Online Technical Appendix to “Measuring Aggregate Price Indexes with Taste Shocks: Theory and Evidence for CES Preferences” (Not for Publication)

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A.1 Introduction

This online appendix contains technical derivations, additional information about the data, and supplementary empirical results.

Section A.2 derives the exact CES price index from Section 2.3 of the paper and compares it to the Sato-Vartia index. Section A.3 characterizes the consumer-valuation bias and shows that a positive taste shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all other goods. Section A.4 derives the elasticity of substitution implied by the Sato-Vartia index under its assumption of time-invariant tastes for each common good. Section A.5 considers our robustness test in which we rule out a pure change in consumer tastes by requiring that a generalized mean of order-$r$ of the consumer taste parameters is constant.

Section A.6 develops the extension to non-homothetic CES preferences from Section 3.1 of the paper. Section A.7 provides further details for the extension to nested CES preferences from Section 3.2 of the paper.
Section A.8 develops the generalization to mixed CES with heterogeneous groups of consumers from Section 3.3 of the paper. Section A.9 shows that our unified approach to the demand system and the unit expenditure function also can be applied to the closely-related logit and mixed logit preferences, as discussed in Section 3.4 of the paper.

Section A.10 shows that our main insight that the demand system can be inverted to construct a money-meteric price index with time-varying taste shocks is not specific to CES, but also holds for the flexible functional forms of translog and almost ideal demand system (AIDS) preferences, as discussed in Section 3.5 of the paper. We show that the Törnqvist index for translog preferences exhibits a similar consumer valuation bias as the Sato-Vartia index for CES preferences.

Section A.11 provides further details on the Feenstra (1994) estimator used to estimate the elasticity of substitution. Section A.12 develops our joint specification test of the assumption of CES demand and our normalization that tastes have a constant geometric mean across common goods. Section A.13 contains the data appendix, which reports summary statistics for each of the product groups in our data. Section A.14 reports additional empirical results discussed in Sections 5.2, 5.5 and 7 of the paper.

### A.2 Derivation of Exact CES Price Index

In this section of the online appendix, we derive the expression for the exact CES price index in terms of taste-adjusted prices in equation (10) in Section 2.4 of the paper. From the common goods expenditure share in equation (5) in the paper, we can express the change in the common goods price index as:

\[
\frac{P_t^*}{P_{t-1}^*} = \frac{(p_{kt}/\varphi_{kt}) / (p_{kt-1}/\varphi_{kt-1})}{(s_{kt}^* / s_{kt-1}^*)^{1/\sigma}}. \tag{A.1}
\]

Taking logs of both sides, and rearranging, we have:

\[
\ln \left( \frac{P_t^*}{P_{t-1}^*} \right) - \ln \left( \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \right) = \frac{1}{\sigma - 1}. \tag{A.2}
\]

If we now multiply both sides of this equation by \(s_{kt}^* - s_{kt-1}^*\) and sum across all common goods, we obtain:

\[
\sum_{k \in \Omega_t^*} (s_{kt}^* - s_{kt-1}^*) \ln \left( \frac{P_t^*}{P_{t-1}^*} \right) - \ln \left( \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \right) = 0 \tag{A.3}
\]

or

\[
\sum_{k \in \Omega_t^*} \left( \frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*} \right) \ln \left( \frac{P_t^*}{P_{t-1}^*} \right) = \sum_{k \in \Omega_t^*} \left( \frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*} \right) \ln \left( \frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}} \right). \tag{A.4}
\]
Re-writing this expression, we obtain the log change in our exact CES price index in equation (10) in the paper:

\[
\ln \left( \frac{P_t^*}{P_{t-1}^*} \right) = \left[ \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) \right] - \left[ \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \right], \quad (A.5)
\]

\[
\omega_{kt}^* \equiv \frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}, \quad \sum_{k \in \Omega_t^*} \omega_{kt}^* = 1. \quad (A.6)
\]

We now show that the exact CES price index in equation (A.5) is equal to the unified price index in equation (8) in the paper. Using our inversion of the demand system from equation (12) in the paper and our normalization that the taste shocks are mean zero across common goods \(\left( \ln \left( \bar{\phi}_t / \bar{\phi}_{t-1} \right) = 0 \right)\), we can substitute for the taste shocks \(\left( \varphi_{kt} / \varphi_{kt-1} \right)\) in equation (A.5) to obtain:

\[
\ln \left( \frac{P_t^*}{P_{t-1}^*} \right) = \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) + \frac{1}{\sigma - 1} \ln \left( \frac{s_t^*}{s_{t-1}^*} \right) - \frac{1}{\sigma - 1} \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right), \quad (A.7)
\]

where a tilde above a variable denotes a geometric mean across common goods such that \(\tilde{x}_t = \left( \prod_{k \in \Omega_t^*} x_{kt} \right)^{1/N_t^*}\) for the variable \(x_{kt}\). Using the definition of the Sato-Vartia weights \(\left( \omega_{kt}^* \right)\) from equation (A.6) above, the final term in equation (A.7) is equal to zero, so that equation (A.7) reduces to the CES common goods unified price index:

\[
\ln \left( \frac{P_t^*}{P_{t-1}^*} \right) = \ln \Phi_t^{CCG} = \ln \left( \frac{\hat{p}_t}{\hat{p}_{t-1}} \right) + \frac{1}{\sigma - 1} \ln \left( \frac{s_t^*}{s_{t-1}^*} \right). \quad (A.8)
\]

Finally, using equations (A.5) and (A.8) together with the definition of the Sato-Vartia index (the special case of equation (10) in the paper in which \(\varphi_{kt} / \varphi_{kt-1} = 1\) for all \(k \in \Omega_t^*\)), we can express our common goods exact CES price index as equal to the Sato-Vartia index minus an additional term that we refer to as the consumer valuation bias, as in equation (13) in the paper:

\[
\ln \left( \frac{P_t^*}{P_{t-1}^*} \right) = \ln \Phi_t^{CCG} = \ln \Phi_t^{SV} - \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right). \quad (A.9)
\]

### A.3 Consumer-Valuation Bias

As discussed in Section 2.5 of the paper, the Sato-Vartia index is only unbiased if the taste shocks \(\left( \varphi_{kt} / \varphi_{kt-1} \right)\) are orthogonal to the expenditure-share weights \(\left( \omega_{kt}^* \right)\); it is upward-biased if they are positively correlated with these weights; and it is downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the taste shocks \(\left( \ln \left( \varphi_{kt} / \varphi_{kt-1} \right) \right)\) and the expenditure-share weights \(\left( \omega_{kt}^* \right)\) is possible, depending on the underlying correlation between taste and price shocks. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are functions of the taste shocks. In this section of the online appendix, we show that a positive
taste shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all other goods.

Note that the Sato-Vartia common goods expenditure share weights \((\omega_{kt}^*)\) can be written as:

\[
\omega_{kt}^* = \frac{\xi_{kt}^*}{\sum_{l \in \Omega_t} \xi_{lt}^*},
\]

where

\[
\xi_{kt}^* = \frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*},
\]

Note also that tastes, prices and expenditure shares at time \(t - 1\) \((\varphi_{kt-1}, p_{kt-1}, s_{kt-1})\) are pre-determined at time \(t\). To evaluate the impact of a positive taste shock for good \(k\) \((\varphi_{kt} / \varphi_{kt-1} > 1)\), we consider the effect of an increase in tastes at time \(t\) for a good \((\varphi_{kt})\) given its tastes at time \(t - 1\) \((\varphi_{kt-1})\). Using the definitions (A.10)-(A.12), we have following two results:

\[
\frac{d\omega_{kt}^*}{ds_{kt}^*} \frac{\xi_{kt}^*}{\omega_{kt}^*} = (1 - \omega_{kt}^*) > 0,
\]

\[
\frac{d\omega_{lt}^*}{d\xi_{kt}^*} \frac{\xi_{kt}^*}{\omega_{lt}^*} = -\omega_{kt}^* < 0, \quad \ell \neq k,
\]

where we have used the fact that percentage changes are larger in absolute magnitude than logarithmic changes and hence:

\[
\frac{s_{kt-1}^* - s_{kt}^*}{s_{kt}^*} > \ln \left( \frac{s_{kt-1}^*}{s_{kt}^*} \right) > 0 \quad \text{for} \quad s_{kt-1}^* > s_{kt}^*,
\]

\[
\frac{s_{kt-1}^* - s_{kt}^*}{s_{kt}^*} < \ln \left( \frac{s_{kt-1}^*}{s_{kt}^*} \right) < 0 \quad \text{for} \quad s_{kt-1}^* < s_{kt}^*.
\]

We also have the following third result:

\[
\frac{ds_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} = (\sigma - 1) (1 - s_{kt}^*) > 0, \quad \frac{ds_{lt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{lt}^*} = -(\sigma - 1) s_{kt}^* < 0.
\]

Together (A.13), (A.14), (A.15) and (A.16) imply that a positive taste shock for good \(k\) increases the Sato-Vartia expenditure share weight for that good \((\omega_{kt}^*)\):

\[
\frac{d\omega_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{\omega_{kt}^*} = \left( \frac{d\omega_{kt}^*}{d\xi_{kt}^*} \frac{\xi_{kt}^*}{\omega_{kt}^*} \right) \left( \frac{d\xi_{kt}^*}{ds_{kt}^*} \frac{s_{kt}^*}{\xi_{kt}^*} \right) \left( \frac{ds_{kt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{kt}^*} \right) > 0,
\]

and reduces the Sato-Vartia expenditure share weight for all other goods \(\ell \neq k\) \((\omega_{lt}^*)\):

\[
\frac{d\omega_{lt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{\omega_{lt}^*} = \left( \frac{d\omega_{lt}^*}{d\xi_{lt}^*} \frac{\xi_{lt}^*}{\omega_{lt}^*} \right) \left( \frac{d\xi_{lt}^*}{ds_{lt}^*} \frac{s_{lt}^*}{\xi_{lt}^*} \right) \left( \frac{ds_{lt}^*}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{lt}^*} \right) < 0.
\]
A.4 Elasticity of Substitution Implied by the Sato-Vartia Index

In this section of the online appendix, we show that the Sato-Vartia index’s assumption of time-invariant tastes for each common good implies that the elasticity of substitution can be recovered from the observed data on prices and expenditure shares with no estimation. We first show that under this assumption there exists an infinite number of approaches to recovering the elasticity of substitution, each of which uses different weights for each common good. If tastes for all common goods are indeed constant (including no changes in tastes, quality, measurement error or specification error), all of these approaches will recover the same elasticity of substitution. We next show that if consumer tastes for some common good change over time, but a researcher falsely assumes time-invariant tastes for all common goods, these alternative approaches will return different values for the elasticity of substitution, depending on which weights are used.

Under the Sato-Vartia assumption of constant tastes for each common good ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_t^s$ and $t$), the common goods expenditure share is:

$$s_{kt}^* = \frac{(p_{kt}/\varphi_k)^{1-\sigma}}{\sum_{\ell \in \Omega_t^s} (p_{\ell t}/\varphi_\ell)^{1-\sigma}}.$$  \hspace{1cm} (A.19)

Dividing the expenditure share by its geometric mean across common goods, we get:

$$\frac{s_{kt}^*}{\bar{s}_t^*} = \left(\frac{p_{kt}/\varphi_k}{\bar{p}_t/\bar{\varphi}}\right)^{1-\sigma},$$  \hspace{1cm} (A.20)

where a tilde above a variable denotes a geometric mean across common goods. Taking logarithms in (A.20), we obtain:

$$\ln \left(\frac{s_{kt}^*}{\bar{s}_t^*}\right) = (1-\sigma) \ln \left(\frac{p_{kt}}{\bar{p}_t}\right) + (\sigma - 1) \ln \left(\frac{\varphi_k}{\bar{\varphi}}\right).$$  \hspace{1cm} (A.21)

Taking differences in (A.21), we have:

$$\Delta \ln \left(\frac{s_{kt}^*}{\bar{s}_t^*}\right) = (1-\sigma) \Delta \ln \left(\frac{p_{kt}}{\bar{p}_t}\right).$$  \hspace{1cm} (A.22)

Multiplying both sides of (A.22) by $\omega_{kt}^*$ and summing across common goods, we get:

$$\sum_{k \in \Omega_t^s} \omega_{kt}^* \Delta \ln \left(\frac{s_{kt}^*}{\bar{s}_t^*}\right) = (1-\sigma) \sum_{k \in \Omega_t^s} \omega_{kt}^* \Delta \ln \left(\frac{p_{kt}}{\bar{p}_t}\right),$$  \hspace{1cm} (A.23)

where $\omega_{kt}^*$ are the Sato-Vartia weights:

$$\omega_{kt}^* = \frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}.$$  \hspace{1cm} (A.24)

Equation (A.23) yields the following closed-form solution for $\sigma^V$:

$$\sigma^V = 1 + \frac{\sum_{k \in \Omega_t^s} \omega_{kt}^* \ln \left(\frac{s_{kt}^*}{\bar{s}_{kt-1}^*}\right) - \ln \left(\frac{s_{kt}^*}{\bar{s}_{kt-1}^*}\right)}{\sum_{k \in \Omega_t^s} \omega_{kt}^* \ln \left(\frac{p_{kt}}{\bar{p}_{kt-1}}\right) - \ln \left(\frac{p_{kt}}{\bar{p}_{kt-1}}\right)},$$  \hspace{1cm} (A.25)

which establishes that the elasticity of substitution ($\sigma$) is identified from observed changes in prices and expenditure shares with no estimation under the Sato-Vartia index’s assumption of time-invariant tastes for
all common goods \((q_{kt} = q_{kt-1} = \tilde{q}_k\) for all \(k \in \Omega_i^t\) and \(t\)). Note that we could have instead multiplied both sides of (A.22) by any positive finite share that sums to one across common goods:

\[
\sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \Delta \ln \left( \frac{s_{kt}^*}{s_t^*} \right) = (1 - \sigma) \sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \Delta \ln \left( \frac{p_{kt}}{\tilde{p}_t} \right), \quad \sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* = 1,
\]  

(A.25)

and obtained another expression for \(\sigma\) given observed prices and expenditure shares:

\[
\sigma^{ALT} = 1 + \frac{\sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \left[ \ln \left( \frac{s_{kt}^*}{s_{kt-1}^*} \right) - \ln \left( \frac{s_{kt-1}^*}{s_{kt-1}^*} \right) \right]}{\sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \left[ \ln \left( \frac{p_t}{\tilde{p}_{t-1}} \right) - \ln \left( \frac{p_{t-1}}{\tilde{p}_{t-1}} \right) \right]}.
\]  

(A.26)

Therefore, there exists a continuum of approaches to measuring \(\sigma\), each of which weights prices and expenditure shares with different non-negative weights that sum to one. Under the Sato-Vartia index’s assumption of constant tastes for each good \((q_{kt} = q_{kt-1} = \tilde{q}_k\) for all \(k \in \Omega_i^t\) and \(t\)), each of these alternative approaches returns the same value for \(\sigma\), since all are derived from equation (A.22).

