The Simple Theory of Temptation and Self-Control†

Faruk Gul

and

Wolfgang Pesendorfer

Princeton University

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Abstract

We analyze a two period model of temptation for a finite choice setting. We formalize the idea that temptation depends only on the most tempting alternatives and provide two representations of such preferences. The representation is an ordinal analogue of the self-control preferences in Gul and Pesendorfer (2001).

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1. Introduction

In this paper, we analyze a two period model of temptation. In contrast to earlier work which analyzes preferences over lotteries, we consider a setting with an arbitrary finite set of alternatives. Our goal is to formulate a simple model of temptation for that framework.

Following Kreps (1979), we analyze preferences over sets of alternatives. The interpretation is that the agent must take an action in period 0 that constrains the feasible choices in period 1. Period 0 behavior is described by a preference over sets of alternatives.

We consider individuals who may benefit from commitment, i.e., adding an alternative to a set may make the agent strictly worse off. In our model that occurs if the added alternative is a *temptation* that alters choice behavior or requires costly self control. If $A \succ A \cup \{x\}$ we say that x is more tempting than y for $y \in A$. This definition assumes that only the most tempting alternatives in a set can reduce the welfare of the agent. We assume that "more tempting" is an acyclic relation, i.e., if x_n is more tempting than x_{n+1} for $n = 1, \ldots, N$ then x_{N+1} is not more tempting than x_1. Acyclicity is equivalent to the assumption that there are maximally tempting alternatives in every set. Hence, our model of temptation can be paraphrased as *"only the most tempting alternatives matter."*

Suppose adding x to a set A makes the agent strictly better off. We interpret this to mean that x is the unique optimal choice (in period 1) from the set $A \cup \{x\}$. This interpretation assumes that the agent has no "preference for flexibility" as analyzed in Kreps (1979) and Dekel, Lipman and Rusticchini (2001).[1] We say that x is a better choice than y if $A \cup \{x\} \succ A$ for some A that contains y and assume that "better choice" is an acyclic binary relation. Hence, we assume that there exist second period choice function that are consistent (i.e., maximize) the better-choice relation inferred from period 0 behavior.

Our assumptions yield the following representation. There are utility functions w, v and an aggregator u such that

$$W(A) = u(\max_{x \in A} w(x), \max_{y \in A} v(y))$$

[1] Kreps (1979) and Dekel, Lipman and Rustichini (2001) analyze models where agents are uncertain in period 0 about their period 1 preference. In that case, $A \cup \{x\} \succ A$ does not imply that x is the only possible choice in period 1.

represents the preference. The aggregator u is non-decreasing in the first argument and non-increasing in the second argument. We also provide a stronger axiom that guarantees that u is strictly increasing in its first and strictly decreasing in its second argument. We can interpret the utility function w as representing the optimal choice in the second period and the utility function v as representing the temptation ranking of alternatives.

For a setting with lotteries, Gul and Pesendorfer (2001) obtained the representation

$$W(A) = \max_{x \in A}(u(x) + v(x)) - \max_{y \in A} v(y)$$

Setting $w(x) = U(x) + V(x)$ and $v(x) = V(x)$ it is easily seen that the representation here generalizes the earlier representation. In particular, the representation derived in this paper allows for a non-additive aggregator u.

In Gul and Pesendorfer (2001) we assume that the objects are lotteries and that the agent is an expected utility maximizer. The expected utility hypothesis together with set-betweenness ($A \succeq B$ implies $A \succeq A \cup B \succeq B$) are shown to imply the representation above. The assumption of acyclic temptation and choice is stronger than set-betweenness. Hence, to get an ordinal version of this result we must strengthen set-betweenness to acyclic choice and temptation.

Strotz (1956) analyzes a model of consistent planning when tastes are changing over time. In Gul and Pesendorfer (2005) we show that, in a two-period setting, Strotz' formulation is equivalent to axiom NC below:

$$A \sim A \cup B \text{ or } B \sim A \cup B \tag{NC}$$

Axiom NC is stronger than acyclic choice and temptation and therefore, the model analyzed here includes Strotz' model as a special case.

