Redistribution and Pork in Two-Party Competition

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Abstract

Why might citizens vote against redistributive policies from which they would seem to benefit? Many scholars focus on “wedge” issues such as religion or race, but another explanation might be geographically-based patronage or pork. We examine the tension between redistribution and patronage with a model that combines partisan elections across multiple districts with legislation in spatial and divide-the-dollar environments. There are two types of voters, poor and rich, each of whom have natural ideological allegiances with the left and right parties, respectively. The model yields a unique equilibrium that allows us to explore the circumstances under which poor voters support right-wingers parties that oppose redistributive transfers. The model predicts that poor voters may support right-wing parties even when a majority of districts are poor, but rich voters do not support left-wing parties when a majority of districts are rich. We also examine the model’s implications for the level of redistribution, platform competition, party discipline, and districting.

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1 Introduction

Why might citizens vote against redistributive policies from which they would seem to benefit? Many scholars focus on “wedge” issues such as religion or race, that create a second dimension that trumps economic interests (e.g., Roemer 1998). But another explanation might be geographically-based patronage or pork. Key (1984), for instance, attributes the weakness of pre-World War II southern Republicans at the state level in part to their ability to win patronage as part of the winning coalition at the national level. But since the quantity of pork is endogenous to the political process and finite, the appeal of pork must be limited. This paper examines how the tension between redistributive preferences, on one hand, and patronage-based preferences, on the other, can shape voting incentives of poor and rich voters. Under what circumstances will poor voters support right-wing parties that favor low taxes and low levels of redistribution, or will rich voters support left-wing parties that favor high taxes and high levels of redistribution?

Our answer to this question emphasizes the role of government transfers that are not based on income. Although governments often have programs that tax the rich to benefit the poor — the central focus of standard “tax and transfer” models in the Meltzer and Richard (1981) tradition — they also have a plethora of programs that target the distribution of government revenues based on criteria unrelated to income. We focus specifically on geographically-targeted programs, or “pork.” Such programs take such a wide range of forms — including the siting of government offices, hospitals, universities, military bases, and national industries, as well as targeted programs, such as road construction and other public works projects — and their size can be comparable to that of redistributive programs. A clear understanding of how voters make choices based on personal economic benefits therefore requires one to consider the effect of targeted transfers in addition to the effect of redistributive programs.

Our model combines partisan elections across multiple districts with legislative bargaining to determine final policy outcomes. There are two types of voters, poor and rich, and there are also two kinds of government spending, pork and redistributive transfers. Pork is transferred directly to the districts through legislative bargaining, and redistributive transfers are shared equally among only
the poor voters. The government is a majority-rule legislature composed of the winning candidates from two parties. The parties commit to platforms that describe the proportion of the budget that goes to redistribution and pork, as well as an ideological position of the party. The left-wing party prefers that a higher level of government revenues be spent on redistribution than does the right-wing party. It also has a more left-wing ideological position on policies unrelated to redistribution. Poor voters therefore prefer the redistributive policy and the ideological policy position of the left-wing party, while the rich voters prefer the right-wing party on these dimensions. After elections determine the majority party, the majority party determines the proportion of the budget that is used for pork, and legislative bargaining determines how the pork is distributed across districts.

Our central focus is on the equilibrium levels of “cross-over voting,” which occurs when the poor support the right-wing party or the rich support the left-wing party. Party discipline in the legislature is one of the most important factors affecting cross-over voting. When parties are “strong” (i.e., highly disciplined), the majority party can exclude members of the minority party from pork, creating incentives for voters to support the winning party in order to obtain pork in their district.\(^1\) Consequently, when voters expect the party with with “bad” redistributive policies to win at the national level, they may nonetheless cross-over vote for this party in their district in order to obtain local pork.

We show that although cross-over voting by rich and poor can occur in equilibrium, there is an asymmetry that advantages the rich voters and right-wing party. Since cross-over voting occurs in order to obtain pork, and since the right-wing party spends a lower proportion of the budget on redistribution, and hence a higher proportion of the budget on pork, incentives for the poor to cross-over vote for the right-wing party are greater than are the incentives for the rich to cross-over vote for the left-wing party. Our model therefore provides an intuition for why right-wing parties should have an advantage over left-wing ones in plurality rule electoral systems.

This advantage, however, depends on the existence of strong parties. When parties are “weak,” the distribution of pork is the outcome of a bargaining process where all legislators are free to bargain across party lines, with no advantage to members of the majority party. This environment

\(^1\)As an example, the Liberal Democratic Party in Japan is widely believed to direct government pork for political purposes (e.g., Curtis 1992, Ramseyer and Rosenbluth 1995, Fukui and Fukai 1996).
might describe a system such as the U.S., as opposed to many parliamentary systems. Voters’ incentives in this model are much simpler, since there is no longer any reason to “bandwagon” with a winning party in order to take advantage of its pork. Voters therefore always support the party that they favor on the redistributive and ideological dimensions.

Although the main focus of our analysis assumes that party positions are fixed, in order to link our work with some of the main concerns of the elections literature, we explore two extensions of the model, one where Downsian parties adopt platforms to maximize seat shares, and one where parties control electoral districting. When parties maximize their number of legislative seats, Downsian competition will drive anticipated losers to adopt the ideal points of their natural constituents. This minimizes (but does not eliminate) the extent of cross-over voting to the winning party. While losing parties are ideological purists, winning parties will typically deviate from their constituents’ preferred policy somewhat in order to draw cross-over votes. When parties control districting, there can be stark implications for the geographic distribution of public money, but only when parties are strong. Since a planner who allocates the distribution of voters needs only to place a majority of voters in a majority of districts, she can achieve large swings in distributive outcomes under strong parties even when the number of sympathetic voters is relatively small.

Our work joins an extensive list of models that have considered the interaction between elections and government policy. With respect to model structure, our analysis is perhaps most closely related to Dixit and Londregan (1995), who examine political competition among parties with fixed ideological platforms and the ability to commit to transfer payments to groups within a single electorate. Our model is also similar in structure to Milesi-Ferretti, Perotti and Rostagno (2002), who allow parties to compete on transfers to specific groups (such as the poor or the aged) and to geographic constituencies. Their paper assumes that the distribution of groups within each district is the same in majoritarian systems, and thus cannot examine the question of cross-over voting that is central here. Snyder and Ting (2003) have partisan elections with fixed platforms, but the subsequent legislation is on a spatial dimension. While there is no election in their model, Jackson and Moselle (2002) consider the problem of simultaneous bargaining over spatial policy and pork in a legislature. The lack of a simple equilibrium solution in their work motivates our simplifying
assumption that these two issues are considered separately in our legislature.\footnote{Other models of electoral systems and redistribution that distinguish between universal and targetable programs include Persson and Tabellini (1999) and Lizerri and Persico (2001).}

Our model also joins a line of recent research that examines how redistribution is affected not by a second dimension that is orthogonal to economic self-interest, but by the ability of governments to target transfers to specific groups on a basis other than income. Levy (2005), for example, examines the formation of electoral coalitions between the rich (who receive low taxes) and those poor who value education (who receive higher educational spending).\footnote{See also the related model in Fernández and Levy (2008).} Austen-Smith and Wallerstein (2006) examine how the ability to target transfers based on race affects redistribution. And Huber and Stanig (2008) examine how state-based support for transfers through religious organizations can create electoral coalitions between the religious poor and the rich. Although none of these models shares the institutional structure or the focus on cross-over voting that is central here, like our model, they each underscore the fact that the distribution of government resources occurs along pathways other than income-based redistribution. Further, they show that the existence of such pathways can affect the formation of electoral coalitions based on income, and the amount of income-based redistribution that occurs.

