

Princeton/Stanford Working Papers in Classics

CHAPTER 1 of *The City-State Commensurate: Plato and Pythagorean Political Philosophy*: “Aristotle’s Description of Mathematical Pythagoreanism in the 4th Century BCE”

Version 1.0

May 2010

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Abstract: Scholars of the history of ancient philosophy have been hesitant to attribute particular characteristics to those Pythagoreans called “mathematical” by Aristotle. Aristotle himself, to be sure, not only felt it important to distinguish this type of Pythagorean from the more traditional “acousmatic” type, but he also invested in this distinction the basic tenets of his own philosophical methodology regarding the pursuit of knowledge from first principles. In this chapter, I describe the philosophical system (*pragmateia*) of the mathematical Pythagoreans by analyzing and comparing the accounts of Pythagoreanism in both the surviving treatises of Aristotle (especially *Metaphysics*) and the fragmentary works on the Pythagoreans preserved in Iamblichus’ *On the General Mathematical Science* and *On the Pythagorean Way of Life*. This is the newest version of the first chapter of a book-length study in which I describe the philosophical and political history of the mathematical Pythagoreans and their influence on Plato’s later thought.

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CHAPTER 1: ARISTOTLE'S DESCRIPTION OF MATHEMATICAL PYTHAGOREANISM IN THE 4TH CENTURY BCE

In order to define the influence of the mathematical Pythagoreans on Plato's philosophy, we must first describe the philosophical system of the mathematical Pythagoreans. We cannot advance upon this problem without investigating the complex record of their philosophical and political activities, preserved by ancient philosophers, doxographers, and historians, which spans Classical Greece to Late Antiquity. It goes without saying that this challenge is indeed daunting: we are required to classify and evaluate evidence that is often fragmentary, selective, and biased. It may be ill-founded as well. Of course, in pursuit of an understanding of mathematical Pythagoreanism, we need to evaluate not only the information, but also the reliability of the evidence that survives. As Charles Kahn fittingly notes, the history of Pythagorean philosophy is indebted to analysis of written documents.¹ This presents a problem for us, since a primary mode of transmission of wisdom among archaic philosophical communities is likely to have been oral. Most of the information transmitted orally has been lost, but our written sources – in spite of the complex interpretive problems that attend them – are still of great value in the reconstruction of an early history of Pythagoreanism. Such written documents, often fragmentary, come in the form of philosophical dialogues (such as those of Plato and the Academy, Heraclides of Pontus, Aristotle, and even Cicero), the philosophical treatises of Aristotle and his school (notably Theophrastus, Aristoxenus, Dicaearchus, and Eudemus of Rhodes), doxographies (such as those transmitted by Aetius and Pseudo-Plutarch), the Middle and Neo-Platonic texts (such as those of Plutarch, Nicomachus, Iamblichus, and Porphyry), and

¹ Kahn 2001: 23.

even 4th Century BCE literary texts (such as comedies and tragedies, speeches of the Attic orators, etc.).² Our knowledge of the history of Pythagoreanism can also be deduced, importantly, from other kinds of written sources, such as the historical writings of Timaeus of Tauromenium, Polybius, Diodorus Siculus, and Strabo, and illumination of this source material is made possible, in some cases, by appeal to material evidence, especially archaeological and epigraphical.³ The multiple accounts of historical Pythagoreanism that survive are reflected in the complexity of the various modes of evidentiary transmission. Because we are forced to deal with such a variety of source materials and evidence, scholars of Pythagoreanism must be “polymaths” of the sort that was insultingly ascribed to Pythagoras by Heraclitus; we can only hope, however, that such polymathy (πολυμαθία) does not produce the “artful knavery” (κακοτεχνία) that Heraclitus associates with such an inquiry (ἵστορίη).⁴

In this first chapter, we will attempt to describe the kinds of Pythagoreans that may have existed and their activities by appeal to the evidence preserved in both the Corpus and fragments of Aristotle. Our goal is to identify the characteristics that distinguished the mathematical Pythagorean *pragmateia*, which we may tentatively describe here (for Aristotle) as both the object of a philosophical inquiry and the treatment of that same object, from the *pragmateia* of the rival acousmatic Pythagorean brotherhood in Magna Graecia during the late 6th and 5th Centuries BCE. Our claim is that Aristotle, especially in *Metaphysics A* and the lost writings on the Pythagoreans (preserved in a fragmentary state without significant modifications in Iamblichus’ *On the General Mathematical Science*), establishes this distinction by appeal to the different philosophical methodologies of each group: the mathematical Pythagoreans, who are

² The most comprehensive study of the doxography of the Pythagoreans remains Burkert 1972, but a better organized discussion and summary of the material is Riedweg 2005.

³ Musti (2005: 103-203) provides an excellent example of how integration of historical, archaeological and epigraphical material can help us to construct a history of Pythagoreanism.

⁴ DK 22 B129 = D.L. 8.6.

the same as the “so-called Pythagoreans” in *Metaphysics A*, employ superordinate mathematical sciences in establishing demonstrations that explain the “reason why” (τὸ δῖότι) they hold their philosophical positions, whereas the acousmatic Pythagoreans, who are distinguished from the “so-called Pythagoreans” in *Metaphysics A*, only appeal to basic, empirically derived “fact” (τὸ ὄτι) in defense of their doctrines. Furthermore, we suggest, Aristotle criticizes the *pragmateia* of the mathematical Pythagoreans for improper methodological procedure: while the proofs offered by the mathematical Pythagoreans represent a significant philosophical innovation over the simple acceptance of so-called “facts” by the acousmatic Pythagoreans, it is the mathematical Pythagoreans’ activity of hasty “analogizing” across ontological categories that leads to confusions that are not mediated by proper proportioning, both in terms of logic and metaphysics. Analysis of the extant fragments of Philolaus of Croton corroborates Aristotle’s criticism of the mathematical Pythagoreans, and it becomes likely that the targets of Aristotle’s disapproval are those Pythagoreans who applied the superordinate mathematical sciences and their principles to their understanding the human sciences, most importantly, politics.

The Pythagoreans of the 5th Century BCE probably did not see themselves as a community unified by philosophical and political doctrines. Rather, insofar as we can reconstruct their history, there arose an internal conflict among the Pythagoreans who were living in South Italy, one that may have effected a split between the ascetic Pythagoreans who may have lived in the western part of Italy (and fled to Asia Minor) and the intellectualist Pythagoreans who occupied the eastern part of the Italian peninsula, nearby Tarentum.⁵

Differences in approach to the philosophical “life” and its activities can be detected in the comic

⁵ Here, I refrain from using the terms “acousmatic” and “mathematical” because the terminology employed by the historical writers was not exactly the same as Aristotle’s, although the bifurcation into ascetics and intellectualists is broadly analogous to the bifurcation into acousmatic and mathematical. On the historical evidence pertaining to the split of Pythagoreans along west/east lines in Italy, see Chapter 2.

fragments that survive from the early part of the 4th Century BCE, as Christoph Riedweg has shown.⁶ With Aristotle, however, we find a rather elaborate division of the early Pythagoreans into two groups – traditional acousmatics (ἀκουσματικοί) and progressive mathematicians (μαθηματικοί). At some later time, perhaps in the 3rd or 2nd Centuries BCE, there is apparently a further subsection into three groups.⁷ Whether or not this triad represented a substructure of the Pythagoreans – or of which group (acousmatic or mathematical) – cannot be deduced from the available evidence, and it will be more productive for our argument to focus on the division into two groups as described by Aristotle.⁸ While most scholars have been willing to admit the categorical difference between acousmatic and mathematical Pythagoreans of the 5th Century BCE, they nevertheless assume that certain contradictory elements within their own constructed “Pythagoreanism” are often the misinterpretation of critics like Aristoxenus, Dicaearchus, or Timaeus of Tauromenium, all Hellenistic commentators on Pythagoreanism whose accounts are at least somewhat derivative from the descriptions of Pythagoreanism in the writings of Aristotle.⁹ We should not be so hasty. Since Aristotle, our best source for a history of

⁶ Riedweg 2005: 108-9.

⁷ Burkert 1972: 192-3 with n. 6. That the division into “acousmatic” and “mathematical” is original with Aristotle is followed by most scholars, including Kahn (2001: 15), Riedweg (2005: 106-8), and Huffman (2006). For a useful description of the state of the question, see Huffman 2006.

⁸ It is extremely difficult to correlate the bifurcation into “acousmatic” and “mathematical” Pythagorean with the tripartite subsections. Armand Delatte took seriously the possibility of the tripartite organization, to which earlier and later traditions as well as the so-called Hellenistic Pythagorean writings adhere closely. Minar 1942: 34-5 with n.79 criticizes Delatte (1922: 24-6) for seeing a correspondence between the three classes listed by Photius and the Scholiast to Theocritus (μαθηματικοί, σεβαστικοί, and πολιτικοί) and those offices mentioned by Iamblichus at *VP* 72 and 89. Minar overlooks Delatte’s care in distinguishing between, and not drawing immediate analogies among, the classes; at any rate, we take issue with Minar’s claim that “Iamblichus as usual transcribes the phrase quite mechanically in section 89 *fin.*” Yet again, Iamblichus is unclear whether he wishes to distinguish three classes (as he does in 89) or define one class (as he does in 74). Burkert (193 n. 6) suggests that the triad is a sub-categorization structured according to the terms Πυθαγορικοί – Πυθαγόρειοι – Πυθαγοριστάι and correspondent with the “pupils, pupils of pupils, foreign advocates” (Anon. Phot. 538b32ff; *Schol. Theocr.* 14.5). Zhmud (1997: 100-13 with n. 48) considers these distinctions to be dated much later.

⁹ Most recent scholars accept the distinction between acousmatic and mathematical Pythagoreans as original with Aristotle, e.g. Burkert (1972: 192-207), Huffman (1993: 11-12 and 2006), Kahn (2001: 15), McKirahan (1994: 89-93), and Riedweg (2005: 106-8). Among recent scholars, only Zhmud (1992: CITE) considers this division to be later than Aristotle. Of these critics, however, only McKirahan has placed a strong emphasis on the importance of

Pythagoreanism, identified the distinctions along methodological lines, we should be sensitive to the possibility that such contradictory elements are reflective of divisions among the Pythagoreans. Indeed, the primary criterion for distinguishing acousmatic from mathematical Pythagoreans, as we will see, is each group's *pragmateia* (πραγματεία), a term that must be further contextualized in order to make sense of precisely how Aristotle is drawing the line.