In related work, Dekel, Lipman and Rustichini (2005) provide a generalization of Gul and Pesendorfer (2001) in the lottery setting. Their model maintains the expected utility hypothesis but allows for the possibility that utility is lowered by alternatives other than the most tempting ones. By contrast, this paper allows for a general (finite) choice structure but maintains the assumption that only the most tempting alternative can lower the utility of a set.

2. Revealed Choice and Revealed Temptation

Let X denote the finite nonempty set of alternatives and $\mathcal{K}$ denote the set of all nonempty subsets of X. The individual is identified with a preference relation $\succeq$ on $\mathcal{K}$. That is, $\succeq$ is complete and transitive. The preference $\succeq$ represents the individuals ranking of choice problems (in period 0) with the understanding that (in period 1) one alternative from the set must be chosen for consumption.

We model a decision-maker who must deal with temptations. This means that adding an alternative x to a choice problem A may make the agent strictly worse off. If $A \succ A \cup \{x\}$ we conclude that x is more tempting than $y \in A$. If adding an alternative makes the agent better off, i.e., $A \cup \{x\} \succ A$, we conclude that x will be chosen from $A \cup \{x\}$ and that x is a better choice than $y \in A$.

Definition: *(i) The element $x \in X$ is a better choice than $y \in X$ ($x \succ_c y$) if there exists $A \in \mathcal{K}$ such that $y \in A$ and $A \cup \{x\} \succ A$. (ii) The element $x \in X$ is more tempting than $y \in X$ ($x \succ_t y$) if there exists $A \in \mathcal{K}$ such that $y \in A$ and $A \succ A \cup \{x\}$.*

Axiom A below requires that the binary relations $\succ_c$ and $\succ_t$ be acyclic. A binary relation $\succ^*$ on X acyclic if for any $x_1, \ldots, x_n$, $x_i \succ^* x_{i+1}$ for $i = 1, \ldots, n-1$ implies $x_n \not\succ^* x_1$.

Axiom A: *The binary relations $\succ_c$ and $\succ_t$ are acyclic.*

We say that $U : \mathcal{K} \to I\!R$ is a temptation-self-control utility (TSU) if there exists (u, v, w) such that $v : X \to I\!R$, $w : X \to I\!R$ and $u : w(X) \times v(X) \to I\!R$ where u is non-decreasing in its first argument, non-increasing in its second, and

$$U(A) = u(\max_{x \in A} w(x), \max_{y \in A} v(y))$$

We say that $\succeq$ has a TS preference if only if it can be represented by a TSU. When convenient, we identify a TSU with the corresponding (u, w, v) and write $U = (u, w, v)$. We sometimes write $\max w(A), \max v(A)$ rather than $\max_{x \in A} w(x), \max_{y \in A} v(y)$.

Theorem 1: *The preference relation $\succeq$ satisfies axiom A if and only if it is TS preference.*

Let (u, v, w) be a TSU. The utility function w can be interpreted as a possible second period objective function since $x \succ_c y$ implies $w(x) > w(y)$. Hence, whenever choice can be inferred from period 0 behavior it follows that the choice maximizes w. Similarly, we can interpret v to be a possible representation of the temptation ranking since $x \succ_t y$ implies $v(x) > v(y)$.

Note that the representation need not be strictly monotone. Hence, it may occur that $U(A \cup \{x\}) = U(A)$ with $w(x) > \max w(A)$ and $v(x) \leq \max v(A)$. In this case, the period 0 preference suggests that the optimal choice from A remains optimal when x is added but the function w has x as the unique maximizer from $A \cup \{x\}$. Hence, w does not capture all possible optimal choices but rather a selection of optimal choices. In the next section, we provide a stronger axiom that yields a strictly monotone TSU representation. In that case, the utility functions w and v can be interpreted as the choice and temptation utilities.

Suppose the agent prefers x to y if he is committed to a single alternative, i.e., $U(\{x\}) > U(\{y\}$. In that case, weak monotonicity of the representation implies that $w(x) > w(y)$ or $v(y) > v(x)$. Thus, if commitment to x is preferred to commitment to y then either x is chosen over y or y is chosen and y is more tempting than x. Hence, if choice behavior does not maximize the commitment preference it follows that the chosen alternative (the $w-$ maximizer) is more tempting than the best alternatives for the commitment preference.

