A Introduction

In this section of the online appendix, we report the detailed derivations for the results reported in the paper and further supplementary results. In Section B, we report the proofs of the propositions in the paper. In Section C, we consider the neoclassical trade model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. In Section D, we examine the special case of this model that falls within the class of models considered by Arkolakis, Costinot and Rodriguez-Clare (2012, henceforth ACR), which satisfy the four primitive assumptions of (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions;
and (iv) perfect or monopolistic competition; as well as the three macro restrictions of (i) a constant elasticity import demand system, (ii) a constant share of profits in income, and (iii) balanced trade, as discussed in Section 3 of the paper. Although for convenience of exposition we focus in the paper and Section D of this online appendix on the single-sector Armington model, in Section E we show the same income and welfare exposure measures apply for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these exposure measures in the Krugman (1980) model.

In Section F, we consider a number of extensions of our baseline friend-enemy exposure measures from Section 5 of the paper. In Subsection F.1, we derive the corresponding friend-enemy exposure measures allowing for both productivity and trade cost shocks. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system. In Section F.4, we show that our results generalize to a multi-sector version of the constant elasticity Armington model. In Section F.5, we further generalize this multi-sector specification to allow for heterogeneous sector trade elasticities. In Section F.6, we show that we obtain analogous results to those in Section F.4 in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.7, we further extend the multi-sector Armington model from Section F.4 to introduce input-output linkages following Caliendo and Parro (2015).

In Section G, we provide further details on the empirical specifications reported in the paper, as well as additional supplementary empirical results. Section H contains the data appendix.

B Proofs of Propositions

B.1 Proof of Lemmas 1 and 2

Lemma 1 imposes trade balance, with market clearing conditions \( w_i \ell_i = \sum_n s_{ni} w_n \ell_n \). Define \( q_i \equiv \frac{w_i \ell_i}{\sum_n w_n \ell_n} \); we can rewrite market clearing condition as \( q_i = \sum_n s_{ni} q_n \), and, in matrix form, \( q' = q'S \), proving that \( q \) is a left-eigenvector of \( S \) with eigenvalue 1. That \( q' \) is the unique positive left-eigenvector of \( S \) follows from Perron-Frobenius theorem. Under free-trade, every row of \( S \) is identical, and \( \sum_n q_n s_{ni} = s_{1i} \) for all \( i \). The vector \( [s_{11}, s_{12}, \cdots, s_{1N}] \) is therefore also left-eigenvector of \( S \) with eigenvalue 1, and since its entries are all positive, it must be equal to \( q \). Likewise, market-clearing can also be written as \( w_n \ell_n = \sum_i t_{in} w_i \ell_i \), which is equivalent to, in matrix form, \( q' = q'T \). The remaining claims about \( T \) follow analogously.

Lemma 2 introduces trade imbalances to the market clearing conditions, with \( w_i \ell_i = \sum_n [s_{ni} w_n \ell_n + \bar{d}_n] \). Let \( q_i \equiv \frac{w_i \ell_i}{(\sum_n w_n \ell_n)} \) and \( e_i \equiv \frac{(w_i \ell_i + \bar{d}_n)}{(\sum_n w_n \ell_n)} \). Dividing the market clearing condition by \( (\sum_j w_j \ell_j) \), we have

\[
\frac{w_i \ell_i}{\sum_j w_j \ell_j} = \sum_n \left[ \frac{s_{ni} w_n \ell_n + \bar{d}_n}{\sum_j w_j \ell_j} \right] \iff q_i = \sum_n s_{ni} e_n \iff q' = e'S.
\]

Let \( d_i = q_i/e_i \) and \( D \equiv \text{Diag}(d) \). Note \( q' = e'D \) and \( q'D^{-1} = e' \); thus the market clearing condition can be re-written as

\[
e'D = e'S \iff e' = e'SD^{-1}
\]

and

\[
q' = e'S \iff q' = q'D^{-1}S.
\]
q is therefore the unique positive left-eigenvector of \( D^{-1}S \) with eigenvalue 1, and \( e' \) is the unique positive left-eigenvector of \( SD^{-1} \) with eigenvalue one. The remaining claims about \( T \) follow analogously.

### B.2 Proof of Lemma 2

Note that \( Z = T + \theta TS \) is a row-stochastic matrix and represents a Markov chain; its eigenvector \( q \) represents the stationary distribution of the Markov chain. Invertibility of \((I - V)\) follows from convergence of the power series \( \sum_{k=0}^{\infty} V^k = \sum_{k=0}^{\infty} (Z - Q)^k \), which we show now. By construction, \( QZ = Q \) and \( ZQ = Q \). Using these two relations, we can show by induction that \((Z - Q)^k = Z^k - Q\) for any integer \( k > 0 \). That \( \|Z^k - Q\| \leq c \cdot |\mu|^k \), where \( \mu \) is the largest eigenvalue of \( V \) in terms of absolute value (and the second-largest eigenvalue of \( Z \)), follows from standard results on Markov chains (e.g., see Rosenthal (1995)).

### B.3 Proof of Proposition 4

We repeatedly apply the following approximations:

\[
\ln (1 + x) \approx x - \frac{x^2}{2}, \quad x - 1 \approx \ln x + \frac{(\ln x)^2}{2}
\]

\[
\ln \left( \sum_i p_i x_i \right) \approx \sum_i p_i (x_i - 1) - \frac{\left( \sum_i p_i (x_i - 1) \right)^2}{2}
\]

\[
\approx \sum_i p_i \left( \ln x_i + \frac{(\ln x_i)^2}{2} \right) - \frac{\left( \sum_i p_i \ln x_i \right)^2}{2}
\]

\[
= E_p [\ln x_i] + \frac{\nabla_p (\ln x_i)}{2}
\]

Let \( \hat{x} \equiv \ln \hat{x} \). The hat-algebra with only TFP shocks can be written as

\[
\hat{w}_i = \sum_n T_m \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}}, \quad \text{where} \quad \hat{c}_i \equiv \hat{w}_i / \hat{z}_i
\]

Taking logs,

\[
\hat{w}_i = \ln \left( \sum_n T_m \hat{w}_n \frac{\hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right)
\]

\[
= \mathbb{E}_T \left[ \ln \left( \frac{\hat{w}_n \hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right) + \frac{\nabla T_i \left( \ln \left( \frac{\hat{w}_n \hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right) \right)}{2} \right]
\]

\[
= \mathbb{E}_T \left[ \hat{w}_i - \theta \hat{c}_i + \mathbb{E}_S_n [\theta \hat{c}_k] - \frac{\nabla T_i \left[ \frac{\hat{w}_n \hat{c}_i^{-\theta}}{\sum_k s_{nk} \hat{c}_k^{-\theta}} \right]}{2} + \frac{\nabla T_i \left( \hat{w}_i + \mathbb{E}_S_n [\theta \hat{c}_k] \right)}{2} \right]
\]

\[
= -\theta \hat{c}_i + \mathbb{E}_T [\hat{w}_i] + \mathbb{E}_S_n [\theta \hat{c}_k] - \frac{\nabla T_i \mathbb{E}_S_n [\theta \hat{c}_k]}{2} + \frac{\nabla T_i \left( \hat{w}_i + \mathbb{E}_S_n [\theta \hat{c}_k] \right)}{2}
\]

To re-write the second-order terms explicitly as a function of the productivity shocks—thereby deriving the Hessian—we express \( \theta \hat{c}_k \) and \( \hat{w}_i + \mathbb{E}_S_n [\theta \hat{c}_k] \) in terms of productivity shocks to first-order. To do so, note that the first-order approximation is

\[
\hat{w} = T \hat{w} + \theta (TS - I) (\hat{w} - \hat{z})
\]

\[
\implies \hat{w} = -\frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) \hat{z}
\]
where \( V = \frac{T + \theta TS}{1 + \theta} - Q \). We can therefore rewrite

\[
\theta(\bar{w} - \bar{z}) = -\theta \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + I \right) \bar{z} = -\theta \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) + (I - V)^{-1} (I - V) \right) \bar{z} = -\frac{\theta}{\theta + 1} (I - V)^{-1} (I - T) \bar{z}
\]

Where we have used the fact \( q' \bar{z} = 0 \), which follows from CRTS and our normalization that world GDP is constant, to drop \( Q \bar{z} \) from the RHS.

We further have

\[
\bar{w} + \theta S(\bar{w} - \bar{z}) = -\frac{\theta}{\theta + 1} (I - V)^{-1} (TS - I) \bar{z} - S \left( \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T + Q) \right) \bar{z} = -\frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T + Q) \right\} \bar{z}.
\]

Further, to reduce notational clutter, let \( A \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T) \) and \( B \equiv \frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\} \), \( y \equiv \theta(\bar{w} - \bar{z}) \), \( x \equiv \bar{w} - \theta S(\bar{w} - \bar{z}) \), thus

\[
y = -A \bar{z}, \quad x = +B \bar{z}
\]

We can now re-write the second-order terms as (let \( D(x) \equiv diag(x) \) denote the diagonalization operator)

\[
E_{T_i} V_{S_n} [\theta \hat{c}_k] - V_{T_i} (\bar{w}_n + E_{S_n} [\theta \hat{c}_k]) = E_{T_i} V_{S_n} [y_k] - V_{T_i} (x_n)
\]

\[
= \sum_n T_{in} \left[ \sum S_{nk} y_k^2 - \left( \sum S_{nk} y_k \right)^2 \right] - \left( \sum_n T_{in} x_n^2 - \left( \sum_n T_{in} x_n \right)^2 \right)
\]

\[
= y'D([M + I_i])y - y'S'D(T_i)Sy - (x'D(T_i)x - x'T'Tx)
\]

\[
= \bar{z}'A'D([M + I_i])A \bar{z} - z'A'S'D(T_i)SA \bar{z} - (\bar{z}'B'D(T_i)B \bar{z} - \bar{z}'B'T'TB \bar{z})
\]

\[
= \bar{z}'(A'([M + I_i]) - S'D(T_i)S)A - B'(D(T_i) - T_i'T_i)B \bar{z}.
\]

Hence we have \( \epsilon_i \equiv \bar{z}'H_{f_i} \bar{z} \), where

\[
H_{f_i} = -\frac{1}{2} A^T \left( diag([M + I_i]) - S'diag(T_i)S \right) A - B^T (\left( diag(T_i) - T_i'T_i \right) B).
\]

B.4 Proof of Proposition 5

Lemma 3. \( (I - S) A = -(I - T) B \), where \( A \equiv \frac{\theta}{\theta + 1} (I - V)^{-1} (I - T) \) and \( B \equiv \frac{\theta}{\theta + 1} \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\} \).

Proof. By the definition of \( A \) and \( B \):

\[
(I - S) A = \frac{\theta}{\theta + 1} (I - S) (I - V)^{-1} (I - T)
\]

\[
(I - T) B = \frac{\theta}{\theta + 1} (I - T) \left\{ (I - V)^{-1} (TS - I) + S (I - V)^{-1} (I - T) \right\}
\]

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We have
\[
\frac{\theta + 1}{\theta} ((I - S)A + (I - T)B) = (I - S)(I - V)^{-1}(I - T)
\]
\[
+ (I - T)(I - V)^{-1}(TS - I) + (I - T)S(I - V)^{-1}(I - T)
\]
\[
= (I - S + (I - T)S)(I - V)^{-1}(I - T) + (I - T)(I - V)^{-1}(TS - I)
\]
\[
= (I - TS)(I - V)^{-1}(I - T) + (I - T)(I - V)^{-1}(TS - I)
\]
\[
= (I - TS) \left( \frac{\theta}{\theta + 1} (I - V)^{-1}(TS - I) + I \right)
\]
\[
+ \left( \frac{\theta}{\theta + 1} (TS - I)(I - V)^{-1} + I \right)(TS - I)
\]
\[
= \left( \frac{\theta}{\theta + 1} (I - TS)(I - V)^{-1}(TS - I) + (I - TS) \right)
\]
\[
+ \left( \frac{\theta}{\theta + 1} (TS - I)(I - V)^{-1}(TS - I) + (TS - I) \right)
\]
\[
= 0.
\]

\[
\square
\]

**Proof of Proposition 5** To show that the second-order terms average to zero across countries, note
\[
q^\prime \epsilon = \tilde{z} \left( \sum q_i H_{f_i} \right) \tilde{z}
\]
\[
= -\frac{1}{2} \tilde{z} \left( \begin{array}{cc}
A' & (d - S'dS)A - B'(d - T'dT)B
\end{array} \right) \tilde{z},
\]
where \( d \equiv \text{diag}(q) \). We next show \( \left( A' (d - S'dS)A - B'(d - T'dT)B \right) \) is a zero matrix. Using Lemma 1, we have
\[
2 \left( A' (d - S'dS)A - B'(d - T'dT)B \right)
\]
\[
= A'(I - S')d(I + S)A - B'(I + T')d(I - T)B
\]
\[
+ A'(I + S')d(I - S)A - B'(I - T')d(I + T)B
\]
\[
\text{(the next line follows from Lemma 1)}
\]
\[
= -B'(I - T')d(I + S)A + B'(I + T')d(I - S)A
\]
\[
- A'(I + S')d(I - T)B + A'(I - S')d(I + T)B
\]
\[
= -(B' - B'T')d(A + SA) + (B' + B'T')d(A - SA)
\]
\[
- (A' + A'S')d(B - TB) + (A' - A'S')d(B + BT)
\]
\[
= 2(B'(T'd - dS)A + A'(dT - S'd)B)
\]
\[
= 0,
\]
where the last equality follows from \( T'd = dS \) and \( dT = S'd \) (recall \( T_inq_i = S_mq_m \) and \( d \equiv \text{diag}(q) \)).

**B.5 Proof of Proposition 6**

The fact that \( H_{f_i} \) is real and symmetric, \( \mu_{\text{max},i} \) is the largest eigenvalue and \( \tilde{z}_{\text{max},i} \) is the corresponding eigenvector implies that
\[
|\mu_{\text{max},i}| \equiv \max_z \frac{z^T H_{f_i} z}{z^T z}, \quad \tilde{z}_{\text{max},i} \equiv \arg \max_z \frac{z^T H_{f_i} z}{z^T z}.
\]
B.6 Proof of Proposition 7

The fact that $\mu$ is the spectral norm implies that for all $z$, 

$$\mu^4 \|z\|_2^4 \geq g(z) = \frac{1}{N} \sum_{i=1}^{N} (z'H_fz)^2 = \frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2(z)$$

Hence $\sqrt{\frac{1}{N} \sum_{i=1}^{N} \epsilon_i^2(z)} \leq \sqrt{\mu^2 \|z\|_2^2}$, as desired.

B.7 Proof of Proposition 11

To economize on notation we let $x \equiv d\ln w$ and $y \equiv d\ln z$, and we derive the sensitivity of $x$ to $\epsilon$, holding $y$ fixed. Using our assumption of constant returns to scale in production, it is without loss of generality to normalize the weighted mean of productivity shocks $q'y = 0$ and thus $Qy = 0$. Note $(I - V)x = -\frac{\theta}{\theta + 1}My$, $W = -\frac{\theta}{\theta + 1} (I - V)^{-1}M$, $x = (W + Q)y$, and $(W + Q)$ is invertible. Differentiating and noting $dV = dM (= \epsilon O)$, we obtain

$$-dVx + (I - V)dx = -\frac{\theta}{\theta + 1} dM y$$

$$\Rightarrow dx = (I - V)^{-1}dV(x - y)$$

$$\Rightarrow dx = (I - V)^{-1}dV(I - (W + Q)^{-1})x$$

By the Cauchy-Schwarz inequality,

$$\|dx\| \leq \|(I - V)^{-1}\| \|dV\| \|I - (W + Q)^{-1}\| \|x\|.$$ 

We obtain the proposition by substituting $\|dV\| = \frac{\theta}{\theta + 1} \epsilon$ and $\|dx\|/\|x\| = \lim_{\epsilon \to 0} \frac{\|d\ln w - d\ln w\|}{\epsilon}.$

C Neoclassical Trade Model

In this Section of the online appendix, we consider the neoclassical trade model with a general homothetic utility function in which goods are differentiated by country of origin from Section 2 of the paper. We consider a world consisting of many countries indexed by $i, n \in \{1, \ldots, N\}$. Each country has an exogenous supply of $\ell_n$ workers, who are endowed with one unit of labor that is supplied inelastically.

C.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following homothetic indirect utility function:

$$u_n = \frac{w_n}{P(p_n)},$$  \hspace{1cm} \text{(C.1)}$$

where $p_n$ is the vector of prices in country $n$ of the goods produced by each country $i$ with elements $p_{ni} = \tau_{ni}w_i/z_i$; $w_n$ is the income of the representative consumer in country $n$; and $P(p_n)$ is a continuous and twice differentiable function that captures the ideal price index. From Roy’s Identity, country $n$’s demand for the good produced by country $i$ is:

$$c_{ni} = c_{ni}(p_n) = -\frac{\partial (1/P(p_n))}{\partial p_{ni}} w_n P(p_n).$$  \hspace{1cm} \text{(C.2)}$$
### C.2 Expenditure Shares

Country $n$’s expenditure share on the good produced by country $i$ is:

$$ s_{ni} = \frac{p_n c_{ni}(p_n)}{\sum_{\ell=1}^{N} p_{\ell} c_{n\ell}(p_n)} = \frac{x_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)}. \quad (C.3) $$

Totally differentiating this expenditure share equation, we get:

$$ ds_{ni} = \frac{dx_{ni}(p_n)}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} - \frac{x_{ni}(p_n)}{\left[\sum_{\ell=1}^{N} x_{n\ell}(p_n)\right]^2} \sum_{k=1}^{N} dx_{nk}(p_n), $$

$$ ds_{ni} = \frac{1}{\sum_{\ell=1}^{N} x_{n\ell}(p_n)} \sum_{h=1}^{N} \frac{\partial x_{ni}(p_n)}{\partial p_{nh}} dp_{nh} - \frac{x_{ni}(p_n)}{\left[\sum_{\ell=1}^{N} x_{n\ell}(p_n)\right]^2} \sum_{k=1}^{N} \frac{\partial x_{nk}(p_n)}{\partial p_{nh}} dp_{nh}, $$

$$ \frac{ds_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \frac{\partial x_{ni}(p_n) p_{nh}}{x_{ni}} - \sum_{k=1}^{N} \frac{\partial x_{nk}(p_n) p_{nh}}{x_{nk}} \right] \frac{dp_{nh}}{p_{nh}}, \quad (C.4) $$

$$ d \ln s_{ni} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] d \ln p_{nh}, $$

where

$$ \theta_{nih} = \frac{\partial x_{ni}(p_n) p_{nh}}{\partial p_{nh} x_{ni}} = \frac{\partial \ln x_{ni}}{\partial \ln p_{nh}}. $$

Totally differentiating prices in equation (3) in the paper, we have:

$$ \frac{dp_{ni}}{p_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i}, \quad (C.5) $$

$$ d \ln p_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i. $$

### C.3 Market Clearing

Market clearing implies:

$$ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \quad (C.6) $$

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

$$ \frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} ds_{ni} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n, $$

$$ \frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), $$

$$ \frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), $$

$$ \frac{dw_i}{w_i} = \sum_{n=1}^{N} \ell_{ni} \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), $$

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where we have defined $t_{in}$ as the share of country $i$’s income from market $n$:

$$t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}.$$  

Using our result for the derivatives of expenditure shares (C.4), we have:

$$\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}} \right] \right),$$

Using our results for the derivatives of prices, we have:

$$\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \left[ \frac{d\tau_{nh}}{\tau_{nh}} + \frac{dw_h}{w_h} - \frac{dz_h}{z_h} \right] \right] \right),$$

which can be written as:

$$d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left[ \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \left[ d \ln \tau_{nh} + d \ln w_h - d \ln z_h \right] \right) \right),$$

which corresponds to equation (8) in the paper.

### C.4 Welfare

Totally differentiating welfare, we have:

$$du_n = dw_n \left( \frac{1}{P} (p_n) \right) + d \left( \frac{1}{P} (p_n) \right) w_n,$$

$$du_n = dw_n \left( \frac{1}{P} (p_n) \right) + \sum_{i=1}^{N} w_n \frac{\partial \left( \frac{1}{P} (p_n) \right)}{\partial \tau_{ni}} dp_{ni},$$

$$du_n = \frac{dw_n}{w_n} w_n \left( \frac{1}{P} (p_n) \right) + \sum_{i=1}^{N} w_n \frac{\partial \left( \frac{1}{P} (p_n) \right)}{\partial \tau_{ni}} \tau_{ni} \frac{dp_{ni}}{p_{ni}},$$

$$du_n = \frac{dw_n}{w_n} + \sum_{i=1}^{N} \frac{\partial \left( \frac{1}{P} (p_n) \right)}{\partial \tau_{ni}} \frac{P (p_n) \tau_{ni}}{\tau_{ni}} \frac{dp_{ni}}{p_{ni}}.$$

Now recall from equation (C.2) that:

$$c_{ni} = - \frac{\partial \left( \frac{1}{P} (p_n) \right)}{\partial \tau_{ni}} w_n \frac{P (p_n)}{\tau_{ni}}.$$

Using this result in our total derivative for welfare above, we obtain:

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{i=1}^{N} c_{ni} \frac{dp_{ni}}{p_{ni}},$$

$$\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{i=1}^{N} s_{ni} \frac{dp_{ni}}{p_{ni}},$$

which can be equivalently written as:

$$d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d \ln \tau_{ni} + d \ln w_i - d \ln z_i \right],$$

which corresponds to equation (10) in the paper.
D Constant Elasticity Armington

In this section of the online appendix, we report the derivations for our baseline constant elasticity of substitution Armington model in Section 3 of the paper. This model falls within the class of quantitative trade models considered by Arkolakis, Costinot and Rodriguez-Clare (2012), which satisfy the three macro restrictions of (i) a constant elasticity import demand system, (ii) profits are a constant share of income, and (iii) trade is balanced. In Section E of this online appendix, we show that our analysis also holds for other models within this class, including those of perfect competition and constant returns to scale with Ricardian technology differences as in Eaton and Kortum (2002), and those of monopolistic competition and increasing returns to scale, in which goods are differentiated by firm, as in Krugman (1980) and Melitz (2003) with an untruncated Pareto productivity distribution. The world economy consists of many countries indexed by \( i, n \in \{1, \ldots, N\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

D.1 Consumer Preferences

The preferences of the representative consumer in country \( n \) are characterized by the following indirect utility function:

\[
U_n = \frac{w_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,
\]  

(D.1)

where \( w_n \) is the wage; \( p_n \) is the consumption goods price index; \( p_{ni} \) is the price in country \( n \) of the good produced by country \( i \); and we focus on the case in which countries’ goods are substitutes (\( \sigma > 1 \)).

D.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that \( \tau_{ni} \geq 1 \) units must be shipped from country \( i \) to country \( n \) in order for one unit to arrive (where \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \)). Therefore, the consumer in country \( n \) of purchasing a good \( \vartheta \) from country \( i \) is:

\[
p_{ni} = \frac{\tau_{ni} w_i}{z_i},
\]  

(D.2)

where \( z_i \) captures productivity in country \( i \) and iceberg variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \).