Now suppose that some common good experiences a taste shock \((q_{kt} \neq q_{kt-1}\) for some \(k \in \Omega_i^t\) and \(t\)), but a researcher falsely assumes that tastes for all common goods are constant. Dividing the common goods expenditure share by its geometric mean, we get:

\[
\frac{s_{kt}^*}{s_t^*} = \left( \frac{p_{kt}}{q_{kt}} \right)^{(1-\sigma)},
\]  

(A.27)

where a tilde above a variable again denotes a geometric mean across common goods.

Taking logarithms in (A.27) and taking differences, we obtain:

\[
\Delta \ln \left( \frac{s_{kt}^*}{s_t^*} \right) = (1 - \sigma) \Delta \ln \left( \frac{p_{kt}}{\tilde{p}_t} \right) + (\sigma - 1) \Delta \ln q_{kt},
\]  

(A.28)

where we have used our normalization that the geometric mean of consumer tastes is constant such that \(\ln (\tilde{q}_t / \tilde{q}_{t-1}) = 0\). Multiplying both sides of (A.28) by \(\omega_{kt}^*\) and summing across common goods, we get:

\[
\sum_{k \in \Omega_i^t} \omega_{kt}^* \Delta \ln \left( \frac{s_{kt}^*}{s_t^*} \right) = (1 - \sigma) \sum_{k \in \Omega_i^t} \omega_{kt}^* \Delta \ln \left( \frac{p_{kt}}{\tilde{p}_t} \right) + (\sigma - 1) \sum_{k \in \Omega_i^t} \omega_{kt}^* \Delta \ln q_{kt}.
\]  

(A.29)

Rearranging (A.29), we obtain:

\[
\sigma_{\omega, q} = 1 + \frac{\sum_{k \in \Omega_i^t} \omega_{kt}^* \left[ \ln \left( \frac{s_{kt-1}^*}{s_{kt-1}^*} \right) - \ln \left( \frac{s_{kt-1}^*}{s_{kt-1}^*} \right) \right]}{\sum_{k \in \Omega_i^t} \omega_{kt}^* \left[ \ln \left( \frac{p_t}{\tilde{p}_{t-1}} \right) - \ln \left( \frac{p_{t-1}}{\tilde{p}_{t-1}} \right) + \ln \left( \frac{p_{kt}}{q_{kt}} \right) \right]},
\]  

(A.30)

Note that we could have instead multiplied both sides of (A.28) by any positive finite share that sums to one across common goods:

\[
\sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \Delta \ln \left( \frac{s_{kt}^*}{s_t^*} \right) = (1 - \sigma) \sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \Delta \ln \left( \frac{p_{kt}}{\tilde{p}_t} \right) + (\sigma - 1) \sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* \Delta \ln q_{kt},
\]  

(A.31)

where

\[
\sum_{k \in \Omega_i^t} \tilde{\xi}_{kt}^* = 1.
\]
and obtained another expression for the elasticity of substitution ($\sigma$):

$$\sigma_{\varphi_k^*} = 1 + \frac{\sum_{k \in \Omega_i^*} \xi_{kt}^* \left[ \ln \left( \frac{s_{kt}^*}{\tilde{s}_{kt}^*} \right) - \ln \left( \frac{s_{kt}^*}{\tilde{s}_{kt-1}^*} \right) \right]}{\sum_{k \in \Omega_i^*} \xi_{kt}^* \left[ \ln \left( \frac{p_k}{p_{kt-1}} \right) - \ln \left( \frac{p_k}{p_{kt-1}} \right) + \ln \left( \frac{\varphi_k}{\varphi_{kt-1}} \right) \right]}, \quad (A.32)$$

Note that equations (A.30) and (A.32) both return the same value for $\sigma$, because both are derived from equation (A.28). However, suppose that a researcher falsely assumes that tastes for all common goods are constant ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_i^*$ and $t$) and uses equations (A.24) and (A.26) to measure $\sigma$ (instead of equations (A.30) and (A.32)). Under this false assumption, equations (A.24) and (A.26) will return different values for $\sigma$, because in general:

$$\sum_{k \in \Omega_i^*} \omega_{kt}^* \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \neq \sum_{k \in \Omega_i^*} \xi_{kt}^* \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \quad \text{for} \quad \omega_{kt}^* \neq \xi_{kt}^*.$$

Therefore, when tastes for some common good change over time ($\varphi_{kt} \neq \varphi_{kt-1}$ for some $k \in \Omega_i^*$ and $t$), but a researcher falsely assumes that tastes for all common goods are constant ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_i^*$ and $t$), the use of different weights for prices and expenditure shares ($\omega_{kt}^*$ versus $\xi_{kt}^*$) in general returns different elasticities of substitution ($\sigma^{SV} \neq \sigma^{ALT}$).

### A.5 Robustness to Alternative Normalizations for Consumer Tastes

In this section of the online appendix, we develop our robustness test in which we rule out a pure change in consumer tastes by requiring that a generalized mean of order-\(r\) of the consumer taste parameters is constant. From equations (3) and (5) in the paper for expenditure shares in period $t$, we have:

$$\varphi_{kt} P_t = p_{kt} \left( s_{kt}^* \right)^{\frac{1}{r+1}} \lambda_i^{\frac{1}{r+1}}. \quad (A.33)$$

Taking the mean of order $r$ of equation (A.33) across common goods, we obtain:

$$\left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} \varphi_{kt} \right]^\frac{1}{r} P_t = \left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} p_{kt} \left( s_{kt}^* \right)^{\frac{1}{r+1}} \right]^\frac{1}{r} \lambda_i^{\frac{1}{r+1}}. \quad (A.34)$$

Similarly, for period $t-1$, we have:

$$\left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} \varphi_{kt-1} \right]^\frac{1}{r} P_{t-1} = \left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} p_{kt-1} \left( s_{kt-1}^* \right)^{\frac{1}{r+1}} \right]^\frac{1}{r} \lambda_i^{\frac{1}{r+1}}. \quad (A.35)$$

Using the normalization that a generalized mean of order-\(r\) of the consumer taste parameters is constant, we have:

$$\left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} \varphi_{kt} \right]^\frac{1}{r} = \left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} \varphi_{kt-1} \right]^\frac{1}{r}. \quad (A.36)$$

Taking the ratio of equations (A.34) and (A.35), and using our normalization (A.36), we obtain:

$$\frac{P_t}{P_{t-1}} = \left[ \frac{1}{N_i^t} \sum_{k \in \Omega_i^*} p_{kt} \left( s_{kt}^* \right)^{\frac{1}{r+1}} \right]^\frac{1}{r} \left( \frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{r+1}},$$

which corresponds to equation (16) in the paper.
A.6 Non-Homothetic CES

In this section of the online appendix, we derive the generalization of our common goods unified price index (CUPI) for non-homothetic CES preferences from Section 3.1 of the paper.

A.6.1 Preferences

In particular, we generalize our analysis to the non-separable class of CES functions in Sato (1975), which satisfy implicit additivity in Hanoch (1975), as recently used in the macroeconomics literature in Comin, Lashkari and Mestieri (2015). We suppose that we observe data on households indexed by \( h \in \{1, \ldots, H\} \) that differ in income and total expenditure \( (E^h_t) \). The non-homothetic CES consumption index for household \( h \) \( (C^h_t) \) is defined by the following implicit function:

\[
\sum_{k \in \Omega_t} \left( \frac{\varphi^h_{kt} c^h_{kt}}{(C^h_t)^{\frac{\varepsilon_k}{1 - \varepsilon}} / (1 - \varepsilon)} \right)^{\frac{\varepsilon_k}{1 - \varepsilon}} = 1, \tag{A.38}
\]

where \( c^h_{kt} \) denotes household \( h \)'s consumption of good \( k \) at time \( t \); \( \varphi^h_{kt} \) is household \( h \)'s taste parameter for good \( k \) at time \( t \); \( \sigma \) is the constant elasticity of substitution between varieties; \( \varepsilon_k \) is the constant elasticity of consumption of good \( k \) with respect to the consumption index \( (C^h_t) \) that allows preferences to be non-homothetic. Assuming that goods are substitutes \( (\sigma > 1) \), we require \( \varepsilon_k < \sigma \) for the consumption index \( (A.38) \) to be globally monotonically increasing and quasi-concave, and hence to correspond to a well-defined utility function. Our baseline homothetic CES specification corresponds to the special case of equation\( (A.38) \) in which \( \varepsilon_k = 1 \) for all \( k \in \Omega_t \).

A.6.2 Expenditure Minimization

The Lagrangian for the utility maximization problem for household \( h \) is:

\[
\mathcal{L} = C^h_t + \rho^h \left( 1 - \sum_{k \in \Omega_t} \left( \frac{\varphi^h_{kt} c^h_{kt}}{(C^h_t)^{\frac{\varepsilon_k}{1 - \varepsilon}} / (1 - \varepsilon)} \right)^{\frac{\varepsilon_k}{1 - \varepsilon}} \right) + \lambda^h \left( E^h_t - \sum_{k \in \Omega_t} p^h_{kt} c^h_{kt} \right), \tag{A.39}
\]

where we assume for simplicity that all households face the same prices for a given good \( (p^h_{kt}) \). The first-order condition with respect to consumption of each good \( (c^h_{kt}) \) can be written as:

\[
p^h_{kt} c^h_{kt} = \frac{\rho^h}{\lambda^h} \left( 1 - \frac{\varepsilon_k}{\sigma} \right) \kappa^h_{kt}, \tag{A.40}
\]

where we define \( \kappa^h_{kt} \) as:

\[
\kappa^h_{kt} \equiv \left( \frac{\varphi^h_{kt} c^h_{kt}}{(C^h_t)^{\frac{\varepsilon_k}{1 - \varepsilon}} / (1 - \varepsilon)} \right)^{\frac{\varepsilon_k}{1 - \varepsilon}}. \tag{A.41}
\]

From the first-order condition \( (A.40) \) and utility function \( (A.38) \), total expenditure by household \( h \) is given by:

\[
E^h_t = \sum_{k \in \Omega_t} p^h_{kt} c^h_{kt} = \frac{1 - \sigma}{\sigma} \frac{\rho^h}{\lambda^h}. \tag{A.42}
\]
Using this result in the first-order condition (A.40), we find that $\kappa_{kt}^h$ equals the share of good $k$ in the expenditure of household $h$ at time $t$:

$$s_{kt}^h = \frac{p_{kt}c_{kt}^h}{E_t^h} = \kappa_{kt}^h = \left( \frac{q_{kt}^h c_{kt}}{C_t^h} \right)^{\frac{1}{\sigma}}.$$

Re-arranging this relationship, we obtain the demand function for good $k$:

$$c_{kt}^h = (q_{kt}^h)^{\sigma-1} \left( \frac{p_{kt}}{E_t^h} \right)^{-\sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} = \left( \frac{p_{kt}}{P_t^h} \right)^{-\sigma} \left( C_t^h \right)^{\epsilon_k},$$

which highlights that $\epsilon_k$ controls the elasticity of demand for good $k$ with respect to the real consumption index ($C_t^h$). Using this demand function (A.44), the expenditure share (A.43) can be re-written as:

$$s_{kt}^h = \left( \frac{q_{kt}^h}{P_t^h} \right)^{\sigma-1} \left( \frac{p_{kt}}{P_t^h} \right)^{1-\sigma} \left( C_t^h \right)^{\epsilon_k - 1}.$$  

Additionally, using the CES demand function (A.44) in utility in equation (A.38), we can solve for the expenditure function for household $h$:

$$E_t^h = P_t^h C_t^h = \left[ \sum_{k \in \Omega_h} \left( \frac{p_{kt}}{q_{kt}^h} \right)^{1-\sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} \right]^{\frac{1}{1-\sigma}}.$$  

Therefore the price index for household $h$ is given by:

$$p_t^h = \frac{1}{C_t^h} \left[ \sum_{k \in \Omega_h} \left( \frac{p_{kt}}{q_{kt}^h} \right)^{1-\sigma} \left( C_t^h \right)^{\epsilon_k - \sigma} \right]^{\frac{1}{1-\sigma}},$$

or equivalently:

$$p_t^h = \left[ \sum_{k \in \Omega_h} \left( \frac{p_{kt}}{q_{kt}^h} \right)^{1-\sigma} \left( \frac{E_t^h}{P_t^h} \right)^{\epsilon_k - 1} \right]^{\frac{1}{1-\sigma}}.$$  

Combining equations (A.45) and (A.48), the share of good $k$ in expenditure for household $h$ at time $t$ can be written as:

$$s_{kt}^h = \frac{(p_{kt} / q_{kt}^h)^{1-\sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1}}{\sum_{k \in \Omega_h} \left( p_{kt} / q_{kt}^h \right)^{1-\sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1}} = \frac{(p_{kt} / q_{kt}^h)^{1-\sigma} \left( E_t^h / P_t^h \right)^{\epsilon_k - 1}}{(P_t^h)^{1-\sigma}}.$$  

Equations (A.48) and (A.49) correspond to equations (18) and (19) in Section 3.1 of the paper respectively.

