \( y, d, h \).

CLD\textsubscript{sy} is the thermal cooling demand for subsector \( s \) in time \( y, d, h \).

HTD\textsubscript{sy} is the thermal heating demand for subsector \( s \) in time \( y, d, h \).

The DELTA model is fixed for certain time periods, with \( ND_d \) being the number of days per year of day type \( d \).

**Equation Description**

As in standard with linear programming-type of formulations, we divide our description of the equations into two parts: first a description of the objective function and then the constraints.

**Objective Function**

The objective function is to minimize the sum of discounted (over the time period of the model back to 1980 dollars) the annual costs of operation and capacity of electric generation, cooling, heating, and the costs of thermal storage. The mathematical description of the objective function is:

\[
\min \left\{ \sum_{y=1}^{Y} \left( \text{DISC}_y \cdot \sum_{d} \sum_{h} \left( \sum_{n} \text{ELG}_{nydh} \cdot C(ELG) + \sum_{s} \text{COG}_{sydh} \cdot C(COG) + \sum_{s} \text{THC}_{sydh} \cdot C(THC) + \text{ECD}_{sydh} \cdot C(ECD) \right) + \text{TEC}_{sy} \cdot C(TEC) \right) \right\}
\]

**Equation Set**

There are eight different types of equations in the DELTA model: electric demands, cooling energy demands, heating energy demands, capacity availability, thermal storage capacity, thermal storage availability, reserve margin, and maintenance scheduling.

**ELECTRIC DEMANDS**

Electric demands must be met in hour \( h \) of day type \( d \) in each year \( y \). The electric air-conditioning demand (ECD) needs to be divided by the air conditioner’s coefficient of performance (3.0). Electric demands are satisfied with centralized generation equipment and cogenerators:

\[
\sum_{n} \text{ELG}_{nydh} \cdot \sum_{s} \text{COG}_{sydh} \cdot \sum_{s} \text{ECD}_{sydh} / 3.0 + \text{ELD}_{sydh} \text{ for all } y, d, h.
\]

**COOLING ENERGY DEMANDS**

For each subsector, the cooling demand CLD must be satisfied by the output of the electric cooling devices (ECD) and thermal cooling devices (TCD):

\[
\text{ECD}_{sydh} + \text{TCD}_{sydh} \geq \text{CLD}_{sydh} \text{ for all } s, y, d, h.
\]

**HEATING ENERGY DEMANDS**

For each subsector, the combined output of the thermal generators (THG), cogenerator (COG), and thermal storage (TOT) must meet or exceed the in-
puts to thermal storage (TIN) plus heating demands and thermal cooling devices (TCD). The cogeneration output is multiplied by the ratio of steam to electricity (1.29) and the thermal cooling output is multiplied by its coefficient of performance (.67):

\[ \text{THC}_{\text{sydh}} + (\text{COG}_{\text{sydh}} \times 1.29) + \text{TOT}_{\text{sydh}} \geq \text{TIN}_{\text{sydh}} + \text{HTD}_{\text{sydh}} + \frac{(\text{TCD}_{\text{sydh}})(0.67)}{} \text{ for all } s, y, d, h. \]

**CAPACITY AVAILABILITY**

The output of each electrical and thermal generator must not exceed its available capacity. This available capacity is defined as 1980 capacity plus additions in future years minus the amount of capacity removed for maintenance (MAN). In order to sum up all capacity additions, the mathematical shorthand uses the subscript i, ranging from 0 to the value of the year subscript y. There are five types of equations for capacity:

- **Electric capacity:**
  \[ \text{ELC}_{\text{nydh}} \leq A_n \times \sum_{i=0}^{y} (\text{ELC}_{ni} - \text{MAN}_{ni}) \text{ for all } n, y, d, h \]
  \text{ELC}_{n0} \text{ is set at initial 1980 capacity (input)}

- **Thermal capacity:**
  \[ \text{THC}_{\text{nydh}} \leq (0.95) \times \sum_{i=0}^{y} \text{THC}_{ui} \text{ for all } s, y, d, h \]
  \[ \text{THC}_{s0} = 0 \text{ for all } s \text{ (Initial 1980 capacity is set at zero)} \]

- **Cogeneration capacity:**
  \[ \text{COG}_{\text{nydh}} \leq (0.95) \times \sum_{i=0}^{y} \text{COG}_{ui} \text{ for all } s, y, d, h \]
  \[ \text{COG}_{s0} = 0 \text{ for all } s \text{ (Initial 1980 capacity is set at zero)} \]

- **Electric cooling capacity:**
  \[ \text{ECD}_{\text{nydh}} \leq (0.95) \times \sum_{i=0}^{y} \text{ECC}_{si} \text{ for all } s, y, d, h \]
  \[ \text{ECC}_{s0} = 0 \text{ for all } s \]

  **Thermal cooling capacity:**
  \[ \text{TCD}_{\text{nydh}} \leq (0.95) \times \sum_{i=0}^{y} \text{TCC}_{ui} \text{ for all } s, y, d, h \]
  \[ \text{TCC}_{s0} = 0 \text{ for all } s \]

**THERMAL STORAGE CAPACITY**

Thermal storage capacity (TEC) equals or exceeds daily storage multiplied by the efficiency of the storage (0.90) for each subsector s:

\[ (0.90) \times \sum_{h} \text{TIN}_{\text{sydh}} \leq \sum_{h} \text{TEC}_{ui} \text{ for all } s, y, d \]

**THERMAL STORAGE AVAILABILITY**

Thermal energy output must not exceed storage during that particular day multiplied by the storage efficiency (0.90):

\[ \sum_{h} \text{TOT}_{\text{sydh}} \leq (0.90) \times \sum_{h} \text{TIN}_{\text{sydh}} \text{ for all } s, y, d \]

**RESERVE MARGIN**

Centralized electric capacity plus cogeneration capacity equals or exceeds peak demands times 1.20 (a 20 percent reserve margin):

\[ \sum_{s} \sum_{y} \left( \text{ELC}_{ni} - \text{MAN}_{nydh} \right) + \sum_{s} \sum_{i=0}^{y} \text{COG}_{ui} \geq (1.20) \times \sum_{s} \left( \text{ECD}_{\text{sydh}}/3.0 \right) + \text{ELD}_{\text{sydh}} \text{ for peak hours, peak days, and all } y \]

**MAINTENANCE SCHEDULING**

The scheduled maintenance of baseload and intermediate generation capacity equals or exceeds the scheduled maintenance required annually. Each capacity type needs to be maintained for 10 percent of the year, or 876 hours:

\[ 24 \times \sum_{d} (\text{NO}_{d} \times \text{MAN}_{nydh}) \geq 876 \times \sum_{i=0}^{y} \text{ELC}_{ni} \text{ for all } y \text{ and } n = 1, 2 \]