A.1 Introduction

This web appendix contains technical derivations, the proofs of propositions, additional information about the data, and supplementary empirical results.

Section A.2 derives the expression for the overall CES price index in the presence of entry and exit. We decompose the overall change of cost of living \((P_t/P_{t-1})\) into the change in the share of expenditure on common goods \((\lambda_{t,t-1}/\lambda_{t-1,t})\) and the change in the price index for these common goods \((P^*_t/P^*_{t-1})\), as discussed in Section 2.2 of the paper. Section A.3 derives the exact CES price index in terms of demand-adjusted prices and characterizes its relationship with the Sato-Vartia price index, as examined in Section 2.3 of the paper.

Section A.4 characterizes the consumer-valuation bias and shows that a positive demand shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all other goods. Section A.5 derives the elasticity of substitution implied by the Sato-Vartia price index under its assumption of time-invariant demand for each common good. Section A.6 derives the three equivalent expressions for the CES price index: the unified price index in equation (13) in the paper, the forward difference of the unit expenditure function in equation (21) in the paper, and the backward difference of the unit expenditure function in equation (22) in the paper.

Section A.7 characterizes the relationship between our CES unified price index and the major existing economic and statistical index numbers, such as the Laspeyres, Paasche, Fisher and Törnqvist indexes. Each of these existing index numbers assumes no entry and exit of goods and no time-varying demand shocks for surviving goods. Section A.8 provides further details on the Feenstra (1994) estimator discussed in Section 2.5.1 of the paper and used as a robustness check in Section 5.1 of the paper.

Section A.9 derives the expressions for the forward and backward aggregate demand shifters in equation (25) in the paper. Section A.10 demonstrates that our reverse-weighting (RW) estimator generalizes to allow for a Hicks-neutral demand shifter that is common across goods. Section A.11 shows that the reverse-weighting (RW) estimator minimizes the difference between the change in the cost of living using (i) tastes
in both periods and inverting the demand system to express these tastes in terms of observed prices and expenditure shares, (ii) tastes in the initial period, and (iii) tastes in the final period.

Section A.12 proves that the RW estimator consistently estimates the elasticity of substitution as demand shocks become small (Proposition 2 in the paper). Section A.13 shows that our assumption that the forward and backward differences of the unit expenditure function are money metric (equation (27) in the paper) is satisfied up to a first-order approximation. Section A.14 proves that the RW estimator consistently estimates the elasticity of substitution if the number of common goods becomes large and demand shocks are uncorrelated with price shocks for each good and independently and identically distributed across goods.

Section A.15 of the web appendix shows that the RW estimator belongs to the class of M-estimators, and uses results from Newey and McFadden (1994) and Wooldridge (2002) to show that the RW estimates are asymptotically normal. Section A.16 derives the GRW estimator from Section 2.5.3 of the paper. Section A.17 characterizes the asymptotic bias in the RW estimator when demand and price shocks for a given good are correlated with one another (Proposition 4 in the paper).

Section A.18 shows that the GRW estimator is consistent when demand and price shocks are correlated with one another for each good but are independently and identically distributed across goods (Proposition 5 in the paper). Section A.19 uses our inversion of the demand system to provide bounds for the elasticity of substitution ($\sigma$) regardless of the correlation between demand and price shocks. Assuming that demand and price shocks are correlated with one another for each good but are independently and identically distributed across goods, we show that the true elasticity of substitution necessarily lies within the identified set as the number of common goods becomes large. Section A.20 presents Monte Carlo evidence on the finite sample performance of our RW, GRW and bounds estimators.

Section A.21 develops the extension to non-homothetic CES preferences considered in Section 3.1 of the paper. We derive the generalizations of our common goods unified price index and reverse-weighting estimation procedure for non-homothetic CES. Section A.22 provides further details for the extension to nested CES preferences considered in Section 3.2 of the paper. Section A.23 develops the generalization to mixed CES with heterogeneous groups of consumers considered in Section 3.3 of the paper. Section A.24 shows that our unified approach to the demand system and the unit expenditure function also can be applied to the closely-related logit and mixed logit preferences, as widely used in applied microeconometrics.

Section A.25 shows that our main insight that the demand system can be inverted to construct a money-metric price index with time-varying demand shocks is not specific to CES, but also holds for the flexible functional form of translog preferences. Furthermore, the consumer-valuation bias is again present for this flexible functional form, because a price index that rules out demand shocks by assumption cannot capture the potential for consumers to increase welfare by substituting towards goods that experience reductions in demand-adjusted prices from increases in demand.

Section A.26 contains the data appendix which reports summary statistics for each of the product groups in our data. Section A.27 reports additional empirical results discussed in the paper.
A.2 Entry and Exit

In this section of the web appendix, we derive the expression for the change in the cost of living in equation (5) in Section 2.2 of the paper in terms of the change in the share of expenditure on common goods \( (\lambda_{t, t-1} / \lambda_{t-1, t}) \) and the change in the price index for these common goods \( (P_t^s / P_{t-1}^s) \).

We start by expressing the change in the cost of living from \( t-1 \) to \( t \) as the ratio between the unit expenditure functions (equation (1) in the paper) in the two periods:

\[
\Phi_{t-1, t} = \frac{P_t}{P_{t-1}} = \left[ \frac{\sum_{k \in \Omega_t} (p_{kt}/\phi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1}/\phi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}. \tag{A.56}
\]

Summing the expenditure share in equation (3) in the paper across common goods, we can express expenditure on all common goods as a share of total expenditure in periods \( t \) and \( t-1 \) respectively as:

\[
\lambda_{t, t-1} \equiv \frac{\sum_{k \in \Omega_{t-1}} (p_{kt}/\phi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_t} (p_{kt}/\phi_{kt})^{1-\sigma}}, \quad \lambda_{t-1, t} \equiv \frac{\sum_{k \in \Omega_{t-1}} (p_{kt-1}/\phi_{kt-1})^{1-\sigma}}{\sum_{k \in \Omega_t} (p_{kt-1}/\phi_{kt-1})^{1-\sigma}}, \tag{A.57}
\]

where \( \lambda_{t, t-1} \) is equal to the total sales of continuing goods in period \( t \) divided by the sales of all goods available in time \( t \) evaluated at current prices. Its maximum value is one if no goods enter in period \( t \) and will fall as the share of new goods rises. Similarly, \( \lambda_{t-1, t} \) is equal to total sales of continuing goods as share of total sales of all goods in the past period evaluated at \( t-1 \) prices. It will equal one if no goods cease being sold and will fall as the share of exiting goods rises.

Multiplying the numerator and denominator of the fraction inside the square parentheses in equation (A.56) by the summation \( \sum_{k \in \Omega_{t-1}} (p_{kt}/\phi_{kt})^{1-\sigma} \) over common goods at time \( t \), and using the definition of \( \lambda_{t, t-1} \) in equation (A.57), the change in the cost of living can be re-written as:

\[
\Phi_{t-1, t} = \left[ \frac{1}{\lambda_{t-1, t}} \sum_{k \in \Omega_{t-1}} (p_{kt-1}/\phi_{kt-1})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{A.58}
\]

Multiplying the numerator and denominator in equation (A.58) by the summation \( \sum_{k \in \Omega_{t-1}} (p_{kt-1}/\phi_{kt-1})^{1-\sigma} \) over common goods at time \( t-1 \), and using the definition of \( \lambda_{t-1, t} \) in equation (A.57), we obtain the decomposition of the change in the overall exact CES price index \( (P_t/P_{t-1}) \) into the change in the share of expenditure on common goods \( (\lambda_{t, t-1} / \lambda_{t-1, t}) \) and the change in the price index for these common goods \( (P_t^s / P_{t-1}^s) \) in equation (5) of the paper:

\[
\Phi_{t-1, t} = \left[ \frac{\lambda_{t-1, t}}{\lambda_{t, t-1}} \sum_{k \in \Omega_{t-1}} (p_{kt-1}/\phi_{kt-1})^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left( \frac{\lambda_{t, t-1}}{\lambda_{t-1, t}} \right)^{\frac{1}{1-\sigma}} \frac{P_t^s}{P_{t-1}^s}. \tag{A.59}
\]

A.3 Derivation of Exact CES Price Index

In this section of the web appendix, we derive the expression for the exact CES price index in terms of demand-adjusted prices in equation (8) in Section 2.3 of the paper. From the common goods expenditure share (7), we can express the change in the common goods price index as:
\[ \frac{P_{i}^{*}}{P_{i-1}^{*}} = \frac{(p_{kt}/\varphi_{kt}) / (p_{kt-1}/\varphi_{kt-1})}{(s_{kt}^{*}/s_{kt-1}^{*})^{1/\sigma}} \]  

(A.60)

Taking logs of both sides, and rearranging, produces:

\[ \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) - \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \frac{1}{\sigma - 1}. \]  

(A.61)

If we now multiply both sides of this equation by \( s_{kt}^{*} - s_{kt-1}^{*} \) and sum across all common goods, we obtain:

\[ \sum_{k \in \Omega_{kt-1}} \left( s_{kt}^{*} - s_{kt-1}^{*} \right) \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \sum_{k \in \Omega_{kt-1}} \left( s_{kt}^{*} - s_{kt-1}^{*} \right) \ln \left( \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \right). \]  

(A.62)

or

\[ \sum_{k \in \Omega_{kt-1}} \left( s_{kt}^{*} - s_{kt-1}^{*} \right) \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \sum_{k \in \Omega_{kt-1}} \left( s_{kt}^{*} - s_{kt-1}^{*} \right) \ln \left( \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \right). \]  

(A.63)

Re-writing this expression, we obtain the log change in our exact CES price index in equation (8) in the paper:

\[ \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \left[ \sum_{k \in \Omega_{kt-1}} \omega_{kt} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \right] - \left[ \sum_{k \in \Omega_{kt-1}} \omega_{kt} \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \right], \]  

(A.64)

\[ \omega_{kt}^{*} = \frac{s_{kt}^{*} - s_{kt-1}^{*}}{\ln s_{kt}^{*} - \ln s_{kt-1}^{*}}, \quad \sum_{k \in \Omega_{kt-1}} \omega_{kt}^{*} = 1. \]  

(A.65)

We now show that the exact CES price index in equation (A.64) is equal to the unified price index in equation (14) in the paper. Using our inversion of the demand system from equation (11) in the paper and our result that the demand shocks are mean zero across common goods (\( \ln \left( \bar{\phi}_{t} / \bar{\phi}_{t-1} \right) = 0 \)) to substitute for the demand shocks (\( \varphi_{kt} / \varphi_{kt-1} \)) in equation (A.64), we obtain:

\[ \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \ln \left( \frac{\bar{p}_{t}}{\bar{p}_{t-1}} \right) + \frac{1}{\sigma - 1} \ln \left( \frac{s_{kt}^{*}}{s_{kt-1}^{*}} \right) - \frac{1}{\sigma - 1} \sum_{k \in \Omega_{kt-1}} \omega_{kt}^{*} \ln \left( \frac{s_{kt}^{*}}{s_{kt-1}^{*}} \right), \]  

(A.66)

where a tilde above a variable denotes a geometric mean across common goods such that \( \bar{x}_{t} = \left( \prod_{k \in \Omega_{kt-1}} x_{kt} \right)^{1/N_{kt-1}} \) for the variable \( x_{kt} \). Using the definition of the Sato-Vartia weights (\( \omega_{kt}^{*} \)) from equation (A.65) above, the final term in equation (A.66) is equal to zero, so that equation (A.66) reduces to the CES common goods unified price index:

\[ \ln \left( \frac{P_{i}^{*}}{P_{i-1}^{*}} \right) = \ln \Phi_{i-1,t}^{CCG} = \ln \left( \frac{\bar{p}_{t}}{\bar{p}_{t-1}} \right) + \frac{1}{\sigma - 1} \ln \left( \frac{s_{kt}^{*}}{s_{kt-1}^{*}} \right). \]  

(A.67)
Finally, using equations (A.64) and (A.67) together with the definition of the Sato-Vartia price index in equation (10) in the paper, we can express the common goods exact CES price index as equal to the Sato-Vartia price index minus an additional term that we refer to as the consumer valuation bias, as in equation (16) in the paper:
\[
\ln \left( \frac{P^*_t}{P^*_{t-1}} \right) = \ln \Phi^*_{t-1,t} = \ln \Phi^*_{t-1,t} - \sum_{k \in \Omega_{t-1}} \omega^*_k \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right),
\]
where the time-invariant component of demand ($\varphi_{kt}$) differences out such that $\ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) = \ln \left( \theta_{kt}/\theta_{kt-1} \right)$.

### A.4 Consumer-Valuation Bias

As discussed in Section 2.4 of the paper, the Sato-Vartia index is only unbiased if the demand shocks ($\ln \left( \theta_{kt}/\theta_{kt-1} \right)$) are orthogonal to the expenditure-share weights ($\omega^*_kt$); it is upward-biased if they are positively correlated with these weights; and it is downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the demand shocks ($\ln \left( \theta_{kt}/\theta_{kt-1} \right)$) and the expenditure-share weights ($\omega^*_kt$) is possible, depending on the underlying correlation between demand and price shocks. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are functions of the demand shocks. In this section of the web appendix, we show that a positive demand shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all other goods.

Note that the Sato-Vartia common goods expenditure share weights ($\omega^*_kt$) can be written as:
\[
\omega^*_kt = \frac{\xi^*_kt}{\sum_{\ell \in \Omega_{kt-1}} \xi^*_\ell t},
\]
\[
\xi^*_kt = \frac{s^*_kt - s^*_kt-1}{\ln s^*_kt - \ln s^*_kt-1},
\]
where
\[
s^*_kt = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{\sum_{\ell \in \Omega_{kt-1}} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma}}.
\]
Note also that demand, prices and expenditure shares at time $t-1$ ($\varphi_{kt-1}, p_{kt-1}, s_{kt-1}$) are pre-determined at time $t$. To evaluate the impact of a positive demand shock for good $k$ ($\theta_{kt}/\theta_{kt-1} > 1$ and hence $\varphi_{kt}/\varphi_{kt-1} > 1$), we consider the effect of an increase in demand at time $t$ for that good ($\varphi_{kt}$) given its demand at time $t-1$ ($\varphi_{kt-1}$). Using the definitions (A.69)-(A.71), we have the following two results:
\[
\frac{d\omega^*_kt}{d\xi^*_kt} \xi^*_kt = \frac{d\omega^*_lt}{d\xi^*_kt} \omega^*_kt = (1 - \omega^*_kt) > 0, \quad \frac{d\omega^*_kt}{d\xi^*_kt} \omega^*_kt = -\omega^*_kt < 0.
\]
\[
\frac{d\xi^*_kt}{s^*_kt} s^*_kt = \frac{1}{\ln \left( s^*_kt-1/s^*_kt \right)} - \frac{1}{{(s^*_kt-1 - s^*_kt)}/s^*_kt} > 0,
\]
where we have used the fact that percentage changes are larger in absolute magnitude than logarithmic changes and hence:
\[
\frac{s^*_kt-1 - s^*_kt}{s^*_kt} > \ln \left( \frac{s^*_kt-1}{s^*_kt} \right) > 0 \quad \text{for} \quad s^*_kt-1 > s^*_kt.
\[
\frac{s_{kt}^* - s_{kt}^1}{s_{kt}^1} < \ln \left( \frac{s_{kt}^*}{s_{kt}^1} \right) < 0 \quad \text{for} \quad s_{kt}^1 < s_{kt}^*.
\]

We also have the following third result:

\[
\frac{ds_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} = (\sigma - 1) (1 - s_{kt}^*) > 0, \quad \frac{ds_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} = - (\sigma - 1) s_{kt}^* < 0. \tag{A.74}
\]

From our specification of demand in equation (2) in the paper, we have:

\[
\frac{d\varphi_{kt}}{d\theta_{kt}} = 1. \tag{A.75}
\]

Using this result in equation (A.74), we obtain:

\[
\frac{ds_{kt}^*}{d\theta_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} = (\sigma - 1) (1 - s_{kt}^*) > 0, \quad \frac{ds_{kt}^*}{d\theta_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} = - (\sigma - 1) s_{kt}^* < 0. \tag{A.76}
\]

Together (A.72), (A.73), (A.74) and (A.76) imply that a positive demand shock for good \( k \) increases the Sato-Vartia expenditure share weight for that good (\( \omega_{kt}^* \)):

\[
\frac{d\omega_{kt}^*}{d\theta_{kt}} \frac{\omega_{kt}^*}{\omega_{kt}^*} = \frac{d\omega_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{\omega_{kt}^*} = \left( \frac{d\omega_{kt}^*}{d\omega_{kt}^*} \frac{\omega_{kt}^*}{\omega_{kt}^*} \right) \left( \frac{d\omega_{kt}^*}{d\omega_{kt}^*} \frac{\omega_{kt}^*}{\omega_{kt}^*} \right) > 0, \tag{A.77}
\]

and reduces the Sato-Vartia expenditure share weight for all other goods \( \ell \neq k (\omega_{l\ell}^*) 

\[
\frac{d\omega_{l\ell}^*}{d\theta_{kt}} \frac{\omega_{l\ell}^*}{\omega_{l\ell}^*} = \frac{d\omega_{l\ell}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{\omega_{l\ell}^*} = \left( \frac{d\omega_{l\ell}^*}{d\omega_{l\ell}^*} \frac{\omega_{l\ell}^*}{\omega_{l\ell}^*} \right) \left( \frac{d\omega_{l\ell}^*}{d\omega_{l\ell}^*} \frac{\omega_{l\ell}^*}{\omega_{l\ell}^*} \right) < 0. \tag{A.78}
\]

### A.5 Elasticity of Substitution Implied by Sato-Vartia Price Index

In this section of the web appendix, we show that the Sato-Vartia price index’s assumption of time-invariant demand for each common good implies that the elasticity of substitution can be recovered from the observed data on prices and expenditure shares with no estimation. We first show that under this assumption there is an infinite number of approaches to measuring the elasticity of substitution, each of which uses different weights for each common good. If demand for all common goods is indeed constant (including no changes in tastes, quality, measurement error or specification error), all of these approaches will recover the same elasticity of substitution. We next show that if demand for some common goods changes over time, but a researcher falsely assumes time-invariant demand for all common goods, these alternative approaches will return different values for the elasticity of substitution, depending on which weights are used.

Under the Sato-Vartia assumption of constant demand for each common good (\( \varphi_{kt} = \varphi_{kt-1} = \varphi_k \) for all \( k \in \Omega_{t,t-1} \) and \( t \)), the common goods expenditure share is:

\[
\frac{p_{kt}}{\varphi_k} \frac{1-\sigma}{1-\sigma}.
\]

Dividing the expenditure share by its geometric mean across common goods, we get:

