Online Appendix for "The Spatial Distribution of Economic Activities Within Cities " (Not for Publication)

Stephen J. Redding*
Princeton University, NBER and CEPR

January 2023

A Introduction

In this online appendix, we present a more mathematical treatment of the baseline quantitative urban model discussed in the paper.

B Baseline Quantitative Urban Model

We begin by developing a baseline quantitative urban model of internal city structure based on Ahlfeldt, Redding, Sturm and Wolf (2015). This model incorporates agglomeration and dispersion forces and an arbitrary number of heterogeneous locations within a city, while remaining tractable and amenable to analytical analysis.

We consider a city embedded within a wider economy. The city consists of a set of discrete locations or blocks, which are indexed by $i = 1, ..., S$. Each block has an effective supply of floor space H_i. Floor space can be used commercially or residentially, and we denote the endogenous fractions of floor space allocated to commercial and residential use by θ_i and $1 - \theta_i$, respectively.

The city is populated by an endogenous measure of $\bar{L}$ workers, who are perfectly mobile within the city and the larger economy, which provides a reservation level of utility $\bar{U}$. Workers decide whether or not to move to the city before observing idiosyncratic utility shocks for each possible pair of residence and employment blocks within the city. If a worker decides to move to the city, she observes these realizations for idiosyncratic utility, and picks the pair of residence and employment blocks within the city that maximizes her utility. Firms produce a single final good, which is costlessly traded within the city and the larger economy, and is chosen as the numeraire ($p_i = p = 1$). Markets are perfectly competitive.

Blocks differ in terms of their final goods productivity, residential amenities, supply of floor space and access to the transport network, which determines travel times between any two blocks in the city. Productivity depends on production externalities, which are determined by the surrounding density of workers, and production fundamentals, such as topography and proximity

*Dept. Economics and SPIA, JRR Building, Princeton, NJ 08544. Email: reddings@princeton.edu.

to natural supplies of water. Amenities depend on residential externalities, which are determined by the surrounding density of residents, and residential fundamentals, such as access to forests and lakes. Congestion forces are governed by the elasticity of supply of floor with respect to its price and commuting costs that increase with travel time.

The analysis remains tractable despite the large number of asymmetric locations, because of the introduction of a stochastic formulation of workers commuting decisions. In the baseline model, workers are *ex ante* homogenous but *ex post* heterogeneous, because their draw an idiosyncratic preference shock for each pair of workplace and residence locations.

B1 Preferences

Worker preferences are defined over consumption of a single tradeable final good and residential floor space. We assume that these preferences take the Cobb-Douglas form, such that the indirect utility for a worker ω residing in n and working in i is:[1]

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1, \tag{B.1}$$

where we suppress the time subscript; P_n is the price of the tradeable final good; Q_n is the price of residential floor space; w_i is the wage; $\kappa_{ni} = e^{\kappa \tau_{ni}} \in [1, \infty)$ is an iceberg commuting cost that is increasing in the bilateral trade time between residence and workplace (τ_{ni}) with elasticity $\rho > 0$; B_n captures residential amenities that are common across all workers and could be endogenous to the surrounding concentration of economic activity through agglomeration forces; and $b_{ni}(\omega)$ is an idiosyncratic amenity draw that captures all the idiosyncratic factors that can cause an individual to live and work in particular locations within the city.

We assume that idiosyncratic amenities ($b_{ni}(\omega)$) are drawn from an independent extreme value (Fréchet) distribution for each residence-workplace pair and worker:

$$G(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{B.2}$$

where we normalize the Fréchet scale parameter in equation (B.2) to one, because it enters the worker choice probabilities isomorphically to common amenities B_n in equation (B.1); the smaller the Fréchet shape parameter ϵ, the greater the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables.[2]

[1] For empirical evidence using U.S. data in support of the constant housing expenditure share implied by the Cobb-Douglas functional form, see Davis and Ortalo-Magne (2011).

[2] Modeling idiosyncratic preferences using the extreme value distribution has a long tradition in transportation economics, dating back to McFadden (1974). A related literature models workers' migration decisions using extreme value distributed preferences, as in Grogger and Hanson (2011) and Kennan and Walker (2011).

Using the properties of for extreme value distributions, the probability that a worker chooses to reside in n and work in i is given by:

$$\lambda_{ni} = \frac{L_{ni}}{L_N} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.3}$$

where L_{ni} is the measure of commuters from n to i; L_N is the measure of workers that choose the city; and the full derivation is reported in Section C of this online appendix.