D.3 Expenditure Shares

Using the properties of the CES demand function, country \( n \)’s share of expenditure on goods produced in country \( i \) is:

\[
s_{ni} = \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}.
\]  

(D.3)

Totally differentiating this expenditure share equation (D.3), we get:

\[
\frac{ds_{ni}}{s_{ni}} = - (\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} \frac{dp_{nh}}{p_{nh}} \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = - (\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} \frac{dp_{nh}}{p_{nh}} \frac{p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} p_{nm}^{1-\sigma}}.
\]
\[
\frac{ds_{ni}}{s_{ni}} = - (\sigma - 1) \frac{dp_{ni}}{p_{ni}} + (\sigma - 1) \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}},
\]
\[
\frac{ds_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right),
\]
\[
d \ln s_{ni} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln p_{nh} - d \ln p_{ni} \right),
\]

where, from the definition of \(p_{ni}\) in equation (D.2) above, we have:
\[
dl p_{ni} = \sum_{m=1}^{N} \frac{d\tau_{nm}}{\tau_{ni}} + \sum_{m=1}^{N} \frac{d\tau_{nm}}{\tau_{ni}} + \sum_{m=1}^{N} \frac{d\tau_{nm}}{\tau_{ni}} - d \ln p_{ni},
\]
\[
d \ln p_{ni} = d \ln \tau_{ni} + d \ln w_{i} - d \ln z_{i}.
\]

**D.4 Price Indices**

Totally differentiating the consumption goods price index in equation (D.1), we have:
\[
d p_{n} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}} \left[ \sum_{m=1}^{N} p_{nm}^{1-\sigma} \right],
\]
\[
d p_{n} = \sum_{m=1}^{N} \frac{dp_{nm}}{p_{nm}} \frac{p_{nm}^{1-\sigma}}{\sum_{h=1}^{N} p_{nh}^{1-\sigma}},
\]
\[
d p_{n} = \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},
\]
\[
d \ln p_{n} = \sum_{m=1}^{N} s_{nm} d \ln p_{nm}.
\]

**D.5 Market Clearing**

Market clearing requires that income in each country equals expenditure on the goods produced in that country:
\[
w_{i} \ell_{i} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n}.
\]

Totally differentiating this market clearing condition (D.7), holding labor endowments constant, we have:
\[
d w_{i} = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} w_{n} \ell_{n} + \sum_{n=1}^{N} s_{ni} \frac{dw_{n}}{w_{n}} w_{n} \ell_{n},
\]
\[
d w_{i} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right),
\]
\[
d w_{i} = \sum_{n=1}^{N} \frac{w_{n}}{w_{i}} s_{ni} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right).
\]

Using our result for the derivative of expenditure shares in equation (D.4) above, we can rewrite this as:
\[
d w_{i} = \sum_{n=1}^{N} \frac{s_{ni} w_{n} \ell_{n}}{w_{i} \ell_{i}} \left( \frac{dw_{n}}{w_{n}} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right).
\]
\[
\frac{d w_i}{w_i} = \sum_{n=1}^{N} t_{i n} \left( \frac{d w_n}{w_n} + (\sigma - 1) \left( \sum_{h \in N} s_{n h} \frac{d p_{n h}}{p_{n h}} - \frac{d p_{n i}}{p_{n i}} \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{i n} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h \in N} s_{n h} \ln p_{n h} - d \ln p_{n i} \right) \right),
\]

where we have defined \( t_{i n} \) as the share of country \( i \)'s income derived from market \( n \):

\[
t_{i n} = \frac{s_{n i} w_n}{w_i \ell_i}.
\]

### D.6 Utility Again

Returning to our expression for indirect utility, we have:

\[
u_n = w_n p_n.
\]

Totally differentiating indirect utility (D.9), we have:

\[
\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \frac{d p_n}{p_n},
\]

Using our total derivative of the sectoral price index in equation (D.6) above, we get:

\[
\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{m=1}^{N} s_{n m} \frac{d p_{n m}}{p_{n m}},
\]

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{n m} d \ln p_{n m}.
\]

### D.7 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[
d \ln \tau_{n i} = 0, \quad \forall \ n, i \in N.
\]

We start with our expression for the log change in wages from equation (D.8) above:

\[
d \ln w_i = \sum_{n=1}^{N} t_{i n} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h \in N} s_{n h} \ln p_{n h} - d \ln p_{n i} \right) \right),
\]

Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this expression for the log change in wages as:

\[
d \ln w_i = \sum_{n=1}^{N} t_{i n} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{n h} \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right),
\]

\[
\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{m=1}^{N} s_{n m} \frac{d p_{n m}}{p_{n m}},
\]

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{n m} d \ln p_{n m}.
\]

\[
\frac{d w_n}{w_n} + (\sigma - 1) \left( \sum_{h \in N} s_{n h} \frac{dp_{n h}}{p_{n h}} - \frac{dp_{n i}}{p_{n i}} \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{i n} \left( d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{n h} \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right) \right),
\]

\[
\frac{d u_n}{u_n} = \frac{d w_n}{w_n} - \sum_{m=1}^{N} s_{n m} \frac{d p_{n m}}{p_{n m}},
\]

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{n m} d \ln p_{n m}.
\]
which can be re-written as:

$$d \ln w_i = \sum_{n=1}^{N} t_{in} \ d \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \ [d \ln w_n - d \ln z_n] \right),$$

$$m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=1},$$

which has the following matrix representation in the paper:

$$d \ln w = T d \ln w + \theta M (d \ln w - d \ln z), \quad (D.13)$$

$$\theta = \sigma - 1.$$

We solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (D.13) by $\theta + 1$, we have:

$$\frac{1}{\theta + 1} d \ln w = \frac{1}{\theta + 1} T d \ln w + \frac{\theta}{\theta + 1} M (d \ln w - d \ln z),$$

$$\frac{1}{\theta + 1} (I - T - \theta M) d \ln w = - \frac{\theta}{\theta + 1} M d \ln z.$$

Now using $M = TS - I$, we have:

$$\frac{1}{\theta + 1} (I - T - \theta TS + \theta I) d \ln w = - \frac{\theta}{\theta + 1} M d \ln z,$$

$$\left( I - \frac{T + \theta TS}{\theta + 1} \right) d \ln w = - \frac{\theta}{\theta + 1} M d \ln z.$$

Using our choice of world GDP as numeraire, which implies $Q d \ln w = 0$, we have:

$$\left( I - \frac{T + \theta TS}{\theta + 1} + Q \right) d \ln w = - \frac{\theta}{\theta + 1} M d \ln z,$$

which can be re-written as:

$$(I - V) d \ln w = - \frac{\theta}{\theta + 1} M d \ln z,$$

$$V \equiv \frac{T + \theta TS}{\theta + 1} - Q,$$

which yields the following solution of the change in wages in response to a productivity shock:

$$d \ln w = - \frac{\theta}{\theta + 1} (I - V)^{-1} M d \ln z,$$

which can be re-written as:

$$d \ln w = W d \ln z,$$

where $W$ is our friend-enemy income exposure measure:

$$W \equiv - \frac{\theta}{\theta + 1} (I - V)^{-1} M. \quad (D.14)$$
D.8 Welfare and Productivity Shocks

From equation (D.10), the log change in utility is given by:

\[ \frac{d \ln u}{\ln w_n} = \sum_{m=1}^{N} s_{nm} \frac{d \ln p_{nm}}{\ln w_n}, \]

Using the total derivative of prices (D.5) and our assumption of constant bilateral trade costs (D.11), we can write this log change in utility as:

\[ \frac{d \ln u}{\ln w_n} = \frac{d \ln w}{\ln w_n} - \sum_{m=1}^{N} s_{nm} \left[ \frac{d \ln w}{\ln w_m} - \frac{d \ln z}{\ln z_m} \right], \quad (D.15) \]

which has the following matrix representation in the paper:

\[ \frac{d \ln u}{\ln w_n} = \frac{d \ln w}{\ln w} - S \left( \frac{d \ln w}{\ln w} - \frac{d \ln z}{\ln z} \right), \quad (D.16) \]

We can re-write the above relationship as:

\[ \frac{d \ln u}{\ln w_n} = (I - S) \frac{d \ln w}{\ln w} + S \frac{d \ln z}{\ln z}, \]

which using our solution for \( \frac{d \ln w}{\ln w} \) from above, can be further re-written as:

\[ \frac{d \ln u}{\ln w_n} = (I - S) W \frac{d \ln z}{\ln z} + S \frac{d \ln z}{\ln z}, \]

\[ \frac{d \ln u}{\ln w_n} = (I - S) W \frac{d \ln z}{\ln z} + S \frac{d \ln z}{\ln z}, \]

where \( U \) is our friend-enemy welfare exposure measure:

\[ U \equiv (I - S) W + S. \quad (D.17) \]

D.9 Market-size and Cross-Substitution Effects

We define the cross-substitution effect as the wage vector that solves equation (D.13) replacing the term \( T \frac{d \ln w}{\ln w} \) with \( Q \frac{d \ln w}{\ln w} \):

\[ \frac{d \ln w}{\ln w}^{\text{Sub}} = Q \frac{d \ln w}{\ln w}^{\text{Sub}} + \theta \cdot M \left( \frac{d \ln w}{\ln w}^{\text{Sub}} - \frac{d \ln z}{\ln z} \right), \]

where we use the superscript \( \text{Sub} \) to indicate the cross-substitution effect.

We solve for our friend-enemy measure of income exposure due to the cross-substitution effect using matrix inversion. Dividing both sides of the above equation by \( \theta + 1 \), we have:

\[ \frac{1}{\theta + 1} \frac{d \ln w}{\ln w}^{\text{Sub}} = \frac{1}{\theta + 1} Q \frac{d \ln w}{\ln w}^{\text{Sub}} + \frac{\theta}{\theta + 1} M \left( \frac{d \ln w}{\ln w}^{\text{Sub}} - \frac{d \ln z}{\ln z} \right), \]

\[ \frac{1}{\theta + 1} (I - Q - \theta M) \frac{d \ln w}{\ln w}^{\text{Sub}} = -\frac{\theta}{\theta + 1} M \frac{d \ln z}{\ln z}. \]

Now using \( M = TS - I \), we have:

\[ \frac{1}{\theta + 1} (I - Q - \theta TS + \theta I) \frac{d \ln w}{\ln w}^{\text{Sub}} = -\frac{\theta}{\theta + 1} M \frac{d \ln z}{\ln z}, \]

\[ \left( I - \frac{Q + \theta TS}{\theta + 1} \right) \frac{d \ln w}{\ln w}^{\text{Sub}} = -\frac{\theta}{\theta + 1} M \frac{d \ln z}{\ln z}. \]
Using our choice of world GDP as numeraire, which implies $Q_{\text{Sub}} = 0$, we have:

$$
\left( I - \frac{Q + \theta TS}{\theta + 1} + Q \right) d \ln w_{\text{Sub}} = - \frac{\theta}{\theta + 1} M d \ln z,
$$

$$
\left( I - \frac{\theta Q + \theta TS}{\theta + 1} \right) d \ln w_{\text{Sub}} = - \frac{\theta}{\theta + 1} M d \ln z.
$$

Now using $M = TS - I$, we have:

$$
\left( I - \theta Q + \theta TS \theta + 1 \right) d \ln w_{\text{Sub}} = - \frac{\theta}{\theta + 1} M d \ln z,
$$

$$(I - (\theta Q + \theta M)) d \ln w_{\text{Sub}} = - \theta M d \ln z,
$$

which yields the following solution of the change in wages in response to a productivity shock:

$$
d \ln w_{\text{Sub}} = - \theta (I - (\theta Q + \theta M))^{-1} M d \ln z,
$$

which can be re-written as:

$$
d \ln w_{\text{Sub}} = W_{\text{Sub}} d \ln z,
$$

where $W_{\text{Sub}}$ is our friend-enemy measure of income exposure due to the cross-substitution effect:

$$
W_{\text{Sub}} \equiv - \theta (I - (\theta Q + \theta M))^{-1} M.
$$

**D.10 Relationship to the ACR Gains from Trade Formula**

From equation (D.10), the log change in utility is given by:

$$
\text{d} \ln u_n = \text{d} \ln w_n - \sum_{m=1}^{N} s_{nm} \text{d} \ln p_{nm}.
$$

(D.18)

Choosing country $n$’s wage as the numeraire and assuming no changes in its productivity or domestic trade costs, we have:

$$
\text{d} \ln w_n = 0, \quad \text{d} \ln z_n = 0, \quad \text{d} \ln \tau_{nn} = 0, \quad \text{d} \ln p_{nn} = 0.
$$

The import demand system in equation (D.3) implies:

$$
\text{d} \ln s_{nm} - \text{d} \ln s_{nn} = - (\sigma - 1) (\text{d} \ln p_{nm} - \text{d} \ln p_{nn}).
$$

Using this result in equation (D.18), the log change in welfare can be written as:

$$
\text{d} \ln u_n = \sum_{m=1}^{N} s_{nm} \frac{(\text{d} \ln s_{nm} - \text{d} \ln s_{nn})}{(\sigma - 1)}.
$$

Using $\sum_{m=1}^{N} s_{nm} = 1$ and $\sum_{m=1}^{N} d s_{nm} = 0$, we obtain the ACR welfare gains from trade formula for small changes:

$$
\text{d} \ln u_n = - \frac{\text{d} \ln s_{nn}}{\sigma - 1}.
$$

(D.19)

Integrating both sides of equation (D.19), we get:

$$
\int_{u_n^0}^{u_n} \frac{d u_n}{u_n} = - (\sigma - 1) \int_{s_{nn}^1}^{s_{nn}} \frac{d s_{nn}}{s_{nn}}.
$$
\[
\ln \left( \frac{u_n^1}{w_n} \right) = - (\sigma - 1) \ln \left( \frac{s_n^1}{s_n^0} \right),
\]

which corresponds to the ACR welfare gains from trade formula for large changes.

E Single-Sector Isomorphisms

While for convenience of exposition we focus on the single-sector Armington model in the paper, we obtain the same income and welfare exposure measures for all models in the ACR class with a constant trade elasticity. In Subsection E.1, we derive our exposure measures in a version of the Eaton and Kortum (2002) model. In Subsection E.2, we derive these measures in the Krugman (1980) model.

The Armington model in the paper and the Eaton and Kortum (2002) model assume perfect competition and constant returns to scale, whereas the Krugman (1980) model assumes monopolistic competition and increasing returns to scale. In the Eaton and Kortum (2002) model, the trade elasticity corresponds to the Fréchet shape parameter. In contrast, in the Armington model and the Krugman (1980) model, the trade elasticity corresponds to the elasticity of substitution between varieties. Nevertheless, our income and welfare exposure measures hold across all three models, because they all feature a constant trade elasticity.

In Subsection E.2, we focus on the representative firm Krugman (1980) model of monopolistic competition for simplicity, but our income and welfare exposure measures also hold in the heterogeneous firm model of Melitz (2003) with an untruncated Pareto productivity distribution. In Subsection E.2, we also show that our income and welfare exposure measures with monopolistic competition and increasing returns to scale take a similar form whether we consider shocks to the variable or fixed components of production costs.

E.1 Eaton and Kortum (2002)

We consider a version of Eaton and Kortum (2002) with labor as the sole factor of production. Trade arises because of Ricardian technology differences; production technologies are constant returns to scale; and markets are perfectly competitive. The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

E.1.1 Consumer Preferences

The preferences of the representative consumer in country \( n \) are characterized by the following indirect utility function:

\[
\ln \left( \frac{u_n^1}{w_n} \right) = - (\sigma - 1) \ln \left( \frac{s_n^1}{s_n^0} \right),
\]

where \( w_n \) is the wage and \( p_n \) is the consumption goods price index, which is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

\[
p_n = \left[ \int_0^1 p_n(\vartheta)^{1-\sigma} \, d\vartheta \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,
\]

where \( p_n(\vartheta) \) denotes the price of good \( \vartheta \) in country \( n \).
E.1.2 Production Technology

Goods are produced with labor according to a constant returns to scale production technology. These goods can be traded between countries subject to iceberg variable costs of trade, such that \( \tau_{ni} \geq 1 \) units must be shipped from country \( i \) to country \( n \) in order for one unit to arrive (where \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \)). Therefore, the price for consumers in country \( n \) of purchasing a good \( \vartheta \) from country \( i \) is:

\[
p_{ni}(\vartheta) = \frac{\tau_{ni} w_i}{z_i a_i(\vartheta)},
\]

where \( z_i \) captures common determinants of productivity across goods within country \( i \) and \( a_i(\vartheta) \) captures idiosyncratic determinants of productivity for each good \( \vartheta \) within that country. Iceberg variable trade costs satisfy \( \tau_{ni} > 1 \) for \( n \neq i \) and \( \tau_{nn} = 1 \). Productivity for each good \( \vartheta \) in each sector \( k \) and each country \( i \) is drawn independently from the following Fréchet distribution:

\[
F_i(a) = \exp(-a - \theta), \quad \theta > 1,
\]

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( z_i \).

E.1.3 Expenditure Shares

Using the properties of this Fréchet distribution, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[
s_{ni} = \left( \frac{\tau_{ni} w_i}{z_i} \right)^{-\theta} = \frac{(\rho_{ni})^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm})^{-\theta}},
\]

where \( \rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i} \).

Totally differentiating this expenditure share equation (E.5) we get:

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{dp_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} \frac{\theta \frac{d\rho_{nh}}{\rho_{nh}} (\rho_{nh})^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm})^{-\theta}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = -\theta \frac{dp_{ni}}{\rho_{ni}} + \sum_{h=1}^{N} s_{nh} \theta \frac{d\rho_{nh}}{\rho_{nh}},
\]

\[
\frac{ds_{ni}}{s_{ni}} = \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right),
\]

\[
d \ln s_{ni} = \theta \left( \sum_{h=1}^{N} s_{nh} d \ln \rho_{nh} - d \ln \rho_{ni} \right),
\]

where, from the definition of \( \rho_{ni} \) in equation (E.6) above, we have:

\[
\frac{d\rho_{ni}}{\rho_{ni}} = \frac{d\tau_{ni}}{\tau_{ni}} + \frac{dw_i}{w_i} - \frac{dz_i}{z_i},
\]

\[
d \ln \rho_{ni} = d \ln \tau_{ni} + d \ln w_i - d \ln z_i.
\]
E.1.4 Price Indices

Using the properties of the Fréchet distribution (E.4), the consumption goods price index is given by:

\[ p_n = \gamma \left[ \sum_{m=1}^{N} (\rho_{nm})^{-\theta} \right]^{-\frac{1}{\theta}}, \]  

(E.9)

where

\[ \gamma \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{-\frac{1}{\theta}}, \]

and \( \Gamma(\cdot) \) is the Gamma function. Totally differentiating this price index (E.9), we have:

\[ dp_n = \sum_{m=1}^{N} \gamma \frac{d\rho_{nm}}{\rho_{nm}} \left( \sum_{h=1}^{N} (\rho_{nh})^{-\theta} \right)^{-\frac{1}{\theta}}, \]

\[ \frac{dp_n}{p_n} = \sum_{m=1}^{N} \frac{d\rho_{nm}}{\rho_{nm}} \left( \sum_{h=1}^{N} (\rho_{nh})^{-\theta} \right), \]

\[ dp_n \equiv \sum_{m=1}^{N} s_{nm} d\rho_{nm}, \]  

(E.10)

\[ d \ln p_n = \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}. \]

E.1.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \]  

(E.11)

Totally differentiating this market clearing condition (E.11), holding labor endowments constant, we have:

\[ \frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n, \]

\[ \frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right), \]

\[ \frac{dw_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_i \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right). \]

Using our result for the derivative of expenditure shares in equation (E.7) above, we can rewrite this as:

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_i \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right) \right), \]

\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh} \frac{d\rho_{nh}}{\rho_{nh}} - \frac{d\rho_{ni}}{\rho_{ni}} \right) \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} d \ln \rho_{nh} - d \ln \rho_{ni} \right) \right), \]  

(E.12)

where we have defined \( t_{in} \) as country \( n \)'s expenditure on country \( i \)'s income:

\[ t_{in} \equiv \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \]
E.1.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n}. \]  
\[(E.13)\]

Totally differentiating indirect utility (E.13), we have:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n} \]
\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}. \]

Using our total derivative of the sectoral price index in equation (E.10) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \frac{d \rho_{nm}}{\rho_{nm}}. \] 
\[(E.14)\]

\[ d \ln u_n = d \ln w_n - N \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}. \]
\[(E.15)\]

\[ d \ln w = \mathbf{T} d \ln \mathbf{w} + \theta \mathbf{M} ( d \ln \mathbf{w} - d \ln \mathbf{z} ), \]
\[ \theta = \sigma - 1. \]  
\[(E.17)\]

E.1.7 Wages and Common Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[ d \ln \tau_{ni} = 0, \quad \forall n, i \in N. \]  
\[(E.15)\]

We start with our expression for the log change in wages from equation (E.12) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right) \right), \]
\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right), \]
\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \theta \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} (d \ln w_h - d \ln z_h) - (d \ln w_i - d \ln z_i) \right). \]  
\[(E.16)\]

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} (d \ln w_n - d \ln z_n) \right), \]
\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]

which has the same matrix representation as in equation (15) in the paper:
E.1.8 Utility and Common Productivity Shocks

From equation (E.14), the log change in utility is given by:

$$d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln \rho_{nm}.$$ 

Using the total derivative of the price term (E.8) and our assumption of constant bilateral trade costs (E.15), we can write this log change in utility as:

$$d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ \frac{d \ln w_m - d \ln z_m}{2} \right],$$

which has the same matrix representation as in equation (20) in the paper:

$$d \ln u = d \ln w - S \left( d \ln w - d \ln z \right). \quad (E.18)$$

E.2 Krugman (1980)

We consider a version of Krugman (1980) with labor as the sole factor of production, in which markets are monopolistically competitive, and trade arises from love of variety and increasing returns to scale. The world economy consists of a set of countries indexed by $i, n \in \{1, \ldots, N \}$. Each country $n$ has an exogenous supply of labor $\ell_n$.

E.2.1 Consumer Preferences

The preferences of the representative consumer in country $n$ are characterized by the following indirect utility function:

$$u_n = \frac{w_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} \int_{0}^{M_i} p_{ni} (j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,$$

where $w_n$ is the wage; $p_n$ is the consumption goods price index; $p_{ni} (j)$ is the price in country $n$ of a variety $j$ produced in country $i$; $M_i$ is the endogenous mass of varieties; and varieties are substitutes ($\sigma > 1$).