\[
\frac{p_{kt}}{\bar{p}_t} \frac{1-\sigma}{1-\sigma}.
\]

```
where a tilde above a variable denotes a geometric mean across common goods. Taking logarithms in (A.80), we obtain:

\[
\ln \left( \frac{s^*_k}{s^*_l} \right) = (1 - \sigma) \ln \left( \frac{p_{kt}}{\bar{p}_t} \right) + (\sigma - 1) \ln \left( \frac{\varphi_k}{\varphi_l} \right),
\]  
(A.81)

Taking differences in (A.81), we have:

\[
\Delta \ln \left( \frac{s^*_k}{s^*_l} \right) = (1 - \sigma) \Delta \ln \left( \frac{p_{kt}}{\bar{p}_t} \right).
\]  
(A.82)

Multiplying both sides of (A.82) by \( \omega^*_k \) and summing across common goods, we get:

\[
\sum_{k \in \Omega_{t-1}} \omega^*_k \Delta \ln \left( \frac{s^*_k}{s^*_l} \right) = (1 - \sigma) \sum_{k \in \Omega_{t-1}} \omega^*_k \Delta \ln \left( \frac{p_{kt}}{\bar{p}_t} \right),
\]  
(A.83)

where \( \omega^*_k \) are the Sato-Vartia weights:

\[
\omega^*_k = \frac{s^*_l - s^*_{l-1}}{\ln s^*_l - \ln s^*_{l-1}} - \frac{s^*_l - s^*_{l-1}}{\ln s^*_l - \ln s^*_{l-1}}.
\]

Equation (A.83) yields the following closed-form solution for \( \sigma \):

\[
\sigma^{SV} = 1 + \frac{\sum_{k \in \Omega_{t-1}} \omega^*_k \left[ \ln \left( \frac{s^*_k}{s^*_{l-1}} \right) - \ln \left( \frac{s^*_l}{s^*_k} \right) \right]}{\sum_{k \in \Omega_{t-1}} \omega^*_k \left[ \ln \left( \frac{p_{kt}}{\bar{p}_t} \right) - \ln \left( \frac{p_{l-1}}{\bar{p}_t} \right) \right]},
\]  
(A.84)

which establishes that the elasticity of substitution (\( \sigma \)) is uniquely identified from observed changes in prices and expenditure shares with no estimation under the Sato-Vartia assumption of time-invariant demand for all common goods (\( \varphi_{kt} = \varphi_{kt-1} = \bar{\theta}_k \) for all \( k \in \Omega_{t-1} \) and \( t \)). Note that we could have instead multiplied both sides of (A.82) by any positive finite share that sums to one across common goods:

\[
\sum_{k \in \Omega_{t-1}} \zeta^*_k \Delta \ln \left( \frac{s^*_k}{s^*_l} \right) = (1 - \sigma) \sum_{k \in \Omega_{t-1}} \zeta^*_k \Delta \ln \left( \frac{p_{kt}}{\bar{p}_t} \right), \quad \sum_{k \in \Omega_{t-1}} \zeta^*_k = 1, \tag{A.85}
\]

and obtained another expression for \( \sigma \) given observed prices and expenditure shares:

\[
\sigma^{ALT} = 1 + \frac{\sum_{k \in \Omega_{t-1}} \zeta^*_k \left[ \ln \left( \frac{s^*_k}{s^*_{l-1}} \right) - \ln \left( \frac{s^*_l}{s^*_k} \right) \right]}{\sum_{k \in \Omega_{t-1}} \zeta^*_k \left[ \ln \left( \frac{p_{kt}}{\bar{p}_t} \right) - \ln \left( \frac{p_{l-1}}{\bar{p}_t} \right) \right]},
\]  
(A.86)

Therefore there exists a continuum of approaches to measuring \( \sigma \), each of which weights prices and expenditure shares with different non-negative weights that sum to one. Under the Sato-Vartia assumption of constant demand for each good (\( \varphi_{kt} = \varphi_{kt-1} = \varphi_k \) for all \( k \in \Omega_{t-1} \) and \( t \)), each of these alternative approaches returns the same value for \( \sigma \), since all are derived from equation (A.82).

Now suppose that some common good experiences a demand shock (\( \theta_{kt} \neq \theta_{kt-1} \) and hence \( \varphi_{kt} \neq \varphi_{kt-1} \) for some \( k \in \Omega_{t-1} \) and \( t \)), but a researcher falsely assumes that demand for all common goods is constant. Dividing the common goods expenditure share by its geometric mean, we get:

\[
\frac{s^*_k}{s^*_l} = \left( \frac{p_{kt}}{\bar{p}_t/\bar{t}} \right)^{1-\sigma}, \tag{A.87}
\]
where a tilde above a variable again denotes a geometric mean across common goods.

Taking logarithms in (A.87) and taking differences, we obtain:

\[
\Delta \ln \left( \frac{s_{kt}^*}{s_{t1}^*} \right) = (1 - \sigma) \Delta \ln \left( \frac{p_{kt}}{p_{t1}} \right) + (\sigma - 1) \Delta \ln \theta_{kt},
\]

where we have used our result that \( \ln (\tilde{\theta}_t / \tilde{\theta}_{t-1}) = 0 \). Multiplying both sides of (A.88) by \( \omega_{kt}^* \) and summing across common goods, we get:

\[
\sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \Delta \ln \left( \frac{s_{kt}^*}{s_{t1}^*} \right) = (1 - \sigma) \sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \Delta \ln \left( \frac{p_{kt}}{p_{t1}} \right) + (\sigma - 1) \sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \Delta \ln \theta_{kt},
\]

Rearranging (A.89), we obtain:

\[
\sigma_{\theta, \omega^*} = 1 + \frac{\sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \left[ \ln \left( \frac{s_{kt}^*}{s_{t1}^*} \right) - \ln \left( \frac{s_{kt}}{s_{t1}} \right) \right]}{\sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \left[ \ln \left( \frac{p_{kt}}{p_{t1}} \right) - \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \right]},
\]

Note that we could have instead multiplied both sides of (A.88) by any positive finite share that sums to one across common goods:

\[
\sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \Delta \ln \left( \frac{s_{kt}^*}{s_{t1}^*} \right) = (1 - \sigma) \sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \Delta \ln \left( \frac{p_{kt}}{p_{t1}} \right) + (\sigma - 1) \sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \Delta \ln \theta_{kt},
\]

where

\[
\sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* = 1,
\]

and obtained another expression for the elasticity of substitution (\( \sigma \)):

\[
\sigma_{\theta, \tilde{\omega}^*} = 1 + \frac{\sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \left[ \ln \left( \frac{s_{kt}^*}{s_{t1}^*} \right) - \ln \left( \frac{s_{kt}}{s_{t1}} \right) \right]}{\sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \left[ \ln \left( \frac{p_{kt}}{p_{t1}} \right) - \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \right]},
\]

Note that both of the equations (A.90) and (A.92) return the same value for \( \sigma \), because both are derived from (A.88). However, suppose that a researcher falsely assumes that demand for each good is constant (\( \varphi_{kt} = \varphi_{kt-1} = \varphi_k \) for all \( k \in \Omega_{t,t-1} \) and \( t \)) and uses equations (A.84) and (A.86) to measure \( \sigma \) (instead of equations (A.90) and (A.92)). Under this false assumption, equations (A.84) and (A.86) will return different values for \( \sigma \), because in general:

\[
\sum_{k \in \Omega_{t1-1}} \omega_{kt}^* \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \neq \sum_{k \in \Omega_{t1-1}} \tilde{\omega}_{kt}^* \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \quad \text{when} \quad \omega_{kt}^* \neq \tilde{\omega}_{kt}^*.
\]

Therefore, when demand for goods changes over time (\( \theta_{kt} \neq \theta_{kt-1} \) and hence \( \varphi_{kt} \neq \varphi_{kt-1} \) for some \( k \in \Omega_{t,t-1} \) and \( t \)) but a researcher falsely assumes that demand for each good is constant (\( \varphi_{kt} = \varphi_{kt-1} = \varphi_k \) for all \( k \in \Omega_{t,t-1} \) and \( t \)), the use of different weights for prices and expenditure shares (\( \omega_{kt}^* \) versus \( \tilde{\omega}_{kt}^* \)) returns different elasticities of substitution in general (\( \sigma^{SV} \neq \sigma^{ALT} \)).
A.6 Equivalent Expressions for the CES Price Index

In this section of the web appendix, we derive the three equivalent expressions for the CES price index: (i) the unified price index in equation (13) in the paper, (ii) the forward difference of the unit expenditure function in equation (21) in the paper, and (iii) the backward difference of the unit expenditure function in equation (22) in the paper. We begin with the expression for the change in the unit expenditure function going forward in time from period $t - 1$ to $t$:

$$
\Phi_{t-1,t} = \frac{P_t}{P_{t-1}} = \left[ \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}}. \tag{A.93}
$$

**Derivation of $\Phi^F_{t-1,t}$ in (21):** Multiplying the numerator and denominator of the term inside the square parentheses in (A.93) by the summation $\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}$ over common goods at time $t$, we obtain:

$$
\Phi^F_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}} \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}},
$$

which using the share of expenditure on common goods (A.57) can be re-written as:

$$
\Phi^F_{t-1,t} = \left[ \frac{1}{\lambda_{t,t-1}} \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}}.
$$

Multiplying the numerator and denominator of the term inside the square parentheses by the summation $\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}$ over common goods at time $t - 1$, we have:

$$
\Phi^F_{t-1,t} = \left[ \frac{1}{\lambda_{t,t-1}} \frac{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}},
$$

which using the share of expenditure on common goods (A.57) can be expressed as:

$$
\Phi^F_{t-1,t} = \left[ \frac{\lambda_{t-1,t}}{\lambda_{t,t-1}} \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t,t-1}} \right)^{\frac{1}{\sigma}} \frac{P_t^*}{P_{t-1}^*}, \tag{A.94}
$$

which using the share of each common good in expenditure on common goods (7) at time $t - 1$ becomes:

$$
\Phi^F_{t-1,t} = \left[ \frac{\lambda_{t-1,t}}{\lambda_{t,t-1}} \sum_{k \in \Omega_{t-1}} \left[ \frac{(p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right] \right]^{\frac{1}{\sigma}},
$$

$$
\Phi^F_{t-1,t} = \left[ \frac{\lambda_{t-1,t}}{\lambda_{t,t-1}} \sum_{k \in \Omega_{t-1}} \left[ \frac{(p_{kt} / \varphi_{kt})^{1-\sigma}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} \right] \right]^{\frac{1}{\sigma}},
$$

$$
\Phi^F_{t-1,t} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t,t-1}} \right)^{\frac{1}{\sigma}} \left[ \sum_{k \in \Omega_{t-1}} \left[ \frac{1}{s_{kt-1}} \left( \frac{p_{kt} / \varphi_{kt}}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma} \right] \right]^{\frac{1}{\sigma}}.
$$
Using our specification for demand from equation (2) in the paper, which implies \( \varphi_{kt}/\varphi_{kt-1} = \theta_{kt}/\theta_{kt-1} \), we obtain:

\[
\Phi_{t-1,t}^F = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^\frac{1}{\sigma - 1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1} \left( \frac{p_{kt}/\theta_{kt}}{p_{kt-1}/\theta_{kt-1}} \right)^{1-\sigma} \right]^\frac{1}{1-\sigma},
\]

which corresponds to equation (21) in the paper.

**Derivation of \( \Phi_{t-1,t}^F \) in (22):** From equation (A.94), using the share of each common good in expenditure on common goods at time \( t \) in equation (7) in the paper, the change in the unit expenditure function going backwards in time from period \( t \) to period \( t - 1 \) can be re-written as follows:

\[
\Phi_{t-1,t}^B = \frac{P_{t-1}}{P_t} = \left[ \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt-1}/\varphi_{kt-1}}{p_{kt}/\varphi_{kt}} \right)^{1-\sigma} \right]^\frac{1}{\sigma - 1},
\]

\[
\Phi_{t-1,t}^B = \left[ \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt-1}/\varphi_{kt-1}}{p_{kt}/\varphi_{kt}} \right)^{1-\sigma} \right]^\frac{1}{\sigma - 1},
\]

\[
\Phi_{t-1,t}^B = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^\frac{1}{\sigma - 1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt-1}/\varphi_{kt-1}}{p_{kt}/\varphi_{kt}} \right)^{1-\sigma} \right]^\frac{1}{1-\sigma}.
\]

Using our specification for demand from equation (2) in the paper, which implies \( \varphi_{kt}/\varphi_{kt-1} = \theta_{kt}/\theta_{kt-1} \), we obtain:

\[
\Phi_{t-1,t}^F = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^\frac{1}{\sigma - 1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt-1}/\theta_{kt-1}}{p_{kt}/\theta_{kt}} \right)^{1-\sigma} \right]^\frac{1}{1-\sigma},
\]

which corresponds to equation (22) in the paper.

**Derivation of \( \Phi_{t-1,t}^{CUPI} \) in (14):** Using the share of each common good in expenditure on common goods at times \( t \) and \( t - 1 \) in equation (7) in the paper, the change in the unit expenditure function going forward in time from period \( t - 1 \) to period \( t \) (A.94) also can be expressed as:

\[
\Phi_{t-1,t}^{CUPI} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^\frac{1}{\sigma - 1} \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right)^\frac{1}{\sigma - 1},
\]

Taking logs of both sides we have:

\[
\ln \Phi_{t-1,t}^{CUPI} = \frac{1}{\sigma - 1} \ln \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right) + \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) + \frac{1}{\sigma - 1} \ln \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right).
\]

Taking means of both sides across the set of common goods, we obtain:

\[
\ln \Phi_{t-1,t}^{CUPI} = \frac{1}{\sigma - 1} \ln \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right) + \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) + \frac{1}{\sigma - 1} \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right),
\]
Rewriting (A.99), we obtain:

\[
\Phi_{t-1,t}^{CUPI} = \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \left( \frac{1}{\prod_{k \in \Omega_{t-1}} \frac{p_{kt}}{P_{kt-1}}} \right)^{\frac{1}{N_{t-1}}} \left( \prod_{k \in \Omega_{t-1}} \frac{s_{kt}^*}{s_{kt-1}^*} \right)^{\frac{1}{N_{t-1}}} \left( \frac{1}{\prod_{k \in \Omega_{t-1}} \frac{P_{kt}}{P_{kt-1}}} \right)^{\frac{1}{N_{t-1}}}, \tag{A.100}
\]

Using our specification for demand from equation (2) in the paper and our result that demand shocks are mean zero in logs \((\frac{1}{N_{t-1}} \sum_{k=1}^{N_{t-1}} \ln \left( \frac{p_k}{p_{kt-1}} \right) = 0\) and hence \(\bar{\Phi}_t / \bar{\Phi}_{t-1} = 1\), we obtain:

\[
\Phi_{t-1,t}^{CUPI} = \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \left( \frac{1}{\prod_{k \in \Omega_{t-1}} \frac{p_{kt}}{p_{kt-1}}} \right)^{\frac{1}{N_{t-1}}} \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right)^{\frac{1}{N_{t-1}}},
\]

which corresponds to equation (14) in the paper.

### A.7 Relationship with Conventional Price Indexes

In this section of the web appendix, we use the forward and backward differences of the CES unit expenditure function in equations (21) and (22) in the paper to relate our CES unified price index (CUPI) to existing economic and statistical price indexes. Under our assumption of CES preferences, we show that the CUPI coincides with these existing price indexes (including Laspeyres, Paasche, Fisher and Törnqvist indexes) for specific parameter values and assumptions about the entry and exit of goods and changes in demand for surviving goods. Nevertheless, there are of course other ways of rationalizing these existing price indexes using alternative functional form assumptions on preferences (e.g. the Laspeyres index is exact for Leontief preferences).

According to an International Labor Organization (ILO) survey of 68 countries around the world, the Dutot (1738) index is still the most prominent one for measuring price changes (Stoevska (2008)). This index is the ratio of a simple average of prices in two periods:

\[
\Phi_{t-1,t}^D = \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \frac{p_{kt}}{P_{kt-1}} = \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \frac{p_{kt-1} - p_{kt} - p_{kt-1}}{p_{kt-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right) \tag{A.101}
\]

As the above formula shows, this index is simply a price-weighted average change in prices, which does not have a clear rationale in terms of economic theory.

A price-weighted average of price changes is a sufficiently problematic way of measuring changes in the cost of living that most statistical agencies do not just compute unweighted averages of prices in two periods, but select their sample of price quotes based on the largest selling products in the first period. If we think that the probability that a statistical agency picks a product for inclusion in its sample of prices is based on its purchase frequency \(\frac{C_{t-1,k} / \sum_{k \in \Omega_{t-1}} C_{t-1,k}}{\sum_{k \in \Omega_{t-1}} C_{t-1,k}}\), then the Dutot index, as it is typically implemented, becomes the more familiar Laspeyres index, as used in U.S. import and export price indexes:

\(^{1}\)41 percent of countries use this index although historically its popularity was much higher. For example, all U.S. inflation data prior to 1999 is based on this index, and Belgian, German, and Japanese data continues to be based on it. The ILO report can be accessed here: [http://www.ilo.org/public/english/bureau/stat/download/cpi/survey.pdf](http://www.ilo.org/public/english/bureau/stat/download/cpi/survey.pdf)
\[ \Phi_{t-1,t}^L = \frac{\sum_{k \in \Omega_{t-1}} c_{k,t-1} p_{kt}}{\sum_{k \in \Omega_{t-1}} c_{k,t-1} p_{kt-1}} = \sum_{k \in \Omega_{t-1}} \frac{c_{k,t-1} p_{kt-1}}{\sum_{k \in \Omega_{t-1}} c_{k,t-1} p_{kt-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right) = \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right). \quad (A.102) \]

Written this way, it is clear that the Laspeyres index can be derived from the forward difference of the CES unit expenditure function in equation (21) in the paper (which equals our CUPI in equation (14) in the paper) using the assumptions that the utility gain of new goods is exactly offset by the loss from disappearing goods \( (\lambda_{t-1,t} = 1) \), the elasticity of substitution equals zero and demand for each good is constant \( (\varphi_{kt} = 1) \).

The Carli index, used by 19 percent of countries, is another popular index that can be thought of as a variant of the Laspeyres index. The formula for the Carli index is

\[ \Phi_{t-1,t}^C = \sum_{k \in \Omega_{t-1}} \frac{1}{N_{kt-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right) \quad (A.103) \]

This index is identical to the Laspeyres if all goods have equal expenditure shares. However, as with the Dutot, it is important to remember that statistical agencies are more likely to select a good for inclusion in the sample if it has a high sales share \( (s_{kt-1}^*) \). In this case, the Carli index also collapses back to the Laspeyres formula.

Similarly, the Paasche index is closely related to the Laspeyres index with the only difference that it weights price changes from \( t - 1 \) to \( t \) by their expenditure shares in the end period \( t \):

\[ \Phi_{t-1,t}^P = \frac{\sum_{k \in \Omega_{t-1}} p_{kt} c_{kt}}{\sum_{k \in \Omega_{t-1}} p_{kt-1} c_{kt}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right) \right]^{-1}. \quad (A.104) \]

We can derive the Paasche index from the backward difference of the CES unit expenditure function in equation (22) in the paper (which equals our CUPI in equation (14) in the paper) by making the same assumptions as used to derive the Laspeyres index.\(^2\)

Finally, the Jevons index, which forms the basis of the lower level of the U.S. Consumer Price Index, is the second-most popular index currently in use, with 37 percent of countries building their measures of changes in the cost of living based on it.\(^3\) The index is constructed by taking an unweighted geometric mean of price changes from \( t - 1 \) to \( t \):

\[ \Phi_{t-1,t}^J = \prod_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1/N_{kt-1}} = \frac{\hat{p}_t}{\hat{p}_{t-1}}. \quad (A.105) \]

As we discussed earlier, this formula can be derived from our CES unified price index (CUPI) in equation (14) by taking the limit in which \( \sigma \to \infty \) and using the assumptions that the utility gain of new goods is exactly offset by the loss from disappearing goods \( (\lambda_{t-1,1} = 1) \) and demand for each good is constant \( (\varphi_{kt} = 1) \). It is also related to the unified price index through another route. Statistical agencies typically choose products based on their historic sales shares. In this case the Jevons index becomes:

\(^2\)To derive (A.104) from equation (22) in the paper, we use \( \Phi_{t-1,t} = 1/\Phi_{t,1-t}, \) assume \( \lambda_{t-1,t}/\lambda_{t-1,t-1} = 1 \) and \( \varphi_{kt}/\varphi_{kt-1} = 1 \) for all \( k \), and set \( \sigma = 0. \)

\(^3\)The percentages do not sum to 100 because 3 percent of sample respondents used other formulas.
which Konyus (Konüs) and Byushgens (1926) proved was exact for the Cobb-Douglas (1928) functional form. This price index is a special case of the exact CES price index in which the elasticity of substitution equals one, demand for each good is constant, and there are no changes in variety.

Under our assumption of CES preferences, our CUPI also can be related to “superlative” price indexes (Fisher and Törnqvist) that provide an arbitrarily close local approximation to any continuous and twice-differentiable expenditure function. Taking the geometric mean of the forward and backward differences of the CES unit expenditure function in equations (21) and (22) in the paper, we obtain the following quadratic mean of order $2(1 - \sigma)$ price index (Diewert 1976):

$$
\Phi_{t-1,t} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right) \frac{1}{1 - \sigma} \left[ \frac{\sum_{k \in \Omega_{t-1}} s_{kt}^{*} - \sum_{k \in \Omega_{t-1}} s_{kt}^{*}}{\sum_{k \in \Omega_{t-1}} s_{kt}} \right]^{1/(1 - \sigma)}, \tag{A.107}
$$

The Fisher index is the geometric mean of the Laspeyres (A.102) and Paasche (A.104) price indexes, and can be derived from equation (A.107) using the assumptions that $\sigma = 0$, the utility gain from new goods is exactly offset by the loss from disappearing goods ($\lambda_{t,t-1}/\lambda_{t-1,t} = 1$), and demand for each good is constant ($\varphi_{kt}/\varphi_{kt-1} = 1$):

$$
\Phi_{t-1,t}^F = \left( \Phi_{t-1,t}^{L} \Phi_{t-1,t}^{P} \right)^{1/2}. \tag{A.108}
$$

Closely related to the Fisher index is the Törnqvist index, which can be derived from equation (A.107) by taking the limit in which $\sigma \to 1$ and using the assumptions that the utility gain from new goods is exactly offset by the loss from disappearing goods ($\lambda_{t,t-1}/\lambda_{t-1,t} = 1$), and demand for each good is constant ($\varphi_{kt}/\varphi_{kt-1} = 1$):

$$
\Phi_{t-1,t}^T = \prod_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1/2} \left( s_{kt-1}^{*} + s_{kt}^{*} \right). \tag{A.109}
$$

Another way of looking at the Törnqvist index is to realize that it is just a geometric average of Cobb-Douglas price indexes defined in equation (A.106) evaluated at times $t - 1$ and $t$.

Figure A.1 summarizes the relationship between our UPI and all major price indexes. Under our assumption of CES preferences, we can derive most existing price indexes (such as the Dutot, Carli, Laspeyres, Paasche, Jevons, Cobb-Douglas, Sato-Vartia-CES, Feenstra-CES) as special cases of the UPI for particular parameter values and assumptions about the entry and exit of goods and changes in demand for surviving goods. Nonetheless, there are other potential ways of rationalizing these existing price indexes using alternative functional form assumptions on preferences (e.g. the Törnqvist index is exact for translog preferences).
Figure A.1: Relation Between Existing Indexes and the UPI

- **Quadratic Mean of Order** \( r = 2(1 - \sigma) \)

- **Fisher**
- **Törnqvist**

- **Sato Vartia CES**
  - \( \sigma \neq 1 \)
  - \( \lambda_t/\lambda_{t-1} \neq 1 \)

- **Cobb-Douglas**
  - PFW

- **Feenstra CES**
  - \( \varphi_{k,t}/\varphi_{k,t-1} \neq 1 \)
  - \( \sigma \neq \infty \)

- **Unified Price Index**
  - Aggregation
  - \( \varphi_{k,t}/\varphi_{k,t-1} \neq 1 \)
  - \( \sigma \neq 0 \)
  - \( \lambda_t/\lambda_{t-1} \neq 1 \)

- **Laspeyres**
  - PFW

- **Carli**
  - PFW

- **Paasche**
  - PFW

- **Dutot**

---

**Key**

- \( \sigma \) : Elasticity of Substitution
- PFW: Purchase Frequency Weighting
- \( \varphi_{k,t}/\varphi_{k,t-1} = 1 \): No Demand Shifts
- \( \lambda_t/\lambda_{t-1} = 1 \): No Change in Variety
Finally, we now show that the assumption that the demand parameters for each good are constant is also central to existing continuous time index numbers, such as the Divisia index. Given a constant set of goods ($\Omega$) and a unit expenditure function that depends on the vector of prices and demand parameters for each good ($P^* (p_t, \varphi_t)$), and assuming that demand for each good $k \in \Omega$ remains constant ($\varphi_k = \varphi$), the Divisia index can be derived as follows:

$$d \ln P^* (p_t, \varphi) = \sum_{k \in \Omega} \frac{d \ln P^* (p_t, \varphi)}{d \ln p_{kt}} d \ln p_{kt},$$

$$d \ln P^* (p_t, \varphi) = \sum_{k \in \Omega} \left( \frac{d P^* (p_t, \varphi)}{d p_{kt}} \frac{p_{kt}}{P^* (p_t, \varphi)} \right) d \ln p_{kt},$$

$$d \ln P^* (p_t, \varphi) = \sum_{k \in \Omega} s_{kt}^* d \ln p_{kt}.$$

$$\ln P^* (p_0, p_t, \varphi) = \int_{p_0}^{p_1} \sum_{k \in \Omega} s_{kt}^* d \ln p_{kt}.$$

This derivation of the Divisia index makes explicit the assumption of constant demand for each good. In contrast, if demand for each good were time-varying, there would be additional terms in $d \ln \varphi_{kt}$ in the first line of the derivation above.

### A.8 Feenstra (1994) Estimator

In this section of the web appendix, we provide further details on the Feenstra (1994) estimator discussed in Section 2.5.1 of the paper and used as a robustness check in Section 5.1 of the paper. We estimate separate elasticities of substitution for each product group in our data. We develop the estimator for a given product group below, where to simplify notation we omit the subscript $g$ for product group.

We start with the double-differenced expenditure share from the CES demand system in equation (19) in the paper:

$$\Delta \ln s_{kt}^* = \beta_0 + \beta_1 \Delta \ln p_{kt} + u_{kt}, \quad (A.110)$$

where the first difference is over time and the second differences is from the geometric mean across common goods; $\Delta$ denotes the time-difference operator such that $\Delta \ln \bar{p}_{kt} = \ln (\bar{p}_{kt} / \bar{p}_{kt-1})$; a bar above a variable indicates that it is normalized by its geometric mean across common goods such that $\ln (\bar{p}_{kt}) = \ln (p_{kt} / \bar{p}_t)$; and the time-invariant component of demand ($\varphi_k$) has differenced out between the two time periods to leave only the change in the time-varying component of demand ($\Delta \ln \theta_{kt}$); and we have used our result that demand shocks average out across common goods ($\ln (\bar{\varphi}_t / \bar{\varphi}_{t-1}) = 0$).

We combine this relationship from the CES demand system in equation (A.110) above with an analogous supply-side relationship:

$$\Delta \ln s_{kt}^* = \delta_0 + \delta_1 \Delta \ln p_{kt} + w_{kt}. \quad (A.111)$$

The identifying assumption of the Feenstra (1994) estimator is that the double-differenced demand and supply shocks ($u_{kt}, w_{kt}$) are orthogonal and heteroskedastic. The orthogonality assumption defines a rectangular hyperbola for each good in the space of the demand and supply elasticities. The heteroskedasticity...
assumption implies that these rectangular hyperbolas for different goods do not lie on top of another. With two goods, the intersection of these rectangular hyperbolas exactly identifies the elasticity of substitution. With more than two goods, the model is overidentified.

In particular, following Broda and Weinstein (2006), the orthogonality of the double-differenced demand and supply shocks defines a set of moment conditions (one for each good within a product group):

\[ G(\xi) = \mathbb{E}_T [\xi_{kt}(\xi)] = 0, \]

where \( \xi = \left( \frac{\beta_1}{\delta_1} \right); \xi_{kt} = u_{kt} w_{kt}; \) and \( \mathbb{E}_T \) is the expectations operator over time. We stack the moment conditions for all goods within a product group to form the GMM objective function and obtain:

\[ \hat{\xi} = \arg \min \left\{ G^S(\xi)' W G^S(\xi) \right\}, \]

where \( G^S(\xi) \) is the sample analog of \( G(\xi) \) stacked over all goods within a given product group and \( W \) is a positive definite weighting group. As in Broda and Weinstein (2010), we weight the data for each good by the number of raw buyers for that good to ensure that our objective function is more sensitive to goods purchased by larger numbers of consumers.

### A.9 Forward and Backward Aggregate Demand Shifters

In this section of the web appendix, we derive the equalities between equivalent expressions for the change in the cost of living in equations (23) and (24) in the paper, as well as the expressions for the forward and backward aggregate demand shifters in equation (25) in the paper. We start with our three expressions for the change in the cost of living in equations (23) and (24) in the paper, as well as the expressions for the forward and backwards differences of the CES unit expenditure function (equations (21) and (22) in the paper):