A first key implication of the extreme value specification for idiosyncratic amenities is that bilateral commuting flows in equation (B.3) satisfy a gravity equation. Therefore, the probability of commuting between residence n and workplace i depends on the characteristics of that residence n, the attributes of that workplace i and bilateral commuting costs and amenities ("bilateral resistance"). Furthermore, this probability also depends on the characteristics of all residences k, all workplaces ℓ and all bilateral commuting costs ("multilateral resistance"). A large reduced-form literature in urban economics provides empirical evidence that the gravity equation provides a good approximation to commuting flows, as reviewed in Fortheringham and O'Kelly (1989) and McDonald and McMillen (2010).

Summing across workplaces i in equation (B.3), we obtain the probability that a worker chooses to live in residence n ($\lambda_n^R = R_n/\bar{L}$):

$$\lambda_n^R = \frac{R_n}{\bar{L}} = \frac{\sum_{i \in N} (B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.4}$$

where R_n is the measure of residents that choose to live in location n and $\bar{L}$ is the measure of residents that choose to live somewhere in the city.

Similarly, summing across residences n in equation (B.3), we obtain the probability that a worker chooses workplace i ($\lambda_i^L = L_i/\bar{L}$):

$$\lambda_i^L = \frac{L_i}{\bar{L}} = \frac{\sum_{n \in N} (B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.5}$$

where L_i is the measure of residents that choose to work in location i and $\bar{L}$ is defined above.

A second key implication of the extreme value specification for idiosyncratic amenities is that each location faces an upward-sloping curve for labor. Other things equal, in order to attract additional workers (higher λ_i^L), a location must offer a higher wage (w_i) relative to other locations (w_k) in equation (B.5). Although individual workers experience idiosyncratic random preference draws for locations, with a continuous measure of workers, there is no uncertainty in the supply of workers to each location.

We can also evaluate the probability that a worker commutes to location i conditional on having chosen to live in location n, which takes the following form:

$$\lambda^R_{ni|n} = \frac{\lambda_{ni}}{\lambda^R_n} = \frac{(w_i/\kappa_{ni})^\epsilon}{\sum_{\ell\in N}(w_\ell/\kappa_{n\ell})^\epsilon}. \tag{B.6}$$

Commuter market clearing requires that the measure of workers employed in each location i (L_i) equals the sum across all locations n of their measures of residents (R_n) times their conditional probabilities of commuting to i ($\lambda^R_{ni|n}$):

$$\begin{aligned} L_i &= \sum_{n\in N}\lambda^R_{ni|n}R_n \\ &= \sum_{n\in N}\frac{(w_i/\kappa_{ni})^\epsilon}{\sum_{\ell\in N}(w_\ell/\kappa_{n\ell})^\epsilon}R_n. \end{aligned} \tag{B.7}$$

Expected worker income conditional on living in location n equals the wages in all possible workplace locations weighted by the probabilities of commuting to those locations conditional on living in n:

$$\begin{aligned} \bar{v}_n &= \mathbb{E}\left[w|n\right] \\ &= \sum_{i\in N}\lambda^R_{ni|n}w_i, \\ &= \sum_{i\in N}\frac{(w_i/\kappa_{ni})^\epsilon}{\sum_{\ell\in N}(w_\ell/\kappa_{n\ell})^\epsilon}w_i, \end{aligned} \tag{B.8}$$

where $\mathbb{E}$ denotes the expectations operator and the expectation is taken over the distribution for idiosyncratic amenities. Intuitively, expected worker income is high in locations that have low commuting costs (low κ_{ni}) to high-wage employment locations.

A third key implication of the extreme value specification is that expected utility is equalized across all pairs of residences and workplaces within the city and is equal to the reservation level of utility in the wider economy:

$$\bar{U} = \vartheta\left[\sum_{k\in N}\sum_{\ell\in N}(B_k w_\ell)^\epsilon\left(\kappa_{k\ell}P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}, \tag{B.9}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\vartheta \equiv \Gamma((\epsilon-1)/\epsilon)$; and $\Gamma(\cdot)$ is the Gamma function.

The intuition for this second result is that bilateral commutes with attractive economic characteristics (high workplace wages and low residence cost of living) attract additional commuters with lower idiosyncratic amenities, until expected utility (taking into account idiosyncratic amenities) is the same across all bilateral commutes and equal to the reservation utility. A closely related implication is that workplaces and residences face upward-sloping supply functions in real wages for workers and residents respectively (as captured in the choice probabilities (B.3)). To obtain additional workers, a location must pay higher wages to attract workers with lower realizations for idiosyncratic amenities for that workplace. Similarly, to acquire additional residents, a location must offer a lower cost of living to entice residents with lower realization for idiosyncratic amenities for that residence.