The remainder of the paper is organized as follows. Section 2 describes the basic structure of the model. Section 3 examines the unique equilibrium when parties are strong, as well as the factors that affect the level of cross-over voting. Section 4 examines the unique equilibrium when parties are weak, where no cross-over voting occurs. We consider the extensions on endogenous party platforms with Downsian parties and endogenous districting in section 5. The final section discusses the implications of our results.

## 2 The Model

Our model combines partisan elections across multiple districts with legislative policy choices in both spatial and divide-the-dollar environments. All election candidates and legislators belong to one of two (non-strategic) parties, denoted $P_L$ and $P_R$, which respectively represent “Left” and “Right.” There is a continuum of voters who are divided into $n$ districts, denoted $S_1, \ldots, S_n$. The
set of all voters has measure \( n \), where \( n \) is an odd integer, and each district has measure 1. For the election in each district, each party has a candidate, and a winner is chosen by plurality rule. Within each party, candidates are identical across districts.

A central variable in the model is the distribution of voters in each district. There are two types of voters, denoted by \( t \in \{ P, R \} \), which correspond informally to “poor” and “rich,” respectively. The poor voters qualify for means-tested income support and pay no taxes, whereas the rich pay taxes and receive no means-tested transfers. Let \( p^t_k \) be the proportion of voters of type \( t \) in \( S_k \). The total number of type \( t \) voters in society is then \( n^t = \sum_{k=1}^{n} p^t_k \). We refer to a district as “rich” or “poor” if the respective types are a majority of its population, and let \( d^R \) and \( d^P \) represent the number of rich and poor districts, respectively. The legislature is composed of the \( n \) winning candidates.

Each party, \( P_j \), has an exogenous platform, \( \lambda_j \in [0,1] \), that is adopted by all of its candidates, and to which the parties credibly commit. A platform, \( \lambda_j \), describes two perfectly correlated elements of a party’s policy intentions. First, \( \lambda_j \) is party \( j \)’s ideal point in a general ideological policy space. We assume that voters care about electing a representative in their district who is close to them in this ideological space. This may simply be an expressive preference, or there may be non-financial issues that individual legislators can influence independently in their districts. The second element is the party’s commitment to the two types of government financial transfers that exist in the model: pork and redistribution. “Pork” is tax revenues that are transferred directly to the districts through legislative bargaining. These transfers benefit all members of the district, regardless of their income. The proportion of total spending that \( P_j \) pledges to devote to pork is simply \( \lambda_j \). “Redistribution” is a means-tested income support or welfare program that benefits only the poor, and that is shared equally among all poor. This program consumes the entire non-pork portion of government revenues, and thus the proportion of total spending that \( P_j \) pledges to redistribution to the poor is \( 1 - \lambda_j \). We assume that \( P_R \) prefers a smaller welfare system than \( P_L \), which implies \( \lambda_R \geq \lambda_L \).

The legislature is represented by an \( n \)-vector \( x \) of the platform positions of the winning candidates, and median value \( x \) of \( x \) (i.e., the majority platform) determines the majority’s ideological
policy, as well as how the legislature disperses money across society. The proportion of government revenues allotted to pork is the majority platform, $x$. Pork is allocated to districts through an indivisible grant at the district level, and is denoted by an $n$-vector $y$ of district allocations. The proportion $1 - x$ is devoted to redistribution to the poor, and is spread equally among all poor voters.

The model allows party platforms and the government’s budget to be linked. Let $c \in [0,1]$ denote an exogenous constraint on total government spending. The government budget is $b(x) = 1 + (1 - c)(1 - x)$. At $c = 1$, the tax constraint is at its maximum: the government’s resources are fixed at 1 and any increases in welfare spending must come at the expense of pork. For $c < 1$, the government budget increases as the proportion of revenues devoted to welfare spending increase. At $c = 0$, the tax constraint is at its minimum: redistribution is funded entirely by incremental tax dollars. The parameter $c$ therefore represents an exogenous constraint — such as debt, the state of the economy, or the feasibility of tax increases — on the size of government. Note that the total quantity of pork, $x b(x)$, is increasing in $x$, and the total quantity of welfare spending, $(1 - x) b(x)$, is decreasing in $x$. The budget is financed by a flat tax on all rich voters.

Voters have Euclidean preferences over ideological policy and quasilinear utility over money. For the former, each type $t$ of voter has a common ideal point over her representative’s position-taking preferences, where $z^t \in [0,1]$. A key assumption is that voters’ preferences over redistribution and ideological policy are correlated: $t = R (P)$ implies that $z^t \geq (<) \frac{\lambda_L + \lambda_R}{2}$. Thus poor voters are drawn to the ideological position of $P_L$. Clearly, allowing poor voters to prefer the $\lambda_R$ spatial position would create obvious—and trivial—incentives for poor voters to vote against their economic interests. Each voter’s utility function is then:

$$u^t_k(x, y; t) = \begin{cases} u(|z^t - x_k|) + y_k + \frac{(1-x)b(x)}{n} & \text{if } t = P \\ u(|z^t - x_k|) + y_k - \frac{b(x)}{n} & \text{if } t = R, \end{cases}$$

where $u : \mathbb{R}_+ \to \mathbb{R}_-$ is continuous, concave, and decreasing. Legislators simply receive quasilinear utility $y_k$ over pork in their district. Since the model does not address candidate selection of campaign strategies, it is unnecessary to specify utilities for election candidates.
The game begins with simultaneous elections in each district, where voters simultaneously choose between candidates from each party. After the election winner is determined in each district, legislative bargaining determines the level of redistribution and the allocation of pork across districts. As noted above, we assume that legislators from the same party have identical ideological (i.e., “spatial”) preferences regarding redistribution. Under any legislative bargaining arrangement, then, the amount of redistribution will be \( 1 - x = 1 - \lambda_j \), where \( P_j \) is the party that wins a majority of districts. Legislators are also committed to maximizing the proportion of the available pork, \( \lambda_j \), that goes to their districts, pitting legislators against each other, independent of party. We consider two different bargaining processes for the pork, which capture the extremes of party discipline in the legislature.

In the weak party bargaining process, parties do not play a role in how members form coalitions, and legislators are free to bargain with any other legislator, so bargaining over pork involves the entire chamber. The election outcome therefore has no impact on the distribution of pork. In the strong party bargaining process, the majority party has unlimited proposal power and discipline. Because of perfect party discipline, the majority party can pass any legislation that is approved by a majority of its members, and therefore may divide the pork only among districts represented by the party.

We are agnostic about the details of the bargaining game, simply because many bargaining games predict equal \textit{ex ante} expected payoffs in games where all players have equal voting weight. This is true of the noncooperative models of Baron and Ferejohn (1989) and Morelli (1999), and also of power indices based on cooperative game theory, such as Shapley and Shubik (1954) and Banzhaf (1968). Thus the weak party bargaining process implies an \textit{ex ante} expected pork level of \( \frac{x_b(x)}{n} \) for all districts. Likewise, the strong party bargaining process implies an \textit{ex ante} expected pork level of \( \frac{x_b(x)}{\pi} \) in districts represented by the majority party, where \( \pi \) is the total population of such districts. When parties are strong, districts not represented by the majority party receive 0 with certainty.

