Numerical Solutions Technical Appendix for Comparative Advantage and Heterogeneous Firms

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1. Introduction

This technical appendix contains further details on the numerical solutions of the model discussed in the main paper.

2. Numerical Solution Files

Six Matlab files are enclosed with this technical appendix:

File	Description
`HoHet_Autarky_Restud.m`	Solves for autarky equilibrium in the heterogeneous firm model
`HOHet_CT_Restud.m`	Solves for costly trade equilibrium in the heterogeneous firm model
`HK_Autarky_Restud.m`	Solves for autarky equilibrium in the HK benchmark model
`HK_CT_Restud.m`	Solves for costly trade equilibrium in the HK benchmark model
`Costly_Realloc_Restud.m`	Evaluates between-industry, within-industry and total job turnover as the economy moves between autarky and costly trade steady-state equilibria in the heterogeneous firm model
`Costly_Churn_Restud.m`	Evaluates steady-state churning in the heterogeneous firm model

Sections 3., 4. and 5. below discuss the organization of the file `HK_CT_Restud.m` in further detail. The other files are organized analogously. Section 6. below discusses the job turnover code in the file `Costly_Realloc_Restud.m`. Section 7. below discusses the steady-state churning code in the file `Costly_Churn_Restud.m`.

3. Productivity Distribution

The productivity distribution is assumed to be *Pareto-1*:

$$g(\varphi) = ck^c\varphi^{-(c+1)} \quad \text{where} \quad k > 0,\ c > 0 \quad \text{and } \varphi \geq k$$

The corresponding cumulative distribution function for productivity is $G(\varphi) = 1 - \left(\frac{k}{\varphi}\right)^c$. We assume $c > \sigma - 1$, so that the variance of log productivity is finite, in which case the term $\varphi^{\sigma-1}g(\varphi)$ follows a Pareto distribution with parameters $\gamma \equiv c - \sigma + 1$ and k:

$$\begin{aligned} \varphi^{\sigma-1}g(\varphi) &= \xi h(\varphi), \\ \text{where} \quad h(\varphi) &= \gamma k^{\gamma}\varphi^{-(\gamma+1)} \quad \text{and} \quad \xi \equiv ck^{c-\gamma}/\gamma > 0 \end{aligned}$$

The corresponding cumulative distribution function is $H(\varphi) = 1 - \left(\frac{k}{\varphi}\right)^{\gamma}$. Using this result for the distribution of $\varphi^{\sigma-1}g(\varphi)$, we evaluate the integral in the free entry condition (equation (27) in the paper) to derive the expression for the zero-profit productivity cutoff below.

4. Parameters

To focus on comparative advantage, we assume that all industry parameters are the same across countries, and that only factor intensity varies across industries. The numerical solutions appendix in the paper itself contains further discussion of the choice of parameter values.

Fixed production costs: $f_1 = 0.10$, $f_2 = 0.10$.
Sunk entry costs: $f_{e1} = 2$, $f_{e2} = 2$.
Probability of firm death: $\delta = 0.025$.
Industry expenditure shares: $\alpha_1 = \alpha_2 = 0.5$.
Elasticity of substitution: $\sigma = \frac{1}{1-\rho} = 3.8$.
Industry factor intensities: $\beta_1 = 0.6$, $\beta_2 = 0.4$.
Pareto parameters: $k = 0.2$, $c = 3.4$, which since $c > \sigma - 1$ satisfies the requirement for the variance of log productivity to be finite.
Fixed exporting costs: $f_{x1} = 0.10$, $f_{x2} = 0.10$.
Variable trade costs: τ_1 and τ_2 range from 1 to 1.6 in increments of 0.5.
Factor endowments: $\bar{L}^H = 1000$, $\bar{S}^H = 1200$, $\bar{L}^F = 1200$, $\bar{S}^F = 1000$.

5. Costly Trade Equilibrium

5.1. *Conditions for General Equilibrium*

The costly trade equilibrium is referenced by a vector of twenty-six variables in home and foreign taken together: $\{\varphi_1^{*k}, \varphi_2^{*k}, \varphi_{1x}^{*k}, \varphi_{2x}^{*k}, P_1^k, P_2^k, p_1^k(\varphi), p_2^k(\varphi), p_{1x}^k(\varphi), p_{2x}^k(\varphi), w_S^k, w_L^k, R^k\}$ for $k \in \{H, F\}$. All other endogenous variables may be written as functions of these quantities. The equilibrium vector is determined by the following twenty-six equilibrium conditions: firms' pricing rule for each industry and for the domestic and export market separately in the two countries, free entry for each sector in the two countries, the relationship between the two productivity cutoffs for each sector in the two countries, labor market clearing for the two factors in the two countries, the values for the equilibrium price indices implied by consumer and producer optimization for each sector in the two countries, and world expenditure on a country's varieties equals the value of their production for each sector in the two countries.

