



Getting Started in Linear Regression using R

(with some examples in Stata)

(ver. 0.1-Draft)

Oscar Torres-Reyna

Data Consultant

otorres@princeton.edu

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VIGET

R

Stata

Using dataset "Prestige"*

Used in the regression models in the following pages

```
# Dataset is in the following library

library(car)

# If not installed type

install.packages("car")

# Type help(Prestige) to access the codebook

✓ education. Average education of occupational
  incumbents, years, in 1971.

✓ income. Average income of incumbents, dollars, in
  1971.

✓ women. Percentage of incumbents who are women.

✓ prestige. Pineo-Porter prestige score for occupation,
  from a social survey conducted in the mid-1960s.

✓ census. Canadian Census occupational code.

✓ type. Type of occupation. A factor with levels (note:
  out of order): bc, Blue Collar; prof, Professional,
  Managerial, and Technical; wc, White Collar.
```

```
/* Stata version here */

use http://www.ats.ucla.edu/stat/stata/examples/ara/Prestige, clear

/* Renaming/recoding variables to match the
dataset's R version*/

rename educat education
rename percwomn women
rename occ_code census
recode occ_type (2=1 "bc") (4=2 "wc") (3=3
"prof") (else=.), gen(type) label(type)
label variable type "Type of occupation"
drop occ_type
replace type=3 if occtitle=="PILOTS"
gen log2income=log10(income)/log10(2)
```

*Fox, J. and Weisberg, S. (2011) *An R Companion to Applied Regression*, Second Edition, Sage.

NOTE: The R content presented in this document is mostly based on an early version of Fox, J. and Weisberg, S. (2011) *An R Companion to Applied Regression*, Second Edition, Sage; and from class notes from the ICPSR's workshop *Introduction to the R Statistical Computing Environment* taught by John Fox during the summer of 2010.

Linear regression

```
# R automatically process the log base 2 of income
in the equation
```

```
reg1 <- lm(prestige ~ education + log2(income) +
          women, data=Prestige)
```

```
summary(reg1)
```

(See output next page)

```
/* You need to create the log base 2 of income
first, type: */
```

```
gen log2income=log10(income)/log10(2)
```

```
/* Then run the regression */
```

```
regress prestige education log2income women
```

Linear regression (robust, controlling for heteroskedasticity)

```
reg1.robust <- rlm(prestige ~ education +
                  log2(income) + women, data=Prestige)
```

```
summary(reg1.robust)
```

(See output next page)

```
regress prestige education log2income women,
        robust
```

Predicted values/Residuals

```
# After running the regression
```

```
prestige_hat <- fitted(reg1) # predicted values
as.data.frame(prestige_hat)
```

```
Prestige_resid <- residuals(reg1) # residuals
as.data.frame(prestige_resid)
```

```
/* After running the regression */
```

```
predict prestige_hat /* Predicted values */
```

```
predict prestige_resid /* Residuals */
```

NOTE: For output interpretation (linear regression) please see <http://dss.princeton.edu/training/Regression101.pdf>

Linear regression (output)

```
> reg1 <- lm(prestige ~ education + log2(income) + women, data=Prestige)
> summary(reg1)
```

```
Call:
lm(formula = prestige ~ education + log2(income) + women, data = Prestige)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-17.3639  -4.4293  -0.1010   4.3160  19.1793
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -110.9658    14.8429  -7.476 3.27e-11 ***
education     3.7305     0.3544  10.527 < 2e-16 ***
log2(income)  9.3147     1.3265   7.022 2.90e-10 ***
women         0.0469     0.0299   1.568   0.12
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 7.093 on 98 degrees of freedom
Multiple R-squared:  0.8351,    Adjusted R-squared:  0.83
F-statistic: 165.4 on 3 and 98 DF,  p-value: < 2.2e-16
```

```
. regress prestige education log2income women
```

Source	SS	df	MS
Model	24965.5409	3	8321.84695
Residual	4929.88524	98	50.3049514
Total	29895.4261	101	295.994318

```
Number of obs =      102
F( 3, 98) =      165.43
Prob > F       =      0.0000
R-squared      =      0.8351
Adj R-squared  =      0.8300
Root MSE      =      7.0926
```

prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
education	3.730508	.354383	10.53	0.000	3.027246	4.433769
log2income	9.314667	1.326515	7.02	0.000	6.682241	11.94709
women	.0468951	.0298989	1.57	0.120	-.0124382	.1062285
_cons	-110.9658	14.84293	-7.48	0.000	-140.4211	-81.51052

R**Stata****Dummy regression with no interactions (analysis of covariance, fixed effects)**

```
reg2 <- lm

```

```
Stata 11.x*

regress prestige education log2income i.type

Stata 10.x

xi: regress prestige education log2income i.type

*See http://www.stata.com/help.cgi?whatsnew10to11
```

Dummy regression with no interactions (interpretation, see output next page)

	bc	wc	prof
Intercept	-81.2	-81.2-1.44 = -82.64	-81.2 + 6.75 = -74.45
log2(income)	7.27	7.27	7.27
education	3.28	3.28	3.28