### A.6.3 Non-homothetic CES Unified Price Index

We now show that our unified approach to the demand system and the price index can be extended to this case of non-homothetic CES preferences. As for the homothetic CES specification in Section 2 of the paper, the price index (A.48) depends on taste-adjusted prices ($p_{kt} / q_{kt}^h$) rather than observed prices ($p_{kt}$). An additional challenge relative to the homothetic CES case is that the overall CES price index ($P_t^h$) enters the numerator of the expenditure share in equation (A.49). To overcome this additional challenge, we work with the share of
each good in overall expenditure \((s_{kt}^h)\) rather than the common goods expenditure share \((s_{kt}^{hs})\) in our earlier notation). In particular, re-arranging the overall expenditure share in equation (A.49) for an individual common good, we have:

\[
P_{ht} = \frac{p_{kt}}{\varphi_{kt}} \left( s_{kt}^h \right)^{\frac{1}{\sigma-1}} \left( E_{ht}^h / P_{ht}^h \right)^{\frac{\sigma-1}{\sigma}}.
\]  

(A.50)

Taking logarithms yields:

\[
\ln P_{ht} = \ln p_{kt} - \ln \varphi_{kt} + \frac{1}{\sigma-1} \ln s_{kt}^h + \left( \frac{\epsilon_k - 1}{1 - \sigma} \right) \ln \left( E_{ht}^h / P_{ht}^h \right).
\]  

(A.51)

Averaging across the common goods, we obtain:

\[
[1 + \theta] \ln P_{ht} = \ln \bar{p}_t + \frac{1}{\sigma-1} \ln \bar{s}_t^h + \theta \ln \left( E_{ht}^h / P_{ht}^h \right),
\]  

(A.52)

where a tilde above a variable denotes an average across common goods such that \(\bar{p}_t = \left( \prod_{k \in \Omega_t} p_{kt} \right)^{1/N_t^*}\); we have used our normalization that the average taste shock across common goods is equal to zero \((\ln \left( \bar{q}_t / \bar{q}_{t-1} \right) = 0)\); the derived parameter \(\theta\) captures the average across the common goods of the elasticity of expenditure with respect to the consumption index \((\epsilon_k)\) relative to the elasticity of substitution \((\sigma)\). Rearranging terms in equation (A.52) and exponentiating, we obtain the following closed-form solution for the overall CES unit expenditure function:

\[
P_{ht} = (\bar{p}_t)^{\frac{1}{1+\theta}} \left( \bar{s}_t^h \right)^{\frac{1}{\sigma-1+1+\theta}} \left( E_{ht}^h / P_{ht}^h \right)^{\frac{\phi}{1+\theta}}.
\]  

(A.53)

Taking ratios between the two time periods, we obtain our generalization of our CES unified price index to the non-homothetic case for each household \(h\):

\[
\frac{P_{ht}}{P_{ht-1}} = \left( \frac{\bar{p}_t}{\bar{p}_{t-1}} \right)^{\frac{1}{1+\theta}} \left( \frac{\bar{s}_t^h}{\bar{s}_{t-1}^h} \right)^{\frac{1}{\sigma-1+1+\theta}} \left( \frac{E_{ht}^h / E_{ht-1}^h}{P_{ht}^h / P_{ht-1}^h} \right)^{\frac{\phi}{1+\theta}},
\]  

(A.54)

which corresponds to equation (21) in the paper. From this expression, the change in the household’s cost of living \((P_{ht}^h / P_{ht-1}^h)\) now depends directly on the change in income (and hence total expenditure) for parameter values for which preferences are non-homothetic \((\epsilon_k \neq 1\) for some \(k\) and hence \(\theta \neq 0)\).

### A.7 Nested CES

In our baseline specification in Section 2 of the paper, we focus for simplicity on a single CES tier of utility. In this section of the online appendix, we generalize our approach to a nested CES demand system with multiple tiers of utility. For simplicity, we illustrate this generalization for two tiers of utility (an upper tier defined across sectors and a lower tier defined across goods within sectors), but as discussed in the paper our analysis goes through for any number of tiers of utility.
A.7.1 Preferences

We assume that the aggregate unit expenditure function is a constant elasticity function of the unit expenditure function for each sector \( g \in \Omega^G \) as follows:

\[
P_t = \left[ \sum_{g \in \Omega^G} \left( \frac{p_{g t}^G}{\varphi_{g t}^G} \right)^{1-\sigma^G} \right]^{\frac{1}{1-\sigma^G}}, \quad \sigma^G > 1,
\]

(A.55)

where \( \sigma^G \) is the elasticity of substitution across sectors; \( p_{g t}^G \) is the unit expenditure function for each sector; \( \varphi_{g t}^G \) is the taste parameter for each sector; we assume for simplicity that the set of sectors is constant over time and denote the number of elements in this set by \( N^G = |\Omega^G| \).

The unit expenditure function for each sector is a constant elasticity function of the consumption of goods \( k \in \Omega^K_{g t} \) within that sector as follows:

\[
p_{g t}^G = \left[ \sum_{k \in \Omega^K_{g t}} \left( \frac{p_{k t}^K}{\varphi_{k t}^K} \right)^{1-\sigma^K_g} \right]^{\frac{1}{1-\sigma^K_g}}, \quad \sigma^K_g > 1,
\]

(A.56)

where \( \sigma^K_g \) is the elasticity of substitution across goods within each sector and can differ across sectors; \( p_{k t}^K \) is the price for each good; \( \varphi_{k t}^K \) is the taste parameter for each good; we allow the set of goods within each sector to change over time and denote the number of elements within this set by \( N^K_{g t} = |\Omega^K_{g t}| \); we require that both elasticities of substitution (\( \sigma^G \) and \( \sigma^K_g \)) are greater than one, but do not otherwise restrict their values relative to one another.

A.7.2 Aggregate Price Index

Applying Shephard’s Lemma to the aggregate unit expenditure function (A.55), the share of aggregate expenditure on each sector \( s_{g t}^G \) is:

\[
s_{g t}^G = \frac{\left( \frac{p_{g t}^G}{\varphi_{g t}^G} \right)^{1-\sigma^G}}{\sum_{m \in \Omega^G} \left( \frac{p_{m t}^G}{\varphi_{m t}^G} \right)^{1-\sigma^G}} = \frac{\left( \frac{p_{g t}^G}{\varphi_{g t}^G} \right)^{1-\sigma^G}}{N^G}.
\]

(A.57)

Rearranging this expenditure share, and taking logarithms, we obtain the following expression for the aggregate unit expenditure function:

\[
\ln P_t = \ln P_{g t}^G - \ln \varphi_{g t}^G + \frac{1}{\sigma^G - 1} \ln s_{g t}^G.
\]

(A.58)

Differencing over time, and averaging across sectors, the change in the aggregate cost of living can be expressed in the form of our exact CES price index:

\[
\Delta \ln P_t = \frac{1}{N^G} \sum_{g \in \Omega^G} \Delta \ln P_{g t}^G + \frac{1}{\sigma^G - 1} \frac{1}{N^G} \sum_{g \in \Omega^G} \Delta \ln s_{g t}^G,
\]

(A.59)

where we normalize the taste parameters for each sector such that they have a constant geometric mean across sectors, which implies:

\[
\frac{1}{N^G} \sum_{g \in \Omega^G} \ln \left( \frac{\varphi_{g t}^G}{\varphi_{g t-1}^G} \right) = 0.
\]

(A.60)
A.7.3 Sectoral Price Index

We now solve for the change in the unit expenditure function for each sector ($\Delta \ln P^G_{gt}$) in equation (A.59) as a function of the characteristics of the goods within that sector. First, we can decompose the change in the sectoral unit expenditure function between a pair of periods $t$ and $t-1$ into a variety correction term for the entry and exit of goods ($\frac{1}{\sigma_g} \ln \left( \frac{\Lambda^G_{gt}}{\Lambda^G_{gt-1}} \right)$) and the change in the price index for common goods ($\frac{P^G_{gt}}{P^G_{gt-1}}$):

$$\frac{P^G_{gt}}{P^G_{gt-1}} = \left( \frac{\Lambda^G_{gt}}{\Lambda^G_{gt-1}} \right)^{\frac{1}{\sigma_g} - 1} \frac{P^G_{gt}}{P^G_{gt-1}}. \quad (A.61)$$

The sectoral unit expenditure function for common goods ($P^G_{gt}$) takes the same form as in equation (A.56) but the summation is only over common goods $k \in \Omega^K_{gt}$:

$$P^G^*_{gt} = \left[ \sum_{k \in \Omega^K_{gt}} \left( \frac{p^K_{kt}}{\varphi^K_{kt}} \right)^{1-\sigma_g} \right]^{\frac{1}{1-\sigma_g}}, \quad (A.62)$$

and the share of each individual common good in all expenditure on common goods ($s^K_{kt}$) is:

$$s^K_{kt} = \frac{(p^K_{kt} / \varphi^K_{kt})^{1-\sigma_g}}{\sum_{\ell \in \Omega^K_{gt}} (p^K_{\ell t} / \varphi^K_{\ell t})^{1-\sigma_g}} = \left( \frac{p^K_{kt} / \varphi^K_{kt}}{P^G^*_{gt}} \right)^{\sigma_g - 1}. \quad (A.63)$$

Rearranging this common goods expenditure share, and taking logarithms, we obtain the following expression for the sectoral common goods unit expenditure function:

$$\ln P^G^*_{gt} = \ln p^K_{kt} - \ln \varphi^K_{kt} + \frac{1}{\sigma_g - 1} \ln s^K_{kt}. \quad (A.64)$$

Differencing over time, and averaging across common goods, the change in the sectoral common goods unit expenditure function also can be expressed in the form of our exact CES price index:

$$\Delta \ln P^G^*_{gt} = \frac{1}{N^K_{gt}} \sum_{k \in \Omega^K_{gt}} \Delta \ln p^K_{kt} + \frac{1}{\sigma_g - 1} \frac{1}{N^K_{gt}} \sum_{k \in \Omega^K_{gt}} \Delta \ln s^K_{kt}, \quad (A.65)$$

where we normalize the taste parameters for each good such that they have a constant geometric mean across common goods within each sector, which implies:

$$\frac{1}{N^K_{gt}} \sum_{k \in \Omega^K_{gt}} \ln \left( \frac{\varphi^K_{kt}}{\varphi^K_{kt-1}} \right) = 0. \quad (A.66)$$

A.7.4 Nested CES Unified Price Index

Our CES unified price index (CUPI) for each tier of utility is defined over the mean of the logs of the prices and expenditure shares for that tier of utility. As the mean is a linear operator, we can apply this operator recursively across the tiers of utility to express the change in the aggregate cost of living in terms of means across both sectors and goods within each sector. In particular, using equations (A.61) and (A.65) for each sector in the aggregate cost of living in equation (A.59), we obtain equation (23) in the paper:
\[
\ln \left( \frac{P_t}{P_{t-1}} \right) = \frac{1}{N_G} \sum_{g \in \Omega_G} \frac{1}{N_K_g} \sum_{k \in \Omega_{G_g}} \ln \left( \frac{p_{kt}^g}{p_{kt-1}^g} \right) + \frac{1}{N_G} \sum_{g \in \Omega_G} \frac{1}{\sigma^{Kg} - 1} \sum_{k \in \Omega_{G_g}} \ln \left( \frac{s_{gt}^{Kg}}{s_{gt-1}^{Kg}} \right) 
\]

This expression decomposes the change in the aggregate cost of living into four terms: (i) the average log change in prices across sectors and common goods within each sector; (ii) the average log change in common goods expenditure shares across both sectors and common goods within each sector; (iii) the average variety correction across sectors for the entry and exit of goods; and (iv) the average log change in expenditure shares across sectors.

Although, for simplicity, we focus on two tiers of utility here, this procedure can be extended for any number of tiers of utility, from the highest to the lowest. In general, we can estimate the elasticity of substitution recursively for each tier of utility. However, conventional measures of the overall cost of living often aggregate categories using expenditure-share weights. Therefore, we assume that the upper tier of utility across sectors is Cobb-Douglas (\(\sigma^G = 1\)), and estimate the elasticity of substitution across barcodes within sectors (\(\sigma^{Kg}_g\)), separately for each sector.

A.8 Mixed CES

In this section of the online appendix, we show that our results also generalize to a mixed CES specification, in which there are multiple groups of heterogeneous consumers indexed by \(h \in \{1, \ldots, H\}\). For simplicity, we return to the case of a single tier of utility, although this mixed CES generalization can be combined with a nesting structure. In the non-homothetic specification in Section A.6 of this appendix, the only source of heterogeneity in expenditure shares across consumers is differences in income. In contrast, in this mixed CES specification, we allow both the elasticity of substitution (\(\sigma^h\)) and the taste parameter for each good (\(\phi^h_{kt}\)) to vary across the heterogeneous groups of consumers.