We choose the skilled wage in home as the numeraire, $w_S^H = 1$, and solve for the remaining twenty-five variables in the equilibrium vector from the system of twenty-five equations that determine general equilibrium (by Walras Law, one equation is redundant).

In deriving the system of twenty-five equations that determine general equilibrium, the paper makes a number of substitutions. To make the Matlab code as transparent as possible, we explicitly write out many of these substitutions to yield a larger system of sixty-five equations. We solve this larger system of sixty-five equations to determine sixty-five endogenous variables that include the costly trade equilibrium vector itself and variables that are functions of the costly trade equilibrium vector.

In listing the system of sixty-five equations below, we use the notation from the paper. In the Matlab code, upper case letters are used to denote home variables and lower case letters are used to denote foreign variables.

5.2. Non-linear Equation System

Home unskilled labor market clearing:

$$w_L^H = \frac{(1-\beta_1)R_1^H + (1-\beta_2)R_2^H}{\bar{L}^H} \tag{1}$$

Foreign unskilled labor market clearing:

$$w_L^F = \frac{(1-\beta_1)R_1^F + (1-\beta_2)R_2^F}{\bar{L}^F} \tag{2}$$

Foreign skilled labor market clearing:

$$w_S^F = \frac{\beta_1 R_1^F + \beta_2 R_2^F}{\bar{S}^F} \tag{3}$$

Home unskilled labor allocation to sector 1:

$$L_1^H = \frac{(1-\beta_1)R_1^H}{w_L^H} \tag{4}$$

Home unskilled labor allocation to sector 2:

$$L_2^H = \frac{(1-\beta_2)R_2^H}{w_L^H} \tag{5}$$

Home skilled labor allocation to sector 1:

$$S_1^H = \frac{\beta_1 R_1^H}{w_S^H} \tag{6}$$

Home skilled labor allocation to sector 2:

$$S_2^H = \bar{S}^H - S_1^H \tag{7}$$

Foreign unskilled labor allocation to sector 1:

$$L_1^F = \frac{(1-\beta_1)R_1^F}{w_L^F} \tag{8}$$

Foreign unskilled labor allocation to sector 2:

$$L_2^F = \frac{(1-\beta_2)R_2^F}{w_L^F} \tag{9}$$

Foreign skilled labor allocation to sector 1:

$$S_1^F = \frac{\beta_1 R_1^F}{w_S^F} \tag{10}$$

Foreign skilled labor allocation to sector 2:

$$S_2^F = \frac{\beta_2 R_2^F}{w_S^F} \tag{11}$$

Home equilibrium relationship between the productivity cutoffs in sector 1:

$$\Lambda_1^H = \tau_1 \left(\frac{P_1^H}{P_1^F}\right)\left(\frac{R^H f_{x1}}{R^F f_1}\right)^{\frac{1}{\sigma-1}} \tag{12}$$

Home equilibrium relationship between the productivity cutoffs in sector 2:

$$\Lambda_2^H = \tau_2 \left(\frac{P_2^H}{P_2^F}\right)\left(\frac{R^H f_{x2}}{R^F f_2}\right)^{\frac{1}{\sigma-1}} \tag{13}$$

Foreign equilibrium relationship between the productivity cutoffs in sector 1:

$$\Lambda_1^F = \tau_1 \left(\frac{P_1^F}{P_1^H}\right)\left(\frac{R^F f_{x1}}{R^H f_1}\right)^{\frac{1}{\sigma-1}} \tag{14}$$

Foreign equilibrium relationship between the productivity cutoffs in sector 2:

$$\Lambda_2^F = \tau_2 \left(\frac{P_2^F}{P_2^H}\right)\left(\frac{R^F f_{x2}}{R^H f_2}\right)^{\frac{1}{\sigma-1}} \tag{15}$$

Home zero-profit productivity cutoff in sector 1 (free entry condition):

$$\varphi_1^{*H} = \left[f_1 + f_{1x}\left(\Lambda_1^H\right)^{-c}\right]^{\frac{1}{c}} \left(\frac{1}{f_{e1}}\right)^{\frac{1}{c}} \left[\frac{1}{\delta}\left(\frac{c}{\gamma}-1\right)k^c\right]^{\frac{1}{c}} \tag{16}$$

Home zero-profit productivity cutoff in sector 2 (free entry condition):

$$\varphi_2^{*H} = \left[f_2 + f_{2x}\left(\Lambda_2^H\right)^{-c}\right]^{\frac{1}{c}} \left(\frac{1}{f_{e2}}\right)^{\frac{1}{c}} \left[\frac{1}{\delta}\left(\frac{c}{\gamma}-1\right)k^c\right]^{\frac{1}{c}} \tag{17}$$

Foreign zero-profit productivity cutoff in sector 1 (free entry condition):

$$\varphi_1^{*F} = \left[f_1 + f_{1x}\left(\Lambda_1^F\right)^{-c}\right]^{\frac{1}{c}} \left(\frac{1}{f_{e1}}\right)^{\frac{1}{c}} \left[\frac{1}{\delta}\left(\frac{c}{\gamma}-1\right)k^c\right]^{\frac{1}{c}} \tag{18}$$