\[ \frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^{1/\alpha} \frac{\bar{P}_t}{\bar{P}_{t-1}} \left( \frac{s_t^*}{s_{t-1}^*} \right)^{1/\alpha}, \]

\[ \frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^{1/\alpha} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma} \right]^{1/\alpha}, \]

\[ \frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t-1,t}}{\lambda_{t-1,t}} \right)^{1/\alpha} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt-1} / \varphi_{kt-1}}{p_{kt} / \varphi_{kt}} \right)^{1-\sigma} \right]^{1/\alpha}, \]

where our specification of demand in equation (2) in the paper implies \( \varphi_{kt} / \varphi_{kt-1} = \theta_{kt} / \theta_{kt-1} \). Combining these three equations (A.114), (A.115) and (A.116), we obtain:

\[ \Theta_{t-1,t}^F \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right]^{1/\sigma} = \frac{\bar{P}_t}{\bar{P}_{t-1}} \left( \frac{s_t^*}{s_{t-1}^*} \right)^{1/\alpha}, \]

\[ \left( \Theta_{t-1,t}^B \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{-1/\sigma} = \frac{\bar{P}_t}{\bar{P}_{t-1}} \left( \frac{s_t^*}{s_{t-1}^*} \right)^{1/\alpha}, \]
where $\Theta^F_{t-1,j}$ and $\Theta^B_{t,t-1}$ are aggregate demand shifters that are defined as:

$$
\Theta^F_{t-1,j} = \left[ \frac{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P_{kt-1}}{P_{kt}} \right)^{1-\sigma} \left( \frac{\varphi_{kt-1}}{\varphi_{kt}} \right)^{\sigma-1} }{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P_{kt-1}}{P_{kt}} \right)^{1-\sigma}} \right]^{\frac{1}{\sigma}},
\Theta^B_{t,t-1} = \left[ \frac{\sum_{k \in \Omega_{j,t-1}} s^B_{kt} \left( \frac{P_{kt-1}}{P_{kt}} \right)^{1-\sigma} \left( \frac{\varphi_{kt-1}}{\varphi_{kt}} \right)^{\sigma-1} }{\sum_{k \in \Omega_{j,t-1}} s^B_{kt} \left( \frac{P_{kt-1}}{P_{kt}} \right)^{1-\sigma}} \right]^{\frac{1}{\sigma}}, \quad (A.119)
$$

where the variety correction terms for entry/exit have cancelled from both sides of the equations (A.117) and (A.118), and recall that $\varphi_{kt}/\varphi_{kt-1} = \theta_{kt}/\theta_{kt-1}$. Now note the following results:

$$
s^*_{kt} = \left( \frac{P_{kt}}{\varphi_{kt}} \right)^{1-\sigma}, \quad (A.120)
$$

which implies:

$$
\left( \frac{P_{kt}}{\varphi_{kt}} \right)^{1-\sigma} = s^*_{kt} \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\sigma}, \quad (A.121)
$$

$$
\left( \frac{P_{kt}}{\varphi_{kt}} \right)^{(1-\sigma)} = s^*_{kt} \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\sigma}, \quad (A.122)
$$

$$
\left( \frac{P_{kt}}{\varphi_{kt}} \right)^{1-\sigma} = s^*_{kt} \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\sigma} - \varphi_{kt}/\varphi_{kt-1} \quad (A.123)
$$

$$
\left( \frac{P_{kt}}{\varphi_{kt}} \right)^{(1-\sigma)} = s^*_{kt} \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\sigma} - \varphi_{kt}/\varphi_{kt-1} \quad (A.124)
$$

We now use these results to rewrite the aggregate demand shifters. First, using equation (A.121) in the numerator of $\Theta^F_{t-1,t}$ in equation (A.119), we obtain:

$$
\Theta^F_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma} \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\sigma} }{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma}} \right]^{\frac{1}{\sigma}}, \quad (A.125)
$$

Using equation (A.123) in the denominator of equation (A.125), we have:

$$
\Theta^F_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma} }{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma} \left( \frac{\varphi_{kt-1}}{\varphi_{kt}} \right)^{(1-\sigma)}} \right]^{\frac{1}{\sigma}}, \quad (A.126)
$$

which simplifies to equation (25) in the paper:

$$
\Theta^F_{t-1,t} = \left[ \frac{1}{\sum_{k \in \Omega_{j,t-1}} s^F_{kt-1} \left( \frac{\varphi_{kt-1}}{\varphi_{kt-1}} \right)^{(1-\sigma)}} \right]^{\frac{1}{\sigma}} = \left[ \sum_{k \in \Omega_{j,t-1}} s^F_{kt} \left( \frac{\varphi_{kt-1}}{\varphi_{kt}} \right)^{(1-\sigma)} \right]^{\frac{1}{\sigma}}, \quad (A.127)
$$

where $\varphi_{kt}/\varphi_{kt-1} = \theta_{kt}/\theta_{kt-1}$. Second, using equation (A.122) in the numerator of $\Theta^B_{t,t-1}$ in equation (A.119), we obtain:

$$
\Theta^B_{t,t-1} = \left[ \frac{\sum_{k \in \Omega_{j,t-1}} s^B_{kt} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma} }{\sum_{k \in \Omega_{j,t-1}} s^B_{kt} \left( \frac{P^*_{kt-1}}{P^*_t} \right)^{1-\sigma}} \right]^{\frac{1}{\sigma}}, \quad (A.128)
$$
Using equation (A.124) in the denominator of equation (A.128), we have:

\[
\Theta_{t,t-1}^B \equiv \left[ \frac{\sum_{k \in \Omega_{t,t-1}} S_{kt} (\frac{p_{kt}}{p_{kt-1}})^{(1-\sigma)} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}},
\]

which simplifies to the expression in equation (25) in the paper:

\[
\Theta_{t,t-1}^B \equiv \left[ \frac{1}{\sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)}} \right]^{\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} = \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{\tilde{s}_t}{\tilde{s}_{t-1}} \right)^{\frac{1}{1-\sigma}},
\]

where \( \frac{G_{kt}}{G_{kt-1}} = \theta_{kt} \). We now use the definition of the forward aggregate demand shifter \( \Theta_{t-1,t}^F \) in equation (A.119) to rewrite our equality between the forward difference of the unit expenditure function and the unified price index in equation (A.117) as follows:

\[
\Theta^F \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} = \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{\tilde{s}_t}{\tilde{s}_{t-1}} \right)^{\frac{1}{1-\sigma}}.
\]

where \( \Theta^F \) is defined as:

\[
\Theta^F \equiv \left[ \frac{\sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}}.
\]

From our inversion of the demand system in equation (11) in the paper, we have:

\[
\ln \left( \frac{G_{kt}}{G_{kt-1}} \right) = \ln \left( \frac{\hat{p}_{kt}}{\hat{p}_{kt-1}} \right) + \frac{1}{\sigma-1} \ln \left( \frac{S_{kt}}{S_{kt-1}} \right),
\]

where we have used our result that \( \ln (\hat{G}_t/\hat{G}_{t-1}) = 0 \). Using equation (A.133) to substitute for the unobserved demand shocks \( \theta_{kt}/\theta_{kt-1} \) in equation (A.131), we obtain the following key result:

\[
\Theta^F \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}} = 1.
\]

Now we use the definition of the backward aggregate demand shifter \( \Theta_{t-1,t}^B \) in equation (A.119) to rewrite our equality between the backward difference of the unit expenditure function and the unified price index in equation (A.118) as follows:

\[
\left( \Theta^B \right)^{-\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{-\frac{1}{1-\sigma}} \left[ \sum_{k \in \Omega_{t,t-1}} S_{kt-1} (\frac{G_{kt}}{G_{kt-1}})^{(1-\sigma)} \right]^{-\frac{1}{1-\sigma}} = \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{\tilde{s}_t}{\tilde{s}_{t-1}} \right)^{\frac{1}{1-\sigma}},
\]
where $\Theta^B$ is defined as:

$$\Theta^B = \frac{\left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)^{-\sigma} \right]^{1/\sigma}}{\left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{1/\sigma}}.$$  (A.136)

Using equation (A.133) to substitute for the unobserved demand shocks ($\varphi_{kt} / \varphi_{kt-1}$) in equation (A.131), we obtain another key result:

$$\left( \Theta^B \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)^{-\sigma} \right]^{1/\sigma} = 1.$$  (A.137)

Combining equations (A.134) and (A.137), we obtain:

$$\Theta^F = \Theta^B = \mathcal{O}.$$  (A.138)

Additionally, equations (A.119) and (A.127) together imply:

$$\Theta^F_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)^{-\sigma} \right]^{1/\sigma} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{1/\sigma},$$  (A.139)

and equations (A.119) and (A.130) together imply:

$$\Theta^B_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)^{-\sigma} \right]^{1/\sigma} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{1/\sigma}.$$  (A.140)

Combining the definitions of $\Theta^F$ and $\Theta^B$ in equations (A.132) and (A.136) with these results for $\Theta^F$, $\Theta^B$, $\Theta^F_{t-1,t}$ and $\Theta^B_{t-1,t}$ in equations (A.138), (A.139) and (A.140), we obtain the following relationship between these variables:

$$\Theta^F = \Theta^B = \mathcal{O} = \Theta^F_{t-1,t} \Theta^B_{t-1,t}.$$  (A.141)

Using this result in equation (A.134), or equivalently using this result in equation (A.137), we obtain equation (26) in the paper:

$$\left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{1/\sigma} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{-1/\sigma} = \Theta^F_{t-1,t} \Theta^B_{t-1,t} = \mathcal{O}.$$  (A.142)

which must hold for any value of the elasticity of substitution and any constellation of demand and price shocks. Finally, this relationship in equation (A.142) implies that if the equality between the forward difference of the unit expenditure function and the UPI in equation (A.117) is satisfied, the equality between the backward difference of the unit expenditure function and the UPI in equation (A.118) must also be satisfied, and vice versa. To see this, suppose first that equation (A.117) is satisfied. Using equation (A.142) to substitute for $\Theta^F_{t-1,t} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{1/\sigma}$ in equation (A.117), we obtain equation (A.118). Suppose second that
equation (A.118) is satisfied. Using equation (A.142) to substitute for \( \left( \Theta^B_{t,t-1} \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right]^{-1} \left( \frac{1}{\pi} \right)^{\frac{1}{1-\sigma}} \) in equation (A.118), we obtain equation (A.117). Therefore, there is a single value of the elasticity of substitution (\( \sigma \)) that satisfies equations (A.117) and (A.118) for any constellation of demand and price shocks.

### A.10 Hicks-Neutral Shifter

In this Section of the web appendix, we show that our reverse-weighting (RW) estimator generalizes to allow for a Hicks-neutral demand shifter \( (Y_t) \) that is common across goods. In the presence of this Hicks-neutral shifter, the unit expenditure function in equation (1) in the paper becomes:

\[
P_t = \left[ \sum_{k \in \Omega_t} \left( \frac{p_{kt}}{Y_t \varphi_{kt}} \right)^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}}, \tag{A.143}
\]

and the expenditure share in equation (3) in the paper can be written as:

\[
s_{kt} = \frac{(p_{kt} / (Y_t \varphi_{kt}))^{1-\sigma}}{p_t^{1-\sigma}}. \tag{A.144}
\]

Using equations (A.143) and (A.144), we obtain the following generalizations of the three equivalent expressions for the change in the cost of living in equations (14), (21) and (22) in the paper:

\[
\Phi^U_{t-1,t} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \frac{P_t^*}{P_{t-1}^*} = \frac{Y_{t-1}}{Y_t} \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt} / \varphi_{kt}}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{A.145}
\]

\[
\Phi^F_{t-1,t} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \frac{P_t^*}{P_{t-1}^*} = \frac{Y_{t-1}}{Y_t} \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt-1} / \varphi_{kt-1}}{p_{kt} / \varphi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{A.146}
\]

\[
\Phi^B_{t,t-1} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \frac{P_t^*}{P_{t-1}^*} = \frac{Y_{t-1}}{Y_t} \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt-1} / \varphi_{kt-1}}{p_{kt} / \varphi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{A.147}
\]

Equating (A.145) and (A.146), and combining (A.145) and the inverse of (A.147), the change in the Hicks-neutral shifter \( (Y_t / Y_{t-1}) \) cancels from both sides of the equation:

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt} / \varphi_{kt}}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[ \frac{\tilde{p}_{t-1}^* \left( \tilde{s}_{t-1}^* \right)^{\frac{1}{\sigma-1}}}{\tilde{p}_t^* \left( \tilde{s}_t^* \right)^{\frac{1}{\sigma-1}}} \right], \tag{A.148}
\]

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt-1} / \varphi_{kt-1}}{p_{kt} / \varphi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[ \frac{\tilde{p}_{t-1}^* \left( \tilde{s}_{t-1}^* \right)^{\frac{1}{\sigma-1}}}{\tilde{p}_t^* \left( \tilde{s}_t^* \right)^{\frac{1}{\sigma-1}}} \right]. \tag{A.149}
\]

Using our specification for demand from equation (2) in the paper, which implies \( \varphi_{kt} / \varphi_{kt-1} = \theta_{kt} / \theta_{kt-1} \), and the definition of the forward and backward aggregate demand shifters in equation (25) in the paper, we obtain:
Using the CES common goods expenditure share at time $t$, which enables us to rewrite the change in the cost of living using period $t-1$ tastes as:

$$
\Theta_{t-1,t}^{\circ} = \sum_{k \in \Omega_{t-1}} s_{kl}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{s_{kl}^*}{s_{kl-1}^*} \right)^{1-\sigma},
$$

(A.150)

$$
\left( \Theta_{t-1,t}^{\circ} \right)^{-1} = \sum_{k \in \Omega_{t-1}} s_{kl}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\sigma)} \left( \frac{s_{kl}^*}{s_{kl-1}^*} \right)^{-\frac{1}{\sigma}},
$$

(A.151)

which corresponds to equations (23) and (24) in the paper. Therefore, the reverse-weighting estimator remains unchanged in the presence of a Hicks-neutral demand shifter ($Y_t$), as in equations (28)-(29) in the paper.

### A.11 Reverse-Weighting (RW) Estimator and the Change in the Cost of Living Using Tastes in Each Period

In this section of the web appendix, we show that the reverse-weighting (RW) estimator minimizes the sum of squared deviations between (i) the unified price index evaluated using tastes in both periods (inverting the demand system), (ii) the change in the cost of living evaluated using period $t-1$ tastes, and (iii) the change in the cost of living using period $t$ tastes. This property relates to the results of Fisher and Shell (1972), which uses the tastes of the initial or final period to bound the change in the cost of living. Here, we show that the elasticity of substitution itself can be chosen to minimize the difference between change in the cost of living using the tastes of the initial or final period.

First, the change in the cost of living for common goods evaluated using period $t-1$ tastes is:

$$
\Phi_{t-1,t} |_{\varphi_{t-1}} = \left[ \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{\varphi_{kt-1}} \right)^{1-\sigma} \right] \left( \frac{s_{kt-1}^*}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma} = \left[ \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{\varphi_{kt}} \right)^{1-\sigma} \right] \left( \frac{s_{kt-1}^*}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma}.
$$

(A.152)

Using the CES common goods expenditure share at time $t-1$, we have:

$$
\frac{s_{kt-1}^*}{(p_{kt-1} / \varphi_{kt-1})^{1-\sigma}} = \frac{1}{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_{kt})^{1-\sigma}}
$$

(A.153)

which enables us to rewrite the change in the cost of living using period $t-1$ tastes as:

$$
\Phi_{t-1,t} |_{\varphi_{t-1}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right] \left( \frac{s_{kt}^*}{p_{kt-1} / \varphi_{kt-1}} \right)^{1-\sigma}.
$$

(A.154)

Second, the change in the cost of living for common goods evaluated using period $t$ tastes is:

$$
\Phi_{t-1,t} |_{\varphi_t} = \left[ \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{\varphi_t} \right)^{1-\sigma} \right] \left( \frac{s_{kt}^*}{p_{kt-1} / \varphi_t} \right)^{1-\sigma} = \left[ \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{\varphi_t} \right)^{1-\sigma} \right] \left( \frac{s_{kt}^*}{p_{kt-1} / \varphi_t} \right)^{1-\sigma}.
$$

(A.155)

Using the CES common goods expenditure share at time $t$, we have:

$$
\frac{s_{kt}^*}{(p_{kt} / \varphi_t)^{1-\sigma}} = \frac{1}{\sum_{k \in \Omega_{t-1}} (p_{kt} / \varphi_t)^{1-\sigma}}
$$

(A.156)
which enables us to rewrite the change in the cost of living using period $t$ tastes as:

$$
\Phi_{t-1,t|\varphi_t} = \left[ \sum_{k \in \Omega_t} s^*_k \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\sigma)} \right]^{\frac{1}{1-\sigma}}. \tag{A.157}
$$

Third, the change in the cost of living for common goods from the unified price index (using the demand system to substitute for tastes in periods $t-1$ and $t$) is given by equation (13) in the paper. Finally, from equations (28) and (29), the reverse-weighting estimator minimizes an objective given by the following sum of squared deviations:

$$
\left\{ \frac{1}{1-\sigma} \ln \left[ \sum_{k \in \Omega_{t-1}} s^*_{k,t-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right] - \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{s^*_{kt}}{s^*_{kt-1}} \right)^{\frac{1}{1-\sigma}} \right] \right\}^2.
$$

From equation (13) in the paper and equations (A.154) and (A.157) above, this objective function corresponds to the sum of squared deviations between (i) the unified price index, (ii) the change in the cost of living evaluated using period $t-1$ tastes, and (iii) the change in the cost of living evaluated using period $t$ tastes.

**A.12 Proof of Proposition 2 (Small Demand Shocks)**

*Proof.* If this section of the web appendix, we prove that as demand shocks become small ($(\theta_{kt}/\theta_{kt-1}) \to 1$), the reverse-weighting (RW) estimator consistently estimates the true elasticity of substitution ($\delta_{\text{RW}} \to \sigma^D$). Recall from equations (A.119), (A.127) and (A.140) in Section A.9 of this web appendix that the forward and backward aggregate demand shifters are:

$$
\Theta^F_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\theta_{kt-1}}{\theta_{kt}} \right)^{\sigma-1}}{\sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{p_{kt-1}}{p_{kt}} \right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} = \left[ \sum_{k \in \Omega_{t-1}} s^*_{kt} \left( \frac{\theta_{kt-1}}{\theta_{kt}} \right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}, \tag{A.158}
$$

$$
\Theta^B_{t-1,t} = \left[ \frac{\sum_{k \in \Omega_{t-1}} s^*_{kt} \left( \frac{p_{kt-1}}{p_{kt}} \right)^{1-\sigma} \left( \frac{\theta_{kt-1}}{\theta_{kt}} \right)^{\sigma-1}}{\sum_{k \in \Omega_{t-1}} s^*_{kt} \left( \frac{p_{kt-1}}{p_{kt}} \right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} = \left[ \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}. \tag{A.159}
$$

As $(\theta_{kt}/\theta_{kt-1}) \to 1$, equations (A.158) and (A.159) imply that:

$$
\Theta^F_{t-1,t} \overset{P}{\to} 1, \quad \Theta^B_{t-1,t} \overset{P}{\to} 1. \tag{A.160}
$$

As $\Theta^F_{t-1,t} \overset{P}{\to} 1$ and $\Theta^B_{t-1,t} \overset{P}{\to} 1$, we have the following results:

$$
\frac{1}{1-\sigma} \ln \left[ \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right] - \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{s^*_{kt}}{s^*_{kt-1}} \right)^{\frac{1}{1-\sigma}} \right] \overset{P}{\to} 0, \tag{A.161}
$$

$$
- \frac{1}{1-\sigma} \ln \left[ \sum_{k \in \Omega_{t-1}} s^*_{kt} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)} \right] - \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \left( \frac{s^*_{kt}}{s^*_{kt-1}} \right)^{\frac{1}{1-\sigma}} \right] \overset{P}{\to} 0. \tag{A.162}
$$
which implies that the moment condition in equation (28) in the paper is satisfied and the reverse-weighting estimator converges to the true elasticity of substitution: $\hat{\sigma}^{RW} \xrightarrow{P} \sigma^D$. We now show that there exists a unique value for $\sigma$ that solves these two equations. We begin with equation (A.161), which can be re-written as:

$$
- \frac{1}{\sigma - 1} \ln \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt-1}}{p_{kt}} \right)^{\sigma-1} \right] = \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \right] + \frac{1}{\sigma - 1} \ln \left[ \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right) \right],
$$

(A.163)

or equivalently

$$
\Lambda_t^F = \Lambda_t^D,
$$

(A.164)

$$
\Lambda_t^F \equiv - \ln \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt-1}}{p_{kt}} \right)^{\sigma-1} \right],
$$

(A.165)

$$
\Lambda_t^D \equiv (\sigma - 1) \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \right] + \ln \left[ \frac{s_{kt}^*}{s_{kt-1}^*} \right].
$$

(A.166)

First, we differentiate $\Lambda_t^F$ in equation (A.165) to obtain:

$$
\frac{d\Lambda_t^F}{d (\sigma - 1)} = - \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \ln \left( \frac{p_{kt-1}}{p_{kt}} \right) \left( \frac{p_{kt-1}}{p_{kt}} \right)^{\sigma-1},
$$

(A.167)

where we have used $d (a^x) / dx = (\ln a) a^x$. Now note that the common goods expenditure share (7) and the CES price index for common goods (6) imply that as $q_{kt} / q_{kt-1} \rightarrow 1$:

$$
\left( \frac{p_{kt-1}}{p_{kt}} \right)^{\sigma-1} = \frac{s_{kt}^*}{s_{kt-1}^*} \left( \frac{p_{kt-1}}{p_{kt}} \right)^{1-\sigma}, \quad k \in \Omega_{t,t-1}.
$$

(A.168)

Using this result in (A.167), re-arranging terms and noting that $\sum_{k \in \Omega_{t-1}} s_{kt-1}^* = 1$, we obtain:

$$
\frac{d\Lambda_t^F}{d (\sigma - 1)} = \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right).
$$

(A.169)

Note that $s_{kt}^* > 0$; $\ln (p_{kt} / p_{kt-1}) < 0$ for $p_{kt} < p_{kt-1}$; and $\ln (p_{kt} / p_{kt-1}) > 0$ for $p_{kt} > p_{kt-1}$. Therefore, depending on the values of the expenditure shares ($s_{kt}^*$), $\frac{d\Lambda_t^F}{d (\sigma - 1)}$ can be either positive or negative, and is independent of $\sigma - 1$. Second, we differentiate $\Lambda_t^D$ in equation (A.166) to obtain:

$$
\frac{d\Lambda_t^D}{d (\sigma - 1)} = \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \right].
$$

(A.170)

Note that $\frac{d\Lambda_t^D}{d (\sigma - 1)} > 0$ for $\hat{p}_t > \hat{p}_{t-1}$ and $\frac{d\Lambda_t^D}{d (\sigma - 1)} < 0$ for $\hat{p}_t < \hat{p}_{t-1}$. Therefore, depending on the values of prices ($p_{kt}$), $\frac{d\Lambda_t^D}{d (\sigma - 1)}$ can be either positive or negative, and is independent of $\sigma - 1$. Together equations (A.169) and (A.170) imply that both $\frac{d\Lambda_t^F}{d (\sigma - 1)}$ and $\frac{d\Lambda_t^D}{d (\sigma - 1)}$ can be either positive or negative and are independent of $\sigma - 1$. Assuming that:

$$
\sum_{k \in \Omega_{t-1}} s_{kt}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \neq \sum_{k \in \Omega_{t-1}} \frac{1}{N_{t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) = \ln \left[ \frac{\hat{p}_t}{\hat{p}_{t-1}} \right],
$$

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we have:
\[
\frac{d\Lambda_i^B}{d(\sigma - 1)} \neq \frac{d\Lambda_i^D}{d(\sigma - 1)}.
\] (A.171)

Note that the derivatives in (A.171) differ from one another and are independent of \((\sigma - 1)\). Therefore \(\Lambda_i^B\) and \(\Lambda_i^D\) exhibit a single-crossing property such that there exists a unique value of \((\sigma - 1)\) that satisfies (A.161), as shown in Figure A.2.

We next turn to equation (A.162), which can be re-written as:
\[
\frac{1}{\sigma - 1} \ln \left[ \sum_{k \in \Omega_{i,t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{\sigma - 1} \right] = \ln \left[ \frac{\bar{p}_t}{\bar{p}_{t-1}} \right] + \frac{1}{\sigma - 1} \ln \left[ \left( \frac{s_t^*}{s_{t-1}^*} \right) \right],
\] (A.172)

or equivalently
\[
\Lambda_i^B = \Lambda_i^D,
\] (A.173)
\[
\Lambda_i^B \equiv \ln \left[ \sum_{k \in \Omega_{i,t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{\sigma - 1} \right],
\] (A.174)
\[
\Lambda_i^D = (\sigma - 1) \ln \left[ \frac{\bar{p}_t}{\bar{p}_{t-1}} \right] + \ln \left[ \frac{s_t^*}{s_{t-1}^*} \right].
\] (A.175)

First, we differentiate \(\Lambda_i^B\) in equation (A.174) to obtain:
\[
\frac{d\Lambda_i^B}{d(\sigma - 1)} = \frac{\sum_{k \in \Omega_{i,t-1}} s_{kt}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \left( \frac{p_{kt}}{p_{kt-1}} \right)^{\sigma - 1}}{\sum_{k \in \Omega_{i,t-1}} s_{kt}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{\sigma - 1}},
\] (A.176)

where we have again used \(d(a^x)/dx = (\ln a) a^x\). Using the relationship between relative prices and relative common goods expenditure shares as \(\theta_{kt}/\theta_{kt-1} \to 1\) from equation (A.168), re-arranging terms, and noting that \(\sum_{k \in \Omega_{i,t-1}} s_{kt}^* = 1\), we obtain:
\[
\frac{d\Lambda_i^B}{d(\sigma - 1)} = \sum_{k \in \Omega_{i,t-1}} s_{kt-1}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right).
\] (A.177)

Note that \(s_{kt-1}^* > 0\); \(\ln \left( \frac{p_{kt}}{p_{kt-1}} \right) < 0\) for \(p_{kt} < p_{kt-1}\); and \(\ln \left( \frac{p_{kt}}{p_{kt-1}} \right) > 0\) for \(p_{kt} > p_{kt-1}\). Therefore, depending on the values of the expenditure shares \(\{s_{kt-1}^*\}\), \(\frac{d\Lambda_i^B}{d(\sigma - 1)}\) can be either positive or negative, and is independent of \((\sigma - 1)\). Second, we differentiate \(\Lambda_i^D\) in equation (A.175) to obtain:
\[
\frac{d\Lambda_i^D}{d(\sigma - 1)} = \ln \left[ \frac{\bar{p}_t}{\bar{p}_{t-1}} \right].
\] (A.178)

Therefore both \(\frac{d\Lambda_i^B}{d(\sigma - 1)}\) and \(\frac{d\Lambda_i^D}{d(\sigma - 1)}\) can be either positive or negative and are independent of \((\sigma - 1)\). Assuming that:
\[
\sum_{k \in \Omega_{i,t-1}} s_{kt-1}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \neq \sum_{k \in \Omega_{i,t-1}} \frac{1}{N_{i,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) = \ln \left[ \frac{\bar{p}_t}{\bar{p}_{t-1}} \right],
\]

we have:
\[
\frac{d\Lambda_i^B}{d(\sigma - 1)} \neq \frac{d\Lambda_i^D}{d(\sigma - 1)}.
\] (A.179)
Note that the derivatives in (A.179) differ from one another and are independent of \((\sigma - 1)\). Therefore \(\Lambda^B_t\) and \(\Lambda^D_t\) exhibit a single-crossing property such that there exists a unique value of \((\sigma - 1)\) that satisfies (A.162), as shown in Figure A.2. At this unique value of \(\sigma\), both \(\Lambda^F_t\) and \(\Lambda^B_t\) equal \(\Lambda^D_t\), as also shown in Figure A.2.

![Figure A.2: Single crossing between \(\Lambda^F\), \(\Lambda^B\) and \(\Lambda^D\)](image)

### A.13 First-order and Second-Order Approximations

In this section of the web appendix, we show that our assumption that the forward and backward differences of price index are money metric in equation (27) in the paper is satisfied up to a first-order approximation. From equation (25) in the paper, we have:

\[
\ln \Theta^F_{t-1,t} = \frac{1}{1 - \sigma} \ln \left[ \frac{\sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma_{kt-1}} \right)^{\sigma-1}} {\sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma}} \right].
\]

Taking a Taylor-series expansion of \(\ln \Theta^F_{t-1,t}\) around \((p_{kt}/p_{kt-1}) = 1\) and \((\theta_{kt}/\theta_{kt-1}) = 1\), we obtain:

\[
\ln \Theta^F_{t-1,t} = \sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} - 1 \right) - \sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kt-1}} - 1 \right)
- \sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{p_{kt}}{p_{kt-1}} - 1 \right) + O^2_F(s,p),
\]

\[
\ln \Theta^F_{t-1,t} = - \sum_{k \in \Omega_{t,t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kt-1}} - 1 \right) + O^2_F(s,p), \quad (A.180)
\]
where bold math font denotes a vector; the initial common goods expenditure shares \($s^*_{kt-1}\) are pre-determined at time \(t - 1\); and \(O^2_F(s, p)\) denotes the second-order and higher terms such that:

\[
O^2_F(s, p) = (2 - \sigma) \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left[ \frac{\theta_{kt}}{\theta_{kl-1}} - 1 \right] \left[ \frac{\theta_{kl}}{\theta_{kt-1}} - 1 \right] - 2 (1 - \sigma) \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left[ \frac{p_{kt}}{p_{kt-1}} - 1 \right] \left[ \frac{\theta_{kl}}{\theta_{kt-1}} - 1 \right] - (1 - \sigma) \sum_{k \in \Omega_{t-1}} \sum_{\ell \in \Omega_{t-1}} s^*_{kl-1} s^*_{lt-1} \left[ \frac{\theta_{kt}}{\theta_{kt-1}} - 1 \right] \left[ \frac{\theta_{lt}}{\theta_{lt-1}} - 1 \right] + 2 (1 - \sigma) \sum_{k \in \Omega_{t-1}} \sum_{\ell \in \Omega_{t-1}} s^*_{kl-1} s^*_{lt-1} \left[ \frac{p_{kt}}{p_{kt-1}} - 1 \right] \left[ \frac{\theta_{lt}}{\theta_{lt-1}} - 1 \right] + O^3_F(s, p) .
\]

where \(O^3_F(s, p)\) denotes the third-order and higher terms. From equation (25) in the paper, we also have:

\[
\ln \Theta^B_{t-1} = \frac{1}{\sigma - 1} \ln \left[ \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kl-1}} \right)^{\sigma - 1} \right]