A.8.1 Preferences and Expenditure Shares

In particular, the unit expenditure function (\(P^h_t\)) and expenditure share (\(s^h_{kt}\)) for a household from group \(h\) are given by:

\[
P^h_t = \left[ \sum_{k \in \Omega_h} \left( \frac{p_{kt}}{\phi^h_{kt}} \right)^{1-\sigma^h} \right]^{\frac{1}{1-\sigma^h}},
\]

\[
s^h_{kt} = \frac{\left( p_{kt} / \phi^h_{kt} \right)^{1-\sigma^h}}{\sum_{\ell \in \Omega_h} \left( p_{\ell t} / \phi^h_{\ell t} \right)^{1-\sigma^h}} = \frac{\left( p_{kt} / \phi^h_{kt} \right)^{1-\sigma^h}}{\left( P^h_t \right)^{1-\sigma^h}},
\]

where \(s^h_{kt}\) is a share of product \(k\) in the expenditure of group \(h\) at time \(t\); we assume for simplicity that all groups face the same prices (\(p_{kt}\)); we also assume that the set of products available (\(\Omega_t\)) is the same for all groups; but we allow for the possibility that some groups do not consume some products, which we interpret as corresponding to the limiting case in which the taste parameter converges to zero for that group and
product (\(\lim \varphi_{kt}^h \to 0\) for some \(k\) and \(h\)); these groups of consumers could in principle differ by income and/or other demographic characteristics.

A.8.2 Properties of Mixed CES

The presence of heterogeneity across groups relaxes the independence of irrelevant alternatives (IIA) assumption of CES, because the differences in substitution and taste parameters across groups imply that the relative expenditure shares of two goods in two different markets depend on the relative size of the groups in those markets. In particular, the expenditure share of product \(k\) at time \(t\) can be written as:

\[
s_{kt} = \frac{x_{kt}}{x_t} = \frac{\sum_{h=1}^{H} X_{ht}^h}{x_t} = \frac{\sum_{h=1}^{H} X_{ht}^h s_{kt}^h}{x_t} = \frac{\sum_{h=1}^{H} f_t^h s_{kt}^h},
\]

where \(x_{ht}^h\) is expenditure by group \(h\) on product \(k\) at time \(t\); \(x_{kt}\) is expenditure on product \(k\) at time \(t\); \(x_t^h\) is overall expenditure by group \(h\) at time \(t\); \(x_t\) is total expenditure at time \(t\); \(s_{kt}\) is the share of product \(k\) in overall expenditure at time \(t\); \(s_{kt}^h\) is the share of product \(k\) in group \(h\)'s expenditure at time \(t\); and \(f_t^h\) is the share of group \(h\) in overall expenditure at time \(t\). From equation (A.70), the expenditure shares of each product \(k\) \((s_{kt})\) depend not only on their expenditure shares for each group \(h\) \((s_{kt}^h)\), but also on the relative importance of the different groups \((f_t^h)\) in total expenditure, because of the different preferences of these groups.

Similarly, this heterogeneity across groups relaxes the symmetric cross-substitution properties of CES, because the elasticity of expenditure on one variety with respect to a change in the price of another variety in two different markets also depends on group composition. To demonstrate this role for group composition, we begin by writing total expenditure on product \(k\) as the sum across groups \(h\) of their expenditure on that product:

\[
x_{kt} = \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_{t}^h \left( \frac{p_t^h}{p_t} \right)^{\sigma^h-1}.
\]

Differentiating expenditure on product \(k\) with respect to the price of another product \(\ell\), we obtain:

\[
\frac{\partial x_{kt}}{\partial p_{t\ell}} = \sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_{t}^h \left( \frac{p_t^h}{p_t} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) \frac{\partial P_t^h}{\partial p_{t\ell}} \frac{1}{P_t^h}.
\]

Rearranging this equation, we obtain the following elasticity of expenditure on product \(k\) with respect to the price of another product \(\ell\):

\[
\frac{\partial x_{kt} p_{t\ell}}{\partial p_{t\ell} x_{kt}} = \frac{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_{t}^h \left( \frac{p_t^h}{p_t} \right)^{\sigma^h-1} \left( \sigma^h - 1 \right) \frac{\partial P_t^h}{\partial p_{t\ell}} \frac{p_{t\ell}}{P_t^h}}{\sum_{h=1}^{H} \left( \frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma^h} x_{t}^h \left( \frac{p_t^h}{p_t} \right)^{\sigma^h-1}}.
\]
which can be re-written as follows:

\[
\frac{\partial x_{kt} p_{lt}}{\partial p_{lt} x_{kt}} = \sum_{h=1}^{H} \frac{\left(p_{kt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}}{\sum_{h=1}^{H} \left(p_{kt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}} \left(\sigma^h - 1\right) f_t^h \frac{\partial p_{lt}^{h}}{\partial p_{lt}^{h}},
\]

\[
= \sum_{h=1}^{H} \frac{\left(p_{kt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}}{\sum_{h=1}^{H} \left(p_{kt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}} \left(\sigma^h - 1\right) s_{kt}^{h},
\]

\[
= \sum_{h=1}^{H} \sum_{\ell \in \Omega_h} \frac{\left(p_{lt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}}{\sum_{h=1}^{H} \left(p_{kt} / q_{kh}^{h}\right)^{1-\sigma^h} x_{kt}^{h} (P_{lt}^{h})^{\sigma^h-1}} \left(\sigma^h - 1\right) s_{kt}^{h},
\]

\[
= \sum_{h=1}^{H} \frac{1}{s_{kt}^{h}} \left(\sigma^h - 1\right) s_{kt}^{h},
\]

Rearranging the final line, we obtain equation (26) in the paper:

\[
\frac{\partial x_{kt} p_{lt}^{h}}{\partial p_{lt}^{h} x_{kt}} = \frac{1}{s_{kt}^{h}} \sum_{h=1}^{H} f_t^h \left(\sigma^h - 1\right) s_{kt}^{h}. \tag{A.71}
\]

### A.8.3 Entry and Exit

We now show that our results for entry and exit and the change in the cost of living hold for each group of consumers separately. Partitioning goods into entering, exiting and common goods, the change in the overall cost of living for group \( h \) between periods \( t - 1 \) and \( t \) can be expressed in terms of the change in the share of expenditure on common goods \( (\lambda_t^{h}/\lambda_{t-1}^{h}) \) and the change in the cost of living for these common goods \( (P_{lt}^{h^{*}}/P_{lt-1}^{h^{*}}) \):

\[
\Phi_t^{h} = \left(\frac{P_{lt}^{h}}{P_{lt-1}^{h}}\right) = \left(\frac{\lambda_t^{h} / \lambda_{t-1}^{h}}{\lambda_t^{h} / \lambda_{t-1}^{h}}\right)^{1-\sigma^h} \frac{P_{lt}^{h^{*}}}{P_{lt-1}^{h^{*}}}, \tag{A.72}
\]

where \( (\lambda_t^{h}, \lambda_{t-1}^{h}) \) take the same form as in equation (4) in the paper but are defined for each group separately. We again use an asterisk to denote the value of a variable for the common set of goods, such that \( P_{lt}^{h^{*}} \) and \( P_{lt-1}^{h^{*}} \) are the unit expenditure functions for common goods:

\[
P_{lt}^{h^{*}} \equiv \left[ \sum_{h \in \Omega_t} \left(\frac{p_{kt} / q_{kh}^{h}}{p_{kt} / q_{kh}^{h}}\right)^{1-\sigma^h} \right]^{1/(1-\sigma^h)}. \tag{A.73}
\]
In addition to the aggregate shares of common goods in total expenditure \((\lambda_{t}^{h}, \lambda_{t-1}^{h})\), we can also define the share of an individual common good \(k \in \Omega_{t}^{h}\) in expenditure on all common goods \((s_{kt}^{h*})\) for household \(h\):

\[
s_{kt}^{h*} = \frac{(p_{kt} / \varphi_{kt}^{h*})^{1-\sigma^{h}}}{\sum_{t \in \Omega_{t}^{h}} (p_{kt} / \varphi_{kt}^{h*})^{1-\sigma^{h}}} = \frac{(p_{kt} / \varphi_{kt}^{h})^{1-\sigma^{h}}}{(P_{t}^{h})^{1-\sigma^{h}}}, \quad k \in \Omega_{t}^{h}.
\]  

(A.74)

### A.8.4 Exact Price indexes

All our results for the exact CES price index in Section 2.4 of the paper also hold for each group of consumers separately. Using equations (A.73) and (A.74), the log change in group \(h\)'s cost of living for common goods \((\ln \Phi_{t}^{h*})\) between periods \(t - 1\) and \(t\) can be expressed in the following form:

\[
\ln \Phi_{t}^{h*} = \sum_{k \in \Omega_{t}^{h}} \omega_{kt}^{h*} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega_{t}^{h}} \omega_{kt}^{h*} \ln \left( \frac{\varphi_{kt}^{h}}{\varphi_{kt-1}^{h}} \right),
\]  

(A.75)

where the weights \(\omega_{kt}^{h*}\) are the logarithmic mean of common goods expenditure shares \((s_{kt}^{h*})\) in periods \(t\) and \(t - 1\) and sum to one for each group,\n
\[
\omega_{kt}^{h*} = \frac{s_{kt}^{h*} - s_{kt-1}^{h*}}{\ln s_{kt}^{h*} - \ln s_{kt-1}^{h*}},
\]  

(A.76)

where the derivation is the same as that for a single group in Section A.2 of this online appendix.

We use the invertibility of the CES demand system for each group to express the unobserved time-varying taste parameter for that group \((\varphi_{kt}^{h})\) in terms of observed prices \((p_{kt})\) and common goods expenditure shares \((s_{kt}^{h*})\). In particular, taking logarithms in the common goods expenditure share (5), differencing over time, and then differencing from the mean across common goods within each time period for each group separately, we obtain the following closed-form expression for the log change in tastes that is analogous to the expression for a single group in equation (12) in the paper:

\[
\ln \left( \frac{\varphi_{kt}^{h}}{\varphi_{kt-1}^{h}} \right) = \ln \left( \frac{p_{kt} / \hat{p}_{t}}{p_{kt-1} / \hat{p}_{t-1}} \right) + \frac{1}{\sigma^{h} - 1} \ln \left( \frac{s_{kt}^{h*} / s_{kt-1}^{h*}}{s_{kt}^{h*} / s_{kt-1}^{h*}} \right),
\]  

(A.77)

where a tilde denotes a geometric mean across the set of common goods, such that \(\tilde{x}_{t} = \left( \prod_{k \in \Omega_{t}^{h}} x_{kt} \right)^{1/N_{t}^{h}}\) for the variable \(x_{kt}\); and we normalize the tastes for each good to have a constant geometric mean across common goods for each group of consumers: \(\bar{\varphi}_{t}^{h} / \bar{\varphi}_{t-1}^{h} = 1\).

Using equation (A.77) to substitute for the taste shocks in equation (A.75), we obtain an exact CES common goods price index (CCG) for each group separately:

\[
\ln \Phi_{t}^{h*} = \frac{1}{N_{t}^{h}} \sum_{k \in \Omega_{t}^{h}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \frac{1}{\sigma^{h} - 1} \frac{1}{N_{t}^{h}} \sum_{k \in \Omega_{t}^{h}} \ln \left( \frac{s_{kt}^{h*}}{s_{kt-1}^{h*}} \right).
\]  

(A.78)

Substituting this common goods price index into our earlier expression for the overall price index (A.72), we have our exact CES unified price index (CUP) for each group separately as in equation (27) in the paper:

\[
\ln \Phi_{t}^{h} = \frac{1}{\sigma^{h} - 1} \ln \left( \frac{\lambda_{t}^{h}}{\lambda_{t-1}^{h}} \right) + \frac{1}{N_{t}^{h}} \sum_{k \in \Omega_{t}^{h}} \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) + \frac{1}{\sigma^{h} - 1} \frac{1}{N_{t}^{h}} \sum_{k \in \Omega_{t}^{h}} \ln \left( \frac{s_{kt}^{h*}}{s_{kt-1}^{h*}} \right).
\]  

(A.79)
A.9 Logit Specification

In the discrete choice literature, a well-known result is that CES preferences can be derived as the aggregation of the choices of individual consumers with extreme-value-distributed idiosyncratic preferences, as shown in Anderson de Palma and Thisse (1992) and Train (2009). In this section of the online appendix, we use this result to show that our CES unified price index (CUPI) holds for logit preferences, as widely used in applied microeconometric research.

Following McFadden (1974), we suppose that the utility of an individual consumer $i$ who consumes $c_{ikt}$ units of product $k$ at time $t$ is given by:

$$U_{it} = u_{ikt} + z_{ikt}, \quad u_{ikt} \equiv \ln q_{ikt} + \ln c_{ikt} \tag{A.80}$$

where $q_{ikt}$ captures common consumer tastes for each product; $z_{ikt}$ captures idiosyncratic consumer tastes for each product that are drawn from an independent Type-I Extreme Value distribution:

$$G(z) = e^{-e^{(z/n+k)}}, \tag{A.81}$$

where $n$ is the scale parameter of the extreme value distribution and $k \approx 0.577$ is the Euler-Mascheroni constant.