E.2.2 Production Technology

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of $F$ units of labor and a constant variable cost in terms of labor that depends on a country’s productivity $z_i$. Therefore the total amount of labor ($l_i (j)$) required to produce $x_i (j)$ units of variety $j$ in country $i$ is:

$$l_i (j) = F_i + \frac{x_i (j)}{z_i}.$$ 

(E.20)

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country $i$ to country $n$ in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Profit maximization and zero profits imply that equilibrium prices are a constant markup over marginal cost:

$$p_{ni} (j) = \left( \frac{\sigma}{\sigma - 1} \right) \rho_{ni}, \quad \rho_{ni} \equiv \frac{\tau_{ni} w_i}{z_i},$$

and equilibrium employment for each variety is equal to a constant:

$$l_i (j) = \bar{l} = (\sigma - 1) F_i.$$ 

(E.22)
Given this constant equilibrium employment for each variety, labor market clearing implies that the total mass of varieties supplied by each country is proportional to its labor endowment:

\[ M_i = \frac{\ell_i}{\sigma F_i}. \]  

(E.23)

From the definition of \( \rho_{ni} \) in equation (E.21) above, we have:

\[ \frac{d \rho_{ni}}{\rho_{ni}} = \frac{d \tau_{ni}}{\tau_{ni}} + \frac{d w_i}{w_i} - \frac{d z_i}{z_i}, \]

(d.24)

\[ \ln \rho_{ni} = \ln \tau_{ni} + \ln w_i - \ln z_i. \]

Totally differentiating in the labor market clearing condition (E.23), holding country endowments constant, we have:

\[ \frac{d M_i}{M_i} = -\frac{d F_i}{F_i}, \]

(E.25)

\[ \ln M_i = -\ln F_i. \]

E.2.3 Expenditure Shares

Using the symmetry of equilibrium prices and the properties of the CES demand function, country \( n \)'s share of expenditure on goods produced in country \( i \) is:

\[ s_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_m p_m^{1-\sigma}} = \frac{(\ell_i/F_i) \rho_{ni}^{1-\sigma}}{\sum_{m=1}^{N} (\ell_m/F_m) \rho_{nm}^{1-\sigma}}. \]

(E.26)

Totally differentiating this expenditure share equation (E.26) we get:

\[ \frac{d s_{ni}}{s_{ni}} = \left[ \frac{d F_i}{F_i} + (\sigma - 1) \frac{d \rho_{ni}}{\rho_{ni}} \right] + \sum_{h=1}^{N} s_{nh} \left[ \frac{d F_h}{F_h} + (\sigma - 1) \frac{d \rho_{nh}}{\rho_{nh}} \right], \]

(E.27)

\[ \ln s_{ni} = -\left[ \ln F_i + (\sigma - 1) \ln \rho_{ni} \right] + \sum_{h=1}^{N} s_{nh} \left[ \ln F_h + (\sigma - 1) \ln \rho_{nh} \right]. \]

E.2.4 Price Indices

Using the symmetry of equilibrium prices, the price index (E.19) can be re-written as:

\[ p_n = \left[ \sum_{i=1}^{N} M_i p_{ni}^{1-\sigma} \right]^{1/(1-\sigma)}. \]

Totally differentiating this price index, we have:

\[ \frac{d p_n}{p_n} = \sum_{i=1}^{N} \left[ \frac{1}{1-\sigma} \frac{d M_i}{M_i} + \frac{d p_{ni}}{p_{ni}} \right] M_i p_{ni}^{1-\sigma} \left[ \sum_{h=1}^{N} M_h p_h^{1-\sigma} \right]^{1/(1-\sigma)}, \]

\[ \frac{d p_{ni}}{p_{ni}} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{1-\sigma} \frac{d M_i}{M_i} + \frac{d p_{ni}}{p_{ni}} \right] \frac{p_{ni}^{1-\sigma}}{\sum_{h=1}^{N} p_h^{1-\sigma}}, \]

which using the equilibrium pricing rule (E.21) and the labor market clearing condition (E.23) can be written as:

\[ \frac{d p_n}{p_n} = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \frac{d F_i}{F_i} + \frac{d \rho_{ni}}{\rho_{ni}} \right], \]

(E.28)

\[ \ln p_n = \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} \ln F_i + \ln \rho_{ni} \right]. \]
E.2.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n. \] (E.29)

Totally differentiating this market clearing condition (E.29), holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} \frac{ds_{ni}}{s_{ni}} s_{ni} w_n \ell_n + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n,
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_{ni}}{s_{ni}} \right) + \sum_{n=1}^{N} s_{ni} \frac{dw_n}{w_n} w_n \ell_n.
\]

Using our result for the derivative of expenditure shares in equation (E.27) above, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} \right) + \left( \sigma - 1 \right) \left( \sum_{n=1}^{N} s_{ni} \frac{dp_n}{\rho_n} - \frac{dp_n}{\rho_n} \right)
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_n}{w_n} \right) + \left( \sigma - 1 \right) \sum_{n=1}^{N} \frac{d \ln w_n}{w_n} \rho_n d \ln \rho_n - \frac{d \ln \rho_n}{\rho_n}
\]

where we have defined \( t_{in} \) as the share of country \( i \)'s income derived from market \( n \):

\[ t_{in} = \frac{s_{ni} w_n \ell_n}{w_i \ell_i}. \]

E.2.6 Utility Again

Returning to our expression for indirect utility, we have:

\[ u_n = \frac{w_n}{p_n}. \] (E.31)

Totally differentiating indirect utility (E.31), we have:

\[
\frac{du_n}{w_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n},
\]

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \frac{dp_n}{p_n}.
\]

Using our total derivative of the sectoral price index in equation (E.28) above, we get:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} \frac{dF_i}{F_i} + \frac{d \ln \rho_n}{\rho_n} \right], \] (E.32)

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_n \right].
\]
E.2.7 Wages and Productivity Shocks

We consider small shocks to productivity, holding constant bilateral trade costs and fixed costs:

$$d \ln r_{ni} = 0, \quad d \ln F_i = 0, \quad \forall n, i \in N.$$  \hfill (E.33)

We start with our expression for the log change in wages from equation (E.30) above:

$$d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \left( d \ln F_h - d \ln F_i \right) \right) \right),$$

which can be re-written as:

$$d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \left[ d \ln w_h - d \ln z_h \right] - \left[ d \ln w_i - d \ln z_i \right] \right),$$

which has the same matrix representation as in equation (15) in the paper:

$$d \ln w = T d \ln w + \theta M ( d \ln w - d \ln z ),$$

$$\theta = \sigma - 1.$$  \hfill (E.35)

E.2.8 Welfare and Productivity Shocks

From equation (E.32), the log change in utility is given by:

$$d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_{ni} \right],$$

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant fixed costs (E.33), we can write this log change in utility as:

$$d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ d \ln w_i - d \ln z_i \right],$$

which has the same matrix representation as in equation (20) in the paper:

$$d \ln u = d \ln w - S ( d \ln w - d \ln z ).$$  \hfill (E.37)
E.2.9 Wages and Fixed Cost Shocks

We next consider small shocks to fixed costs, holding constant bilateral trade costs and productivity:

\[ d \ln \tau_{ni} = 0, \quad d \ln z_i = 0, \quad \forall \ n, i \in N. \quad (E.38) \]

We start with our expression for the log change in wages from equation (E.30) above:

\[
\begin{align*}
& d \ln w_i = \sum_{n=1}^{N} t_i \ln \left( d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \ d \ln F_h - d \ln F_i \right) \right), \\
& d \ln F_i = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \ d \ln \rho_{ih} - d \ln \rho_{ni} \right).
\end{align*}
\]

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this expression for the log change in wages as:

\[
\begin{align*}
& d \ln w_i = \sum_{n=1}^{N} t_i \ln \left( d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \ d \ln F_h - d \ln F_i \right) \right), \\
& d \ln F_i = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \ d \ln \rho_{ih} - d \ln \rho_{ni} \right).
\end{align*}
\]

which can be re-written as:

\[
\begin{align*}
& d \ln w_i = \sum_{n=1}^{N} t_i \ln \left( d \ln w_n + \left( \sum_{h=1}^{N} s_{nh} \ d \ln F_h - d \ln F_i \right) \right), \\
& m_{in} = \sum_{h=1}^{N} t_i s_{hn} - 1_{n=i},
\end{align*}
\]

which has a similar matrix representation to that for productivity shocks above:

\[
\begin{align*}
& d \ln u_n = T d \ln w + \theta M \left( d \ln w + \frac{1}{\theta} d \ln F \right), \quad (E.40) \\
& \theta = \sigma - 1.
\end{align*}
\]

E.2.10 Welfare and Fixed Cost Shocks

From equation (E.32), the log change in utility is given by:

\[
\begin{align*}
& d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln \rho_{ni} \right], \\
& d \ln F_i = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \ d \ln \rho_{ih} - d \ln \rho_{ni} \right).
\end{align*}
\]

Using the total derivative of the price term (E.24) and our assumptions of constant bilateral trade costs and constant productivity (E.38), we can write this log change in utility as:

\[
\begin{align*}
& d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln w_i \right], \quad (E.41) \\
& d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left[ \frac{1}{\sigma - 1} d \ln F_i + d \ln w_i \right],
\end{align*}
\]

which has a similar matrix representation to that for productivity shocks above:

\[
\begin{align*}
& d \ln u = d \ln w - S \left( d \ln w + \frac{1}{\theta} d \ln F \right), \quad (E.42) \\
& \theta = \sigma - 1.
\end{align*}
\]
F Extensions

In Subsection F.1, we derive the corresponding friend-enemy matrix representations with both productivity and trade cost shocks, as discussed in Section 5.1 of the paper. In Section F.2, we relax one of the ACR macro restrictions to allow for trade imbalance, as discussed in Section 5.2 of the paper. In Section F.3, we relax another of the ACR macro restrictions to consider small deviations from a constant elasticity import demand system, as discussed in Section 5.3 of the paper. In Section F.4, we show that our results generalize to a multi-sector Armington model with a single constant trade elasticity, as discussed in Section 5.4 of the paper. In Section F.6, we show that our results also hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). In Section F.7, we further extend the multi-sector specification to introduce input-output linkages following Caliendo and Parro (2015), as discussed in Section 5.5 of the paper.

F.1 Trade Cost Reductions

We now show that we obtain similar results incorporating trade cost reductions. We start with our expression for the log change in wages from equation (D.8) above:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \left( \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \ln p_{nh} - \ln p_{ni} \right) \right), \]

which can be re-written as follows:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \left[ \ln w_h + \ln \tau_{nh} - \ln z_h \right] - \left[ \ln w_i + \ln \tau_{ni} - \ln z_i \right] \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \sum_{n=1}^{N} t_{in} \left( \sum_{h=1}^{N} s_{nh} \left[ \ln w_h + \ln \tau_{nh} - \ln z_h \right] - \left[ \ln w_i + \ln \tau_{ni} - \ln z_i \right] \right), \]

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{ih} s_{hn} \left[ \ln w_h + \ln \tau_{nh} - \ln z_h \right] - \sum_{n=1}^{N} t_{in} \left[ \ln w_i + \ln \tau_{ni} - \ln z_i \right] \right). \]

We now define inward and outward measures of trade costs as:

\[ d \ln \tau_{ni}^{\text{in}} \equiv \sum_i s_{ni} \ln \tau_{ni}, \]

\[ d \ln \tau_{ni}^{\text{out}} \equiv \sum_{n=1}^{N} t_{in} \ln \tau_{ni}. \]

Using these definitions of inward and outward trade costs, we can rewrite the above proportional change in wages as follows:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{ih} s_{hn} \left[ \ln w_h - \ln z_h \right] - \left[ \ln w_i - \ln z_i \right] \right), \]

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} m_{in} \left[ \ln w_n - \ln z_n \right] + \sum_{n=1}^{N} t_{in} \ln \tau_{ni}^{\text{in}} - \ln \tau_{ni}^{\text{out}} \right), \]

\[ m_{in} = \sum_{h=1}^{N} t_{ih} s_{hn} - 1_{n=i}, \]
which has the following matrix representation from equation (28) in the paper:

\[
d \ln \mathbf{w} = \mathbf{T} d \ln \mathbf{w} + \theta \left[ \mathbf{M} \left( d \ln \mathbf{w} - d \ln \mathbf{z} \right) + \mathbf{T} d \ln \tau^\text{in} - d \ln \tau^\text{out} \right],
\]

\[
\theta = \sigma - 1.
\]

We next consider our expression for the log change in utility from equation (D.10) above:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm},
\]

which can be re-written as follows:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left[ d \ln w_m + d \ln \tau_{nm} - d \ln z_m \right].
\]

Using our definition of inward trade costs from equation (F.1), we can re-write this proportional change in welfare as:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left( d \ln w_m - d \ln z_m \right),
\]

which has the following matrix representation from equation (29) in the paper:

\[
d \ln \mathbf{u} = d \ln \mathbf{w} - \mathbf{S} \left( d \ln \mathbf{w} - d \ln \mathbf{z} \right) - d \ln \tau^\text{in}.
\]

**F.2 Trade Imbalance**

In this section of the online appendix, we relax another of the ACR macro restrictions to allow for trade imbalance. In particular, we consider the constant elasticity Armington model from Section 3 of the paper, but allow expenditure to differ from income.

**F.2.1 Preferences and Expenditure Shares**

We measure the instantaneous welfare of the representative agent as the real value of expenditure:

\[
u_n = \frac{w_n \ell_n + \bar{d}_n}{p_n}, \quad p_n = \left[ \sum_{i=1}^{N} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \sigma > 1,
\]

where \( \bar{d}_n \) is the nominal trade deficit. Expenditure shares take the same form as in equation (12) in Section 3 of the paper.

**F.2.2 Market Clearing**

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[
w_i \ell_i = \sum_{n=1}^{N} s_{ni} \left[ w_n \ell_n + \bar{d}_n \right].
\]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{d w_i}{w_i} \ell_i = \sum_{n=1}^{N} s_{ni} \left[ \frac{d s_{ni}}{s_{ni}} \right] + \sum_{n=1}^{N} s_{ni} \left[ \frac{d w_n}{w_n} \ell_n + \sum_{n=1}^{N} s_{ni} \left( \frac{d \bar{d}_n}{d \bar{d}_n} + \frac{d \bar{d}_n}{w_n} \right) \right] + \sum_{n=1}^{N} s_{ni} \left( \frac{d s_{ni}}{s_{ni}} + \frac{d \bar{d}_n}{d \bar{d}_n} \right),
\]

\[
= \sum_{n=1}^{N} s_{ni} \left( \frac{d s_{ni}}{s_{ni}} \right) \ell_n + \sum_{n=1}^{N} s_{ni} \left( \frac{d w_n}{w_n} \ell_n \right) + \sum_{n=1}^{N} s_{ni} \left( \frac{d \bar{d}_n}{d \bar{d}_n} \right) + \sum_{n=1}^{N} s_{ni} \left( \frac{d s_{ni}}{s_{ni}} + \frac{d \bar{d}_n}{d \bar{d}_n} \right),
\]

\[
= \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \bar{d}_n \right) \left( \frac{d s_{ni}}{s_{ni}} + \frac{d w_n}{w_n} \right) + \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \bar{d}_n \right) \left( \frac{d \bar{d}_n}{w_n \ell_n + \bar{d}_n} + \frac{d \bar{d}_n}{d \bar{d}_n} \right).
\]
\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{d s_{ni}}{s_{ni}} + \frac{d w_n}{w_n} \right) + \sum_{n=1}^{N} s_{ni} \left( w_n \ell_n + \tilde{d}_n \right) \left( \frac{d s_{ni}}{s_{ni}} + \frac{d \tilde{d}_n}{\tilde{d}_n} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left( \frac{d s_{ni}}{s_{ni}} + \frac{d w_n}{w_n} \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \left( \frac{d s_{ni}}{s_{ni}} + \frac{d \tilde{d}_n}{\tilde{d}_n} \right),
\]

where we have defined \( t_{in} \) as the share of country \( i \)'s income from market \( n \):

\[
t_{in} = \frac{s_{ni} \left( w_n \ell_n + \tilde{d}_n \right)}{w_i \ell_i}
\]

and \( d_n \) as country \( n \)'s ratio of income to expenditure:

\[
d_n = \frac{w_n \ell_n}{w_n \ell_n + d_n}.
\]

Using our result for the derivative of expenditure shares above, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^{N} t_{in} d_n \left( \frac{d w_n}{w_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) \left( \frac{d d_n}{d_n} + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right).
\]

We can re-write this expression as:

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d_n d \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} d \ln p_{nh} - d \ln p_{ni} \right) \right) + \sum_{n=1}^{N} t_{in} (1 - d_n) d \ln \tilde{d}_n.
\]

### F.2.3 Welfare

Returning to our expression for welfare (F.6), we have:

\[
u_n = \frac{w_n \ell_n + \tilde{d}_n}{p_n}.
\]

Totally differentiating welfare, holding labor endowments constant, we have:

\[
d \ln \frac{d u_n}{u_n} = \frac{d w_n}{w_n} \ell_n + \frac{d \tilde{d}_n}{\tilde{d}_n} \left[ \frac{d w_n}{w_n} \ell_n + \tilde{d}_n \right] - \frac{d p_n}{p_n} \left[ \frac{d w_n}{w_n} \ell_n + \tilde{d}_n \right],
\]

\[
\frac{d u_n}{u_n} = \frac{w_n \ell_n}{w_n \ell_n + d_n} \frac{d w_n}{w_n} + \frac{\tilde{d}_n}{d_n} \frac{d d_n}{d_n} - \frac{d p_n}{p_n},
\]

\[
= d_n \frac{d w_n}{w_n} + (1 - d_n) \frac{d d_n}{d_n} - \frac{d p_n}{p_n}.
\]

Using the total derivative of the sectoral price index, we get:

\[
\frac{d u_n}{u_n} = d_n \frac{d w_n}{w_n} + (1 - d_n) \frac{d d_n}{d_n} - \sum_{m=1}^{N} s_{nm} \frac{dp_{nm}}{p_{nm}},
\]

which can be re-written as:

\[
d \ln u_n = d_n d \ln w_n + (1 - d_n) d \ln \tilde{d}_n - \sum_{m=1}^{N} s_{nm} d \ln p_{nm}.
\]
F.2.4 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[ \frac{d \ln \tau_{ni}}{n, i \in N}, \quad (F.10) \]

and we assume that trade deficits remain constant in terms of our numeraire of world GDP:

\[ \frac{d \ln \bar{d}_n}{n \in N} = 0. \]

Under these assumptions, the change in prices is:

\[ \frac{d \ln p_{ni}}{n, i \in N} = \frac{d \ln w_i - d \ln z_i}{n, i \in N}. \]

We start with our expression for the log change in wages in equation (F.8) above. With constant trade deficits \( \frac{d \ln \bar{d}_n}{n \in N} = 0 \), this expression simplifies to:

\[ \frac{d \ln w_i}{n, i \in N} = \sum_{n=1}^{N} t_{in} \left( d_n \ln w_n + (\sigma - 1) \left( \sum_{n=1}^{N} s_{nh} \frac{d \ln p_{nh}}{n, i \in N} - d \ln p_{ni} \right) \right). \]

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), this further simplifies to:

\[ \frac{d \ln w_i}{n, i \in N} = \sum_{n=1}^{N} d_n t_{in} \ln w_n + (\sigma - 1) \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} s_{nh} \frac{d \ln w_h}{n, i \in N} - \frac{d \ln w_i}{n, i \in N} \right). \quad (F.11) \]

which has the following matrix representation:

\[ \frac{d \ln w}{n, i \in N} = \theta M \left( \frac{d \ln w}{n, i \in N} - \frac{d \ln z}{n, i \in N} \right), \quad (F.12) \]

where the matrices \( T \) and \( M \) are defined in Section D of this online appendix. The diagonal matrix \( D \) captures trade deficits through the ratio of income to expenditure:

\[ D = \begin{pmatrix}
\frac{d_1}{w_n \ell_n + \bar{d}_n} & 0 & 0 & 0 \\
0 & \frac{d_2}{w_n \ell_n + \bar{d}_n} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \frac{d_N}{w_n \ell_n + \bar{d}_n} \\
\end{pmatrix}_{N \times N}, \quad d_n = \frac{w_n \ell_n}{w_n \ell_n + \bar{d}_n}. \]
F.2.5 Welfare and Productivity Shocks

We start with our expression for the log change in welfare in equation (F.9) above. With constant trade deficits \( (d\ln d_n = 0) \), this simplifies to:

\[
d\ln u_n = d_n \ln w_n - \sum_{m=1}^{N} s_{nm} d\ln p_{nm}. \tag{F.13}
\]

Using the total derivative of prices and our assumption of constant bilateral trade costs (F.10), we can write this proportional change in utility as:

\[
d\ln u_n = d_n \ln w_n - \sum_{m=1}^{N} s_{nm} (d\ln w_m - d\ln z_m), \tag{F.13}
\]

which has the following matrix representation:

\[
d\ln u = D\ln w - S (d\ln w - d\ln z), \tag{F.14}
\]

where the matrix \( S \) is defined in Section D of this online appendix and the diagonal matrix \( D \) is defined in the previous subsection of this online appendix.

F.3 Deviations from Constant Elasticity Import Demand

In this section of the online appendix, we consider the generalization of the constant elasticity Armington model in the previous section to incorporate small deviations from a constant elasticity import demand system, as discussed in Section 5.3 of the paper.

F.3.1 Expenditure Shares

We start from the total derivative for expenditure shares in the Armington model with a general homothetic consumption goods price index in equation (C.4) in Section C of this online appendix, as reproduced below:

\[
\frac{d\ln s_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nih} - \sum_{k=1}^{N} s_{nk}\theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}}, \tag{F.15}
\]

where \( \theta_{nih} \) is the elasticity of expenditure in country \( n \) on the good produced by country \( i \) with respect to the price of the good produced by country \( h \). In the special case of a constant elasticity import demand, we have:

\[
\theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) & \text{if } i = h \\ s_{nh}(\sigma - 1) & \text{otherwise} \end{cases}. \tag{F.16}
\]

Using this result in equation (F.15), we obtain the total derivative for expenditure shares with a constant elasticity import demand system in equation (D.4) in Section D of this online appendix, as reproduced below:

\[
\frac{d\ln s_{ni}}{s_{ni}} = (\sigma - 1) \left( \sum_{h=1}^{N} s_{nh} \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right). \tag{F.17}
\]

We now allow for deviations from a constant elasticity import demand system by considering the following generalization of the specification in equation (F.16):

\[
\theta_{nih} = \begin{cases} (s_{nh} - 1)(\sigma - 1) + o_{nih} & \text{if } i = h \\ s_{nh}(\sigma - 1) + o_{nih} & \text{otherwise} \end{cases}. \tag{F.18}
\]
We assume that this generalized demand specification remains homothetic, which implies:

\[
0 = \sum_{k} x_{nk} \theta_{nkh},
\]

\[
= \sum_{k=1}^{N} x_{nk} \left( (\sigma - 1) s_{nh} + o_{nkh} \right) - (\sigma - 1) x_{nh},
\]

\[
= \sum_{k=1}^{N} x_{nk} o_{nkh},
\]

where \( x_{ni} \) denotes country \( n \)’s expenditure on the goods produced by country \( i \). Therefore, homotheticity implies:

\[
\sum_{k=1}^{N} s_{nk} o_{nkh} = 0.
\]

Using our generalized demand specification (F.18) in the total derivative of expenditure shares in equation (F.15), we obtain:

\[
\frac{ds_{ni}}{s_{ni}} = \sum_{h=1}^{N} \left[ \theta_{nhi} - \sum_{k=1}^{N} s_{nk} \theta_{nkh} \right] \frac{dp_{nh}}{p_{nh}},
\]

\[
= (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[ \left( s_{nh} + \frac{o_{nhi}}{\sigma - 1} - 1_{i=h} \right) - \sum_{k=1}^{N} s_{nk} \left( s_{nh} + \frac{o_{nkh}}{\sigma - 1} \right) - s_{nh} \right] \frac{dp_{nh}}{p_{nh}} \right\},
\]

\[
= (\sigma - 1) \left\{ \sum_{h=1}^{N} \left[ s_{nh} + \frac{o_{nhi}}{\sigma - 1} - \sum_{k=1}^{N} s_{nk} \frac{o_{nkh}}{\sigma - 1} \right] \frac{dp_{nh}}{p_{nh}} - (1 + o_{nii}) \frac{dp_{ni}}{p_{ni}} \right\}.
\]

Using our implication of homotheticity in equation (F.19), this expression simplifies to:

\[
\frac{ds_{ni}}{s_{ni}} = \left\{ \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] \frac{dp_{nh}}{p_{nh}} - [(\sigma - 1) + o_{nii}] \frac{dp_{ni}}{p_{ni}} \right\}.
\]  

(F.20)

### F.3.2 Market Clearing

Market clearing again requires that income in each country equals expenditure on goods produced in that country:

\[
w_{i} \ell_{i} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n}.
\]

(F.21)

Totally differentiating this market clearing condition, we obtain:

\[
\frac{dw_{i}}{w_{i}} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \frac{ds_{ni}}{s_{ni}} \right).
\]

(F.22)

Using our result for the derivative of expenditure shares (F.20) in the above equation, we obtain:

\[
\frac{dw_{i}}{w_{i}} = \sum_{n=1}^{N} s_{ni} w_{n} \ell_{n} \left( \frac{dw_{n}}{w_{n}} + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]

\[
\frac{dw_{i}}{w_{i}} = \sum_{n=1}^{N} t_{in} \left( \frac{dw_{n}}{w_{n}} + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] \frac{dp_{nh}}{p_{nh}} - \frac{dp_{ni}}{p_{ni}} \right) \right),
\]

\[
d \ln w_{i} = \sum_{n=1}^{N} t_{in} \left( d \ln w_{n} + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nhi} \right] d \ln p_{nh} - d \ln p_{ni} \right) \right),
\]

(F.23)

where we have defined \( t_{in} \) as the share of country \( i \)’s income derived from market \( n \):

\[
t_{in} \equiv \frac{s_{ni} w_{n} \ell_{n}}{w_{i} \ell_{i}}.
\]
F.3.3 Welfare

From equation (C.8) in the Armington model with a general homothetic consumption goods price index, we also have the following expression for the total derivative of welfare:

\[
d \ln u_n = d \ln w_n - \sum_{i=1}^{N} s_{ni} \left( d \ln \tau_{ni} + d \ln w_i - d \ln z_i \right),
\]  