\]

Taking a Taylor-series expansion of \(\ln \Theta^B_{t-1}\) around \((p_{kt}/p_{kt-1}) = 1\) and \((\theta_{kt}/\theta_{kt-1}) = 1\), we obtain:

\[
\ln \Theta^B_{t-1,k} = \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kl-1}} - 1 \right) + O^2_B(s, p) ,
\]

where the initial common goods expenditure shares \((s^*_{kt-1})\) are pre-determined at time \(t - 1\) and \(O^2_B(s, p)\) denotes the second-order and higher terms such that:

\[
O^2_B(s, p) = -(2 - \sigma) \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left[ \frac{\theta_{kt}}{\theta_{kl-1}} - 1 \right] \left[ \frac{\theta_{kl}}{\theta_{kt-1}} - 1 \right] + (1 - \sigma) \sum_{k \in \Omega_{t-1}} \sum_{\ell \in \Omega_{t-1}} s^*_{kl-1} s^*_{lt-1} \left[ \frac{\theta_{kt}}{\theta_{kt-1}} - 1 \right] \left[ \frac{\theta_{lt}}{\theta_{lt-1}} - 1 \right] + O^3_B(s, p) .
\]

where \(O^3_B(s, p)\) denotes the third-order and higher terms. The second-order terms in equations (A.180) and (A.182) depend on \((\theta_{kl}/\theta_{kt-1} - 1)\) \((\theta_{kl}/\theta_{kt-1} - 1)\) \((\theta_{kl}/\theta_{kt-1} - 1)\) \((\theta_{kl}/\theta_{kt-1} - 1)\), while the third-order terms depend on higher powers of \((\theta_{kl}/\theta_{kt-1} - 1)\) and \((\theta_{kl}/\theta_{kt-1} - 1)\). For small changes in prices and demand for each good \((p_{kt}/p_{kt-1}) \approx 0\) and \((\theta_{kt}/\theta_{kt-1} - 1) \approx 0\), these second-order and higher terms in equations (A.180) and (A.182) converge to zero \((O^2_F(s, p) \to 0\) and \(O^2_B(s, p) \to 0\). Therefore, to a first-order approximation, the forward and backward aggregate demand shifters satisfy time reversibility:

\[
\ln \Theta^F_{t-1,t} \approx - \ln \Theta^B_{t-1,t} \approx - \sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kl-1}} - 1 \right) .
\]

Noting that \(\sum_{k \in \Omega_{t-1}} s^*_{kt-1} = 1\) and \((\theta_{kt}/\theta_{kt-1} - 1) \approx 0\) for all \(k \in \Omega_{t-1}\), the following weighted average is also necessarily small:

\[
\sum_{k \in \Omega_{t-1}} s^*_{kt-1} \left( \frac{\theta_{kt}}{\theta_{kl-1}} - 1 \right) \approx 0,
\]

and hence

\[
\ln \Theta^F_{t-1,t} \approx - \ln \Theta^B_{t-1,t} \approx 0 .
\]
A.14 Proof of Proposition 3 (Large Number of Common Goods)

Proof. In this section of the web appendix, we prove that the reverse-weighting (RW) estimator consistently estimates the elasticity of substitution $(\hat{\gamma}_{RW}^{\text{RW}} \xrightarrow{p} \sigma^{D})$ if the number of common goods becomes large and demand shocks are uncorrelated with price shocks for each good and independently and identically distributed across goods. Recall from Section A.9 of this web appendix that the forward and backward aggregate demand shifters are given by:

$$
\Theta_{t-1,I}^F = \left[ \sum_{k \in \Omega_{t-1}, s_{kl-1}^*} \left( \frac{p_{kl-1}}{p_{kl}} \right)^{1-\sigma} \left( \frac{p_{kl}}{p_{kl-1}} \right)^{-1} \right]^{1/\sigma - 1} \left[ \sum_{k \in \Omega_{t-1}, s_{kl-1}^*} \theta_{kl-1} \left( \frac{\theta_{kl-1}}{\theta_{kl}} \right)^{1-\sigma} \right]^{1/\sigma - 1},
$$

(A.187)

$$
\Theta_{t-1,B}^B = \left[ \sum_{k \in \Omega_{t-1}, s_{kl-1}^*} \left( \frac{p_{kl-1}}{p_{kl}} \right)^{1-\sigma} \left( \frac{p_{kl}}{p_{kl-1}} \right)^{-1} \right]^{1/\sigma - 1} \left[ \sum_{k \in \Omega_{t-1}, s_{kl-1}^*} \theta_{kl-1} \left( \frac{\theta_{kl-1}}{\theta_{kl}} \right)^{1-\sigma} \right]^{1/\sigma - 1},
$$

(A.188)

We define the following change of variables and weighted means and covariances:

$$
X_{kt}^F = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma}, \quad Y_{kt}^F = \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right)^{\sigma - 1},
$$

$$
X_{kt}^B = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{\text{-(1-}\sigma)}, \quad Y_{kt}^B = \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right)^{-\text{(}\sigma - 1\text{)}}.
$$

$$
\mathbb{E} \left[ X_{kt}^F \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* X_{kt}^F, \quad \mathbb{E} \left[ X_{kt}^B \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* X_{kt}^B,
$$

$$
\mathbb{E} \left[ Y_{kt}^F \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* Y_{kt}^F, \quad \mathbb{E} \left[ Y_{kt}^B \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* Y_{kt}^B,
$$

$$
C \left[ X_{kt}^F, Y_{kt}^F \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* \left( X_{kt}^F - \mathbb{E} \left[ X_{kt}^F \right] \right) \left( Y_{kt}^F - \mathbb{E} \left[ Y_{kt}^F \right] \right),
$$

$$
C \left[ X_{kt}^B, Y_{kt}^B \right] = \sum_{k \in \Omega_{t-1}} s_{kl-1}^* \left( X_{kt}^B - \mathbb{E} \left[ X_{kt}^B \right] \right) \left( Y_{kt}^B - \mathbb{E} \left[ Y_{kt}^B \right] \right),
$$

Using these definitions, the aggregate forward and backward demand shifters can be rewritten as:

$$
\Theta_{t-1,I}^F = \left[ \frac{\mathbb{E} \left[ X_{kt}^F Y_{kt}^F \right]}{\mathbb{E} \left[ X_{kt}^F \right]} \right]^{1/\sigma - 1} = \left[ \frac{C \left[ X_{kt}^F Y_{kt}^F \right] + \mathbb{E} \left[ X_{kt}^F \right] \mathbb{E} \left[ Y_{kt}^F \right]}{\mathbb{E} \left[ X_{kt}^F \right]} \right]^{1/\sigma - 1} = \left[ \frac{1}{\mathbb{E} \left[ Y_{kt}^F \right]} \right]^{1/\sigma - 1},
$$

(A.189)

$$
\Theta_{t-1,B}^B = \left[ \frac{\mathbb{E} \left[ X_{kt}^B Y_{kt}^B \right]}{\mathbb{E} \left[ X_{kt}^B \right]} \right]^{1/\sigma - 1} = \left[ \frac{C \left[ X_{kt}^B Y_{kt}^B \right] + \mathbb{E} \left[ X_{kt}^B \right] \mathbb{E} \left[ Y_{kt}^B \right]}{\mathbb{E} \left[ X_{kt}^B \right]} \right]^{1/\sigma - 1} = \left[ \frac{1}{\mathbb{E} \left[ Y_{kt}^B \right]} \right]^{1/\sigma - 1}.
$$

(A.190)

As $N_{t-1} \to \infty$, our assumption that the demand shocks are independently and identically distributed ($(\theta_{kt}/\theta_{kt-1}) \sim \text{i.i.d.} \left(1, \chi^2_{q}\right)$ for $(\theta_{kt}/\theta_{kt-1}) \in (0, \infty)$) implies:

$$
C \left[ X_{kt}^F Y_{kt}^F \right] = C \left[ X_{kt}^B Y_{kt}^B \right] \xrightarrow{p} 0.
$$

(A.191)
Using this result in equations (A.189) and (A.190), time reversibility is satisfied:

\[
\Theta^F_{t-1,t} \xrightarrow{P} \frac{1}{\Theta^B_{t,t-1}} = \left[ \sum_{k \in \Omega_{t,t-1}} s^k_{t-1} \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma}}, \tag{A.192}
\]

\[
\Theta^B_{t,t-1} \xrightarrow{P} \frac{1}{\Theta^F_{t,t-1}} = \left[ \sum_{k \in \Omega_{t,t-1}} s^k_t \left( \frac{\theta_{kt-1}}{\theta_{kt}} \right)^{\sigma-1} \right]^{\frac{1}{\sigma}}. \tag{A.193}
\]

We now determine the asymptotic properties of the following term from equation (A.192):

\[
\Xi^F_{t-1,t} = \sum_{k \in \Omega_{t,t-1}} s^k_{t-1} \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right)^{\sigma-1}. \tag{A.194}
\]

We have assumed:

\[\varepsilon_{kt} = \frac{\theta_{kt}}{\theta_{kt-1}} \sim \text{i.i.d.} (1, \chi^2_\sigma).\]

Using the Central Limit Theorem, we obtain:

\[
\left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} (\varepsilon_{kt} - 1) \right] \sim \mathcal{N} \left( 0, \frac{\chi^2_\sigma}{N_{t,t-1}} \right).
\]

From the Delta method, a sequence of random variables that satisfies:

\[
\sqrt{n} (Y_n - \mu) \xrightarrow{d} \mathcal{N} \left( 0, \chi^2 \right)
\]

implies:

\[
\sqrt{n} \left[ \mathbb{E} \left[ g \left( Y_n \right) - g \left( \mu \right) \right] \right] \xrightarrow{d} \mathcal{N} \left( 0, \chi^2 \left[ g' \left( \mu \right) \right]^2 \right).
\]

Applying this result to equation (A.194), where \( g \left( \cdot \right) = \left( \cdot \right)^{\sigma-1}, \mu = 1, \chi^2 = \chi^2_\sigma, g \left( \mu \right) = 1 \) and \( g' \left( \mu \right) = \sigma - 1, \) we have:

\[
\left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} (\varepsilon_{kt}^{\sigma-1} - 1) \right] \xrightarrow{d} \mathcal{N} \left( 0, \frac{\chi^2_\sigma}{N_{t,t-1}} \left( \sigma - 1 \right)^2 \right).
\]

Therefore, as \( N_{t,t-1} \to \infty, \) we have:

\[
\left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} (\varepsilon_{kt}^{\sigma-1} - 1) \right] \xrightarrow{P} 0. \tag{A.195}
\]

We will use this result (A.195) in equation (A.194) later. First, note that equation (A.194) can be re-written as:

\[
\Xi^F_{t-1,t} = \sum_{k \in \Omega_{t,t-1}} s^k_{t-1} \left( \varepsilon_{kt}^{\sigma-1} - 1 \right) + 1, \tag{A.196}
\]

where

\[
\left| \left\{ N_{t,t-1} \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} s^k_{t-1} \left( \varepsilon_{kt}^{\sigma-1} - 1 \right) \right] \right\} \right| \leq \left| \left\{ N_{t,t-1} \sup \left\{ s^k_{t-1} \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( \varepsilon_{kt}^{\sigma-1} - 1 \right) \right] \right\} \right| \right|. \tag{A.197}
\]
and

\[
N_{t-1} \sup \{ s_{kt-1} \} = \frac{N_{t-1} \sup \left\{ \left( \frac{p_{1t-1}}{\varphi_{1t-1}} \right)^{1-\sigma}, \ldots, \left( \frac{p_{N_{t-1}t-1}}{\varphi_{N_{t-1}t-1}} \right)^{1-\sigma} \right\}}{\sum_{k=1}^{N_{t-1}} \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right)^{1-\sigma}},
\]

\[= \sup \left\{ \left( \frac{p_{1t-1}}{\varphi_{1t-1}} \right)^{1-\sigma}, \ldots, \left( \frac{p_{N_{t-1}t-1}}{\varphi_{N_{t-1}t-1}} \right)^{1-\sigma} \right\} \frac{1}{N_{t-1} \sum_{k=1}^{N_{t-1}} \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right)^{1-\sigma}} < \infty.
\]

Using the results (A.195), (A.197) and (A.198) in equation (A.196), we obtain the result that as \( N_{t-1} \to \infty \):

\[
\Xi_{t-1}^F \xrightarrow{p} 1.
\]

Therefore, assuming \( (\theta_{kt}/\theta_{kt-1}) \sim \text{i.i.d.} \left( 1, \chi^2_q \right) \) and as \( N_{t-1} \to \infty \), we have:

\[
\Theta_{t-1,t}^F = \left[ \Xi_{t-1}^F \right]^{1/2} \xrightarrow{p} 1,
\]

\[
\Theta_{t-1}^B = \frac{1}{\Theta_{t-1,t}^F} = \left[ \frac{1}{\Xi_{t-1}^F} \right]^{1/2} \xrightarrow{p} 1.
\]

\[\square\]

A.15 Asymptotic Properties of the RW Estimator

In this section of the web appendix, we show that the reverse-weighting (RW) estimator belongs to the class of M-estimators, as characterized in Newey and McFadden (1994) and Wooldridge (2002). We use this result to derive the asymptotic properties of the RW estimator.

We define \( X \equiv (s_t, s_{t-1}, p_t, p_{t-1}) \) as the vector of random variables with some distribution in the population. Let \( W \) denote the set of \( \mathbb{R}^4 \) representing the possible values of \( X \). We use \( \sigma^D \) to denote the true value of the elasticity of substitution \( \sigma \in \Xi \subset \mathbb{R}^+ \), with \( \Xi = [\underline{\sigma}, \bar{\sigma}] \), where \( \underline{\sigma} > 1 \) and \( \bar{\sigma} < \infty \). We assume that our data come as a random sample of size \( N \) from the population. We label this random sample \( \{ X_k : k = 1, 2, \ldots \} \), where each \( X_k \) corresponds to the vector of observed data on prices and expenditure shares for a good \( k \) in the two time periods.

Let \( q(X_k, \sigma) \) be a function of the random vector \( X \) and the parameter \( \sigma \). An M-estimator of \( \sigma^D \) solves the problem:

\[
\min_{\sigma \in \Xi} \frac{1}{N} \sum_{k=1}^{N} q(X_k, \sigma),
\]

where the true parameter \( \sigma^D \) solves the population problem:

\[
\min_{\sigma \in \Xi} \mathbb{E} \left[ q(X, \sigma) \right].
\]

Our RW estimator in equation (29) solves a problem of this form, because it can be written as:

\[
\min_{\sigma \in \Xi} \left\{ \left[ \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} m^F(X_k, \sigma) \right]^2 + \left[ \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} m^B(X_k, \sigma) \right]^2 \right\}
\]

\[
m^F(X_k, \sigma) = \frac{1}{1-\sigma} \ln \left[ \sum_{t \in \Omega_{t-1}} s_{kt-1}^n \left( \frac{p_{lt}}{p_{lt-1}} \right)^{1-\sigma} \right] - \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \frac{1}{\sigma - 1} \ln \left( \frac{s_{kt}}{s_{kt-1}} \right),
\]

29
\[ m^B(X_k, \sigma) = -\frac{1}{1 - \sigma} \ln \left[ \sum_{t \in \Omega_{t-1}} s^*_{lt} \left( \frac{p_{lt}}{p_{lt-1}} \right)^{-(1 - \sigma)} \right] - \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \frac{1}{\sigma - 1} \ln \left( \frac{s^*_{kt}}{s^*_{kt-1}} \right) \] (A.205)

The consistency of an M-estimator requires two sets of conditions to be satisfied: (i) identification and (ii) uniform convergence in probability. First, identification requires:

\[ \mathbb{E} \left[ q \left( X, \sigma^D \right) \right] < \mathbb{E} \left[ q \left( X, \sigma \right) \right], \quad \forall \sigma \in \Xi, \; \sigma \neq \sigma^D, \] (A.206)

which is satisfied for the RW estimator as demand shocks become small (Proposition 2 in the paper) or as the number of common goods becomes large and demand shocks are uncorrelated with price shocks for each good and independently and identically distributed across goods (Proposition 3 in the paper). Second, uniform convergence in probability in probability requires:

\[ \max_{\theta \in \Xi} \left| \frac{1}{N} \sum_{k=1}^{N} q \left( X_k, \sigma \right) - \mathbb{E} \left[ q \left( X, \sigma \right) \right] \right| \overset{P}{\to} 0. \] (A.207)

We provide conditions for this property to be satisfied using the following uniform weak law of large numbers result from Newey and McFadden (1994).

**Proposition 7. (Uniform Weak Law of Large Numbers)** Let \( X \) be a random vector taking values in \( \mathcal{W} \subseteq \mathbb{R}^4 \), let \( \Xi \) be a subset of \( \mathbb{R} \), and let \( q : \mathcal{W} \times \Xi \to \mathbb{R} \) be a real-valued function. Assume that (a) \( \Xi \) is compact; (b) for each \( \sigma \in \Xi \), \( q(\cdot, \sigma) \) is Borel measurable on \( \mathcal{W} \); (c) for each \( X \in \mathcal{W} \), \( q(X, \cdot) \) is continuous on \( \Xi \); and (d) \( |q(X, \sigma)| \leq b(X) \) for all \( \sigma \in \Xi \), where \( b \) is a non-negative function on \( \mathcal{W} \) such that \( \mathbb{E}[b(X)] < \infty \). Under these assumptions, the M-estimator in equation (A.201) satisfies uniform convergence in probability in equation (A.207).

**Proof.** See Newey and McFadden (1994), Lemma 2.4, and Wooldridge (2002), Theorem 12.1. \( \square \)

Our RW estimator satisfies conditions (a)-(d) of Proposition 7 above. Condition (a) is satisfied under our assumption that \( \sigma \in \Xi \subset \mathbb{R}^+ \), with \( \Xi = [\underline{\sigma}, \bar{\sigma}] \), where \( \underline{\sigma} > 1 \) and \( \bar{\sigma} < \infty \). Condition (b) is satisfied because \( m^F(X_k, \sigma) \) and \( m^B(X_k, \sigma) \) in equations (A.204) and (A.205) are Borel-measurable. Condition (c) is satisfied, because \( m^F(X_k, \sigma) \) and \( m^B(X_k, \sigma) \) are continuous in \( \sigma \in \Xi \). Finally, the expenditure shares \((s_t, s_{t-1})\) are bounded between zero and one, and we restrict the distribution of price changes \((p_t, p_{t-1})\) such that the expected absolute value of \( m^F(X_k, \sigma) \) and \( m^B(X_k, \sigma) \) is bounded across \( \sigma \in \Xi \).

Under the assumption that equation (A.206) and conditions (a)-(d) in Proposition 7 are satisfied, the M-estimator is consistent as summarized by the following proposition.

**Proposition 8. (Consistency of M-Estimators)** Under the assumptions of Proposition 7, assume that the identification assumption in equation (A.206) holds. Then, a random variable \( \hat{\sigma} \) solves problem (A.201) and \( \hat{\sigma} \to \sigma^D \).

**Proof.** See Newey and McFadden (1994), Theorem 2.1, and Wooldridge (2002), Theorem 12.2. \( \square \)

As discussed above, our RW estimator satisfies conditions (a)-(d) of Proposition 7 and satisfies the identification assumption in equation (A.206) as demand shocks become small (Proposition 2 in the paper) or as
the number of common goods becomes large and demand shocks are uncorrelated with price shocks for each good and independently and identically distributed across goods (Proposition 3 in the paper).

We now use these general properties of M-estimators to characterize the asymptotic properties of the RW estimator. We begin by defining the score of the objective function:

$$S (X, \sigma) = \frac{\partial q (X, \sigma)}{\partial \sigma},$$

and the Hessian of the objective function:

$$H (X, \sigma) = \frac{\partial^2 q (X, \sigma)}{\partial^2 \sigma}.$$

Using these definitions, we have the following asymptotic normality result from Newey and McFadden (1994) and Wooldridge (2002).

**Proposition 9. (Asymptotic Normality of M-estimators)** In addition to the assumptions in Proposition 8, assume that (a) \( \sigma^D \) is in the interior of \( \Xi \); (b) \( S (X, \sigma) \) is continuously differentiable on the interior of \( \Xi \) for all \( X \in \mathcal{W} \); (c) Each element of \( H (X, \sigma) \) is continuously differentiable in absolute value by a function \( b (X) \), where \( E [b (X)] < \infty \); (d) \( A^D = E [H (X, \sigma^D)] \) is positive definite; (e) \( E [S (X, \sigma^D)] = 0 \); and (f) each element of \( S (X, \sigma^D) \) as finite second moment. Then:

$$\sqrt{N} \left( \hat{\sigma} - \sigma^D \right) \overset{d}{\rightarrow} N \left( 0, \left( A^D \right)^{-1} B^D \left( A^D \right)^{-1} \right),$$

where

$$A^D = E [H (X, \sigma^D)],$$

and

$$B^D = E \left[ S (X, \sigma^D) S (X, \sigma^D)^\top \right] = \text{Var} \left[ S (X, \sigma^D) \right].$$

Thus

$$\text{Avar} \hat{\sigma} = \frac{(A^D)^{-1} B^D (A^D)^{-1}}{N}.$$

**Proof.** See Newey and McFadden (1994) and Wooldridge (2002), Theorem 12.3. \( \square \)

Condition (a) is satisfied by our RW estimator for \( \underline{\sigma} < \sigma^D < \overline{\sigma} \). Condition (b) is also satisfied, because \( m^F (X_k, \sigma) \) and \( m^B (X_k, \sigma) \) in equations (A.204) and (A.205) are twice continuously differentiable. For condition (c), we use the fact that the expenditure shares \( (s_t, s_{t-1}) \) are bounded between zero and one, and restrict the distribution of price changes \( (p_t, p_{t-1}) \) such that this condition is satisfied. Condition (d) is satisfied for our single parameter of the elasticity of substitution \( \sigma \). Condition (e) is satisfied as demand shocks become small (Proposition 2 in the paper) or as the number of common goods becomes large and demand shocks are uncorrelated with price shocks for each good and independently and identically distributed across goods (Proposition 3 in the paper). For condition (f), we again use the fact that the expenditure shares \( (s_t, s_{t-1}) \) are bounded between zero and one, and restrict the distribution of price changes \( (p_t, p_{t-1}) \) such that this condition is satisfied.

Therefore, we have established that the reverse-weighting (RW) estimator belongs to the class of M-estimators, and hence inherits the asymptotic normality properties of this class of estimators.
A.16 Derivation of the GRW Estimator

In this section of the web appendix, we provide additional derivations for the generalized-reverse-weighting (GRW) estimator in Section 2.5.3 of the paper.

A.16.1 Demand System

We have the following structural CES demand system:

\[ \ln \left( \frac{s_{kt}/s_{t-1}}{s_{kt-1}/s_{t-1}} \right) = (1 - \sigma) \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) + (\sigma - 1) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right), \tag{A.208} \]

where we have used our result that \( \ln (\bar{\theta}_t/\bar{\theta}_{t-1}) = (1/N_{t,t-1}) \sum_{k \in \Omega_{t-1}} \ln (\theta_{kt}/\theta_{kt-1}) = 0 \). We assume that demand shocks for each good can be partitioned into a component that is correlated with prices and an idiosyncratic component that is uncorrelated with prices for each good and independently and identically distributed across goods:

\[ \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) = \gamma \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) + \ln \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right), \tag{A.209} \]

where \( \gamma \) is the projection coefficient for demand shocks on price shocks with:

\[ \gamma = \frac{\chi_\theta}{\chi_p}, \quad \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right) \perp \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right), \tag{A.210} \]

where we use the symbol \( \chi \) to denote variances and covariances and the symbol \( \rho \) to denote correlations such that:

\[ \chi_\theta^2 = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right), \tag{A.211} \]

\[ \chi_p^2 = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right), \]

\[ \chi_s^2 = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{s_{kt}/s_{t-1}}{s_{kt-1}/s_{t-1}} \right) \ln \left( \frac{s_{kt}/s_{t-1}}{s_{kt-1}/s_{t-1}} \right), \]

\[ \chi_{p\theta} = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right), \]

\[ \chi_{p\epsilon} = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) \ln \left( \frac{s_{kt}/s_{t-1}}{s_{kt-1}/s_{t-1}} \right), \]

\[ \rho = \frac{\chi_{p\theta}}{\chi_p \chi_\theta}, \]

where we have used \( \ln (\bar{\theta}_t/\bar{\theta}_{t-1}) = 0 \) and

\[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) \ln \left( \frac{p_{kt}/\bar{p}_t}{p_{kt-1}/\bar{p}_{t-1}} \right) = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{p_{kt-1}} \right). \]
From the structural demand system (A.208) and using the definition (A.210), we have the following relationship between the projection coefficient of demand shocks on price shocks ($\gamma$), the observed moments ($\chi_{ps}$, $\chi_p^2$) and the elasticity of substitution ($\sigma$):

$$\chi_{ps} = (1 - \sigma) (1 - \gamma) \chi_p^2. \quad (A.212)$$

Additionally, the structural demand system (A.208) and the definition (A.210) imply the following relationship for the variance of sales:

$$\chi_s^2 = (1 - \sigma)^2 \left[ \chi_p^2 + \chi_o^2 - 2\rho \chi_o \chi_p \right], \quad (A.213)$$

$$= (1 - \sigma)^2 \left[ \chi_p^2 + \chi_o^2 - 2\gamma \chi_p^2 \right],$$

$$= (1 - \sigma)^2 \left[ (1 - 2\gamma) \chi_p^2 + \chi_o^2 \right].$$

Using (A.212), we can derive a closed-form expression for the projection coefficient scaled by the elasticity of substitution ($\gamma$) as a function of the observed moments ($\chi_{ps}$, $\chi_p^2$) and the elasticity of substitution ($\sigma$):

$$\gamma = \gamma(\sigma) = \frac{1}{\sigma - 1} \left[ \sigma - 1 + \frac{\chi_{ps}}{\chi_p^2} \right]. \quad (A.214)$$

Similarly, using equations (A.212) and (A.213), we can solve in closed-form for the variance of demand shocks ($\chi_o^2$) as a function of the observed moments ($\chi_{ps}$, $\chi_p^2$, $\chi_s^2$) and the elasticity of substitution ($\sigma$):

$$\chi_o^2 = \frac{\chi_s^2}{(1 - \sigma)^2} + \chi_p^2 + \frac{2\chi_{ps}}{\sigma - 1}, \quad (A.215)$$

where we have the definitional restriction $\chi_o(\sigma) \geq 0$. Finally, combining equations (A.214) and (A.215), we can obtain a closed-form solution for the correlation between price and demand shocks ($\rho$) as a function of the observed moments ($\chi_{ps}$, $\chi_p^2$, $\chi_o^2$) and the elasticity of substitution ($\sigma$):

$$\rho = \frac{\chi_p \theta}{\chi_o \chi_{ps}} = \frac{\gamma \chi_p}{\chi_o} = \frac{1 + \frac{1}{\sigma - 1} \frac{\chi_{ps}}{\chi_p^2}}{\chi_p^2 + \frac{2\chi_{ps}}{\sigma - 1}}, \quad (A.216)$$

where we have the definitional restriction $|\rho(\sigma)| \leq 1$. Therefore, as the variance of demand shocks ($\chi_o^2$) is observed, we have solved for the variance and correlation terms ($\chi_o^2$, $\rho$) in terms of observed moments ($\chi_{ps}$, $\chi_p^2$, $\chi_o^2$) and the elasticity of substitution ($\sigma$).

### A.16.2 Log Normal Derivation

We now show that the specification for demand shocks in equation (A.209) in the previous section can be derived from a joint log normal distribution of price and demand shocks with a general variance-covariance matrix:

$$\begin{pmatrix}
\ln \left( \frac{\theta_p}{\theta_{p-1}} \right) \\
\ln \left( \frac{\theta_o}{\theta_{p-1}} \right)
\end{pmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \pi_{\theta} \\ \pi_p \end{bmatrix}, \begin{bmatrix} \chi_o^2 & \rho \chi_{ps} \chi_p \\ \rho \chi_{ps} \chi_p & \chi_p^2 \end{bmatrix} \right), \quad (A.217)$$

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where recall that \( \pi_\theta = 0 \).

Note that this specification imposes additional structure on the stochastic process for demand shocks. In Section 2.1 of the paper, we derived the property that demand shocks are mean zero (\( \pi_\theta = 0 \)) from the assumption that the time-varying component of demand (\( \theta_{kt} \)) is independently and identically distributed across goods. Now we make additional assumptions about the stochastic processes for demand and prices by requiring that the distributions of price and demand shocks ((\( p_{kt} / p_{kt-1} \)), (\( \theta_{kt} / \theta_{kt-1} \))) are joint log normally distributed with a general variance-covariance matrix.

Using this distributional assumption, we can partition the joint log normal distribution (A.217) into a marginal distribution for price shocks and a conditional distribution for demand shocks given price shocks:

\[
\ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \sim N \left( \pi_p, \chi_p^2 \right),
\]

\[
\ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \sim N \left( \pi_\theta + \frac{\chi_\theta}{\chi_p} \rho \ln \left( \frac{p_{kt}}{p_{kt-1}} \right), (1 - \rho^2) \chi_\theta^2 \right),
\]

where recall that \( \pi_\theta = 0 \). Using this distributional assumption, we can write price and demand shocks as follows:

\[
\ln \left( \frac{p_{kt}}{p_{kt-1}} \right) = \chi_p \ln \left( \frac{u_{kt}}{u_{kt-1}} \right),
\]

\[
\ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) = \rho \frac{\chi_\theta}{\chi_p} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \sqrt{(1 - \rho^2)} \chi_\theta \ln \left( \frac{v_{kt}}{v_{kt-1}} \right),
\]

where \( \ln \left( \frac{u_{kt}}{u_{kt-1}} \right) \) and \( \ln \left( \frac{v_{kt}}{v_{kt-1}} \right) \) have independent standard normal distributions:

\[
\ln \left( \frac{u_{kt}}{u_{kt-1}} \right) \sim N (0, 1),
\]

\[
\ln \left( \frac{v_{kt}}{v_{kt-1}} \right) \sim N (0, 1).
\]

Note that equation (A.221) takes exactly the same form as equation (A.209), confirming that this specification for demand shocks in equation (A.209) can be derived from a joint log normal distribution for price and demand shocks. Under this distributional assumption, the variances and correlation (\( \chi_\theta^2, \chi_p^2, \rho \)) and the projection coefficient (\( \gamma = \rho \chi_\theta / \chi_p \)) are all parameters. We showed in Section A.16.1 above that these parameters can be expressed in terms of the elasticity of substitution (\( \sigma \)) and the observed moments (\( \chi_{ps}, \chi_p^2, \chi_\theta^2 \)).

### A.16.3 Reverse-Weighting (RW) Estimator

As shown in Section 2.5.2 of the paper, the reverse-weighting (RW) estimator combines the demand system from Section A.16.1 with the CES unit expenditure function. Using the identifying assumption of a money-metric utility function, the two RW moment conditions are:

\[
\Delta^{RW}(\sigma, X) = \begin{pmatrix}
\frac{1}{\sigma} \ln \left( \frac{\sum_{c \in \Omega_{ts-1}} \frac{1}{\rho_{m-1}} \ln \left( \frac{\sum_{c \in \Omega_{ts-1}} s_{kt}^e \left( \frac{p_{kt-1}}{p_{kt-1}} \right)^{1-\sigma}}{\rho_{m-1}} \right) - \ln \left( \frac{\rho_{m-1}}{\rho_{m-1}} \right) - \frac{\rho_{m-1}}{\rho_{m-1}} \ln \left( \frac{\rho_{m}}{\rho_{m-1}} \right)}{\rho_{m-1}} \right) \bigg| \frac{1}{\rho_{m-1}} \ln \left( \frac{\sum_{c \in \Omega_{ts-1}} s_{kt}^e \left( \frac{p_{kt-1}}{p_{kt-1}} \right)^{1-\sigma}}{\rho_{m-1}} \right) - \ln \left( \frac{\rho_{m-1}}{\rho_{m-1}} \right) - \frac{\rho_{m-1}}{\rho_{m-1}} \ln \left( \frac{\rho_{m}}{\rho_{m-1}} \right) \bigg| \bigg) \\
\end{pmatrix}.
\]

These RW moment conditions are defined over price shocks (\( p_{kt} / p_{kt-1} \)) and initial and end-period expenditure shares (\( s_{kt}^e, s_{kt-1}^e \)), but do not directly include demand shocks (\( \theta_{kt} / \theta_{kt-1} \)). Under the conditions specified