NOTE: "type" is a categorical or factor variable with three options: bc (blue collar), prof (professional, managerial, and technical) and wc (white collar). R automatically recognizes it as factor and treat it accordingly. In Stata you need to identify it with the "i." prefix (in Stata 10.x or older you need to add "xi:")

NOTE: For output interpretation (linear regression) please see <http://dss.princeton.edu/training/Regression101.pdf>

NOTE: For output interpretation (fixed effects) please see <http://dss.princeton.edu/training/Panel101.pdf>

R

Stata

Dummy regression with interactions (output)

```
> reg2 <- lm(prestige ~ education + log2(income) + type, data = Prestige)
> summary(reg2)
```

```
Call:
lm(formula = prestige ~ education + log2(income) + type, data = Prestige)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-13.511  -3.746   1.011   4.356  18.438
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -81.2019   13.7431  -5.909 5.63e-08 ***
education      3.2845    0.6081   5.401 5.06e-07 ***
log2(income)  7.2694    1.1900   6.109 2.31e-08 ***
typewc       -1.4394    2.3780  -0.605  0.5465
typeprof      6.7509    3.6185   1.866  0.0652 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 6.637 on 93 degrees of freedom
(4 observations deleted due to missingness)
Multiple R-squared: 0.8555, Adjusted R-squared: 0.8493
F-statistic: 137.6 on 4 and 93 DF, p-value: < 2.2e-16
```

```
. regress prestige education log2income i.type
```

Source	SS	df	MS
Model	24250.5893	4	6062.64731
Residual	4096.2858	93	44.0460839
Total	28346.8751	97	292.235825

```
Number of obs = 98
F( 4, 93) = 137.64
Prob > F = 0.0000
R-squared = 0.8555
Adj R-squared = 0.8493
Root MSE = 6.6367
```

prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
education	3.284486	.608097	5.40	0.000	2.076926	4.492046
log2income	7.269361	1.189955	6.11	0.000	4.906346	9.632376
type						
2	-1.439403	2.377997	-0.61	0.546	-6.161635	3.282828
3	6.750887	3.618496	1.87	0.065	-.434729	13.9365
_cons	-81.20187	13.74306	-5.91	0.000	-108.4929	-53.91087

R**Stata****Dummy regression with interactions**

```
reg3 <- lm(prestige ~ type*(education +
log2(income)), data = Prestige)

summary(reg3)

(See output next page)

# Other ways to run the same model

reg3a <- lm(prestige ~ education + log2(income) +
  type + log2(income):type + education:type,
  data = Prestige)

reg3b <- lm(prestige ~ education*type +
  log2(income)*type, data = Prestige)
```

```
Stata 11.x*

regress prestige i.type##c.education
  i.type##c.log2income

Stata 10.x

xi: regress prestige i.type*education
  i.type*log2income

*See http://www.stata.com/help.cgi?whatsnew10to11
```

Dummy regression with interactions (interpretation, see output next page)

	bc	wc	prof
Intercept	-120.05	-120.05 + 30.24 = -89.81	-120.05 + 85.16 = -34.89
log2(income)	11.08	11.08 - 5.653 = 5.425	11.08 - 6.536 = 4.542
education	2.34	2.34 + 3.64 = 5.98	2.34 + 0.697 = 3.037

NOTE: "type" is a categorical or factor variable with three options: bc (blue collar), prof (professional, managerial, and technical) and wc (white collar). R automatically recognizes it as factor and treat it accordingly. In Stata you need to identify it with the "i." prefix (in Stata 10.x or older you need to add "xi:")

NOTE: For output interpretation (linear regression) please see <http://dss.princeton.edu/training/Regression101.pdf>

NOTE: For output interpretation (fixed effects) please see <http://dss.princeton.edu/training/Panel101.pdf>

Dummy regression with interactions (output)

```
> reg3 <- lm(prestige ~ type*(education + log2(income)), data = Prestige)
> summary(reg3)
```

```
Call:
lm(formula = prestige ~ type * (education + log2(income)), data = Prestige)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-13.970  -4.124   1.206   3.829  18.059
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -120.0459    20.1576  -5.955 5.07e-08 ***
typewc       30.2412     37.9788   0.796 0.42800
typeprof     85.1601     31.1810   2.731 0.00761 **
education     2.3357      0.9277   2.518 0.01360 *
log2(income) 11.0782      1.8063   6.133 2.32e-08 ***
typewc:education 3.6400     1.7589   2.069 0.04140 *
typeprof:education 0.6974     1.2895   0.541 0.58998
typewc:log2(income) -5.6530     3.0519  -1.852 0.06730 .
typeprof:log2(income) -6.5356     2.6167  -2.498 0.01434 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 6.409 on 89 degrees of freedom
(4 observations deleted due to missingness)
Multiple R-squared: 0.871,    Adjusted R-squared: 0.8595
F-statistic: 75.15 on 8 and 89 DF,  p-value: < 2.2e-16
```

```
. regress prestige i.type##c.education i.type##c.log2income
```