Because the outcome of the bargaining process can be reduced to an expected payoff, the model is effectively a simultaneous-move game amongst voters. We assume that voters choose as if they
were pivotal in choosing their district’s legislator. This implies that voters of the same type in a given district always vote the same way; thus, let \(v^t_k\) be the vote by type \(t\) in \(S_k\). While Nash equilibria in this game are typically not unique, we can derive a unique prediction by considering coalition-proof Nash equilibria (CPNE). CPNE rule out Nash equilibria in which subsets of players may credibly deviate from a Nash equilibrium. The concept is weaker than that of a strong Nash equilibrium, which rules out any Nash equilibrium in which a subset of players may profitably deviate. By contrast, CPNE only rules out equilibria with self-enforcing deviations. A coalition of deviators is self-enforcing if no subset thereof would receive strictly higher payoffs from deviating in turn from the coalition’s proposed alternative strategy profile. As Bernheim, Peleg, and Whinston (1987) show, all strong Nash equilibria are CPNE, but CPNE are not generally guaranteed to exist. As we show in Proposition 1 below, CPNE is sufficient to guarantee a unique configuration of election returns.

3 Strong Parties

We begin by examining the equilibrium in the most interesting case; that of strong parties. In this environment, the incentives for joining a winning coalition are especially strong, as districts receive no pork if they do not elect a representative from the majority party.

3.1 Main Result

Poor voters always prefer a \(P_L\) victory on ideological policy and redistribution, and rich voters always prefer \(P_R\) on these two dimensions. However, both types of voters may have incentives to cross-over and support the “wrong” party if so doing ensures access to a sufficient level of pork. To characterize the equilibrium levels of support for each party, then, it is useful to characterize the circumstances under which the rich and the poor will engage in cross-over voting.

We first consider the optimal voting strategies in a single district \(k\). Let \(w_{-k}\) represent the number of districts excluding \(k\) that are expected to vote for \(P_R\) and suppose that the district’s pivotal voter is rich. When would such a voter cross-over and support \(P_L\)? If she the district is
pivotal for the election outcome (i.e., \(w_k = \frac{n-1}{2}\)), then her utility from supporting the candidate from party \(j\) is:

\[
u(|z^R - \lambda_j|) + \frac{\lambda_j b(\lambda_j)}{(n+1)/2} - \frac{b(\lambda_j)}{n^R}. \tag{1}
\]

Since \(u(|z^R - \lambda_R|) > u(|z^R - \lambda_L|)\), \(\frac{\lambda_R b(\lambda_R)}{(n+1)/2} > \frac{\lambda_L b(\lambda_L)}{(n+1)/2}\) and \(\frac{b(\lambda_R)}{n^R} \leq \frac{b(\lambda_L)}{n^R}\), the rich voter will never support \(P_L\). That is, voting for the left would yield lower ideological utility, less pork, and potentially higher taxes than voting for the right, so cross-over voting will not occur.

If a rich voter lives in a non-pivotal district and a majority of districts are expected to support \(P_R\) (i.e., \(w_k \geq \frac{n+1}{2}\)), then the rich voter will cross-over and support the Left Party only if

\[
u(|z^R - \lambda_R|) + \frac{\lambda_R b(\lambda_R)}{w_k + 1} - \frac{b(\lambda_R)}{n^R} > u(|z^R - \lambda_L|) - \frac{b(\lambda_R)}{n^R}.
\]

Since the rich voter gets more pork and higher ideological utility by supporting \(P_R\) (with no implications for taxes), this expression can obviously never be satisfied. Consequently, a rich voter might support \(P_L\) only if she lives in a non-pivotal district, and if a majority of districts are expected to support \(P_L\) (i.e., when \(w_k < \frac{n-1}{2}\)). In this case, a rich voter supports \(P_L\) only if:

\[
u(|z^R - \lambda_R|) - u(|z^R - \lambda_L|) < \frac{\lambda_L b(\lambda_L)}{n - w_k}. \tag{2}
\]

When \(w_k < \frac{n-1}{2}\), voting for \(P_L\) yields lower policy utility but more pork. The rich voter will vote for \(P_L\) if the loss in ideological utility is small relative to the gain in pork (i.e., if \(u(|z^R - \lambda_R|) - u(|z^R - \lambda_L|)\) is small relative to \(\frac{\lambda_L b(\lambda_L)}{n - w_k}\)). Thus, cross-over voting by the rich can only occur if a majority of districts are expected to vote for \(P_L\) and the ideological utility losses of voting left are small relative to the pork gains. This cross-over voting behavior by voters in rich districts is summarized in Remark 1.

\textbf{Remark 1} Rich voters will support \(P_L\) if and only if \(w_k < \frac{n-1}{2}\) and (2) holds. \hfill \blacksquare

Next consider cross-over voting by poor voters. In non-pivotal districts, their cross-over voting incentives are similar to the cross-over voting incentives of rich voters. If \(P_L\) is expected to win a majority of districts (i.e., \(w_k < \frac{n-1}{2}\)), then the poor voters in a non-pivotal district cannot
influence the level of redistribution, and they obtain a worse policy outcome and less pork by supporting \( P_R \). That is, the poor voters will cross-over and vote \( P_R \) only if:

\[
u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{n - w_{-k}} < u(|z^P - \lambda_R|), \tag{3}\]

which can never be satisfied.

If poor voters are in a non-pivotal district and \( P_R \) is expected to win a majority of districts \((w_{-k} \geq \frac{n+1}{2})\), then they cannot influence the level of redistribution. In casting their vote, like the rich voter when \( P_L \) is expected to win a majority of districts, they must weigh a trade-off between ideological policy and pork. Supporting \( P_L \) yields better ideological policy but less pork. The poor voters in this case will cross-over and vote for \( P_R \) if:

\[
u(|z^P - \lambda_L|) - u(|z^P - \lambda_R|) < \frac{\lambda_R b(\lambda_R)}{w_{-k} + 1}. \tag{4}\]

The central difference between poor voters and rich voters occurs in pivotal districts. Recall that rich voters in a pivotal district will never vote for \( P_L \) because so doing results in worse ideological policy, less pork and potentially higher taxes. Poor voters in a pivotal district, by contrast, face a trade-off. If they support \( P_R \), they receive more pork but less redistribution and worse ideological policy, so the poor voter in a pivotal district will cross-over and support \( P_R \) if:

\[
u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{(n + 1)/2} + \frac{(1 - \lambda_L)b(\lambda_L)}{n^P} < u(|z^P - \lambda_R|) + \frac{\lambda_R b(\lambda_R)}{(n + 1)/2} + \frac{(1 - \lambda_R)b(\lambda_R)}{n^P}. \tag{5}\]

Note that (5) implies (4) for \( w_{-k} = \frac{n-1}{2} \).

Remark 2 summarizes the cross-over voting conditions for poor voters:

**Remark 2** Poor voters will support \( P_R \) if either \( w_{-k} \geq \frac{n+1}{2} \) and (4) holds, or \( w_{-k} = \frac{n-1}{2} \) and (5) holds.  