in Propositions 2 and 3 in the paper, these end-period expenditure shares ($s_{kt}^*$) are not systematically influenced by these demand shocks, either because these demand shocks are close to one ($\theta_{kt}/\theta_{kt-1} \rightarrow 1$), or because demand and price shocks for a given good are uncorrelated with one another ($\text{Cov} \left[ \left( \theta_{kt}/\theta_{kt-1} \right), \left( p_{kt}/p_{kt-1} \right) \right] = 0$). However, if demand shocks are large ($\theta_{kt}/\theta_{kt-1} \neq 1$) and correlated with price shocks for a given good ($\text{Cov} \left[ \left( \theta_{kt}/\theta_{kt-1} \right), \left( p_{kt}/p_{kt-1} \right) \right] \neq 0$), the true direct effect of these price shocks on the end-period expenditure shares through the elasticity of substitution ($\sigma$) is obscured by their correlation with demand shocks.

### A.16.4 Generalized-Reverse-Weighting (GRW) Estimator

We now develop our GRW estimator that allows for large demand shocks that are correlated with price shocks for a given good, but are independently distributed across goods. As already established in Proposition 2, in a counterfactual world in which only prices change between periods $t - 1$ and $t$ and there are no demand shocks ($\theta_{kt}/\theta_{kt-1} = 1$ for all $k \in \Theta_{t,t-1}$), the elasticity of substitution ($\sigma$) can be consistently estimated from the following two moment conditions:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Theta_{t,t-1}} \left[ \frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Theta_{t,t-1}} s_{\ell t}^* \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right)^{1-\sigma} \right) - \frac{1}{\sigma-1} \ln \left( \frac{S_{kt}^*}{s_{kt-1}^*} \right) \right] = 0, \quad (A.225)
\]

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Theta_{t,t-1}} \left[ -\frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Theta_{t,t-1}} S_{\ell t}^* \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right)^{-1-(\sigma)} \right) - \frac{1}{\sigma-1} \ln \left( \frac{S_{kt}^*}{s_{kt-1}^*} \right) \right] = 0, \quad (A.226)
\]

where $S_{kt}^*$ denote the end-period common goods expenditure shares in this counterfactual world in which only prices change.

If we could observe the demand shocks ($\theta_{kt}/\theta_{kt-1}$), we could construct these counterfactual end-period expenditure shares ($S_{kt}^*$) by using the CES demand system to remove the entire effect of demand shocks on the observed end-period expenditure shares ($s_{kt}^*$):

\[
S_{kt}^* = \frac{\left( \theta_{kt-1}/\theta_{kt} \right)^{\sigma-1} s_{kt}^*}{\sum_{\ell \in \Theta_{t,t-1}} \left( \theta_{\ell t-1}/\theta_{\ell t} \right)^{\sigma-1} s_{\ell t}^*}. \quad (A.227)
\]

Although we do not observe the demand shocks ($\theta_{kt}/\theta_{kt-1}$), we can use equation (A.209) to remove the component of demand shocks that is correlated with price shocks. In particular, we partition demand shocks into a component that is correlated with price shocks and an idiosyncratic component and use equation (A.209) to re-write equation (A.227) as follows:

\[
S_{kt}^* = \frac{\left( \epsilon_{kt-1}/\epsilon_{kt} \right)^{\sigma-1} \left( p_{kt-1}/p_{kt} \right)^{\gamma(\sigma-1)} s_{kt}^*}{\sum_{\ell \in \Theta_{t,t-1}} \left( \epsilon_{\ell t-1}/\epsilon_{\ell t} \right)^{\sigma-1} \left( p_{\ell t-1}/p_{\ell t} \right)^{\gamma(\sigma-1)} s_{\ell t}^*}. \quad (A.228)
\]

To remove the component of demand shocks that is correlated with prices from the end-period expenditure shares, we use equation (A.228) omitting the orthogonal component of demand shocks ($\epsilon_{kt}/\epsilon_{kt-1}$):

\[
S_{kt}^*(\sigma) = \frac{\left( p_{kt-1}/p_{kt} \right)^{\gamma(\sigma-1)} s_{kt}^*}{\sum_{\ell \in \Theta_{t,t-1}} \left( p_{\ell t-1}/p_{\ell t} \right)^{\gamma(\sigma-1)} s_{\ell t}^*}. \quad (A.229)
\]
Using equation (A.229) in equations (A.225) and (A.226), we construct the following two moment conditions for the Generalized-Reverse-Weighting (GRW) estimator:

\[
M_{\text{GRW}}(\sigma, \chi) = \left( \begin{array}{c}
\frac{1}{n_{t-1}} \Sigma_{k \in N_{t-1}} \left[ \frac{1}{n_{t-1}} \ln \left( \frac{\Sigma_{i \in N_{t-1}} s_{it}^* (\sigma / (\sigma - 1))^{-1} - \ln \left( \frac{s_{it}^*}{s_{it-1}^*} \right) - \ln \left( \frac{\tilde{s}_{it}}{\tilde{s}_{it-1}} \right) }{\frac{s_{it}^*}{s_{it-1}^*} - \frac{\tilde{s}_{it}}{\tilde{s}_{it-1}}} \right) \right] = \left( 0 \right),
\end{array} \right)
\] (A.230)

where \( S_{kt}^* (\sigma) \) depends on \( \gamma (\sigma) = \rho (\sigma) \chi_0 (\sigma) / \chi_p \) from equations (A.214) and (A.229); and we have the definitional restrictions \( \chi_0 (\sigma) \geq 0 \) and \( |\rho (\sigma)| \leq 1 \).

### A.17 Proof of Proposition 4 (RW Estimator with Large and Correlated Shocks)

**Proof.** We start with our two equalities between equivalent expressions for the change in the cost of living from equations (14), (21) and (22) in the paper:

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{p_{kt} / \tilde{p}_t}{p_{kt-1} / \tilde{p}_{t-1}} \right)^{\gamma (\sigma - 1)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{\sigma - 1} \right]^{\frac{1}{1-\sigma}} = \tilde{p}_t \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*},
\] (A.231)

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\sigma)} \left( \frac{p_{kt} / \tilde{p}_t}{p_{kt-1} / \tilde{p}_{t-1}} \right)^{-\gamma (\sigma - 1)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{-(\sigma - 1)} \right]^{\frac{1}{1+\sigma}} = \tilde{p}_t \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*},
\] (A.232)

Using our specification for demand shocks in equation (30) in the paper, we obtain:

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\gamma (1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{\sigma - 1} \right]^{\frac{1}{1-\sigma}} = \tilde{p}_t \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*},
\] (A.233)

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\gamma) (1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{-(\sigma - 1)} \right]^{\frac{1}{1+\sigma}} = \tilde{p}_t \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*},
\] (A.234)

which can be re-written as:

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)(1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{\sigma - 1} \right]^{\frac{1}{1-\sigma}} = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\gamma} \left( \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*} \right)^{\frac{1}{1-\sigma}},
\] (A.235)

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\sigma)(1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{-(\sigma - 1)} \right]^{\frac{1}{1+\sigma}} = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\gamma} \left( \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*} \right)^{\frac{1}{1+\sigma}},
\] (A.236)

and hence as:

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{(1-\sigma)(1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{\sigma - 1} \right]^{\frac{1}{1-\sigma}} = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*} \right)^{\frac{1}{1-\sigma}},
\] (A.237)

\[
\left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\sigma)(1-\gamma)} \left( \frac{\epsilon_{kt}}{\epsilon_{kt-1}} \right)^{-(\sigma - 1)} \right]^{\frac{1}{1+\sigma}} = \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\tilde{s}_{kt-1}^*}{\tilde{s}_{kt-1}^*} \right)^{\frac{1}{1+\sigma}},
\] (A.238)
which can be further re-written as:

$$
\hat{\Theta}^F_{t-1,t} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\tilde{\sigma}} \right]^{\frac{1}{1-\tilde{\sigma}}} = \left( \frac{\hat{p}_t}{\bar{p}_{t-1}} \right) \left( \frac{s^*_t}{s^*_{t-1}} \right), \quad (A.239)
$$

$$
\left( \hat{\Theta}^B_{t-1,t} \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-(1-\tilde{\sigma})} \right]^{-\frac{1}{1-\tilde{\sigma}}} = \left( \frac{\hat{p}_t}{\bar{p}_{t-1}} \right) \left( \frac{s^*_t}{s^*_{t-1}} \right)^{\frac{1}{1-\tilde{\sigma}}}, \quad (A.240)
$$

where the caron (') above a variable denotes an adjusted value, such that \( \tilde{\sigma} \) is the elasticity of substitution adjusted for the correlation between demand and price shocks:

$$
1 - \tilde{\sigma} = (1 - \sigma) (1 - \gamma), \quad (A.241)
$$

or equivalently,

$$
\tilde{\sigma} = \sigma - \gamma (\sigma - 1), \quad (A.242)
$$

and \( \hat{\Theta}^F_{t-1,t} \) and \( \left( \hat{\Theta}^B_{t-1,t} \right)^{-1} \) are adjusted aggregate demand shifters defined as:

$$
\hat{\Theta}^F_{t-1,t} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\tilde{\sigma}} \left( \frac{s^*_t}{s^*_{t-1}} \right)^{\tilde{\sigma}-1} \right]^{\frac{1}{\tilde{\sigma}}}, \quad (A.243)
$$

$$
\left( \hat{\Theta}^B_{t-1,t} \right)^{-1} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^* \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-(1-\tilde{\sigma})} \left( \frac{s^*_t}{s^*_{t-1}} \right)^{-(\tilde{\sigma}-1)} \right]^{-\frac{1}{1-\tilde{\sigma}}}, \quad (A.244)
$$

where

$$
\tilde{\epsilon}_{kt} = \epsilon_{kt}^{1/(1-\gamma)}. \quad (A.245)
$$

Using equations (A.239) and (A.240) and the identifying assumption of \( \hat{\Theta}^F_{t-1,t} = \left( \hat{\Theta}^B_{t-1,t} \right)^{-1} = 1 \), we can construct the following reverse-weighting (RW) moment condition:

$$
M^{RW}(\sigma, \mathbf{X}) = \begin{pmatrix}
\frac{1}{n_{t-1}} \sum_{k \in \Omega_{t-1}} \left[ \frac{1}{1-\tilde{\sigma}} \ln \left( \sum_{j \in \Omega_{t-1}} s^*_j \left( \frac{p_{jt}}{p_{jt-1}} \right)^{1-\tilde{\sigma}} \right) - \ln \left( \frac{\hat{p}_t}{\bar{p}_{t-1}} \right) - \frac{1}{1-\tilde{\sigma}} \ln \left( \frac{s^*_t}{s^*_{t-1}} \right) \right] \\
\frac{1}{n_{t-1}} \sum_{k \in \Omega_{t-1}} \left[ -\frac{1}{1-\tilde{\sigma}} \ln \left( \sum_{j \in \Omega_{t-1}} s^*_j \left( \frac{p_{jt}}{p_{jt-1}} \right)^{1-\tilde{\sigma}} \right) - \ln \left( \frac{\hat{p}_t}{\bar{p}_{t-1}} \right) - \frac{1}{1-\tilde{\sigma}} \ln \left( \frac{s^*_t}{s^*_{t-1}} \right) \right]
\end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (A.246)
$$

Note that these RW moment conditions take exactly the same form as in equation (28) in the paper, but with \( \tilde{\sigma} \) replacing \( \sigma \). Furthermore, the error terms (\( \tilde{\epsilon}_{kt} / \tilde{\epsilon}_{kt-1} \)) in the forward and backward aggregate demand shifters (\( \hat{\Theta}^F_{t-1,t} \) and \( \left( \hat{\Theta}^B_{t-1,t} \right)^{-1} \)) in equations (A.243) and (A.244) are orthogonal to price shocks for a given good and independently and identically distributed across goods, and hence satisfy the conditions for the RW estimator to be consistent. Therefore, if a researcher uses the RW moment conditions to estimate the elasticity of substitution (\( \sigma \)) when demand and price shocks are correlated, she consistently estimates \( \tilde{\sigma} \) rather than \( \sigma \). From equation (A.242), noting that \( \sigma > 1 \), we have \( \tilde{\sigma} < \sigma \) if and only if \( \gamma > 0 \), and \( \tilde{\sigma} > \sigma \) if and only if \( \gamma < 0 \). It follows that the RW estimator is asymptotically downward biased (\( \text{plim} (\hat{\sigma}^{RW}) < \sigma \)) if price and demand shocks are positively correlated (\( \gamma > 0 \)), and is asymptotically upward biased (\( \text{plim} (\hat{\sigma}^{RW}) > \sigma \)) if demand and price shocks are negatively correlated (\( \gamma < 0 \)), which establishes the proposition.
A.18 Proof of Proposition 5 (GRW Estimator)

Proof. As already established in Proposition 2, in a counterfactual world in which only prices change between periods \( t - 1 \) and \( t \) and there are no demand shocks \((\theta_{kt}/\theta_{kt-1} = 1 \text{ for all } k \in \Omega_{t,t-1})\), the elasticity of substitution \((\sigma)\) can be consistently estimated from the following two moment conditions:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ \frac{1}{1-\sigma} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} s^*_{\ell t-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right) - \frac{1}{\sigma - 1} \ln \left( \frac{s^*_{kt}}{s^*_{kt-1}} \right) \right] = 0, \tag{A.247}
\]

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ -\frac{1}{1-\sigma} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} S^*_{\ell t} \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right)^{-(1-\sigma)} \right) - \frac{1}{\sigma - 1} \ln \left( \frac{S^*_{kt}}{S^*_{kt-1}} \right) \right] = 0, \tag{A.248}
\]

where \( S^*_{kt} \) denote the end-period common goods expenditure shares in this counterfactual world in which only prices change.

As discussed in Section A.16.4 above, if we could observe the demand shocks \((\theta_{kt}/\theta_{kt-1})\), we could construct these counterfactual end-period expenditure shares \((S^*_{kt})\) by using the CES demand system to remove the entire effect of demand shocks on the end-period expenditure shares \((s^*_{kt})\):

\[
S^*_{kt} = \frac{(\theta_{kt-1}/\theta_{kt})^{\sigma-1} s^*_{kt}}{\sum_{\ell \in \Omega_{t,t-1}} (\theta_{\ell t-1}/\theta_{\ell t})^{\sigma-1} s^*_{\ell t}}. \tag{A.249}
\]

Although we do not observe the demand shocks \((\theta_{kt}/\theta_{kt-1})\), we can use equation (A.209) to remove the component of demand shocks that is correlated with price shocks. In particular, we partition demand shocks into a component that is correlated with price shocks and an idiosyncratic component and use equation (A.209) to re-write equation (A.249) as follows:

\[
S^*_{kt}(\sigma) = \frac{(\epsilon_{kt-1}/\epsilon_{kt})^{\sigma-1} (p_{kt-1}/p_{kt})^{\gamma(\sigma-1)} s^*_{kt}}{\sum_{\ell \in \Omega_{t,t-1}} (\epsilon_{\ell t-1}/\epsilon_{\ell t})^{\sigma-1} (p_{\ell t-1}/p_{\ell t})^{\gamma(\sigma-1)} s^*_{\ell t}}, \tag{A.250}
\]

where from equation (A.210) we have a closed-form solution for the projection coefficient \((\gamma)\) in terms of the elasticity of substitution \((\sigma)\) and sample moments \((\chi_{ps}, \chi_{p}^2)\):

\[
\gamma = \frac{1}{\sigma - 1} \left[ \sigma - 1 + \frac{\chi_{ps}}{\chi_{p}^2} \right]. \tag{A.251}
\]

Using our assumption that demand and price shocks are independently distributed across goods, as the number of common goods becomes large \((N_{t,t-1} \to \infty)\), the sample moments \((\chi_{ps}, \chi_{p}^2)\) converge to their population counterparts. Therefore, as the number of common goods becomes large \((N_{t,t-1} \to \infty)\), we recover the true value for \(\gamma\) from equation (A.251) for the true value of the elasticity of substitution \((\sigma)\).

Using equation (A.250) in our two moment conditions in equations (A.247) and (A.248), we obtain:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ \frac{1}{1-\sigma} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} s^*_{\ell t-1} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma} \right) - \frac{1}{\sigma - 1} \ln \left( \frac{\left(\epsilon_{kt-1}/\epsilon_{kt}\right)^{\sigma-1} (p_{kt-1}/p_{kt})^{\gamma(\sigma-1)} s^*_{kt}}{\sum_{\ell \in \Omega_{t,t-1}} \left(\epsilon_{\ell t-1}/\epsilon_{\ell t}\right)^{\sigma-1} (p_{\ell t-1}/p_{\ell t})^{\gamma(\sigma-1)} s^*_{\ell t}} \right) \right] = 0, \tag{A.252}
\]

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Therefore, as

\[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ -\frac{1}{1-\sigma} \ln \left( \sum_{k \in \Omega_{t,t-1}} \left( \frac{(e_{it-1}/e_{it})^{\sigma-1}(p_{it-1}/p_{it})^{\gamma(\sigma-1)s_{it}}}{\sum_{k \in \Omega_{t,t-1}} (e_{it-1}/e_{it})^{\sigma-1}(p_{it-1}/p_{it})^{\gamma(\sigma-1)s_{it}^*}} \right) \right) \right] = 0. \]

(A.253)

Under our assumptions that the idiosyncratic component of demand is orthogonal to price shocks for a given good and is independently and identically distributed across goods, it follows that the idiosyncratic demand shock going backwards in time from \( t \) to \( t-1 \) \((e_{kt-1}/e_{kt})\) is orthogonal to both the price shock going backwards in time from \( t \) to \( t-1 \) \((p_{kt-1}/p_{kt})\) and expenditure shares at time \( t \) \((s_{kt}^*)\). Using these properties, we have the following result:

\[ \mathbb{E} \left[ \left( \frac{e_{it-1}}{e_{it}} \right)^{\sigma-1} \left( \frac{p_{it-1}}{p_{it}} \right)^{\gamma(\sigma-1)} s_{it}^* \right] = \mathbb{E} \left[ \left( \frac{e_{it-1}}{e_{it}} \right)^{\sigma-1} \right] \mathbb{E} \left[ \left( \frac{p_{it-1}}{p_{it}} \right)^{\gamma(\sigma-1)} s_{it}^* \right]. \]

(A.254)

Replacing the population expectations with their sample counterparts, and taking the limit as the number of common goods becomes large \((N_{t,t-1} \to \infty)\), we have:

\[ \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( \frac{e_{kt-1}}{e_{kt}} \right)^{\sigma-1} \left( \frac{p_{kt-1}}{p_{kt}} \right)^{\gamma(\sigma-1)} s_{kt}^* \right] = \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( \frac{e_{it-1}}{e_{it}} \right)^{\sigma-1} \right] \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( \frac{p_{it-1}}{p_{it}} \right)^{\gamma(\sigma-1)} s_{it}^* \right]. \]

(A.255)

Using our assumption that demand shocks are independent across goods, the Central Limit Theorem implies:

\[ \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( \frac{e_{kt-1}}{e_{kt}} - 1 \right) \right] \sim N \left( 0, \frac{\chi^2}{N_{t,t-1}} \right). \]

(A.256)

Therefore, as \( N_{t,t-1} \to \infty \), we have:

\[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \frac{e_{kt-1}}{e_{kt}} = \mathbb{E} \left[ \frac{e_{kt-1}}{e_{kt}} \right] \overset{p}{\to} 1. \]

(A.257)

From the Delta method, a sequence of random variables that satisfies:

\[ \sqrt{n} \left( Y_n - \mu \right) \overset{d}{\to} \mathcal{N} \left( 0, \chi^2 \right) \]

implies:

\[ \sqrt{n} \left[ g \left( Y_n \right) - g \left( \mu \right) \right] \overset{d}{\to} \mathcal{N} \left( 0, \chi^2 \left[ g' \left( \mu \right) \right]^2 \right) \]

(A.259)

Applying this result for \( g \left( \cdot \right) = \left( \cdot \right)^{\sigma-1}, \mu = 1, \chi^2 = \chi^2, g \left( \mu \right) = 1 \) and \( g' \left( \mu \right) = \sigma - 1 \), we have:

\[ \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( e_{kt-1}^{\sigma-1} - 1 \right) \right] \overset{d}{\to} \mathcal{N} \left( 0, \frac{\chi^2}{N_{t,t-1}} (\sigma - 1)^2 \right). \]

(A.260)

Therefore, as \( N_{t,t-1} \to \infty \), we have:

\[ \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left( e_{kt-1}^{\sigma-1} - 1 \right) \right] \overset{p}{\to} 0. \]

(A.261)
Using this result from equation (A.261) in equation (A.255), we obtain:

\[
\left[ \frac{1}{N_{t,t-1}} \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{\epsilon_{\ell t}}{\epsilon_{\ell t}} \right)^{\sigma-1} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right] = \left[ \frac{1}{N_{t,t-1}} \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{\epsilon_{\ell t}}{\epsilon_{\ell t}} \right)^{\sigma-1} \left[ \frac{1}{N_{t,t-1}} \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right] \right],
\]

(A.62)

which cancelling the terms in \( N_{t,t-1} \) implies:

\[
\left[ \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{\epsilon_{\ell t}}{\epsilon_{\ell t}} \right)^{\sigma-1} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right] \overset{p}{\rightarrow} \left[ \frac{1}{N_{t,t-1}} \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right].
\]

(A.63)

Using this result from equation (A.63) in equations (A.252) and (A.253), and taking the limit as \( N_{t,t-1} \rightarrow \infty \), we have:

\[
\lim_{N_{t,t-1} \to \infty} \left\{ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ \frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} s_{\ell t-1}^* \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right)^{\gamma(\sigma-1)} \right) - \ln \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right) - \frac{1}{\sigma-1} \ln \left( \frac{\sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right) s_{\ell t-1}^* \right] \right\} \overset{p}{\rightarrow} 0,
\]

(A.64)

\[
\lim_{N_{t,t-1} \to \infty} \left\{ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ -\frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right) \right] - \ln \left( \frac{p_{\ell t}}{p_{\ell t-1}} \right) - \frac{1}{\sigma-1} \ln \left( \frac{\sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right) s_{\ell t-1}^* \right\} \overset{p}{\rightarrow} 0.
\]

(A.65)

Among the terms in equations (A.64) and (A.65), note that:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right) = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{\epsilon_{kt-1}}{\epsilon_{kt}} \right) + \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \frac{1}{\sigma-1} \ln \left( \frac{p_{kt-1}}{p_{kt}} \right) s_{kt}^* \]

\[
- \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \frac{1}{\sigma-1} \ln \left( \sum_{\ell \in \Omega_{t,t-1}} \left( \frac{p_{\ell t-1}}{p_{\ell t}} \right)^{\gamma(\sigma-1)} s_{\ell t}^* \right) \overset{p}{\rightarrow} 0.
\]

(A.66)

Applying the Delta method from equations (A.258) and (A.259) for \( g(\cdot) = \ln(\cdot) \), \( \mu = 1 \), \( \chi^2 = \chi_1^2 \), \( g'(\mu) = 1 \), and \( g'(\mu) = \mu^{-1} \), we have:

\[
\left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{\epsilon_{kt-1}}{\epsilon_{kt}} \right) \right] \sim N \left( 0, \frac{\chi_1^2}{N_{t,t-1}} \right).
\]

(A.67)

Therefore, as \( N_{t,t-1} \rightarrow \infty \), we have:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{\epsilon_{kt-1}}{\epsilon_{kt}} \right) \overset{p}{\rightarrow} 0,
\]

(A.68)
which implies:

\[
\frac{1}{N_{t,1}} \sum_{k \in \Omega_{t,1}} \frac{1}{\sigma - 1} \ln \left[ \frac{(z_{kt-1})^{\gamma-1} (p_{kt-1} / p_{t1-1})^{\gamma\sigma-1} s_{kt}^{\gamma\sigma}}{\sum_{\ell \in \Omega_{t,1-1}} (p_{kt-1} / p_{\ell t-1})^{\gamma\sigma-1} s_{\ell t}^{\gamma\sigma}} \right] \quad \xrightarrow{p} \quad \frac{1}{N_{t,1}} \sum_{k \in \Omega_{t,1}} \frac{1}{\sigma - 1} \ln \left[ \frac{(p_{kt-1} / p_{t1-1})^{\gamma\sigma-1} s_{kt}^{\gamma\sigma}}{\sum_{\ell \in \Omega_{t,1-1}} (p_{kt-1} / p_{\ell t-1})^{\gamma\sigma-1} s_{\ell t}^{\gamma\sigma}} \right]
\]  

(A.269)

Using this result from equation (A.269) in equations (A.264) and (A.265), and taking the limit as \( N_{t,1} \to \infty \), we have:

\[
\lim_{N_{t,1} \to \infty} \left\{ \frac{1}{N_{t,1}} \sum_{k \in \Omega_{t,1}} \left[ \frac{1}{\sigma - 1} \ln \left( \frac{\sum_{\ell \in \Omega_{t,1-1}} s_{\ell t}^{\gamma\sigma-1} (p_{kt-1} / p_{\ell t-1})^{1-\sigma} s_{kt}^{\gamma\sigma}}{\sum_{\ell \in \Omega_{t,1-1}} (p_{kt-1} / p_{\ell t-1})^{\gamma\sigma-1} s_{\ell t}^{\gamma\sigma}} \right) \right] \right\} \quad \xrightarrow{p} \quad 0. 
\]  

(A.270)

\[
\lim_{N_{t,1} \to \infty} \left\{ \frac{1}{N_{t,1}} \sum_{k \in \Omega_{t,1}} \left[ \frac{1}{\sigma - 1} \ln \left( \frac{\sum_{\ell \in \Omega_{t,1-1}} (p_{kt-1} / p_{\ell t-1})^{\gamma\sigma-1} s_{\ell t}^{\gamma\sigma-1} (p_{kt-1} / p_{t1-1})^{1-\sigma} s_{kt}^{\gamma\sigma-1}}{\sum_{\ell \in \Omega_{t,1-1}} (p_{kt-1} / p_{\ell t-1})^{\gamma\sigma-1} s_{\ell t}^{\gamma\sigma-1}} \right) \right] \right\} \quad \xrightarrow{p} \quad 0. 
\]  

(A.271)

Equations (A.270) and (A.271) correspond to the GRW moment conditions in equation (35) in the paper. Therefore, as \( N_{t,1} \to \infty \), these moment conditions yield consistent estimates of the elasticity of substitution \((\sigma)\).

The GRW estimator, like the RW estimator, belongs to the class of M-estimators (Newey and McFadden 1994 and Wooldridge 2002) discussed in Section A.15 of this web appendix. Therefore, the GRW estimator inherits the same asymptotic normality properties as shown for the RW estimator in Section A.15 above.

### A.19 Bounding the Elasticity of Substitution

In this section of the web appendix, we show that our inversion of the CES demand system and the assumption of joint log normality can be used to provide upper and lower bounds for the true elasticity of substitution regardless of the correlation between demand and price shocks. From the CES demand system (11), we have the following expressions for the covariance between log price shocks (\(\ln ((p_{kt} / \hat{p}_{t}) / (p_{kt-1} / \hat{p}_{t-1}))\)) and log sales shocks (\(\ln ((\hat{s}_{kt} / \hat{s}_{t}) / (s_{kt-1} / \hat{s}_{t-1}))\)) and the variance of log sales shocks:

\[
\chi_{ps} = (1 - \sigma) \left[ \chi_{p}^2 - \chi_{p\theta} \right], 
\]  

(A.272)

\[
\chi_{s}^2 = (1 - \sigma)^2 \left[ \chi_{p}^2 + \chi_{\theta}^2 - 2\chi_{p\theta} \right], 
\]  

(A.273)

where we have used \(q_{kt} / q_{kt-1} = \theta_{kt} / \theta_{kt-1} = 1\) and \(\hat{\theta}_{t-1} = 1; (\chi_{ps}, \chi_{p}, \chi_{\theta}, \chi_{s}, \chi_{p\theta})\) are standard deviations and covariances, as defined in equation (A.211) above. Under our assumption of joint log normality, the covariance between price and demand shocks \((\chi_{p\theta})\) and the variance of demand shocks \((\chi_{s}^2)\) are both parameters.
Using equation (A.272) to substitute for $\lambda_p \theta$ in equation (A.273), we obtain the following relationship that implicitly defines the elasticity of substitution ($\sigma$) as a function of the observed moments ($\lambda_{ps}, \lambda_p^2, \lambda_q^2$) for each assumed value for the variance of demand shocks ($\lambda_{\theta}^2$):

$$\lambda_{\theta}^2 = \frac{\lambda_q^2}{(\sigma - 1)^2} + \lambda_p^2 + \frac{2}{\sigma - 1} \lambda_{ps}. \quad (A.274)$$

As discussed in the paper, our model requires $\sigma > 1$ to ensure positive utility given the entry and exit of goods with positive demand ($\varphi_{kt} > 0$). Therefore, our lower bound for the elasticity of substitution is one ($\sigma = 1$). We now show that a necessary and sufficient condition for the elasticity of substitution ($\sigma$) implied by equation (A.274) to be monotonically decreasing in the assumed variance of demand shocks ($\lambda_{\theta}^2$) is that the variance of demand shocks exceeds the variance of price shocks ($\lambda_{\theta}^2 > \lambda_p^2$). We start by rewriting equation (A.274) as the following implicit function:

$$\Lambda (\sigma, \lambda_{\theta}^2) = \lambda_{\theta}^2 - \frac{\lambda_q^2}{(\sigma - 1)^2} - \lambda_p^2 - \frac{2}{\sigma - 1} \lambda_{ps} = 0. \quad (A.275)$$

Applying the implicit function theorem, we obtain the following expression that determines the derivative of the elasticity of substitution with respect to the variance of demand shocks:

$$\frac{d (\sigma - 1)}{d \lambda_{\theta}^2} = -\frac{\frac{d \Lambda (\sigma, \lambda_{\theta}^2)}{d \lambda_{\theta}^2}}{\frac{d \Lambda (\sigma, \lambda_{\theta}^2)}{d (\sigma - 1)}} = -\frac{1}{\frac{\lambda_q^2}{(\sigma - 1)^2} + \frac{\lambda_{ps}}{\sigma - 1}}. \quad (A.276)$$

Noting that $\sigma > 1$ and $\frac{d \sigma}{d \lambda_{\theta}^2}$, with $\frac{d \sigma}{d (\sigma - 1)} > 0$, the elasticity of substitution is decreasing in the variance of demand shocks if and only if the following condition is satisfied:

$$\frac{d \sigma}{d \lambda_{\theta}^2} < 0 \quad \Leftrightarrow \quad \frac{\lambda_q^2}{(\sigma - 1)^2} + \frac{\lambda_{ps}}{\sigma - 1} > 0. \quad (A.277)$$

To determine the circumstances under which this condition is satisfied, note that equation (A.274) implies:

$$\frac{\lambda_q^2}{(\sigma - 1)^2} + \frac{2}{\sigma - 1} \lambda_{ps} = \lambda_{\theta}^2 - \lambda_p^2. \quad (A.278)$$

Noting that $\lambda_{\theta}^2 > 0$ and $\lambda_p^2 > 0$, equations (A.277) and (A.278) together imply that a necessary and sufficient condition for the elasticity of substitution ($\sigma$) to be monotonically decreasing in the assumed variance of demand shocks ($\lambda_{\theta}^2$) is that the variance of demand shocks exceeds the variance of prices:

$$\frac{d \sigma}{d \lambda_{\theta}^2} < 0, \quad \Leftrightarrow \quad \lambda_{\theta}^2 > \lambda_p^2. \quad (A.279)$$

Evaluating equation (A.274) using our sample moments ($\lambda_s^2, \lambda_{ps}^2, \lambda_{ps}$) for each of our product groups, we find that the implied value of $\sigma$ is indeed monotonically decreasing in the assumed value of $\lambda_{\theta}^2$, which implies that this necessary and sufficient condition in equation (A.279) is satisfied in our data. Using these properties
that the variance of demand shocks exceeds the variance of price shocks ($\chi_\theta^2 > \chi_p^2$) and the implied elasticity of substitution is decreasing in the assumed variance of demand shocks ($\chi_\theta^2$), our upper bound for the elasticity of substitution ($\bar{\sigma}$) is obtained by solving equation (A.274) for the lowest possible value for the variance of demand shocks ($\chi_\theta^2 = \chi_p^2$).