Source	SS	df	MS	Number of obs = 98		
Model	24691.4782	8	3086.43477	F(8, 89) =	75.15	
Residual	3655.3969	89	41.0718753	Prob > F =	0.0000	
				R-squared =	0.8710	
				Adj R-squared =	0.8595	
Total	28346.8751	97	292.235825	Root MSE =	6.4087	

prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
type						
2	30.24117	37.97878	0.80	0.428	-45.22186	105.7042
3	85.16011	31.181	2.73	0.008	23.20414	147.1161
education	2.335673	.927729	2.52	0.014	.492295	4.179051
type#						
c.education						
2	3.640038	1.758948	2.07	0.041	.1450456	7.13503
3	.6973987	1.289508	0.54	0.590	-1.864827	3.259624
log2income	11.07821	1.806298	6.13	0.000	7.489136	14.66729
type#						
c.log2income						
2	-5.653036	3.051886	-1.85	0.067	-11.71707	.410996
3	-6.535558	2.616708	-2.50	0.014	-11.7349	-1.336215
_cons	-120.0459	20.1576	-5.96	0.000	-160.0986	-79.99318

R

Diagnostics for linear regression (residual plots, see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type,
data = Prestige)

residualPlots(reg1)

      Test stat Pr(>|t|)
education   -0.684   0.496
income     -2.886   0.005
type                NA     NA
Tukey test  -2.610   0.009

# Using 'income' as is.
# Variable 'income' shows some patterns.

# Other options:

residualPlots(reg1, ~ 1, fitted=TRUE) #Residuals
vs fitted only

residualPlots(reg1, ~ education, fitted=FALSE) #
Residuals vs education only
```

```
library(car)

regla <- lm(prestige ~ education + log2(income) +
type, data = Prestige)

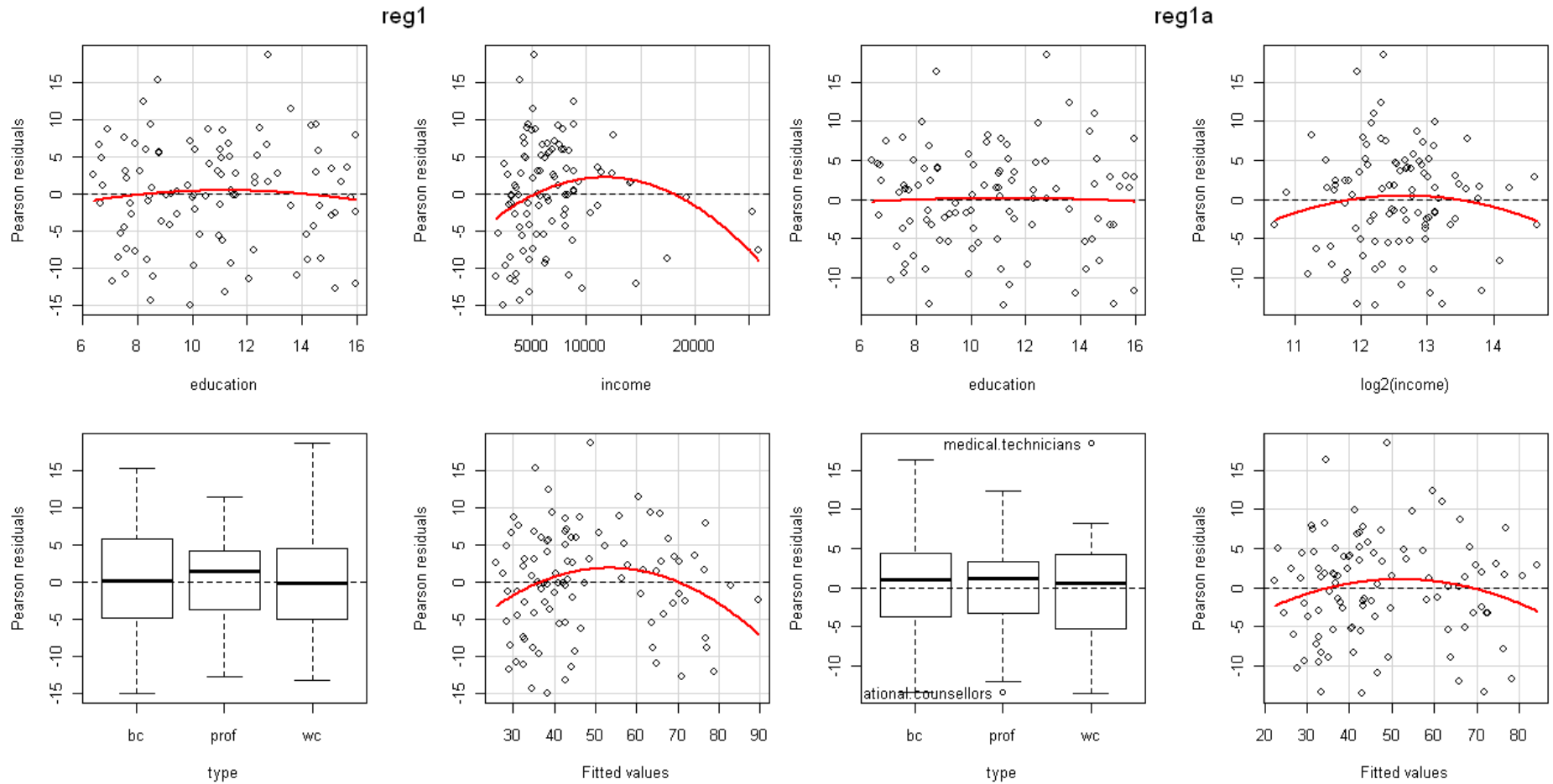
residualPlots(regla)