Foreign zero-profit productivity cutoff in sector 2 (free entry condition):

$$\varphi_2^{*F} = \left[f_2 + f_{2x}\left(\Lambda_2^F\right)^{-c}\right]^{\frac{1}{c}} \left(\frac{1}{f_{e2}}\right)^{\frac{1}{c}} \left[\frac{1}{\delta}\left(\frac{c}{\gamma}-1\right)k^c\right]^{\frac{1}{c}} \tag{19}$$

Exporting productivity cutoff in home in sector 1:

$$\varphi_{1x}^{*H} = \Lambda_1^H \varphi_1^{*H} \tag{20}$$

Exporting productivity cutoff in home in sector 2:

$$\varphi_{2x}^{*H} = \Lambda_2^H \varphi_2^{*H} \tag{21}$$

Exporting productivity cutoff in foreign in sector 1:

$$\varphi_{1x}^{*F} = \Lambda_1^F \varphi_1^{*F} \tag{22}$$

Exporting productivity cutoff in foreign in sector 2:

$$\varphi_{2x}^{*F} = \Lambda_2^F \varphi_2^{*F} \tag{23}$$

Home weighted average productivity in sector 1:

$$\tilde{\varphi}_1^H = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_1^{*H} \tag{24}$$

Home weighted average productivity in sector 2:

$$\tilde{\varphi}_2^H = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_2^{*H} \tag{25}$$

Foreign weighted average productivity in sector 1:

$$\tilde{\varphi}_1^F = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_1^{*F} \tag{26}$$

Foreign weighted average productivity in sector 2:

$$\tilde{\varphi}_2^F = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_2^{*F} \tag{27}$$

Home weighted average productivity in the export market in sector 1:

$$\tilde{\varphi}_{1x}^H = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_{1x}^{*H} \tag{28}$$

Home weighted average productivity in the export market in sector 2:

$$\tilde{\varphi}_{2x}^H = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_{2x}^{*H} \tag{29}$$

Foreign weighted average productivity in the export market in sector 1:

$$\tilde{\varphi}_{1x}^F = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_{1x}^{*F} \tag{30}$$

Foreign weighted average productivity in the export market in sector 2:

$$\tilde{\varphi}_{2x}^F = \left(\frac{c}{\gamma}\right)^{\frac{1}{\sigma-1}} \varphi_{2x}^{*F} \tag{31}$$

Home probability of exporting in sector 1:

$$\chi_1^H = \left(\frac{\varphi_1^{*H}}{\varphi_{1x}^{*H}}\right)^c \tag{32}$$

Home probability of exporting in sector 2:

$$\chi_2^H = \left(\frac{\varphi_2^{*H}}{\varphi_{2x}^{*H}}\right)^c \tag{33}$$

Foreign probability of exporting in sector 1:

$$\chi_1^F = \left(\frac{\varphi_1^{*F}}{\varphi_{1x}^{*F}}\right)^c \tag{34}$$

Foreign probability of exporting in sector 2:

$$\chi_2^F = \left(\frac{\varphi_2^{*F}}{\varphi_{2x}^{*F}}\right)^c \tag{35}$$

Home average firm revenue in sector 1:

$$\bar{r}_1^H = \left(\frac{\tilde{\varphi}_1^H}{\varphi_1^{*H}}\right)^{\sigma-1} \sigma f_1 \left(w_S^H\right)^{\beta_1} \left(w_L^H\right)^{1-\beta_1} + \chi_1^H \left(\frac{\tilde{\varphi}_{1x}^H}{\varphi_{1x}^{*H}}\right)^{\sigma-1} \sigma f_{1x} \left(w_S^H\right)^{\beta_1} \left(w_L^H\right)^{1-\beta_1} \tag{36}$$

Home average firm revenue in sector 2:

$$\bar{r}_2^H = \left(\frac{\tilde{\varphi}_2^H}{\varphi_2^{*H}}\right)^{\sigma-1} \sigma f_2 \left(w_S^H\right)^{\beta_2} \left(w_L^H\right)^{1-\beta_2} + \chi_2^H \left(\frac{\tilde{\varphi}_{2x}^H}{\varphi_{2x}^{*H}}\right)^{\sigma-1} \sigma f_{2x} \left(w_S^H\right)^{\beta_2} \left(w_L^H\right)^{1-\beta_2} \tag{37}$$

Foreign average firm revenue in sector 1:

$$\bar{r}_1^F = \left(\frac{\tilde{\varphi}_1^F}{\varphi_1^{*F}}\right)^{\sigma-1} \sigma f_1 \left(w_S^F\right)^{\beta_1} \left(w_L^F\right)^{1-\beta_1} + \chi_1^F \left(\frac{\tilde{\varphi}_{1x}^F}{\varphi_{1x}^{*F}}\right)^{\sigma-1} \sigma f_{1x} \left(w_S^F\right)^{\beta_1} \left(w_L^F\right)^{1-\beta_1} \tag{38}$$