B2 Production

Production of the tradeable final good occurs under conditions of perfect competition and constant returns to scale. For simplicity, we assume that the production technology takes the Cobb-Douglas form, so that output of the final good in location i (y_i) is:

$$y_i = A_i L_i^{\beta} \left(\theta_i H_i\right)^{1-\beta}, \qquad 0 < \beta < 1, \tag{B.10}$$

where A_i is final goods productivity; L_i is employment; H_i is the supply of floor space; and θ_i is the fraction of floor space used commercially.

Firms choose their location of production and their inputs of workers and commercial floor space to maximize profits, taking as given final goods productivity A_i, the distribution of idiosyncratic utility, goods and factor prices, and the location decisions of other firms and workers. Profit maximization implies that equilibrium employment in location i is increasing in productivity (A_i), decreasing in the wage (w_i), and increasing in the supply of commercial floor space ($\theta_i H_i$):

$$L_i = \left(\frac{\beta A_i}{w_i}\right)^{\frac{1}{1-\beta}} \theta_i H_i, \tag{B.11}$$

where the equilibrium wage is determined by the requirement that the demand for workers in each employment location (B.11) equals the supply of workers choosing to commute to that location (B.7).

From the first-order conditions for profit maximization and zero profits, equilibrium commercial floor prices (q_i) in each block with positive employment must satisfy the following zero-profit condition:

$$q_i = (1-\beta)\left(\frac{\beta}{w_i}\right)^{\frac{\beta}{1-\beta}} A_i^{\frac{1}{1-\beta}}. \tag{B.12}$$

Intuitively, firms in blocks with higher productivity (A_i) and/or lower wages (w_i) are able to pay higher commercial floor prices and still make zero profits.

B3 Land Market Clearing

Land market equilibrium requires no-arbitrage between the commercial and residential use of floor space after the tax equivalent of land use regulations. The share of floor space used commercially (θ_i) is:

$$\begin{array}{llll} \theta_i & = 1 & \text{if} & q_i > \xi_i Q_i, \\ \theta_i & \in [0,1] & \text{if} & q_i = \xi_i Q_i, \\ \theta_i & = 0 & \text{if} & q_i < \xi_i Q_i, \end{array} \tag{B.13}$$

where $\xi_i \geq 1$ captures one plus the tax equivalent of land use regulations that restrict commercial land use relative to residential land use. We allow this wedge between commercial and residential floor prices to vary across blocks.

We follow the standard approach in the urban literature of assuming that floor space H_i is supplied by a competitive construction sector that uses land K_i and capital M_i as inputs. Following Combes, Duranton and Gobillon (2018) and Epple, Gordon and Sieg (2010), we assume that the production function takes the Cobb-Douglas form:

$$H_i = M_i^{\mu} K_i^{1-\mu}. \tag{B.14}$$

Therefore, profit maximization and zero profits in the construction sector implies that the price for commercial floor space in blocks with positive commercial land use satisfies:

$$q_i = \mu^{-\mu}(1-\mu)^{-(1-\mu)}\mathbb{P}^{\mu}\mathbb{R}_i^{1-\mu}.$$

where $\mathbb{P}$ is the common price for capital across all blocks, and $\mathbb{R}_i$ is the price for land. Since the price for capital is the same across all locations, the relationships between the quantities and prices of floor space and land can be summarized as:

$$\begin{aligned} H_i &= \varphi_i K_i^{1-\mu} \\ q_i &= \chi \mathbb{R}_i^{1-\mu}, \end{aligned} \tag{B.15}$$

where we refer to $\varphi_i = M_i^{\mu}$ as the density of development (since it determines the relationship between floor space and land area) and χ is a constant.