What does the term *pragmateia* mean for Aristotle? It is a difficult question to answer succinctly, and I hope that a more thorough understanding of the concept will unfold as this chapter progresses. But it will be useful to start with an operating definition, which can then be developed in the course of our argument: in Aristotle's usage, the *pragmateia* of a philosopher or philosophical group is both the *object* of their philosophical inquiry and the unique *treatment* of that object in their philosophy.¹⁰ *LSJ* lists possible meanings for Aristotle as "system" (*Metaph.* I.6, 987a30 and I.5, 986a8), "manner of dealing with" (*Rh.* I.15, 1376b4), "philosophical argument or treatise" (*Top.* I.1, 100a18 and I.2, 101a26; *Phys.* II.3, 194b18; *EN* II.2, 1103b26), and "subject of such a treatise" (*Phys.* II.7, 198a30). We can assume some semantic overlap, in the sense that for Aristotle, there was a fluid relationship between these meanings for the term *pragmateia*. It is apparently first used in a technical manner *vis-à-vis* philosophy by the mathematical Pythagorean Archytas of Tarentum, who posits it as the "treatment" or "investigation into" an object of mathematics:

Logistic (ἡ λογιστική) seems to be far superior indeed to the other arts in regard to wisdom and in particular to deal with (πραγματεύεσθαι) what it wishes more clearly than geometry. Again in those respects in which geometry is deficient, logistic puts demonstrations into effect

the division for our understanding of the *philosophical methodologies* and consequences thereby of these various groups.

¹⁰ Some other scholars' definitions of Aristotelian *pragmateia*: "rei alicuius tractatio via ac ratiouae instituta" (Bonitz); "treatment" (Klein), as in "general treatment" (*katholou pragmateia*); "philosophic activity" (Burkert/Minar); "Wissengebiet" or "Forschungsgebiet" (Zhmud, in reference to similar uses by Aristoxenus). Unfortunately, Aristotle nowhere explicitly defines *pragmateia*.

(ἀποδείξιαις ἐπιτελεῖ) and equally, if there is any *pragmateia* of shapes (εἰ μὲν εἰδέων τεὰ πραγματεία), [logistic puts demonstrations into effect] with respect to what concerns shapes as well.

(F 4 Huffman = Stob. *Proem.*, Translated by Huffman)

Since we can detect a discrepancy between Archytas and Plato in the use of the term *pragmateia*, namely that Archytas' use parallels Aristotle's, whereas Plato's more generally means "the business of" (e.g. *Grg.* 453a, *Theaet.* 161e)¹¹, it is likely that Aristotle inherited this special use of the term *pragmateia* from Archytas himself.¹² This conceptual inheritance is very important, since, as I will argue, Aristotle himself uses the term *pragmateia* as a marker that distinguishes the mathematical Pythagoreans from the acousmatic Pythagoreans. The larger implications of the difference between the *pragmateia* of the mathematical and acousmatic Pythagoreans have a direct significance for our study, since the figure credited with distinguishing the *pragmateia* of the mathematical Pythagoreans, Hippasus of Metapontum (ca. 520? – 440 BCE?), also apparently catalyzed the political factionalization that occurred in the Pythagorean community.¹³

That Hippasus of Metapontum was a mathematical Pythagorean (μαθηματικός) is well-established and accepted, thanks to the work of Walter Burkert in his influential study *Lore and Science in Ancient Pythagoreanism*, who is nevertheless not entirely confident about what that term means.¹⁴ This provides an opportunity to develop Burkert's model beyond what he, or any

¹¹ Noted by Huffman (2005: 251).

¹² There is one place (*R.* 528d1-3) where Plato uses the term *pragmateia* in relation to mathematics. Glaucon asks Socrates if the "geometry" is to be considered the "study of the plane" (τοῦ ἐπιπέδου πραγματεία). It is possible that Glaucon is using a term inherited from Pythagorean mathematics here.

¹³ Iambl. *de Comm. Math.* 76.19-77.22. I will discuss this passage in Chapter 2.

¹⁴ Burkert 1972: 192-201 *et passim*. Similarly followed by Huffman (2005), Riedweg (2005), and Kahn (2001). *Contra* Silvestre 2000: 418-9 with n. 27, but her position places too much emphasis on the testimony concerning "Soul-number" that, as I have shown above, derives from the early academy and cannot be easily extricated from their own Platonist assumptions.

other scholar of Pythagoreanism, has done.¹⁵ Burkert synthesizes the available material in order to demonstrate two significant points: first, that all followers of Pythagoras were adherents of the *acusmata*, also called *symbola*, a set of orally transmitted sayings passed down from Pythagorean teacher to students in a traditional mode; second, that what distinguished the ascetic acousmatic Pythagoreans (ἀκουσματικοί) from the progressive mathematical Pythagoreans (μαθηματικοί) was each group's unique philosophical and political *pragmateia*:

Aristotle recognizes among the Pythagoreans a twofold πραγματεία: on the one hand, the Πυθαγορικοί μῦθοι, metempsychosis, the Pythagoras legend, and the *acusmata*, and on the other a philosophy of number connected with mathematics, astronomy, and music, which he never tries to trace back to Pythagoras himself and whose chronology he leaves in abeyance.¹⁶

Burkert demonstrates that Aristotle categorized the Pythagorean *acusmata* according to whether or not they answered these three questions: τί ἔστι (what is?), τί μάλιστα (what is to the greatest degree?), and τί πρακτέον (what is to be done?).¹⁷ While I will concern myself throughout my study with this important tripartite categorization, for the sake of this chapter, I would like to focus on the last group, namely those things that fall under τί πρακτέον, in part because I think it elucidates how ethics and political activity relate to the object of the Pythagorean investigations (i.e. *pragmateia*) in Aristotle's characterizations. In Burkert's study, he explicates those *acusmata* that fall under the category "what is to be done" by focusing,

¹⁵ Riedweg's account (2005: 106-8) is probably the best synthetic account outside of Burkert (1972), although we should recognize the care with which Burnyeat (2005) examined the philosophical context in Aristotle (without analysis of the political aspects of the reported schism). Burnyeat thus leads the way for my study.

¹⁶ Burkert 1972: 197. A useful description is also given by McKirahan 1994: 114: "For the Pythagoreans (more precisely, the *mathematikoi*) this clear knowledge [of the *kosmos*] is not simply a matter of parroting a set of beliefs, saying a catechism of fixed doctrine without understanding. It involves the study of mathematics and the *kosmos*. The numerical basis of the *kosmos* implies that the *kosmos* is comprehensible to humans, and the knowledge of it which benefits our soul demands thought and understanding. Our soul becomes orderly (*kosmios*) when it understands the order (*kosmos*) of the universe [cf. *Pl. R.* 500c, cited by the author]. This is the inspiration that underlay the developments in Pythagorean thought and which gives the Pythagoreans much common ground with their Ionian predecessors as well as their successors in mathematics, science, and philosophy."

¹⁷ See Burkert 1972: 167-9, with Iambli. *VP* 82, and Delatte 1915: 274-307. Burkert rightly reminds us that these "orally transmitted maxims and sayings" were also called *symbola*. Recently, Peter Struck has done a comprehensive study on symbolic or enigmatic communication in antiquity, although his book also fails to treat the third kind of *acusma*. See Struck 2004: 96-110.

almost entirely, on ethical imperatives and ritual activity.¹⁸ By way of an ingenious classification of the *acusmata*, he demonstrates the significance of these prescriptions in the establishment of a Pythagorean way of life, an “amazing, inextricable tangle of religious and rational ethics.”¹⁹ This is a valuable and historically contingent approach to understanding one important aspect of the ethical positions ascribed to the Pythagoreans, in no small part because it reveals the religious semantics of the concept of *pragmateia*. Such an approach is appropriate, since, as Iamblichus argues (in the Aristotelian analysis of the “what is to be done” injunctions that follows upon their listing), what is divine (τὸ θεῖον) is the first principle and origin (ἀρχή).²⁰ But there is much more to this passage than Burkert discusses, especially in illuminating how Aristotle defined the *pragmateia* of the Pythagoreans:

All such *acusmata*, however (μέντοι), which define what is to be done or what is not to be done (περὶ τοῦ πράττειν ἢ μὴ πράττειν), are directed toward the divine (ἑστόχασται πρὸς τὸ θεῖον), and this is a first principle (καὶ ἀρχὴ αὕτη ἐστὶ), and their whole way of life is arranged for following God (ὁ βίος ἅπας συντέτακται πρὸς τὸ ἀκολουθεῖν τῷ θεῷ), and this is the rationale (λόγος) of their philosophy. For human beings act ridiculously in seeking the good anywhere else than from the gods, just like someone who pays court to a subordinate governor of the citizens in a country ruled by a king, neglecting him who is the ruler of all (ἀμελήσας αὐτοῦ τοῦ πάντων ἄρχοντος); for (γὰρ) just so do they think humans behave. *For since* (ἐπεὶ γὰρ) *there is a god, and he is lord of all* (πάντων κύριος), *it is agreed one ought to ask for the good from the lord. For all give good things to those whom they cherish and with whom they are pleased, but to those toward whom they are oppositely disposed, they give the opposite.*²¹

¹⁸ Burkert 1972: 174-92. Similarly followed by Kahn (2001: 9-10) and Riedweg (2005: 63-7).

¹⁹ Burkert 1972: 185.

²⁰ Iambl. *VP* 86. Note that Aristotle explicitly states a similar claim at *Metaph.* I.2, 983a6-11, on which see Nightingale 2004: 236-7.

²¹ Translated by Dillon and Hershbell. This passage is repeated, with a few changes in words, at *VP* 137, with regard to the *pragmateia*, which he calls ἡ μαντική, or “the means of interpreting the thought of the gods” (ἑρμηνεία τῆς παρὰ τῶν θεῶν διανοίας). Generally, throughout this chapter and in regard to those passages of Iamblichus that preserve Aristotelian material, I will put in **bold** those passages I consider to originate with Aristotle and *italicize* those passages I consider to be Iamblichus’ exegesis. For passages about which I am unsure, I will leave them in plain, unadorned text. This model follows that of Hutchinson and Johnson in their forthcoming edition of Aristotle’s *Protrepticus*.