      Test stat Pr(>|t|)
education   -0.237   0.813
log2(income) -1.044   0.299
type                NA     NA
Tukey test  -1.446   0.148

# Using 'log2(income)'.
# Model looks ok.
```

```
# What to look for: No patterns, no problems.
# All p's should be non-significant.
# Model ok if residuals have mean=0 and variance=1 (Fox, 316)
# Tukey test null hypothesis: model is additive.
```

Diagnostics for linear regression (residual plots graph)



R

Influential variables - Added-variable plots (see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

avPlots(reg1, id.n=2, id.cex=0.7)

# id.n - id most influential observation
# id.cex - font size for id.

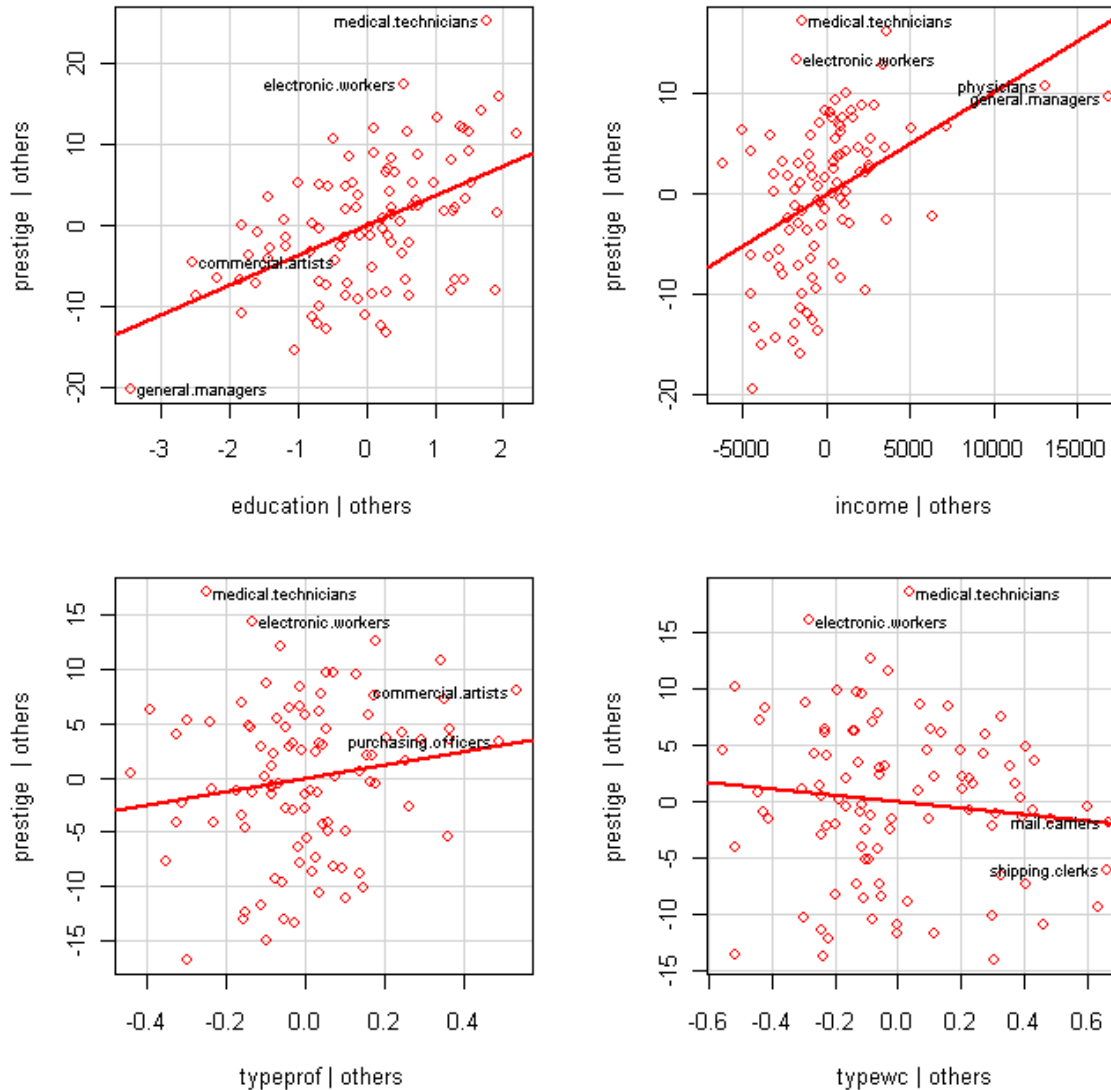
# Graphs outcome vs predictor variables holding the rest constant (also called partial-regression
plots)
# Help identify the effect (or influence) of an observation on the regression coefficient of the
predictor variable
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

R

Added-variable plots – Influential variables (graph)

Added-Variable Plots



R

Outliers – QQ-Plots (see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

qqPlot(reg1, id.n=3)

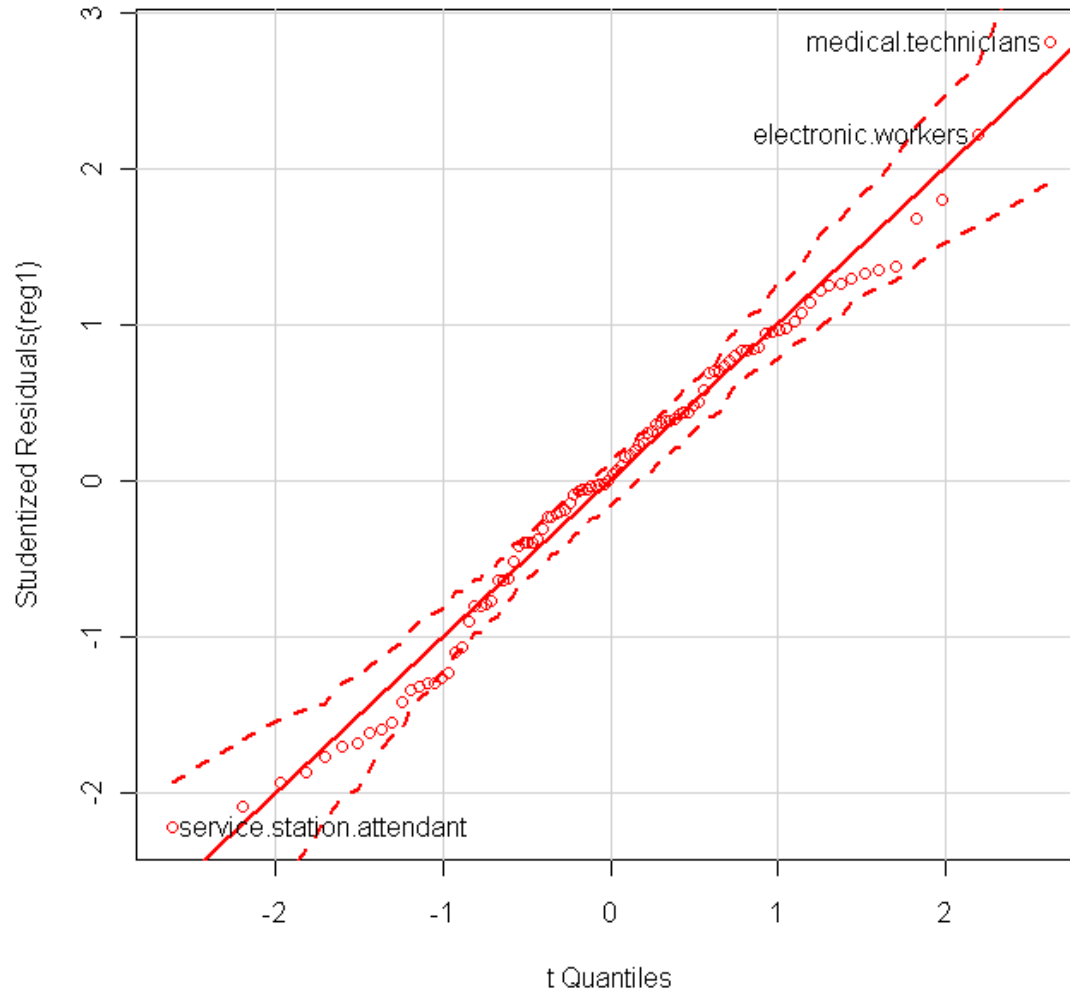
[1] "medical.technicians"      "electronic.workers"
[3] "service.station.attendant"

# id.n - id observations with high residuals
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

R

Added-variable plots – Influential variables (graph)



R

Outliers – Bonferonni test

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

outlierTest(reg1)

No Studentized residuals with Bonferonni p < 0.05
Largest |rstudent|:
               rstudent unadjusted p-value Bonferonni p
medical.technicians 2.821091          0.0058632          0.57459

# Null for the Bonferonni adjusted outlier test is the observation is an outlier. Here observation
  related to 'medical.technicians' is an outlier.
```

High leverage (*hat*) points (graph next page)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

influenceIndexPlot(reg1, id.n=3)

# Cook's distance measures how much an observation influences the overall model or predicted values
# Studentized residuals are the residuals divided by their estimated standard deviation as a way to
  standardized
# Bonferroni test to identify outliers
# Hat-points identify influential observations (have a high impact on the predictor variables)