Taken together, Remarks 1 and 2 suggest two important implications of the model. First, Remark 1 indicates that rich voters will never cross-over and support \( P_L \) when a majority of
districts support $P_R$, and Remark 2 indicates that poor voters will never cross-over and support $P_R$ when there are a majority of districts supporting $P_L$. Thus, in any Nash equilibrium, the winning party must carry all like-minded districts. If $P_L$ wins a legislative majority in equilibrium, then all districts for which poor voters are a majority must vote for $P_L$. Similarly, if $P_R$ wins, all rich districts must vote for $P_R$. Second, the remarks suggest an important advantage that right-wing parties enjoy in pivotal districts. If a pivotal district has a majority of rich voters, then since these voters receive no redistribution and know that $P_L$ offers less pork than $P_R$, they never face a trade-off between ideological policy, pork and redistribution. Rich voters in pivotal districts will therefore always support the right-wing party, ensuring a $P_R$ victory. By contrast, if a pivotal district has a majority of poor voters, then by (5), it is not certain that $P_L$ will win because the poor voters may face a tradeoff: supporting $P_R$ will often yield more pork, but supporting $P_L$ will yield superior outcomes in ideological policy and redistribution. If the value of pork from $P_R$ is relatively large, poor voters may cross-over and support $P_R$ in a pivotal district.

We can now characterize the equilibrium levels of voter support for each party. It is useful to define explicitly the number of districts that will cross-over if the “wrong” party is expected to win. If $P_R$ is expected to win a majority of seats, then Remark 2 indicates that the number of poor districts supporting $P_R$ is:

$$w = \max \left\{ w \mid u(|z^P - \lambda_L|) - u(|z^P - \lambda_R|) < \frac{\lambda_R b(\lambda_R)}{d^R + w + 1} \right\}. \tag{6}$$

Intuitively, $w$ is the size of the largest collection of poor districts such that poor voters contained within are willing to support $P_R$. Since $\frac{\lambda_R b(\lambda_R)}{d^R + w + 1}$ is obviously decreasing in $w$, $w$ is uniquely defined. For obvious reasons, we set $w$ at 0 and $d^P$ when the expression (6) implies a value less than 0 or greater than $d^P$, respectively.

Similarly, let the number of non-pivotal rich districts that would support $P_L$ if $P_L$ is expected to win be (uniquely) defined as:

$$w = \max \left\{ w \mid u(|z^R - \lambda_R|) - u(|z^R - \lambda_L|) < \frac{\lambda_L b(\lambda_L)}{d^P + w + 1} \right\}. \tag{7}$$
As with \( w \), we bound \( w \) between 0 and \( d^R \) in the obvious way.

Let \( w^* \) be the number of districts supporting the winning party. We can use \( w \) and \( w^* \) to characterize the unique winner and \( w^* \) in a coalition-proof Nash equilibrium. The CPNE are unique up to combinations of the districts supporting each party, and we therefore use the term “unique” in this context.

**Proposition 1** There is a unique coalition-proof Nash equilibrium, where:

(i) If \( d^R \geq \frac{n+1}{2} \) then \( P_R \) wins and \( w^* = d^R + w \).

(ii) If \( d^R < \frac{n+1}{2} \) and (5) is satisfied, then \( P_R \) wins and \( w^* = d^R + w \).

(iii) If \( d^R < \frac{n+1}{2} \) and (5) is not satisfied, then \( P_L \) wins and \( w^* = d^P + w \).

**Proof.** Notationally, let \( W \) denote a generic winning coalition, and \( C \) a subcoalition of deviators from a prescribed strategy profile.

Observe first that by (6), the number of poor districts supporting \( P_R \) in a Nash equilibrium cannot be greater than \( w \) (otherwise, a poor district supporting \( P_R \) would prefer switching to \( P_L \)) or less than \( w \) (otherwise, a one poor district supporting \( P_L \) would prefer switching to \( P_R \)). Thus, only \( w \) poor districts can support \( P_L \) in a Nash equilibrium. Likewise, by (7), the number of rich districts supporting \( P_L \) in a Nash equilibrium can only be \( w \). Thus, for any configuration of districts and voter preferences, a profile of voting strategies in which each citizen votes as if she were pivotal is a Nash equilibrium only if: \( P_R \) wins and \( w^* = d^R + w \) (with all rich districts voting for \( P_R \)), or \( P_L \) wins and \( w^* = d^P + w \) (with all poor districts voting for \( P_L \)). We now consider possible CPNE for each case.

(i) There cannot be an equilibrium majority for \( P_L \) because for any such majority of size \( |W| \), any subcoalition \( C \) of \( |W| - \frac{n-1}{2} \) rich districts would prefer to defect collectively to \( P_R \). All such defectors would then receive \( u(|z^R - \lambda_R|) + \frac{\lambda_P b(\lambda_R)}{(n+1)/2} - \frac{b(\lambda_R)}{n^R} \). A proper subset of \( C \) of size \( d \) would then receive \( u(|z^R - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{d^R(n-1)/2} - \frac{b(\lambda_L)}{n^R} \) by deviating back to \( P_L \) from this defection. Clearly, no profitable deviation exists, and so \( P_L \) cannot win.
To show that $P_R$ wins in equilibrium, observe first that by Remarks 1, 2, and (6), the strategy profile under which all rich districts and $\pi$ poor districts vote for $P_R$ is a Nash equilibrium.

To show coalition proofness, consider any potential subcoalition of deviators $C$. $C$ cannot contain any rich districts, since any profitable deviation must result in a $P_L$ victory, and by (1) a subcoalition of $C$ of rich districts would deviate to form a minimal-winning $P_R$ coalition. Thus $C$ can consist only of poor districts, and cannot affect $P_R$’s victory. By (6), no subcoalition of poor districts in $W$ can do better by switching to $P_L$, and no subcoalition of districts outside $W$ can do better by switching to $P_R$. Finally, any $C$ containing members of both $W$ and the losing coalition cannot strictly increase the payoffs of all members of $C$. The unique CPNE therefore has $w^* = d^R + \pi$ districts supporting $P_R$.

(ii) We first show that $P_L$ cannot win in equilibrium when (5) is satisfied. Suppose otherwise. Because $|W| = \frac{n+1}{2}$ would imply that poor voters in pivotal districts vote for $P_L$ in violation of (5), it follows that $|W| > \frac{n+1}{2}$. The maximum payoff that poor voters from poor districts could therefore ever receive from a $P_L$ majority for would occur when $|W| = \frac{n+3}{2}$. Such a majority would yield poor voters in $W$ a utility of:

$$u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{(n+3)/2} + \frac{(1 - \lambda_L)b(\lambda_L)}{n^P} < u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{(n+1)/2} + \frac{(1 - \lambda_L)b(\lambda_L)}{n^P}.$$

But since by (5) we have $u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{(n+3)/2} + \frac{(1 - \lambda_L)b(\lambda_L)}{n^P} < u(|z^P - \lambda_R|) + \frac{\lambda_R b(\lambda_R)}{(n+1)/2} + \frac{(1 - \lambda_R)b(\lambda_R)}{n^P}$, the utility to voters in poor districts from a minimal winning majority for $P_R$ is greater than the utility from any possible $P_L$ majority. From this it follows that if a majority of size $|W| \geq \frac{n+3}{2}$ formed for $P_L$, there would exist a subcoalition $C$ of $|W| - \frac{n+1}{2}$ poor districts that would prefer to defect collectively to $P_R$, thus inducing a minimum winning $P_R$ majority. To show that $C$ is self-enforcing, note again that by (5), there is no subset of $C$ that would prefer to defect back and support $P_L$, because all members prefer a minimum winning coalition for $P_R$ to any winning coalition for $P_L$. This contradicts the existence of a CPNE where $P_L$ wins.