Each consumer has the same expenditure $E_t$ and chooses their preferred product given the observed realizations for idiosyncratic tastes. Therefore the consumer’s budget constraint implies:

$$c_{ikt} = \frac{E_t}{p_{ikt}} \tag{A.82}$$

The probability that individual $i$ chooses product $k$ at time $t$ is:

$$x_{ikt} = \text{Prob} \left( u_{ikt} + z_{ikt} > u_{i\ell t} + z_{i\ell t}, \forall \ell \neq k \right),$$

$$= \text{Prob} \left( z_{i\ell t} < z_{ikt} + v_{ikt} - v_{i\ell t}, \forall \ell \neq k \right).$$

Therefore, using the distribution of idiosyncratic tastes (A.81), we have:

$$x_{ikt} \mid z_{ikt} = \prod_{\ell \neq k} e^{-e^{(z_{ikt} + u_{ikt} - u_{i\ell t})/n + k}}.$$

Integrating across the probability density function for $z_{ikt}$, we have:

$$x_{ikt} = \int_{-\infty}^{\infty} \left( \prod_{\ell \neq k} e^{-e^{(y + u_{ikt} - u_{i\ell t})/n + k}} \right) \frac{1}{\mu} e^{-y/v + x} e^{-e^{(-y/v + x)}} dy.$$

Noting that $u_{ikt} - u_{ikt} = 0$, this expression can be re-written as:

$$x_{ikt} = \int_{-\infty}^{\infty} \left( \prod_{\ell \in \Omega_t} e^{-e^{(y + u_{ikt} - u_{i\ell t})/n + k}} \right) \frac{1}{\mu} e^{-y/v + x} dy,$$

which can be in turn re-written as:

$$x_{ikt} = \int_{-\infty}^{\infty} \exp \left( - \sum_{\ell \in \Omega_t} e^{-(y + u_{ikt} - u_{i\ell t})/n + k} \right) \frac{1}{\mu} e^{-y/v + x} dy,$$
and hence:

\[ x_{ikt} = \int_{-\infty}^{\infty} \exp \left( -e^{-y/v + \kappa} \sum_{t \in \Omega_t} e^{-u_{ikt} - u_{ikt}}/v \right) \frac{1}{\mu} e^{-y/v + \kappa} \, dy. \]

Now define the following change of variable:

\[ h = \exp (-y/v + \kappa), \]

where

\[ -\frac{1}{v} \exp (-y/v + \kappa) \, dy = dh. \]

As \( y \to \infty \), we have \( h \to 0 \). As \( y \to -\infty \), we have \( h \to \infty \). Using this change of variable, we have:

\[ x_{ikt} = \int_{0}^{\infty} \exp \left( -h \sum_{t \in \Omega_t} e^{-u_{ikt} - u_{ikt}}/v \right) - dh, \]

or equivalently:

\[ x_{ikt} = \int_{0}^{\infty} \exp \left( -h \sum_{t \in \Omega_t} e^{-u_{ikt} - u_{ikt}}/v \right) dh, \]

which yields:

\[ x_{ikt} = \left[ \exp \left( -h \sum_{t \in \Omega_t} e^{-u_{ikt} - u_{ikt}}/v \right) \right]_{0}^{\infty}, \]

and hence:

\[ x_{ikt} = \frac{1}{\sum_{t \in \Omega_t} e^{-u_{ikt} - u_{ikt}}/v}. \]

The probability that individual \( i \) chooses product \( k \) at time \( t \) is therefore:

\[ x_{ikt} = \frac{e^{u_{ikt}}/v}{\sum_{t \in \Omega_t} e^{u_{ikt}}/v}, \]

which from the definition of \( u_{ikt} \) in (A.80) and the consumer’s budget constraint in (A.82) becomes:

\[ s_{ikt} = s_{kt} = \frac{(p_{kt} / q_{kt})^{-1/v}}{\sum_{t \in \Omega_t} (p_{kt} / q_{kt})^{-1/v}}, \tag{A.83} \]

which makes clear that our consumer taste shocks \((q_{kt} / q_{kt-1})\) correspond to shifts in the common component of tastes for each good for all consumers \((q_{kt})\). As shown in Anderson, De Palma and Thisse (1992), the expected utility of consumer \( i \) at time \( t \) is:

\[ \mathbb{E} [U_{it}] = \mathbb{E} \left[ \max \{ u_{i1t} + z_{i1t}, \ldots, u_{iNt} + z_{iNt} \} \right] = v \ln \left( \sum_{t \in \Omega_t} \exp \left( \frac{u_{ikt}}{v} \right) \right). \tag{A.84} \]

Using the definition of \( u_{ikt} \) in (A.80) and the consumers budget constraint in (A.82), expected utility can be written as:

\[ \mathbb{E} [U_{it}] = \frac{E_t}{P_t}, \tag{A.85} \]
where $P_t$ is the unit expenditure function:

$$P_t = \left[ \sum_{k \in \Omega} \left( \frac{p_{kt}}{q_{kt}} \right)^{-1/v} \right]^{-v}. \quad (A.86)$$

Total expenditure on product $k$ across all consumers $i$ at time $t$ is:

$$E_t = \sum_i E_{ikt} = \sum_i s_{kt} E_{it} = s_{kt} E_t, \quad (A.87)$$

where we have used the fact that each consumer has the same expenditure $E_t$. Combining equations (A.83) and (A.87), total expenditure on product $k$ at time $t$ can be written as:

$$E_{kt} = \left( \frac{p_{kt}}{q_{kt}} \right)^{-1/v} P_t^{1/v} E_t, \quad (A.88)$$

where $P_t$ is again the unit expenditure function (A.86).

Note that equations (A.85), (A.86) and (A.88) take the same form as in our baseline CES specification in Section 2 of the paper, where $1/v = \sigma - 1$. Therefore, our unified price index (CUPI) can be applied for the closely-related logit model. Additionally, in the same way that our baseline CES specification can be generalized to accommodate mixed CES (as in Section A.8 of this online appendix), this baseline logit model can be generalized to accommodate a mixed logit specification, as in McFadden and Train (2000).

### A.10 Flexible Functional Forms

In this section of the online appendix, we show that our approach also holds for the flexible functional forms of homothetic translog preferences and the non-homothetic almost ideal demand system (AIDS). We first present results for the homothetic translog case, before turning to the non-homothetic AIDS specification.

#### A.10.1 Homothetic Translog Preferences

Homothetic translog preferences provide an arbitrary close local approximation to any continuous and twice-differentiable homothetic expenditure function. Following a similar approach to that used for CES preferences in the paper, we show that the translog demand system can be inverted to solve for unobserved time-varying tastes in terms of observed prices and expenditure shares. We use this result to derive an exact price index for translog preferences in terms of only prices and expenditure shares. We compare this exact price index to the conventional Törnqvist index, which is exact for translog preferences under the assumption of time-invariant tastes for each good. We show that this conventional Törnqvist index for translog is subject to a similar consumer-valuation bias as the Sato-Vartia index for CES in the presence of time-varying taste shocks.

We consider the following translog unit expenditure function defined over the price $(p_{kl})$ and taste parameter $(q_{kl})$ for a constant set of goods $k \in \Omega$ with number of elements $N = |\Omega|$:

$$\ln P_t = \ln P(p_t, q_t, \sigma) = \ln a_0 + \sum_{k \in \Omega} \alpha_k \ln \left( \frac{p_{kt}}{q_{kt}} \right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{l \in \Omega} \beta_{k'l} \ln \left( \frac{p_{kt}}{q_{kt}} \right) \ln \left( \frac{p_{lt}}{q_{lt}} \right), \quad (A.89)$$

where the parameters $\beta_{k'l}$ control substitution patterns between goods; symmetry between goods requires $\beta_{k'l} = \beta_{l'k}$; and symmetry and homotheticity together imply $\sum_{k \in \Omega} \alpha_k = 1$ and $\sum_{k \in \Omega} \beta_{k'l} = \sum_{l \in \Omega} \beta_{l'k} = 0$. 
We begin by deriving the demand system from the unit expenditure function \((A.89)\), which can be re-written as:

\[
\ln P_t = \ln a_0 + \sum_{k \in \Omega} \alpha_k \ln p_{kt} - \sum_{k \in \Omega} \alpha_k \ln \varphi_{kt}
+ \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kt} \ln p_{\ell t} - \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kt} \ln \varphi_{\ell t}
- \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln p_{\ell t} + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln \varphi_{\ell t}.
\]

Differentiating with respect to \(p_{mt}\), we have:

\[
\frac{\partial P_t}{\partial p_{mt}} = \frac{\alpha_m}{p_{mt}} + \frac{1}{2} \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln p_{\ell t} - \frac{1}{2} \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln \varphi_{\ell t}
+ \frac{1}{2} \sum_{k \in \Omega} \frac{\beta_{km}}{p_{mt}} \ln p_{kt} - \frac{1}{2} \sum_{k \in \Omega} \frac{\beta_{km}}{p_{mt}} \ln \varphi_{kt}.
\]

Assuming symmetry \((\beta_{m\ell} = \beta_{km})\), this simplifies to:

\[
\frac{\partial P_t}{\partial p_{mt}} = \frac{\alpha_m}{p_{mt}} + \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln p_{\ell t} - \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln \varphi_{\ell t},
\]

which implies:

\[
\frac{\partial P_t}{\partial p_{mt}} \cdot \frac{p_{mt}}{P_t} = \alpha_m + \sum_{\ell \in \Omega} \beta_{m\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right),
\]

and hence:

\[
s_{kt} = \alpha_k + \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right). \tag{A.90}
\]

We assume that a good’s expenditure share is decreasing in its own taste-adjusted price \((\beta_{kk} < 0)\), and increasing in the taste-adjusted price of other goods \((\beta_{k\ell} > 0 \text{ for } \ell \neq k)\). This assumption ensures that the demand system satisfies the “connected substitutes” conditions from Berry, Gandhi and Haile (2013), which rule out the possibility that some goods are substitutes while others are complements.

We next use the unit expenditure function \((A.89)\) to derive the exact price index for translog preferences. Consider any quadratic function of the following form:

\[
F(z_t) = a_0 + \sum_{k \in \Omega} a_k z_{kt} + \sum_{k \in \Omega} \sum_{\ell \in \Omega} a_{k\ell} z_{kt} z_{\ell t}, \tag{A.91}
\]

where bold font is used to denote a matrix or vector. Under the assumption that the parameters of this quadratic function \(\{a_0, a_k, a_{k\ell}\}\) are constant, the following result holds exactly:

\[
F(z_t) - F(z_{t-1}) = \frac{1}{2} \sum_{k \in \Omega} \left[ \frac{\partial F(z_{kt})}{\partial z_{kt}} + \frac{\partial F(z_{kt-1})}{\partial z_{kt-1}} \right] (z_{kt} - z_{kt-1}). \tag{A.92}
\]

Now note that the homothetic translog unit expenditure function \((A.89)\) corresponds to such a quadratic function where:

\[
F(z_t) = \ln P_t, \quad z_{kt} = \ln p_{kt}, \quad \frac{\partial F(z_t)}{\partial z_{kt}} = \frac{\partial \ln P_t}{\partial \ln p_{kt}} = \frac{\partial P_t}{\partial p_{kt}} \cdot \frac{p_{kt}}{P_t}.
\]
Applying the result (A.92) for this homothetic translog unit expenditure function, we obtain:

$$\ln P_t - \ln P_{t-1} = \frac{1}{2} \sum_{k \in \Omega} \left( \frac{\partial P_t}{\partial p_{kt}} p_{kt} + \frac{\partial P_{t-1}}{\partial p_{kt-1}} p_{kt-1} \right) \left( \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) - \ln \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right) \right),$$  \hspace{1cm} (A.93)

which using the properties of the unit expenditure function can be re-written as:

$$\ln P_t - \ln P_{t-1} = \sum_{k \in \Omega} \frac{1}{2} \left( s_{kt} + s_{kt-1} \right) \left( \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) - \ln \left( \frac{p_{kt-1}}{\varphi_{kt-1}} \right) \right),$$  \hspace{1cm} (A.94)

which corresponds to the exact price index for translog ($\ln \Phi_T^{TR}$) in equation (30) in the paper:

$$\ln \Phi_t^{TR} = \ln \left( \frac{P_t}{P_{t-1}} \right) = \sum_{k \in \Omega} \frac{1}{2} \left( s_{kt} + s_{kt-1} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega} \frac{1}{2} \left( s_{kt} + s_{kt-1} \right) \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right),$$  \hspace{1cm} (A.95)

where the weights are the arithmetic mean of expenditure shares in the two time periods ($1/2 \left( s_{kt} + s_{kt-1} \right)$) and hence necessarily sum to one.