(Iamblichus, *On the Pythagorean Way of Life* 86-7)

One of the great challenges of this passage is to extrapolate from it what is genuinely Aristotelian, so far as possible. We may never be absolutely certain. But there is good reason to believe that Iamblichus stops excerpting from Aristotle when the shift occurs from analysis to exegesis: at the introduction of the notion of god as “lord of all” (πάντων κύριος), a term that smacks of Middle Platonism.²² Moreover, in the passage that immediately precedes this one, the attempt to define a “first principle” (ἀρχή) and a “reason” or “rationale” (λόγος) for the Pythagorean philosophy as related to the first principle is characteristic of Aristotle’s method of describing and critiquing earlier philosophical systems. We might, for example, recall the beginning of the *Nicomachean Ethics* (I.4, 1095a31-b14), where Aristotle questions whether it is better to employ arguments (λόγοι) that *derive from* first principles (ἀπὸ τῶν ἀρχῶν) or those that *lead to* first principles (ἐπὶ τὰς ἀρχάς), a question that, so Aristotle claims, Plato had raised. There, Aristotle distinguishes his own deductive method from that of Plato by arguing that we should begin from what is already known to us, namely, the “what is” or “fact” (τὸ ὄτι), which he calls a “first principle” (ἀρχή). Further development along these lines occurs in the *Posterior Analytics* (II.19, 100b5-17) and the *Physics* (I.1, 184a10-b10), where Aristotle sketches out a complex epistemology in which scientific knowledge proceeds *from* first principles, obtained by means of “intuition” (νοῦς), *to* particulars.²³ With regard to first principles, Aristotle’s epistemology stands in contrast to the epistemology attributed to the Pythagoreans in

²² Note the accumulation of explanation words (γὰρ, ἐπεὶ γὰρ), which are marks of exegesis and authorial intervention in Iamblichus’ writing. On the subject of how Iamblichus quotes ancient sources such as Plato and Aristotle, see the deft and meticulous analysis of Hutchinson and Johnson 2005. Similar Middle Platonist uses of the term “lord” appear in the writings of Philo of Alexandria (e.g. *Leg. All.* 3.198), doubtlessly influenced by similar uses in the Septuagint, and Plutarch (e.g. *Alex.* 30.5, *Is. et Os.* 355e), influenced by Persian and Egyptian religions.

²³ This is a general – indeed far too general – sketch of Aristotelian epistemology, and I only make reference to it in order to provide context for the Aristotelian passage preserved in *VP* 86-7. A full discussion of Aristotelian epistemology is beyond the scope of this chapter. In general, I agree with the analysis of Cherniss (1944: 76-80).

On the Pythagorean Way of Life 86-7, which attributes to Pythagoreans the sorts of λόγοι that lead up to, and follow, the first principle, namely the divine.²⁴

But which Pythagoreans, acousmatics or mathematicians, was Aristotle describing in this passage? Or was he talking about the *pragmateia* of all the Pythagoreans? There is no standard scholarly position on this question, in part because scholars have been unclear about which sections to attribute to Aristotle.²⁵ It is likely, however, that he is referring to the Pythagoreans in general, and not to a particular faction, in this description of their *pragmateia*. While the distinction between acousmatic and mathematical Pythagoreans immediately precedes this passage, there are three reasons for interpreting this passage as referring to Pythagoreans more generally. First, Iamblichus separates the earlier passage (*On the Pythagorean Way of Life* 86), where he discusses the distinctions between the so-called “esoteric” and “exoteric” Pythagoreans, by a poignant “however” (μέντοι), suggesting that he has completed discussion of the split between the “esoteric” and “exoteric” Pythagoreans and is returning to the issue of first principles.²⁶ Second, there is nothing specific that suggests to us to identify the system of religious order described as acousmatic or mathematical: this is unsurprising, since it is generally agreed that Aristotle describes the mathematical Pythagoreans as accepting the philosophy of the acousmatic Pythagoreans, which consisted mostly of religious and ethical precepts.²⁷ Finally, when Iamblichus returns to discussing the *acusmata* later in the treatise (*On the Pythagorean Way of Life* 137), he repeats this passage and describes it as illustrating the principles of religious

²⁴ In this way, the Aristotelian passage preserved in *VP* 86-7 may have formed the basis for (or referred to the same system described by) Aristoxenus’ account of the *Pythagorean Precepts*, especially F 33(= Iambli. *VP* 174-5) and F 34 (= Stob. *Ecl.* IV.25.45), which describe the ontological stratification of being for the “Pythagoreans.” Cf. Huffman 2006: 112 and 2008: 107-8.

²⁵ Cf. Burkert 1972: 196 n. 17.

²⁶ Is this an insertion of another authority by Iamblichus, or is it the continuation of citation from Aristotle? Burkert (1972: 196 n.17) doubts whether it is Iamblichus’ own, and the focus on distinguishing “esoteric” from “exoteric” shows signs of derivation from from Timaeus of Tauromenium. On these terms, see below.

²⁷ Cf. Huffman 2006, Riedweg 2005: 106-7, and Kahn 2001:15.

worship of the gods as attributed to Pythagoras and to his followers (Πυθαγόρας τε καὶ οἱ ἀπ’ αὐτοῦ ἄνδρες). Therefore, the Aristotelian description of the *pragmateia* of the Pythagoreans (as preserved in *On the Pythagorean Way of Life* 86-7) focuses on two important aspects that we will continue to discuss in our argument: the hierarchy of the universe, which is honored by means of proper understanding that the divine is the first principle which humans must pursue in order to attain the Good; and the hierarchy of a political organization, which is analogous. In this way, when Aristotle (*apud* Iamblichus) characterizes the Pythagorean *pragmateia* in the most general terms, he appeals both to the religious and the political.

Close attention to the *philosophical methodologies* attributed to the Pythagoreans by Aristotle, which has not been attempted by Burkert or any scholar who has followed him, might give us a better insight into the understanding the rationales (λόγοι) that distinguished the *pragmateiai* of the acousmatic and mathematical Pythagoreans. When he describes the rationale (λόγος) for the maxims that answer the question τί πρακτέον, Iamblichus (*VP* 85-6) distinguishes the use of rationales by the more conservative Pythagoreans from the use by those people he claims are “non-Pythagoreans” (οὐκ εἰσὶ Πυθαγορικάί), who are also considered “outsiders” (ἐξώθεν). We are reminded of the term employed by Aristotle in the *Metaphysics* and elsewhere to refer to some Pythagoreans, whom he calls the “so-called Pythagoreans” (οἱ καλούμενοι Πυθαγόρειοι), and who, he claims, were the first to develop the science of mathematics as a science of first principles.²⁸ Are those figures designated “non-Pythagoreans” and “outsiders” the same as the “so-called Pythagoreans”? The evidence concerning the “esoteric” and “exoteric” Pythagoreans as preserved in Iamblichus’ *On the Pythagorean Way of*

²⁸ See esp. Arist. *Metaph.* I.4, 985b24ff.

Life alone is ambivalent, and we cannot be sure that Aristotle is the source here.²⁹ One possibility is that the source for information in the passages that distinguish “exoteric” from “esoteric” Pythagoreans is the 3rd Century BCE historian Timaeus of Tauromenium, who was apparently interested in the division between those Pythagoreans who were more advanced in their learning (esoteric) and those who had not proceeded beyond a certain level (exoteric). These exoteric Pythagoreans, so the authority suggests, differ from the real Pythagoreans because they “attempt to give a likely rationale/account” (πειρωμένων προσάπτειν εἰκότα λόγον) for the injunctions that constitute the *acusmata*.³⁰ The “likely account” (εἰκοτολογία) which Iamblichus or his source attributes to those people who are “non-Pythagoreans” or “exoteric” in this passage represents a more sophisticated approach to wisdom traditions such as those of Pythagoras or the Seven Sages, but it is not “mathematical” in the strong sense, at least if we are to judge by the examples given. Indeed, the sorts of “likely account” given by the “exoteric” Pythagoreans are focused on practical – even political – reasoning in a way not unlike the εἰκὼς λόγος given by the quasi-Pythagorean Timaeus of Epizephyrian Locri in Plato’s *Timaeus*.³¹ Those Pythagoreans who, from early on, attached a *logos* to the *acusma*, spoke enigmatically: “one should not break bread because it is not advantageous for judgment in Hades.” On the

²⁹ For a useful study of the relationship between the terms “exoteric/esoteric” and “acousmatic/mathematical” Pythagorean, see von Fritz 1960: 10ff.

³⁰ The term εἰκὼς λόγος, which is technical, receives a great number of conflicting treatments in antiquity. In Plato’s *Timaeus* (30b8), it refers to the “likely story” that cannot, on Morgan’s reading (2000: 275), be verifiable by appeal to empirical knowledge. We might suggest that it is also not verifiable by appeal to theoretical reasoning alone. For Aristotle in the *Prior Analytics* (II.27, 70a3-7), it refers to a generally-accepted premise, i.e. to something that is known by people to be or to happen a certain way, e.g. that envious people are malevolent. The two uses are not incommensurable, but they are not the same either. It is interesting to contrast these with the description of an argument from probability described in the Aristotelian *Rhetoric to Alexander* (7, 1429a15-18) which, by appealing to emotions with which humans will be sympathetic, diverts the audience away from rational calculation (*logismos*). Moreover, there is Theophrastus’ (F 142 Fortenbaugh = Simpl. *In Arist. Phys.* 184a16-b14) description of φυσιολογία as εἰκοτολογία (if that word is to be attributed to him), which means something less than proper demonstration (ἀπόδειξις), but is not thereby dismissable, since human beings themselves are limited. It is interesting to note that Ps-Archytas’ *On Intelligence and Perception* (F 1 Thesleff = Stob. I.41.5) refers to εἰκοτολογία in reference to political treatises, namely things that deal with “affairs” (πράξις).

³¹ Cf. Burnyeat 2005b, who emphasizes the reasonableness or appropriateness (the ‘ought’: δεῖ) that constitutes the goal to which the practitioner of the εἰκὼς λόγος aims.

other hand, those who were described as “non-Pythagoreans” or “exoteric” gave different types of *logoi*: cultural-historical explanation (“one should not break bread” because, in the past, people used to come together in order to eat a single loaf of bread, as foreigners do), or normative (“one should not break bread” because one ought not to establish the sort of omen that occurs at the beginning of the meal by means of breaking and crushing bread).³² Such examples demonstrate that the exoteric “non-Pythagoreans” whose *pragmateia* involved cultural-historical or normative types of *logos* were responsive to contemporary (i.e. 5th Century BCE) sorts of inquiry (ἱστορία), such as those we find in the writings of Herodotus or the writers of the Hippocratic Corpus.³³ They appear, in this account, to resemble something more like clever men who know how to make various devices (ἐπισοφίζομένων) than highly regarded practitioners of wisdom, at least in regard to the *acusmata*.³⁴

But if we consider these questions in the light of another passage definitely derived from Aristotle’s lost works on the Pythagoreans, as preserved by Iamblichus, we get a sense of what

³² It is worth noting that the information preserved here is almost exactly the same as that attributed by Diogenes Laertius to Aristotle’s *On the Pythagoreans* (F 195 Rose = D.L. VIII. 33-5). It is possible, then, that Iamblichus was looking at Aristotle’s text while recording this information or, for that matter, that Timaeus of Tauromenion (?) had access to Aristotle’s text while drawing up his list of the *acusmata*. The evidence preserved in these passages also might have a more immediate source Alexander Polyhistor’s *Pythagorean Notebooks*, on which see especially Long: *forthcoming* CITE.