It remains to show that a $P_R$ victory with $w^* = d^R + \pi$ represents a CPNE. Observe first that (5), (6), and Remarks 1 and 2 establish that it is a Nash equilibrium for all rich districts and exactly
\( \pi \geq \frac{n+1}{2} - d^R \) poor districts to vote for \( P_R \). Thus the strategy profile supporting a \( P_R \) victory is a Nash equilibrium.

To show that this equilibrium is coalition-proof, there are three cases. First, suppose that a potential defecting coalition \( C \) is composed only of districts not in \( W \). Then (6) clearly implies that these districts (which must be poor) cannot benefit by defecting. Next, suppose that \( C \) is composed only of districts in \( W \). Let \( d = w^* - \frac{n+1}{2} \). Any \( C \) must satisfy \( |C| > d \); otherwise, districts in \( C \) would not cause \( P_L \) to win and would therefore receive no pork for defecting. Now suppose that there exists a \( C \) with \( |C| > d \) that prefers to defect to \( P_L \). Then there exists a proper subset of \( C \) that would give \( P_R \) a minimum winning coalition by defecting back to \( P_R \). By an argument identical to that in the proof that \( P_L \) cannot win, this outcome is better for poor districts in \( C \) than any winning coalition for \( P_L \) when (5) is satisfied. It must therefore also be better for any rich district in \( C \). Finally, suppose that \( C \) contains districts both in \( W \) and not in \( W \). There is clearly no \( C \) that strictly improves all members’ payoffs if \( P_R \) continues to win. But if \( P_L \) wins, then by an argument identical to that of the previous case there exists a subcoalition of \( C \) that would restore defect back to a minimum-winning \( P_R \) coalition.

Thus there does not exist a self-enforcing \( C \). We conclude that in the unique CPNE, \( w^* = d^R + \pi \).

(iii) This proof is symmetric to that of case (i) and is therefore omitted.

Proposition 1 demonstrates that when parties are strong, cross-over voting incentives exist for rich and poor alike, as voters in both income groups may have incentives to support the “wrong” party in order to ensure access to pork. The extent to which this occurs, which we examine in the next section, depends on how voters weigh trade-offs between ideological policy, taxes, redistribution, and pork.

It is important to highlight, however, an asymmetry between cross-over voting incentives for rich and poor. Because the right-wing party wants to limit redistribution, it has more government revenues available for pork. This implies that poor voters have a greater incentive to cross-over and support the right-wing party than do rich voters to cross-over and support the left-wing party. As the Proposition 1 describes, in the unique equilibrium, if a majority of districts have poor majorities, poor voters may still ensure a majority for the right-wing party. But if rich voters control a
majority of districts, the left-wing party will never win. The analysis therefore suggests a different explanation for why we might expect right-wing parties to have an advantage in majoritarian systems (e.g., Iversen and Soskice 2006).

A final source of asymmetry between cross-over voting patterns is the extent to which different voter types join the opposing party’s coalition. Given an equal number of districts, the two parties will differ in their attractiveness to voters who are not their natural constituents. Since $P_R$ spends less on redistribution (which follows from the fact that $\lambda_j b(\lambda_j)$ is increasing in $\lambda_j$), a rich governing coalition will have more pork to distribute. This allows it to attract more cross-over voters than $P_L$. The following remark therefore follows immediately from (6) and (7).

**Remark 3** If $d^R = d^P$, then $w \geq w$. ■

One implication of this result is that a symmetric partisan swing in voter types can have different implications depending on its direction. Due to Proposition 1(ii), a swing may not shift partisan control at all if (5) is satisfied. But otherwise, a swing from rich to poor will induce a relatively small $P_L$ majority, while a swing from poor to rich will induce a large $P_R$ majority.

### 3.2 Determinants of Cross-over Voting

**Cross-over voting by the rich.** We can use Proposition 1 to explore the factors that affect cross-over voting by the rich and poor. We first consider cross-over voting by the rich. From Proposition 1, two necessary conditions for the rich to support $P_L$ are that a majority of districts are poor ($d^R < \frac{n+1}{2}$) and that poor voters in pivotal districts support $P_L$. When these conditions are satisfied, then $w$ rich districts will cross-over and support $P_L$.

What affects the size of $w$? From (7), the number of rich districts supporting $P_L$ will increase as it becomes easier to satisfy the following inequality:

$$u(|z^R - \lambda_R|) - u(|z^R - \lambda_L|) < \frac{\lambda_L b(\lambda_L)}{d^P + w + 1}.$$  (8)

Party system polarization, which occurs either when $\lambda_R$ increases (which we will call “$P_R$-polarization”) or $\lambda_L$ decreases (“$P_L$-polarization”), is one factor that influences cross-over voting.
by the rich. Since the Left Party must win a majority if cross-over voting by the rich occurs, $P_R$-polarization affects only the ideological utility of rich voters. If rich voters are relatively centrist ideologically (i.e., $z_R < \lambda_R$), then as $P_R$ becomes more extreme, centrist rich voters receive worse ideological policy and will become more inclined to support $P_L$. On the other hand, if the rich are ideologically extreme ($z_R > \lambda_R$), then $P_R$-polarization results in better ideological policy, and cross-over voting will be less attractive.

$P_L$-polarization, by contrast, always makes it more difficult to satisfy the inequality in (8). As $\lambda_L$ becomes smaller, $u(|z_R - \lambda_L|)$ declines, increasing the left-hand side of the inequality in (8). In addition, as $P_L$-polarization occurs, the left party devotes fewer resources to pork, and the budget increases, decreasing the value of supporting $P_L$ (i.e., $\frac{\partial}{\partial \lambda_L} \left[ \frac{\lambda_L b(\lambda_L)}{d^P + w + 1} \right] = \frac{\lambda_L (2c - 1) + 2 - c}{d^P + w + 1} > 0 \forall c \in [0, 1]$). $P_L$-polarization therefore increases the right-hand side of the inequality in (8).

Two other factors that affect cross-over voting levels by the rich are the number of majority-poor districts ($d^P$), and the effect of redistribution on taxes ($c$). As $d^P$ increases, the value of the pork distributed by $P_L$ decreases, decreasing the value of cross-over voting. Similarly, as $c$ increases, the total budget becomes smaller, and thus the proportion of the budget going to pork is smaller, decreasing the value of cross-over voting. Rich voters therefore have the greatest incentive to cross-over vote when taxes are highest. Formally, $\frac{\partial}{\partial c} \left[ \frac{\lambda_L b(\lambda_L)}{d^P + w + 1} \right] = \frac{\lambda_L (\lambda_L - 1)}{d^P + w + 1} < 0$, implying it is more difficult to satisfy (8) as $c$ becomes larger.