In the same way that our CES unified price index (CUPI) is a generalization of the Sato-Vartia price index to allow for taste shocks for each good, so the translog exact price index in equation (A.95) is a generalization of the Törnqvist index ($\ln \Phi_t^{TO}$), which corresponds to the special case in which tastes are assumed to be constant for all goods ($\left( \frac{\varphi_{kt}}{\varphi_{kt-1}} = 1 \right)$ for all $k \in \Omega$):

$$\ln \Phi_t^{TO} = \ln \left( \frac{P_t}{P_{t-1}} \right) = \sum_{k \in \Omega} \frac{1}{2} \left( s_{kt} + s_{kt-1} \right) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right).$$  \hspace{1cm} (A.96)

From equations (A.95) and (A.96), the exact translog price index with time-varying taste shocks differs from the conventional Törnqvist index that assumes time-invariant tastes by a correction term that we term the consumer-valuation bias:

$$\ln \Phi_t^{TR} = \ln \Phi_t^{TO} - \left[ \sum_{k \in \Omega} \frac{1}{2} \left( s_{kt} + s_{kt-1} \right) \ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \right].$$  \hspace{1cm} (A.97)

Comparing equation (A.97) for translog with equation (13) in the paper for CES, this consumer-valuation bias takes a similar form as for CES, except that the taste shock for each good is weighted by the arithmetic mean of expenditure shares in the two time periods rather than the logarithmic mean of these expenditure shares. The Törnqvist index index is unbiased if the demand shocks ($\ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)$) are orthogonal to the expenditure-share weights ($\frac{1}{2} \left( s_{kt} + s_{kt-1} \right)$), upward-biased if they are positively correlated with these weights, and downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the taste shocks ($\ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)$) and the expenditure-share weights ($\frac{1}{2} \left( s_{kt} + s_{kt-1} \right)$) is possible, depending on the underlying correlation between taste shocks ($\ln \left( \frac{\varphi_{kt}}{\varphi_{kt-1}} \right)$) and price shocks ($\ln \left( \frac{p_{kt}}{p_{kt-1}} \right)$) in the two time periods. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are endogenous to the taste shocks, as for our baseline CES specification in Section 2.5 of the paper. In particular, a positive taste shock for a good mechanically increases the expenditure-share weight for that good and reduces the expenditure-share weight for all
other goods in the demand system (A.90). We therefore obtain the following result for translog preferences, which is analogous to our result in equation (14) in Section 2.5 of the paper for CES preferences.

**Proposition.** A positive taste shock for a good $k$ (i.e., $\ln (q_{kt} / q_{k\ell t-1}) > 0$ for some $k \in \Omega$) increases the expenditure share for that good $k$ at time $t$ ($s_{kt}$) and reduces the expenditure share for all other goods $\ell \neq k$ at time $t$ ($s_{\ell t}$).

**Proof.** Note that tastes, prices and expenditure shares at time $t - 1$ ($q_{k\ell t-1}$, $p_{kt-1}$, $s_{kt-1}$) are pre-determined at time $t$. To evaluate the impact of a positive tastes shock for good $k$ ($\ln (q_{kt} / q_{k\ell t-1}) > 0$), we consider the effect of an increase in tastes at time $t$ for that good ($q_{kt}$) given its tastes parameter at time $t - 1$ ($q_{k\ell t-1}$). From the expenditure share (A.90), and using our assumption that the parameters $\{\beta_{k\ell}\}$ satisfy “connected substitutes,” we have:

$$
\frac{ds_{kt}}{d q_{kt}} \frac{q_{kt}}{s_{kt}} = -\beta_{kk} s_{kt} > 0, \quad \text{since} \quad \beta_{kk} < 0,
$$

$$
\frac{ds_{\ell t}}{d q_{kt}} \frac{q_{kt}}{s_{\ell t}} = -\beta_{\ell k} s_{\ell t} < 0, \quad \text{since} \quad \beta_{\ell k} > 0, \quad \ell \neq k.
$$

\[ \Box \]

For both translog and CES preferences, the source for the consumer-valuation bias is the failure to take account that an increase in taste for a good is analogous to a fall in its price. This failure induces a systematic overstatement of the increase in the cost of living, because consumers substitute towards goods that become more desirable. Therefore, other things equal, goods experiencing an increase in tastes (for which the change in observed prices is greater than the true change in taste-adjusted prices) receive a higher expenditure-share weight than goods experiencing a decrease in tastes (for which the change in observed prices is smaller than the true change in taste-adjusted prices).

As for our CES specification in Section 2 of the paper, the challenge in implementing the exact price index (A.95) empirically is that taste-adjusted prices ($p_{kt} / q_{kt}$) are not directly observed in the data. Again we overcome this challenge by inverting the demand system to solve for the taste parameters ($q_{kt}$) as a function of the observed prices and expenditure shares ($p_{kt}$, $s_{kt}$). Differentiating over time in the demand system (A.90), we obtain the following expression for the change in the expenditure share for each good, which corresponds to equation (32) in the paper,

$$
\Delta s_{kt} = \sum_{\ell \in \Omega} \beta_{k\ell} [\Delta \ln (p_{\ell t}) - \Delta \ln q_{\ell t}] .
$$

(A.98)

We solve for the unobserved taste shocks ($\Delta \ln (q_{\ell t})$) by inverting the demand system in equation (A.98). This demand system (A.98) consists of a system of equations for the change in the expenditure shares ($\Delta s_{kt}$) of the $N$ goods that is linear in the change in the log price ($\Delta \ln p_{kt}$) and log tastes parameter ($\Delta \ln q_{kt}$) for each good. This demand system can be written in the following matrix form:

$$
\Delta s_t = \beta \Delta \ln p_t - \beta \Delta \ln q_t ,
$$

(A.99)

where we use bold math font to denote a vector or matrix.
In this demand system (A.99), the changes in expenditure shares ($\Delta s_t$) must sum to zero across goods. Furthermore, under our assumptions of symmetry and homotheticity, the rows and columns of the symmetric matrix $\beta$ must each sum to zero. Therefore, without loss of generality, we omit the equation for the first good. We nevertheless recover the taste shock for all goods (including the omitted one) using our result that the taste shocks are mean zero across goods ($\frac{1}{N} \sum_{k \in \Omega} \Delta \ln (\varphi_{kt}) = 0$). In particular, we define the following augmented variables:

$$\Delta \hat{s}_t \equiv \begin{pmatrix} 0 \\ \Delta s_t^- \end{pmatrix}, \quad \hat{\beta} \equiv \begin{pmatrix} 0, \ldots, 0 \\ \beta^- \end{pmatrix}, \quad \hat{\gamma} \equiv \begin{pmatrix} 1, \ldots, 1 \\ \beta^- \end{pmatrix},$$  \quad (A.100)

where $\Delta s_t^-$ denotes the vector of changes in expenditure shares omitting the first good; and $\beta^-$ denotes the symmetric matrix of substitution parameters omitting the first row. Using this notation, the demand system (A.99) can be written in the following form:

$$\Delta \hat{s}_t = \hat{\beta} \Delta \ln p_t - \hat{\gamma} \Delta \ln \varphi_t,$$  \quad (A.101)

which can be inverted to solve for the vector of taste shocks ($\Delta \ln \varphi_t$). We thus obtain the unobserved taste shock for each good in terms of observed prices and expenditure shares:

$$\Delta \ln \varphi_{kt} = S_{kt}^{-1} (\Delta \hat{s}_t, \Delta \ln p_t, \{\beta_{kl}\}).$$  \quad (A.102)

Substituting for the unobserved taste shock in equation (A.95), we obtain the following exact price index for translog preferences with time-varying taste parameters:

$$\ln \Phi_t^{TCG} = \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) \ln \left( \frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega} \frac{1}{2} (s_{kt} + s_{kt-1}) S_{kt}^{-1} (\Delta \hat{s}_t, \Delta \ln p_t, \{\beta_{kl}\}).$$  \quad (A.103)

This exact translog common goods price index ($\Phi_t^{TCG}$) is the analog of our exact CES common goods price index ($\ln \Phi_t^{CCG}$) in equation (9) in the paper.

Therefore, our main insight that the demand system can be unified with the unit expenditure function to construct an exact price index that allows for time-varying taste shocks for individual goods is not specific to CES, but also holds for flexible functional forms. Furthermore, the consumer-valuation bias is again present, because a conventional price index that assumes time-variant tastes interprets all movements in expenditure shares as reflecting changes in prices, and hence does not take into account that these movements in expenditure shares are also influenced by the time-varying demand residual.

### A.10.2 Non-Homothetic Almost Ideal Demand System (AIDS)

The non-homothetic almost ideal demand system (AIDS) provides an arbitrary first-order linear approximation to the demand system and is based on translog functions. In particular, the AIDS expenditure function is defined over the price ($p_{ikt}$) and taste parameter ($\varphi_{ikt}$) for a constant set of goods $k \in \Omega$ with number of elements $N = |\Omega|$:}

$$\ln E(p_t, \varphi_t, u_t) = \ln \alpha_0 + \sum_{k \in \Omega} \alpha_k \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{l \in \Omega} \beta_{kl} \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) \ln \left( \frac{p_{lt}}{\varphi_{lt}} \right) + u_t \gamma_0 \prod_{k \in \Omega} \left( \frac{p_{kt}}{\varphi_{kt}} \right)^{\gamma_k}, \quad (A.104)$$
where $u$ denotes the level of utility and linear homogeneity in $p_{kt}$ requires $\sum_{k \in \Omega} a_k = 1$, $\sum_{k \in \Omega} \beta_{kt} = \sum_{\ell \in \Omega} \beta_{\ell k} = 0$, and $\sum_{k \in \Omega} \gamma_k = 0$. Using Shephard’s Lemma, we differentiate with respect to $p_{kt}$ in equation (A.104) to obtain the following expression for the expenditure share for each good $k$:

$$s_{kt} = a_k + \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right) + \gamma_k \ln \left( \frac{E_j}{P_t} \right), \quad (A.105)$$

where $P_t$ is a price index defined by:

$$\ln P_t = \ln a_0 + \sum_{k \in \Omega} a_k \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \tilde{\beta}_{k\ell} \ln \left( \frac{p_{kt}}{\varphi_{kt}} \right) \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right), \quad (A.106)$$

where

$$\tilde{\beta}_{k\ell} = \frac{1}{2} (\beta_{kt} + \beta_{\ell k}) = \tilde{\beta}_{\ell k}. \quad (A.107)$$

Taking the derivative of the expenditure share ($s_{kt}$) of a good $k$ in equation (A.105) with respect to the price of any good $j$, we have:

$$\frac{ds_{kt}}{dp_{jt}} \frac{p_{jt}}{s_{kt}} = \frac{\beta_{kj}}{s_{kt}} - \frac{\gamma_j a_j}{s_{kt}} + \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left( \frac{p_{\ell t}}{\varphi_{\ell t}} \right), \quad (A.108)$$

where connected substitutes requires $\frac{ds_{kt}}{dp_{jt}} \frac{p_{jt}}{s_{kt}} < 0$ for all $k$ and $\frac{ds_{kt}}{dp_{jt}} \frac{p_{jt}}{s_{kt}} > 0$ for all $j \neq k$.

Assuming that connected substitutes is satisfied, we can again invert the demand system (A.105) to solve for unique values for tastes ($\varphi_{kt}$) up to a normalization for the geometric mean for consumer tastes ($\frac{1}{N} \sum_{k \in \Omega} \ln \varphi_{kt} = 0$):

$$\varphi_{kt} = S_{kt}^{-1} (s_t, p_t). \quad (A.109)$$

Using these solutions for consumer tastes ($\varphi_{kt} = S_{kt}^{-1} (s_t, p_t)$) in the expenditure function (A.104), we obtain:

$$\ln E_t = \ln a_0 + \sum_{k \in \Omega} a_k \ln \left( \frac{p_{kt}}{S_{kt}^{-1} (s_t, p_t)} \right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left( \frac{p_{kt}}{S_{kt}^{-1} (s_t, p_t)} \right) \ln \left( \frac{p_{\ell t}}{S_{kt}^{-1} (s_t, p_t)} \right)$$

$$+ u_t \gamma_0 \prod_{k \in \Omega} \left( \frac{p_{kt}}{S_{kt}^{-1} (s_t, p_t)} \right)^{\gamma_k}, \quad (A.110)$$

which can be used to compute the change welfare between any pair of time periods.

### A.11 Feenstra (1994) Estimator

In this section of the online appendix, we provide further details on the Feenstra (1994) estimator used to estimate the elasticity of substitution. We estimate a separate elasticity of substitution for each product group, but suppress the subscript on parameters for product groups to simplify notation. We start by taking logarithms and double-differences in the CES demand system for common goods in equation (5) in the paper:

$$\Delta \ln s_{kt} = \beta_0 + \beta_1 \Delta \ln p_{kt} + u_{kt}, \quad (A.111)$$
where the first difference is over time and the second difference is from the geometric mean across common goods; \( \Delta \) denotes the time-difference operator such that \( \Delta \ln \tilde{p}_{kt} = \ln (\tilde{p}_{kt} / \tilde{p}_{kt-1}) \); a bar above a variable indicates that it is normalized by its geometric mean across common goods, such that \( \ln (\bar{p}_{kt}) = \ln (p_{kt} / \tilde{p}_{t}) \); the regression error \((u_{kt})\) includes the time-varying taste shock \((\Delta \ln \tilde{\varphi}_{kt})\); and any time-invariant component of tastes is differenced out between the two time periods.

We combine this relationship from the CES demand system in equation (A.111) above with an analogous supply-side relationship:

\[
\Delta \ln \tilde{s}^*_{kt} = \delta_0 + \delta_1 \Delta \ln \tilde{p}_{kt} + w_{kt}. \tag{A.112}
\]

The identifying assumption of the Feenstra (1994) estimator is that the double-differenced demand and supply shocks \((u_{kt}, w_{kt})\) are orthogonal and heteroskedastic. The orthogonality assumption defines a rectangular hyperbola for each good in the space of the demand and supply elasticities. The heteroskedasticity assumption implies that these rectangular hyperbolas for different goods do not lie on top of one another. With two goods, the intersection of these rectangular hyperbolas exactly identifies the elasticity of substitution. With more than two goods, the model is overidentified.