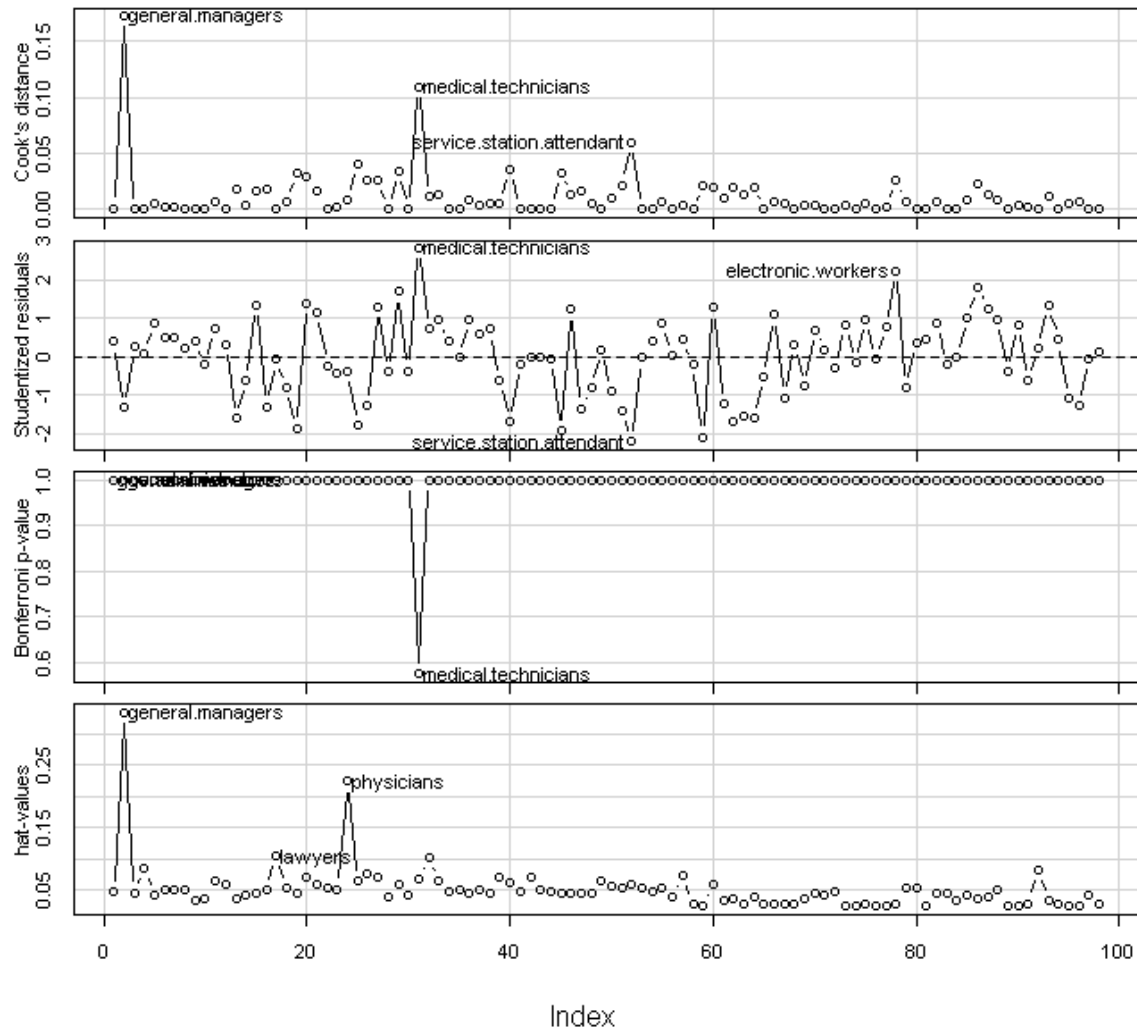
NOTE: If an observation is an outlier and influential (high leverage) then that observation can change the fit
  of the linear model, it is advisable to remove it. To remove a case(s) type
regla <- update(prestige.reg4, subset=rownames(Prestige) != "general.managers")
reglb <- update(prestige.reg4, subset= !(rownames(Prestige) %in% c("general.managers","medical.technicians")))
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

R

High leverage (*hat*) points (graph)