We thus obtain set identification for the elasticity of substitution ($\sigma$) using the CES demand system and joint log normality, given the sample moments for the variances and covariance of price and sales shocks ($\chi_{ps}$, $\chi_p^2$, $\chi_s^2$). Using our assumption that demand and price shocks are independent across goods, as the number of common goods becomes large ($N_{it-1} \to \infty$), the sample moments ($\chi_{ps}$, $\chi_p^2$, $\chi_s^2$) converge to their population counterparts. Therefore, the true elasticity of substitution ($\sigma$) necessarily lies within this identified set as the number of common goods becomes large. We now prove Proposition 6 in the paper, which is reproduced below.

**Proposition 10.** Assume that demand and price shocks can be correlated with one another for each good but are independently and identically distributed across goods. As the number of common goods becomes large ($N_{it-1} \to \infty$), equation (A.274) identifies the set of possible values for elasticity of substitution $\sigma \in (1, \bar{\sigma})$ consistent with the observed data on prices and expenditure shares ($p_{kt}$, $s_{kt}^*$) under our assumptions of CES demand and joint log normality. As the number of common goods becomes large ($N_{it-1} \to \infty$), the true elasticity of substitution ($\sigma$) necessarily lies within this identified set.

**Proof.** From our CES demand system in equation (11) in the paper, we have the following closed-form expression for changes in common goods expenditure shares as a function of the price and demand shocks:

$$
\ln \left( \frac{s_{kt}^* / s_{kt-1}^*}{\tilde{s}_{kt-1}^* / \tilde{s}_{kt-1}^*} \right) = (1 - \sigma) \ln \left( \frac{p_{kt} / \tilde{p}_t}{p_{kt-1} / \tilde{p}_{t-1}} \right) + (\sigma - 1) \ln \left( \frac{\theta_{kt} / \tilde{\theta}_t}{\theta_{kt-1} / \tilde{\theta}_{t-1}} \right),
$$

(A.280)

where the tilde above a variable denotes a sample geometric mean:

$$
\ln \tilde{p}_t = \frac{1}{N_{it-1}} \sum_{k \in \Omega_{it-1}} \ln p_{kt},
$$

$$
\ln \tilde{\theta}_t = \frac{1}{N_{it-1}} \sum_{k \in \Omega_{it-1}} \ln \theta_{kt},
$$

$$
\ln \tilde{s}_{kt}^* = \frac{1}{N_{it-1}} \sum_{k \in \Omega_{it-1}} \ln s_{kt}^*,
$$

and recall $\ln \left( \frac{\tilde{\theta}_t / \tilde{\theta}_{t-1}}{\tilde{\theta}_t / \tilde{\theta}_{t-1}} \right) = 0$. In part (a) of the proof, we show that the sample variances and covariances of observed prices and expenditure shares converge in probability to their population counterparts as the number of common goods becomes large. In part (b) of the proof, we show that these observed moments can be used to provide bounds on the elasticity of substitution and that the true elasticity of substitution lies within these bounds as the number of common goods becomes large.

(a) Under our assumption that price and demand shocks are independently and identically distributed across goods, as the number of common goods becomes large ($N_{it-1} \to \infty$), the weak law of large numbers implies that the sample means of price and demand shocks converge in probability to their population counterparts:

$$
P \{ -\epsilon < \ln \tilde{p}_t - \mathbb{E} \left[ \ln p_{kt} \right] < \epsilon \} \geq 1 - \delta,
$$

(A.281)
\[ P \left[ -\epsilon < \ln \hat{\theta}_t - \mathbb{E} \left[ \ln \theta_{kt} \right] < \epsilon \right] \geq 1 - \delta, \quad (A.282) \]

for arbitrarily small values of \( \epsilon \) and \( \delta \). We now write the sample variances and covariances of price and demand shocks as follows:

\[
\hat{\chi}_p^2 = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right), \quad (A.283)
\]

\[
\hat{\chi}_\theta^2 = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) - \ln \left( \hat{\theta}_t \ln \hat{\theta}_t \right), \quad (A.284)
\]

\[
\hat{\chi}_{p\theta} = \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) - \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) \ln \hat{\theta}_t, \quad (A.285)
\]

where we have used:

\[
\frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \left[ \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) \right] \left[ \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) \right]
\]

\[
= \left[ \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \right] - \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right),
\]

and we use the hat to denote the sample value of a variance or covariance and recall that \( \ln \left( \hat{\theta}_t / \hat{\theta}_{t-1} \right) = 0 \). As the number of common goods becomes large \( (N_{t,t-1} \rightarrow \infty) \), we have already established in equations (A.281) and (A.282) that the sample means of price shocks \( (\ln \hat{p}_t) \) and demand shocks \( (\ln \hat{\theta}_t) \) converge in probability to their population counterparts. As the number of common goods becomes large \( (N_{t,t-1} \rightarrow \infty) \), the weak law of large numbers also implies:

\[
P \left[ -\epsilon < \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \mathbb{E} \left[ \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \right] < \epsilon \right] \geq 1 - \delta, \quad (A.286)
\]

\[
P \left[ -\epsilon < \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) - \mathbb{E} \left[ \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \right] < \epsilon \right] \geq 1 - \delta, \quad (A.287)
\]

\[
P \left[ -\epsilon < \frac{1}{N_{t,t-1}} \sum_{k \in \Omega_{t,t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) - \mathbb{E} \left[ \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \right] < \epsilon \right] \geq 1 - \delta, \quad (A.288)
\]

for arbitrarily small values of \( \epsilon \) and \( \delta \). Together equations (A.281)-(A.288) establish that the sample variances and covariances of price shocks \( (p_{kt} / p_{kt-1}) \) and demand shocks \( (\theta_{kt} / \theta_{kt-1}) \) converge in probability to their population counterparts as the number of common goods becomes large \( (N_{t,t-1} \rightarrow \infty) \):

\[
P \left[ -\epsilon < \hat{\chi}_p^2 - \chi_p^2 < \epsilon \right] \geq 1 - \delta, \quad (A.289)
\]

\[
P \left[ -\epsilon < \hat{\chi}_\theta^2 - \chi_\theta^2 < \epsilon \right] \geq 1 - \delta, \quad (A.290)
\]

\[
P \left[ -\epsilon < \hat{\chi}_{p\theta} - \chi_{p\theta} < \epsilon \right] \geq 1 - \delta, \quad (A.291)
\]

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for arbitrarily small values of $\epsilon$ and $\delta$. Using these results from equations (A.289)-(A.291) in equations (A.272) and (A.273), it follows that the sample variances and covariances of sales shocks also converge in probability to their population counterparts as the number of common goods becomes large ($N_{t,t-1} \to \infty$):

$$P[-\epsilon < \hat{\chi}_s^2 - \chi_s^2 < \epsilon] \geq 1 - \delta,$$  \hspace{1cm} (A.292)

$$P[-\epsilon < \hat{\chi}_{ps} - \chi_{ps} < \epsilon] \geq 1 - \delta,$$  \hspace{1cm} (A.293)

for arbitrarily small values of $\epsilon$ and $\delta$.

(b) In order to ensure positive utility given the entry and exit of goods with positive demand ($q_{kt} > 0$), our model implies an elasticity of substitution that is greater than one ($s > 1$), which establishes a lower bound for the elasticity of substitution of one ($\sigma = 1$). Using the properties established earlier in this section that the sample variance of demand shocks exceeds the variance of price shocks ($\hat{\chi}_p^2 > \hat{\chi}_s^2$) and the implied elasticity of substitution is decreasing in the assumed sample variance of demand shocks ($\hat{\chi}_p^2$), our upper bound for the elasticity of substitution ($\bar{\sigma}$) is obtained by solving equation (A.274) given the sample moments ($\hat{\chi}_p^2, \hat{\chi}_s^2, \hat{\chi}_{ps}$) for the lowest possible value for the variance of demand shocks ($\hat{\chi}_q^2 = \hat{\chi}_p^2$). As the number of common goods becomes large ($N_{t,t-1}$), the sample moments ($\hat{\chi}_p^2, \hat{\chi}_s^2, \hat{\chi}_{ps}$) converge to their population counterparts ($\chi_p^2, \chi_s^2, \chi_{ps}$). Therefore, as the number of common goods becomes large ($N_{t,t-1} \to \infty$), the true elasticity of substitution ($\sigma$) lies within this identified set.

\[\square\]

### A.20 Monte Carlo Evidence

In this section of the web appendix, we provide Monte Carlo evidence on the finite sample performance of our estimators. In each Monte Carlo, we first assume true values for the elasticity of substitution ($\sigma$), demand ($q_{kt}$) and prices ($p_{kt}$). We next solve for equilibrium expenditure shares ($s_{kt}$). Finally, we suppose that a researcher only observes data on these prices and expenditure shares and implements our estimation approach. For each combination of parameters, we undertake 50 replications of the model. We compare the mean and standard deviation of the parameter estimates across these replications with the true parameter values.

As the RW estimator uses only the subset of common goods, we focus on this subset, and are not required to make assumptions about entering and exiting goods. We focus for simplicity on a single pair of time periods $t-1$ and $t$. We consider numbers of common goods ranging from 10 to 1,000. We set the elasticity of substitution equal to 4, which is consistent with estimates using U.S. data in Bernard, Eaton, Jensen and Kortum (2004). We set the time-invariant component of demand and price for each good equal to zero and draw the time-varying components of demand and price from a joint log normal distribution:

$$\begin{pmatrix}
\ln \theta_{kt} \\
\ln p_k
\end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix}
0 & \chi_p^2 & \rho \chi_p \chi_s \\
0 & \rho \chi_p \chi_s & \chi_s^2
\end{pmatrix} \right),$$  \hspace{1cm} (A.294)

where again $\chi_p^2$ is the variance of demand shocks, $\chi_p^2$ is the variance of price shocks, and $\rho$ is the correlation between demand and price shocks. This specification allows demand and price to be correlated for each good (if $\rho \neq 0$), but assumes that they are independently and identically distributed across goods. Since the difference
of normally distributed random variables is also normally distributed, log-demand shocks \( \left( \ln \left( \frac{q_{kt}}{q_{kt-1}} \right) \right) \) and log price shocks \( \left( \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \right) \) both inherit a normal distribution.

In our first set of quantitative exercises, we examine the finite sample performance of the RW estimator when demand and price shocks are uncorrelated. We start by varying the standard deviation of demand shocks (as in Proposition 2 in the paper). We assume a standard deviation of prices of \( \chi_p = 1 \), a standard deviation of demand of \( \chi_{\theta} \in [0.001, 1] \), a correlation of \( \rho = 0 \), and 1,000 common goods. In the top panel of Figure A.3, we show the mean of the RW estimate \( \hat{\theta}^{\text{RW}} \) across the 50 replications (solid black line), the 95 percent confidence intervals (gray shading), and the true parameter value (red dashed line). We find that the mean RW estimate is always close to the true parameter value and we are unable to reject the null hypothesis of the true parameter at conventional levels of significance for all values of the dispersion of demand. As we reduce the dispersion of demand, the standard deviation of the RW estimate across the replications falls, as reflected in the narrowing of the confidence intervals.

Figure A.3: Reverse-weighting Estimates with Independent Demand and Price Shocks

We next vary the number of common goods (as in Proposition 3 in the paper). We assume \( \chi_p = \chi_{\theta} = 1 \), \( \rho = 0 \) and vary the number of common goods from 10 to 1,000. In the bottom panel of Figure A.3, we show the mean of the RW estimate \( \hat{\theta}^{\text{RW}} \) across the 50 replications (solid black line), the 95 percent confidence intervals (gray shading), and the true parameter value (red dashed line). We find relatively small differences between the RW estimate and the true parameter value even for small numbers of common goods (such as 10 or 25). For all number of common goods, we are unable to reject the null hypothesis of the true parameter value at conventional levels of significance. As the number of common goods increases, the standard deviation of the RW estimate across the replications again declines.
In our second set of quantitative exercises, we examine the finite sample performance of our estimators when demand and price shocks are correlated. In all of these remaining exercises, we assume \( \chi_P = \chi_\theta = 1 \), a correlation between demand and price shocks of \( \rho \in [-0.5, 0.5] \), and 1,000 common goods. We begin with the RW estimator. In the top panel of Figure A.4, we show the mean of the RW estimate (\( \hat{\sigma}^{RW} \)) across the 50 replications (solid black line), the 95 percent confidence intervals (gray shading), and the true parameter value (red dashed line). Consistent with the RW estimator providing a first-order approximation to the data, we find that the mean estimate remains relatively close to the true parameter value. Hence, we are unable to reject the null hypothesis of the true parameter value for correlations as large as 0.25 in absolute value. As these correlations increase towards 0.50 in absolute value, these differences become statistically significant at conventional critical values. Consistent with Proposition 4 in the paper, the mean RW estimator is above the true parameter value when demand and price shocks are negatively correlated and below it when demand and price shocks are positively correlated.

Figure A.4: Reverse-Weighting (RW) and Generalized-Reverse-Weighting (GRW) Estimates with Correlated Demand and Price Shocks

We next turn to the GRW estimator. In the middle panel of Figure A.4, we show the mean of the GRW estimate (\( \hat{\sigma}^{GRW} \)) across the 50 replications (solid black line), the 95 percent confidence intervals (gray shading), and the true parameter value (red dashed line). Perhaps unsurprisingly, given the additional structure imposed on the data, we find that the GRW estimator is less precisely estimated than the RW estimator. In the finite samples considered here, we find that the mean GRW estimate can differ from the true parameter values, but it is closer to the true parameter value than the RW estimator for most values of the correlation between demand and price shocks. Additionally, we find that the mean GRW estimate is relatively flat across alternative values.
for the correlation between demand and price shocks. For each value of this correlation, we are unable to reject the null hypothesis of the true parameter value at conventional levels of statistical significance.

Taken together, the results from these Monte Carlos confirm that the RW estimator performs well in finite samples when its identifying assumptions are satisfied (Figure A.3 above) and provides a good approximation to the data when demand and price shocks are correlated (Figure A.4 above). The mean GRW estimator remains close to the true parameter value regardless of the correlation between demand and price shocks, although it less precisely estimated than the RW estimator (Figure A.4 above).

### A.21 Non-Homothetic CES

In this section of the web appendix, we report additional derivations for the non-homothetic CES specification in Section 3.1 of the paper. We derive the generalization of our common goods unified price index (CUPI) for non-homothetic CES. We show that our reverse-weighting (RW) estimation procedure also can be generalized to the non-homothetic case to estimate both the elasticity of substitution between goods ($\sigma$) and the elasticity of consumption of each good with respect to the aggregate consumption index ($\epsilon_k$).

#### A.21.1 Preferences

In particular, we generalize our analysis to the non-separable class of CES functions in Sato (1975), which satisfy implicit additivity in Hanoch (1975), as recently used in the macroeconomics literature in Comin, Lashkari and Mestieri (2015). We suppose that we observe data on households indexed by $h \in \{1, \ldots, H\}$ that differ in income and total expenditure ($E_h^t$). The non-homothetic CES consumption index for household $h$ ($C^h_t$) is defined by the following implicit function:

\[
\sum_{k \in \Omega_t} \left( \frac{q^h_{kt} c^h_{kt}}{(C^h_t)^{(\epsilon_k - \sigma)/(1-\sigma)}} \right)^{\frac{\sigma - 1}{\sigma}} = 1, \tag{A.295}
\]

where $c^h_{kt}$ denotes household $h$’s consumption of good $k$ at time $t$; $q^h_{kt}$ is household $h$’s demand parameter for good $k$ at time $t$, which evolves according to equation (2) in the paper; $\sigma$ is the constant elasticity of substitution between varieties; $\epsilon_k$ is the constant elasticity of consumption of good $k$ with respect to the consumption index ($C^h_t$) that allows preferences to be non-homothetic. Assuming that goods are substitutes ($\sigma > 1$), we require $\epsilon_k < \sigma$ for the consumption index (A.295) to be globally monotonically increasing and quasi-concave, and hence to correspond to a well-defined utility function. Our baseline homothetic CES specification corresponds to the special case of equation (A.295) in which $\epsilon_k = 1$ for all $k \in \Omega_t$.

#### A.21.2 Expenditure Minimization

The Lagrangian for the utility maximization problem for household $h$ is:

\[
\mathcal{L} = C^h_t + \rho^h \left( 1 - \sum_{k \in \Omega_t} \left( \frac{q^h_{kt} c^h_{kt}}{(C^h_t)^{(\epsilon_k - \sigma)/(1-\sigma)}} \right)^{\frac{\sigma - 1}{\sigma}} \right) + \lambda^h \left( E^h_t - \sum_{k \in \Omega_t} p^h_{kt} c^h_{kt} \right), \tag{A.296}
\]
where we assume for simplicity that all households face the same prices for a given good \((p_{kt})\). The first-order condition with respect to consumption of each good \((c_{kt}^h)\) can be written as:

\[
p_{kt}c_{kt}^h = \frac{\rho^h}{\lambda^h} \left( \frac{1 - \sigma}{\sigma} \right) k_{kt}^h,
\]

where we define \(k_{kt}^h\) as:

\[
k_{kt}^h = \left( \frac{\phi_{kt}^h c_{kt}^h}{\left(C_t^h \right)^{(\epsilon_k - \sigma)/(1 - \sigma)}} \right)^{\frac{\sigma - 1}{\sigma}}.
\]

From the first-order condition (A.297) and utility function (A.300), total expenditure by household \(h\) is given by:

\[
E_t^h = \sum_{k \in \Omega} p_{kt} c_{kt}^h = \frac{1 - \sigma}{\sigma} \rho^h \lambda^h.
\]

Using this result in the first-order condition (A.297), we find that \(k_{kt}^h\) equals the share of good \(k\) in the expenditure of household \(h\) at time \(t\):

\[
s_{kt}^h = \frac{p_{kt} c_{kt}^h}{E_t^h} = k_{kt}^h = \left( \frac{\phi_{kt}^h c_{kt}^h}{\left(C_t^h \right)^{(\epsilon_k - \sigma)/(1 - \sigma)}} \right)^{\frac{\sigma - 1}{\sigma}}.
\]

Re-arranging this relationship, we obtain the demand function for good \(k\):

\[
c_{kt}^h = \left( \phi_{kt}^h \right)^{\sigma - 1} \left( \frac{p_{kt}}{E_t^h} \right)^{-\sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} = \left( \phi_{kt}^h \right)^{\sigma - 1} \left( \frac{p_{kt}}{P_t^h} \right)^{-\sigma} \left( C_t^h \right)^{\epsilon_k},
\]

which highlights that \(\epsilon_k\) controls the elasticity of demand for good \(k\) with respect to the real consumption index \((C_t^h)\). Using this demand function (A.301), the expenditure share (A.300) can be re-written as:

\[
s_{kt}^h = \left( \phi_{kt}^h \right)^{\sigma - 1} \left( \frac{p_{kt}}{P_t^h} \right)^{1 - \sigma} \left( C_t^h \right)^{\epsilon_k - 1}.
\]

Additionally, using the CES demand function (A.301) in utility in equation (A.295), we can solve for the expenditure function for household \(h\):

\[
E_t^h = P_t^h C_t^h = \left[ \sum_{k \in \Omega_t} \left( \frac{p_{kt}}{\phi_{kt}^h} \right)^{1 - \sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} \right]^{\frac{1}{1 - \sigma}}.
\]

Therefore the price index for household \(h\) is given by:

\[
p_t^h = \left( \frac{1}{C_t^h} \right) \left[ \sum_{k \in \Omega_t} \left( \frac{p_{kt}}{\phi_{kt}^h} \right)^{1 - \sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} \right]^{\frac{1}{1 - \sigma}},
\]

or equivalently:

\[
p_t^h = \left[ \sum_{k \in \Omega_t} \left( \frac{p_{kt}}{\phi_{kt}^h} \right)^{1 - \sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1} \right]^{\frac{1}{1 - \sigma}}.
\]

Combining equations (A.302) and (A.304), the share of good \(k\) in expenditure for household \(h\) at time \(t\) can be written as:

\[
s_{kt}^h = \left( \frac{p_{kt}}{\phi_{kt}^h} \right)^{1 - \sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1} = \left( \frac{p_{kt}}{\phi_{kt}^h} \right)^{1 - \sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1} \left( P_t^h \right)^{1 - \sigma}.
\]
A.21.3 Non-homothetic CES Unified Price Index

We now show that our unified approach to the demand system and the price index can be extended to this case of non-homothetic CES preferences. As for the homothetic specification in Section 2 of the paper, the price index (A.304) depends on demand-adjusted prices \( p_{kt}/q_{kt}^h \) rather than observed prices \( p_{kt} \). One challenge relative to the homothetic CES case is that the overall CES price index \( P_{ht} \) enters the numerator of the expenditure share in equation (A.305). To overcome this challenge, we work with the share of each good in overall expenditure \( s_{ht}^k \) rather than the common goods expenditure share \( s_{ht}^\ast \) in our earlier notation, but we still take averages across the common goods, because only those common goods are supplied in both time periods. In particular, re-arranging the overall expenditure share in equation (A.305) for an individual common good, we have:

\[
P_{ht} = \frac{p_{kt}}{q_{kt}} \left( s_{ht}^k \right)^{\frac{1}{\sigma-1}} \left( \frac{E_t^h}{P_t^h} \right)^{\frac{\epsilon_k - 1}{1-\sigma}}.
\]  
(A.306)

Taking logarithms yields:

\[
\ln P_{ht} = \ln p_{kt} - \ln q_{kt} + \frac{1}{\sigma-1} \ln s_{ht}^k + \left( \frac{\epsilon_k - 1}{1-\sigma} \right) \ln \left( \frac{E_t^h}{P_t^h} \right).
\]  
(A.307)

Averaging across the common goods, we obtain:

\[
[1 + \theta] \ln P_{ht} = \ln \bar{p}_t + \frac{1}{\sigma-1} \ln \bar{s}_t^k + \theta \ln \left( \frac{E_t^h}{P_t^h} \right),
\]  
(A.308)

where a tilde above a variable denotes an average across common goods such that \( \bar{p}_t = \left( \prod_{k \in \Omega_{t-1}} p_{kt} \right)^{1/N_{t-1}} \); we have used our result that the average of the demand shocks across common goods is equal to zero \( \ln (\bar{p}_t/\bar{q}_t) = 0 \); the derived parameter \( \theta \) captures the average across the common goods of the elasticity of expenditure with respect to the consumption index \( \epsilon_k \) relative to the elasticity of substitution \( \sigma \).

Rearranging terms in equation (A.308) and exponentiating, we obtain the following closed-form solution for the overall CES price index:

\[
P_{ht} = (\bar{p}_t)^{\frac{1}{1+\theta}} \left( \bar{s}_t^k \right)^{\frac{1}{\sigma-1+\theta}} \left( \frac{E_t^h}{P_t^h} \right)^{\frac{\epsilon_k - 1}{1+\theta}}.
\]  
(A.309)

Taking ratios between the two time periods, we obtain our generalization of our CES unified price index to the non-homothetic case for each household \( h \):

\[
\frac{P_{ht}^h}{P_{ht-1}^h} = \left( \frac{\bar{p}_t}{\bar{p}_{t-1}} \right)^{\frac{1}{1+\theta}} \left( \frac{\bar{s}_t^h}{\bar{s}_{t-1}^h} \right)^{\frac{1}{\sigma-1+\theta}} \left( \frac{E_t^h}{E_{t-1}^h} \right)^{\frac{\epsilon_k - 1}{1+\theta}},
\]  
(A.310)

which corresponds to equation (43) in the paper. From this expression, the change in the household’s cost of living \( P_{ht}^h/P_{ht-1}^h \) now depends directly on the change in income (and hence total expenditure) for parameter values for which preferences are non-homothetic \( (\epsilon_k \neq 1 \text{ for some } k \text{ and hence } \theta \neq 0) \).
A.21.4 Non-Homothetic-Reverse-Weighting (NHRW) Estimator

We now show that our reverse-weighting (RW) estimation procedure can be generalized to estimate the elasticity of substitution between goods ($\sigma$) and the elasticity of consumption of each good with respect to the consumption index ($e_k$). We begin by defining the change in the cost of living for common goods ($P_{t}^{h_{s}} / P_{t-1}^{h_{s}}$) for household $h$ between periods $t - 1$ and $t$:

$$
\frac{P_{t}^{h_{s}}}{P_{t-1}^{h_{s}}} = \left[ \frac{\sum_{k \in \Omega_{t-1}} (p_{kt} / \phi_{kt}^{h})^{1-\sigma} (E_{t}^{h} / P_{t}^{h})^{\epsilon_{k} - 1}}{\sum_{k \in \Omega_{t-1}} (p_{kt-1} / \phi_{kt-1}^{h})^{1-\sigma} (E_{t-1}^{h} / P_{t-1}^{h})^{\epsilon_{k} - 1}} \right]^{\frac{1}{1-\sigma}},
$$

(A.311)

and the share of an individual common good in all expenditure on common goods ($s_{kt}^{h_{s}}$):

$$
s_{kt}^{h_{s}} = \frac{(p_{kt} / \phi_{kt}^{h})^{1-\sigma} (E_{t}^{h} / P_{t}^{h})^{\epsilon_{k} - 1}}{\sum_{l \in \Omega_{t-1}} (p_{lt} / \phi_{lt}^{h})^{1-\sigma} (E_{t-1}^{h} / P_{t-1}^{h})^{\epsilon_{l} - 1}},
$$

(A.312)

where the asterisk indicates the value of a variable for common goods. In both equations (A.311) and (A.312), the summations are only across common goods. But both expressions include the overall CES price index ($P_{t}^{h}$), as determined in equation (A.309) above.

Using the common goods expenditure share (A.312), we can re-write the change in the cost of living for common goods (A.311) as the following forward and backward differences respectively:

$$
\frac{P_{t}^{h_{s}}}{P_{t-1}^{h_{s}}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{1-\sigma} \left( \frac{\phi_{kt}^{h}}{\phi_{kt-1}^{h}} \right)^{\epsilon_{k} - 1} \left( \frac{E_{t}^{h} / P_{t}^{h}}{E_{t-1}^{h} / P_{t-1}^{h}} \right) \right]^{\frac{1}{1-\sigma}},
$$

(A.313)

$$
\frac{P_{t}^{h_{s}}}{P_{t-1}^{h_{s}}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{(1-\sigma)} \left( \frac{\phi_{kt}^{h}}{\phi_{kt-1}^{h}} \right)^{(1-\sigma)} \left( \frac{E_{t}^{h} / P_{t}^{h}}{E_{t-1}^{h} / P_{t-1}^{h}} \right)^{-(\epsilon_{k} - 1)} \right]^{\frac{1}{1-\sigma}},
$$

(A.314)

which correspond to generalizations of the Lloyd-Moulton indexes discussed in Section 2.5.2 of the paper to allow for both time-varying demand shocks for each good and non-homotheticity. Using our specification for demand from equation (2) in the paper, which implies $\phi_{kt} / \phi_{kt-1} = \theta_{kt} / \theta_{kt-1}$, we obtain:

$$
\frac{P_{t}^{h_{s}}}{P_{t-1}^{h_{s}}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{1-\sigma} \left( \frac{\theta_{kt}^{h}}{\theta_{kt-1}^{h}} \right)^{\epsilon_{k} - 1} \left( \frac{E_{t}^{h} / P_{t}^{h}}{E_{t-1}^{h} / P_{t-1}^{h}} \right) \right]^{\frac{1}{1-\sigma}},
$$

(A.315)

$$
\frac{P_{t}^{h_{s}}}{P_{t-1}^{h_{s}}} = \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{(1-\sigma)} \left( \frac{\theta_{kt}^{h}}{\theta_{kt-1}^{h}} \right)^{(1-\sigma)} \left( \frac{E_{t}^{h} / P_{t}^{h}}{E_{t-1}^{h} / P_{t-1}^{h}} \right)^{-(\epsilon_{k} - 1)} \right]^{\frac{1}{1-\sigma}}.
$$

(A.316)

Equating these two expressions, we obtain the following key equality for the change in the cost of living for common goods:

$$
\theta_{t-1}^{h_{s}} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{1-\sigma} \left( \frac{\phi_{kt}^{h}}{\phi_{kt-1}^{h}} \right)^{1-\sigma} \left( p_{kt-1} / p_{kt} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left( \frac{\phi_{t-1}^{h_{s}}}{\phi_{t-1}^{h_{s}}} \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt-1}^{h_{s}} (p_{kt} / p_{kt-1})^{-(1-\sigma)} \left( \frac{\phi_{kt}^{h}}{\phi_{kt-1}^{h}} \right)^{(1-\sigma)} \left( p_{kt-1} / p_{kt} \right)^{(1-\sigma)} \right]^{\frac{1}{1-\sigma}},
$$

(A.317)

where $\Theta_{t-1,j}^{h_{s}}$ and $\Theta_{t-1,j}^{h_{B}}$ are forward and backward aggregate demand shifters that are defined as:
\[ \Theta^{Fh}_{t,L,t} \equiv \left[ \sum_{k \in \Omega_{t-1}} s^{h}_{kt-1} \left( \frac{p_{k_L}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\phi^{h}_{kt}}{\phi^{h}_{kt-1}} \right)^{\sigma-1} \left( \frac{E^{h}_{kt}}{E^{h}_{kt-1}} \right) \right]^{\frac{1}{1-\sigma}}, \quad (A.318) \]