(F.24)

F.3.4 Wages and Productivity Shocks

We consider small productivity shocks, holding constant bilateral trade costs:

\[
d \ln \tau_{ni} = 0, \quad \forall n, i \in N.  
\]

(F.25)

We start with our expression for the log change in wages in equation (F.23) above. Using the total derivative of prices and our assumption of constant bilateral trade costs (F.25), we can write this expression for the log change in wages as:

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \left( d \ln w_n + \left( \sum_{h=1}^{N} \left[ (\sigma - 1) s_{nh} + o_{nih} \right] \left( d \ln w_h - d \ln z_h \right) \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} \left[ (\sigma - 1) s_{nh} + o_{nih} \right] \left( d \ln w_h - d \ln z_h \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} \left[ (\sigma - 1) s_{nh} + o_{nih} \right] \left( d \ln w_h - d \ln z_h \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} \left[ (\sigma - 1) s_{nh} + o_{nih} \right] \left( d \ln w_h - d \ln z_h \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} d \ln w_n + \left( \sum_{h=1}^{N} \sum_{n=1}^{N} t_{in} \left[ (\sigma - 1) s_{nh} + o_{nih} \right] \left( d \ln w_h - d \ln z_h \right) \right),
\]

which has the following matrix representation in equation (38) in the paper:

\[
d \ln w = T d \ln w + (\theta M + O)(d \ln w - d \ln z),
\]

(F.26)

\[\theta = (\sigma - 1).\]

F.3.5 Welfare and Productivity Shocks

Using our assumption of constant bilateral trade costs (F.25) in our expression for the log change in welfare (F.24) above, we obtain:

\[
d \ln u_n = d \ln w_n - \sum_{m=1}^{N} s_{nm} \left( d \ln w_m - d \ln z_m \right),
\]

(F.27)

which the following matrix representation in equation (39) in the paper:

\[
d \ln u = d \ln w - S (d \ln w - d \ln z).
\]

(F.28)
F.4 Multiple Sectors

In this section of the online appendix, we show that our friends-and-enemies exposure measures extend naturally to a multi-sector version of the constant elasticity Armington model, as discussed in Section 5.4 of the paper. The world economy consists of many countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors indexed by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

F.4.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[
 u_n = \frac{w_n}{\prod_{k=1}^{K} (p_{kn})^{\alpha_{kn}}}, \quad \sum_{k=1}^{K} \alpha_{kn} = 1. \tag{F.29}
\]

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

\[
 p_{kn}^k = \left[ \sum_{i=1}^{N} (p_{ni}^k)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma > 1. \tag{F.30}
\]

F.4.2 Production Technology

Goods are produced with labor under conditions of perfect competition, such that the cost to a consumer in country \( n \) of purchasing the variety of country \( i \) within sector \( k \) is:

\[
 p_{ni}^k = \tau_{ni}^k \frac{w_i}{z_i^k}, \tag{F.31}
\]

where \( z_i^k \) captures productivity and iceberg trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

F.4.3 Expenditure Shares

Using the properties of CES demand, country \( n \)’s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[
 s_{ni}^k = \frac{(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}. \tag{F.32}
\]

Totally differentiating the expenditure share equation (F.32), we get:

\[
 ds_{ni}^k = \frac{\theta dp_{ni}^k (p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}} - \theta \sum_{h=1}^{N} \frac{\theta dp_{nh}^k (p_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}},
\]

\[
 \frac{ds_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{ni}^k}{p_{ni}^k} + \theta \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \theta \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k},
\]

\[
 \frac{ds_{ni}^k}{s_{ni}^k} = -\theta \frac{dp_{ni}^k}{p_{ni}^k} + \theta \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k},
\]

\[
 \frac{ds_{ni}^k}{s_{ni}^k} = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right),
\]

\[
 ds_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln p_{nh}^k - d \ln p_{ni}^k \right),
\]

\[
 d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln p_{nh}^k - d \ln p_{ni}^k \right), \tag{F.33}
\]
where, from equilibrium prices in equation (F.31), we have:

\[
\frac{dp_n^k}{p_{ni}^k} = \frac{d\tau_{ni}^k}{\tau_{ni}^k} + \frac{dw_i}{w_i} - \frac{dz_i^k}{z_i^k},
\]

\(d \ln p_{ni}^k = d \ln \tau_{ni}^k + d \ln w_i - d \ln z_i^k \).

### F.4.4 Price Indices

Totally differentiating the sectoral price index (F.30), we have:

\[
dp_n^k = \sum_{m=1}^{N} \frac{dp_{nm}^k}{p_{nm}^k} \left( \frac{p_{nm}^k}{p_{nm}^k} \right)^{-\theta} \left[ \sum_{h=1}^{N} \left( \frac{p_{nh}^k}{p_{nh}^k} \right)^{-\theta} \right]^{-\frac{1}{\theta}},
\]

\[
dp_n^k = \sum_{m=1}^{N} \frac{dp_{nm}^k}{p_{nm}^k} \left( \frac{p_{nm}^k}{p_{nm}^k} \right)^{-\theta} \left[ \sum_{h=1}^{N} \left( \frac{p_{nh}^k}{p_{nh}^k} \right)^{-\theta} \right]^{-\frac{1}{\theta}},
\]

\[
dp_n^k = \sum_{m=1}^{N} s_{nm}^k \frac{dp_{nm}^k}{p_{nm}^k},
\]

\(d \ln p_n^k = \sum_{m=1}^{N} s_{nm}^k d \ln p_{nm}^k \).

### F.4.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[w_i \ell_i = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_{n,ni}^k s_{ni}^k w_n \ell_n.\]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
dw_i \frac{w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_{n,ni}^k s_{ni}^k w_n \ell_n \left( \frac{dw_i}{w_i} + \frac{ds_{ni}^k}{s_{ni}^k} \right),
\]

\[
dw_i \frac{w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_{n,ni}^k s_{ni}^k w_n \ell_n \left( \frac{dw_i}{w_i} + \frac{ds_{ni}^k}{s_{ni}^k} \right).
\]

Using our result for the derivative of expenditure shares in equation (F.33) above, we can rewrite this as:

\[
dw_i \frac{w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \alpha_{n,ni}^k s_{ni}^k w_n \ell_n \left( \frac{dw_i}{w_i} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right) \right),
\]

\[
dw_i \frac{w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} \ell_n^k \left( \frac{dw_i}{w_i} + \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln p_{nh}^k - d \ln p_{ni}^k \right) \right),
\]

where we have defined \(\ell_n^k\) as the share of country \(i\)'s income derived from market \(n\) and industry \(k\):

\[\ell_n^k = \frac{\alpha_{n,ni}^k s_{ni}^k w_n \ell_n}{w_i \ell_i}.\]
F.4.6 Utility Again

Returning to our expression for indirect utility in equation (F.29), we have:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}. \]

Totally differentiating indirect utility, we have:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} \prod_{k=1}^{K} (p_n^k)^{\alpha_n^k} - \sum_{k=1}^{K} \alpha_n^k \frac{dp_n^k}{p_n^k}. 
\]

Using our total derivative of the sectoral price index in equation (F.35) above, we get:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \ln p_{nm}^k,
\]

(F.38)

\[
d \ln u_n = \frac{d \ln w_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \ln p_{nm}^k.
\]

F.4.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[
d \ln z_i^k = d \ln z_i, \quad \forall = k \in K, \quad i \in N, \\
d \ln \tau_{ni}^k = 0, \quad \forall, n, i \in N. 
\]

(F.39)

We start with our expression for the log change in wages from equation (F.37) above:

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k [\ln w_h - \ln z_h] - [\ln w_i - \ln z_i] \right) \right).
\]

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this expression for the log change in wages as:

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k [\ln w_h - \ln z_h] - [\ln w_i - \ln z_i] \right) \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \ln w_n + \theta \left( \sum_{h=1}^{N} \sum_{n=1}^{K} t_{ih}^k s_{nh}^k [\ln w_h - \ln z_h] - [\ln w_i - \ln z_i] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \ln w_n + \theta \left( \sum_{h=1}^{N} \sum_{n=1}^{K} t_{ih}^k s_{nh}^k [\ln w_h - \ln z_h] - [\ln w_i - \ln z_i] \right),
\]

\[
d \ln w_i = \sum_{n=1}^{N} t_{in} \ln w_n + \theta \left( \sum_{n=1}^{N} m_{in} [\ln w_n - \ln z_n] \right),
\]

\[
t_{in} = \sum_{k=1}^{K} t_{in}^k,
\]

\[
m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} t_{ih}^k s_{hn}^k - 1_{n=i},
\]

33
which has the following matrix representation in the paper:

$$
\frac{d \ln w}{\theta + 1} = T \frac{d \ln w}{\theta + 1} + \frac{\theta}{\theta + 1} M (d \ln w - d \ln z),
$$

(F.40)

We again solve for our friend-enemy income exposure measure by matrix inversion. Dividing both sides of equation (F.40) by $\theta + 1$, we have:

$$
\frac{1}{\theta + 1} \left( I - M \right) \frac{d \ln w}{\theta + 1} = \frac{\theta}{\theta + 1} M d \ln z.
$$

Now using $M = TS - I$, we have:

$$
\left( I - \frac{T + \theta TS}{\theta + 1} + Q \right) \frac{d \ln w}{\theta + 1} = \frac{\theta}{\theta + 1} M d \ln z,
$$

which can be re-written as:

$$
(I - V) \frac{d \ln w}{\theta + 1} = \frac{\theta}{\theta + 1} M d \ln z,
$$

where $V$ is our friend-enemy income exposure measure:

$$
V \equiv -\frac{\theta}{\theta + 1} \left( I - V \right)^{-1} M.
$$

(F.42)

**F.4.8 Welfare and Common Productivity Shocks**

We start with our expression for the log change in utility in equation (F.38) above:

$$
\frac{d \ln u}{\theta + 1} = \frac{d \ln w}{\theta + 1} - \sum_{k=1}^{K} \alpha_k \sum_{m=1}^{N} s_{nm} d \ln p_{nm},
$$

or equivalently:

$$
\frac{d \ln u}{\theta + 1} = \frac{d \ln w}{\theta + 1} - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_k s_{nm} d \ln p_{nm}.
$$

Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can write this change in log utility as:

$$
\frac{d \ln u}{\theta + 1} = \frac{d \ln w}{\theta + 1} - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_k s_{nm} (d \ln w_m - d \ln z_m),
$$

(F.43)
which has the following matrix representation in the paper:

\[ \ln u = \ln w - S (\ln w - \ln z). \quad (F.44) \]

We can re-write the above relationship as:

\[ \ln u = (I - S) \ln w + S \ln z, \]

which, using our solution for \( \ln w \) from equation (F.41), can be further re-written as:

\[ \ln u = (I - S) W \ln z + S \ln z, \]

\[ \ln u = U \ln z, \quad (F.45) \]

where \( U \) is our friend-enemy welfare exposure measure:

\[ U \equiv [(I - S) W + S]. \quad (F.46) \]

**F.4.9 Industry-Level Sales Exposure**

In this multi-sector model, our approach also yields bilateral friend-enemy measures of income exposure to global productivity shocks for each sector. Labor income in each sector and country equals value-added, which in turn equals expenditure on goods produced in that sector and country:

\[ w_i^k = y_i^k = \sum_{n=1}^{N} \alpha_{kn}^{k} s_{ni}^{k} w_n^l. \]

Totally differentiating this industry market clearing condition, we have:

\[ \frac{dy_i^k}{y_i^k} w_i^l = \sum_{n=1}^{N} \alpha_{kn}^{k} \frac{ds_{ni}^{k}}{s_{ni}^{k}} w_n^l \alpha_{kn}^{k} + \sum_{n=1}^{N} \alpha_{kn}^{k} s_{ni}^{k} \frac{dw_n^l}{w_n^l} w_n^l, \]

\[ \frac{dy_i^k}{y_i^k} w_i^l = \sum_{n=1}^{N} \alpha_{kn}^{k} s_{ni}^{k} w_n^l \left( \frac{ds_{ni}^{k}}{s_{ni}^{k}} + \frac{dw_n^l}{w_n^l} \right), \]

\[ \frac{dy_i^k}{y_i^k} = \sum_{n=1}^{N} \frac{ds_{ni}^{k}}{s_{ni}^{k}} + \frac{dw_n^l}{w_n^l}, \]

\[ \frac{dy_i^k}{y_i^k} = \sum_{n=1}^{N} \frac{ds_{ni}^{k}}{s_{ni}^{k}} + \frac{dw_n^l}{w_n^l}. \]

Using the total derivative of expenditure shares in equation (F.33), we can rewrite this as:

\[ \frac{dy_i^k}{y_i^k} = \sum_{n=1}^{N} \left( \frac{dw_n^l}{w_n^l} + \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right) \right), \]

which can be re-written as:

\[ \ln y_i^k = \sum_{n=1}^{N} \left( \ln w_n^l + \theta \left( \sum_{h=1}^{N} s_{nh}^k \ln p_{nh}^k - \ln p_{ni}^k \right) \right). \]
Using the total derivative of prices (F.34) and our assumptions of common productivity shocks and no change in bilateral trade costs in equation (F.39), we can re-write this change in sector value-added as:

$$\text{d} \ln y^k_i = \sum_{n=1}^{N} \psi_{in}^k \left( \text{d} \ln w_n + \theta \left( \sum_{h=1}^{N} s_{nh}^k \left( \text{d} \ln w_h - \text{d} \ln z_h \right) - \left( \text{d} \ln w_i - \text{d} \ln z_i \right) \right) \right),$$

which has the following matrix representation:

$$\text{d} \ln \mathbf{Y}^k = \mathbf{T}^k \text{d} \ln \mathbf{w} + \theta \mathbf{M}^k \left( \text{d} \ln \mathbf{w} - \text{d} \ln \mathbf{z} \right). \quad (F.47)$$

Using our solution for changes in wages as a function of productivity shocks in equation (F.40) above, we have:

$$\text{d} \ln \mathbf{Y}^k = \mathbf{T}^k \mathbf{W} \text{d} \ln \mathbf{z} + \theta \mathbf{M}^k \left( \text{W} - \mathbf{I} \right) \text{d} \ln \mathbf{z},$$

which can be re-written as:

$$\text{d} \ln \mathbf{Y}^k = \left[ \mathbf{T}^k \mathbf{W} + \theta \mathbf{M}^k \left( \mathbf{W} - \mathbf{I} \right) \right] \text{d} \ln \mathbf{z},$$

which corresponds to our friends-and-enemies measure of sector value-added exposure to productivity shocks.

We now show that our aggregate friends-and-enemies measure of income exposure ($\mathbf{W}$) in equation (F.42) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure in equation (F.49).

Note that aggregate income equals aggregate value-added:

$$w_i \ell_i = \sum_{k=1}^{K} y^k_i.$$

which implies:

$$\text{d} w_i \ell_i = \sum_{k=1}^{K} \frac{d y^k_i}{y^k_i} y^k_i \text{d} \ell_i,$$

$$\text{d} w_i = \sum_{k=1}^{K} \frac{y^k_i}{w_i \ell_i} d y^k_i,$$

$$\text{d} w_i = \sum_{k=1}^{K} r^k_i d y^k_i,$$

$$\text{d} \ln w_i = \sum_{k=1}^{K} r^k_i \text{d} \ln y^k_i \quad (F.50)$$

where $r^k_i \equiv \frac{y^k_i}{w_i \ell_i}$ is the value-added share of industry $k$. Together, equations (F.41), (F.48) and (F.50) imply:

$$\mathbf{W}_i \text{d} \ln \mathbf{z} = \sum_{k=1}^{K} r^k_i \mathbf{W}^k_i \text{d} \ln \mathbf{z},$$

where $\mathbf{W}_i$ is the income exposure vector for country $i$ with respect to productivity shocks in its trade partners and $\mathbf{W}^k_i$ is the sector value-added exposure vector for country $i$ and sector $k$ with respect to productivity shocks in those partners.
Now consider a shock to productivities, holding all else constant:

\[ W_i = \sum_k r^k_i W^k_i. \]

Therefore, we can decompose our aggregate income exposure measure into the contributions of the income exposure measures of particular industries, and how much of that income exposure of particular industries is explained by various terms (market-size, cross-substitution etc within the industry).

**F.5 Heterogeneous Sector Trade Elasticities**

For simplicity, we have so far assumed a common elasticity of substitution (\( \sigma \)) and trade elasticity (\( \theta \)) across sectors through this section of the online appendix. We now further generalize our analysis to allow for heterogeneous trade elasticities across sectors. The specification remains as in the previous subsection, except that the elasticity of substitution (\( \sigma^k \)) and trade elasticity (\( \theta^k \)) now vary across sectors \( k \). Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} w_n \ell_n. \]  

(F.51)

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} w_n \ell_n \frac{dw_n}{w_n} + \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} \frac{ds_n^k}{s_n^k} w_n \ell_n \frac{dw_n}{w_n} + \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} \frac{d\sigma_n^k}{\sigma_n^k} w_n \ell_n \frac{dw_n}{w_n},
\]

\[
\frac{dw_i}{w_i} w_i \ell_i = \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} w_n \ell_n \left( \frac{dw_n}{w_n} + \frac{ds_n^k}{s_n^k} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} \frac{dw_n}{w_n} \ell_n + \theta^k \left( \frac{\sum_{n=1}^N \alpha^k_n s_{ni} \frac{dp_n^k}{p_n^k} - \frac{dp_n^k}{p_n^k}}{w_n} \right),
\]

(F.52)

Using the derivative of expenditure shares, we can rewrite this as:

\[
\frac{dw_i}{w_i} = \sum_{n=1}^N \sum_{k=1}^K \alpha^k_n s_{ni} w_n \ell_n \frac{dw_n}{w_n} \left( \frac{dp_n^k}{p_n^k} - \frac{dp_n^k}{p_n^k} \right),
\]

\[
\frac{dw_i}{w_i} = \sum_{n=1}^N \sum_{k=1}^K \ell_n^k \left( \frac{d \ln w_n}{w_n} + \theta^k \left( \frac{\sum_{n=1}^N \alpha^k_n s_{ni} \frac{dp_n^k}{p_n^k} - \frac{dp_n^k}{p_n^k}}{w_n} \right) \right),
\]

where we have defined \( \ell_n^k \) as the share of country \( i \)’s income derived from market \( n \) and industry \( k \):

\[
\ell_n^k = \frac{\alpha^k_n s_{ni} w_n \ell_n}{w_i \ell_i}.
\]

Now consider a shock to productivities, holding all else constant:

\[
d \ln w_i = \sum_{n=1}^N \sum_{k=1}^K \ell_n^k \left( d \ln w_n + \theta^k \left( \sum_{h=1}^N \frac{s_{nh}^k}{s_{nh}} [d \ln p_n^h - d \ln z_h] - [d \ln w_i - d \ln z_i] \right) \right).
\]
We can equivalently re-write this expression as:

\[
d\ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{ik}^k \ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{K} \frac{\theta^k}{\theta^{k}} t_{ih}^k s_{hn}^{k} \left[ d\ln w_h - d\ln z_h \right] - \left[ d\ln w_i - d\ln z_i \right],
\]

where we have multiplied and divided by \( \theta \). We can further rewrite this expression as:

\[
d\ln w_i = \sum_{n=1}^{N} \sum_{h=1}^{K} t_{ih}^k \ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{K} \frac{\theta^k}{\theta^{k}} t_{ih}^k s_{hn}^{k} \left[ d\ln w_h - d\ln z_h \right] - \left( \sum_{k=1}^{K} \frac{\theta^k t_{ih}^k}{\theta} \right) \left[ d\ln w_i - d\ln z_i \right],
\]

\[
d\ln w_i = \sum_{n=1}^{N} \sum_{h=1}^{K} t_{ih}^k \ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{K} \frac{\theta^k}{\theta^{k}} t_{ih}^k s_{hn}^{k} \left[ d\ln w_h - d\ln z_h \right] - \left( \sum_{k=1}^{K} \frac{\theta^k t_{ih}^k}{\theta} \right) \left[ d\ln w_i - d\ln z_i \right],
\]

\[
d\ln w_i = \sum_{n=1}^{N} t_{in} d\ln w_n + \theta \sum_{n=1}^{N} \sum_{h=1}^{K} \frac{\theta^k}{\theta^{k}} t_{ih}^k s_{hn}^{k} \left[ d\ln w_h - d\ln z_h \right] - \left( \sum_{k=1}^{K} \frac{\theta^k t_{ih}^k}{\theta} \right) \left[ d\ln w_i - d\ln z_i \right]
\]

\[
m_{in} = \sum_{h=1}^{K} \frac{\theta^k}{\theta^{k}} t_{ih}^k s_{hn}^{k} - \left( \sum_{k=1}^{K} \frac{\theta^k t_{ih}^k}{\theta} \right) \left[ d\ln w_i - d\ln z_i \right],
\]

where the definition of \( m_{in} \) now depends on the ratio of the sectoral trade elasticity (\( \theta^k \)) to the common parameter (\( \theta \)). Equation (F.53) has the following matrix representation as in the paper:

\[
d\ln w = T d\ln w + \theta M \left( d\ln w - d\ln z \right),
\]

where only the definitions and interpretation of \( \theta \) and \( M \) differ. Totally differentiating the indirect utility function, we again have:

\[
d\ln u_n = d\ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^{k} d\ln p_{nm}^k.
\]

Now consider a shock to productivities, holding all else constant:

\[
d\ln u_n = d\ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^k s_{nm}^{k} \left( d\ln w_m - d\ln z_m \right),
\]

which has the following matrix representation as in the paper:

\[
d\ln u = d\ln w - S \left( d\ln w - d\ln z \right).
\]

**F.6 Multi-Sector Isomorphism**

For the convenience of exposition, we focus in Section 3 of the paper and the previous section of this online appendix on a multi-sector version of the constant elasticity Armington model. In this section, we show that the same results hold in a multi-sector version of the Eaton and Kortum (2002) model following Costinot, Donaldson and Komunjer (2012). The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors indexed by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).
F.6.1 Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_{kn}^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1. \]  
\( \text{(F.57)} \)

Each sector contains a fixed continuum of goods that enter the sectoral price index according to the following CES functional form:

\[ p_n^k = \left[ \int_0^1 p_{n}^k (\vartheta)^{1-\sigma^k} d\vartheta \right]^{\frac{1}{1-\sigma^k}}, \quad \sigma^k > 1. \]  
\( \text{(F.58)} \)

F.6.2 Production Technology

Goods are produced with labor and can be traded subject to iceberg variable trade costs, such that the cost to a consumer in country \( n \) of purchasing a good \( \vartheta \) from country \( i \) is:

\[ p_{ni}^k (\vartheta) = \frac{\tau_{ni}^k w_i z_k^i}{a_k^i (\vartheta)}, \]  
\( \text{(F.59)} \)

where \( z_k^i \) captures determinants of productivity that are common across all goods within a country \( i \) and sector \( k \) and \( a_k^i (\vartheta) \) captures idiosyncratic determinants of productivity for each good within that country and sector. Iceberg trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \). Productivity for each good \( \vartheta \) in each sector \( k \) and each country \( i \) is drawn independently from the following Fréchet distribution:

\[ F_k^i (a) = \exp \left( -a - \theta \right), \quad \theta > 1, \]  
\( \text{(F.60)} \)

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to \( z_k^i \).