Diagnostic Plots



R

Influence Plots (see next page for a graph)

```
library(car)
```

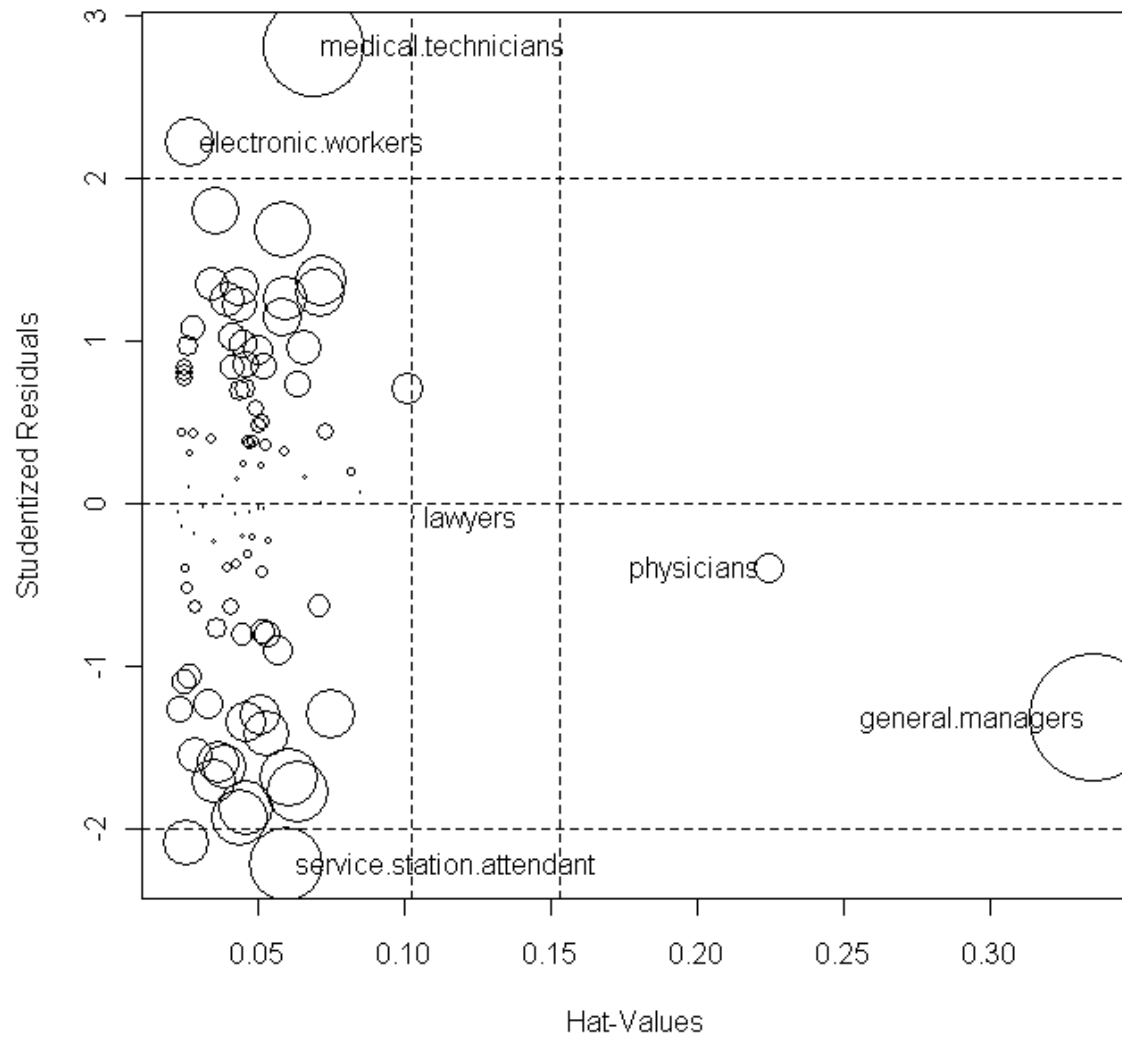
```
reg1 <- lm(prestige ~ education + income + type, data = Prestige)
```

```
influencePlot(reg1, id.n=3)
```

```
# Creates a bubble-plot combining the display of Studentized residuals, hat-values, and Cook's distance (represented in the circles).
```

R

Influence plot



R

Testing for normality (see graph next page)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