Residential land market clearing implies that the demand for residential floor space equals the supply of floor space allocated to residential use in each location:

$$(1-\alpha)\frac{\bar{v}_i R_i}{Q_i} = (1-\theta_i) H_i. \tag{B.16}$$

Commercial land market clearing requires that the demand for commercial floor space equals the supply of floor space allocated to commercial use in each location: $\theta_i H_i$. Using the first-order conditions for profit maximization, this commercial land market clearing condition can be written as follows:

$$\frac{1-\beta}{\beta}\frac{w_i L_i}{q_i} = \theta_i H_i. \tag{B.17}$$

Finally, we require that the overall land market clears such that the demand for residential floor space (B.16) plus the demand for commercial floor space (B.17) equals the total supply of floor space from equation (B.15):

$$(1-\theta_i) H_i + \theta_i H_i = H_i = \varphi_i K_i^{1-\mu}. \tag{B.18}$$

B4 Agglomeration Forces

The attractiveness of a location for residence and production can depend both on exogenous natural advantages (locational fundamentals) and endogenous agglomeration forces.

Production Agglomeration Forces We allow final goods productivity to depend on production fundamentals (a_i) and production externalities ($\mathbb{A}_i$). Production fundamentals capture features of physical geography that make a location more or less productive independently of the surrounding density of economic activity (for example access to natural water). Production externalities impose structure on how the productivity of a given block is affected by the characteristics of other blocks. Specifically, we follow the standard approach in urban economics of modeling these externalities as depending on the travel-time weighted sum of workplace employment density in surrounding blocks:

$$A_i = a_i \mathbb{A}_i^{\eta^L}, \qquad \mathbb{A}_i \equiv \sum_{n \in N} e^{-\delta^L \tau_{in}} \left(\frac{L_n}{K_n}\right), \tag{B.19}$$

where L_n/K_n is workplace employment density per unit of land area; production externalities decline with travel time (τ_{in}) through the iceberg factor $e^{-\delta^L \tau_{in}} \in (0,1]$; δ^L determines their rate of spatial decay; and η^L controls their relative importance in determining overall productivity.

Residential Agglomeration Forces We model the externalities in workers' residential choices analogously to the externalities in firms' production choices. We allow residential amenities to depend on residential fundamentals (b_i) and residential externalities ($\mathbb{B}_i$). Residential fundamentals capture features of physical geography that make a location a more or less attractive place to live independently of the surrounding density of economic activity (for example green areas). Residential externalities again impose structure on how the amenities in a given block are affected by the characteristics of other blocks. Specifically, we adopt a symmetric specification as for production externalities, and model residential externalities as depending on the travel-time weighted sum of residential employment density in surrounding blocks:

$$B_i = b_i \mathbb{B}_i^{\eta^R}, \qquad \mathbb{B}_i \equiv \sum_{n \in N} e^{-\delta^R \tau_{in}} \left(\frac{R_n}{K_n} \right), \tag{B.20}$$

where R_n/K_n is residence employment density per unit of land area; residential externalities decline with travel time (τ_{in}) through the iceberg factor $e^{-\delta^R \tau_{in}} \in (0,1]$; δ^R determines their rate of spatial decay; and η^R controls their relative importance in overall residential amenities. The parameter η^R captures the net effect of residence employment density on amenities, including negative spillovers such as air pollution and crime, and positive externalities through the availability of urban amenities.

B5 General Equilibrium

Given the model's parameters $\{\alpha,\ \beta,\ \mu,\ \epsilon,\ \rho,\ \eta^L,\ \delta^L,\ \eta^R,\ \delta^R\}$, the reservation level of utility in the wider economy $\bar{U}$ and exogenous location characteristics $\{\tau_{ni},\ a_i,\ b_i,\ \varphi_i,\ K_i,\ \xi_i\}$, the general equilibrium of the model is referenced by the following seven endogenous variables in each location $\{\ \lambda_i^L,\ \lambda_i^R,\ Q_i,\ q_i,\ w_i,\ \theta_i,\ H_i\}$ and total city population $\bar{L}$. These eight components of the equilibrium vector are determined by the following system of eight equations: the residential choice probability (B.4), the workplace choice probability (B.5), population mobility (B.9), profit maximization and zero profits (B.12), no-arbitrage between alternative uses of land (B.13), residential land market clearing (B.16), commercial land market clearing (B.17), and overall land market clearing (B.18), where productivity and amenities satisfy (B.19) and (B.20).

In general, there can be a unique equilibrium or multiple equilibria in the model, depending on the strength of agglomeration and dispersion forces. Ahlfeldt, Redding, Sturm and Wolf (2015) establish the existence of a unique equilibrium in the absence of agglomeration forces. Allen, Arkolakis and Li (2021) provide conditions for the existence of a unique equilibrium in the presence of agglomeration forces, which require that these agglomeration forces are sufficiently weak relative to the dispersion forces in the model.

B6 Residential Choices

We now use the characterization of commuting choices in Section B1 of this online appendix to derive the partial equilibrium representation of residential choices in Figure 4a in the paper.