In particular, following Broda and Weinstein (2006), the orthogonality of the double-differenced demand and supply shocks defines a set of moment conditions (one for each good within a product group):

\[
G (\zeta) = \mathbb{E}_T [\tilde{\zeta}_{kt} (\zeta)] = 0, \tag{A.113}
\]

where \( \zeta = \begin{pmatrix} \beta_1 \\ \delta_1 \end{pmatrix} \); \( \tilde{\zeta}_{kt} = u_{kt}w_{kt} \); and \( \mathbb{E}_T \) is the expectations operator over time. We stack the moment conditions for all goods within a product group to form the GMM objective function and obtain:

\[
\hat{\zeta} = \arg \min \left\{ G^S (\zeta)' W G^S (\zeta) \right\}, \tag{A.114}
\]

where \( G^S (\zeta) \) is the sample analog of \( G (\zeta) \) stacked over all goods within a given product group and \( W \) is a positive definite weighting matrix. As in Broda and Weinstein (2010), we weight the data for each good by the number of raw buyers for that good to ensure that our objective function is more sensitive to goods purchased by larger numbers of consumers.

### A.12 Specification Check Using a Subset of Common Goods

In this section of the online appendix, we discuss the joint specification test of our assumption of CES demand and our normalization that tastes have a constant geometric mean from Section 5.5 of the paper. We use the independence of irrelevant alternatives (IIA) property of CES, which implies that the change in the cost of living can be computed either (i) using all common goods and an entry/exit term or (ii) choosing a subset of common goods and adjusting the entry/exit term for the omitted common goods. If preferences are CES and taste shocks average out across goods such that the geometric mean of tastes is constant for both definitions of common goods, we should obtain the same change in the cost of living from these two different specifications.
We start with the following expression for the change in the cost of living under CES preferences:

\[
\frac{P_t}{P_{t-1}} = \left[ \frac{\sum_{l \in \Omega_k} (p_{lt}/\varphi_{lt})^{1-\sigma}}{\sum_{l \in \Omega_{k-1}} (p_{lt-1}/\varphi_{lt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}}. \tag{A.115}
\]

Our first approach to analyzing the change in the cost of living uses the full set of common goods and rewrites equation (A.115) as:

\[
\frac{P_t}{P_{t-1}} = \left[ \frac{\sum_{l \in \Omega_k} (p_{lt}/\varphi_{lt})^{1-\sigma} \sum_{l \in \Omega_k} (p_{lt-1}/\varphi_{lt-1})^{1-\sigma}}{\sum_{l \in \Omega_k} (p_{lt}/\varphi_{lt})^{1-\sigma} \sum_{l \in \Omega_k} (p_{lt-1}/\varphi_{lt-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}}, \tag{A.116}
\]

where

\[
\Omega_t^* = \Omega_t \cap \Omega_{t-1} \tag{A.117}
\]

is the set of common goods and

\[
\lambda_t = \frac{\sum_{l \in \Omega_k} (p_{lt}/\varphi_{lt})^{1-\sigma}}{\sum_{l \in \Omega_k} (p_{lt}/\varphi_{lt})^{1-\sigma}} , \quad \lambda_{t-1} = \frac{\sum_{l \in \Omega_k} (p_{lt-1}/\varphi_{lt-1})^{1-\sigma}}{\sum_{l \in \Omega_k} (p_{lt-1}/\varphi_{lt-1})^{1-\sigma}} \tag{A.118}
\]

are the shares of expenditure on common goods in total expenditure in the two time periods, as in Feenstra (1994). We can also define the share of an individual common good in all expenditure on common goods:

\[
s_{kt}^* = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{\sum_{l \in \Omega_k^*} (p_{lt}/\varphi_{lt})^{1-\sigma}} = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{(P_t^*)^{1-\sigma}}, \tag{A.119}
\]

where

\[
P_t^* = \left[ \sum_{l \in \Omega_k^*} (p_{lt}/\varphi_{lt})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{A.120}
\]

is the unit expenditure function for common goods and we use an asterisk to denote the value of a variable for common goods $k \in \Omega_k^*$. Rearranging equation (A.119), taking logarithms, then taking means across common goods, and exponentiating, we obtain the following equivalent expression for the unit expenditure function for common goods:

\[
P_t^* = \frac{\hat{p}_t}{\bar{\varphi}_t} \left( \tilde{s}_{kt}^* \right)^{\frac{1}{\sigma-1}}, \tag{A.121}
\]

where a tilde denotes a geometric mean across common goods. Using the final line of equation (A.116) and this equivalent expression for the unit expenditure function for common goods (A.121), we obtain the following expression for the change in the cost living:

\[
\frac{P_t}{P_{t-1}} = \left( \frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{\sigma-1}} \frac{\hat{p}_t}{\bar{\varphi}_{t-1}} \frac{\tilde{s}_{kt}^*}{\tilde{s}_{kt-1}^*} \left( \frac{\tilde{s}_{kt}^*}{\tilde{s}_{kt-1}^*} \right)^{\frac{1}{\sigma-1}}. \tag{A.122}
\]
Our second approach to measuring the change in the cost of living uses a subset of common goods and rewrites equation (A.115) as follows:

\[
\frac{P_t}{P_{t-1}} = \left[ \frac{\sum_{\ell \in \Omega_t} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma} \cdot \sum_{\ell \in \Omega_{t-1}^*} (p_{\ell t-1}/\varphi_{\ell t-1})^{1-\sigma}}{\sum_{\ell \in \Omega_t} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma} \cdot \sum_{\ell \in \Omega_{t-1}} (p_{\ell t-1}/\varphi_{\ell t-1})^{1-\sigma}} \right]^{\frac{1}{\sigma}},
\]

(A.123)

where

\[
\Omega_{t}^{*} \subset \Omega_{t} = \Omega_{t} \cap \Omega_{t-1},
\]

(A.124)
is a subset of common goods and

\[
\mu_{t} = \frac{\sum_{\ell \in \Omega_{t}^{*}} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma}}{\sum_{\ell \in \Omega_t} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma}}, \quad \mu_{t-1} = \frac{\sum_{\ell \in \Omega_{t-1}^{*}} (p_{\ell t-1}/\varphi_{\ell t-1})^{1-\sigma}}{\sum_{\ell \in \Omega_{t-1}} (p_{\ell t-1}/\varphi_{\ell t-1})^{1-\sigma}},
\]

(A.125)

are the shares of expenditure on this subset of common goods in total expenditure in the two time periods. We can also define the share of an individual good from this subset in all expenditure on this subset:

\[
s_{kt}^{**} = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{\sum_{\ell \in \Omega_{t}^{*}} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma}} = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{(P_{t}^{**})^{1-\sigma}},
\]

(A.126)

where

\[
P_{t}^{**} = \left[ \sum_{\ell \in \Omega_{t}^{*}} (p_{\ell t}/\varphi_{\ell t})^{1-\sigma} \right]^{\frac{1}{\sigma}},
\]

(A.127)
is the unit expenditure function for this subset of common goods and we use a double asterisk to denote the value of a variable for this subset of common goods. Rearranging equation (A.123), taking logarithms, then taking means across the subset of common goods, and exponentiating, we obtain the following equivalent expression for the unit expenditure function for this subset of common goods:

\[
P_{t}^{**} = \frac{\tilde{p}_{t}}{\tilde{\varphi}_{t}} (\tilde{s}_{t}^{**})^{\frac{1}{\sigma - 1}},
\]

(A.128)

where a double tilde denotes a geometric mean across this subset of common goods. Using the final line of equation (A.123) and this equivalent expression for the unit expenditure function for common goods (A.128), we obtain the following alternative expression for the change in the cost of living:

\[
\frac{P_{t}}{P_{t-1}} = \left( \frac{\mu_{t}}{\mu_{t-1}} \right) ^{\frac{1}{\sigma - 1}} \frac{\tilde{p}_{t}/\tilde{\varphi}_{t}}{\tilde{p}_{t-1}/\tilde{\varphi}_{t-1}} \left( \frac{\tilde{s}_{t}}{\tilde{s}_{t-1}} \right)^{\frac{1}{\sigma - 1}}.
\]

(A.129)

Our two approaches to measuring the change in the cost of living in equations (A.122) and (A.129) provide the basis for a joint specification test of the sensitivity of our results to our assumption of CES demand and our normalization of a constant geometric mean of consumer tastes. If preferences are CES and the geometric mean of consumer tastes is constant across both (i) all common goods ($\tilde{\varphi}_{t} = \tilde{\varphi}_{t-1}$) and (ii) this subset of common goods ($\tilde{\varphi}_{t} = \tilde{\varphi}_{t-1}$), we should obtain the same change in the cost of living whether we use all common goods in equation (A.122) or this subset of common goods in equation (A.129):
\[
\frac{P_t}{P_{t-1}} = \left( \frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{\Delta t}} \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \left( \frac{s_{t}^*}{s_{t-1}^*} \right)^{\frac{1}{\Delta t}} = \left( \frac{\mu_t}{\mu_{t-1}} \right)^{\frac{1}{\Delta t}} \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \left( \frac{s_{t}^{**}}{s_{t-1}^{**}} \right)^{\frac{1}{\Delta t}}, \tag{A.130}
\]
which corresponds to equation (35) in the paper.

**A.13 Data Appendix**

In this data appendix, we report our full list of product groups and summary statistics for each product group in Table A.1, as a supplement to Table 1 in the paper. Consistent with our discussion for the full sample in Section 4 of the paper, we find pervasive entry and exit for all product groups, combined with substantial variation across these product groups in the share of products that enter and exit and the share of common goods in expenditure in period \( t \) relative to period \( t - 1 \).
## Table A.1: Descriptive Statistics by Product Group

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Number of UPCs</th>
<th>Mean $\lambda_{t-1}$</th>
<th>Percent of UPCs that Enter in a Year</th>
<th>Percent of UPCs that Exit in a Year</th>
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Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode (UPC) on the total value sold, total quantity sold, and average price; $\lambda_t$ and $\lambda_{t-1}$ are the shares of expenditure on common goods in total expenditure in time $t$ and $t-1$ respectively. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Number of UPCs</th>
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<th>Percent of UPCs that Enter in a Year</th>
<th>Percent of UPCs that Exit in a Year</th>
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Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode (UPC) on the total value sold, total quantity sold, and average price; $\lambda_t$ and $\lambda_{t-1}$ are the shares of expenditure on common goods in total expenditure in time $t$ and $t-1$ respectively. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
A.14 Additional Empirical Results

In this section of the online appendix, we report additional empirical results for the specifications discussed in Sections 5.2, 5.5 and 7 of the paper. First, we examine the relationship between our estimated demand residuals and separate measures of Brand Asset Values (BAVs) from consumer surveys. Second, we examine the sensitivity of our measured changes in the cost of living to the Feenstra (1994) estimated elasticities. Third, we illustrate the relevance of our results for official measures of the consumer price index (CPI). Fourth, we demonstrate the robustness of our results to the treatment of goods with smaller expenditure shares for which measurement error could be relatively more important.

A.14.1 Relationship Between Estimated Tastes and Measured Brand Asset Values

To provide additional empirical evidence on the extent to which our estimates capture consumer preferences rather than specification or measurement error, we obtained survey data on consumer evaluations of brands from Young and Rubicam (the U.S. subsidiary of the world’s largest marketing firm WPP). In particular, Young and Rubicam (Y&R) conducts an annual survey of approximately 17,000 U.S. consumers in which they try to ascertain “brand asset values” (BAVs) based on how surveyed consumers respond to a large number of questions about brands. We then matched the Y&R brands with the Nielsen brands. One difficulty we faced is that the definition of “brand” is not standardized across sources. For example, while both Nielsen and Young and Rubicam agree that Diet Coke and Coke Zero are different brands, Breakstone’s butter, cottage cheese, and yogurt count as three brands in Nielsen data but only one brand (“Breakstone’s”) in Young and Rubicam’s survey data. Young and Rubicam have fewer brands, so in the end we used a many-to-one match of 11,178 Nielsen brands to 1,327 Young and Rubicam brands.

Young and Rubicam aggregate consumer responses about brand perceptions into four basic factors, each of which is designed to capture a brand attribute that causes consumers to purchase that brand instead of another brand. They identify four factors as important in assessing a brand’s value. “Energized differentiation” or “differentiation” is a measure of perceptions of the uniqueness or innovativeness of a product. Consumers rate brands with high levels of differentiation when they feel loyalty to those products and are likely to choose them despite price premia over other products with similar physical characteristics. In Young & Rubicam data, brands with high levels of differentiation are iPhones, Bose headphones, Trader Joe’s products, Mountain Dew, Listerine, and Ben&Jerry’s ice cream. In contrast, brands like DonQ rum, Mazola corn oil, and Cheer detergent have low levels of energized differentiation, reflecting the fact that consumers state that they are not willing to choose these products when their price is relatively high.

A second important brand characteristic is “relevance,” which is a measure of whether consumers feel that the brand is relevant for them. For example, tobacco brands tend to have low relevance because many consumers do not smoke and so are unlikely to purchase the product regardless of price. Other low-relevance brands are Botox, NuvaRing, and Nicorettes. On the other hand, high-relevance brands contain products that
many consumers think are “necessary.” Examples of high-relevance brands are Band-Aids, Heinz ketchup, and Kleenex.

“Esteem” is a third characteristic that captures the perceived prestige of the brand. For example, Lucky Strike cigarettes, Schlitz Malt Liquor, and Method dish soap are brands that consumers hold in low esteem. Consumers are unlikely to purchase these products in order to impress people. By contrast, Duracell batteries, Band-Aid brand bandaids, and Ziploc bags are high esteem brands because consumers think of them as the best in their classes.

Finally, and most relevant for many economic models of advertising, “knowledge” measures how familiar a consumer is with the brand. If consumers feel that they have a good understanding of the characteristics of a brand, then the brand obtains a high knowledge score. Whether they like the products or not, consumers report having a excellent understanding of exactly what they are buying when they purchase Coca-Cola, M&Ms, and Hershey’s bars.