³³ For Herodotean ἱστορία and its contexts, see Lateiner 1989: 15-17 and Thomas 2000: 21ff.; for Presocratic and Hippocratic ἱστορία, see Schiefky 2005: 19-35; more generally, for philosophically related uses of ἱστορία before Plato, see Riedweg 2005: 94-5 and Darbo-Peschanski 2007.

³⁴ The term ἐπισοφίζομαι occurs in Iamblichus and in post-Iamblichean texts, but it is also attested in the Hippocratic corpus (*Art.* 14) with reference to clever doctors who demonstrate their cleverness by attaching a piece of lead to a fractured bone in order to stabilize it. Cf. Burkert 1972: 174 with n. 64 and 200. I would add, however, that such “cleverness” is attached to the Tarentine Pythagoreans whose rhetorical *logoi* are satirized in two plays, both entitled *The Tarentines*, written by the 4th Century BCE comedians Alexis of Thuri (F 223 K-A: Πυθαγορισμοὶ καὶ λόγοι / λεπτοὶ διεσμιλευμένοι τε φροντίδες / τρέφουσ’ ἐκείνους) and Cratinus the Younger (F 7 K-A: ἔθος ἔστιν αὐτοῖς...διαπειρώμενον / τῆς τῶν λόγων ῥωμῆς ταραττεῖν καὶ κυκᾶν / τοῖς ἀντιθέτοις, τοῖς πέρασι, τοῖς παρισώμασιν, / τοῖς ἀποπλάνοις, τοῖς μεγέθεσιν, νουβιστικῶς). We can thus posit a popular tradition, not necessarily derived from Aristotle, that attributes sophisms of a rhetorical sort to the Tarentine Pythagoreans. Note, too, that Cratinus employs terms both rhetorical and mathematical, such as πέρας and μέγεθος, translated by Edmonds as “end” and “sublimity”. The former is attested in a rhetorical sense in the Aristotelian *Rhetoric to Alexander* (32, 1439a38), where it is described as the conclusion that rounds off an exhortation. The latter appears in Aristotle’s *Rhetoric* (III.9, 1409a36), with reference to periodic sentences that can be measured, as well as in Dionysius of Halicarnassus (*Comp.* 17) as “sublimity.” It is difficult to know precisely what Cratinus the Younger intended their meaning to be.

Aristotle, for his part, considered fundamentally different about the philosophy of the acousmatic and the mathematical Pythagoreans:

(A) There are two types of the Italic, also called the Pythagorean (καλουμένης Πυθαγορικῆς), philosophy. For there were also two kinds of people who treated it (γένη τῶν μεταχειριζομένων), namely the acousmatics and the mathematicians. Of these two, the acousmatics were recognized to be Pythagoreans by the others [the mathematicians], but they did not recognize the mathematicians [as Pythagoreans], nor did they think that the *pragmateia* [of the mathematicians] derived from Pythagoras, but rather that it derived from Hippiasus. Some say that Hippiasus was from Croton, while others say from Metapontum. And, of the Pythagoreans, those who concern themselves with learning (μαθήματα)³⁵ recognize that the others (i.e. the acousmatics) are Pythagoreans, and they declare that they themselves are even more [Pythagorean], and that the things they say (ἃ λέγουσιν) are true. And *our sources say that the reason for such differentiation is this:*

(B) “Pythagoras came from Ionia, more precisely from Samos, at the time of the tyranny of Polycrates, when Italy was at its height, and the first men of the city-states became his associates. The older of these [men] he addressed in a simple style, since they, who had little leisure on account of their being occupied in political affairs, had trouble when he conversed with them in terms of learning (μαθήματα) and demonstrations (ἀποδείξεις). He thought that they would fare no worse if they knew what to do (εἰδότας τί δεῖ πράττειν), even if they lacked the reason (ἄνευ τῆς αἰτίας) for it, just as people under medical care fare no worse when they do not additionally hear the reason why they are to do (διὰ τὶ πρακτέον) each thing in their treatment. The younger of these [men], however, who had the ability to endure the education, he conversed with in terms of demonstrations and learning. So, then, these men [i.e. the mathematicians] are descended from the latter group, as are the others [i.e. the acousmatics] from the former group.”

(C) And concerning Hippiasus, *our sources say that* while he was one of the Pythagoreans, he was drowned at sea for committing heresy, on account of being the first to publish, in written form (διὰ τὸ ἐξευεγκεῖν καὶ γράψασθαι), the sphere, which was constructed from twelve pentagons. He gained his reputation from discovering this, but all such

³⁵ This term is extremely difficult to translate, and no translation will do justice. Alternatives include “sciences” or “mathematics,” but I think Burkert (1972: 195 and 207 n. 80) is correct in defining this term as the branches of learning the Greeks called arithmetic, geometry, astronomy, and music.

discoveries were from “that man” – so they call Pythagoras, and they do not call him by name.

(Iamblichus, *On the General Mathematical Science* 76.19-77.22)

This passage of Iamblichus, which Burkert has recognized as being the authoritative version of the story of the factionalization of the Pythagorean brotherhood reported by Aristotle, further supports our claim that what distinguished the acousmatic and mathematical Pythagoreans was, primarily, the rationale (λόγος) that led to their *pragmateia*, that is, the object of their philosophical inquiry. It is apparently divided into three sections: (A), which, while not direct quotation, is nonetheless derived, in great part (if not wholly), from Aristotle’s lost writings on the Pythagoreans; (B), which is apparently direct quotation from Aristotle; and (C), which may or may not be from Aristotle.³⁶ In the section apparently quoted directly from one of Aristotle’s lost works on the Pythagoreans (B), what distinguishes the acousmatic from the mathematical Pythagoreans is *type of knowledge*: the acousmatic Pythagoreans only have knowledge of the “the fact” (τὸ ὅτι) of “what one is to do” (τί δεῖ πράττειν), but the mathematical Pythagoreans, whose knowledge is advanced, understand the “reason why they are to do” (διὰ τὶ πρακτέον) what they should do. This methodological distinction between “fact” (ὅτι) and “reason why” (διὰ τί) is originally Aristotelian, and it secures the authenticity of this passage for Aristotle. Indeed, the distinction between the “fact” (ὅτι) and the “reason why” (διότι) is central in Aristotle’s descriptions of mathematicians in the *Posterior Analytics*:

The reason why (τὸ διότι) is superior to the fact (διαφέρει τοῦ ὅτι) in another way, when each is considered by means of a different science. Such is the case with things that are related to one another in such a way that one is subordinate to the other, e.g. optics to geometry, mechanics to stereometry, harmonics to arithmetic, and star-gazing to astronomy. Some of these sciences bear almost the same name, e.g. mathematical and nautical astronomy are called “astronomy”, and mathematical and

³⁶ Where (C) might have arisen, I will discuss at the beginning of Chapter 2.

acoustical harmonics are called “harmonics.” In these cases it is for those who concern themselves with perception to have knowledge of the facts (τὸ ὅτι εἰδέναι), whereas it is for the mathematicians to have knowledge of the reason why (τὸ διότι εἰδέναι). For the latter are able to make demonstrations of the causes (τῶν αἰτίων τὰς ἀποδείξεις), and they often do not understand the facts (τὸ ὅτι), just like people who study the universal often do not know some of the particular instances for lack of noticing them. The objects of their study are the sort that, although they are something different in substance (ἢ οὐσία), deals with forms (κέχρηται τοῖς εἶδεσιν). For mathematics is concerned with forms; its objects do not exist according to some substrate.

(Aristotle, *Posterior Analytics* I.13, 78b34-79a8)

This description of the so-called “subalternate” sciences develops a useful analogue for how acousmatic Pythagoreans differ from the mathematical type. The mathematicians described in the *Posterior Analytics* have knowledge of the “reason why” and are thus able to understand and create “demonstrations” of the causes of the objects of their study.³⁷ In making these demonstrations, those who employ the superordinate mathematical sciences (e.g. geometry, stereometry, arithmetic, and astronomy) employ Aristotelian (not Platonic) forms, which Richard McKirahan usefully describes, in regard to this passage, as “what remains when abstraction is made of the material substrate.”³⁸ Aristotle’s characterization of mathematicians as people who make use of demonstrations in their philosophical *pragmateia* parallels that of the mathematical Pythagoreans in the Aristotelian passage quoted at *On the General Mathematical Science* 76.19-

³⁷ Further explication of this passage, and contextualization of its place in Aristotelian methodology, is offered by Richard McKirahan (1992: 76), who describes the role that axioms play in demonstrations superordinate sciences: “The doctrine of the qualified identity of subject genera of superior and subordinated sciences also explains how axioms apply in more than one science. The relation between quantity in general and spatial magnitude is the same as the relation between spatial magnitude and the subject genus of optics. Just as visual lines are geometrical lines, but not all geometrical lines, and are treated qua having geometrical properties and visual properties as well, so spatial magnitudes are quantities, but not all quantities, and geometry treats them qua having properties of quantities in general and geometrical ones as well. Each axiom of a science is a proper principle or a conclusion of a superior science. This is guaranteed by the fact that axioms are common to more than one science and are common in virtue of a common character of their subject genera, and so have a place in the science whose genus is appropriately general.” McKirahan points to Eudoxus’ theory of proportion as an example of such a superordinate science. Johnson (2009: 332-8), moreover, has shown that Aristotle himself applies the methodology of the mathematicians in his explanation of the halo in the *Meteorology*.

³⁸ McKirahan 1978: 204.

77.22, although, importantly, there is no reference to Aristotle's peculiar understanding of mathematical "forms" or "substance" in the Aristotelian text. If the work quoted from was composed very early in Aristotle's career, before he undertook to propose new approaches to ontology in the *Categories*, it would not be surprising that we do not hear about such problems. Regardless, the evidence suggests that while the mathematical Pythagoreans described in Aristotle's lost works are not exactly the same as the mathematicians described in the *Posterior Analytics*, they do share in the knowledge of the "reason why" and ability to carry out proof by means of demonstrations.³⁹

The establishment of sections (A) and (B) from Iamblichus' *On the General Mathematical Science* as genuinely Aristotelian is very important for our understanding of mathematical Pythagoreanism, as Aristotle constructed it, because it confirms a claim that has often been suggested but never explicitly demonstrated by scholars⁴⁰: that the "so-called Pythagoreans" (οἱ καλούμενοι Πυθαγόρειοι), to which Aristotle refers in *Metaphysics A* (I.5, 985b24 and I.8, 989b29), *On the Heavens* (II.1, 284b7 and II.13, 293a20), and *Meteorology* (I.6, 342b30 and I.8, 345a14) are, indeed, one and the same with the mathematical Pythagoreans.⁴¹ Let us examine a famous passage from the first book of Aristotle's *Metaphysics*, which we must assume (with Jaeger, Ross, and Owens)⁴² was written rather early in Aristotle's career, when he was still heavily under Platonic influence:

³⁹ Although, if we follow McKirahan (1992: 76) in believing that Aristotle's point of reference here is the geometer Eudoxus, who studied under Archytas (cf. Huffman 2005: 7), the links to mathematical Pythagoreanism are explicit.

