

Quantum Control and Quantum Estimation Theory

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Preliminaries

- 0 Quantum mechanics background
 - 0.1 Hilbert space
 - 0.2 Density matrix
 - 0.3 Discrete vs continuous systems
 - 0.4 Observables
 - 0.5 Schrodinger (coherent) evolution
 - 0.6 Heisenberg picture
 - 0.7 Solving the Schrodinger eqn (numerical)

Part I: Quantum optimal control theory

Chapter 1. *Control system background/definitions*

- 1.1 Linear/bilinear control systems
- 1.2 Controllability
 - 1.2.1 Operator
 - 1.2.2 Pure state
 - 1.2.3 Mixed state/multiobservable
 - 1.2.4 Exact time controllability
 - 1.2.5 Infinite dimensional systems

Chapter 2. *Quantum optimal control theory*

- 2.1 Euler-Lagrange equations
- 2.2 Pontryagin's maximum principle
- 2.3 Control objective functionals
- 2.4 Analytical solutions
 - 2.4.1 Quadratic cost on control
 - 2.4.2. Uniformly bounded control
 - 2.4.3 Time optimal control

- 2.5 Control Mechanisms (constructive/destructive interferences)
 - 2.5.1. Bang-bang and singular controls
 - 2.5.2. Resonant controls
- 2.6 Open systems
- 2.7 Infinite-dimensional, chaotic systems

Chapter 3. *Quantum control landscapes*

- 3.1 Topology (regular extremals)
 - 3.1.1 Observables
 - 3.1.2 Gates
- 3.2 Singular extremals in quantum control
 - 3.2.1 Local surjectivity Gramian
- 3.3 Optimization complexity
 - 3.3.1 Bounds on the Gramian
 - 3.3.2 Linear programming complexity

Chapter 4. *Numerical methods for quantum control*

- 4.1 Stochastic search algorithms
 - 4.1.1 Genetic, evolutionary algorithms (GA, EA)
 - 4.1.2 Multiobjective evolutionary algorithms (MOEA)
- 4.2 Deterministic search algorithms
 - 4.2.1 Steepest ascent/Conjugate gradient
 - 4.2.2 Iterative algorithms
 - 4.2.3 Homotopy tracking (continuation) algorithms

Chapter 5. *Applications of (open loop) quantum optimal control*

- 5.1 Single observable control
 - 5.1.1 Single observable maximization (controlled fragmentation)
- 5.2 Multiobservable/multiobjective control
 - 5.2.1 Pareto optimal control
 - 5.2.2 Chemical reaction selectivity/Optimal dynamical discrimination
- 5.3 Gate control (quantum computation)
- 5.4 Example problems

Chapter 6. *Experimental methods for (open loop) quantum optimal control*

- 6.1 Radiofrequency nuclear spin control
- 6.2 Femtosecond laser control of molecular dynamics
- 6.3 Open loop architectures

Part II: Quantum estimation theory and stochastic control

Intro: Transition from deterministic to stochastic control

Chapter 7. *Concepts of quantum probability*

- 7.1 Noncommutative probability spaces
- 7.2 Quantum de Finetti representation
- 7.3 Quantum Fisher information
- 7.4 Quantum Bayesian Decision Theory
- 7.5 Quantum state estimation (stationary)

Chapter 8. *Quantum stochastic processes*

- 8.1 Quantum Wiener processes (quantum noise)
- 8.2 Ito/Stratnovich quantum stochastic differential equations (QSDE)
- 8.3 Master equation (Markov)
- 8.4 Equivalence with completely positive maps
- 8.5 Quantum non-Markovian stochastic processes

Chapter 9. *Quantum measurement*

- 9.1 Continuous (generalized) quantum measurement/detection, back action
- 9.2 Sequential measurements: time-ordered correlation fns
- 9.3 Imprecise measurements

Chapter 10. *Quantum filtering and forecasting*

- 10.1 Observability
 - 10.1.1 Duality with controllability
 - 10.1.2 Stochastic observability
 - 10.1.3 Sequential measurements on same or diff systems
 - 10.1.4 Coherent observability
- 10.2 Kalman filter; maximum likelihood estimate (frequentist)
- 10.3 Filtering of dynamical parameters, latent variables
- 10.4 Bayesian vs frequentist filtering: accuracy, efficiency
- 10.5 Forecasting: Bayesian and frequentist
- 10.6 Example: two-level systems; quantum optics

Chapter 11. *Closed loop quantum feedback*

- 11.1 Classical vs coherent feedback controllers
- 11.2 Optimal feedback control
 - 11.2.1 Feedback control law: Hamilton-Jacobi-Bellman

- equations (dynamic programming)
- 11.2.2 Perturbative feedback control
- 11.2.3 Regulator synthesis, stability; Riccati eqns
- 11.2.4 Coherent feedback
- 11.3 Stochastic feedback control theory
 - 11.3.1 Connection to quantum filtering
 - 11.3.2 Continuous feedback with measurements
 - 11.3.3 Average behavior of optimal control system
 - 11.3.4 Feedback time delay and non-Markov processes
- 11.4 Examples: discrete (spin control; error correction) and continuous systems

Chapter 12. *Numerical methods for quantum estimation*

- 12.1 Maximum likelihood estimation (frequentist)
- 12.2 Markov Chain Monte Carlo (Bayesian)
- 12.3 Complexity: optimization vs conditional simulation
- 12.4 Dynamic programming algorithms

Appendix

- A.1 Standard approximations useful for analytical solutions
- A.2 Lie groups and Lie algebras
- A.3 Review of classical probability
- A.4 Classical stochastic processes
 - A.4.1 Classical Wiener processes
 - A.4.2 Langevin equations, stochastic differential equations
 - A.4.3 Markov processes
 - A.4.4 Kolmogorov forward equation
- A.5 Open quantum systems
 - A.5.1 Lindblad condition (conservation of probability)
 - A.5.2 Completely positive maps
- A.6 Matrix Riccati equations

References

Index