Foreign average firm revenue in sector 2:

$$\bar{r}_2^F = \left(\frac{\tilde{\varphi}_2^F}{\varphi_2^{*F}}\right)^{\sigma-1} \sigma f_2 \left(w_S^F\right)^{\beta_2} \left(w_L^F\right)^{1-\beta_2} + \chi_2^F \left(\frac{\tilde{\varphi}_{2x}^F}{\varphi_{2x}^{*F}}\right)^{\sigma-1} \sigma f_{2x} \left(w_S^F\right)^{\beta_2} \left(w_L^F\right)^{1-\beta_2} \tag{39}$$

Home domestic varieties pricing in sector 1:

$$p_{1d}^H(\tilde{\varphi}_1^H) = \left(\frac{1}{\rho\tilde{\varphi}_1^H}\right) \left(w_S^H\right)^{\beta_1} \left(w_L^H\right)^{1-\beta_1} \tag{40}$$

Home domestic varieties pricing in sector 2:

$$p_{2d}^H(\tilde{\varphi}_2^H) = \left(\frac{1}{\rho\tilde{\varphi}_2^H}\right) \left(w_S^H\right)^{\beta_2} \left(w_L^H\right)^{1-\beta_2} \tag{41}$$

Foreign domestic varieties pricing in sector 1:

$$p_{1d}^F(\tilde{\varphi}_1^F) = \left(\frac{1}{\rho\tilde{\varphi}_1^F}\right) \left(w_S^F\right)^{\beta_1} \left(w_L^F\right)^{1-\beta_1} \tag{42}$$

Foreign domestic varieties pricing in sector 2:

$$p_{2d}^F(\tilde{\varphi}_2^F) = \left(\frac{1}{\rho\tilde{\varphi}_2^F}\right) \left(w_S^F\right)^{\beta_2} \left(w_L^F\right)^{1-\beta_2} \tag{43}$$

Home mass of firms in sector 1:

$$M_1^H = \frac{R_1^H}{\bar{r}_1^H} \tag{44}$$

Home mass of firms in sector 2:

$$M_2^H = \frac{R_2^H}{\bar{r}_2^H} \tag{45}$$

Foreign mass of firms in sector 1:

$$M_1^F = \frac{R_1^F}{\bar{r}_1^F} \tag{46}$$

Foreign mass of firms in sector 2:

$$M_2^F = \frac{R_2^F}{\bar{r}_2^F} \tag{47}$$

Home price index in sector 1:

$$P_1^H = \left[M_1^H p_{1d}^H(\tilde{\varphi}_1^H)^{1-\sigma} + \chi_1^F M_1^F \tau_1^{1-\sigma} \left(\frac{\tilde{\varphi}_1^F}{\tilde{\varphi}_{1x}^F}\right)^{1-\sigma} p_{1d}^F(\tilde{\varphi}_1^F)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{48}$$

Home price index in sector 2:

$$P_2^H = \left[M_2^H p_{2d}^H(\tilde{\varphi}_2^H)^{1-\sigma} + \chi_2^F M_2^F \tau_2^{1-\sigma} \left(\frac{\tilde{\varphi}_2^F}{\tilde{\varphi}_{2x}^F}\right)^{1-\sigma} p_{2d}^F(\tilde{\varphi}_2^F)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{49}$$

Foreign price index in sector 1:

$$P_1^F = \left[M_1^F p_{1d}^F(\tilde{\varphi}_1^F)^{1-\sigma} + \chi_1^H M_1^H \tau_1^{1-\sigma} \left(\frac{\tilde{\varphi}_1^H}{\tilde{\varphi}_{1x}^H}\right)^{1-\sigma} p_{1d}^H(\tilde{\varphi}_1^H)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{50}$$

Foreign price index in sector 2:

$$P_2^F = \left[M_2^F p_{2d}^F(\tilde{\varphi}_2^F)^{1-\sigma} + \chi_2^H M_2^H \tau_2^{1-\sigma} \left(\frac{\tilde{\varphi}_2^H}{\tilde{\varphi}_{2x}^H} \right)^{1-\sigma} p_{2d}^H(\tilde{\varphi}_2^H)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{51}$$

Aggregate home expenditure on home varieties in sector 1:

$$E_1^H = \left(P_1^H\right)^{\sigma-1} p_{1d}^H(\tilde{\varphi}_1^H)^{1-\sigma} \alpha R^H M_1^H \tag{52}$$

Aggregate home expenditure on home varieties in sector 2:

$$E_2^H = \left(P_2^H\right)^{\sigma-1} p_{2d}^H(\tilde{\varphi}_2^H)^{1-\sigma} (1-\alpha) R^H M_2^H \tag{53}$$

Aggregate foreign expenditure on foreign varieties in sector 1:

$$E_1^F = \left(P_1^F\right)^{\sigma-1} p_{1d}^F(\tilde{\varphi}_1^F)^{1-\sigma} \alpha R^F M_1^F \tag{54}$$