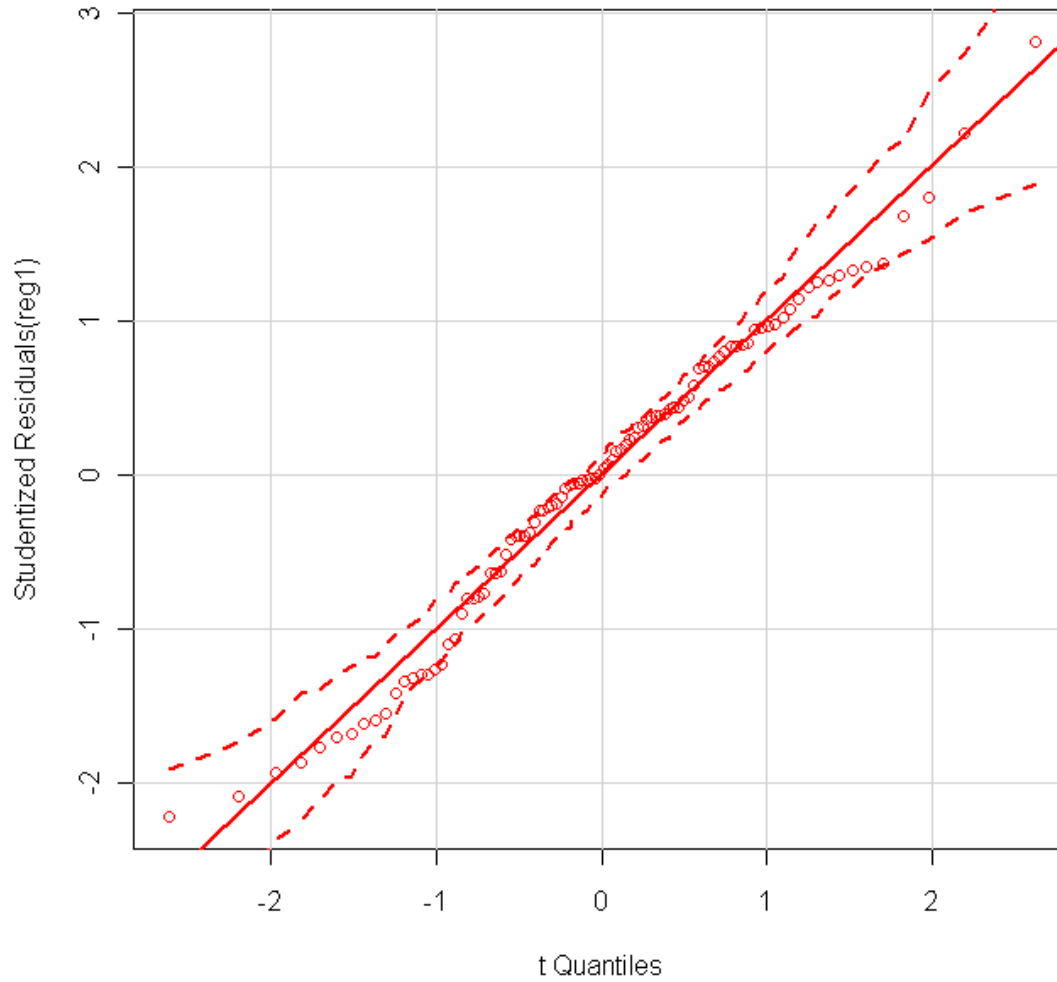
qqPlot(reg1)

# Look for the tails, points should be close to the line or within the confidence intervals.
# Quantile plots compare the Studentized residuals vs a t-distribution
# Other tests: shapiro.test(), mshapiro.test() in library(mvnormtest)-library(ts)
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

R

Influence plot



R

Testing for heteroskedasticity

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

ncvTest(reg1)

      Non-constant Variance Score Test
      Variance formula: ~ fitted.values
      Chisquare = 0.09830307    Df = 1    p = 0.7538756

# Breush/Pagan and Cook/Weisberg score test for non-constant error variance. Null is constant variance
# See also residualPlots(reg1).
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

R

Testing for multicollinearity

```
library(car)
```

```
reg1 <- lm(prestige ~ education + income + type, data = Prestige)
```

```
vif(reg1)
```

	GVIF	Df	GVIF^(1/(2*Df))
education	5.973932	1	2.444163
income	1.681325	1	1.296659
type	6.102131	2	1.571703

```
# A gvif > 4 suggests collinearity.
```

```
# "When there are strong linear relationships among the predictors in a regression analysis, the  
precision of the estimated regression coefficients in linear models declines compared to what it  
would have been were the predictors uncorrelated with each other" (Fox:359)
```

NOTE: For Stata version please see <http://dss.princeton.edu/training/Regression101.pdf>

References/Useful links

- DSS Online Training Section <http://dss.princeton.edu/training/>
- Princeton DSS Libguides <http://libguides.princeton.edu/dss>
- John Fox's site <http://socserv.mcmaster.ca/jfox/>
- Quick-R <http://www.statmethods.net/>
- UCLA Resources to learn and use R <http://www.ats.ucla.edu/stat/R/>
- UCLA Resources to learn and use Stata <http://www.ats.ucla.edu/stat/stata/>
- DSS - Stata http://dss/online_help/stats_packages/stata/
- DSS - R http://dss.princeton.edu/online_help/stats_packages/r

References/Recommended books

- *An R Companion to Applied Regression*, Second Edition / John Fox , Sanford Weisberg, Sage Publications, 2011
- *Data Manipulation with R* / Phil Spector, Springer, 2008
- *Applied Econometrics with R* / Christian Kleiber, Achim Zeileis, Springer, 2008
- *Introductory Statistics with R* / Peter Dalgaard, Springer, 2008
- *Complex Surveys. A guide to Analysis Using R* / Thomas Lumley, Wiley, 2010
- *Applied Regression Analysis and Generalized Linear Models* / John Fox, Sage, 2008
- *R for Stata Users* / Robert A. Muenchen, Joseph Hilbe, Springer, 2010
- *Introduction to econometrics* / James H. Stock, Mark W. Watson. 2nd ed., Boston: Pearson Addison Wesley, 2007.
- *Data analysis using regression and multilevel/hierarchical models* / Andrew Gelman, Jennifer Hill. Cambridge ; New York : Cambridge University Press, 2007.
- *Econometric analysis* / William H. Greene. 6th ed., Upper Saddle River, N.J. : Prentice Hall, 2008.
- *Designing Social Inquiry: Scientific Inference in Qualitative Research* / Gary King, Robert O. Keohane, Sidney Verba, Princeton University Press, 1994.
- *Unifying Political Methodology: The Likelihood Theory of Statistical Inference* / Gary King, Cambridge University Press, 1989
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