F.6.3 Expenditure Shares

Using the properties of this Fréchet distribution, country \( n \)'s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[ s_{ni}^k = \frac{(r_{ni}^k w_i / z_k^i)^{-\theta}}{\sum_{m=1}^{N} (r_{nm}^k w_m / z_k^m)^{-\theta}} = \frac{(\rho_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}}, \]  
\( \text{(F.61)} \)

where we have defined the following price term:

\[ \rho_{ni}^k \equiv \frac{r_{ni}^k w_i}{z_k^i}. \]  
\( \text{(F.62)} \)

Totally differentiating the expenditure share equation (F.61), we get:

\[ ds_{ni}^k = \theta \frac{\hat{\rho}_{ni}^k (\rho_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}} + \sum_{h=1}^{N} \frac{\theta \frac{\hat{\rho}_{nh}^k (\rho_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (\rho_{nm}^k)^{-\theta}}}{s_{ni}^k}, \]  
\( \frac{d}{ds_{ni}^k} \)

\[ \frac{d}{ds_{ni}^k} \]
Using the properties of the Fréchet distribution (F.60), the sectoral price index is given by:

\[ p_n^k = \gamma^k \left( \sum_{m=1}^{N} \left( \rho_{nm}^k \right)^{-\theta} \right)^{-\frac{1}{\theta}}, \tag{F.65} \]

where

\[ \gamma^k \equiv \left[ \Gamma \left( \frac{\theta + 1 - \sigma^k}{\theta} \right) \right] \frac{1}{(1 - \theta)^{1 - \sigma^k}}, \]

and \( \Gamma(\cdot) \) denotes the Gamma function. Totally differentiating this sectoral price index (F.65), we have:

\[
\begin{align*}
\frac{dp_n^k}{\rho_n^k} & = \sum_{m=1}^{N} \gamma^k \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \sum_{h=1}^{N} \left( \rho_{nh}^k \right)^{-\theta} \right)^{-\frac{1}{\theta}} - \frac{\theta}{\theta - 1} \frac{1}{\theta - 1} \frac{1}{\theta - 1} \left( \sum_{m=1}^{N} \left( \rho_{nm}^k \right)^{-\theta} \right)^{-\frac{1}{\theta}} \\
\frac{dp_n^k}{p_n^k} & = \sum_{m=1}^{N} \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \rho_{nm}^k \right)^{-\theta} - \frac{\theta}{\theta - 1} \frac{1}{\theta - 1} \frac{1}{\theta - 1} \left( \sum_{m=1}^{N} \left( \rho_{nm}^k \right)^{-\theta} \right)^{-\frac{1}{\theta}} \\
\frac{dp_n^k}{p_n^k} & = \sum_{m=1}^{N} \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \frac{\rho_{nm}^k}{\rho_{nm}^k} \right)^{-\theta} \\
\frac{dp_n^k}{p_n^k} & = \sum_{m=1}^{N} \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \frac{\rho_{nm}^k}{\rho_{nm}^k} \right)^{-\theta} \\
\frac{dp_n^k}{p_n^k} & = \sum_{m=1}^{N} \frac{dp_{nm}^k}{\rho_{nm}^k} \left( \frac{\rho_{nm}^k}{\rho_{nm}^k} \right)^{-\theta}.
\end{align*}
\tag{F.66} \]

\[ d \ln p_n^k = \sum_{m=1}^{N} s_{nm}^k \ln \frac{\rho_{nm}^k}{\rho_{nm}^k}. \]

F.6.5 Market Clearing

Market clearing requires that income in each country equals expenditure on goods produced in that country:

\[ w_i \ell_i = \sum_{n=1}^{N} s_{ni}^k w_n \ell_n. \tag{F.67} \]

Totally differentiating this market clearing condition, holding labor endowments constant, we have:

\[
\begin{align*}
\frac{dw_i}{w_i} & = \sum_{n=1}^{N} s_{ni}^k \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} \frac{dp_{nh}^k}{\rho_{nh}^k} \left( \frac{\rho_{nh}^k}{\rho_{nm}^k} \right)^{-\theta} \right), \\
\frac{dw_i}{w_i} & = \sum_{n=1}^{N} s_{ni}^k \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} \frac{dp_{nh}^k}{\rho_{nh}^k} \left( \frac{\rho_{nh}^k}{\rho_{nm}^k} \right)^{-\theta} \right), \\
\frac{dw_i}{w_i} & = \sum_{n=1}^{N} s_{ni}^k \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} \frac{dp_{nh}^k}{\rho_{nh}^k} \left( \frac{\rho_{nh}^k}{\rho_{nm}^k} \right)^{-\theta} \right).
\end{align*}
\]

Using our result for the derivative of expenditure shares in equation (F.63) above, we can rewrite this as:

\[
\begin{align*}
\frac{dw_i}{w_i} & = \sum_{n=1}^{N} \frac{dw_n}{w_n} + \theta \left( \sum_{h=1}^{N} \frac{dp_{nh}^k}{\rho_{nh}^k} \left( \frac{\rho_{nh}^k}{\rho_{nm}^k} \right)^{-\theta} \right),
\end{align*}
\]
\[ \frac{dw_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \frac{dw_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh}^k \frac{d\rho_{nh}^k}{\rho_{nh}} - \frac{d\rho_{ni}^k}{\rho_{ni}} \right) \right) , \]  

\( \text{(F.68)} \)

\[ \frac{d \ln w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \frac{d \ln w_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh}^k \frac{d \ln \rho_{nh}^k}{\rho_{nh}} - \frac{d \ln \rho_{ni}^k}{\rho_{ni}} \right) \right) , \]

where we have defined \( t_{in}^k \) as country \( n \)'s expenditure on country \( i \) in industry \( k \) as a share of country \( i \)'s income:

\[ t_{in}^k = \frac{\alpha_{n}^k s_{in}^k w_n}{w_i} . \]

### F.6.6 Utility Again

Returning to our expression for indirect utility in equation (F.57), we have:

\[ u_n = \prod_{k=1}^{K} \frac{1}{(p_n^k)^{\alpha_{n}^k}} . \]

Totally differentiating indirect utility, we have:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \frac{\alpha_{n}^k}{(p_n^k)^{\alpha_{n}^k}} \frac{dp_n^k}{p_n^k} . \]

Using our total derivative of the sectoral price index in equation (F.66) above, we get:

\[ \frac{du_n}{u_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_{n}^k \sum_{m=1}^{N} s_{nm}^k \frac{d \ln \rho_{nm}^k}{\rho_{nm}^k} , \]  

\( \text{(F.69)} \)

### F.6.7 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[ \frac{d \ln z_i^k}{z_i} = \frac{d \ln z_i}{z_i}, \quad \forall k \in K, \ i \in N, \]

\[ \frac{d \ln \tau_{ni}^k}{\tau_{ni}} = 0, \quad \forall n, i \in N. \]  

\( \text{(F.70)} \)

We start with our expression for the log change in wages from equation (F.68) above:

\[ \frac{d \ln w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \frac{d \ln w_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh}^k \left( d \ln w_h - d \ln z_h \right) - \left( d \ln w_i - d \ln z_i \right) \right) \right) . \]

Using the total derivative of prices (F.64) and our assumption of constant bilateral trade costs (F.70), we can write this log change in wages as:

\[ \frac{d \ln w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( \frac{d \ln w_n}{w_n} + \theta \left( \sum_{h \in N} s_{nh}^k \left( d \ln w_h - d \ln z_h \right) \right) \right) , \]

\[ \frac{d \ln w_i}{w_i} = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^k \left( d \ln w_n + \theta \sum_{h \in N} s_{nh}^k \left( d \ln w_h - d \ln z_h \right) \right) , \]

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\[ d \ln w_i = \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^{k} \ d \ln w_n + \theta \left( \sum_{h=1}^{N} \sum_{n=1}^{N} \sum_{k=1}^{K} t_{in}^{k} s_{nh}^{k} \left( d \ln w_h - d \ln z_h \right) - \left( d \ln w_i - d \ln z_i \right) \right), \]  

(F.71)

which can be re-written as:

\[ d \ln w_i = \sum_{n=1}^{N} t_{in} \ d \ln w_n + \theta \left( \sum_{n=1}^{N} m_{in} \left[ d \ln w_n - d \ln z_n \right] \right), \]

\[ t_{in} = \sum_{k=1}^{K} t_{in}^{k}, \]

\[ m_{in} = \sum_{h=1}^{N} \sum_{k=1}^{K} s_{ih}^{k} s_{hn}^{k} - 1_{n=i}, \]

which has the same matrix representation as in equation (43) in the paper:

\[ d \ln w = T d \ln w + \theta M \left( d \ln w - d \ln z \right). \]  

(F.72)

**F.6.8 Utility and Common Productivity Shocks**

We start with our expression for the log change in utility in equation (F.69) above:

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^{k} \sum_{m=1}^{N} s_{nm}^{k} d \ln \rho_{nm}^{k}, \]

or equivalently:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^{k} s_{nm}^{k} d \ln \rho_{nm}^{k}, \]

Using the total derivative of prices (F.64) and our assumption of constant bilateral trade costs (F.70), we can write this change in utility as:

\[ d \ln u_n = d \ln w_n - \sum_{m=1}^{N} \sum_{k=1}^{K} \alpha_n^{k} s_{nm}^{k} \left( d \ln w_m - d \ln z_m \right), \]

which has the same matrix representation as in equation (44) in the paper:

\[ d \ln u = d \ln w - S \left( d \ln w - d \ln z \right). \]  

(F.73)

**F.7 Multiple Sectors and Input-Output Linkages**

In this section of the online appendix, we report the derivations for a version of the model with multiple sectors and input-output linkages following Caliendo and Parro (2015), henceforth CP. In particular, we consider a generalization of the multi-sector Armington model in Section F.4 of this online appendix to incorporate input-output linkages, as discussed in Section 5.5 of the paper. The world economy consists of a set of countries indexed by \( i, n \in \{1, \ldots, N\} \) and a set of sectors referenced by \( k \in \{1, \ldots, K\} \). Each country \( n \) has an exogenous supply of labor \( \ell_n \).

**F.7.1 Notations**

We use \( i, n, o, r \) to index for countries and \( j, k, l \) for industries. We refer to the varieties in industry \( k \) produced in country \( i \) as "goods \( ik \)". We use subscripts to denote countries and superscripts to denote industries. Let \( I_{NK} \) denote the identity matrix with dimension \( NK \times NK \).
F.7.2  Consumer Preferences

Consumer preferences are defined across sectors according to the following Cobb-Douglas indirect utility function:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}, \quad \sum_{k=1}^{K} \alpha_n^k = 1. \]  \hspace{1cm} (F.74)

Each sector is characterized by constant elasticity of substitution preferences across country varieties:

\[ p_n^k = \left[ \sum_{i=1}^{N} \left( p_{ni}^k \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \quad \theta = \sigma - 1, \quad \sigma^k > 1. \]  \hspace{1cm} (F.75)

F.7.3  Production Technology

Goods are produced with labor and can be traded subject to iceberg trade costs, such that the cost to a consumer in country \( n \) of purchasing country \( i \)'s variety within sector \( k \) is:

\[ p_{ni}^k (\omega) = \tau_{ni}^k \alpha_i^k, \quad c_i^k = \left( \frac{w_i}{z_i^k} \right)^{\gamma_i^k} \prod_{j=1}^{K} \left( p_j^i \right)^{\gamma_i^j}, \quad \sum_{k=1}^{K} \gamma_i^k = 1 - \gamma_i^k, \]  \hspace{1cm} (F.76)

where \( c_i^k \) denotes the unit cost function within that country and sector; \( \gamma_i^k \) is the share of labor in production costs; \( \gamma_i^{k,j} \) is the share of materials from sector \( j \) used in sector \( k \); \( z_i^k \) captures determinants of productivity that are common across all goods within a country \( i \) and sector \( k \); and it proves convenient to define this common component of productivity in value-added terms (such that it augments labor). Iceberg variable trade costs satisfy \( \tau_{ni}^k > 1 \) for \( n \neq i \) and \( \tau_{nn}^k = 1 \).

F.7.4  Expenditure Shares

Using the properties of CES demand, country \( n \)'s share of expenditure on goods produced in country \( i \) within sector \( k \) is given by:

\[ s_{ni}^k = \frac{(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}. \]  \hspace{1cm} (F.77)

Totally differentiating this expenditure share equation, we get:

\[ \frac{ds_{ni}^k}{s_{ni}^k} = \frac{\theta d(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}} + \frac{N \theta d(p_{ni}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}} \frac{\theta d(p_{nh}^k)^{-\theta}}{\sum_{m=1}^{N} (p_{nm}^k)^{-\theta}}, \]

\[ \frac{ds_{ni}^k}{s_{ni}^k} = \frac{\theta dp_{ni}^k}{p_{ni}^k} + \frac{N \theta dp_{nh}^k}{p_{nh}^k}, \]

\[ \frac{ds_{ni}^k}{s_{ni}^k} = \theta \frac{dp_{ni}^k}{p_{ni}^k} + \sum_{h=1}^{N} \frac{\theta dp_{nh}^k}{p_{nh}^k}, \]

\[ \frac{ds_{ni}^k}{s_{ni}^k} = \theta \left( \sum_{h=1}^{N} s_{nh}^k \frac{dp_{nh}^k}{p_{nh}^k} - \frac{dp_{ni}^k}{p_{ni}^k} \right), \]

so that

\[ d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k d \ln p_{nh}^k - d \ln p_{ni}^k \right), \]  \hspace{1cm} (F.78)
where, from equilibrium prices in equation (F.76), we have:

$$
\frac{dp^k_{ni}}{p_{ni}} = \frac{d\tau^k_{ni}}{\tau_{ni}} + \gamma^k_i \left( \frac{dw_i}{w_i} - \frac{dz^k_i}{z^k_i} \right) + \sum_{j=1}^{K} \gamma^k_{i,j} \frac{dp^j_i}{p^j_i},
$$

(F.79)

$$
d\ln p^k_{ni} = d\ln \tau^k_{ni} + \gamma^k_i \left( d\ln w_i - d\ln z^k_i \right) + \sum_{j=1}^{K} \gamma^k_{i,j} d\ln p^j_i.
$$

F.7.5 Price Indices

Totally differentiating the sectoral price index (F.75), we have:

$$
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} \gamma^k_m \frac{dp^k_{nm}}{p^k_{nm}} \left( \frac{p^k_{nm}}{p^k_{nm}} \right)^{-\theta} \left[ \sum_{m=1}^{N} \left( p^k_{nm} \right)^{-\theta} \right]^{-\frac{1}{\theta}},
$$

$$
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} \frac{dp^k_{nm}}{p^k_{nm}} \left( \frac{p^k_{nm}}{p^k_{nm}} \right)^{-\theta} \sum_{h=1}^{N} \left( p^k_{nh} \right)^{-\theta},
$$

$$
\frac{dp^k_n}{p^k_n} = \sum_{m=1}^{N} \gamma^k_{nm} \frac{dp^k_{nm}}{p^k_{nm}},
$$

(F.80)

$$
d\ln p^k_n = \sum_{m=1}^{N} \gamma^k_{nm} d\ln p^k_{nm}.
$$

F.7.6 Labor Market Clearing

The labor market clearing condition is:

$$
\ell_n = \sum_{j=1}^{K} \gamma^j_n y^j_n.
$$

(F.81)

where $y^j_n$ is total sales by country $n$’s industry $j$. Totally differentiating this labor market clearing condition, holding endowments constant, we have:

$$
\frac{dw_n}{w_n} \ell_n = \sum_{j=1}^{K} \gamma^j_n \frac{dy^j_n}{y_n} y^j_n,
$$

$$
\frac{dw_n}{w_n} = \sum_{j=1}^{K} \left( \frac{\gamma^j_n y^j_n}{w_n L_n} \right) \frac{dy^j_n}{y_n},
$$

$$
\frac{dw_n}{w_n} = \sum_{j=1}^{K} \frac{\xi^j_n}{y_n} \frac{dy^j_n}{y_n},
$$

(F.82)

where $\xi^j_n$ is the share of sector $j$ in country $n$’s total income:

$$
\xi^j_n = \frac{\gamma^j_n y^j_n}{w_n L_n}.
$$

F.7.7 Goods Market Clearing

Goods market clearing requires that income in each country and sector equals expenditure on goods produced in that country and sector:

$$
y^k_i = \sum_{n=1}^{N} s^k_{ni} x^k_{ni},
$$

(F.83)
where expenditure in country $n$ in sector $k$ is:

$$
x_n^k = \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_{n}^{k,j} y_{n}^j, \quad (F.84)
$$

and recall that $\gamma_{n}^{j,k}$ is the share of materials from sector $k$ used in sector $j$. Combining these two relationships and the labor market clearing ($F.81$), we obtain the following market clearing condition:

$$
y_{i}^k = \sum_{n=1}^{N} s_{ni}^k \left[ \alpha_n^k w_n \ell_n + \sum_{j=1}^{K} \gamma_{n}^{k,j} y_{n}^j \right],
$$

$$
= \sum_{n=1}^{N} s_{ni}^k \left[ \alpha_n^k \sum_{j=1}^{K} \gamma_{n}^{j} y_{n}^j + \sum_{j=1}^{K} \gamma_{n}^{k,j} y_{n}^j \right],
$$

$$
= \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \left[ \alpha_n^k \gamma_{n}^{j} + \gamma_{n}^{k,j} \right] y_{n}^j.
$$

Totally differentiating this market clearing condition, we have:

$$
\frac{dy_{i}^k}{y_{i}^k} = \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{d s_{ni}^k}{s_{ni}^k} s_{ni}^k \left[ \alpha_n^k \gamma_{n}^{j} + \gamma_{n}^{k,j} \right] y_{n}^j.
$$

Let $\vartheta_{i}^{j,k}$ denote the fraction of $ik$’s revenue derived from selling to consumers in country $n$; $\Theta$ denote an $NK \times NK$ matrix with entries $\Theta_{nm}^{ij}$ capturing the fraction of $ik$’s revenue derived from selling to producers in country $n$ industry $j$; and $\Delta$ denote the Leontief-inverse of $\Theta$, such that $\Delta \equiv (I_{NK} - \Theta)^{-1}$, with the $(ik, nj)$-th entry, $\Delta_{ij}^{k,n}$, capturing the network-adjusted fraction of $ik$’s revenue derived from market $nj$, either directly or indirectly through customers of customers, ad infinitum. The above market clearing can be re-written as

$$
d \ln y_{i}^k = \sum_{n=1}^{N} \frac{d \ln s_{ni}^k}{s_{ni}^k} s_{ni}^k \alpha_n^k \sum_{j=1}^{K} \gamma_{n}^{j} y_{n}^j + \sum_{n=1}^{N} \sum_{j=1}^{K} \frac{d \ln s_{ni}^k}{s_{ni}^k} s_{ni}^k \gamma_{n}^{k,j} y_{n}^j
$$

$$
+ \sum_{n=1}^{N} s_{ni}^k \alpha_n^k w_n \ell_n \sum_{j=1}^{K} \gamma_{n}^{j} y_{n}^j \frac{d \ln y_{n}^j}{y_{n}^j} + \sum_{n=1}^{N} \sum_{j=1}^{K} s_{ni}^k \gamma_{n}^{k,j} y_{n}^j \frac{d \ln y_{n}^j}{y_{n}^j},
$$

$$
= \sum_{n=1}^{N} \vartheta_{i}^{k,n} d \ln w_n + \sum_{n=1}^{N} \left( \vartheta_{i}^{k,n} + \sum_{j=1}^{K} \Theta_{i}^{k,j,n} \right) d \ln s_{ni}^k + \sum_{n=1}^{N} \sum_{j=1}^{K} \Theta_{i}^{k,j,n} d \ln y_{n}^j,
$$

where in the second equality we used equations ($F.81$) and ($F.82$). Subtracting the latest term on the right hand side from both sides of the equations and taking the Leontief-inverse of $\Theta_{i}^{k,n}$, we obtain:

$$
d \ln y_{i}^k = \sum_{n=1}^{N} \sum_{j=1}^{K} \Delta_{i}^{k} \left[ \vartheta_{i}^{k,n} d \ln w_n + \sum_{n=1}^{N} \left( \vartheta_{i}^{k,n} + \sum_{j=1}^{K} \Theta_{i}^{k,j,n} \right) d \ln s_{ni}^k \right]. \quad (F.85)
$$
Combining this result with (F.82), we get

\[
\frac{d \ln w_i}{d x} = \sum_k \xi_k^k \sum_{a=1}^N \sum_{l=1}^K \Delta_{kl} \sum_{n=1}^N \frac{\partial^l_{on}}{\partial x_n} d \ln w_n + \sum_{n=1}^N \left( \sum_{j=1}^K \Theta_{jn}^j \right) \frac{d \ln s_{no}^j}{d x}.
\]  

(F.86)

F.7.8 Wages and Common Productivity Shocks

We consider small productivity shocks for each country that are common across sectors, holding constant bilateral trade costs:

\[
\begin{align*}
\frac{d \ln z_i^k}{d x} &= \frac{d \ln z_i}{d x}, \quad \forall k \in K, \; i \in N, \\
\frac{d \ln \gamma_i^k}{d x} &= 0, \quad \forall n, \; i \in N.
\end{align*}
\]

We start with our expression for the change in prices above:

\[
\frac{d \ln p_{ni}^k}{d x} = \frac{d \ln \tau_{ni}^k}{d x} + \gamma_i^k \left( \frac{d \ln w_i}{d x} - \frac{d \ln z_i^k}{d x} \right) + \sum_{j=1}^K \gamma_i^{k,j} \frac{d \ln p_j^j}{d x},
\]

\[
= \gamma_i^k \left( \frac{d \ln w_i}{d x} - \frac{d \ln z_i}{d x} \right) + \sum_{j=1}^K \gamma_i^{k,j} \frac{d \ln p_j^j}{d x}.
\]

Using our result for the total derivative of price indices (F.80), we can rewrite this expression for the change in prices as:

\[
\frac{d \ln p_{ni}^k}{d x} = \gamma_i^k \left( \frac{d \ln w_i}{d x} - \frac{d \ln z_i}{d x} \right) + \sum_{j=1}^K \gamma_i^{k,j} \sum_{m=1}^N s_{im}^j \frac{d \ln p_{im}^j}{d x}.
\]

We use \( \Sigma_{im}^{kj} = \gamma_i^{k,j} s_{im}^j \) to denote expenditure in country \( i \) and sector \( k \) on the goods produced by country \( m \) and sector \( j \) as a share of revenue in country \( i \) and sector \( k \). Using this notation, we can rewrite the above expression for the change in prices as:

\[
\frac{d \ln p_{ni}^k}{d x} = \gamma_i^k \left( \frac{d \ln w_i}{d x} - \frac{d \ln z_i}{d x} \right) + \sum_{j=1}^K \sum_{m=1}^N \Sigma_{im}^{kj} \frac{d \ln p_{im}^j}{d x} + \sum_{j=1}^K \sum_{m=1}^N \Sigma_{im}^{kj} \frac{d \ln \tau_{im}}{d x}.
\]  

(F.87)

Let \( \Sigma \) denote the \( NK \times NK \) matrix with entries \( \Sigma_{im}^{kj} \) capturing the input cost share (relative to revenue) on goods \( mj \) by producer \( ik \). Let us also define the Leontief inverse \( \Gamma \equiv (I_{(NK)} - \Sigma)^{-1} \), with the \( (nj, ik) \)-th entry, \( \Gamma_{ni}^{jk} \), capturing the network-adjusted share of \( nj \)'s revenue spent on inputs \( ik \), either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Finally, let \( \Lambda_{ni}^j = \sum_{k=1}^K \gamma_i^{k,j} \Gamma_{ni}^{jk} \) denote the network-adjusted input cost share of \( nj \)'s revenue on value-added (labor) in country \( i \); note that \( \sum_{i=1}^N \Lambda_{ni}^j = 1 \) for all \( nj \) due to constant returns to scale. Equation (F.87) can be re-written as:

\[
\frac{d \ln p_{ni}^k}{d x} = \sum_{l=1}^N \Lambda_{nl}^k \left( \frac{d \ln w_l}{d x} - \frac{d \ln z_l}{d x} \right).
\]  

(F.88)

We can now use (F.88) to re-write the linearized expenditure shares from equation (F.78) as:

\[
\frac{d \ln s_{ni}^k}{d x} = \theta \left( \sum_{h=1}^N \sum_{r=1}^N \Lambda_{hr}^k \left( \frac{d \ln w_r}{d x} - \frac{d \ln z_r}{d x} \right) - \sum_{r} \Lambda_{ir}^k \left( \frac{d \ln w_r}{d x} - \frac{d \ln z_r}{d x} \right) \right),
\]

\[
= \theta \sum_{r=1}^N \left( \sum_{h=1}^N \Lambda_{hr}^k - \Lambda_{ir}^k \right) \left( \frac{d \ln w_r}{d x} - \frac{d \ln z_r}{d x} \right).
\]

(F.89)

Substitute this into (F.86), we get
To simplify notation, let us now define \( \Pi^l_{io} \equiv \sum_{k=1}^K \xi^k \Delta_{io}^k \) to be the network-adjusted share of income in country \( i \) derived from selling to country \( o \) industry \( l \). Also denote \( Y^l_{nor} \equiv \sum_{h=1}^N s_{nh}^l \Lambda_{hr}^l - \Lambda_{or}^l \); \( \theta Y^l_{nor} \) is the elasticity of \( n \)’s expenditure on goods \( lo \) with respect to \( r \)’s factor cost. Then the above expression can be re-written as:

\[
\frac{\ln w_i}{\ln w_n} = \sum_{n=1}^N \left( \sum_{o=1}^N \sum_{l=1}^K \Pi^l_{io} \vartheta^l_{on} \right) \frac{\ln w_n}{\ln w_n} \\
+ \theta \sum_{n=1}^N \left( \sum_{r=1}^N \sum_{o=1}^N \sum_{l=1}^K \Pi^l_{io} \left( \vartheta^l_{or} + \sum_{j=1}^K \Theta^l_{or} \right) Y^l_{ron} \right) \frac{\ln w_n - \ln w_n}{\ln w_n} \tag{F.90}
\]

Finally, we define \( T \) as an \( N \times N \) matrix with entries \( T_{in} \equiv \sum_{o=1}^N \sum_{l=1}^K \Pi^l_{io} \vartheta^l_{on} \). Element \( T_{in} \) captures the network-adjusted share of \( i \)’s income derived from selling to consumers in country \( n \); it sums across the network-adjusted income share that country \( i \) derives from selling to country-industry \( ol \), times the revenue share that \( ol \) derives from selling to consumers in country \( n \). We define \( M \) as an \( N \times N \) matrix with entries \( M_{in} \):

\[
M_{in} \equiv \sum_{r=1}^N \sum_{o=1}^N \sum_{l=1}^K \Pi^l_{io} \left( \vartheta^l_{or} + \sum_{j=1}^K \Theta^l_{or} \right) Y^l_{ron}.
\]

To interpret, \( \theta Y^l_{ron} \) captures how \( r \)’s expenditure on goods \( ol \) responds to factor cost in \( n \). Element \( M_{in} \) sums the cross-substitution effects across \( i \)’s exposure to all markets through network linkages: \( \Pi^l_{io} \) is \( i \)’s network-adjusted income share derived from selling to producers in country \( o \) industry \( l \); goods \( ol \) are then exposed to substitution due to changes in \( n \)’s factor costs through markets that \( ol \) supplies to, including consumers \( (\vartheta^l_{on}) \) and producers \( (\sum_{j=1}^K \Theta^l_{or}) \) in all countries \( r \).