Aggregate foreign expenditure on foreign varieties in sector 2:

$$E_2^F = \left(P_2^F\right)^{\sigma-1} p_{2d}^F(\tilde{\varphi}_2^F)^{1-\sigma} (1-\alpha) R^F M_2^F \tag{55}$$

Aggregate home expenditure on foreign varieties in sector 1:

$$G_1^H = (P_1^H)^{\sigma-1} \tau_1^{1-\sigma} \left(\frac{\tilde{\varphi}_1^F}{\tilde{\varphi}_{1x}^F} \right)^{1-\sigma} p_{1d}^F(\tilde{\varphi}_1^F)^{1-\sigma} \alpha R^H \chi_1^F M_1^F \tag{56}$$

Aggregate home expenditure on foreign varieties in sector 2:

$$G_2^H = (P_2^H)^{\sigma-1} \tau_2^{1-\sigma} \left(\frac{\tilde{\varphi}_2^F}{\tilde{\varphi}_{2x}^F} \right)^{1-\sigma} p_{2d}^F(\tilde{\varphi}_2^F)^{1-\sigma} (1-\alpha) R^H \chi_2^F M_2^F \tag{57}$$

Aggregate foreign expenditure on home varieties in sector 1:

$$G_1^F = (P_1^F)^{\sigma-1} \tau_1^{1-\sigma} \left(\frac{\tilde{\varphi}_1^H}{\tilde{\varphi}_{1x}^H} \right)^{1-\sigma} p_{1d}^H(\tilde{\varphi}_1^H)^{1-\sigma} \alpha R^F \chi_1^H M_1^H \tag{58}$$

Aggregate foreign expenditure on home varieties in sector 2:

$$G_2^F = (P_2^F)^{\sigma-1} \tau_2^{1-\sigma} \left(\frac{\tilde{\varphi}_2^H}{\tilde{\varphi}_{2x}^H} \right)^{1-\sigma} p_{2d}^H(\tilde{\varphi}_2^H)^{1-\sigma} (1-\alpha) R^F \chi_2^H M_2^H \tag{59}$$

Home industry revenue in sector 1:

$$R_1^H = E_1^H + G_1^F \tag{60}$$

Home industry revenue in sector 2:

$$R_2^H = E_2^H + G_2^F \tag{61}$$

Foreign industry revenue in sector 1:

$$R_1^F = E_1^F + G_1^H \tag{62}$$

Foreign industry revenue in sector 2:

$$R_2^F = E_2^F + G_2^H \tag{63}$$

Home aggregate revenue, which equals aggregate expenditure, equals aggregate income:

$$R^H = w_S^H \bar{S}^H + w_L^H \bar{L}^H \tag{64}$$

Foreign aggregate revenue, which equals aggregate expenditure, equals aggregate income:

$$R^F = w_S^F \bar{S}^F + w_L^F \bar{L}^F \tag{65}$$

5.3. Solving the Non-linear Equation System

The Matlab code solves for the costly trade equilibrium of the model for each value of variable trade costs. For a given value of variable trade costs, we choose an arbitrary vector of non-zero starting values for the 65 endogenous variables in equations (1)-(65). We next solve the system of equations (1)-(65) to obtain a vector of end values for the 65 endogenous variables. We then create a new vector of starting values equal to a weighted average of the previous vector of starting values and the vector of end values. Finally, we repeatedly solve the system of equations in this way until the starting and end values of the endogenous variables converge to equal those in the unique costly trade equilibrium of the model.

After solving the model for each value of variable trade costs, the Matlab code evaluates other variables of interest that can be written as functions of the 65 endogenous variables in equations (1)-(65). Matrices containing the variables of interest for each value of variable trade costs are then saved to datasets. All figures in the paper were created from these datasets using Microsoft Excel.

6. Job Creation and Job Destruction in the Movement Between Steady-State Equilibria

The Matlab file `Costly_Realloc_Restud.m` evaluates between-industry, within-industry and total job turnover as the economy moves from the autarky steady-state equilibrium to the costly trade steady-state equilibrium with variable trade costs equal to 1.2 (Table 1). To simplify notation, we omit the country superscript throughout this section. The superscript A is used to indicate autarky and the superscript CT is used to denote costly trade. In the interests of brevity, we only derive results for skilled employment, but those for unskilled employment are directly analogous.

6.1. Total Job Turnover

When trade costs fall from infinity under autarky to a finite positive value under costly trade, the zero-profit productivity cutoff in each industry rises from φ_i^{*A} to φ_i^{*CT}, and the exporting productivity cutoff falls from infinity to a finite positive value φ_i^{*CT}.

The change in skilled employment in each industry following the opening of costly trade can be decomposed into four components: (a) the change in skilled employment used in the sunk costs of entry; (b) the change in skilled employment for domestic production due to firm exit; (c) the change in skilled employment for domestic production at continuing firms; (d) the change in skilled employment due to new production for the export market.