The proof of Theorem 1 uses the following two Lemmas which are proven in the appendix.

Lemma 1: *If $\succ^*$ is an acyclic binary relation on X then there exists a function $f : X \to \mathbb{R}$ such that $x \succ^* y$ implies $f(x) > f(y)$.*

Lemma 2: *Let $N = \{1, \ldots, n\}, M = \{1, \ldots, m\}$. Let $D \subset M \times N$ and let $f : D \to \mathbb{R}$ be strictly increasing (non-decreasing). Then, there exists a strictly increasing (non-decreasing) function $F : M \times N \to \mathbb{R}$ with $f(i, j) = F(i, j)$ for $(i, j) \in D$.*

Proof of Theorem 1: To prove that the axioms are necessary for the representation, assume that such u, w, v exist. First, we show that $x \succ_c y$ implies $w(x) > w(y)$. To see this, note that $w(y) \geq w(x)$ then for all A such that $y \in A$, $\max w(A \cup \{x\}) = \max w(A)$ and

$\max v(A \cup \{x\}) \geq \max v(A)$. Therefore, since u is nondecreasing in its second argument, we have $u(\max w(A \cup \{x\}), \max v(A \cup \{x\})) \leq u(\max w(A), \max v(A))$ whenever $y \in A$. Hence, $x \not\succ_c y$. Now, suppose for some $x_1, \ldots, x_n$ we have $x_i \succ_c x_{i+1}$ for all $i = 1, \ldots, n-1$. Then, the above argument ensures that $w(x_i) > w(x_{i+1})$ for all $i = 1, \ldots n-1$ and therefore $w(x_1) > w(x_n)$, which again by the above argument ensures that $x_n \not\succ_c x_1$. The proof of the acyclicity of $\succ_t$ follows from a symmetric argument.

Next, we prove that the axioms imply the representation. Define $w : X \to I\!R, v : X \to I\!R$ such that $x \succ_c y$ implies $w(x) > w(y)$ and $x \succ_t y$ implies $v(x) > v(y)$. Lemma 1 implies that such functions w, v exist. Without loss of generality assume that $w(X) = \{1, \ldots, n\}, v(X) = \{-n, \ldots, -1\}$.

Claim 1: $A \sim \{x\} \cup \{y\}$ if $x, y \in A$ such that $w(x) = \max w(A)$ and $v(y) = \max v(A)$. To prove claim 1 let $B = \{x\} \cup \{y\}$ for $x, y \in A$ such that $w(x) = \max w(A)$ and $v(y) = \max v(A)$. If $B = A$ there is nothing to prove. Otherwise, note that by our choice of w and v it follows that $z_n \not\succ_c x$ and $z_n \not\succ_t y$ for all $z_n \in \{z_1, \ldots, z_N\} = A \backslash B$. Hence, $B \sim B \cup \{z_1\}$. Continuing in this fashion we get $B \cup \{z_1\} \sim B \cup \{z_1, z_2\}$ etc. Therefore $A \sim B$.

Claim 2: $A \succeq B$ if $\max w(A) \geq \max w(B)$ and $\max v(A) \leq \max v(B)$.
To prove claim 2, let $x \in A$ be such that $w(x) = \max w(A)$ and let $y \in B$ be such that $v(y) = \max v(B)$. Since $w(x) \geq w(y)$ it follows that $y \not\succ_c x$ and hence $A \succeq A \cup \{y\}$ and since $v(x) \leq v(y)$ it follows that $x \not\succ_t y$ and hence $B \cup \{x\} \succeq B$. By Step 1 we have $A \cup \{y\} \sim B \cup \{x\} \sim \{x\} \cup \{y\}$ and therefore

$$A \succeq A \cup \{y\} \sim B \cup \{x\} \succeq B$$

which proves Claim 2.