We have thus obtained the same matrix representation as in the paper:

\[
\frac{\ln w}{\ln w} = T \ln w + \theta M (\ln w - \ln z) \tag{F.91}
\]

We again solve for our friend-enemy income exposure measure by matrix inversion:

\[
\frac{\ln w}{\ln w} = W (\ln w - \ln z) \tag{F.92}
\]

where

\[
W \equiv -\frac{\theta}{\theta + 1} (I - V)^{-1} M \tag{F.93}
\]

and, using our choice of world GDP as numeraire, which implies \( Q \ln w = 0 \),

\[
V = \frac{T + \theta TS}{\theta + 1} - Q.
\]
F.7.9 Welfare

Returning to our expression for indirect utility (F.74), we have:

\[ u_n = \frac{w_n}{\prod_{k=1}^{K} (p_n^k)^{\alpha_n^k}}. \]

Totally differentiating this expression for indirect utility, we have:

\[
\frac{du_n}{u_n} = \frac{dw_n}{w_n} \prod_{k=1}^{K} (p_n^k)^{\alpha_n^k} - \sum_{k=1}^{K} \alpha_n^k \frac{dp_n^k}{p_n^k} w_n,
\]

\[
\frac{du_n}{w_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \frac{dp_n^k}{p_n^k}.
\]

Using our total derivative of the sectoral price index above, we get:

\[ \frac{du_n}{w_n} = \frac{dw_n}{w_n} - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \ln p_{nm}^k, \quad \text{(F.94)} \]

\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \ln p_{nm}^k. \]

Plugging (F.88) into the above, we get

\[ d \ln u_n = d \ln w_n - N \sum_{i=1}^{N} \left( \sum_{k=1}^{K} \alpha_n^k \sum_{m=1}^{N} s_{nm}^k \Lambda_{mi}^k \right) \left( d \ln w_i - d \ln z_i \right). \]

We define \( S \) as an \( N \times N \) matrix with entries \( S_{ni} = \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha_n^k s_{nm}^k \Lambda_{mi}^k \). Element \( S_{ni} \) captures the network-adjusted expenditure share of consumer \( n \) on value-added by country \( i \); it sums across the expenditure share of consumer \( n \) on goods \( mk \), times the network-adjusted input cost share of \( mk \) on factor \( i \), captured by \( \Lambda_{mi}^k \). We have thus obtained the same matrix representation for welfare as in the paper:

\[ d \ln u_n = d \ln w_n - S \left( d \ln w - d \ln z \right). \quad \text{(F.95)} \]

We can re-write the above relationship as:

\[ d \ln u = (I - S) d \ln w + S d \ln z, \]

which, using our solution for \( d \ln w \) from equation (F.92), can be further re-written as:

\[ d \ln u = (I - S) W d \ln z + S d \ln z, \]

\[ = [(I - S) W + S] d \ln z, \]

\[ = U d \ln z, \]

where \( U \) is our friend-enemy welfare exposure measure:

\[ U \equiv [(I - S) W + S]. \quad \text{(F.96)} \]
F.7.10 Industry-Level Sales Exposure

Similarly to the multi-sector model without input linkages, our approach also yields bilateral friend-enemy measures of sales exposure to global productivity shocks for each sector. From equations (F.85) and (F.89) we obtain the following expression for changes in the sales of industry \( k \) in country \( i \):

\[
d \ln y^k_i = \sum_{n=1}^{N} \left( \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{lo}^{ki} \theta_{on} \right) d \ln w_n ,
\]

\[
+ \theta \sum_{n=1}^{N} \left( \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{lo}^{ki} \left( \theta_{or} + \sum_{j=1}^{K} \Theta_{j or} \right) \right) Y^l_{ron} \left( d \ln w_n - d \ln z_n \right) .
\]

We define \( T^k \) as an \( N \times N \) matrix with entries \( T^k_{in} = \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{lo}^{ki} \theta_{on} \). Element \( T^k_{in} \) captures the network-adjusted share of sector \( ik \)’s income derived from selling to consumers in country \( n \); it sums across the network-adjusted income share that sector \( ik \) derives from selling to country-industry \( ol \), times the revenue share that \( ol \) derives from selling to consumers in country \( n \). We define \( M^k \) as an \( N \times N \) matrix with entries \( M^k_{in} = \sum_{r=1}^{N} \sum_{o=1}^{N} \sum_{l=1}^{K} \Delta_{lo}^{ki} \left( \theta_{or} + \sum_{j=1}^{K} \Theta_{j or} \right) Y^l_{ron} \).

To interpret, \( \theta Y^l_{ron} \) captures how \( r \)’s expenditure on goods \( ol \) responds to factor cost in \( n \). Element \( M^k_{in} \) sums the cross-substitution effects across sector \( ik \)’s exposure to all markets through network linkages: \( \Delta_{lo}^{ki} \) is \( ik \)’s network-adjusted income share derived from selling to producers in country \( o \) industry \( l \); goods \( ol \) are then exposed to substitution due to changes in \( n \)’s factor costs through markets that \( ol \) supplies to.

We get the following matrix representation:

\[
d \ln Y^k = T^k d \ln w + \theta M^k \left( d \ln w - d \ln z \right) , \quad (F.97)
\]

\[
= \left[ T^k W + \theta M^k \left( W - I \right) \right] d \ln z ,
\]

which can be re-written as:

\[
d \ln Y^k = W^k d \ln z , \quad (F.98)
\]

\[
W^k = T^k W + \theta M^k \left( W - I \right) , \quad (F.99)
\]

which corresponds to our friends-and-enemies measure of sector sales exposure to productivity shocks. Note that \( W^k \) also captures sector value-added exposure to productivity shocks, as value added is a constant share of revenues in this model. Finally, recall from equation (F.82) that

\[
d \ln w_n = \sum_{j=1}^{K} \xi^k_n d \ln y^k_n ,
\]

where \( \xi^k_n \) is the value-added share of industry \( k \) in country \( n \)’s total income. Together, this relationship and equations (F.92) and (F.98) imply that:

\[
W_i d \ln z = \sum_{k=1}^{K} \xi^k_n W^k_i d \ln z ,
\]

which is the value-added share of industry \( k \) in country \( n \)’s total income.
where \( \mathbf{W}_i \) is the income exposure vector for country \( i \) with respect to productivity shocks in its trade partners and \( \mathbf{W}_i^k \) is the sector value-added exposure vector for country \( i \) and sector \( k \) with respect to productivity shocks in those trade partners. It follows that our aggregate friends-and-enemies measure of income exposure (\( \mathbf{W}_i \)) is a weighted average of our industry friends-and-enemies measures of sector value-added exposure (\( \mathbf{W}_i^k \)):

\[
\mathbf{W}_i = \sum_{k=1}^{K} \xi_i^k \mathbf{W}_i^k.
\]

Therefore, we can decompose how much of our aggregate income exposure measure is driven by the value-added exposure of particular industries, and how much of that value-added exposure of particular industries is explained by various terms (market-size, cross-substitution, etc. within the industry).

### F.7.11 Isomorphisms

Although for expositional convenience we focus on an extension of the constant elasticity Armington model with multiple sectors and input-output linkages, the same results hold in an extension of the Eaton and Kortum (2002) model to incorporate both multiple sectors following Costinot, Donaldson and Komunjer (2012) and also input-output linkages following Caliendo and Parro (2015).

### F.7.12 Income and Welfare Exposure to Common Productivity and Trade Cost Shocks

We now derive our income and welfare exposure measures in the multi-sector model with input-output linkages allowing for both productivity and trade cost shocks. We consider small productivity and trade cost shocks for each country that are common across sectors \( k \):

\[
d \ln z^k_i = d \ln z_i, \quad \forall k \in K, \ i \in N, \\
d \ln \tau^k_{ni} = d \ln \tau_{ni}, \quad \forall n, i \in N.
\]

We start with our expression for the change in prices above:

\[
d \ln p^k_{ni} = d \ln \tau^k_{ni} + \gamma_i^k (d \ln w_i - d \ln z^k_i) + \sum_{j=1}^{K} \gamma_{i,j}^k d \ln p^j_i,
\]

Using our result for the total derivative of price indices (F.80), we can rewrite this expression for the change in prices as:

\[
d \ln p^k_{ni} - d \ln \tau_{ni} = \gamma_i^k (d \ln w_i - d \ln z_i) + \sum_{j=1}^{K} \gamma_{i,j}^k \sum_{m=1}^{N} s^j_{im} d \ln p^j_{im}.
\]

We use \( \Sigma^k_{im} = \gamma_{i,j}^k s^j_{im} \) to denote expenditure in country \( i \) and sector \( k \) on the goods produced by country \( m \) and sector \( j \) as a share of revenue in country \( i \) and sector \( k \). Using this notation, we can rewrite the above expression for the change in prices as:

\[
d \ln p^k_{ni} - d \ln \tau_{ni} = \gamma_i^k (d \ln w_i - d \ln z_i) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma^k_{im} \left( d \ln p^j_{im} - d \ln \tau_{im} \right) + \sum_{j=1}^{K} \sum_{m=1}^{N} \Sigma^j_{im} d \ln \tau_{im}. \tag{F.100}
\]

Recall \( \Sigma \) is the \( NK \times NK \) matrix with entries \( \Sigma^k_{im} \) capturing the input cost share (relative to revenue) on goods \( mj \) by producer \( ik \), and \( \Gamma \equiv \left( I_{(NK)} - \Sigma \right)^{-1} \) is the Leontief inverse, with the \((nj,ik)\)-th entry, \( \Gamma_{nj,ik}^k \), capturing the network-adjusted share of \( nj \)'s revenue spent on inputs \( ik \), either directly or indirectly through suppliers and suppliers of suppliers, ad infinitum. Equation (F.100) can be re-written as:
\[
\ln p_{ni}^k - \ln \tau_{ni} = \sum_{l=1}^{N} \left( \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{N} \sum_{x=1}^{K} \sum_{l_m} \ln \tau_{lm} + \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{j=1}^{N} \ln \tau_{il} \right) \tag{F.101}
\]

Recall \( \Lambda_{ni}^{k} \equiv \sum_{k=1}^{K} \gamma_{nk} R_{ni}^{k} \) is the network-adjusted input cost share of \( n_j \)'s revenue on value-added labor in country \( i \). Also let
\[
d \ln c_{i}^{k, r} = \sum_{l=1}^{N} \sum_{j=1}^{K} \Gamma_{il}^{kj} \sum_{m=1}^{N} \sum_{x=1}^{K} \sum_{l_m} \ln \tau_{lm},
\]
which captures the change in the cost of good \( ik \) due to changes in trade costs, holding factor costs constant. Equation (F.101) can be rewritten as
\[
\ln p_{ni}^k = d \ln \tau_{ni} + d \ln c_{i}^{k, r} + \sum_{l} \Lambda_{il}^k \left( d \ln w_l - d \ln z_l \right) \tag{F.102}
\]

We can now use (F.102) to re-write the linearized expenditure shares from equation (F.78) as:
\[
d \ln s_{ni}^k = \theta \left( \sum_{h=1}^{N} s_{nh}^k \ln \tau_{nh} + \sum_{r=1}^{N} \Lambda_{nr}^k \left( d \ln w_r - d \ln z_r \right) \right) - \left( \sum_{h=1}^{N} \sum_{r=1}^{N} \Lambda_{nh}^k \left( d \ln w_r - d \ln z_r \right) \right)
\]

\[
= \theta \sum_{r=1}^{N} \left( \sum_{h=1}^{N} s_{nh}^k \ln \tau_{nh} - \Lambda_{nr}^k \right) \left( d \ln w_r - d \ln z_r \right) + \sum_{h=1}^{N} \left( s_{nh}^k - \delta_{h=i} \right) \left( d \ln \tau_{nh} + d \ln c_{i}^{k, r} \right)
\]

Substitute this into (F.86), we get
\[
d \ln w_i = \sum_{k=1}^{K} \xi_i^k \sum_{l=1}^{N} \Delta_{il}^k \sum_{n=1}^{N} \phi_{in} \ln w_n
\]

\[
+ \theta \sum_{k=1}^{K} \xi_i^k \sum_{h=1}^{N} \sum_{l=1}^{K} \Delta_{il}^k \sum_{n=1}^{N} \phi_{in} \ln w_n \Theta_{ih}^l \sum_{j=1}^{K} \Theta_{ij}^l \left( d \ln w_r - d \ln z_r \right)
\]

\[
+ \theta \sum_{k=1}^{K} \xi_i^k \sum_{h=1}^{N} \sum_{l=1}^{K} \Delta_{il}^k \sum_{n=1}^{N} \phi_{in} \ln w_n \Theta_{ih}^l \sum_{j=1}^{K} \Theta_{ij}^l \left( s_{nh}^k - \delta_{h=i} \right) \left( d \ln \tau_{nh} + d \ln c_{i}^{k, r} \right)
\]

Recall we have defined \( \Pi_{ion}^l = \sum_{k=1}^{K} \xi_i^k \Delta_{il}^k \) as the network-adjusted share of income in country \( i \) derived from selling to country \( o \) industry \( l \). Also recall \( \tau_{nor}^l = \sum_{h=1}^{N} s_{nh}^k \ln \tau_{nh} - \Lambda_{nor}^l \); \( \theta \tau_{nor}^l \) is the elasticity of \( n \)'s expenditure on goods \( lo \) with respect to \( r \)'s factor cost. Using these results, we can rewrite income exposure to productivity and trade cost shocks as:
\[
d \ln w_i = \sum_{n=1}^{N} \left( \sum_{l=1}^{K} \Pi_{ion}^l \phi_{in} \right) d \ln w_n
\]

\[
+ \theta \sum_{n=1}^{N} \left( \sum_{l=1}^{K} \Pi_{ion}^l \phi_{in} \right) \left( c_{i}^{k, r} - \Lambda_{ir}^l \right) \left( d \ln w_r - d \ln z_r \right) \tag{F.105}
\]

\[
+ \sum_{n=1}^{N} \sum_{l=1}^{K} \sum_{m=1}^{N} \Pi_{ion}^l \left( \phi_{in} \right) \left( \Theta_{nor}^l \right) \left( s_{nr}^l - \delta_{r=i} \right) \left( d \ln \tau_{nr} + d \ln c_{i}^{k, r} \right)
\]
For welfare, note
\[ d \ln u_n = d \ln w_n - \sum_{k=1}^{K} \alpha^k_n \sum_{m=1}^{N} s^k_{nm} d \ln p^k_{nm}. \]

Plugging equation (F.102) into the above expression, we can write welfare exposure to productivity and trade cost shocks as:
\[
d \ln u_n = d \ln w_n - N \sum_{i=1}^{N} \left( \sum_{k=1}^{K} \sum_{m=1}^{N} \alpha^k_n s^k_{nm} \Lambda^k_{mi} \right) (d \ln w_i - d \ln z_i)
- \sum_{k=1}^{K} \alpha^k_n \sum_{m=1}^{N} s^k_{nm} \left( d \ln \tau_{nm} + d \ln c^k_{m, \tau} \right).
\]

### G Additional Empirical Results

In this section of the online appendix, we report additional empirical results that are discussed in the paper.

#### G.1 Economic Friends and Enemies

We begin by reporting additional empirical results for country income and welfare exposure to productivity and trade cost shocks, as discussed in Section 6 of the paper. In Section G.1.1, we provide further evidence on the quality of the approximation of our linearization to the full non-linear model solution. In Section G.1.2, we provide further evidence on the evolution of global income and welfare exposure over our sample period.

#### G.1.1 Quality of the Approximation

In this subsection, we report further empirical evidence on the quality of the approximation of our linearization to the full non-linear model solution for large productivity shocks, as discussed in Section 6.2 of the paper. First, we discuss our inversion of the full non-linear model to recover the empirical distribution of productivity and trade cost shocks (Paragraph (i)). Second, we show in the paper that our linearization provides an almost-exact approximation to the full non-linear model solution for empirically-reasonable productivity shocks and trade elasticities in actual trade networks. Therefore, we next report the results of an extensive search over simulated trade networks to try to counterexamples where the approximation performs less well for large productivity shocks (Paragraph (ii)). Third, we provide evidence of the quality of the approximation for empirically-reasonable trade cost shocks (Paragraph (iii)). Fourth, we focus in the paper on the quality of our approximation in our baseline single-sector Armington model from Section 3 of the paper, for simplicity and because of this model’s prominence in the existing literature (Paragraph (iv)). Therefore, we next show that our linearization also provides an almost-exact approximation for large productivity shocks in our input-output model from Section 5.5 of the paper.

**(i) Empirical Distribution of Productivity and Trade Cost Shocks** To compare our linearization with exact-hat algebra for empirically-reasonable shocks, we recover the empirical distribution of productivity and trade cost shocks that rationalize the observed trade data within our baseline single-sector constant elasticity Armington model from Section D of this online appendix. Changes in trade costs and productivity are only separately identified up to a normalization or choice of units, because an increase in a country’s productivity is isomorphic to a reduction in its
trade costs with all partners (including itself). We use the normalization that there are no changes in own trade costs over time ($\tau_{ntt} = 1 \forall n, t$), which absorbs common unobserved changes in trade costs across all partners into changes in productivity. But our results are not sensitive to the way in which we recover productivity shocks, as explored in the Monte Carlo simulations discussed in the paper and further below.

Using this normalization, we estimate time-varying bilateral trade cost shocks following Head and Ries (2001). Specifically, let $x_{nit}$ denote the expenditure by country $n$ on goods from country $i$ in year $t$, then

$$\frac{x_{nit}}{x_{nt}} = \frac{w_{it}^{-\theta} z_{it} (\tau_{nt})^{-\theta}}{w_{nt}^{-\theta} z_{nt} (\tau_{nt})^{-\theta}},$$

which implies

$$\left( \frac{x_{nit}}{x_{nt}} \frac{x_{int}}{x_{nt}} \right)^{\frac{1}{2}} = \left( \frac{\tau_{nit} \tau_{int}}{\tau_{nt} \tau_{it}} \right)^{-\frac{\theta}{2}}.$$  

Denoting by $\hat{x}$ relative changes in variable $x$ across periods and using our normalization that $\hat{\tau}_{ntt} = \hat{\tau}_{itt} = 1$,

$$\left( \frac{\hat{x}_{nit}}{\hat{x}_{nt}} \frac{\hat{x}_{int}}{\hat{x}_{nt}} \right)^{\frac{1}{2}} = (\hat{\tau}_{nit} \hat{\tau}_{int})^{-\frac{\theta}{2}}.$$  

Assuming a standard value for the trade elasticity of $\theta = 5$ and that bilateral trade cost shocks are symmetric ($\hat{\tau}_{nit} = \hat{\tau}_{int}$), we can recover all bilateral relative changes in trade costs from the bilateral trade data.

Using the market clearing condition that equates a country’s income with expenditure on its goods, and again assuming a standard value for the trade elasticity of $\theta = 5$, we estimate the changes in productivity in each country ($\hat{z}_{it}$) that exactly rationalize the observed changes in per capita incomes ($\hat{w}_{it}$) and populations ($\hat{\ell}_{it}$), given our estimated changes in bilateral trade costs ($\hat{\tau}_{nit}^{-\theta}$):

$$\hat{w}_{it} \hat{\ell}_{it} w_{it} \hat{\ell}_{it} = \sum_{n=1}^{N} \sum_{t=1}^{T} s_{nit} \hat{\tau}_{nit}^{-\theta} w_{it}^{-\theta} z_{it} \hat{w}_{nt} \hat{\ell}_{nt} w_{nt} \hat{\ell}_{nt},$$  \hspace{1cm} (G.1)

where any omitted changes in trade costs for an importer $n$ that are common across exporters cancel from the numerator and denominator of the fraction on the right-hand side; any omitted changes in trade costs for an exporter $i$ that are common across importers are implicitly absorbed into the estimated changes in productivities ($\hat{z}_{it}$). Note that the fraction on the right-hand side of equation (G.1) corresponds to an expenditure share and is homogenous of degree zero in these changes in productivities ($\hat{z}_{it}$). Therefore, these changes in productivities ($\hat{z}_{it}$) only can be recovered up to a normalization or choice of units, and we use the normalization that the mean of the log changes in productivities across all countries is equal to zero (a geometric mean of changes in productivities of one).