We first determine skilled employment in entry, production for the domestic market and production for the export market in an industry. We next evaluate each of the four components of skilled employment changes listed above. Total skilled job turnover in an industry is the sum of the absolute values of the four components of skilled employment changes:

$$\text{Total}_i^S = |\Delta S_i^e| + \left|(\Delta S_{id}^p)^{\text{Exit}}\right| + \left|(\Delta S_{id}^p)^{\text{Survive}}\right| + \left|(\Delta S_{ix}^p)^{\text{Survive}}\right| \tag{66}$$

6.1.1. Employment in Entry

Skilled employment in entry in each industry is:

$$S_i^e = \frac{\beta_i M_{ei} f_{ei} \left(w_S\right)^{\beta_i} \left(w_L\right)^{1-\beta_i}}{w_S} = \frac{\beta_i M_i \bar{\pi}_i}{w_S} \tag{67}$$

where the second equation on the right-hand side of (67) follows from combining the steady-state stability and free entry conditions.

6.1.2. Firm Employment for Domestic and Export Production

Skilled employment for domestic production at a firm with productivity φ_i is:

$$\begin{aligned} S_{id}^{p}(\varphi_i) &= \frac{\beta_i}{w_S}\left[r_{id}(\varphi)-\pi_{id}(\varphi)\right] \\ &= \frac{\beta_i}{w_S}\left[\left(\frac{\sigma-1}{\sigma}\right)r_{id}(\varphi_i)+f_i\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}\right] \\ &= \frac{\beta_i f_i\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}}{w_S}\left[(\sigma-1)\left(\frac{\varphi}{\varphi_i^*}\right)^{\sigma-1}+1\right] \end{aligned} \tag{68}$$

Skilled employment for export production at a firm with productivity φ_i is:

$$\begin{aligned} S_{ix}^{p}(\varphi_i) &= \frac{\beta_i}{w_S}\left[r_{ix}(\varphi)-\pi_{ix}(\varphi)\right] \\ &= \frac{\beta_i}{w_S}\left[\left(\frac{\sigma-1}{\sigma}\right)r_{ix}(\varphi_i)+f_{ix}\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}\right] \\ &= \frac{\beta_i f_{ix}\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}}{w_S}\left[(\sigma-1)\left(\frac{\varphi}{\varphi_{ix}^*}\right)^{\sigma-1}+1\right] \end{aligned} \tag{69}$$

Total employment for domestic and export production are determined by integrating across firms with different levels of productivity, as shown below.

6.1.3. Changes in Employment in the Sunk Costs of Entry

The change in skilled employment in the sunk costs of entry is:

$$\begin{aligned} \Delta S_i^e &= \left(\frac{\beta_i M_{ei}^{CT} f_{ei}\left(w_S^{CT}\right)^{\beta_i}\left(w_L^{CT}\right)^{1-\beta_i}}{w_S^{CT}}\right)-\left(\frac{\beta_i M_{ei}^{A} f_{ei}\left(w_S^{A}\right)^{\beta_i}\left(w_L^{A}\right)^{1-\beta_i}}{w_S^{A}}\right) \\ &= \left(\frac{\beta_i M_i^{CT}\bar{\pi}_i^{CT}}{w_S^{CT}}\right)-\left(\frac{\beta_i M_i^{A}\bar{\pi}_i^{A}}{w_S^{A}}\right) \end{aligned} \tag{70}$$

6.1.4. Changes in Employment for Domestic Production

Total skilled employment for domestic production is:

$$\begin{aligned} S_{id}^{p} &= \int_0^\infty S_{id}^{p}(\varphi)\,M_i\mu_i(\varphi)\,d\varphi \\ &= \frac{\beta_i f_i\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}\,M_i}{w_S\left[1-G\left(\varphi_i^*\right)\right]}\int_{\varphi_i^*}^{\infty}\left[(\sigma-1)\left(\frac{\varphi}{\varphi_i^*}\right)^{\sigma-1}+1\right]g(\varphi)\,d\varphi \end{aligned} \tag{71}$$

With a Pareto productivity distribution, this simplifies to:

$$S_{id}^{p}=\frac{\beta_i f_i\,(w_S)^{\beta_i}\,(w_L)^{1-\beta_i}\,M_i}{w_S}\left[\frac{(\sigma-1)\,c}{\gamma}+1\right] \tag{72}$$

Therefore, the change in skilled employment for domestic production is:

$$\Delta S_{id}^{p}=S_{id}^{pCT}-S_{id}^{pA} \tag{73}$$

The change in skilled employment for domestic production can be decomposed into the employment change due to firm exit and the employment change at surviving firms, as shown below.