Let $g : \mathcal{K} \to M \times N$ be defined as $g(A) = (\max w(A), \max v(A))$ and let $D := g(\mathcal{K}) \subset M \times N$ be the set of values attained by g. Let $U : \mathcal{K} \to I\!R$ represent the preference $\succeq$. Claim 2 implies that $U(A) = U(B)$ if $\max w(A) = \max w(B)$ and $\max v(A) = \max v(B)$. Therefore, we can define $f : D \to I\!R$ by

$$f(\max w(A), -\max v(A)) = U(A)$$

Note that f is non-decreasing by Claim 2 above and therefore Lemma 2 implies that we can extend f to a non-decreasing function F on $M \times N$. Define $u : M \times N \to I\!R$ by $u(i, -j) := F(i, j)$ and note that $U(A) = f(i, j) = u(i, -j)$ for $(\max w(A), \max v(A)) = (i, -j)$. Hence, (u, v, w) is a TSU representation. $\square$

There are typically multiple, ordinally not equivalent, representations (u, v, w) for a single preference $\succeq$. Theorem 2 below provides a minimal uniqueness result. If (u, v, w) is a TS representation of $\succeq$ then $x \succ_c y$ implies $w(x) > w(y)$ and $x \succ_t y$ implies $v(x) > v(y)$. Hence, the utility functions w, v must represent the relations choice and temptation relations. Theorem 2 also demonstrates the extent of the non-uniqueness: any pair of utility functions w, v such that w represents $\succ_c$ and v represents $\succ_t$ is part of a representation for some aggregator u.

Theorem 2: *Let $\succeq$ a preference relation and $w, v : X \to I\!R$. There exists $u : w(X) \times v(X) :\to I\!R$ such that the TSU (u, w, v) represents $\succeq$ if and only if $x \succ_c y$ implies $w(x) > w(y)$ and $y \succ_t x$ implies $v(y) > v(x)$.*

Proof: To see necessity, note that $x \succ_c y$ implies $U(A \cup \{x\}) > U(A)$ for some A containing y. This in turn implies $w(x) > w(y)$ for any TSU representing $\succeq$. Similarly, $x \succ_t y$ implies $U(A) > U(A \cup \{x\})$ for some A containing y and therefore $v(x) > v(y)$.

To prove sufficiency, note that the only role of Axiom A in the proof of the representation was to yield w, v such that $x \succ_c y$ implies $w(x) > w(y)$ and $y \succ_t x$ implies $v(y) > v(v)$. Hence, starting with any w, v that has these properties the arguments in the proof of Theorem 1 yield the desired u and establish that (u, w, v) is a TSU that represents $\succeq$. $\square$

3. A Strict Representation

Let $C : \mathcal{K} \to \mathcal{K}$ and $T : \mathcal{K} \to \mathcal{K}$ be two choice functions, that is, $C(A) \subset A, T(A) \subset A$. We will interpret $C(A)$ as the set of chosen elements from A and we will interpret $T(A)$ as the set of most tempting alternatives.

A choice function $F : \mathcal{K} \to \mathcal{K}$ satisfies Houthakker's axiom if $x \in F(A) \cap B$ and $y \in A \cap F(B)$ implies $x \in F(B)$. It is well known that a choice function F satisfies Houthakker's axiom if and only if F maximizes some utility function on X.

Property 1: *C and T satisfy Houthakker's Axiom.*

Property 2 says that if B contains a most tempting element from $A \cup B$ then $A \cup B$ is weakly preferred to B. If, in addition, no optimal choice from $A \cup B$ is in B then this preference is strict.

Property 2: *$T(A \cup B) \cap B \neq \emptyset$ implies $A \cup B \succeq B$; if also $C(A \cup B) \cap B = \emptyset$ then $A \cup B \succ B$.*

Property 3 says that if an optimal choice from $A \cup B$ is in A then A is weakly preferred to $A \cup B$. If, in addition, A does not contain a most tempting element then this preference is strict.