\[ \Theta^{Bh}_{t,L,t-1} \equiv \left[ \sum_{k \in \Omega_{t-1}} s^{h}_{kt} \left( \frac{p_{k_L}}{p_{kt-1}} \right)^{-(1-\sigma)} \left( \frac{\phi^{h}_{kt}}{\phi^{h}_{kt-1}} \right)^{-(\sigma-1)} \left( \frac{E^{h}_{kt}}{E^{h}_{kt-1}} \right) \right]^{\frac{1}{1-\sigma}}. \]

The non-homothetic-reverse-weighting estimator (NHRW) imposes the identifying assumption of money-metric utility for each household \( h \):

\[ \Theta^{Fh}_{t,L,t} = \left( \Theta^{Bh}_{t,L,t-1} \right)^{-1} = 1. \quad (A.319) \]

Using this identifying assumption in equation (A.317), we obtain the following sample moment condition for each household \( h \):

\[ h^h(\sigma) - \frac{1}{\sigma-1} \ln \left[ \sum_{k \in \Omega_{t-1}} s^{h}_{kt-1} \left( \frac{p_{k_L}}{p_{kt-1}} \right)^{1-\sigma} \left( \frac{\phi^{h}_{kt}}{\phi^{h}_{kt-1}} \right)^{\sigma-1} \right] - \frac{1}{1-\sigma} \ln \left[ \sum_{k \in \Omega_{t-1}} s^{h}_{kt} \left( \frac{p_{k_L}}{p_{kt-1}} \right)^{-(1-\sigma)} \left( \frac{\phi^{h}_{kt}}{\phi^{h}_{kt-1}} \right)^{-(\sigma-1)} \right] = 0, \quad (A.320) \]

where we have a closed-form solution for the change in the overall cost of living for each household \( h \) from equation (A.310) above, reproduced below:

\[ \frac{p_{h}^{t}}{p_{h}^{t-1}} = \left( \frac{p_{1}}{p_{t-1}} \right)^{\frac{1}{\sigma-1}} \left( \frac{s_{h}^{t}}{s_{h}^{t-1}} \right) \left( \frac{E_{h}^{t}}{E_{h}^{t-1}} \right)^{\frac{\sigma}{\sigma-1}}. \quad (A.321) \]

Stacking the moment conditions for each household (A.320) such that \( M(\sigma) = [M^1(\sigma), \ldots, M^H(\sigma)] \) for \( h \in \{1, \ldots, H\} \), the non-homothetic-reverse-weighting estimator (NHRW) solves:

\[ \hat{\sigma}^{NHRW} = \arg \min \left\{ M(\sigma)' \times I \times M(\sigma) \right\}, \quad (A.322) \]

where we weight each moment condition equally using the identity matrix (I).

In contrast to our baseline RW estimator in Section 2.5 of the paper, the NHRW has one moment condition for each household, and we require the number of households to be larger than the number of goods in order to identify the elasticity of consumption with respect to the consumption index (\( \epsilon_k \)) for each good \( k \) as well as the elasticity of substitution across goods (\( \sigma \)). The NHRW estimator inherits the same properties as the RW estimator in our baseline CES specification, as characterized in Propositions 2 and 3 in the paper.

**A.22 Nested CES**

In our baseline specification in Section 2 of the paper, we focus for simplicity on a single CES tier of utility. In this section of the web appendix, we generalize our approach to a nested CES demand system with multiple tiers of utility. For simplicity, we illustrate this generalization for two tiers of utility (an upper tier defined across sectors and a lower tier defined across barcodes within sectors), but as discussed in the paper our analysis goes through for any number of tiers of utility.
A.22.1 Preferences

We assume that the aggregate unit expenditure function is a constant elasticity function of the unit expenditure function for each sector \( g \in \Omega^G \) as follows:

\[
P_t = \left[ \sum_{g \in \Omega^G} \left( \frac{p^G_{gt}}{\varphi^G_{gt}} \right)^{1-\sigma^G} \right]^{1/1-\sigma^G}, \quad \sigma^G > 1, \tag{A.323}
\]

where \( \sigma^G \) is the elasticity of substitution across sectors; \( p^G_{gt} \) is the unit expenditure function for each sector; \( \varphi^G_{gt} \) is the demand parameter for each sector; we assume for simplicity that the set of sectors is constant over time and denote the number of elements in this set by \( N^G = |\Omega^G| \).

The unit expenditure function for each sector is a constant elasticity function of the consumption of goods \( k \in \Omega^K_{gt} \) within each sector as follows:

\[
p^G_{gt} = \left[ \sum_{k \in \Omega^K_{gt}} \left( \frac{p^K_{kt}}{\varphi^K_{kt}} \right)^{1-\sigma^K} \right]^{1/1-\sigma^K}, \quad \sigma^K > 1, \tag{A.324}
\]

where \( \sigma^K \) is the elasticity of substitution across goods within each sector and is allowed to differ across sectors; \( p^K_{kt} \) is the price for each good; \( \varphi^K_{kt} \) is the demand parameter for each good; we allow the set of goods within each sector to change over time and denote the number of elements within this set by \( N^K_{gt} = |\Omega^K_{gt}| \); we require that both elasticities of substitution (\( \sigma^G \) and \( \sigma^K \)) are greater than one but do not otherwise restrict their values relative to one another.

The demand parameters for each sector (\( \varphi^G_{gt} \)) and good (\( \varphi^K_{kt} \)) take the same form as in equation (2) in the paper, with time-invariant and time-varying components:

\[
\ln \varphi^G_{gt} = \ln \varphi^G_{g} + \ln \theta^G_{gt}, \quad \ln \theta^G_{gt} \sim F^G_{g} \left( \mu^G_{\theta}, (\chi^G_{\theta})^2 \right). \tag{A.325}
\]
\[
\ln \varphi^K_{kt} = \ln \varphi^K_{k} + \ln \theta^K_{kt}, \quad \ln \theta^K_{kt} \sim F^K_{k} \left( \mu^K_{\theta}, (\chi^K_{\theta})^2 \right). \tag{A.326}
\]

We assume that the time-varying components of demand (\( \ln \theta^G_{gt}, \ln \theta^K_{kt} \)) are drawn from distributions that are independent across sectors, good and time periods. We also assume that these time-varying components of demand are not observed until after goods have been supplied to the market. Therefore, applying the same weak law of large numbers argument as used in our baseline CES specification in the paper, the mean time-varying demand components (\( \ln \theta^G_{gt}, \ln \theta^K_{kt} \)) converge in probability to their population values (\( \mu^G_{\theta}, \mu^K_{\theta} \)) as the number of sectors and common goods respectively become large.

A.22.2 Aggregate Price Index

Applying Shephard’s Lemma to the aggregate unit expenditure function (A.323), the share of aggregate expenditure on each sector (\( s^G_{gt} \)) is:

\[
s^G_{gt} = \frac{\left( \frac{p^G_{gt}}{\varphi^G_{gt}} \right)^{1-\sigma^G}}{\sum_{m \in \Omega^G} \left( \frac{p^G_{mt}}{\varphi^G_{mt}} \right)^{1-\sigma^G}} = \left( \frac{p^G_{gt} / \varphi^G_{gt}}{p^G_{t}} \right)^{1-\sigma^G}. \tag{A.327}
\]
Rearranging this expenditure share, and taking logarithms, we obtain the following expression for the aggregate unit expenditure function:

\[ \ln P_t = \ln P_{gt}^G - \ln \phi_{gt}^G + \frac{1}{\sigma_g^G - 1} \ln s_{gt}^G. \]  
(A.328)

Differenting over time, and averaging across sectors, the change in the aggregate cost of living can be expressed in the money-metric form of our unified price index:

\[ \Delta \ln P_t = \frac{1}{N_G^G} \sum_{g \in G} \Delta \ln P_{gt}^G + \frac{1}{\sigma_g^G - 1} \frac{1}{N_G^G} \sum_{g \in G} \Delta \ln s_{gt}^G, \]  
(A.329)

where we have used the fact that the mean demand shock across sectors is equal to zero:

\[ \frac{1}{N_G^G} \sum_{g \in G} \ln \left( \frac{\phi_{gt}^G}{\phi_{gt-1}^G} \right) = \frac{1}{N_G^G} \sum_{g \in G} \ln \left( \frac{\theta_{gt}^G}{\theta_{gt-1}^G} \right) = 0. \]  
(A.330)

### A.22.3 Sectoral Price Index

We now solve for the change in the unit expenditure function for each sector (\( \Delta \ln P_{gt}^G \)) in equation (A.329) as a function of the characteristics of the goods within that sector. First, following the same line of argument as in Section 2.2 of the paper, the change in the sectoral unit expenditure function between a pair of periods \( t \) and \( t - 1 \) can be decomposed into a variety correction term for the entry and exit of goods \((1 / (\sigma_g^K - 1)) \ln \left( \frac{\lambda_{gt-1}^G}{\lambda_{gt}^G} \right)\) and the change in the price index for common goods \((P_{gt}^* / P_{gt-1}^*)\):

\[ \frac{P_{gt}^G}{P_{gt-1}^G} = \left( \frac{\lambda_{gt-1}^G}{\lambda_{gt}^G} \right) \frac{1}{\sigma_g^K - 1} \frac{P_{gt}^*}{P_{gt-1}^*}. \]  
(A.331)

The sectoral unit expenditure function for common goods \((P_{gt}^*\) takes the same form as in equation (A.324) but the summation is only over common goods \( k \in \Omega_{gt,t-1}^K \):

\[ P_{gt}^* = \left[ \sum_{k \in \Omega_{gt,t-1}^K} \left( \frac{p_{kt}^G}{\phi_{kt}^G} \right)^{1-\sigma_g^K} \right]^{1/1-\sigma_g^K}, \]  
(A.332)

and the share of each individual common good in all expenditure on common goods \((s_{kt}^{K*}\) is:

\[ s_{kt}^{K*} = \frac{\left( \frac{p_{kt}^G}{\phi_{kt}^G} \right)^{1-\sigma_g^K}}{\sum_{\ell \in \Omega_{gt,t-1}^K} \left( \frac{p_{\ell t}^G}{\phi_{\ell t}^G} \right)^{1-\sigma_g^K}} = \left( \frac{p_{kt}^G}{\phi_{kt}^G} \right)^{1-\sigma_g^K} \]  
(A.333)

Rearranging this common goods expenditure share, and taking logarithms, we obtain the following expression for the sectoral common goods unit expenditure function:

\[ \ln P_{gt}^* = \ln p_{kt}^K - \ln \phi_{kt}^K + \frac{1}{\sigma_g^K - 1} \ln s_{kt}^{K*}. \]  
(A.334)

Differenting over time, and averaging across common goods, the change in the sectoral common goods unit expenditure function also can be expressed in the money-metric form of our unified price index:

\[ \Delta \ln P_{gt}^* = \frac{1}{N_{gt,t-1}^K} \sum_{k \in \Omega_{gt,t-1}^K} \Delta \ln p_{kt}^K + \frac{1}{\sigma_g^K - 1} \frac{1}{N_{gt,t-1}^K} \sum_{k \in \Omega_{gt,t-1}^K} \Delta \ln s_{kt}^{K*}. \]  
(A.335)
where the mean demand shock across common goods within each sector is equal to zero:

\[
\frac{1}{N^G_{glt-1}} \sum_{k \in \Omega^G_{glt-1}} \ln \left( \frac{\varphi^K_{kt}}{\varphi^K_{kt-1}} \right) = \frac{1}{N^K_{glt-1}} \sum_{k \in \Omega^K_{glt-1}} \ln \left( \frac{\theta^K_{kt}}{\theta^K_{kt-1}} \right) = 0.
\]  

(A.336)

### A.22.4 Nested CES Unified Price Index

Our CES unified price index (CUPI) for each tier of utility is defined over the mean of the logs of the prices and expenditure shares for that tier of utility. As the mean is a linear operator, we can apply this operator recursively across the tiers of utility to express the change in the aggregate cost of living in terms of means across both sectors and goods within each sector. In particular, substituting equations (A.331) and (A.335) for each sector into the change in the aggregate cost of living in equation (A.329), we obtain equation (45) from the paper:

\[
\ln \left( \frac{P_t}{P_{t-1}} \right) = \frac{1}{N^G} \sum_{g \in \Omega^G} \frac{1}{N^K_{glt-1}} \sum_{k \in \Omega^K_{glt-1}} \ln \left( \frac{P^K_{glt}}{P^K_{glt-1}} \right) + \frac{1}{N^G} \sum_{g \in \Omega^G} \sum_{k \in \Omega^K_{glt-1}} \frac{1}{\varphi^K_{kt}} - 1 \frac{1}{N^K_{glt-1}} \sum_{k \in \Omega^K_{glt-1}} \ln \left( \frac{s^K_{gkt}}{s^K_{gkt-1}} \right) \right),
\]  

This expression decomposes the change in the aggregate cost of living into four terms: (i) the average log change in prices across sectors and goods within each sector; (ii) the average log change in common goods expenditure shares across sectors and goods within each sector; (iii) the average variety correction across sectors for the entry and exit of goods; (iv) the average log change in expenditure shares across sectors.

Although, for simplicity, we focus on two tiers of utility here, this procedure can be extended from the highest tier of utility all the way down to the lowest. In general, our RW estimator can be applied recursively to each of these tiers of utility. However, conventional measures of the cost of living aggregate across categories using a Jevons Index weighted by sampling frequencies. In Section A.7 of this web appendix, we show that this corresponds to the assumption of a Cobb-Douglas upper tier of utility. Therefore, to ensure comparability with these conventional measures, we assume that the upper tier of utility across sectors is Cobb-Douglas, and use our RW estimator to estimate the elasticity of substitution across goods within sectors, which yields a separate estimated elasticity for each sector.

### A.23 Mixed CES

In this section of the web appendix, we show that our results also generalize to a mixed CES specification, in which there are multiple groups of heterogeneous consumers indexed by \( h \in \{1, \ldots, H\} \). For simplicity, we return to the case of a single tier of utility, although this mixed CES generalization can be combined with a nesting structure. In the non-homothetic specification in Section A.21 of this appendix, the only source of heterogeneity in expenditure shares across consumers was differences in income. In contrast, in this mixed CES specification, we allow both the elasticity of substitution (\( \sigma^h \)) and the demand parameter for each good (\( \varphi^h_{kl} \)) to vary across groups of consumers.
A.23.1 Preferences and Expenditure Shares

In particular, the unit expenditure function \( P^h_t \) and expenditure share \( s^h_{kt} \) for a household from group \( h \) are given by:

\[
P^h_t = \left[ \sum_{k \in \Omega_t} \frac{p_{kt}}{q^h_{kt}} \right]^{1-e^h},
\]

\( (A.338) \)

\[
s^h_{kt} = \frac{\left( \frac{p_{kt}}{q^h_{kt}} \right)^{1-e^h}}{\sum_{l \in \Omega_t} \left( \frac{p_{lt}}{q^h_{lt}} \right)^{1-e^h}} = \frac{\left( \frac{p_{kt}}{q^h_{kt}} \right)^{1-e^h}}{\left( P^h_t \right)^{1-e^h}},
\]

\( (A.339) \)

where \( s^h_{kt} \) is a share of product \( k \) in the expenditure of group \( h \) at time \( t \); we assume for simplicity that all groups face the same prices \( (p_{kt}) \); we also assume that the set of products available \( ( \Omega_t ) \) is the same for all groups, but we allow for the possibility that some groups do not consume some products, which we interpret as corresponding to the limiting case in which the demand parameter converges to zero for that group and product \( (\lim q^h_{kt} \to 0) \); these groups of consumers could differ by income and/or other demographic characteristics.

We assume that the specification of demand in equation (2) in the paper holds for each group of consumers separately, with time-invariant and time-varying components of demand:

\[
\ln q^h_{kt} = \ln q^h_k + \ln \theta^h_{kt}, \quad \ln \theta^h_{kt} \sim F^h \left( \mu^h_{kt}, \left( \lambda^h_{kt} \right)^2 \right).
\]

\( (A.340) \)

We assume that the time-varying component of demand \( (\ln \theta^h_{kt}) \) is drawn from a distribution that is independent across goods and time periods. We also assume that this time-varying component of demand is not observed until after goods have been supplied to the market. Therefore, applying the same weak law of large numbers argument as used in our baseline CES specification in the paper, the mean time-varying demand component \( (\ln \theta^h_{kt}) \) for each group converges in probability to its population value \( (\mu^h_{kt}) \) as the number of common goods becomes large.

A.23.2 Properties of Mixed CES

The presence of heterogeneity across groups relaxes the independence of irrelevant alternatives (IIRA) assumption of CES, because the differences in substitution and demand parameters across groups imply that the relative expenditure shares of two goods in two different markets depends on the relative size of the groups in those markets. In particular, the expenditure share of product \( k \) at time \( t \) can be written as:

\[
s^h_{kt} = \frac{x^h_{kt}}{x_t} = \sum_{h=1}^{H} \frac{x^h_{kt}}{x^h_t} = \sum_{h=1}^{H} \frac{x^h_{kt}}{x^h_t} \cdot \frac{x^h_t}{x_t} = \sum_{h=1}^{H} f^h_t s^h_{kt},
\]

\( (A.341) \)

where \( x^h_{kt} \) is expenditure by group \( h \) on product \( k \) at time \( t \); \( x_{kt} \) is expenditure on product \( k \) at time \( t \); \( x^h_t \) is overall expenditure by group \( h \) at time \( t \); and \( x_t \) is total expenditure at time \( t \); \( s^h_{kt} \) is the share of product \( k \) in overall expenditure at time \( t \); \( s^h_{kt} \) is the share of product \( k \) in group \( h \)'s expenditure at time \( t \); and \( f^h_t \) is the share of group \( h \) in overall expenditure at time \( t \). From equation (A.341), the expenditure shares of each product \( k \)
Rearranging the final line, we obtain equation (48) in the paper:

\[ x_{kt} = \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}. \]

Differentiating expenditure on product \( k \) with respect to the price of another product \( \ell \), we obtain:

\[ \frac{\partial x_{kt}}{\partial p_{tt}} = \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) \frac{\partial p_t^h}{\partial p_{tt}} \frac{1}{P_t^h}. \]

Rearranging this equation, we obtain the following elasticity of expenditure on product \( k \) with respect to the price of another product \( \ell \):

\[ \frac{\partial x_{kt}}{\partial p_{tt}} x_{kt} = \frac{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) \frac{\partial p_t^h}{\partial p_{tt}} \frac{1}{P_t^h}}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}}, \]

which can be re-written to as follows:

\[ \frac{\partial x_{kt}}{\partial p_{tt}} x_{kt} = \sum_{h=1}^{H} \frac{\left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) \frac{\partial p_t^h}{\partial p_{tt}} \frac{1}{P_t^h}}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}}. \]

\[ = \sum_{h=1}^{H} \frac{\left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) s_{lt}^h}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}}. \]

\[ = \sum_{h=1}^{H} \frac{\left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) s_{kt}^h \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} f_t^h}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}}. \]

\[ = \sum_{h=1}^{H} \frac{\sum_{\ell \in \Omega_k} \left( \frac{p_{kl}}{\varphi_{kl}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1} f_t^h}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_t^h \left( \frac{p_t^h}{P_t^h} \right)^{\sigma^h-1}}. \]

Rearranging the final line, we obtain equation (48) in the paper:

\[ \frac{\partial x_{kt}}{\partial p_{tt}} x_{kt} = \frac{1}{s_{kt}} \sum_{h=1}^{H} f_t^h \left( \sigma^h - 1 \right) s_{kt}^h s_{lt}^h. \]
A.23.3 Entry and Exit

We now show that our results for entry and exit in Section 2.2 of the paper hold for each group of consumers separately. Partitioning goods into entering, exiting and common goods, the change in the overall cost of living for group $h$ between periods $t - 1$ and $t$ can be expressed in terms of the change in the share of expenditure on common goods ($\lambda_{i,t-1}^h / \lambda_{t-1,t}^h$) and the change in the cost of living for these common goods ($P_t^{h\star} / P_{t-1}^{h\star}$):

$$\Phi_{t-1,t}^h = \frac{p_t^h}{p_{t-1}^h} = \left( \frac{\lambda_{i,t-1}^h}{\lambda_{t-1,t}^h} \right)^{\frac{1}{1-\sigma^h}} \frac{P_t^{h\star}}{P_{t-1}^{h\star}}, \quad (A.343)$$

where ($\lambda_{i,t-1}^h, \lambda_{t-1,t}^h$) take the same form as in Section A.2 of this web appendix but are defined for each group separately. We again use an asterisk to denote the value of a variable for the common set of goods, such that $P_t^{h\star}$ and $P_{t-1}^{h\star}$ are the unit expenditure functions for common goods:

$$P_t^{h\star} \equiv \left[ \sum_{k \in \Omega_{t-1}} \left( \frac{p_{kt}}{q_{kt}^h} \right)^{1-\sigma^h} \right]^{\frac{1}{1-\sigma^h}}. \quad (A.344)$$

As well as the shares of common goods in total expenditure ($\lambda_{i,t-1}^h, \lambda_{t-1,t}^h$), we can also define the share of individual common good $k \in \Omega_{t-1}$ in expenditure on all common goods ($s_{kt}^{h\star}$) for each household $h$:

$$s_{kt}^{h\star} = \frac{(p_{kt} / q_{kt}^h)^{1-\sigma^h}}{\sum_{t \in \Omega_{t-1}} (p_{kt} / q_{kt}^h)^{1-\sigma^h}} = \frac{(p_{kt} / q_{kt}^h)^{1-\sigma^h}}{(P_t^{h\star})^{1-\sigma^h}}, \quad k \in \Omega_{t-1}. \quad (A.345)$$

A.23.4 Exact Price indexes

All our results for the exact CES price index in Section 2.3 of the paper also hold for each group of consumers separately. Using equations (A.344) and (A.345), the log change in group $h$’s cost of living for common goods ($\ln \Phi_{t-1,t}^{h\star}$) between periods $t$ and $t - 1$ can be expressed in the following form,

$$\ln \Phi_{t-1,t}^{h\star} = \sum_{k \in \Omega_{t-1}} \omega_{kt}^{h\star} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega_{t-1}} \omega_{kt}^{h\star} \ln \left( \frac{q_{kt}^h}{q_{kt-1}^h} \right), \quad (A.346)$$

where the weights $\omega_{kt}^{h\star}$ are the logarithmic mean of common goods expenditure shares ($s_{kt}^{h\star}$) in periods $t$ and $t - 1$ and sum to one for each group,

$$\omega_{kt}^{h\star} \equiv \frac{s_{kt}^{h\star} - s_{kt-1}^{h\star}}{\ln s_{kt}^{h\star} - \ln s_{kt-1}^{h\star}}, \quad (A.347)$$

where the derivation is the same as that for a single group in Section A.3 of this web appendix.

We use the invertibility of the CES demand system for each group to express the unobserved time-varying demand parameter for that group ($q_{kt}^h$) in terms of observed prices ($p_{kt}$) and common goods expenditure shares ($s_{kt}^{h\star}$). In particular, taking logarithms in the common goods expenditure share (7), differencing over time, and
then differencing from the mean across common goods within each time period for each group separately, we obtain the following closed-form expression for the log change in the demand shifter for each common good for that group:

\[
\ln \left( \frac{\Phi_{kt}^{h}}{\Phi_{kt-1}^{h}} \right) = \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \frac{1}{\sigma^{h} - 1} \ln \left( \frac{s_{kt}^{h}}{s_{kt-1}^{h}} \right),
\]  

(A.348)

where a tilde over a variable denotes a geometric average across the set of common goods, such that \( \tilde{x}_{t} = \left( \prod_{k \in \Omega_{t-1}} x_{kt} \right)^{1/N_{t-1}} \) for the variable \( x_{kt} \).

We now make use of our result that as the number of common goods becomes large (\( N_{t-1} \to \infty \)), the mean of the time-varying component of demand (\( \ln \theta_{kt}^{h} \)) converges towards its population value (\( \mu_{0}^{h} \)) for each group, and hence the mean demand shock (\( \ln \left( \frac{\Phi_{kt}^{h}}{\Phi_{kt-1}^{h}} \right) \)) converges towards zero:

\[
\lim_{N_{t-1} \to \infty} \ln \left( \frac{\Phi_{kt}^{h}}{\Phi_{kt-1}^{h}} \right) = \frac{1}{N_{t-1}} \sum_{k=1}^{N_{t-1}} \ln \left( \frac{\theta_{kt}^{h}}{\theta_{kt-1}^{h}} \right) = 0.
\]  

(A.349)

Using equations (A.348) and (A.349) to substitute for this closed-form solution for the demand shocks into equation (A.346), we obtain an exact CES common goods price index (CCG) for each group separately:

\[
\ln \Phi_{t-1}^{h} = \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \frac{1}{\sigma^{h} - 1} \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{s_{kt}^{h}}{s_{kt-1}^{h}} \right).
\]  

(A.350)

Substituting this common goods price index into our earlier expression for the overall price index (A.343), we obtain our exact CES unified price index (CUPI) for each group separately:

\[
\ln \Phi_{t-1}^{h} = \frac{1}{\sigma^{h} - 1} \ln \left( \frac{\lambda_{t-1}^{h}}{\lambda_{t-1}^{h}} \right) + \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \frac{1}{\sigma^{h} - 1} \frac{1}{N_{t-1}} \sum_{k \in \Omega_{t-1}} \ln \left( \frac{s_{kt}^{h}}{s_{kt-1}^{h}} \right).
\]  

(A.351)

### A.23.5 Reserve-Weighting (RW) Estimator

Our reverse-weighting (RW) estimator can be used to estimate the elasticities of substitution (\( \sigma^{h} \)) for each group separately following the same procedure as for our baseline specification with a single group in Section 2.5.2 of the paper. Again we start with three equivalent expressions for the change in the cost of living from the unified price index, the forward difference of the unit expenditure function and the backward difference of the unit expenditure function. From these three expressions, we obtain the following two key equalities between equivalent ways of writing the change in the cost of living for common goods for each group:

\[
\Theta_{t-1,1}^{hF} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^{h} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{1-\sigma^{h}} \right]^{-\frac{1}{1-\sigma^{h}}} = \frac{\tilde{p}_{t}}{p_{t-1}} \left( \frac{s_{t}^{h}}{s_{t-1}^{h}} \right)^{\frac{1}{\sigma^{h}-1}},
\]  

(A.352)

\[
\left( \Theta_{t-1,1}^{hB} \right)^{-1} \left[ \sum_{k \in \Omega_{t-1}} s_{kt}^{h} \left( \frac{p_{kt}}{p_{kt-1}} \right)^{-\sigma^{h}} \right]^{-\frac{1}{1-\sigma^{h}}} = \frac{\tilde{p}_{t}}{p_{t-1}} \left( \frac{s_{t}^{h}}{s_{t-1}^{h}} \right)^{\frac{1}{\sigma^{h}-1}},
\]  

(A.353)

where \( \Theta_{t-1,1}^{hF} \) and \( \Theta_{t-1,1}^{hB} \) are forward and backward aggregate demand shifters for each group:
A.24 Logit Specification

In the discrete choice literature, a standard result is that CES preferences can be derived as the aggregation of the choices of individual consumers with extreme-value-distributed idiosyncratic preferences, as shown in Anderson de Palma and Thisse (1992) and Train (2009). In this section of the web appendix, we use this result to show that our CES unified price index (CUPI) and reverse-weighting (RW) estimation procedure hold for logit preferences, as widely used in applied microeconometric research.

Following McFadden (1974), we suppose that the utility of an individual consumer \( i \) who consumes \( c_{ik} \) units of product \( k \) at time \( t \) is given by:

\[
U_{it} = u_{kt} + z_{ikt}, \quad u_{kt} = \ln q_{kt} + \ln c_{ikt}
\]  

(A.358)
where \( q_{kt} \) captures common consumer tastes for each product; \( z_{ikt} \) captures idiosyncratic consumer tastes for each product that are drawn from an independent Type-I Extreme Value distribution:

\[
G(z) = e^{-e^{-(z/v + \kappa)}},
\]

(A.359)

where \( v \) is the scale parameter of the extreme value distribution and \( \kappa \approx 0.577 \) is the Euler-Mascheroni constant.

Each consumer has the same expenditure \( E_t \) and chooses their preferred product given the observed realizations for idiosyncratic tastes. Therefore the consumer’s budget constraint implies:

\[
c_{ikt} = \frac{E_t}{p_{ikt}}.
\]

(A.360)

The probability that individual \( i \) chooses product \( k \) at time \( t \) is:

\[
x_{ikt} = \text{Prob}(u_{ikt} + z_{ikt} > u_{ikt} + z_{i\ell t}, \forall \ell \neq k),
\]

\[
= \text{Prob}(z_{i\ell t} < z_{ikt} + v_{ikt} - v_{i\ell t}, \forall \ell \neq k).
\]

Therefore, using the distribution of idiosyncratic tastes (A.359), we have:

\[
x_{ikt} \mid z_{ikt} = \prod_{\ell \neq k} e^{-e^{-(z_{ikt} + u_{ikt} - u_{i\ell t})/v + \kappa}}.
\]

Integrating across the probability density function for \( z_{ikt} \), we have:

\[
x_{ikt} = \int_{-\infty}^{\infty} \left( \prod_{\ell \neq k} e^{-e^{-(y + u_{ikt} - u_{i\ell t})/v + \kappa}} \right) \frac{1}{\mu} e^{-y/v + \kappa} e^{-e^{-(y + \kappa)}/v} dy.
\]

Noting that \( u_{ikt} - u_{ikt} = 0 \), this expression can be re-written as:

\[
x_{ikt} = \int_{-\infty}^{\infty} \left( \prod_{\ell \in \Omega_t} e^{-e^{-(y + u_{ikt} - u_{i\ell t})/v + \kappa}} \right) \frac{1}{\mu} e^{-y/v + \kappa} dy,
\]

which can be in turn re-written as:

\[
x_{ikt} = \int_{-\infty}^{\infty} \exp \left( -\sum_{\ell \in \Omega_t} e^{-(y + u_{ikt} - u_{i\ell t})/v + \kappa} \right) \frac{1}{\mu} e^{-y/v + \kappa} dy,
\]

and hence:

\[
x_{ikt} = \int_{-\infty}^{\infty} \exp \left( -e^{-y/v + \kappa} \sum_{\ell \in \Omega_t} e^{-(u_{ikt} - u_{i\ell t})/v} \right) \frac{1}{\mu} e^{-y/v + \kappa} dy.
\]

Now define the following change of variable:

\[
h = \exp \left( -y/v + \kappa \right),
\]

where

\[
- \frac{1}{\nu} \exp \left( -y/v + \kappa \right) dy = dh.
\]
As \( y \to \infty \), we have \( h \to 0 \). As \( y \to -\infty \), we have \( h \to \infty \). Using this change of variable, we have:

\[
x_{ikt} = \int_{0}^{\infty} \exp \left( -h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt} - u_{i\ell t}\right)/v} \right) - dh,
\]

or equivalently:

\[
x_{ikt} = \int_{0}^{\infty} \exp \left( -h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt} - u_{i\ell t}\right)/v} \right) dh,
\]

which yields:

\[
x_{ikt} = \left[ \exp \left( -h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt} - u_{i\ell t}\right)/v} \right) \right]_{0}^{\infty}
\]

and hence:

\[
x_{ikt} = \frac{1}{\sum_{\ell \in \Omega_t} e^{-\left(u_{ikt} - u_{i\ell t}\right)/v}}.
\]

The probability that individual \( i \) chooses product \( k \) at time \( t \) is therefore:

\[
x_{ikt} = \frac{\mu_{ikt}/v}{\sum_{\ell \in \Omega_t} e^{\mu_{i\ell t}/v}},
\]

which from the definition of \( \mu_{ikt} \) in (A.358) and the consumer’s budget constraint in (A.360) becomes:

\[
s_{ikt} = s_{kt} = \frac{(p_{kt}/\varphi_{kt})^{-1/v}}{\sum_{\ell \in \Omega_t} (p_{\ell t}/\varphi_{\ell t})^{-1/v}},
\]

which makes clear that our demand shocks \( (\varphi_{kt}/\varphi_{kt-1}) \) correspond to shifts in the common component of tastes for each good for all consumers \( (\varphi_{kt}) \). As shown in Anderson, De Palma and Thisse (1992), the expected utility of consumer \( i \) at time \( t \) is:

\[
E \left[ U_{it} \right] = E \left[ \max \left\{ u_{i1t} + z_{i1t}, \ldots, u_{iNt} + z_{iNt} \right\} \right] = v \ln \left[ \sum_{\ell \in \Omega_t} \exp \left( \frac{u_{ikt}}{v} \right) \right].
\]

Using the definition of \( u_{ikt} \) in (A.358) and the consumers budget constraint in (A.360), expected utility can be written as:

\[
E \left[ U_{it} \right] = \frac{E_t}{P_t}
\]

where \( P_t \) is the unit expenditure function:

\[
P_t = \left[ \sum_{k \in \Omega_t} (p_{kt}/\varphi_{kt})^{-1/v} \right]^{-v}.
\]

Total expenditure on product \( k \) across all consumers \( i \) at time \( t \) is:

\[
E_t = \sum_{i} E_{ikt} = \sum_{i} s_{kt} E_{it} = s_{kt} E_t,
\]

where we have used the fact that each consumer has the same expenditure \( E_t \). Combining equations (A.361) and (A.365), total expenditure on product \( k \) at time \( t \) can be written as:

\[
E_{kt} = (p_{kt}/\varphi_{kt})^{-1/v} P_t^{1/v} E_t,
\]

where \( P_t \) is the unit expenditure function:

\[
P_t = \left[ \sum_{k \in \Omega_t} (p_{kt}/\varphi_{kt})^{-1/v} \right]^{-v}.
\]
where $P_t$ is again the unit expenditure function (A.364).

Note that equations (A.363), (A.364) and (A.366) take exactly the same form as in our baseline CES specification in Section 2 of the paper. Therefore, our unified price index (CUPI) and reverse-weighting (RW) estimator for CES preferences also can be applied for the closely-related logit model. Additionally, in the same way that our baseline CES specification can be generalized to accommodate mixed CES (as in Section A.23 of this web appendix), the baseline logit model in this section can be generalized to accommodate a mixed logit specification, as in McFadden and Train (2000).

### A.25 Translog Preferences

In this section of the web appendix, we show that our approach also holds for the flexible functional form of homothetic translog preferences, which provide an arbitrary close local approximation to any continuous and twice-differentiable homothetic expenditure function. We show that the translog demand system can be inverted to derive a money-metric expression for the change in the cost of living. We show that that the conventional Törnqvist exact price index for translog that assumes time-invariant demand for each good is subject to a consumer-valuation bias in the presence of time-varying demand shocks. Although for simplicity we focus on a homothetic translog specification, our analysis can be generalized to the non-homothetic translog case, in the same way that our baseline homothetic CES specification can be extended to non-homothetic CES, as shown in Section A.21 of this web appendix above.

We consider the following homothetic translog unit expenditure function defined over the price ($p_{kt}$) and demand parameter ($q_{kt}$) for a constant set of goods $k \in \Omega$ with number of elements $N = |\Omega|$:

$$
\ln P_t = \ln P(p_t, q_t, \sigma) = \ln a_0 + \sum_{k \in \Omega} a_k \ln \left( \frac{p_{kt}}{q_{kt}} \right) + \frac{1}{2} \sum_{k, l \in \Omega} \beta_{kl} \ln \left( \frac{p_{kt}}{q_{kt}} \right) \ln \left( \frac{p_{lt}}{q_{lt}} \right),
$$

(A.367)

where the the parameters $\beta_{kl}$ control substitution patterns between goods; symmetry between goods requires $\beta_{kl} = \beta_{lk}$; and symmetry and homotheticity together imply $\sum_{k \in \Omega} a_k = 1$ and $\sum_{k \in \Omega} \beta_{kl} = \sum_{l \in \Omega} \beta_{lk} = 0$.\footnote{Although, for simplicity, we focus on a homothetic translog expenditure function, our analysis also can be extended to the non-homothetic translog case, similar to our generalization to non-homotheticity in the CES case in Section 3.1 above.}

We assume that a good’s expenditure share is decreasing in its own demand-adjusted price ($\beta_{kk} < 0$), and increasing in the demand-adjusted price of other goods ($\beta_{k\ell} > 0$ for $\ell \neq k$). This assumption ensures that the demand system satisfies the “connected substitutes” conditions from Berry, Gandhi and Haile (2013), which rule out the possibility that some goods are substitutes while others are complements.

We begin by using the unit expenditure function (A.367) to derive the exact price index for translog preferences. Consider any quadratic function of the following form:

$$
F(z_t) = a_0 + \sum_{k \in \Omega} a_k z_{kt} + \sum_{k \in \Omega} \sum_{\ell \in \Omega} a_{kl} z_{kt} z_{lt},
$$

(A.368)

where bold font is used to denote a matrix or vector. Under the assumption that the parameters of this quadratic function $\{a_0, a_k, a_{kl}\}$ are constant, the following result holds exactly:
Now note that the homothetic translog unit expenditure function (A.367) corresponds to such a quadratic function where:

\[
F(z_t) = \ln P_t, \quad z_{kt} = \ln p_{kt}, \quad \frac{\partial F(z_t)}{\partial z_{kt}} = \frac{\partial \ln P_t}{\partial \ln p_{kt}} = \frac{\partial P_t}{\partial p_{kt}}. 
\]

Applying the result (A.369) for this homothetic translog unit expenditure function, we obtain:

\[
\ln P_t - \ln P_{t-1} = \frac{1}{2} \sum_{k \in \Omega} \left( \frac{\partial P_t}{\partial p_{kt}} + \frac{\partial P_{t-1}}{\partial p_{kt-1}} \right) \left( \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) - \ln \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right) \right),
\]

which using the properties of the unit expenditure function can be re-written as:

\[
\ln P_t - \ln P_{t-1} = \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \left( \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) - \ln \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right) \right),
\]

which corresponds to the exact price index for translog (\(\ln \Phi_{t-1,t}^{TR}\)) in equation (52) in the paper:

\[
\ln \Phi_{t-1,t}^{TR} = \ln \left( \frac{P_t}{P_{t-1}} \right) = \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right),
\]

where the weights are the arithmetic mean of expenditure shares in the two time periods \((1/2 \ (s_{kt} + s_{kt-1}))\) and hence necessarily sum to one.

In the same way that our CES unified price index (CUPI) is a generalization of the Sato-Vartia price index to allow for demand shocks for each good, so the translog exact price index in equation (A.372) is a generalization of the Törnqvist index (\(\ln \Phi_{t-1,t}^{TO}\)), which corresponds to the special case in which demand is assumed to be constant for all goods ((\(\varphi_{kt}/\varphi_{kt-1} = 1\) for all \(k \in \Omega\))):

\[
\ln \Phi_{t-1,t}^{TO} = \ln \left( \frac{P_t}{P_{t-1}} \right) = \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right).
\]

From equations (A.372) and (A.373), the exact translog price index with time-varying demand shocks differs from the conventional Törnqvist index that assumes time-invariant demand by a correction term that we term the consumer-valuation bias:

\[
\ln \Phi_{t-1,t}^{TR} = \ln \Phi_{t-1,t}^{TO} - \left[ \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{\theta_{kt}}{\theta_{kt-1}} \right) \right],
\]

where we have used our specification for demand from equation (2) in the paper, which implies \(\varphi_{kt}/\varphi_{kt-1} = \theta_{kt}/\theta_{kt-1}\).

Comparing equation (A.374) for translog with equation (16) in the paper for CES, this consumer-valuation bias takes a similar form as for CES, except that the demand shock for each good is weighted by the arithmetic mean of expenditure shares in the two time periods rather than the logarithmic mean of these expenditure shares. This consumer evaluation bias again arises because the true exact CES price index depends on demand-adjusted price changes, whereas the Törnqvist index is based on observed price changes.
Therefore, the Törnqvist index will be unbiased if the demand shocks \( \ln (q_{kt}/q_{kt-1}) \) are orthogonal to the expenditure-share weights \( \frac{1}{2} (s_{kt} + s_{kt-1}) \), upward-biased if they are positively correlated with these weights, and downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the demand shocks \( \ln (q_{kt}/q_{kt-1}) \) and the expenditure-share weights \( \frac{1}{2} (s_{kt} + s_{kt-1}) \) is possible, depending on the underlying correlation between demand shocks \( \ln (q_{kt}/q_{kt-1}) \) and price shocks \( \ln (p_{kt}/p_{kt-1}) \) in the two time periods. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are endogenous to the demand shocks, as for our baseline CES specification in Section 2.4 of the paper. In particular, a positive demand shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all other goods. We therefore obtain the following result for translog preferences, which is analogous to our result in equation (17) in Section 2.4 of the paper for CES preferences.

**Proposition 11.** A positive demand shock for a good \( k \) (i.e., \( \ln (q_{kt}/q_{kt-1}) > 0 \) for some \( k \in \Omega_{t,t-1} \)) increases the expenditure share for that good \( k \) at time \( t \) \( (s_{kt}) \) and reduces the expenditure share for all other goods \( \ell \neq k \) at time \( t \) \( (s_{\ell t}) \).

**Proof.** Note that demand, prices and expenditure shares at time \( t-1 \) \( (q_{kt-1}, p_{kt-1}, s_{kt-1}) \) are pre-determined at time \( t \). To evaluate the impact of a positive demand shock for good \( k \) \( (\ln (q_{kt}/q_{kt-1}) = \ln (q_{kt}/q_{kt-1}) > 0) \), we consider the effect of an increase in demand at time \( t \) for that good \( (q_{kt}) \) given its demand parameter at time \( t-1 \) \( (q_{kt-1}) \). From the expenditure share \( (A.375) \), and using our assumption that the demand parameters \( \{\beta_{kt}\} \) satisfy “connected substitutes,” we have:

\[
\frac{ds_{kt} \theta_{kt}}{d\theta_{kt} s_{kt}} = \frac{d\theta_{kt} q_{kt}}{d\theta_{kt} s_{kt}} = -\frac{\beta_{kk}}{s_{kt}} > 0, \quad \text{since} \quad \beta_{kk} < 0,
\]

\[
\frac{ds_{kt} \theta_{kt}}{d\theta_{kt} s_{kt}} = \frac{d\theta_{kt} q_{kt}}{d\theta_{kt} s_{kt}} = -\frac{\beta_{\ell k}}{s_{\ell t}} < 0, \quad \text{since} \quad \beta_{\ell k} > 0.
\]

Therefore the consumer-valuation bias for translog preferences takes an analogous form as for CES preferences in the paper. An increase in demand for a good is analogous to a reduction in its price. In response to such a fall in the demand-adjusted price for a good, consumers can obtain a higher level of welfare by substituting towards that good and away from other goods. A price index that rules out demand shocks by assumption cannot capture this substitution in response to changes in demand, in the same way that a Laspeyres index cannot capture substitution in response to changes in price.

As for our CES specification in Section 2 of the paper, the challenge in implementing the exact price index \( (A.372) \) empirically is that demand-adjusted prices \( (p_{kt}/q_{kt}) \) are not directly observed in the data. Again we overcome this challenge by inverting the demand system to solve for the demand parameters \( (q_{kt}) \) as a function of the observed prices and expenditure shares \( (p_{kt}, s_{kt}) \). We begin by deriving the demand system...
from the unit expenditure function (A.367), which can be re-written as:

\[
\ln P_t = \ln a_0 + \sum_{k \in \Omega} \alpha_k \ln p_{kt} - \sum_{k \in \Omega} \alpha_k \ln \varphi_{kt},
\]

\[
+ \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kl} \ln p_{\ell t} - \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kl} \ln \varphi_{\ell t},
\]

\[
- \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln p_{\ell t} + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln \varphi_{\ell t},
\]

Differentiating with respect to \( P_{mt} \), we have:

\[
\frac{\partial P_t}{\partial p_{mt}} = \alpha_m p_{mt} + \frac{1}{2} \sum_{\ell \in \Omega} \beta_{m\ell} p_{mt} \ln p_{\ell t} - \frac{1}{2} \sum_{\ell \in \Omega} \beta_{m\ell} p_{mt} \ln \varphi_{\ell t},
\]

\[
+ \frac{1}{2} \sum_{k \in \Omega} \beta_{km} p_{mt} \ln p_{kt} - \frac{1}{2} \sum_{k \in \Omega} \beta_{km} p_{mt} \ln \varphi_{kt}.
\]

Assuming symmetry (\( \beta_{m\ell} = \beta_{km} \)), this simplifies to:

\[
\frac{\partial P_t}{\partial p_{mt}} = \alpha_m \frac{p_{mt}}{p_t} + \sum_{\ell \in \Omega} \beta_{m\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right),
\]

which implies:

\[
\frac{\partial P_t}{\partial p_{mt}} \frac{p_{mt}}{P_t} = \alpha_m + \sum_{\ell \in \Omega} \beta_{m\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right),
\]

and hence:

\[
s_{kt} = \alpha_k + \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right). \tag{A.375}
\]

Differencing over time, we obtain the following expression for the change in the expenditure share for each product, which corresponds to equation (54) in the paper,

\[
\Delta s_{kt} = \sum_{\ell \in \Omega} \beta_{k\ell} [\Delta \ln (p_{\ell t}) - \Delta \ln \theta_{\ell t}], \tag{A.376}
\]

where we have used our specification of demand from equation (2) in the paper, which implies \( \varphi_{kt} / \varphi_{kt-1} = \theta_{kt} / \theta_{kt-1} \).

We solve for the unobserved demand shocks (\( \Delta \ln (\theta_{\ell t}) \)) by inverting the demand system in equation (A.376). This demand system (A.376) consists of a system of equations for the change in the expenditure shares (\( \Delta s_{kt} \)) of the \( N \) goods that is linear in the change in the log price (\( \Delta \ln p_{kt} \)) and log demand parameter (\( \Delta \ln \theta_{kt} \)) for each good. This demand system can be written in the following matrix form:

\[
\Delta s_t = \beta \Delta \ln p_t - \beta \Delta \ln \theta_t, \tag{A.377}
\]

where we use bold math font to denote a vector or matrix.

In this demand system (A.377), the changes in expenditure shares (\( \Delta s_t \)) must sum to zero across goods. Furthermore, under our assumptions of symmetry and homotheticity, the rows and columns of the symmetric matrix \( \beta \) must each sum to zero. Therefore, without loss of generality, we omit the equation for the first good. We nevertheless recover the demand shock for all goods (including the omitted one) using our result.
that the demand shocks are mean zero across goods \((1/N) \sum_{k \in N} \Delta \ln (\theta_{kt}) = 0\). In particular, we define the following augmented variables:

\[
\Delta \tilde{s}_t \equiv \begin{pmatrix} 0 \\ \Delta s^-_t \end{pmatrix}, \quad \tilde{\beta} \equiv \begin{pmatrix} 0, \ldots, 0 \\ \beta^- \end{pmatrix}, \quad \tilde{\gamma} \equiv \begin{pmatrix} 1, \ldots, 1 \\ \beta^- \end{pmatrix}, \quad \text{(A.378)}
\]

where \(\Delta s^-_t\) denotes the vector of changes in expenditure shares omitting the first good; and \(\beta^-\) denotes the symmetric matrix of substitution parameters omitting the first row. Using this notation, the demand system (A.377) can be written in the following form:

\[
\Delta \tilde{s}_t = \tilde{\beta} \Delta \ln p_t - \tilde{\gamma} \Delta \ln \theta_t, \quad \text{(A.379)}
\]

which can be inverted to solve for the vector of demand shocks \((\Delta \ln \theta_t)\). We thus obtain the unobserved demand shock for each good in terms of observed prices and expenditure shares:

\[
\Delta \ln \theta_{kt} = \Delta \ln \varphi_{kt} = S^{-1} (\Delta s_t, \Delta \ln p_t, \{\beta_{kl}\}). \quad \text{(A.380)}
\]

Substituting for the unobserved demand shock in equation (A.372), we obtain the following exact money-metric price index for translog preferences with time-varying demand parameters:

\[
\ln \Phi_{T}^{\text{TCG}}_{t-1,t} = \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) S^{-1} (\Delta s_t, \Delta \ln p_t, \{\beta_{kl}\}). \quad \text{(A.381)}
\]

This exact translog common goods price index \((\Phi_{T}^{\text{TCG}}_{t-1,t})\) is the analog of our exact CES common goods price index \((\ln \Phi_{t-1,t}^{\text{CCG}})\) in equation (13) in the paper.

Therefore, our main insight that the demand system can be unified with the unit expenditure function to construct a price index that both allows for time-varying demand shocks and remains money metric is not specific to CES, but also holds for the flexible functional form of translog preferences. Furthermore, the consumer-valuation bias is again present for this flexible functional form, because a price index that rules out demand shocks by assumption cannot capture the potential for consumers to increase welfare by substituting towards goods that experience reductions in demand-adjusted prices from increases in demand.

### A.26 Data Appendix

In this data appendix, we report our full list of product groups and summary statistics for each product group in Table A.1, as a supplement to Table 1 in the paper. Consistent with our discussion for the full sample in Section 4 of the paper, we find pervasive entry and exit for all product groups, combined with substantial variation across these product groups in the share of products that enter and exit and the share of common goods in expenditure in period \(t\) relative to period \(t - 1\).
Table A.1: Descriptive Statistics by Product Group

<table>
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<tr>
<th>Code</th>
<th>Description</th>
<th>Number of UPCs</th>
<th>Average $\frac{\lambda_{t-1}}{\lambda_{t}}$</th>
<th>Percent of UPCs that Enter in a Year</th>
<th>Percent of UPCs that Exit in a Year</th>
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Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode (UPC) on the total value sold, total quantity sold, and average price; $\lambda_{t-1}$ and $\lambda_{t-1}$ are the shares of expenditure on common goods in total expenditure in time $t$ and $t - 1$ respectively as defined in equation (A.57) in this web appendix. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
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<th>Percent of UPCs that Enter in a Year</th>
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Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode (UPC) on the total value sold, total quantity sold, and average price; $\lambda_{t+1}$ and $\lambda_{t-1}$ are the shares of expenditure on common goods in total expenditure in time $t$ and $t-1$ respectively as defined in equation (A.57) in this web appendix. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
A.27 Additional Empirical Results

In this section of the web appendix, we report additional empirical results for the other robustness checks discussed in Section 5.5.2 of the paper, including the mixed CES specification discussed in Section 3.3, the use of alternative values for the elasticity of substitution, a comparison with official CPI categories, and sensitivity to measurement error for goods with small expenditure shares.

A.27.1 Grid Search over the Elasticity of Substitution

We examine the sensitivity of our consumer-valuation bias to the value of the elasticity of substitution by undertaking a grid search over alternative values for this elasticity. From Table 2 in the paper, our lower bound for the elasticity of substitution is one, and our upper bound ranges from around 10 to 20 across product groups. Therefore, we consider a grid of thirty-eight evenly spaced values for this parameter ranging from 1.5 to 20. For each parameter value on this grid, we first compute our CCG and CUPI for each product group and year. We next compute an overall measure of the cost of living by aggregating across product groups using expenditure share weights.

We present a plot of these results in Figure A.5. Interestingly, despite the fact that the Laspeyres and Fisher indexes registered average changes of 2.0 and 1.6 percent per year over this time period, the aggregate change in the cost of living measured by the CUPI is negative for all values of the elasticity below 20. As one should expect, the change in the cost of living captured by the CCG tends to be lower when the elasticity of substitution is small, because demand shifts matter more for welfare if goods are less substitutable. Similarly, smaller values of the elasticity of substitution are associated with greater gains from variety (and therefore a lower cost of living) because a low elasticity means that new varieties are considered more differentiated and hence more valuable to households. The results suggest that both the CCG and the CUPI register substantial price falls over the full time period when the elasticity of substitution is very small (e.g., less than 3), but the differences in average changes in the cost of living measured by the CUPI vary by only 2.4 percentage points per year if we restrict ourselves to the range of median elasticities we found in Table 2 (4.5 to 7.5).
Figure A.5: Average of Four-Quarter Proportional Changes in the Aggregate Cost of Living \(\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)\) from 2005-1014 for Alternative Elasticities of Substitution

Note: Average of four-quarter proportional changes in the aggregate cost of living from 2005-2014. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \(\left(\frac{P_{st} - P_{st-1}}{P_{st-1}}\right)\) by their expenditure shares. Figure shows the Fisher index from Figure 5 in the paper and the results from recomputing the Feenstra index (equation (15) in the paper), the CCG (equation (13) in the paper), and the CUPI (equation (14) in the paper) for thirty-eight evenly-spaced values of the elasticity ranging from 1.5 to 20. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

A.27.2 Comparison with Official CPI Categories

We demonstrate the relevance of our results for official measures of the consumer price index (CPI) by comparing conventional price indexes computed using the Nielsen data to official CPI price indexes. We were able to map 89 of our 104 product groups into CPI categories. We again aggregate across these price sub-indexes for each of the 89 product groups using expenditure-share weights to construct a measure of the overall change in the cost of living. In Figure A.6, we compare the resulting aggregate price indexes using the Nielsen data and the official CPI sub-indexes.

We find a strong positive and statistically significant correlation of 0.98 between the Laspeyres (based on Nielsen data) and the CPI measures of the change in the overall cost of living. Moreover, the average changes in the cost of living as measured by the Laspeyres index and the CPI are almost identical: 2.30 versus 2.37 percent respectively. The Paasche index (based on the Nielsen data) has the same correlation with the CPI, but has an average change that is only 1.5 percent per year. In other words, annual movements in changes in the cost of living as measured by the BLS for this set of goods can be closely approximated by using a Laspeyres index and the Nielsen data, and the difference between the Laspeyres and the Paasche in the Nielsen data is less than one percentage point per year (consistent with the findings of Boskin et al. 1996). In contrast, we find a substantial bias from abstracting from entry/exit and changes in demand for surviving goods, with our CUPI-RW and CUPI-GRW registering average changes in the cost of living that are more than two percentage points below the CPI.
Figure A.6: Four-Quarter Proportional Changes in the Aggregate Cost of Living \( \left( \frac{P_t - P_{t-1}}{P_{t-1}} \right) \), CPI Matched Sample

Note: This figure shows alternative measures of the four-quarter proportional change in the aggregate cost of living using different price indexes for the 89 out of 104 product groups that we can match to subcategories of the CPI. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \( \left( \frac{P_{gt} - P_{gt-1}}{P_{gt-1}} \right) \) by their expenditure shares. The thick gray line shows the aggregate price index based on the CPI subcategories. The other lines show alternative price indexes computed using the Nielsen data. CCG-RW and CUPI-RW are our exact common goods price index (equation (13)) and unified price index (equation (14)), respectively, computed using our reverse-weighting (RW) estimates of the elasticity of substitution for each product group. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

A.27.3 Measurement Error in Small Expenditure Shares

As discussed in the paper, an advantage of our exact CES common-goods (CCG) and CES unified (CUPI) price indexes is that they are robust to mean-zero log-additive measurement error in prices and expenditure shares, because they depend on the mean of the log of the prices and expenditure shares of common goods. However, one remaining concern about such measurement error is that the CCG depends on an unweighted mean of the log expenditure shares across common goods. Therefore, it could be affected by measurement error for goods with small expenditure shares. To address this concern, we use the property of CES preferences that the price index for all common goods can be rewritten as equal to a price index for a subset of these common goods and the expenditure on this subset as a share of expenditure on all common goods. Therefore, we re-compute our common goods price index, choosing as our subset common goods with above-median expenditure shares:

\[
\frac{P^{*}_{gt}}{P^{*}_{gt-1}} = \left( \frac{\mu^B_{gt}}{\mu^B_{gt-1}} \right)^{\frac{1}{\gamma-1}} \frac{P^{*B}_{gt}}{P^{*B}_{gt-1}}
\]

(A.382)

where \( P^{*}_{gt} \) and \( P^{*}_{gt-1} \) are the unit expenditure function for common goods for product group \( g \) in periods \( t \) and \( t-1 \) respectively; \( P^{*B}_{gt} \) and \( P^{*B}_{gt-1} \) are the corresponding unit expenditure functions for the subset of common goods with above-median expenditure shares; and \( \mu^B_{gt} \) and \( \mu^B_{gt-1} \) are the shares of these common goods with above-median expenditure shares in total expenditure on all common goods. In this alternative expression for the common goods price index in equation (A.382), expenditure on common goods with below-median expenditure shares only enters through the aggregate share of expenditure on goods with above-median expenditure shares (\( \mu^B_{gt} \)).
In Figure A.7, we compare the resulting measures of the CCG and CUPI to those in our baseline specification that does not distinguish between common goods with above-median versus below-median expenditure shares. We first recompute our price indexes for each product group and year. We next aggregate these price indexes across product groups using expenditure-share weights to obtain measures of the change in the aggregate cost of living. As apparent from the figure, we obtain a similar measure of the change in the aggregate cost of living as in our baseline specification. The differences between our alternative and baseline measures of the CCG and the CUPI are small relative to the difference between all of these price indexes and conventional price indexes that abstract from the entry and exit of goods and demand shocks for each common good.

Figure A.7: Robustness of Four-Quarter Proportional Changes in the Aggregate Cost of Living \(( (P_t - P_{t-1}) / P_{t-1})\) to Measurement Error in Small Expenditure Shares

Note: Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \(( (P_{gt} - P_{gt-1}) / P_{gt-1})\) by their expenditure shares. CUPI-RW and CUPI-GRW are our baseline CES unified price indexes from equation (14) in the paper using our reverse-weighting (RW) or generalized-reverse-weighting (GRW) estimates. CUPI-RW-Restricted and CUPI-GRW-Restricted are robustness checks using our RW and GRW parameter estimates. In these robustness checks, we recompute the change in the common goods price index using the change in the price index for the subset of common goods with above-median expenditure shares and the change in the expenditure share of this subset in all expenditure on common goods (equation (A.382) in this web appendix). Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
References