Having recovered these changes in productivities ($\hat{z}_{it}$) implied by the observed data, we compare the predictions from our (first-order) linearization for the impact of productivity shocks on income to those from the full non-linear solution of the model using the exact-hat algebra approach. In particular, we undertake exact-hat algebra counterfactuals, in which we solve for the counterfactual change in per capita income ($\hat{w}_{it}$) and welfare ($\hat{w}_{it}$) in the full non-linear model in response to the changes in productivity alone ($\hat{z}_{it}$), holding trade costs and populations constant. The counterfactual change in income ($\hat{w}_{it}$) in each country in the full non-linear model satisfies the following system of equations:

$$\hat{w}_{it} w_{it} \hat{\ell}_{it} = \sum_{n=1}^{N} \sum_{t=1}^{T} s_{nit} \hat{w}_{it}^{-\theta} z_{it} \hat{w}_{nt} \hat{\ell}_{nt} w_{nt} \hat{\ell}_{nt}. $$  

(G.2)
The corresponding counterfactual change in welfare \( (\hat{u}_{it}) \) in each country in the full non-linear model satisfies the following system of equations:

\[
\ln \hat{u}_i = \ln \hat{w}_i + \frac{1}{\theta} \ln \left[ \sum_{n=1}^{N} s_{in} w_n^{1-\theta} z_{in}^{\theta} \right].
\]

We compare these solutions of the full non-linear model to the predictions of our linearization, which implies the following log change in countries’ per capita incomes and welfare in response to these productivity shocks: \( \ln \hat{w} = W \ln \hat{z} \) and \( \ln \hat{u} = U \ln \hat{z} \). We also undertake an analogous exercise in which we compare the predictions of our linearization for changes in bilateral trade costs to the counterfactual predictions from exact-hat algebra, as discussed further below.

(ii) Quality of the Approximation for Productivity Shocks in Simulated Trade Networks

In Section 6.2 of the paper, we show that our linearization provides an almost-exact approximation to the full non-linear model solution in actual trade for empirically-reasonable productivity shocks and trade elasticities in actual trade networks. We now report the results of an extensive search over simulated trade networks to try to find counterexamples where our approximation performs less well for empirically-reasonable productivity shocks.

In the paper, we report comparisons of our linearization and the full non-linear model solution for the empirical distribution of productivity shocks from 2000-2010. We start from the observed equilibrium in the data in 2000 and shock productivity in each country by the empirical distribution of productivity shocks from 2000-2010. We report these comparisons for both our baseline value of trade elasticity of \( \theta = 5 \) and the empirically-relevant range of values for the trade elasticity from \( \theta = 2 \) to \( \theta = 20 \). Across all of these parameter values, we find that our linearization provides an almost-exact approximation to the full non-linear model solution.

As discussed in Section 4 of the paper, our linearization and the full non-linear model solution are identical in the two limiting cases of autarky and free trade. Furthermore, we find that our linearization also performs well for random trade matrices. We now show that observed trade matrices are well approximated by a weighted average of autarky, free trade, and random noise, which provides an intuition why our approximation works so well for actual trade networks. Under autarky, the expenditure share matrix is an identity \( (S = I) \); under free trade, \( S = Q \) where \( Q_{ni} = q_i \), the size of country \( i \), for all \( n \). Figure G.1 shows the histogram of the entries of the matrix \( (S - (1 - \alpha) Q - \alpha I) \) for the year 2000, where \( \alpha \equiv \frac{1}{N} \sum_{i=1}^{N} S_{ii} \) is the average expenditure share on domestic goods. The mass close to zero shows that the observed trade matrix \( S \) in 2000 is well approximated by a weighted average of autarky, free trade, and random noise. This is a robust pattern that holds across all years in our sample.
Figure G.1: Histogram of the entries in the off-diagonal elements of the matrix \((S - Q)\)

Note: Figure shows the histogram of the entries of the matrix \((S - (1 - \alpha)Q - \alpha I)\) for the year 2000, where \(\alpha \equiv \frac{1}{N} \sum_{i=1}^{N} s_{ii}\) is the average expenditure share on domestic goods.

We now assess quality of the linearized solution in simulated trade networks. We construct several types of artificial trade networks to validate our intuition above. Specifically, we assess the quality of approximation for TFP shocks in four types of simulated networks, each with varying number of simulated countries \(N \in \{10, 20, 50, 100, 150\}\):

1. Networks that are well-approximated by weighted averages of autarky, free-trade, and noise: 
   \[
   \tilde{S}^{(1)} \equiv \alpha I + (1 - \alpha) \tilde{Q} + \tilde{\epsilon};
   \]

2. Networks that are purely noise: 
   \[
   \tilde{S}^{(2)} \equiv \tilde{\epsilon};
   \]

3. Networks that represent circular graphs, with every country \(i\) consuming only goods \(i + 1\) and \(i + 2\), i.e.,
   \[
   S_{ij}^{(3)} = \begin{cases} 
   1/2 & \text{if } (j - i) \mod N \in \{1, 2\} \\
   0 & \text{otherwise};
   \end{cases}
   \]

4. Networks that represent circular graphs with noise: 
   \[
   \tilde{S}^{(4)} \equiv S^{(3)} + \tilde{\epsilon}.
   \]

We use a bar above a variable to denote random draws. We draw the random error \(\tilde{\epsilon}\) (with replacement) from the residuals shown in Figure G.1 above; the simulated free-trade \(\tilde{Q}\) is obtained by drawing (with replacement) a random row vector \(\tilde{q}'\) of country level income shares from the empirical distribution \(q'\) in the year 2000, normalizing such that the total world income sums to one, and then stacking the rows \(N\) times. In every simulation, expenditure shares are rescaled if necessary to ensure that the row sums of \(\tilde{S}\) always equal one.

Simulated networks of type 1 are constructed to imitate the observed actual world trade matrix. Simulated networks of type 3 are constructed as polar opposites of type 1, where the expenditure shares deviate substantially from autarky, free trade, or random noise. Simulated networks of types 2 and 4 are introduced to assess the role of noise that is residualized from the autarky and free trade matrices.

We simulate each network type for each given number of countries 1,000 times. For each simulation we draw (with replacement) country-level TFP shocks from empirical distribution of productivity shocks from 2000-2010, and we compare the counterfactual predictions of our linearization and the non-linear model solution for changes in income (\(\hat{w}_{it}\)) and welfare (\(\hat{u}_{it}\)) in response to productivity shocks (\(\hat{z}_{it}\)).

\footnote{When constructing a circular graph, each country needs to consume at least two goods in order for country-level income to respond to TFP shocks; world income distribution is invariant to TFP shocks in a trade network with 
\[
S_{ij} = \begin{cases} 
1 & \text{if } (j - 1) \mod N = 1 \\
0 & \text{otherwise.}
\end{cases}
\]
In Figures G.2 through G.5, we display histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations for each type of network. The top row shows the histograms for the regression slope coefficient for log changes in income. The second row shows the histograms for the regression slope coefficient for log changes in welfare. The third row shows the histograms for the correlation coefficient for log changes in income. The fourth row shows the histograms for the correlation coefficient for log changes in welfare. The columns of each figure correspond to different assumed numbers of countries. As the regression slope coefficients and correlation coefficients become more concentrated towards one, our linearization provides a closer approximation to the full non-linear solution of the model.

We now summarize the main take-aways from these simulations. First, the approximation quality increases in the number of countries. With 150 countries, approximation quality is almost-exact in simulated networks of types 1, 2, and 4. Second, even with a small number of countries, the approximation quality is extremely high in simulated networks of type 1, which is constructed to imitate the actual trade network in the observed data. The only networks in which our linearization performs substantially less well are those with both a small number of countries and an extreme, circular trade pattern that is quite different from that observed in the data. Even with this extreme trade pattern, we find that as we increase the number of countries, the quality of the approximation improves. This pattern of results using simulated trade networks provides further intuition for why our linearization performs so well using the actual trade network in the observed data: this actual trade network is well-approximated by a weighted average of autarky, free-trade, and noise, and it features a large number of 140 countries.

(iii) Quality of the Approximation for Trade Cost Shocks  In Figure 2a in the paper, we report a comparison of counterfactual predictions for log changes in income in our linearization and the full non-linear model solution for the empirical distribution of productivity shocks from 2000-2010. In Figure G.6 below, we report an analogous comparison of our linearization and exact-hat algebra counterfactuals using the empirical distribution of trade cost shocks from 2000-2010. While we again find a strong relationship between the predictions of our linearization and the exact-hat algebra counterfactuals, it is less strong than for productivity shocks with a regression slope of less than one, because of the bilateral variation in trade cost shocks. Nevertheless, even though the regression slope now differs from one for trade cost shocks, the correlation coefficient between the predicted changes in per capita income using our linearization and exact-hat algebra counterfactuals remains greater than 0.99. In Monte Carlo simulations in which we draw a shock to bilateral trade costs for an exporter-importer pair from the empirical distribution of trade cost shocks from 2000-2010, and compare our linearization and the exact-hat algebra counterfactuals, we find this same pattern of results with regressions slopes that can differ from one, but correlation coefficients that remain close to one. Furthermore, although our linearization works less well in general for bilateral trade cost shocks, we find many examples in which it provides a close approximation to the full non-linear solution of the model, as shown for a shock to US-China bilateral trade costs in Figure G.7.
Figure G.2: Approximation quality for simulated network type 1 (expenditure share matrix is a weighted average of free trade, autarky, and noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure G.3: Approximation quality for simulated network type 2 (expenditure share matrix is pure noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure G.4: Approximation quality for simulated network type 3 (expenditure share matrix represents a circular graph)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
Figure G.5: Approximation quality for simulated network type 4 (expenditure share matrix represents a circular graph with noise)

Note: histograms for regression slope coefficients and correlation coefficients between the linear and non-linear predictions across the 1,000 simulations; the top row shows the histograms for the regression slope coefficient for log changes in income; the second row shows the histograms for the regression slope coefficient for log changes in welfare; the third row shows the histograms for the correlation coefficient for log changes in income; the fourth row shows the histograms for the correlation coefficient for log changes in welfare; the columns correspond to different numbers of countries.
(iv) Multiple Sectors and Input-Output Linkages  In our analysis of the quality of our approximation in Section 6.2 in the paper, we focus on our baseline single-sector Armington model from Section 3 of the paper, for simplicity and because of its prominence in the existing literature. We now show that our linearization also provides an almost-exact approximation to the full non-linear model solution for our extension to multiple sectors and input-output linkages from Section 5.5 of the paper.

We begin by recovering the empirical distribution of productivity and trade cost shocks that rationalizes the observed data for our assumed trade elasticity of $\theta = 5$. Using the resulting empirical distribution of changes in productivities, we compute exact-hat algebra counterfactuals for changes in country income ($\hat{w}_i$) and welfare ($\hat{u}_i$) in response
to these productivity shocks ($\hat{z}_i$). In particular, we start from the observed equilibrium in the data in 2000, and shock productivity in each country by this empirical distribution of productivity shocks from 2000-2010, holding trade costs constant ($\hat{\tau}_{ni} = 1$). We compare the resulting counterfactual predictions for changes in income and welfare from the full non-linear model solution to those of our linearization, in which we pre-multiply the empirical distribution of productivity shocks by our income exposure ($W^{IO}$) and welfare exposure ($U^{IO}$) matrices from the input-output model: $\ln \hat{w} = W^{IO} \ln \hat{z}$ and $\ln \hat{u} = U^{IO} \ln \hat{z}$.

In Figures G.8a and G.8b, we display the two sets of predictions for changes in income and welfare, respectively. Again we find that our linearization provides an almost exact approximation to the full non-linear model solution, with regression slopes close to one and correlation coefficients greater than 0.999. Therefore, regardless of whether we consider the single-sector model or the input-output model, we find that our linearization is not only exact for small shocks, but is also almost exact for empirically-reasonable productivity shocks and trade elasticities.

Figure G.8: Counterfactual Predictions for the Empirical Distribution of Productivity Shocks from 2000-2010 (Trade Elasticity $\theta = 5$, Input-Output Model)

(a) Our Linearization Versus Non-linear Model Solution for Predicted Changes in Income ($\ln \hat{w}_{it}$)

(b) Our Linearization Versus Non-linear Model Solution for Predicted Changes in Welfare ($\ln \hat{u}_{it}$)

Source: NBER World Trade Database and authors’ calculations using our input-output model from Section 5.5 of the paper.

### G.1.2 Global Income and Welfare Exposure

In Section 6.3 of the paper, we provide evidence on the evolution of the global network of income and welfare exposure to productivity shocks over our long historical sample period. We compute bilateral income and welfare exposure for our balanced panel of 143 countries over the 43 years from 1970-2012 ($143 \times 143 \times 43 = 879,307$ bilateral predictions for each variable). In this section of the online appendix, we report additional empirical results on this global network of income and welfare that are discussed in the paper.

In Table 2 in the paper, we show that our income ($W^{IO}$) and welfare ($U^{IO}$) exposure measures in our input-output model cannot be fully captured by simpler measures of trading relationships between countries: (i) the log value of trade; (ii) aggregate import shares (the expenditure share matrix from our single-sector model ($S^{SSM}$)); (iii) the expenditure share matrix from the input-output model ($S^{IO}$); (iv) the income share matrix from the input-output model ($T^{IO}$); and (v) the cross-substitution matrix from the input-output model ($M^{IO}$).

In Table G.1 below, we reproduce the results for income ($W^{IO}$) and welfare ($U^{IO}$) exposure from the paper, and show that their market-size, cross-substitution and price index components also cannot be fully captured by these simpler measures of trading relationships between countries. Across all specifications, we find statistically significant correlations. For both the market-size and cost of living effects, we find regression R-squared of less than
For the cross-substitution effect, we find higher R-squared for the income share matrix \((T^{IO})\) and the cross-substitution matrix \((M^{IO})\), which is consistent with the central role played by these matrices in determining the cross-substitution effect. In all of these specifications, the estimated coefficients in these reduced-form regressions do not have a structural interpretation, which implies that it would be hard to infer income and welfare exposure from these simpler measures of trading relationships without our closed-form solutions.

Table G.1: Correlations of Income \((W^{IO})\) and Welfare \((U^{IO})\) Exposure and their Components with Other Measures of Trading Relationships Between Countries

<table>
<thead>
<tr>
<th>(W^{IO})</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log value</td>
<td>-0.00339***</td>
<td>-0.286***</td>
<td>-2.533***</td>
<td>-2.207***</td>
<td>-2.695***</td>
</tr>
<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0435</td>
<td>0.165</td>
<td>0.152</td>
<td>0.128</td>
<td>0.254</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(U^{IO})</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log value</td>
<td>0.00000571***</td>
<td>0.0120***</td>
<td>0.123***</td>
<td>0.113***</td>
<td>0.0976***</td>
</tr>
<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0290</td>
<td>0.688</td>
<td>0.843</td>
<td>0.781</td>
<td>0.783</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Market Size(^{IO})</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log value</td>
<td>-0.000302***</td>
<td>-0.223**</td>
<td>-1.890*</td>
<td>-1.358*</td>
<td>-2.005***</td>
</tr>
<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0377</td>
<td>0.109</td>
<td>0.0924</td>
<td>0.0529</td>
<td>0.154</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cross Substitution(^{IO})</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log value</td>
<td>-0.000370***</td>
<td>-0.0637***</td>
<td>-0.644***</td>
<td>-0.849***</td>
<td>-0.690***</td>
</tr>
<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0273</td>
<td>0.431</td>
<td>0.517</td>
<td>0.998</td>
<td>0.878</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price Index(^{IO})</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log value</td>
<td>0.000345***</td>
<td>0.298***</td>
<td>2.657***</td>
<td>2.320***</td>
<td>2.793***</td>
</tr>
<tr>
<td>Observations</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
<td>20592</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0443</td>
<td>0.177</td>
<td>0.165</td>
<td>0.139</td>
<td>0.269</td>
</tr>
</tbody>
</table>

Note: Observations are a cross-section of exporting and importing countries in the year 2000; \(W^{IO}\) is our income exposure measure for the input-output model from equation (49); \(U^{IO}\) is our welfare exposure measure for the input-output model from equation (50); log value is the log of one plus the value of bilateral trade; \(S^{SSM}\) is the share of each exporter in aggregate importer expenditure (the expenditure share matrix in the single-sector model); \(S^{IO}\) is the expenditure share matrix in the input-output model; \(T^{IO}\) is the income share matrix in the input-output model; \(M^{IO}\) is the cross-substitution matrix in the input-output model; table reports the regressions of \(W^{IO}\) and \(U^{IO}\) (rows) on alternative measures of trading relationships between countries (columns); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

In the remainder of this section, we report the results of a further validation exercise. We show that our welfare exposure measure captures the large-scale changes in the network of trade relationships that occurred over our
sample period, including regional integration in North America following the North American Free Trade Agreement (NAFTA), the reorientation of European trade relationships following the fall of the Iron Curtain, and the reorganization of trading patterns in East Asia following the emergence of China into the global economy. In Figure G.9 we illustrate global welfare exposure to productivity shocks in 1970, 1985, 2000 and 2012 using a network graph, where the nodes are countries and the edges capture bilateral welfare exposure. For legibility, we display the 50 largest countries in terms of GDP and the 200 edges with the largest absolute values of bilateral welfare exposure.\(^2\) The size of each node captures the importance of each country as a source of productivity shocks (as a source of welfare exposure for other countries); the arrow for each edge shows the direction of bilateral welfare exposure (from the source of the productivity shock to the exposed country); and the thickness of each edge shows the absolute magnitude of the bilateral welfare exposure. Countries are grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random).

At the beginning of our sample period in 1970, the global network of welfare exposure is dominated by the U.S., Germany and other Western industrialized countries (top-left panel). Moving forward to 1985, we see the emergence of Japan and a cluster of Newly Industrialized Countries (NICs) in Asia, and we observe Western Europe increasingly emerging as a separate cluster of interdependent nations. By the time we reach 2000, the separate clusters of countries in Asia and Western Europe become even more apparent, with China beginning to displace Japan at the center of the Asian cluster. By the end of our sample period in 2012, China replaces the U.S. at the center of the global network of welfare exposure, with the US more tightly connected to China and other Asian countries than to the cluster of Western European countries. Therefore, we find substantial changes, not only in the mean and dispersion of welfare exposure, but also in the network of bilateral interdependencies between countries.

\(^2\) All of the bilateral welfare exposure links shown in the figure are positive.
We next provide further evidence in the change in networks of welfare exposure in North America following the North American Free Trade Agreement (NAFTA), in Central Europe following the fall of the Iron Curtain, and in Asia following the emergence of China into the global economy. We use chord or radial network diagrams, as used for example in comparative genomics in Krzywinski et al. (2009) and for bilateral migration in Sander et al. (2014).

In Figure G.10, we show welfare exposure in 1970 and 2012 for U.S., Canada, Mexico, Japan and China, where each country is labelled by its three-letter International Organization for Standardization (ISO) code. These countries are arranged around a circle, where the size of the inner segment for each country shows its overall outward exposure (the effect of its productivity shocks on other countries), and the gap between the inner and outer segments shows its overall inward exposure (the effect of foreign productivity shocks upon it). Arrows emerging from the inner segment for each country show the bilateral impact of its productivity shocks on welfare in other countries. Arrows pointing towards the gap between the inner and outer segments show the bilateral impact of other countries’ productivity growth on its welfare. In 1970, the network is dominated by the effect of US productivity shocks on

---

3To ensure a consistent treatment of countries over time, we manually assign some three-letter codes, such as the code USR for the members of the former Soviet Union.

4We omit own exposure to focus on the impact of foreign productivity shocks on country welfare. Almost all values of our welfare exposure
welfare in the other countries, and Japan is substantially more connected to the network than China. By 2012, following Mexican trade liberalization in 1987, the Canada-US Free Trade Agreement (CUSFTA) in 1988 and the North American Free Trade Agreement (NAFTA) in 1994, we observe much deeper integration between the three North American economies. Additionally, we find a reversal of the relative positions of the two Asian economies, with China substantially more integrated into the network than Japan.

Figure G.10: North American Welfare Exposure, 1970 and 2012

![Figure G.10: North American Welfare Exposure, 1970 and 2012](image)

Source: NBER World Trade Database and authors’ calculations using our baseline constant elasticity Armington model from Section 3.

In Figure G.11, we display welfare exposure for a broader group of Asian countries. Three features stand out. First, we again find a dramatic change in the relative positions of Japan and China. Whereas in 1970 Japan dominated the network of welfare exposure, in 2012 this position is firmly occupied by China. Second, Vietnam becomes both substantially more exposed to foreign productivity shocks and a much more important source of these productivity shocks for other countries, following its trade liberalization. Third, the overall network of welfare exposure is much denser in 2012 than in 1970, consistent with greater trade integration among these Asian countries increasing their economic interdependence on one another.

measure in these diagrams are positive. For ease of interpretation, we add a constant to our welfare exposure measure in each year, such that its minimum value is zero, which implies that these diagrams show the impact of the productivity shock on relative levels of welfare.
In Figure G.11, we show bilateral welfare exposure in Central Europe before and after the fall of the Iron Curtain. In 1988 immediately before this event, we observe strong connections between the countries of the former Soviet Union (USR) and Eastern European nations such as the former Czechoslovakia (CSK). By 2012, these connections have substantially weakened, and we observe growing connections between Western European countries such as Italy and Eastern European nations. Although Germany is here the aggregation of the former and East and West Germanies in all years, we also observe a strengthening of its position at the center of the network of welfare exposure. More broadly, we also find an increase in the overall density of connections over time, consistent with trade liberalization increasing countries’ economic interdependence.
G.2 Economic and Political Friends and Enemies

In this section of the online appendix, we report additional information for our analysis of the relationship between international relations and our exposure measures in Section 7 of the paper. In Subsection G.2.1, we use our measures to provide new evidence on the determinants and effects of preferential trade agreements (PTAs). In Subsection G.2.2, we use our measures to shed new light on the classic debate in international relations and political science about the extent to which increased conflict of economic interests between countries necessarily involves heightened political tension between them.

G.2.1 Preferential Trade Agreements (PTAs)

In Section 7.1 of the paper, we provide a further validation of our exposure measures, by examining whether they have predictive power for which countries self select into PTAs, and whether they detect increased integration between countries following the formation a PTA.

Selection into PTAs We begin by examining selection into PTAs. In Table 3 in the paper, we provide evidence from regressing future membership of a PTA from 1971-2012 on past welfare exposure to bilateral reductions in trade costs ($U_{IO}$) in 1970 for each exporter-importer pair. In Table G.2, we report analogous regressions of future membership of a PTA from 1971-2012 on past income exposure to bilateral reductions in trade costs ($W_{IO}$) in 1970 for each exporter-importer pair. We find a similar pattern of results for income exposure as for welfare exposure in the paper. Countries with larger income gains from bilateral trade cost reductions in 1970 are more likely to form future PTAs from 1971-2012. We find that these results for income exposure are marginally weaker than those for welfare exposure above, which is consistent with a role for the cost of living effect in influencing selection into trade agreements. More broadly, these findings are consistent with the view that political actors place at least some weight on the welfare of their constituents when negotiating PTAs.
Table G.2: Selection into Future Preferential Trade Agreements (PTAs) and Past Income Exposure to Bilateral Trade Cost Reductions ($W^{\tau IO}$)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PTA_{1971}^{2012}$</td>
<td>0.654**</td>
<td>0.566*</td>
<td>0.350</td>
<td>0.566*</td>
<td>0.564*</td>
</tr>
<tr>
<td></td>
<td>(0.232)</td>
<td>(0.218)</td>
<td>(0.240)</td>
<td>(0.216)</td>
<td>(0.216)</td>
</tr>
<tr>
<td>$PTA_{1981}^{2012}$</td>
<td>0.561***</td>
<td>0.536***</td>
<td>0.558***</td>
<td>-0.102***</td>
<td>-0.136***</td>
</tr>
<tr>
<td></td>
<td>(0.0112)</td>
<td>(0.0107)</td>
<td>(0.0112)</td>
<td>(0.0115)</td>
<td>(0.0126)</td>
</tr>
<tr>
<td>Log value 1970</td>
<td>0.0212***</td>
<td>0.0194***</td>
<td>0.0187***</td>
<td>-0.102***</td>
<td>-0.136***</td>
</tr>
<tr>
<td></td>
<td>(0.00127)</td>
<td>(0.00125)</td>
<td>(0.00124)</td>
<td>(0.0115)</td>
<td>(0.0126)</td>
</tr>
<tr>
<td>$SS_{1970}^{SSM}$</td>
<td>2.300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.030)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PTA_{1971}^{1980}$</td>
<td>0.684***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0119)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PTA_{1971}^{1990}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.719***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0125)</td>
</tr>
</tbody>
</table>

Note: Observations are a cross-section of exporter-importer pairs; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $PTA_{1971}^{2012}$ is a dummy variable that is equal to one if an exporter-importer pair is a member of a preferential trade agreement (PTA) from 1971-2012; $PTA_{1971}^{1981}$, $PTA_{1971}^{1991}$, and $PTA_{1971}^{1980}$ are defined analogously; $W^{\tau IO}$ 1970 is welfare exposure to bilateral trade cost reductions in 1970 in the input-output model from equation (50) in the paper; PTA 1970 is a dummy variable that is equal to one if an exporter-importer pair is a member of a PTA in 1970 or earlier; log value 1970 is the log of one plus the value of bilateral trade flows in 1970; $SS_{1970}^{SSM}$ 1970 is the share of each exporter in aggregate importer expenditure in 1970 (the expenditure share matrix in the single-sector model); standard errors in parentheses are heteroskedasticity robust; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

Impact of Trade Agreements on Economic Integration  We next provide evidence on the subsequent impact of PTAs on economic integration between countries, controlling for the non-random selection into these agreements established above.