6.1.5. Changes in Employment for Domestic Production Due to Firm Exit

The change in skilled employment for domestic production due to firm exit following the rise in the zero-profit productivity cutoff from φ_i^{*A} to φ_i^{*CT} is:

$$\begin{aligned}(\Delta S_{id}^{p})^{\text{Exit}} &= -\int_{\varphi_i^{*A}}^{\varphi_i^{*CT}} S_{id}^{pA}(\varphi_i) M_i^A \mu_i^A(\varphi_i) d\varphi_i \\ &= -\frac{\beta_i f_i \left(w_S^A\right)^{\beta_i} \left(w_L^A\right)^{1-\beta_i} M_i^A}{w_S^A \left[1 - G\left(\varphi_i^{*A}\right)\right]} \int_{\varphi_i^{*A}}^{\varphi_i^{*CT}} \left[(\sigma-1)\left(\frac{\varphi}{\varphi_i^{*A}}\right)^{\sigma-1} + 1\right] g(\varphi_i) d\varphi_i\end{aligned} \tag{74}$$

With a Pareto productivity distribution, this simplifies to:

$$(\Delta S_{id}^{p})^{\text{Exit}} = -\frac{\beta_i f_i \left(w_S^A\right)^{\beta_i} \left(w_L^A\right)^{1-\beta_i} M_i^A}{w_S^A} \left[\frac{(\sigma-1)\, c}{\gamma}\left[1 - \left(\frac{\varphi_i^{*CT}}{\varphi_i^{*A}}\right)^{-\gamma}\right] + \left[1 - \left(\frac{\varphi_i^{*CT}}{\varphi_i^{*A}}\right)^{-c}\right]\right] \tag{75}$$

6.1.6. Changes in Employment for Domestic Production at Surviving Firms

Skilled employment for domestic production at surviving firms under costly trade equals total skilled employment for domestic production under costly trade:

$$\begin{aligned}\left(S_{id}^{pCT}\right)^{\text{Survive}} &= \left[\int_{\varphi_i^{*CT}}^{\infty} S_{id}^{pCT}(\varphi_i) M_i^{CT} \mu_i^{CT}(\varphi_i) d\varphi_i\right] \\ &= \frac{\beta_i f_i \left(w_S^{CT}\right)^{\beta_i} \left(w_L^{CT}\right)^{1-\beta_i} M_i^{CT}}{w_S^{CT} \left[1 - G\left(\varphi_i^{*CT}\right)\right]} \int_{\varphi_i^{*CT}}^{\infty} \left[(\sigma-1)\left(\frac{\varphi}{\varphi_i^{*CT}}\right)^{\sigma-1} + 1\right] g\left(\varphi\right) d\varphi\end{aligned} \tag{76}$$

With a Pareto productivity distribution, this simplifies to:

$$\left(S_{id}^{pCT}\right)^{\text{Survive}} = \frac{\beta_i f_i \left(w_S^{CT}\right)^{\beta_i} \left(w_L^{CT}\right)^{1-\beta_i} M_i^{CT}}{w_S^{CT}} \left[\frac{(\sigma-1)\, c}{\gamma} + 1\right] \tag{77}$$

Skilled employment for domestic production at surviving firms under autarky is:

$$\begin{aligned}\left(S_{id}^{pA}\right)^{\text{Survive}} &= \left[\int_{\varphi_i^{*CT}}^{\infty} S_{id}^{pA}(\varphi_i) M_i^A \mu_i^A(\varphi_i) d\varphi_i\right] \\ &= \frac{\beta_i f_i \left(w_S^A\right)^{\beta_i} \left(w_L^A\right)^{1-\beta_i} M_i^A}{w_S^A \left[1 - G\left(\varphi_i^{*A}\right)\right]} \int_{\varphi_i^{*CT}}^{\infty} \left[(\sigma-1)\left(\frac{\varphi}{\varphi_i^{*A}}\right)^{\sigma-1} + 1\right] g\left(\varphi\right) d\varphi\end{aligned} \tag{78}$$

With a Pareto productivity distribution, this simplifies to:

$$\left(S_{id}^{pA}\right)^{\text{Survive}} = \frac{\beta_i f_i \left(w_S^A\right)^{\beta_i} \left(w_L^A\right)^{1-\beta_i} M_i^A}{w_S^A} \left[\frac{(\sigma-1)\, c}{\gamma}\left(\frac{\varphi_i^{*CT}}{\varphi_i^{*A}}\right)^{-\gamma} + \left(\frac{\varphi_i^{*CT}}{\varphi_i^{*A}}\right)^{-c}\right] \tag{79}$$

Therefore, the change in skilled employment for domestic production at surviving firms is:

$$(\Delta S_{id}^{p})^{\text{Survive}} = \left(S_{id}^{pCT}\right)^{\text{Survive}} - \left(S_{id}^{pA}\right)^{\text{Survive}} \tag{80}$$