Each of these brand characteristics are imperfectly correlated with one another. For example, many brands have “energized differentiation,” in the sense that consumers who know about the brands really like them, but are largely unknown (e.g., Pat LaFrieda meats and Kagome drinks). Other brands are held in esteem even if their products are not well known (e.g., Corning and DeWalt).

One issue with these variables is that it is difficult to interpret the raw scores of each variable since they are based on a variety of scaled survey questions (e.g., self-reported familiarity with a brand on 7 point scale). Since it is not obvious that a movement from 1 to 2 in a raw score means the same thing as a movement from 2 to 3, we expressed each variable as a percentile. Thus, a one unit movement in knowledge corresponds to moving one percentile in the distribution of consumer assessment of familiarity.

Since marketing data is reported at the level of the brand rather than the barcode, we estimate the nested CES specification from Section 3.2 of the paper, with product groups and Nielsen brands as our nests. In this specification, consumer taste for a barcode depends on both consumer taste for that barcode relative to other barcodes within the Nielsen brand ($\phi^k_{\text{bar}}$) and consumer taste for the Nielsen brand itself ($\phi^B_{\text{bt}}$). We normalize consumer tastes such that the geometric mean of barcode tastes is constant across common barcodes within each Nielsen brand and the geometric mean of brand tastes is constant across common Nielsen brands within each product group.

We can see the relationship between our estimates of consumer tastes and the Y&R BAV components through binned scatter plots. In Figure A.1 we present a binned scatter plot of the residuals from regressing $\ln \phi^B_{\text{bt}}$ on product-group-time fixed effects against the residuals from regressing each of the BAVs against product-group-time fixed effects. This plot gives us a sense of whether brands that have high estimated consumer tastes also correspond to brands that have characteristics associated high brand asset values. We see that this is generally true for each of the BAV measures, each of which is strongly and positively associated with our estimates of consumer tastes. In Figure A.2, we repeat this exercise, this time adding brand fixed effects to the regression, so the residuals can now be interpreted as how estimated consumer tastes or BAVs shift over time. This specification enables us to see if our estimated consumer taste shifts correspond to shifts in preferences or knowledge as measured in consumer surveys. The binned scatters clearly indicate a strong
Note: In $\ln(\phi_{bt})$ is the log of the estimated brand consumer tastes parameter, where we use the nested CES specification from Section 3.2 of the paper to aggregate from barcodes to brands. Energized Differentiation, Relevance, Esteem, and Knowledge are the components of Brand Asset Value as measured by Young and Rubicam. They are expressed in percentiles ordered so that products with high values for a BAV component have high percentiles. The figure portrays binned scatter plots of the residuals from regressing $\ln(\phi_{bt})$ on product-group-time fixed effects against the residuals for regressing each of our BAVs against product-group-time fixed effects. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

association. There appears to be an almost linear relationship between the two measures of taste shifts.

We also can examine this relationship in a standard regression framework by estimating the following regression specification:

$$\ln(\phi_{bt}) = \alpha_r + \mu_{gt} + \zeta BAV^i_{rt} + \epsilon_{bt},$$

where $\alpha_r$ is a Y&R brand fixed effect corresponding to the set of Nielsen brands ($b$) that are matched to Y&R brand $r$; $\mu_{gt}$ is a product-group-time fixed effect; $BAV^i_{rt}$ is Y&R BAV component $i$ ($i \in \{\text{differentiation, relevance, esteem, knowledge}\}$); and $\zeta$ are the associated coefficients on these variables. We cluster the standard errors by Y&R-brand-time to take account of the fact that the BAV measures take the same value across Nielsen brands within each Y&R-brand-time-period.

In Table A.2, we estimate equation (A.131) without Y&R brand fixed effects ($\alpha_r$) to see whether brands with high estimated taste parameters also score highly on BAV components. We find a positive and statistically significant correlation between our estimates of brand tastes and each of these BAV measures. Inevitably, the BAV measures based on consumer surveys are imperfect proxies for the whole host of characteristics that influence the appeal of a brand to consumers (including physical characteristics, quality, fashion, lifestyle etc). Furthermore, the Nielsen brands ($b$) are measured at a more disaggregated level than the Young and Rubicam brands ($r$). Thus, knowing how consumers perceive Breakstone’s products in general tells us about average Nielsen measured brand appeal (as we saw in Figures A.1 and A.2), but is not informative about the differential perceptions of sub-brands like Breakstone’s butter or Breakstone’s yogurt. For both these
Figure A.2: Partial Regression and Binned Scatter Plot of Estimated Brand Consumer Tastes vs. BAVs After Conditioning on Product-Group-Time and Y&R Brand Fixed Effects

Note: $\ln(\phi_{bt}^B)$ is the log of the estimated brand consumer tastes parameter, where we use the nested CES specification from Section 3.2 of the paper to aggregate from barcodes to brands. Energized Differentiation, Relevance, Esteem, and Knowledge are the components of Brand Asset Value as measured by Young and Rubicam. They are expressed in percentiles ordered so that products with high values for a BAV component have high percentiles. The figure portrays binned scatter plots of the residuals from regressing $\ln(\phi_{bt}^B)$ on product-group-time and Y&R-brand fixed effects against the residuals for regressing each of our BAVs against product-group-time and Y&R-brand fixed effects. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

and other reasons, there remains substantial idiosyncratic variation in estimated consumer tastes that is not captured by the BAV measures. Nevertheless, these results confirm that our estimates of consumer tastes for brands are systematically related to separate measures of the extent to which these brands appeal to consumers from consumer surveys.

In Table A.3 we augment the regression specification with Y&R brand fixed effects, which implies that the estimated coefficients are now identified from the relationship between changes in estimated consumer tastes and changes in BAVs. We find that changes in each of the four BAVs are positively correlated with changes in our estimated consumers tastes, and the coefficients on relevance, esteem, and knowledge are statistically significant at conventional critical values. We also find an important role for the brand fixed effects, which is consistent with both our measures of consumer tastes and the BAV measures capturing persistent characteristics of brands.

Taken together, in both levels and changes, our estimated demand residuals are systematically related to separate measures of brand asset values, consistent with them capturing consumer tastes.
Table A.2: Regressions of Estimated Brand Consumer Tastes on BAVs Including Product-Group-Time Fixed Effects

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<th>(2)</th>
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<td>0.00367***</td>
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<td></td>
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<tr>
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</tr>
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</tr>
<tr>
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<td>(0.0141) (0.0490) (0.0505) (0.0738)</td>
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<td>Yes Yes Yes Yes</td>
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<td></td>
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<td>Brand FE</td>
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</table>

Note: \(\ln(\varphi_{bt}^B)\) is the log of the estimated brand consumer tastes parameter calculated using the Nielsen definition of brand, where we use the nested CES specification from Section 3.2 of the paper to aggregate from barcodes to brands. The dependent variables—Energized Differentiation, Relevance, Esteem, and Knowledge—are the components of Brand Asset Value as measured by Young and Rubicam (Y&R). They are expressed in percentiles ranging from the lowest to the highest. Because there are often several Nielsen brands matched to a Y&R brand, standard errors, reported in parentheses are clustered by Y&R-brand-time. Brands with only one observation are dropped. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

Table A.3: Regressions of Estimated Brand Consumer Tastes on BAVs Including Y&R-Brand and Product-Group-Time Fixed Effects

<table>
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<th>(1)</th>
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<td>0.550*** 0.371*** 0.385*** 0.288***</td>
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<tr>
<td>Brand FE</td>
<td>Yes Yes Yes Yes</td>
<td>Yes Yes Yes Yes</td>
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Note: \(\ln(\varphi_{bt}^B)\) is the log of the estimated brand consumer tastes parameter calculated using the Nielsen definition of brand, where we use the nested CES specification from Section 3.2 of the paper to aggregate from barcodes to brands. The dependent variables—Energized Differentiation, Relevance, Esteem, and Knowledge—are the components of Brand Asset Value as measured by Young and Rubicam (Y&R). They are expressed in percentiles ranging from the lowest to the highest. Because there are often several Nielsen brands matched to a Y&R brand, standard errors, reported in parentheses are clustered by Y&R-brand-time. Brands with only one observation are dropped. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
A.14.2 Grid Search over the Elasticity of Substitution

We now examine the robustness of our findings of a substantial consumer valuation bias to the estimated elasticity of substitution. In particular, we undertake a grid search over the range of plausible values for the elasticity of substitution. We consider a grid of thirty-eight evenly spaced values for this elasticity ranging from 1.5 to 20. For each value on the grid, we compute our CCG and CUPI for each product group and year, and then aggregate across product groups using expenditure-share weights. In Figure A.3, we compare these changes in the cost of living to the Fisher index. A smaller elasticity of substitution implies that varieties are more differentiated, which increases the absolute magnitude of the variety correction term for entering varieties being more desirable than exiting varieties \((1/ (\sigma - 1)) \ln (\lambda_t/\lambda_{t-1}) < 0\). As a result, we find that the CCG and CUPI fall further below the Fisher index as the elasticity of substitution becomes small. Nevertheless, across the entire range of plausible values for this elasticity, we find a quantitatively relevant consumer valuation bias.

Figure A.3: Average of Four-Quarter Proportional Changes in the Aggregate Cost of Living 
\(\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)\) from 2005-2013 for Alternative Elasticities of Substitution

Note: Average of four-quarter proportional changes in the aggregate cost of living from 2005-2013. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \((\frac{P_{kt} - P_{kt-1}}{P_{kt-1}})\) by their expenditure shares. Figure shows the time-averaged values of (i) the Fisher index from Figure 5 in the paper; (ii) the Feenstra (1994) index, which combines the variety correction term with the Sato-Vartia price index for common goods (the special case of equation (10) in the paper in which \(\phi_{kt} = \phi_{kt-1} = 1\) for all \(k \in \Omega^*_t\); (iii) the CCG (equation (9) in the paper); and (iv) the CUPI (equation (8) in the paper) for thirty-eight evenly-spaced values of the elasticity ranging from 1.5 to 20. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

A.14.3 Comparison with Official CPI Categories

We next illustrate the relevance of our estimated changes in the cost of living using the Nielsen data for official measures of the consumer price index (CPI). In particular, we map 89 of our 104 product groups to official CPI categories. For each of these 89 product groups, we compute conventional Laspeyres and Paasche indexes, and aggregate across product groups using expenditure share weights to compute the change in the aggregate cost of living over time. As shown in Figure A.4, we find that conventional price indexes computed using the Nielsen data are remarkably successful in replicating properties of official price indexes, with a positive and statistically significant correlation of 0.99 between the Laspeyres (based on Nielsen data) and
the CPI. Moreover, the average changes in the cost of living as measured by the Laspeyres index and the CPI are almost identical: 2.65 versus 2.35 percent respectively. The Paasche index (based on Nielsen data) has the same correlation with the CPI, but has an average change that is only 1.9 percent per year. In other words, annual movements in changes in the cost of living as measured by the BLS for this set of goods can be closely approximated by using a Laspeyres index and the Nielsen data, and the difference between the Laspeyres and the Paasche indexes in the Nielsen data is less than one percentage point per year (consistent with the findings of the Boskin Commission in Boskin et al. 1996). In contrast, we find a substantial bias from abstracting from entry/exit and taste shocks, with our CUPI more than one percentage point below the CPI.

Figure A.4: Four-Quarter Proportional Changes in the Aggregate Cost of Living \(\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right)\), CPI Matched Sample

![Chart showing four-quarter proportional changes in the aggregate cost of living](image)

Note: This figure shows alternative measures of the four-quarter proportional change in the aggregate cost of living using different price indexes for the 89 out of 104 product groups that we can match to subcategories of the CPI. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \(\left(\frac{P_{g,t} - P_{g,t-1}}{P_{g,t-1}}\right)\) by their expenditure shares. The thick gray line shows the aggregate price index based on the CPI subcategories. The other lines show alternative price indexes computed using the Nielsen data. CCG and CUPI are our exact common goods price index (equation (9)) and unified price index (equation (8)), respectively, using the Feenstra (1994) estimated elasticities of substitution. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

A.14.4 Measurement Error in Small Expenditure Shares

Finally, we examine the sensitivity of our results to measurement error in expenditure shares for goods that account for small shares of expenditure. In particular, we use the property that the change in the cost of living can be computed either (i) using all common goods and an entry/exit term or (ii) choosing a subset of common goods and adjusting the entry/exit term for the omitted common goods, as discussed in Section 5.5 of the paper. Using this property, we recompute the CUPI using the subset of our baseline sample of common goods with above-median expenditure shares. This specification is less sensitive to measurement error for goods that account for small shares of expenditure, because expenditures on goods with below-median expenditure shares only enter the change in the cost of living through the aggregate share of expenditure on goods with above-median expenditure shares. In Figure A.5, we compare the resulting measures of the CCG and CUPI to those in our baseline specification that does not distinguish between common goods with above-median versus below-median expenditure shares. As apparent from the figure, we find a similar change in the ag-
aggregate cost of living as in our baseline specification in the paper. This pattern of results suggests that our results are not sensitive to measurement error in expenditure shares for goods that account for small shares of expenditure.

Figure A.5: Robustness of Four-Quarter Proportional Changes in the Aggregate Cost of Living \((P_t - P_{t-1}) / P_{t-1}\) to Measurement Error in Small Expenditure Shares

Note: Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the product groups in our data \((P_{gt} - P_{gt-1}) / P_{gt-1}\) by their expenditure shares. CUPI is our baseline CES unified price index from equation (8) in the paper using the Feenstra (1994) estimated elasticities. CUPI-Restricted is the robustness check using the subset of common goods with above-median expenditure shares. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.
References