⁴⁰ Cf. Burkert 1972: 30 with nn. 8-9 and 51-2, who is followed by Huffman (1993: 31-5). Huffman's suggestion that others who might be "so-called Pythagoreans" would include Hippiasus, Lysus, and Eurytus is plausible, although I doubt that those who proposed the theory of *sustoicheia* would be included. The most extensive analysis of this problem is Timpanaro Cardini 1964: 6-19, but she concludes erroneously, I would argue, that there is no distinction between the various types of Pythagoreans named in Aristotle's *Metaphysics*.

⁴¹ I will deal primarily with the passages in *Metaphysics A*, for the sake of their strong connections with the fragments of Aristotle's lost works on the Pythagoreans.

⁴² Cf. Owens 1978: 85-9; Jaeger 1948: 171-6; Ross 1924: xv.

The “so-called Pythagoreans” deal with more removed⁴³ first principles (ἀρχαίς) and elements (στοιχείοις) than those of the natural scientists. The reason is that they took their first principles from non-sensible objects (οὐκ ἐξ αἰσθητῶν): for the objects of mathematics (τὰ μαθηματικὰ τῶν ὄντων), except for those of astronomy, are a class of things lacking in motion (ἄνευ κινήσεως ἔστιν). They discuss, however, and wholly make the object of their philosophical inquiry (πραγματεύονται) nature. For they generate heaven, and they observe what happens concerning the parts, attributes, and functions of it, and they lavish these things with first principles and causes, and as such they are in agreement with the natural scientists that what exists is just all that is perceived and that “so-called heaven” contains it. But, as we discussed earlier, they say that the causes and the first principles are able to rise up above the horizon (ἐπαναβῆναι)⁴⁴ to the higher parts of reality; these are better suited for the arguments concerning nature. Nevertheless, they say nothing about how motion will exist, if the only things premised are Limit and Limitless, and Odd and Even, nor about how there can be generation and destruction or the activities of objects that pass along heaven without motion and change (ἄνευ κινήσεως καὶ μεταβολῆς).

And, what’s more, if someone were to grant to them that spatial magnitude derives from these things, or if this were to have been demonstrated by them (δειχθείη τοῦτο), in what way will some bodies be light and others heavy? For, given what they assume and maintain, they are speaking no more about mathematical bodies than about sensible bodies. Hence they have said nothing whatsoever about fire or earth or any other bodies of this sort, since, in my opinion, nothing they say is peculiar to sensible bodies.

Moreover, how is one to understand that the attributes of Number and Number itself are the causes of things that exist and are generated in heaven – both from the beginning and now – and that there is no other number than this Number out of which the cosmos is composed? For, whenever they place Opinion and Opportunity in such and such a region, and Injustice and Separation or Mixture are a bit higher or lower, and they prove by demonstration the fact that (ἀπόδειξιμ λέγωσιν ὅτι) each of these is Number, but there happens to be already a plurality of magnitudes composed [of numbers] in that place because these attributes correspond to each of these places – so is the Number in heaven, which one is to

⁴³ ἔκτοπώτεροις, following Bonitz and *Alexandrii commentaria*. ἔκτοπώτερος is also attested by the *Guilielmi de Moerbeka translatio* (c. 1260-1275 CE) and *Asclepii commentaria*.

⁴⁴ This translation is preferable to Tredennick’s “capable of application to the remoter class of realities” or Ross’s “sufficient to act as steps even up to the higher realms of reality,” neither of which accounts for the technical language of astronomy reported here. In a passage of the *Meteorology* (I.5, 342b30-35), Aristotle describes how the “some of so-called Pythagoreans” believe that Mercury is, like comets, one of the Planets which “does not rise far above the horizon” (τὸ μικρὸν ἐπαναβάνειν), and therefore its appearances are invisible as it is seen in long intervals.

understand as each [of these abstractions], the same [as the one in the lower region], or is it a different number?

(Aristotle, *Metaphysics*, I.8, 989b29-990a29)

Obviously, there is a great deal to unpack in this extended discussion of the “so-called Pythagoreans”. I would like to highlight just a few aspects of Aristotle’s argument. Aristotle seeks a technical language and paradigm in order to respond to what he takes as the fundamental aspects of the “so-called Pythagorean” philosophical system. He points out an apparent absurdity in the “so-called Pythagorean” philosophy, namely that while their first principles are all derived from the mathematical (i.e. non-sensible) science, the objects of their philosophical inquiry (i.e. those things in their *pragmateia*) are things that have been generated and possess motion, namely phenomena. This would be unsurprising, especially if the “so-called Pythagoreans” made the object of their investigations the motions of the heavens, the superordinate science of which would be stereometry. But the example he gives is Number: how can Number, which is a non-sensible entity, both be (a) a superordinate entity (i.e. a “first principle”) that resides in the highest part of reality and (b) correlative with an entity that resides in one of the lower substrates, like Opinion or Opportunity?⁴⁵ In accordance with Aristotle’s

⁴⁵ Compare Aristotle’s extended description of “Pythagorean” number in book N (XIV.3, 1090a20- 35): “As for the Pythagoreans, it was because they saw that many attributes of numbers apply to sensible bodies that they made entities to be numbers – not separate, however, but they made entities derive from numbers. According to what rationale (διὰ τί)? Because (ὅτι) the attributes of numbers apply in musical scale (ἐν ἁρμονίᾳ), both in heaven and in many other things. But those who say that only mathematical Number exists [i.e. Speusippus] cannot make any argument of this sort in accordance with their assumptions; indeed, they [Speusippus] used to say that there cannot be branches of knowledge for those things [i.e. the sensible bodies]. But we say that there can be such branches of knowledge, just as we said before. And it is clear that mathematical objects are not separate, for, if they were separate, their attributes would not apply to bodies. They Pythagoreans are not open to any objection on this score; but by making natural bodies out of numbers – things with lightness of heaviness out of things that do not possess lightness or heaviness – they seem to be talking about a different heaven and bodies than the ones that are sensible.” There are two remarks I have about this passage: first, that I assume that by the time Aristotle was writing *Metaphysics* M and N, he no longer emphasized the distinction between various types of Pythagoreans, and that he was only interested in discussing the mathematical Pythagoreans; second, that Aristotle might be preserving the traces of a mathematical Pythagorean demonstration (ἀπόδειξις) here, when he provides the observed “fact” (i.e. “things are made from numbers”) and then the “reason why” (i.e. “because the attributes of numbers apply in musical scale to things in and below heaven”).

establishment of the subalternate sciences in the *Posterior Analytics*, this approach represents a confusion of the sciences that deal with the “reason why” (τὸ διότι) and the sciences that deal with the “fact” (τὸ ὅτι).

What becomes clear from analysis of this long passage, then, is that the “so-called Pythagoreans” of *Metaphysics A* engaged in demonstrations, which suggests that they are, indeed, the same as the mathematical Pythagoreans as Aristotle had described them in his works on the Pythagoreans. The mathematical Pythagoreans as described in *Metaphysics A* apparently provided at least two demonstrations: (a) that all entities, which are derived from Number, are themselves numbers, on the grounds that all entities possess the attributes of Number; and, possibly, (b) that spatial magnitude is derived from their first principles, namely the objects of mathematics.⁴⁶ Other demonstrations ascribed to the “so-called Pythagoreans” are suggestive, if incomplete, evidence for the topics and methods employed by these philosophers.⁴⁷ In this way, they appear to be distinguished from the acousmatic Pythagoreans, whose philosophical method focused uniquely on the “fact” (τὸ ὅτι), i.e. that which is particular, mutable, and sensible, and known through experience alone.⁴⁸ Aristotle’s focus on the role of the objects of mathematics in the *pragmateia* of the “so-called Pythagoreans” begs the question: if the *pragmateia* of the mathematical Pythagoreans was concerned with the objects of mathematics, which constitute their first principles, how is this system distinguished from the *pragmateia* of the acousmatic Pythagoreans, as it is presented in the *Metaphysics*?

⁴⁶ That Aristotle mentions the proof concerning magnitude suggests it possible that someone could have (or did) try to demonstrate this.

⁴⁷ Among those that I will not discuss further: at *Cael.* II.2, 284b6-8, they have a *logos* – it is unclear how it is demonstrated – that argues that there is a right and left side in heaven; at *Mete.* I.8, 345a14-19, we hear of two kinds of arguments attributed to “so-called Pythagoreans”: the first is mythological (the milky way is a path on the grounds that it is the path of one of the stars that fell at the time of the fall of Phaethon), and the second based on stereometric speculation and natural science (the sun, which was once borne through the circle that is the milky way, created a path when it moved out of this orbit by scorching the region).

⁴⁸ Cf. McKirahan 1992: 242 and Johnson 2009: 336.

One answer to this question arises if we interrogate Aristotle’s descriptions of the first principles of the “so-called Pythagoreans.” In an earlier passage of *Metaphysics A*, where the “so-called Pythagoreans” appear for the first time in the text, we get a more precise description of what Aristotle considered the first principles of the “so-called Pythagoreans”:

In the time of these men [sc. Leucippus and Democritus] and before them the “so-called Pythagoreans” first developed mathematics by applying themselves to it, and by learning it they began to believe⁴⁹ that the first principles of mathematics (ἀρχὰς [τῶν μαθημάτων]) were the first principles of all things in existence (ἀρχὰς τῶν ὄντων πάντων). And since numbers are first among principles⁵⁰ by nature, they seemed to see many homologies (ὁμοιώματα) in numbers with things that exist and things that are coming into being, rather than in fire and earth and water. For example: *this* attribute of numbers was Justice, *that* was Soul and Mind, and another Opportunity, and all the rest, so to speak, in the same way. Moreover, because they saw that the attributes and ratios of musical scales were based on numbers – well, since other things seemed to be homologized (ἀφωμοιῶσθαι) with numbers in their whole nature and numbers seemed to be first-order of all nature, they began to assume that the elements of numbers were the elements of things that exist and that the whole of heaven was musical scale (ἄρμονία) and Number (ἀριθμός). Whatever analogues to the attributes and regions of heaven and the entire order of the cosmos they were able to show (δεικνύνασι) to be in numbers and musical scales, they collected (συνάγοντες) and fit them together (ἐφήρμοστον). And if anything was lacking anywhere, they hastened to supplement it (προσεγγίχοντο)⁵¹ in order that their entire *pragmateia* might be connected. For example, since the Decad is thought to be perfect and to encompass the whole nature of numbers, they assert that there are ten things in heavenly orbit; but since there are only nine that are actually visible throughout the heavens, they invent a tenth, the Counter-Earth.