6.1.7. Changes in Employment for Export Production

In addition to the change in employment for domestic production, there is a change in employment due to entry into export markets following the opening of costly trade. Skilled employment for export production under costly trade is:

$$\begin{aligned} S_{ix}^{pCT} &= \int_0^\infty S_{ix}^{pCT}(\varphi_i) M_{ix}^{CT} \chi_i \mu_i^{CT}(\varphi_i) d\varphi_i \\ &= \frac{\beta_i f_{ix} \left(w_S^{CT}\right)^{\beta_i} \left(w_L^{CT}\right)^{1-\beta_i} \left(\chi_i^{CT} M_i^{CT}\right)}{w_S^{CT} \left[1 - G\left(\varphi_{ix}^{*CT}\right)\right]} \int_{\varphi_{ix}^{*CT}}^\infty \left[(\sigma-1)\left(\frac{\varphi}{\varphi_{ix}^{*CT}}\right)^{\sigma-1} + 1\right] g(\varphi)\, d\varphi \end{aligned} \tag{81}$$

With a Pareto productivity distribution, this simplifies to:

$$S_{ix}^{pCT} = \frac{\beta_i f_{ix} \left(w_S^{CT}\right)^{\beta_i} \left(w_L^{CT}\right)^{1-\beta_i} \left(\chi_i^{CT} M_i^{CT}\right)}{w_S^{CT}} \left[\frac{(\sigma-1)\, c}{\gamma} + 1\right] \tag{82}$$

Therefore, the change in skilled employment due to production for the export market is:

$$\left(\Delta S_{ix}^p\right)^{\text{Survive}} = S_{ix}^{pCT} \tag{83}$$

6.2. Between-Industry Job Turnover

Between-industry job turnover corresponds to net job creation and destruction in the two sectors. Labor market clearing implies that between-industry job turnover for a given factor is opposite in sign and equal in magnitude in the two sectors. Between-industry job turnover for skilled labor is:

$$\text{Between}_1^S = \Delta S_1 = -\Delta S_2 = -\text{Between}_2^S \tag{84}$$

$$\left(\frac{\beta_1 R_1^{CT}}{w_S^{CT}}\right) - \left(\frac{\beta_1 R_1^{A}}{w_S^{A}}\right) = -\left[\left(\frac{\beta_2 R_2^{CT}}{w_S^{CT}}\right) - \left(\frac{\beta_2 R_2^{A}}{w_S^{A}}\right)\right]$$

Between-industry job turnover is the sum of the four components of employment changes (without taking absolute values):

$$\text{Between}_i^S = \Delta S_i^e + \left(\Delta S_{id}^p\right)^{\text{Exit}} + \left(\Delta S_{id}^p\right)^{\text{Survive}} + \left(\Delta S_{ix}^p\right)^{\text{Survive}} \tag{85}$$

6.3. Within-industry Job Turnover

Within-industry job turnover equals total job turnover minus between-industry job turnover:

$$\text{Within}_i^S = \text{Total}_i^S - \text{Between}_i^S \tag{86}$$

Total job turnover exceeds between-industry job turnover (i.e. gross job creation and destruction exceed net job creation and destruction) because the four components of employment changes in equations (66) and (85) have different signs.

7. Steady-State Job Churning

The Matlab file `Costly_Churn_Restud.m` evaluates steady-state job churning in the autarky equilibrium and in costly trade equilibria with variable trade costs ranging from 20% to 60%. To simplify notation, we again omit the country superscript through this section. In the interests of brevity, we only derive results for skilled employment, but those for unskilled employment are directly analogous.

In the steady-state equilibrium of the model, employment of skilled labor in entry each period is:

$$S_i^e = \frac{\beta_i M_{ei} f_{ei} \left(w_S\right)^{\beta_i} \left(w_L\right)^{1-\beta_i}}{w_S} = \frac{\beta_i M_i \bar{\pi}_i}{w_S} \tag{87}$$

Steady-state employment of skilled labor for domestic and export production at the flow of firms who die each period is:

$$
\begin{aligned}
\left(S_i^p\right)^{\text{Die}} &= \frac{\delta M_i \beta_i \left[\bar{r}_i - \bar{\pi}_i\right]}{w_S} \\
&= \frac{\delta M_i \beta_i}{w_S} \left\{ \begin{array}{c} \left[(\sigma-1)\left(\frac{\tilde{\varphi}_i}{\varphi_i^*}\right)^{\sigma-1}+1\right] f_i \left(w_S\right)^{\beta_i}\left(w_L\right)^{1-\beta_i} \\ +\chi_i \left[(\sigma-1)\left(\frac{\tilde{\varphi}_{ix}}{\varphi_{ix}^*}\right)^{\sigma-1}+1\right] f_{ix} \left(w_S\right)^{\beta_i}\left(w_L\right)^{1-\beta_i} \end{array} \right\}
\end{aligned} \tag{88}
$$

where, in the autarkic steady-state equilibrium of the model, the second term inside the parentheses is equal to zero.

Steady-state churning for skilled labor is defined as skilled employment in entry and in production at firms who die divided by the economy's total labor force:

$$
\left(S_i\right)^{\text{Churn}} = \frac{S_i^e + \left(S_i^p\right)^{\text{Die}}}{\bar{S} + \bar{L}} \tag{89}
$$