Cross-over voting by the poor. If a majority of districts are rich, then by (6) more poor districts support $P_R$ as it becomes easier to satisfy:

$$u(|z^P - \lambda_L|) - u(|z^P - \lambda_R|) < \frac{\lambda_R b(\lambda_R)}{d^P + w + 1}. \quad (9)$$

Analogously to the case of the rich voter, $P_L$-polarization can lead to more cross-over voting by the poor when the poor are ideologically centrist ($z^P \geq \lambda_L$), but not if the poor are ideologically extreme ($z^P < \lambda_L$). But unlike for the rich (where $P_L$-polarization always makes cross-over voting less attractive), $P_R$-polarization may make cross-over voting more attractive to the poor. Although such polarization yields worse ideological utility, it also yields more pork. $P_R$-polarization will
increase cross-over voting if:

\[ \frac{\partial}{\partial \lambda_R} \left[ u(|z^P - \lambda_R|) \right] + \frac{\partial}{\partial \lambda_R} \left[ \frac{\lambda_R b(\lambda_R)}{d^R + w + 1} \right] > 0. \]  

(10)

Since \( \frac{\partial}{\partial \lambda_R} \left[ u(|z^P - \lambda_R|) \right] < 0 \) and \( \frac{\partial}{\partial \lambda_R} \left[ \frac{\lambda_R b(\lambda_R)}{d^R + w + 1} \right] > 0 \), whether \( P_R \)-polarization will lead to more cross-over voting by the poor will depend on the functional form of \( u \). As ideological utility becomes less important to the poor, \( P_R \)-polarization is more likely to increase the propensity for cross-over voting.

Polarization, then, affects rich and poor voters differently when the other income group controls a majority of districts. For the rich when the poor control a majority of districts, \( P_L \)-polarization can never lead to more cross-over voting. For the poor when the rich control a majority of districts, \( P_R \)-polarization can lead to more cross-over voting because it increases the expected level of pork.

Another difference in cross-over voting by the rich and poor is that the poor may cross-over vote even if they control a majority of districts, but the rich will never cross-over vote if a majority of districts are rich. As noted in Proposition 1, if a majority of districts are poor but poor voters in pivotal districts prefer voting for \( P_R \), then cross-over voting will occur and \( P_R \) will win. It is therefore useful to consider what factors make it more or less likely that pivotal poor voters support \( P_R \) (i.e., that (5) is satisfied).

The number of poor voters, \( n^P \), is one factor that obviously affects the utility of cross-over voting for pivotal poor voters. Since \( P_L \) redistributes a greater proportion of the budget to the poor, an increase in the number of poor diminishes the value of supporting \( P_L \) and increases the relative value of cross-over voting. The model suggests, then, that in majoritarian systems with a large number of poor voters, some poor voters may obtain more resources from government if they support the right-wing parties. So doing allows them to share a relatively large amount of pork with a relatively small number of others. Such incentives may help explain why right-wing parties are often able to use patronage to become entrenched in relatively poor democracies. Such parties can use pork and lower taxes to construct majorities of the rich and a subset of the poor.

The exogenous constraint on government revenues, \( c \), which determines the effect of increasing
redistribution on the size of government, is another factor that affects cross-over voting incentives for poor voters in pivotal districts. The poor always prefer a smaller $c$ because as $c$ declines, additional taxes paid by the rich increase the size of government. But to determine the effect of $c$ on cross-over voting, we must consider whether changes in $c$ cause a bigger decrease in the utility of supporting $P_L$ versus $P_R$. To this end, let $m = \frac{n+1}{2}$ denote the size of the majority if a poor voter is in a pivotal district, and let $W = \frac{\lambda_j b(\lambda_j)}{m} + \frac{(1-\lambda_j)b(\lambda_j)}{n'}$ denote the aggregate transfers expected by a poor voter in a coalition of size $m$. Since $\frac{\partial W}{\partial c} = \frac{\lambda_j (\lambda_j - 1)}{m} - \frac{(\lambda_j - 1)^2}{n'} < 0$, an increase in $c$ will increase the relative value of cross-over voting by the pivotal poor if $\frac{\partial^2 W}{\partial c \partial \lambda_j} > 0$. That is, when $\frac{\partial^2 W}{\partial c \partial \lambda_j} > 0$, the decrease in utility from an increase in $c$ will be smallest when $\lambda_j$ is largest (and thus the decrease in utility is lower for $\lambda_R$ than for $\lambda_L$). Since $\frac{\partial^2 W}{\partial c \partial \lambda_j} = \frac{2\lambda_j - 1}{m} - \frac{2(\lambda_j - 1)}{n'}$, $\frac{\partial^2 W}{\partial c \partial \lambda_j} > 0$ for all parameter values. To see this, note that if $n^P \geq m$, then $\frac{\partial^2 W}{\partial c \partial \lambda_j} > 0$ if $\lambda_j > \frac{n^P - 2m}{2n' - 2m}$, which is always true given that $n^P - 2m < 0$. If $n^P < m$, then $\frac{\partial^2 W}{\partial c \partial \lambda_j} > 0$ if $\lambda_j < \frac{n^P - 2m}{2n' - 2m}$, which is always true given that $\frac{n^P - 2m}{2n' - 2m} > 1$.

Thus, as the budget constraint becomes stronger, the relative value to poor voters in pivotal districts of supporting $P_R$ increases. This is true because a strong constraint diminishes the relative value of $P_L$’s advantage on redistribution. If redistribution is funded by additional taxes, this makes $P_L$ more attractive to the poor. If redistribution is funded by taking away from pork, it is relatively less valuable, and $P_R$ therefore become more attractive. So the electoral advantage of the right-wing party should be largest among the poor when it is most difficult for left-wing parties to raise taxes to fund redistribution.

Finally, party polarization affects the value to a poor voter in a pivotal district of supporting $P_R$. The ideological considerations for cross-over voting are the same for poor voters in pivotal districts as for poor voters in non-pivotal districts, discussed above. Therefore, to understand the net effect of polarization on voting, we need to examine the how vote choice affects the pork/redistribution tradeoff. Remark 4 describes the circumstances under which party polarization leads to more crossover voting.

**Remark 4** Let $S = \frac{m(3-2\lambda_L)+n^P(2\lambda_L-2)}{2m(1-\lambda_L)+n'(2\lambda_L-1)}$. For a poor voter in a pivotal district, if $j = L, R$, $P_j$-polarization increases crossover voting for $P_R$ if and only if:
\[ \lambda_j \leq \frac{2n_P - 3m}{2n_P - 2m}, \text{ or} \]
\[ \lambda_j > \frac{2n_P - 3m}{2n_P - 2m} \text{ and } c > S. \]

**Proof.** Recall \( W = \frac{\lambda_j b(\lambda_j)}{m} + \frac{(1-\lambda_j) b(\lambda_j)}{n_P} \) is the aggregate transfer to a poor voter in a pivotal district. For \( j = L, R \), \( P_j \)-polarization increases support for \( P_R \) if \( \frac{\partial W}{\partial \lambda_j} > 0 \). We show that the conditions in the proposition are those that ensure \( \frac{\partial W}{\partial \lambda_j} > 0 \).