Property 3: *$C(A \cup B) \cap A \neq \emptyset$ implies $A \succeq A \cup B$. If also $T(A \cup B) \cap A = \emptyset$ then $A \succ A \cup B$.*

Axiom B: *There exist C, T such that $(\succeq, C, T)$ satisfy Properties 1-3.*

We say that $\succeq$ has a strict TSU representation if there exist (u, v, w) such that $v : X \to I\!R, w : X \to I\!R, u : w(X) \times v(X) \to I\!R$ with u strictly increasing in its first argument and strictly decreasing in its second argument, such that

$$U(A) := u(\max_{x \in A} w(x), \max_{y \in A} v(y))$$

represents $\succeq$.

Theorem 3: *The preference $\succeq$ satisfies Axiom B if and only if $\succeq$ has a strict TSU representation.*

Proof: Let (u, v, w) be a strict TSU representation. Then it is straightforward to verify that $(\succeq, C, T)$ satisfy properties 1-3 where

$$\begin{aligned} C(A) &= \{x \in A | w(x) \geq w(y) \forall y \in A\} \\ T(A) &= \{x \in A | v(y) \geq v(y') \forall y' \in A\} \end{aligned} \tag{5}$$

Hence, it remains to prove the existence of a strict TSU representation if $\succeq$ satisfies Properties 1-3.

Let $w, v : X \to I\!R$ be utility functions that satisfy (5).

Claim 3*(i) If x, y are such that $w(x) = \max w(A), v(y) = \max v(A)$ then $\{x, y\} \sim A$. (ii) If $\max w(A) \geq \max w(B), \max v(B) \geq \max v(A)$ the $A \succeq B$. If one of these inequalities is strict then $A \succ B$.*

To prove part (i) of Claim 3 let $B = \{x, y\}$. Note that by Properties 2 and 3, $B \succeq A \cup B = A \succeq B$ and therefore $A \sim B$. To prove part (ii) of Claim 3, note that Property 2 implies that $A \cup B \succeq B$ and property 3 implies that $A \succeq A \cup B$. Therefore, $A \succeq B$. The strict version follows from the second parts of properties 2 and 3.

Since $\succeq$ is complete and transitive we can represent it by a function $U : \mathcal{K} \to I\!R$. Without loss of generality choose m, n so that $w : X \to \{1, \ldots, m\}, v : X \to \{-n, \ldots, -1\}$ and v, w are onto. Let $D \subset M \times N$ be such that $(\max w(A), -\max v(A)) \in D$ for some $A \in \mathcal{K}$. Define $f : D \to I\!R$ by

$$f(i, j) = U(A)$$

for A such that $(\max w(A), -\max v(A)) = (i, j)$. By Claim 3 f is well defined and strictly increasing. Therefore, we can apply Lemma 2 to yield a strictly increasing function $F : M \times N \to I\!R$ that coincides with f on D. Set $u(i, j) = F(i, -j)$ and note that $U(A) = f(i, j) = u(i, -j)$ for $(\max w(A), \max v(A)) = (i, -j)$. Hence, (u, v, w) is a strict TSU representation. $\square$

4. Related Representations

In Gul and Pesendorfer (2001) we analyze preferences over sets of lotteries and provide axioms for a representation of the form:

$$W(A) = \max_{x \in A} U(x) + V(x) - \max_{y \in A} V(y)$$

We refer to preferences that have such a representation as *self-control preferences.* Note that self-control preferences are a special case of TS preferences. To see this, set $w = U + V$, $v = V$ and $u = w - v$. Hence, TS preferences generalizes our earlier model by allowing for a non-additive aggregator of choice utility (w) and temptation utility (v).

In Gul and Pesendorfer (2001) we showed that the above representation obtains if preferences satisfy continuity, a version of the independence axiom and the *Set Betweenness* axiom below.

Axiom C: *(Set Betweenness) The preference relation $\succeq$ satisfies Set Betweenness if $A \succeq B$ implies $A \succeq A \cup B \succeq B$ for all $A, B \in \mathcal{K}$.*

Theorem 4 shows that axiom A (and therefore also axiom B) is a stronger than set betweenness.