In equilibrium, the expected utility for each residence and workplace pair is equal to the reservation level of utility in the wider economy. Using expected utility (B.9) and the residential choice probabilities (B.4), we can write this population mobility condition for each location n in terms of its own characteristics and the probability of residing in that location:

$$\bar{U} = \vartheta \left[\frac{\sum_{\ell \in N} (B_n w_\ell)^{\epsilon} \left(\kappa_{n\ell} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\lambda_n^R} \right]^{\frac{1}{\epsilon}}, \tag{B.21}$$

which can be further re-written as:

$$\bar{U} = \frac{\vartheta \bar{L}^{\frac{1}{\epsilon}} B_n RMA_n}{P_n^{\alpha} Q_n^{1-\alpha} (R_n)^{\frac{1}{\epsilon}}}, \qquad RMA_n \equiv \left[\sum_{i \in N} (w_i / \kappa_{ni})^{\epsilon} \right]^{\frac{1}{\epsilon}}, \tag{B.22}$$

where we have defined RMA_n is a measure of residents commuting market access to employment opportunities in surrounding locations.

Using the fact that the final good is costlessly traded and chosen as our numeraire ($p_n = 1$), and using the residential land market clearing condition (B.16), we can further re-write this expression to obtain the population mobility condition shown in Figure 4a in the paper:

$$\bar{U} = (R_n)^{-\left(1-\alpha+\frac{1}{\epsilon}\right)} \vartheta \bar{L}^{\frac{1}{\epsilon}} B_n RMA_n \left(\frac{(1-\theta_n) H_n}{\bar{v}_n} \right)^{1-\alpha}. \tag{B.23}$$

The left-hand side of this population mobility condition (B.23) is the reservation level of utility in the wider economy ($\bar{U}$), which is shown as the horizontal line in the figure. The right-hand side of this population mobility condition is the expected utility of living in location n, which is shown as the downward-sloping line in the figure. Given the supply of residential floor space ($(1-\theta_n)H_n$) and wages (which determine residents commuting market access (RMA_n) and residents' expected income ($\bar{v}_n$)), an increase in the number of residents in a given location bids up the price of floor space and brings residents with lower idiosyncratic realizations for preferences for that location, thereby reducing expected utility for that location.

The equilibrium number of residents (R_n) is determined by the intersection of the two lines, at which the expected utility of living in location n is equal to the expected utility in the wider economy. Shifts in the supply of residential floor space ($(1-\theta_n)H_n$) and wages (and hence residents commuting market access (RMA_n) and residents expected income ($\bar{v}_n$)) lead to shifts in the downward-sloping line for the expected utility of living in location n, and hence shifts in the number of residents. In general equilibrium, both the supply of residential floor space and wages are endogenously determined, as characterized above.

B7 Labor Demand and Supply

We now use the characterization of commuting choices and production in Sections B1 and B2 of this online appendix to derive the partial equilibrium representation of labor supply and demand in Figure 4b in the paper.

In equilibrium, the demand for labor in each location must equal the supply of labor from workers choosing to commute to that location. We derive the downward-sloping labor demand curve in Figure 4b from the production technology (B.10). From profit maximization in competitive markets subject to this production technology, the requirement that the value marginal product of labor equals the wage (w_n) in location n implies:

$$w_n = (1-\beta) A_n (\theta_n H_n)^{\beta} L_n^{-\beta}. \tag{B.24}$$

Other things equal, an increase in the number of workers (L_n) implies a decline in the value marginal product of labor on the right-hand side of this equation, and hence a decline in the wage (w_n), as shown in the downward-sloping labor demand curve. Changes in productivity (A_n) and the supply of commercial floor space ($\theta_n H_n$) shift this labor demand curve and are endogenously determined in general equilibrium.

We derive the upward-sloping labor supply in Figure 4b from workers commuting choices. Using the workplace choice probability (B.5), expected utility (B.9) and our choice of numeraire, we can write the number of workers choosing to commute to location n as follows:

$$L_n = \bar{L} \left(\frac{\bar{U}}{\delta}\right)^{-\epsilon} \left(\frac{w_n}{WMA_n}\right)^{\epsilon}, \qquad WMA_n = \left[\sum_{k \in N} B_k^{\epsilon} \left(\kappa_{kn} Q_k^{1-\alpha}\right)^{-\epsilon}\right]^{-\frac{1}{\epsilon}}, \tag{B.25}$$

where WMA_n is a measure of workplace market access, which summarizes the access of workplace n to commuters from surrounding locations.