We have treated this subject in greater detail elsewhere.⁵² But the object of our discussion is to learn from them what first principles they

⁴⁹ Taking, with Tredennick, ὥρθησαν as an inceptive aorist. Aristotle’s reference to the past here reflects his interest in creating a developmental narrative of the history of “so-called Pythagorean” philosophy.

⁵⁰ This is the reading if τούτων refers to the direct object ἀρχὰς in the previous sentence, as Tredennick reads the text. However, Ross plausibly reads against the word order and suggests that τούτων refers to μαθημάτων (Ross 1924: *ad loc.*).

⁵¹ This extremely uncommon word also appears at *Metaph.* XIV.3, 1090b31, where Aristotle likewise complains that some Platonists (perhaps Xenocrates or Speusippus? See Annas 1976: 209-10) hasten to apply mathematics to the Forms (προσγλιχόμενοι ταῖς ἰδέαις τὰ μαθηματικά). At *Cael.* II.13, 293a27, Aristotle also accuses some of the “so-called Pythagoreans” of “attracting the phenomena to certain rationales/reasons and opinions of their own (πρὸς τινὰς λόγους καὶ δόξας αὐτῶν τὰ φαινόμενα προσέλκοντες).”

⁵² It has been discussed at *Cael.* II.13, 293a18-b15, an extremely challenging passage that has presented many difficulties for scholars; it may also have been discussed in Aristotle’s lost writings on the Pythagoreans. In general,

posit and how these correspond to the causes we have discussed. Well, then, it appears that these men too believe that Number is a first principle, both as matter to things that exist and as [constituting] their attributes and states. They believe: that the elements of Number are Odd and Even, and that, of these, that the former has been limited (πεπερασμένον), and the latter is unlimited (ἄπειρον); the One is constituted of both of these (since it is both odd and even); that Number is derived from the One; and, as we've already said, that the whole heaven is numbers.

(Aristotle, *Metaphysics*, I.5, 985b24-986a21)

I think it best to follow the analysis of Huffman (1993: 231-61), which is by far the most comprehensive study of this fragment and others related to it. For my purposes, however, I would like to make two points that are not mentioned by Huffman (in so many words) and diverge, at least slightly, from his interpretation: first, that Aristotle apparently distinguishes between **three** types of Pythagoreans in this passage, namely two types of “so-called Pythagoreans” and a simpler set of “Pythagoreans.” Both types of “so-called Pythagoreans” possess knowledge of the “reason why” and are thus able to make demonstrations: the former “do not pursue arguments and causes in conformity with the phenomena” but “attract the phenomena to certain rationales and opinions of their own.” It is important to note that, while Aristotle considers that this type fails from a methodological point of view, they are characterized as making use of phenomena (i.e. “facts” in Aristotle’s mind) in their philosophical *pragmateia*. The latter type of “so-called Pythagoreans,” referred to as “many others,” make no use of phenomena whatsoever. Their arguments are based on establishing a hierarchy from what is “most honorable” to what is “more honorable” etc. Therefore, so Aristotle claims as he apes the arguments of these “so-called Pythagoreans”, fire, which is a more honorable thing, must be at the limit, which is a more honorable place. The limit is at the center. It follows, then, that fire must be at the limit, i.e. at the center. This process Aristotle identifies as “proportioning/creating analogues” (ἀναλογιζόμενοι), and, although deficient too, it is at least conceptual. Burkert (1972: 326-7 n. 16) wishes to identify the latter “so-called Pythagorean” group with Speusippus, but that is unlikely, since there is no reason to believe that the passage he calls on as evidence (Theophr. *Metaph.* II A18-26 = Tarán F 83) is referring to astronomy (cf. Tarán 1981: 445). Both these groups, I maintain, are distinguished from those identified as “the Pythagoreans” who, at no place, are identified as positing arguments for their assumptions. Instead, Aristotle claims that they “call the fire which occupies this place the ‘Watch-tower of Zeus’, as if it were simply (ἀπλῶς) called the center.” Moreover, this simple statement of naming recalls the *acusmata*. My second point concerns the famous problem of the correlation of the second group of “so-called Pythagoreans” with what Simplicius (*In Cael.* 511.25ff. = Arist. F 204 Rose) calls “those who partake of the views (concerning the earth and counter-earth, previously expressed) more genuinely (γνησιώτερον).” Several scholars (notably Cornford 1937: 126-8; for a list, see Burkert 1972: 232 n. 73) ascribed to the position that Aristotle originally identified the second group of “so-called Pythagoreans” with this group, on the assumption that they represented an earlier manifestation of Pythagoreanism. Burkert (1972: 232-3) and Cherniss (1944: 562) have sufficiently established that Simplicius’ source here is not, as Cornford and others assumed, Aristotle, but Iamblichus instead. This is likely, and, moreover, the presence of Neoplatonizing aspects of the description, e.g. that “the fire in the middle is the demiurgic force (δημιουργική δύναμις) which, from the middle, generates life on the whole earth and warms the parts of it that have become cool” and that “the earth is a star insofar as it is an instrument of time (ὄργανον χρόνου)” (trans. Huffman, with minor changes), recall the sorts of doxographical accounts of Hippasus’ philosophy as preserved in Iamblichus. For example, Iamblichus (*in Nicom. Arithm.* 10.20 Pistelli = F 12 Timpanaro Cardini) claims that, for the *acusmatics* who follow Hippasus, “Number is...the discerning tool of God the cosmurge” (κριτικὸν κοσμουργῶ θεοῦ ὄργανον). The language used to establish the doctrinal positions is similar, and might be Middle Platonic, as I will argue in Chapter 2. If this speculative argument is correct, Simplicius might be referring to one doxographic tradition (also preserved by Iamblichus) that has it that the *acusmatics* are those Pythagoreans who partake of views that are “more genuine.” This would represent a confusion of Aristotle’s position *viz.* mathematical or acousmatic Pythagorean, I would argue, as was common among the later Neoplatonist commentators.

Now, it is difficult to infer what came before these lines: there is a significant lacuna in the manuscripts (169 letters by Ross's figuration)⁵³ preceding this passage, which must have discussed the theories of the Monists and the Dualists. Presumably, Aristotle saw the "so-called Pythagoreans" as a group whose first principles provided a bridge between the natural scientists and Plato, because the description of Platonic ideal theory follows directly after the section that discusses the first principles of the "so-called Pythagoreans" and the "Pythagoreans."

This passage gives us a sense of what Aristotle thought the philosophical method and the first principles of the "so-called Pythagoreans" to be. According to Aristotle, the "so-called Pythagoreans" allow their systematization by means of the "reason why" to contaminate their analyses by means of the "fact" by "hastening to supplement" whatever might be lacking in the empirical aspect of their philosophy.⁵⁴ Or, to put it another way, they change the fact (τὸ ὅτι) to fit the explanation (τὸ διότι) in an ad hoc manner. The example given involves the bodies of the heavens: there are nine bodies that can be perceived by the senses, but, since the Pythagoreans premise the Decad as the perfect number, and since all things are Number, there must be ten heavenly bodies. This example reveals the second major criticism that Aristotle levels against the philosophical system of the "so-called Pythagoreans", namely that they make hasty and careless use of analogy and proportioning in order to establish a means between their first principles and observed phenomena. This process of creating an a priori system that flies in the face of observed empirical evidence with appeal to abstruse types of proportioning is considered "fitting together" (εἰσφέρουσιν), a word whose semantics are strongly related to the concept of "musical scale" (ἁρμονία). Of course, we are immediately reminded of the beginning of Philolaus' *On Nature* (F 7 Huffman = Stob. *Ecl.* 1.21.8), in which he claims that

⁵³ Ross 1924: clix.

⁵⁴ Compare Arist. *Metaph.* I.2, 981a12ff., on which see McKirahan 1992: 242.

“the first thing fitted together (τὸ πρῶτον ἄρμωσθέν), the One in the center of the sphere, is called the hearth.” Huffman has shown that Philolaus is to be included among the “so-called Pythagoreans,” and that Aristotle had Philolaus chiefly in mind when discussing these issues.⁵⁵

Now before Aristotle gets too carried away discussing the philosophical method of the “so-called Pythagoreans,” he reminds himself (and the reader) that the topic under discussion is their first principles, which he proceeds to describe. He claims that the “so-called Pythagoreans” posit the objects of mathematics as the first principles and elements of everything. One example he gives is Number, which is primary in two senses: as what Aristotle calls formal and material cause. This concern with formal and material is reinforced, in this passage, by the description of those things that are primary as either first principles (ἀρχαί) or elements (στοιχεῖα). They are first principles *vis-à-vis* their function as formal causes, but they are elements *vis-à-vis* their function as immanent materials from which things in the universe are constructed.⁵⁶ These he defines in according to a stratified hierarchy: the Odd and Even are apparently prior to the One, since the One is constructed out of them, and the One is prior to Number, since Number is constituted of the One. From Number come all entities within phenomenological reality. But this passage also reveals another set of principles ascribed to the “so-called Pythagoreans” that are even apparently prior to Odd and Even: namely, Limit and Unlimited, given that the attributes of Limit are present in the Odd (i.e. it “has been limited” [πεπερασμένον]), and the

⁵⁵ Huffman 1993: 225, with reference to this passage, but without a sufficient discussion of Aristotle’s description of “so-called Pythagorean” methodology. For a very good general analysis of the relationship between Philolaus’ fragments and this passage, see Huffman 1993: 177-93.

⁵⁶ The subject of the relationship between first principles and substances is well-trodden ground, and I don’t wish to pursue this question too far. It does not appear to me that, in *Metaphysics A*, Aristotle distinguishes explicitly between these, nor that he has discovered a clear means of distinguishing them, but that does not mean that they are simply synonymous either. By the time he composed Book Λ (XII.6, 1071b22-6), he had at least distinguished between first principles as external and elements as inherent. It is suggestive in Book Δ (V.1, 1013a8-12) that he defines ἀρχή in several ways, among them (a) the thing as a result of whose immanent presence something first comes into being, and (b) that from which something comes into being, although it is not present in it. When defining στοιχεῖον in the same book (V.3, 1014a25ff.), he clarifies that it is an immanent, indivisible, entity out of which other things are composed.