Theorem 4: *For any preference $\succeq$, axiom A implies axiom C but not the converse.*

Proof: Using the representation, it is straightforward to verify that a TS preference satisfies Set-Betweenness. Let $M_f(A) = \{x \in A \,|\, f(x) \geq f(y) \forall y \in A\}$. Take any TSU (u, w, v). Then, $M_u(A \cup B) \cap A$ implies $A \succeq A \cup B$ and $M_v(A \cup B) \cap A$ implies $A \cup B \succeq A$. Hence, by Theorem 1, Axiom A implies Set Betweenness.

For the failure of the converse consider the following example. Let $X = \{x, y, z\}$. Let $\{x\} \succ \{x, y\} \succ \{y\} \succ \{y, z\} \sim \{x, y, z\} \succ \{z\} \sim \{x, z\}$. To verify that this preference relation satisfies Set Betweenness is straightforward. Note that $x \succ_c y$ since $\{x, y\} \succ \{x\}$. On the other hand, $y \succ_c x$ since $\{x, y, z\} \succ \{x, z\}$. Thus, the example violates Axiom A.$\square$

Next, consider Strotz' model of changing tastes. A Strotz representation obtains if there are functions U, V such that the function W defined by

$$W(A) = \max_{x \in M_V(A)} U(x)$$

represents $\succeq$, where $M_V(A) = \{x \in A \,|\, V(x) \geq V(y) \forall y \in A\}$. The utility function W describes a decision maker who chooses (in period 1) to maximize V but evaluates these choices according to U. Ties are broken in favor of U. The above describes Strotz' model of consistent planning with changing utility functions. In Gul and Pesendorfer (2005) we show that Strotz' model of changing tastes obtains if the preference satisfies the following *"No Compromise"* axiom.

Axiom D: *(No Compromise) $A \sim A \cup B$ or $B \sim A \cup B$ for all $A, B \in \mathcal{K}$.*

Theorem 5 shows that Axiom B (and hence also Axiom A) is a weakening of the Axiom D. Hence, strict TS preferences include Strotz' model.

Theorem 5: *For any preference $\succeq$ axiom D implies axiom B but not the converse.*

Proof: Gul and Pesendorfer (2005) show that if $\succeq$ is a preference relation and satisfies No Compromise, it has a Strotz representation; that is, there are functions U, V such that the function W defined by

$$W(A) = \max_{x \in M_V(A)} U(x)$$

represents $\succeq$, where $M_V(A) = \{x \in A \,|\, V(x) \geq V(y) \forall y \in A\}$. Assume without loss of generality that $U(x) > 0$ for all $x \in X$. Let $\epsilon = 1$ if V is constant. Otherwise, let $\epsilon = \min\{|V(x) - V(y)| \,|\, |V(x) - V(y)| > 0, x, y \in X\}$. Choose $K > 0$ such that $|U(x) - U(y)|/K < \epsilon$ for all $x, y \in X$. Let $w(x) = v(x) = V(x) + \frac{U(x)}{K}$ for all $x \in X$. It is easy to see that $x \succeq_c y$ iff $w(x) \geq w(y)$ and $y \succeq_t x$ iff $v(x) \geq v(y)$ satisfy Properties 1-3.

Let $X = \{x, y\}$ and $\{x\} \succ \{x, y\} \succ \{y\}$. Then, $\succeq$ satisfies axiom B but fails axiom D. $\square$

5. Appendix

Lemma 1: *If $\succ^*$ is an acyclic binary relation on X then there exists a function $f : X \to I\!R$ such that $x \succ^* y$ implies $f(x) > f(y)$.*

Proof: Define $Y_0 = \emptyset$, $X_n = \{x \in X \backslash Y_{n-1} \,|\, z \succ^* x$ implies $x \in Y_{n-1}$ and $Y_n = Y_{n-1} \cup X_n$ for $n > 1$. Note that by acyclicity and the finiteness of X, $X_n \neq \emptyset$ whenever $Y_n \neq X$. Hence, there exists a finite N such that $X_N \neq \emptyset$ and $Y_n = X$. It follows that for every $x \in X$ there exists a unique n such that $x \in X_n$. Define $f(x) = -n$ for this n. Now, suppose $x \succ^* y$ and $f(x) = -n$. It follows that $x \notin Y_i$ for all $i < n$. Therefore $y \notin X_i$ for all $i \leq n$ and hence $f(y) = -i$ for some $i > n$; that is, $f(y) < f(x)$. □