We estimate the event-study specification in equation (51) in the paper, in which we regress importer $n$’s exposure to exporter $i$ at time $t$ on PTA treatment-year interactions, exporter-importer fixed effects, and continent-year fixed effects. The inclusion of the exporter-importer fixed effects controls for selection into PTAs based on time-invariant factors. Therefore, if exporter-importer pairs with high levels of welfare exposure are more likely to form PTAs in all years, this is controlled for in the exporter-importer fixed effect. The key identifying assumption is parallel trends between the treatment and control group within continents. In all specifications, we include the treatment-year interactions for years both before and after joining a PTA, which allows us to check whether a treated exporter-importer pair differs from a control pair even before joining a PTA.

In Figure 14 in the paper, we report the estimated treatment-year interactions and 95 percent confidence intervals for welfare exposure to productivity growth ($U^{\tau IO}_{n,m}$) and bilateral trade cost reductions ($U^{\tau IO}_{\tau,n,m}$) in our input-output model from equation (50). In Figure G.13, we display the analogous estimated treatment-year interactions and 95 percent confidence intervals for each importer’s income exposure to productivity growth ($W^{\tau IO}_{n,m}$) and bilateral trade cost reductions ($W^{\tau IO}_{\tau,n,m}$) in our input-output model from equation (50). We find a similar pattern of results for income
exposure as for welfare exposure in the paper. Again we find that these income exposure results are marginally weaker than those for welfare exposure, which is consistent with the idea that regional integration also increases the absolute magnitude of the cost of living effect between member countries.

Figure G.13: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Income Exposure

(a) Importer Income Exposure to Exporter Productivity Growth ($W_{nit}$)

(b) Importer Income Exposure to Exporter Bilateral Trade Cost Reductions ($W_{r,nit}$)

Note: Estimated treatment-year interactions ($\beta_s$) from the event-study specification in equation (51) in the paper using the two-way fixed effects estimator; Figure G.13a shows results for an importer’s income exposure to productivity growth in an exporter ($W_{nit}$) in our input-output model from equation (50) in the paper; Figure G.13b displays results for an importer’s income exposure to reductions in bilateral trade costs with an exporter ($W_{r,nit}$) in our input-output model from equation (50) in the paper; all specifications include exporter-importer fixed effects and continent-year fixed effects; standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.

In Figure G.14, we show that we find a similar pattern of results for welfare exposure ($U_{nit}$ and $U_{r,nit}$) as reported in Figure 14 in the paper, once we additionally control for the log value of bilateral trade between an exporter-importer in each year. Similarly, we find that our income exposure results in Figure G.13 above are robust to also controlling for the log value of bilateral trade between an exporter-importer in each year (not shown to conserve space). Therefore, we again find that our exposure measures cannot be captured by simpler measures of trading relationships between countries.

Figure G.14: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure (Controlling for Trade Flows)

(a) Importer Welfare Exposure to Exporter Productivity Growth ($U_{nit}$)

(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($U_{r,nit}$)

Note: Estimated treatment-year interactions ($\beta_s$) from the event-study specification in equation (51) in the paper using the two-way fixed effects estimator; Figure G.14a shows results for the welfare exposure of importer n to productivity growth in exporter i at time t ($U_{nit}$) in our input-output model from equation (50) in the paper; Figure G.14b displays results for the welfare exposure of importer n to reductions in bilateral trade costs with exporter i at time t ($U_{r,nit}$) in our input-output model from equation (50) in the paper; both specifications include the log of one plus the value of bilateral trade between an exporter-importer pair in each year, exporter-importer fixed effects and continent-year fixed effects; standard errors clustered by exporter-importer pair to allow for serial correlation in the error term over time.
Finally, the estimates reported in Figure 14 in the paper and in Figures G.13 and G.14 above use the two-way fixed effects estimator. This estimator uses variation over time within already-treated units, which can be hard to interpret in the presence of treatment heterogeneity and variable timing in the treatment, as recently pointed out in Chaisemartin and D’Haultflocouille (2020), Borusyak et al. (2021) and Goodman-Bacon (2021). In Figure G.15, we show that we find the same pattern of results using the alternative difference-in-differences event-study estimator of Chaisemartin and D’Haultflocouille (2020), which only exploits variation from transitions from untreated to treated status.

Figure G.15: Estimated Treatment Effects of Preferential Trade Agreements (PTAs) on Welfare Exposure (Chaisemartin and D’Haultflocouille (2020) Estimator)

(a) Importer Welfare Exposure to Exporter Productivity Growth ($U_{nit}$)

(b) Importer Welfare Exposure to Exporter Bilateral Trade Cost Reductions ($W_{r,nit}$)

Note: Estimated treatment-year interactions ($\beta_s$) from the event-study specification in equation (51) in the paper using the Chaisemartin and D’Haultflocouille (2020) Estimator; Figure G.15a shows results for the welfare exposure of importer $n$ to productivity growth in exporter $i$ at time $t$ ($U_{nit}$) in our input-output model from equation (50) in the paper; Figure G.15b displays results for the welfare exposure of importer $n$ to reductions in bilateral trade costs with exporter $i$ at time $t$ ($W_{r,nit}$) in our input-output model from equation (50) in the paper; both specifications include exporter-importer fixed effects and continent-year fixed effects; standard errors based on 50 bootstrap replications.

G.2.2 Bilateral Political Attitudes

In this section of the online appendix, we provide further details on the data sources and definitions for our political attitudes measures, and report additional empirical results for our analysis of the relationship between bilateral political attitudes and welfare exposure in Section 7.2 of the paper.

We begin by discussing our two main sources of data on bilateral political attitudes following the political science and international relations literature. First, we use data on observed voting behavior in the United Nations General Assembly (UNGA) to reveal the bilateral similarity of countries’ foreign policies. Second we use measures of strategic rivalries, as classified by political scientists, based on contemporary perceptions by political decision makers of whether countries regard one another as competitors, a source of actual or latent threats, or enemies. We next present additional robustness tests for the empirical results reported in the main paper.

United Nations Voting Behavior  We follow a large literature in political science in measuring countries’ bilateral political attitudes towards one another using the similarity of their votes in the United Nations (UN). The ultimate source for our UN voting data is Voeten (2013), which reports non-unanimous plenary votes in the United Nations General Assembly (UNGA) from 1962-2012, and includes on average around 128 votes each year. Countries are recorded as either voting “no” (coded 1), “abstain” (coded 2) or “yes” (coded 3). In particular, we use the bilateral measures of
voting similarity constructed using these data in the Chance-Corrected Measures of Foreign Policy Similarity (FPSIM) database, as reported in Häge (2017).\(^3\) We denote the outcome of vote \(v\) for country \(i\) by \(O_i(v)\):

\[
O_i(v) \in \{1, 2, 3\} \quad v \in \{1, \ldots, V\}. \tag{G.4}
\]

Building on a large literature in international relations, we consider a number of different measures of bilateral voting similarity. Our first and simplest measure is the \(S\)-score of Signorino and Ritter (1999), which measures the extent of agreement between the votes of countries \(n\) and \(i\) as one minus the sum of the squared actual deviation between their votes scaled by the sum of the squared maximum possible deviations between their votes:

\[
S_{ni}^S = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\frac{1}{2} \sum_{v=1}^{V} (d_{\text{max}}(v))^2}, \tag{G.5}
\]

where \((d_{\text{max}}(v))^2 = \left(\sup\{O_n(v) - O_i(v)\}\right)^2\) represents the maximum possible disagreement for each vote and this measure is bounded between minus one (maximum possible disagreement) and one (maximum possible agreement).

A limitation of this \(S\)-score measure is that it does not control for properties of the empirical distribution function of country votes. In particular, country votes may align by chance, such that the frequency with which countries agree on a “yes” depends on the frequency with which countries vote “yes.” Similarly, the frequency with which they agree on each of the other voting outcomes (“no” and “abstain”) depends on the frequencies with which they choose these other voting outcomes. Therefore, we also consider two alternative measures of countries’ bilateral similarity in voting patterns that control in different ways for properties of the empirical distribution of voting outcomes. First, the \(\pi\)-score of Scott (1955) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around average outcomes across the two countries taken together:

\[
S_{ni}^\pi = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\sum_{v=1}^{V} \left(\frac{O_n(v)}{2} - \bar{O}_n\right)^2 + \sum_{v=1}^{V} \left(\frac{O_i(v)}{2} - \bar{O}_i\right)^2}, \tag{G.6}
\]

where \(\bar{O}_i = \frac{1}{V} \sum_{v=1}^{V} O_i(v)\) is the average outcome for country \(i\).

Second, the \(\kappa\)-score of Cohen (1960) adjusts the observed variability of the countries’ bilateral voting outcomes with the variability of each country’s own voting outcomes around its own average outcome and the difference between the two countries’ average outcomes:

\[
S_{ni}^\kappa = 1 - \frac{\sum_{v=1}^{V} (O_n(v) - O_i(v))^2}{\sum_{v=1}^{V} (O_n(v) - \bar{O}_n)^2 + \sum_{v=1}^{V} (O_i(v) - \bar{O}_i)^2 + \sum_{v=1}^{V} (\bar{O}_n - \bar{O}_i)^2}. \tag{G.7}
\]

Both the \(\pi\)-score and \(\kappa\)-score have an attractive statistical interpretation, as discussed further in Krippendorf (1970), Fay (2005) and Häge (2011). In the case of binary (0,1) voting outcomes, these indices reduce to the form of \(1 - (D_o/D_e)\), where \(D_o\) is the observed frequency of agreement and \(D_e\) is the expected frequency of agreement. The \(\pi\)-score estimates the expected frequency of agreement using the average of the two countries marginal distributions of voting outcomes. In contrast, the \(\kappa\)-score estimates the expected frequency of agreement using each country’s own individual marginal distribution of voting outcomes. Whereas our economic measures of exposure are potentially asymmetric, such that \(n\’s\) exposure to \(i\) is not necessarily the same as \(i\’s\) exposure to \(n\), each of these measures of foreign policy similarity is necessarily symmetric.

\(^3\)See https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/ALVXLM
Finally, as Bailey et al. (2017) point out, measures based on dyadic similarity of vote choices—such as the $S$, $\pi$, and $\kappa$ scores—do not account for the heterogeneity in resolutions being voted on. As a result, these measures could incorrectly attribute changes in agenda as changes in state preferences. To resolve this issue, Bailey et al. (2017) apply spatial voting models from the roll call literature to estimate each country’s political preferences embedded in its UN votes. The outcome of this statistical procedure is a time-varying, one-dimensional measure called “ideal points”, which reflects each country’s preference. Bailey et al. (2017) show that ideal points consistently capture the position of states vis-à-vis a US-led liberal order. We derive a measure of bilateral distance by taking the absolute difference between the ideal points of countries $i$ and $j$ in year $t$.

**Strategic Rivalries** Our second set of measures of countries’ bilateral attitudes are indicator variables that pick up whether country $i$ is a strategic rival of $j$ in year $t$, as classified by Thompson (2001) and Colaresi et al. (2010). These rivalry measures capture the risk of conflict with a country of significant relative size and military strength, based on contemporary perceptions by political decision makers, gathered from historical sources on foreign policy and diplomacy. Specifically, rivalries are identified by whether two countries regard each other as competitors, a source of actual or latent threats that pose some possibility of becoming militarized, or enemies. 

Prior research has shown that rivalry occurs much more frequently than actual wars (Colaresi et al. 2010, Aghion et al. 2018). In our sample from 1970-2012, a total of 42 countries have had at least one strategic rival; 74 country-pairs have been strategic rivals at some point; and the total number of country-pair-years that exhibit strategic rivalry is 2,452. China, for instance, is classified as being in strategic rivalry with the U.S. (1970–1972 and 1996–present), India (the entire sample period), Japan (1996–present), the former Soviet Union (1970–1989), and Vietnam (1973–1991).

**Bilateral Political Attitudes and Welfare Exposure** We next report additional empirical results for the relationship between changes in welfare exposure and changes in bilateral political attitudes. As discussed in the paper, we instrument changes in our welfare exposure measure using exogenous changes in the geographical determinants of bilateral trade. In Table 5 in the paper, we report a robustness test using alternative measures of bilateral political attitudes. We find that increases in welfare exposure to productivity growth predicted by our instruments decreases the distance between countries ideal points (Column (1)) and reduces the propensity with which countries are strategic rivals (Column (2)). We find the same negative relationship for each of the different types of strategic rivalries (Columns (3)-(5)), although the estimated coefficient for ideological rivalries is not significant at conventional critical values, which is consistent with the idea that ideological rivalries could be less sensitive to economic factors. In Table G.3 below, we show that these results using alternative measures of bilateral political attitudes are also robust to controlling for the log value of trade as a simpler measures of trading relationships between countries.

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6 Specifically, Bailey et al. (2017)’s methodology identifies preference change over time by exploiting duplicate resolutions that are voted repeatedly in consecutive sessions. This methodology also weights resolutions based on how much they reflect the main policy preference dimension in order to ensure that ideal points are not heavily influenced by resolutions that reflect idiosyncratic factors.

7 Colaresi et al. (2010) further refine the data to distinguish between three types of rivalries: spatial, where rivals contest the exclusive control of a territory; positional, where rivals contest relative shares of influence over activities and prestige within a system or subsystem; and ideological, where rivals contest the relative virtues of different belief systems relating to political, economic or religious activities.
Table G.3: Robustness of Changes in Political and Economic Friends (Five-year Long Differences, Controlling for Trade Flows)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta A_{nit}^D$</td>
<td>-1831.5***</td>
<td>-42.31**</td>
<td>-26.88**</td>
<td>-36.92***</td>
<td>-5.281</td>
</tr>
<tr>
<td></td>
<td>(328.3)</td>
<td>(17.00)</td>
<td>(11.26)</td>
<td>(14.00)</td>
<td>(4.276)</td>
</tr>
<tr>
<td>$\Delta A_{nit}^R$</td>
<td>0.0163***</td>
<td>0.000368**</td>
<td>0.000256**</td>
<td>0.000343**</td>
<td>-0.0000143</td>
</tr>
<tr>
<td></td>
<td>(0.00347)</td>
<td>(0.000174)</td>
<td>(0.000121)</td>
<td>(0.000137)</td>
<td>(0.0000620)</td>
</tr>
</tbody>
</table>

Note: Observations are pooled five-year long differences from 1970 through 2010 for exporter-importer pairs; the first subscript $n$ denotes the importer, the second subscript $i$ corresponds to the exporter, and the third subscript $t$ indexes the five-year difference; each column corresponds to a separate regression, with the left-hand side variable reported at the top of the column and the right-hand side variables listed in the rows; $\Delta A_{nit}^D$ is the five-year change in the ideal-distance measure of political attitude; $\Delta A_{nit}^R$ is the five-year change in the measure of any strategic rivalries; $\Delta A_{nit}^{RP}$ is the five-year change in the measure of positional strategic rivalries; $\Delta A_{nit}^{RS}$ is the five-year change in the measure of spatial strategic rivalries; $\Delta A_{nit}^{RI}$ is the five-year change in the measure of ideological strategic rivalries; $\Delta U_{nit}^{IO}$ is the five-year change in welfare exposure in the input-output model from equation (50); $\Delta \log value_{nit}$ is the five-year change in the log of one plus the value of bilateral trade; the five-year change in welfare exposure ($\Delta U_{nit}^{IO}$) is instrumented by the change in the expenditure share matrix in the input-output model predicted by geographic variables ($\Delta S_{it}^{LOG}$) from a gravity equation (exporter and importer population and bilateral distance with time-varying coefficients); first-stage F-statistic is a test of the statistical significance of the instrument in the first-stage regression; the second-stage R-squared is not reported, because it does not have a meaningful interpretation; standard errors in parentheses are clustered by exporter-importer pair; *** denotes significance at the 1 percent level; ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level.

H Data Appendix

NBER World Trade Database Our data on international trade are from the NBER World Trade Database, which reports values of bilateral trade between countries for around 1,500 4-digit Standard International Trade Classification (SITC) codes. The ultimate source for these data is the United Nations COMTRADE database and we use an updated version of the dataset from Feenstra et al. (2005) for the time period 1970-2012. We augment these trade data with information on countries’ gross domestic product (GDP), population and bilateral distances from the GEODIST and GRAVITY datasets from CEPII. Note that the NBER World Trade Database lacks direct data on a country’s expenditure on domestic goods ($X_{nt}$). Therefore, we compute this domestic expenditure as gross output minus exports plus imports. To measure gross output for each country, we multiply its GDP (value added) by 2.2, which is the mean ratio of gross output to GDP in the EU-KLEMS database (which includes the USA and Japan).

In our multi-sector models, we report results aggregating the products in the NBER World Trade Database to the 20 International Standard Industrial Classification (ISIC) industries listed below. In specifications incorporating input-output linkages, we use a common input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015). We allow some variation in self-expenditure shares across using the following procedure. First, we compute global sector-level expenditure shares that take into account both the final-demand coefficients and the input-output matrix. We then multiply these shares by the imputed gross output of each country to get sectoral expenditure levels for each country. Finally, we allocate these sectoral expenditure levels to various origin countries using the imports data. The residual is treated as sectoral self-expenditure. We show below that this procedure for measuring sectoral self-expenditure generates income and

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8 See https://cid.econ.ucdavis.edu/wix.html.
welfare exposure measures that are strongly correlated with those from the EORA database that reports sectoral self-expenditure shares.

<table>
<thead>
<tr>
<th>Industry Code</th>
<th>Short Name</th>
<th>Long Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>Agriculture forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining</td>
<td>Mining and quarrying</td>
</tr>
<tr>
<td>3</td>
<td>Food</td>
<td>Food products, beverages and tobacco</td>
</tr>
<tr>
<td>4</td>
<td>Textile</td>
<td>Textiles, textile products, leather and footwear</td>
</tr>
<tr>
<td>5</td>
<td>Wood</td>
<td>Wood and products of wood and cork</td>
</tr>
<tr>
<td>6</td>
<td>Paper</td>
<td>Pulp, paper, paper products, printing and publishing</td>
</tr>
<tr>
<td>7</td>
<td>Petroleum</td>
<td>Coke, refined petroleum and nuclear fuel</td>
</tr>
<tr>
<td>8</td>
<td>Chemicals</td>
<td>Chemicals</td>
</tr>
<tr>
<td>9</td>
<td>Plastic</td>
<td>Rubber and plastics products</td>
</tr>
<tr>
<td>10</td>
<td>Minerals</td>
<td>Other nonmetallic mineral products</td>
</tr>
<tr>
<td>11</td>
<td>Basic Metals</td>
<td>Basic metals</td>
</tr>
<tr>
<td>12</td>
<td>Metal Products</td>
<td>Fabricated metal products, except machinery and equipment</td>
</tr>
<tr>
<td>13</td>
<td>Machinery nec</td>
<td>Machinery and equipment n.e.c</td>
</tr>
<tr>
<td>14</td>
<td>Office</td>
<td>Office, accounting and computing machinery</td>
</tr>
<tr>
<td>15</td>
<td>Electrical</td>
<td>Electrical machinery and apparatus, n.e.c.</td>
</tr>
<tr>
<td>16</td>
<td>Communication</td>
<td>Radio, television and communication equipment</td>
</tr>
<tr>
<td>17</td>
<td>Medical</td>
<td>Medical, precision and optical instruments, watches and clocks</td>
</tr>
<tr>
<td>18</td>
<td>Auto</td>
<td>Motor vehicles trailers and semi-trailers</td>
</tr>
<tr>
<td>19</td>
<td>Other Transport</td>
<td>Other transport equipment</td>
</tr>
<tr>
<td>20</td>
<td>Other</td>
<td>Manufacturing n.e.c and recycling</td>
</tr>
</tbody>
</table>

**EORA Database**  Our baseline specification using the NBER World Trade Database has the advantage of including a large number of more than 140 countries over a long time period of more than 40 years and with a relatively disaggregated industry classification of 40 sectors (out of which 20 are tradable sectors). A limitation is that we need to measure self-expenditure shares using the procedure discussed in the previous paragraph, and assume the same input-output matrix for all countries, based on the median input-output coefficients across the country sample in Caliendo and Parro (2015).

To assess the sensitivity of our results to these assumptions, we replicated our analysis using the EORA Global Supply Chain Database (https://www.worldmrio.com/), which contains country-specific input-output tables and can be used to directly measure self-expenditure shares in each sector. Limitations of this dataset are that it considers a much shorter time period from 1990-2015, a more aggregated industry classification of 26 sectors (out of which 11 are tradable sectors), and is constructed with substantial imputation.

We replicate our input-output analysis from Section 5.5 of the paper using the EORA data for this shorter time period and more aggregated industry classification. For comparability with our baseline specification, we use a country-specific but time invariant input-output table for each country in the EORA data. We compare results using the two datasets for the 126 countries in common to both datasets over the time period from 1990-2015 for which both data are available. Recall that welfare exposure ($U^{IO}$) is invariant to our choice of numeraire. To ensure that our results for income exposure ($W^{IO}$) are not sensitive to our choice of numeraire, we normalize income exposure by the income-weighted average for the OECD (excluding the country experiencing the productivity shock).

As shown in Figure H.1, we find a strong, positive and statistically significant correlation between our input-
output exposure measures computed using each dataset, which is equal to 0.78 for relative income exposure \(W^{IO}\) and 0.87 for welfare exposure \(U^{IO}\) in the year 2000. As shown in Figure H.2, we also find a strong, positive and statistically significant correlation between the underlying trade share matrices, which is equal to 0.92 for expenditure shares \(S^{IO}\) and 0.86 for income shares \(T^{IO}\) in the year 2000. Although both figures show results for the year 2000, we find the same high correlation for all years from 1990-2015 for which both data are available. Taken together, this strong correlation between results using the COMTRADE and EORA databases provides evidence that our findings are robust to different assumptions about self-expenditure shares and input-output matrices.

Figure H.1: Comparison of Our Baseline Exposure Measures using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)

![Figure H.1: Comparison of Our Baseline Exposure Measures using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)](image)

Note: Baseline specification uses the NBER World Trade Database; EORA specification uses the EORA Database; Welfare exposure to productivity growth \(U^{IO}\) is measured using our input-output model from Section 5.5 of the paper and is invariant to our choice of numeraire; Relative income exposure equals income exposure to productivity growth \(W^{IO}\) measured using our input-output model from Section 5.5 of the paper normalized by its income-weighted mean across OECD countries (excluding the shocked country), to ensure that results for income exposure are not sensitive to our choice of numeraire.

Figure H.2: Comparison of Our Baseline Expenditure and Income Share Matrices using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)

![Figure H.2: Comparison of Our Baseline Expenditure and Income Share Matrices using the NBER World Trade Database to those using the EORA Database in 2000 (Input-Output Model)](image)

Note: Baseline specification uses the NBER World Trade Database; EORA specification uses the EORA Database; Expenditure share matrix \(S\) is the expenditure share matrix \(S^{IO}\) from our input-output model from Section 5.5 of the paper; Income share matrix \(T\) is the income share matrix \(T^{IO}\) from our input-output model from Section 5.5 of the paper.
References