Even is Unlimited (ἄπειρον).⁵⁷ It is important to note here that a hierarchy of entities, even among first principles, is implicit in the passage, and that the “so-called Pythagoreans” employ demonstration of this hierarchy by means of a rationale. Moreover, the hierarchy of entities apparently is ordered according to the idea that the prior principles act upon those that are given definition by them: in the case of Limit and Unlimited, we have good evidence from the genuine fragments of Philolaus that Aristotle, while he was distorting Philolaus’ thought, was essentially presenting a verifiable account of how some of the more sophisticated Pythagoreans undertook demonstrative proof by employing mathematical objects as principles in demonstrations that involve sensible objects. One example of this is a particularly thorny fragment of Philolaus preserved by Stobaeus:

It is necessary (ἀνάγκη) that the things that are be all either Limiting (περσίνοντα), or Unlimited (ἄπειρα), or both Limiting and Unlimited, but not in every case Unlimited alone. Well then, since it is apparent (φάνεται) that they are neither from Limiting things alone, nor from Unlimited things alone, it is clear that the cosmos and the things in it were fitted together (συναρμόχθη)⁵⁸ from both Limiting and Unlimited things. Things in their functions also make this clear (δηλοῖ δὲ καὶ τὰ ἐν τοῖς ἔργοις). For, some of them from Limiting [constituents] limit, others from both Limiting and Unlimited [constituents] both limit and do not limit, others from Unlimited [constituents] will appear to be Unlimited.

(F 2 Huffman = *Eclogae* I.21.7a; Translated by Huffman, with slight changes)

Careful examination of this fragment demonstrates that Aristotle’s criticism is not off the mark:

Philolaus demonstrates by reducing the objects of mathematics to the sensible world. This is

⁵⁷ This is confirmed at *Metaph.* I.8, 990a8-12.

⁵⁸ The appearance of this term, along with other terms related to ἀρμόζω, in other fragments of Philolaus (e.g. F 1 Huffman = D.L. 8.85: ἡ φύσις δ’ ἐν τῷ κόσμῳ ἀρμόχθη ἐξ ἀπειρῶν τε καὶ περσίνοντων; F 7 Huffman = Stob. *Ecl.* 1.15.7: τὸ πρῶτον ἀρμῶσθεν...ἐστὶ καλεῖται) is suggestive evidence for the correlation between Aristotle’s description of the demonstration of the “so-called Pythagoreans”, which involves “fitting together in addition” (ἐφήρμοστων) the “analogues to the attributes and regions of heaven and the entire order of the cosmos”. Again, this term could also mean “co-harmonized.”

suggested in the first few lines, but it is explicit when Philolaus claims that one can see that Limiters and Unlimited things constitute the cosmos when one detects them in “actions” (τὰ ἔργα), a word which refers to sensible activities.⁵⁹

If I am correct in thinking that the “so-called Pythagoreans” as described in Aristotle’s *Metaphysics A* are one and the same as the mathematical Pythagoreans in his writings on the Pythagoreans, then we should expect to find a description of the *pragmateia* of the acousmatic Pythagoreans, who, so Aristotle claims in the lost writings on the Pythagoreans, did not engage in demonstrations. Indeed, immediately following the long passage I translated just above (*Metaph. I.5, 985b24-986a21*), our hypotheses are corroborated in a concise description of what appears to be the *pragmateia*, or objects of philosophical inquiry and treatment of those objects, of the acousmatic Pythagoreans:

People other than these very people (i.e. the “so-called Pythagoreans”) say that there are ten principles, which they name in two elementary columns of cognates:

Limit	Unlimited
Odd	Even
One	Plurality
Right	Left
Male	Female
Rest	Motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong... ⁶⁰

The Pythagoreans declared how many and what sorts (πόσῃ καὶ τίνας) of contraries there were. Thus, from both of these authorities [i.e. Alcmaeon of Croton and the Pythagoreans] we can gather this much, that the contraries are the first principles of things; and how many and what sorts these are [we can gather] from one of these authorities. But how (πῶς) these principles can be brought together (συνάγειν) and referred to our aforementioned list of causes has not been clearly articulated by them, but they seem to arrange (τάττειν) their elements under the grouping of

⁵⁹ See Huffman 1993: 111-2.

⁶⁰ Following Huffman (1993: 10-11), I have excised anything that deals explicitly and solely with Alcmaeon of Croton, whose status as a “Pythagorean” is questionable.

matter; for they say that substance is composed and fashioned out of these underlying elements.

(Aristotle, *Metaphysics* I.5, 986a22-b8)

Aristotle, then, distinguishes these “Pythagoreans” from the “so-called Pythagoreans” in this passage by arguing that their scientific pursuit only goes so far as to declare the number and types of contraries and put them into an order. There is neither any mention of demonstration nor any evidence of speculation into the “reason why” (τὸ διότι) their so-called “Table of Contraries” exists. Moreover, there is no hastening to connect a holistic system, which was characteristic of the “so-called Pythagorean” philosophical activity. In fact, there is little discussion of any “activity”, other than arrangement, at all. What the first principles and elements of the “Pythagoreans” represent, according to Aristotle’s criticism, is only the material cause, and they have no capacity – as the first principles of the “so-called Pythagoreans” do – as *formal* causes. That is to say, in the case of the “Pythagoreans”, there is no attempt to discuss, for instance, *how* the limiter *limits* things in existence.⁶¹ Because these “Pythagoreans”, who are “other” than the “so-called Pythagoreans,” view the universe as constituted of unchanging elements, they do not concern themselves with the activities or motions that cause generation within the universe. Given the lack of attribution to these “Pythagoreans” any sort of rationalization that involves demonstrations, it is a reasonable speculation that Aristotle considered these “Pythagoreans” to be one and the same with the acousmatic Pythagoreans discussed in his lost writings on the Pythagoreans.

There is a third and final passage in *Metaphysics* A that refers to Pythagoreans of one or the other sort, and we ought to identify (if possible) the group to which it refers and consider what it tells us about them. It is a passage crucial to our understanding of Pythagoreanism, as

⁶¹ Cf. Huffman 1993: 47 n. 1.

reconstructed and appropriated by Aristotle, because it illuminates another way in which Pythagoreans engaged in their *pragmateia*, namely through definitions:

But while the Pythagoreans have declared in the same way that there are two first principles, they made this addition (προσεπέθεσαν), which is peculiar to them, namely that they thought that the Limited and the Unlimited were not uniquely different substances⁶², such as fire and earth and anything else of this sort, but that the Unlimited itself and the One itself were the substance of the things of which they are predicated, and hence (διό) that Number is the substance of all things. Thus, they have made their declarations in this way; and concerning the essence (i.e. the “what is” [περὶ τοῦ τί ἐστίν]), they began to make statements and definitions (λέγειν καὶ ὀρίζεσθαι), but their treatment was too simple (λίαν ἀπλῶς ἐπραγματεύθησαν). For they both defined superficially and thought that the substance of a thing (ἡ οὐσία τοῦ πράγματος) was that to which a stated term would first be predicable, e.g. as if someone were to believe that “double” and “two” were the same because “two” is the first thing of which “double” is predicable. But surely to be “double” and to be “two” are not the same things. If that were to be the case, many things would be One, a consequence which they actually drew.

(Aristotle, *Metaphysics* I.5, 987a13-27)

While it is true that Aristotle refers to this group as “the Pythagoreans,” and not the “so-called Pythagoreans,” it is nonetheless clear that this is a description of a mathematical Pythagorean *pragmateia*.⁶³ This group of Pythagoreans is not simply listing first principles as contraries and assuming them as elemental to all things in existence. According to Aristotle, they “make statements and definitions” and engage in analysis concerning the “what is”, although their “treatment” (ἐπραγματεύθησαν, i.e. the activity of their *pragmateia*) was too simplistic. There also appears to be some preservation of the argumentative technique, where Aristotle claims that these Pythagoreans thought that, since the “Unlimited” and the “One” are the substance (οὐσία)

⁶² Taking φύσις in the sense later defined by Aristotle in Book Δ (V.6, 1014b35ff.), and only because it makes sense of Aristotle’s use of the term “substance” (οὐσία) in the next sentence.

⁶³ There could be a very good reason for this. As Cherniss (1944: 192 with n. 112) points out, this passage appears to have been inserted later by Aristotle. If, as I’ve suggested, Aristotle only distinguished between the “so-called Pythagoreans” (i.e. mathematical) and the “Pythagoreans” (i.e. acousmatic) in his earlier treatments of the history of philosophy, which would include the crucial passage (I.5, 985b24-986b8) that demonstrates the differences, and if later on he only concerned himself with the philosophy of the mathematical Pythagoreans, then it would be unsurprising for him to refer to the mathematical Pythagoreans here as “Pythagoreans” *simpliciter*.

of things of which they are predicated, *therefore* Number is the substance of all things. But how are we to understand this statement? Is it that Number underlies Unlimited and the One, or is it the other way around? The logic is apparently confused, which would not be surprising given Aristotle’s willingness to distort the mathematical Pythagoreans’ arguments in order to demonstrate the impropriety of their philosophical method.⁶⁴

The accumulation of evidence concerning the *pragmateia* of the mathematical Pythagoreans from *Metaphysics A* corroborates and further develops the claim that I have made, namely that Iamblichus in *On the General Mathematical Science* (76.19-77.22) has excerpted a section from Aristotle’s works on the Pythagoreans that accounts for the difference between the *pragmateia*, or object of the philosophical study and its treatment, of the mathematical and the acousmatic Pythagoreans. So much for the treatment of the “so-called Pythagorean” philosophy in *Metaphysics A* which, although it presents remarkable difficulties for interpretation, can be said to preserve the distinction between the mathematical and acousmatic Pythagoreans. But there remains one last passage from *On the General Mathematical Science*, established as genuinely Aristotelian by Walter Burkert⁶⁵ and analyzed closely by Myles Burnyeat⁶⁶, that we ought to investigate in our pursuit of the *pragmateia* of the mathematical Pythagoreans as

⁶⁴ Cf. Huffman 1993: 206-7. The closest point of comparison I can discover for this passage is Philolaus F 6 (= Stob. *Ecl.* I.21.7d), where Philolaus claims:

Concerning nature and harmony the situation is this: the being of things (ἀἴστω τῶν πραγμάτων), which is eternal, and nature in itself admit of divine and not human knowledge, except that it was impossible for any of the things that are and are known by us to have come to be, if the being of the things from which the cosmos came together, both the Limiting and the Unlimited things, did not preexist (μὴ ὑπαρχούσας). But since these beginnings preexisted (ταῖ ἀρχαῖ ὑπάρχον) and were neither alike nor even related, it would have been impossible for them to be ordered, if a harmony had not come upon them, in whatever way it came to be...

Philolaus’ employment of the concept of “preexistence” likely anticipates Aristotle’s theories of predication and substrate, but it is easy to see how Aristotle could have confused things in his attempt to appropriate mathematical Pythagoreanism to his own philosophical systems and technical vocabulary. Of course, the use of the term ὑπαρχεῖν and its cognates in describing some basic sense of preexistence is attested elsewhere in the 5th Century BCE. Cf. Nussbaum 1979: 101 n. 94.