Note that \( \frac{\partial W}{\partial \lambda_j} > 0 \) requires
\[ c(2\lambda_j n^P + 2m - 2\lambda_j m - n^P) > -2\lambda_j m - 2n^P + 2\lambda_j n^P + 3m. \] (11)

To sign the derivative, it is useful to note the following
1. \( 2\lambda_j n^P + 2m - 2\lambda_j m - n^P \geq 0 \) if \( \lambda_j \geq \frac{n^P - 2m}{2n_P - 2m} \).
2. \( -2\lambda_j m - 2n^P + 2\lambda_j n^P + 3m > 0 \) if \( \lambda_j > \frac{2n^P - 3m}{2n_P - 2m} \).
3. \( m < n^P \) implies that \( -2\lambda_j m - 2n^P + 2\lambda_j n^P + 3m < 2\lambda_j n^P + 2m - 2\lambda_j m - n^P \).
4. \( \frac{2n^P - 3m}{2n_P - 2m} > \frac{n^P - 2m}{2n_P - 2m} \).

There are three cases to consider:

Case 1: \( \lambda_j < \frac{n^P - 2m}{2n_P - 2m} \). In this case, (11) is satisfied if \( c < S \) and the conditions imply \( S > 1 \), ensuring that the derivative is positive.

Case 2: \( \lambda_j \in \left[ \frac{n^P - 2m}{2n_P - 2m}, \frac{2n^P - 3m}{2n_P - 2m} \right] \). In this case, (11) is satisfied if \( c > S \) and the conditions imply that \( S < 0 \), ensuring that the derivative is positive.

Case 3: \( \lambda_j > \frac{2n^P - 3m}{2n_P - 2m} \). In this case, (11) is satisfied if \( c > S \) and the conditions imply that \( S < 1 \), so the derivative is positive only if \( c > S \).

Remark 4 shows that there exist non-trivial circumstances under which poor voters benefit from large \( \lambda_j \), and thus that as either party moves to the extreme (with \( P_L \) promising more redistribution and \( P_R \) promising more pork), the value of supporting the right-wing party increases.
4 Weak Parties

Strong parties control the distribution of pork, and thus create cross-over voting incentives by rich and poor. Individuals may vote for the “wrong” party — even when it gives them worse ideological policy, higher taxes (for the rich) and lower levels of redistribution (for the poor) — because so doing allows them to elect a legislator from the winning coalition, ensuring access to pork. We next consider voting behavior when parties are weak.

When parties are weak, the distribution of pork is not controlled by the majority party. Instead, there is an open bargaining process that allows all elected legislators an equal opportunity to gain pork for their districts. Voter strategies are therefore unaffected by incentives to support the candidate that will bring the most pork to the district—all individuals expect \( xb(x)/n \) in pork \( \text{ex ante} \) — and cross-over voting incentives therefore will not exist. Rich voters, for example, receive higher ideological utility from \( \lambda_R \), receive no money (but are taxed) from redistribution, and therefore have a weakly dominant strategy of voting for \( P_R \).

A poor voter in a district that is not pivotal will vote for \( P_L \) to secure the preferred ideological policy benefits from a friendly legislator. Finally, consider a poor voter’s calculation when her district is pivotal. Comparing expected utilities, a poor voter chooses \( P_L \) if:

\[
u(|z^P - \lambda_L|) + \frac{\lambda_L b(\lambda_L)}{n} + \frac{(1 - \lambda_L) b(\lambda_L)}{n^P} \geq u(|z^P - \lambda_R|) + \frac{\lambda_R b(\lambda_R)}{n} + \frac{(1 - \lambda_R) b(\lambda_R)}{n^P}.
\]

(12)

Since \( u(|z^P - \lambda_L|) \geq u(|z^P - \lambda_R|) \) and \( \lambda_L b(\lambda_L) \leq \lambda_R b(\lambda_R) \), the pivotal poor voter chooses \( P_L \) whenever \( n^P < n \), which is always true.

We summarize this result in Proposition 2.

**Proposition 2** When parties are weak, there is no cross-over voting in equilibrium: rich voters support \( P_R \) and poor voters support \( P_L \). ■

Propositions 1 and 2 together make a simple but important point in understanding how income and voting should be related across different types of party systems. If, as we assume, one role of disciplined parties is to control the distribution of government pork, then the linkage between
income and voting should be weaker in systems with strong parties. In such systems, both rich and (especially) poor voters will have incentives to cross-over and vote for the “wrong” party, but these incentives do not exist when parties are weak. This result therefore suggests the difficulty of passing large redistributive programs when parties are weak and poor voters are a minority. Under these conditions, redistribution might have to be less narrowly targeted toward the poor in order to win electoral support.

It is finally worth noting that from a poor voter’s perspective, the appeal of weak parties depends on the leftist party’s electoral prospects. Strong parties tend to benefit anticipated election winners, who will then benefit from the ability to exclude election losers from pork. In this environment, a poor district in an environment where a majority of districts are rich can receive pork benefits only at the expense of an ideologically undesirable representative. By contrast, weak parties tend to benefit anticipated election losers, and thus a poor voter in the position of Proposition 1(iii) can expect a positive share of the pork along with an ideologically compatible representative.

5 Extensions

5.1 Downsian Competition

We have assumed thus far that party platforms are exogenously fixed. While there are many good reasons for believing that this assumption should hold in the short run (for example, because of organizational recruitment), it is natural to consider whether the incentives posed by our model would in fact support divergent party platforms.

To analyze the implications of our model for party platform strategy, we consider a modified version of the strong parties game that begins with an initial step where both parties simultaneously chose $\lambda_L$ and $\lambda_R$. Parties have the simple Downsian motivation for maximizing the number of seats won. Since parties no longer have automatic ideological allies, we assume that poor and rich voters break ties in favor of $P_L$ and $P_R$, respectively.

Given the unique CPNE of Section 3, the extended game reduces to a simple, simultaneous-move game. Unfortunately, we cannot guarantee that pure strategy equilibria exist for all parameter
values. However, we are able to derive several simple statements that must hold in any pure strategy equilibrium.

First, the party that expects to lose in equilibrium must choose a platform equal to the ideal point of its “natural” constituents. This follows directly from expressions (6) and (7): a losing party will minimize the number of districts that cast cross-over votes (i.e., $\overline{w}$ and $\underline{w}$) by maximizing the policy utility of its base constituents. The winning party’s decision is more complex, as it balances its constituents’ utility over policy against that over pork and taxation or redistribution. Thus, we expect that losing parties will tend to be ideological “purists,” while winning parties are “compromisers.”

Second, under a wide range of parameter values, the election winner will not depend on the platform choices. Proposition 1 establishes that when $d_R \geq \frac{n+1}{2}$, $P_R$ must win. But if rich districts are not a majority, then it is possible that the winner will be endogenously determined by the platforms. The key relationship here is (5), which determines whether a pivotal poor district will cross-over to $P_R$. $P_R$ therefore has a clear incentive to choose $\lambda_R$ to satisfy (5), while $P_L$ would like to choose a $\lambda_L$ that would violate it.

In a pure strategy equilibrium, $P_L$ can ensure that (5) is never satisfied simply by choosing $\lambda_L = \lambda_R$. Since poor voters break ties in favor of $P_L$, this ensures that the only possible CPNE is one in which $P_L$ wins. Thus when a majority of districts are poor, $P_R$ must choose $\lambda_R = z^R$, and $P_L$ maximizes its seats won against that platform.