Lemma 2: *Let $N = \{1, \ldots, n\}, M = \{1, \ldots, m\}$. Let $D \subset M \times N$ and let $f : D \to I\!R$ be strictly increasing (non-decreasing). Then, there exists a strictly increasing (non-decreasing) function $F : M \times N \to I\!R$ with $f(i, j) = F(i, j)$ for $(i, j) \in D$.*

Proof: We first prove the Lemma for the strictly increasing case. If f takes on a single value, define $\epsilon = 1$. Otherwise, let ϵ be the minimal non-zero difference between two values of f. Let $\bar{f}$ be the maximal value of f. For $(i, j) \in M \times N$, let $D_{ij} = \{(i', j') \in D | i' \geq i, j' \geq j\}$ and let $\kappa_{ij} = \min_{j'} \min_{i'} \{(i', j') | (i', j') \in D_{ij}\}$, $\tau_{ij} = \min_{i'} \min_{j'} \{(i', j') | (i', j') \in D_{ij}\}$.

For $(i, j) \in M \times N$ define

$$G(i,j) = \begin{cases} \bar{f} + \epsilon \cdot i/m & \text{if } D_{ij} = \emptyset \\ f(i,j) & \text{if } (i,j) \in D \\ f(\kappa_{ij}) - \epsilon + \epsilon i/m & \text{otherwise} \end{cases}$$

and let

$$H(i,j) = \begin{cases} \bar{f} + \epsilon \cdot j/n & \text{if } D_{ij} = \emptyset \\ f(i,j) & \text{if } (i,j) \in D \\ f(\tau_{ij}) - \epsilon + \epsilon j/n & \text{otherwise} \end{cases}$$

Finally, define

$$F(i,j) = \tfrac{1}{2} G(i,j) + \tfrac{1}{2} H(i,j)$$

We will show that G is strictly increasing in its first argument and weakly increasing in its second argument. A symmetric argument for H then proves the lemma.

Let $(i,j),(k,j) \in M \times N$ with $k > i$. Note that $D_{kj} = \emptyset$ implies that $G(k,j) = \bar{f} + \epsilon k/m > \bar{f} + \epsilon i/m \geq G(i,j)$. Since $D_{kj} \subset D_{ij}$ it follows that we are done if either D_{kj} or D_{ij} are empty. Therefore, assume that D_{ij} and D_{kj} are non-empty. In that case, κ_{ij} and κ_{kj} are well defined. If $\kappa_{kj} > \kappa_{ij}$ then $f(\kappa_{kj}) \geq f(\kappa_{ij}) + \epsilon$ and therefore $G(k,j) \geq f(\kappa_{ij})k/m + \epsilon > f(\kappa_{i,j}) \geq G(i,j)$. If $\kappa_{kj} = \kappa_{ij}$ then $G(i,j) = f(\kappa_{ij}) - \epsilon + i/m$ and the result follows since $G(k,j) \geq f(\kappa_{ij}) - \epsilon + k/m$.

Finally, let $(i,j),(i,k) \in M \times N$ with $k > j$. Note that $D_{ik} \subset D_{ij}$ and therefore $\kappa_{ik} \geq \kappa_{ij}$. If $\kappa_{ij} = \kappa_{ik}$ then $G(i,j) = G(i,k)$. If $\kappa_{ik} > \kappa_{ij}$ then $G(i,k) \geq f(\kappa_{ik}) - \epsilon + 1/m > f(\kappa_{ij}) \geq G(i,j)$.

If f is non-decreasing then define

$$F = \begin{cases} \bar{f} & \text{if } D_{ij} = \emptyset \\ f(i,j) & \text{if } (i,j) \in D_{ij} \\ f(\kappa_{ij}) & \text{otherwise} \end{cases}$$

The function F is non-decreasing since $D_{ij} \subset D_{i'j'}$ for $i \geq i', j \geq j'$. □

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