⁶⁵ Burkert 1972: 50 n. 112.

⁶⁶ Burnyeat 2005a: 38-43.

formulated by Aristotle. This passage continues from the earlier passage I discussed above (on page 14) in *On the General Mathematical Science* (76.19-77.22) and is likely to be extracted from the same lost writings of Aristotle:

(A) The Pythagoreans devoted themselves to mathematics. They both admired the accuracy of its reasonings, because it alone among things that humans treat contains demonstrations (εἶχεν ἀποδείξεις ὧν μετεχειρίζοντο), and they saw that general agreement is given in equal measure to theorems concerning attunement, because they are [established] through numbers, and to mathematical studies that deal with vision, because they are [established] through diagrams. This led them to think that these things and their principles are quite generally the causes of existing things. Consequently, these are what anyone who wishes to comprehend things in existence – how they are – should turn their attention to, namely numbers and geometrical forms⁶⁷ of existing things and proportions, because everything is made clear (δηλοῦσθαι) through them. So, consequently, by attaching the powers of each thing to the causes and primaries – only things that were less opportune or less honorable than them – they defined other things too in nearly the same manner. (B) Therefore, their education in numbers and the objects of mathematics (τὰ μαθήματα τῶν πραγμάτων) seemed to come through these subjects and in this general sketch. Such was also the method of demonstrations (ἡ μέθοδος τῶν ἀποδείξεων) among them, which both began from such principles and thus attained fidelity and security in their reasonings.

(Iamblichus, *On the General Mathematical Science* 78.8-26)

Now this is a passage that is extremely difficult to assess because it has been thought to possess spurious insertions in at least two sections.⁶⁸ Burnyeat, however, has argued convincingly that the object of the criticism here is likely Archytas of Tarentum, who engaged in mathematical proof of a more complex sort than, e.g., Philolaus.⁶⁹ This is in keeping with the stylistic traits of Iamblichus when he quotes from Aristotle, as recently analyzed by D.S. Hutchinson and Monte

⁶⁷ The text has γεωμετρούμενα εἶδη τῶν ὄντων, which does, indeed, sound suspicious, and was originally excised by Burkert. But see Burnyeat 2005a: 42, with explicit appeal to Arist. *APo.* I. 13, 79a7-8, which I discussed above.

⁶⁸ For the arguments in defense of the deletions, see Burkert 1972: 447-8.

⁶⁹ Cf. Burnyeat 2005a, followed by Huffman 2005: 563-8. But Huffman maintains, against Burnyeat, that Aristotle is referring to Philolaus or, at any rate, not Archytas. I will discuss this at the beginning of Chapter 2.

Ransome Johnson: he tends to preserve large blocks of material without modifying them.⁷⁰ But what about the summary of information that follows, which I've marked as (B)? There is nothing that necessarily marks its contents as Iamblichean, *e.g.* no extopic explanations or markedly Neoplatonic rationalizations. But Iamblichus refers to the educational curriculum of the mathematical Pythagoreans as dealing with the “objects of mathematics” (τὰ μαθημᾶτα τῶν πραγμάτων), a specific term that is unattested anywhere else in Iamblichus' oeuvre, or, for that matter, in Greek philosophy. We do, however, see something very close to it in Aristotle's description in *Metaphysics A* of the *pragmateia* of Plato⁷¹, which is considered a successor to the philosophical *pragmateia* of the “Italians”⁷², although with some innovations:

So then, these sorts of entities he called Forms, and he believed that all sensible things were called after them and existed in virtue of them. For he held that the plurality of things which possess the same name as their Forms did so in virtue of their participation in them (κατὰ μέθεξιν). With regard to “participation”, he only changed the name: for whereas the Pythagoreans claim that entities exist by means of imitation of numbers (μίμησει τῶν ἀριθμῶν), Plato says by means of participation (μέθεξει), modifying the name. As to what participation or imitation is, however, they left it to us to seek it out together.

Furthermore, Plato claims that besides sensibles and Forms is a middle type of entity, the objects of mathematics (τὰ μαθηματικὰ τῶν πραγμάτων), which differ from sensibles in being eternal and immutable, and from Forms in that many [objects of mathematics] are similar, whereas each Form itself is unique.

(Aristotle, *Metaphysics* I.6, 987b7-18)

Obviously, this is one of the more problematic passages in the history of ancient philosophy, and the task to identify what the objects of mathematics, as intermediaries between the Forms and sensibles, are is not made easier by Aristotle's admitted confusion.⁷³ Part of the problem here is

⁷⁰ Cf. Hutchinson and Johnson 2005: 281-2.

⁷¹ Identified explicitly as such at I.6, 987a30.

⁷² Arist. *Metaph.* I.6, 987a29-31.

⁷³ See Ross's useful discussion of the problems that arise from this passage and for a history of their treatment from antiquity to the early 20th Century (1924:161-8). Jaeger concerned himself with the principle of intermediary,

that the term τὰ μαθηματικὰ τῶν πραγμάτων, like the explicit ascription of a theory of imitation (μίμησις) in numbers to the Pythagoreans, occurs nowhere else in the surviving works of Aristotle, or, for that matter, in surviving Greek literature. It is not clear from this passage whether Aristotle would consider τὰ μαθηματικὰ τῶν πραγμάτων to be distinguished from other terms he uses to describe the objects of mathematics, especially the relatively common simple formulation in τὰ μαθηματικά, which he uses often in reference to the ontological theories of Plato, Speusippus, and Xenocrates.⁷⁴ We have seen, of course, that Aristotle mentioned the “objects of the mathematics” (τὰ μαθηματικὰ τῶν ὄντων) in reference to the first principles of the “so-called Pythagoreans” (*Metaph.* I.8, 989b32). Generally, Aristotle does not distinguish between τὰ πράγματα or τὰ ὄντα in referring to “things” that exist, but the unusual complex formulation of τὰ μαθηματικὰ with either τὰ πράγματα or τὰ ὄντα in the genitive plural is uniquely Aristotelian, and, moreover, localized to discussions of Plato or the mathematical Pythagoreans (and, importantly, not Speusippus or Xenocrates) in *Metaphysics A*. Indeed, the relatively strange formulation τὰ μαθηματικὰ τῶν πραγμάτων resembles the language of Philolaus of Croton (F 6 Huffman = Stob. *Ecl.* I.21.7d), who, when he spoke of the “being of things” (ἃ ἔστι τῶν πραγμάτων), was referring to the existence of Limiters and

without focusing upon the objects of mathematics (1948: 91 with n. 2). Cherniss (1945: 75-8) denied that the objects of mathematics as intermediates existed for Plato and considered the ascription of this by Aristotle possibly to have been a misunderstanding of a passage from Plato’s *Republic* (551a-e). Tarán (1981: 23 n. 120) followed Cherniss but saw the ascription of intermediary objects of mathematics to Plato as a point of contrast to Speusippus’ postulation of separate and unchangeable mathematical numbers/ideas and magnitudes. Burkert (1972: 43-5) plausibly connects “imitation” to Aristotle’s descriptions of “homologies” (ὁμοιώματα) at *Metaph.* I.5, 985b27 and concludes that “again and again it becomes clear that the Pythagorean doctrine cannot be expressed in Aristotle’s terminology.” Annas’ (1975, 1976: 19-21) appeal to the Uniqueness Problem provides a useful critical response to the issue, but does not deal with Aristotle’s focus on participation and its relation to the doctrine of imitation attributed to the Pythagoreans. Amazingly, the issues of intermediary and imitation in this passage are not discussed by Fine (1993).

⁷⁴ E.g. *Metaph.* VII.17, 1042a12; XII.1, 1069a35; XIII.1, 1076a33; XIII.2, 1077a16; XIII.3, 1077b33, et al. Aristotle will speak of mathematical that are “separate from” + genitive (e.g. *Metaph.* XIII.2, 1076a33-4: κεχωρισμένα τῶν αἰσθητῶν) or “intermediate of” + genitive (e.g. *Metaph.* XI.1, 1059b6: μεταξὺ τε τῶν εἰδῶν καὶ τῶν αἰσθητῶν).

Unlimited things, the mathematical principles of his philosophy.⁷⁵ It is therefore probable that Iamblichus, in mentioning τὰ μαθήματα τῶν πραγμάτων in passage (B) from *On the General Mathematical Science*, continued to refer to Aristotle's treatise on the Pythagoreans, which was written contemporaneously with *Metaphysics A* and, importantly, earlier than the treatments of the Pythagoreans in M or N. This is significant, because it suggests that it was Aristotle who credited the mathematical Pythagoreans with having achieved credibility in their method of argumentation, even if they were overzealous in their pursuit of a complete philosophical system.

Let us conclude our study of the *pragmateia* of the mathematical Pythagoreans as described by Aristotle in his early works on the Pythagoreans. Our examination of this question has shown that the fundamental difference between acousmatic and mathematical Pythagoreanism as formulated by Aristotle lies in the latter group's attempts to make use of demonstrative argumentation in order to provide explanations for their ideas. While acousmatic Pythagoreans apparently simply accepted the facts as they were, mathematical Pythagoreans engaged in investigations that employed the principles of mathematics in order to make sense of the world they experienced. Their demonstrations tended to be derived from the fundamental principles of mathematics, including Limiter and Unlimited, as attested in the genuine fragments of Philolaus of Croton. It is also possible, as Burnyeat has argued, that their demonstrations were based on axioms and took the form of diagrams, as in the case of Archytas of Tarentum's approach to optics.⁷⁶ Doubtless there were other types of demonstrative proof that, while satirized by the contemporary authors of Middle Comedy Alexis of Thuri and Cratinus the Younger, have been lost to us, and the plurality of the objects of their study made it difficult for

⁷⁵ The term ἄ ἐστὼ τῶν πραγμάτων has been grounds for dismissal of this fragment as authentic, especially since the term itself is replicated in the spurious Ps.-Archytan Περὶ ἄρχων. But the authenticity of Philolaus' fragment has been defended on various grounds by Nussbaum 1979: 101 with n. 94 and Huffman 1993: 131-2.

⁷⁶ Burnyeat 2005a: 45-51.

Aristotle to simply characterize their philosophical system and to locate it within his history of philosophy. The mathematical Pythagoreans were apparently also prone to establish analogues between numbers and sensible things, and they posited an ontological order that was based on attributes that were related strongly to social organization within the *polis*, such as the notion of “what is more honorable,” which suggests an organic relationship between the terms of political order and of ontological hierarchy. This important aspect of Aristotle’s description of the *pragmateia* of the mathematical Pythagoreans I will investigate in the next two chapters.