These observations are summarized in the following remark.

**Remark 5** In any pure strategy Nash equilibrium of the platform competition game, if $d_R \geq \frac{n+1}{2}$, then $P_R$ wins and $\lambda_L = z^P$, and if $d_R < \frac{n+1}{2}$, then $P_L$ wins and $\lambda_R = z^R$. □

This result suggests that endogenous platforms mitigate the possibility of the “perverse” cross-over equilibrium in which a polity of poor districts choose the right-wing party. This implies that from the perspective of a poor voter, strong but flexible (i.e., mobile) parties will typically have an advantage over weak but inflexible parties when poor districts are a majority, as they allow majority-poor districts to capture most pork benefits. They do not have this advantage when
poor districts are a minority, because they will be excluded from pork unless they choose a $P_R$ representative.

In a world with weak parties, there is a range of pure strategy equilibrium platform positions. One is simple Downsian convergence toward the ideal point of the more numerous district type. To see why, note that either party can win the election outright by choosing a platform closer this point. Under the tie-breaking assumption made above, both parties would then attract only the votes of their natural constituents. Note however that given the winning party’s platform, the losing party is then free to choose any platform that is closer to its natural constituents than the winning party’s platform. Thus there exists a wide range of divergent equilibrium platform configurations.

Interestingly, the weak party case suggests a platform-choice incentive that is opposite to that of the strong party case. In the latter, the anticipated loser tends to converge to its core constituents, while in the former, the anticipated winner converges to its core constituents with certainty. The welfare implications for poor voters are ambiguous. When poor districts are a majority, poor voters benefit ideologically but lose some pork under weak parties. When they are a minority, weak parties reverse these relationships.

5.2 Districting

In some environments, party officials may have control over the allocation of voters to legislative districts. In the U.S., for example, legislative districts in some states can drawn by partisan decision-makers with a preference for specific policy outcomes. It is therefore worth asking how the such a step would affect the results of the model. The extended game simply adds a first stage to the model, in which either $P_R$ or $P_L$ chooses how to allocate the two types of voters across the $n$ districts. As in the previous extension, parties are seat-maximizing.

Under strong parties, our analysis is simplified by the fact that the distribution of district types does not affect whether (5) is satisfied. Under strong parties, Proposition 1 therefore implies that if (5) is not satisfied, then parties are weakly better off by maximizing the number of districts in which their “natural” constituents (i.e., poor voters for $P_L$, rich voters for $P_R$). Note that if $w$ or $\bar{w}$ remained strictly positive under any redistricting plan, then parties would be just as well off
by ensuring that sympathetic districts are a majority. In this case, (6) and (7) imply that due to crossover voting, the ultimate size of the winning coalition is independent of the number of sympathetic districts. By contrast, under weak parties, districts vote “sincerely.” Without the help of crossover voting, parties are always interested in maximizing the number of districts controlled by natural constituents.

Under either assumption about party strength, partisan districting can have a pronounced effect on electoral outcomes. To achieve a majority of voters in a majority of districts, a party can achieve its goal with only $\frac{1}{2}\left(\frac{n+1}{2}\right)$ citizens who are natural constituents. The implications for partisan advantage are summarized by the next result.

**Remark 6** Suppose that $P_j$ allocates voters across districts. If parties are weak or parties are strong and (5) is not satisfied, then $P_R$ wins if $n^P < \frac{n+1}{4}$, $P_L$ wins if $n^R < \frac{n+1}{4}$, and $P_j$ wins otherwise.

Thus, for $n$ large, a party would need only about a quarter of the populace to assure itself of victory. Equivalently, only lopsided distributions of income types would result in electoral outcomes that were immune from the preferences of a districting official. While the ideological and redistributive effects of party control are the same under strong or weak parties, the pork effects are quite different. Because strong parties can control the allocation of pork more tightly, partisan districting can greatly affect the geographical distribution of public money when parties are strong. Note however that in practice there are a variety of constraints (e.g., geography, connectedness) on the ability of a districting official to allocate voters. These constraints would have the effect of raising the threshold of voters required to achieve a party’s majority of districts.

### 6 Discussion

When deciding how to vote, issues unrelated to economic well-being undoubtedly influence citizen choices. Our analysis, however, cautions against assuming that when voters support the “wrong” party on the redistribution issue, it is because the voters are emphasizing issues unrelated to economic well-being. Redistribution from rich to poor is but one way that governments distribute
tax revenues, and voters may maximize their economic well-being by supporting a party that is not their most-preferred on the issue of redistribution from rich to poor.

By focusing on pork-barrel politics, the model here explores one of the central ways that governments redistribute revenues on a basis unrelated to individual income. A central intuition that emerges from the model concerns the importance of party discipline. When parties are weak, voters expect the same level of pork no matter which party they support, and thus redistributive preferences are fundamental to vote choice. When parties are strong, by contrast, voters may need to weigh a trade-off between pork and income-based redistribution. The value of being included in the majority-controlled pork-coalition will often be decisive, resulting in cross-over voting. Our model therefore suggests that cross-over voting levels should be highest in systems where the majority party has opportunities and incentives to concentrate the distribution of pork in the districts that support this majority party.

Since party discipline affects cross-over voting incentives, it also affects party competition and electoral districting. In systems with strong parties, parties will not converge to the median voter in a pure strategy equilibrium because the party that expects to lose at the national level will adopt the ideological policy most preferred by its constituents. When parties are weak, by contrast, the absence of cross-over voting incentives may lead parties to converge to the preferences of the median district. Similarly, both types of parties are interested in concentrating supporters in sympathetic districts in order to maximize their legislative majority. The incentives for doing so may be mildly weaker under strong parties, however, due to the possibility of attracting crossover voters.

The analysis also brings into sharp relief an advantage that rich voters and right-wing parties should have in majoritarian systems with strong parties. When voters have pork-based incentives to elect a legislator from the expected majority party, the right-party benefits because they offer more pork and less redistribution. As a result, rich voters in pivotal districts never support left-wing parties, but poor voters in pivotal districts may support right wing parties. The incentives of poor voters to do so increases as the number of poor voters increases and as constraints on the government budget increase. Party-system polarization will often also lead to more cross-over voting by the poor. These cross-over voting incentives by the poor imply the majorities by the right-
wing party should be larger than majorities for the left-wing party, and that there should therefore be asymmetries in partisan swings, with swings to the right party producing larger majorities than swings to the left-party.

Two limitations of our model suggest avenues for further research. First, the model treats party strength as exogenous. As the analysis above suggests, party strength has implications for party strategies, and it would therefore be interesting to consider ways in which party strength might evolve in response to the electoral incentives posed in our model. Second, the model assumes that eligibility for redistribution — or the identity of the “poor” — is exogenous. Of course, many distributive programs have a strong middle-class component, and politicians determine the cutoff for what constitutes a “poor person.” And many programs are quasi-redistributive, such as social insurance or tax deductions. Our model suggests that under some distributions of voters, such programs might prove to be electorally beneficial when compared to strictly redistributive programs. Analyzing endogenous determination of eligibility of for redistribution programs, or endogenous selection of quasi-redistributive programs would require a richer set of assumptions about voter types, but may make it possible to understand how politicians create electoral coalitions that transcend economic groups.

7 References