Other things equal, a location must offer a higher wage (w_n) in order to attract workers with lower idiosyncratic preferences for that location, and hence increase labor supply (L_n) on the right-hand side of this equation, as shown in the upward-sloping labor supply curve. Changes in total city population ($\bar{L}$), the reservation utility ($\bar{U}$), amenities (B_k), commuting costs (κ_{nk}) and prices of residential floor space (Q_n) shift this labor supply curve. All of these variables, except for commuting costs, are endogenously determined in general equilibrium.

C Derivation of Choice Probabilities and Expected Utility

In this section of the online appendix, we provide the derivation of the worker commuting probabilities and expected utility in the baseline quantitative urban model.

C1 Distribution of Utility

From the indirect utility function in equation (B.1), we have the following monotonic relationship between idiosyncratic amenities ($b_{ni}(\omega)$) and utility ($U_{ni}(\omega)$):

$$b_{ni}(\omega) = \frac{U_{ni}(\omega) \kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}{B_n w_i}. \tag{C.1}$$

We assume that idiosyncratic amenities ($b_{ni}(\omega)$) are drawn from an independent extreme value (Fréchet) distribution for each residence-workplace pair and each worker:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{C.2}$$

where we normalize the Fréchet scale parameter in equation (C.2) to one, because it enters worker choice probabilities isomorphically to the common bilateral amenities parameter B_n.

Together equations (C.1) and (C.2) imply that the distribution of utility for residence n and workplace i is:

$$G_{ni}(u) = e^{-\Psi_{ni} u^{-\epsilon}}, \qquad \Psi_{ni} \equiv (B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}. \tag{C.3}$$

From all possible pairs of residence and workplace, each worker chooses the bilateral commute that offers the maximum utility. Since the maximum of a sequence of Fréchet distributed random variables is itself Fréchet distributed, the distribution of utility across all possible pairs of residence and workplace is:

$$1 - G(u) = 1 - \prod_{k \in N} \prod_{\ell \in N} e^{-\Psi_{k\ell} u^{-\epsilon}},$$

where the left-hand side is the probability that a worker has a utility greater than u, and the right-hand side is one minus the probability that the worker has a utility less than u for all possible pairs of residence and employment locations. Therefore we have:

$$G(u) = e^{-\Psi u^{-\epsilon}}, \qquad \Psi = \sum_{k \in N} \sum_{\ell \in N} \Psi_{k\ell}. \tag{C.4}$$

Given this Fréchet distribution for utility, expected utility is:

$$\mathbb{E}\left[u\right] = \int_0^{\infty} \epsilon \Psi u^{-\epsilon} e^{-\Psi u^{-\epsilon}} du. \tag{C.5}$$

Now define the following change of variables:

$$y = \Psi u^{-\epsilon}, \qquad dy = -\epsilon \Psi u^{-(\epsilon+1)} du. \tag{C.6}$$

Using this change of variables, expected utility can be written as:

$$\mathbb{E}\left[u\right] = \int_0^{\infty} \Psi^{1/\epsilon} y^{-1/\epsilon} e^{-y} dy, \tag{C.7}$$

which can be in turn written as:

$$\mathbb{E}\left[u\right] = \vartheta \Psi^{1/\epsilon}, \qquad \vartheta = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right), \tag{C.8}$$

where $\Gamma(\cdot)$ is the Gamma function. Therefore we obtain the following expression for expected utility:

$$\mathbb{E}\left[u\right] = \vartheta \Psi^{1/\epsilon} = \vartheta \left[\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}\right]^{1/\epsilon}. \tag{C.9}$$

C2 Residence and Workplace Choices

Using the distribution of utility for pairs of residence and employment locations, the probability that a worker chooses the bilateral commute from n to i out of all possible bilateral commutes is:

$$\begin{aligned}\lambda_{ni} &= \Pr\left[u_{ni} \geq \max\{u_{k\ell}\}; \forall k, \ell\right], \\ &= \int_0^\infty \prod_{\ell \neq i} G_{n\ell}(u) \left[\prod_{k \neq n} \prod_{\ell \in N} G_{k\ell}(u)\right] g_{ni}(u) du, \\ &= \int_0^\infty \prod_{k \in N} \prod_{\ell \in N} \epsilon \Psi_{ni} u^{-(\epsilon+1)} e^{-\Psi_{k\ell} u^{-\epsilon}} du, \\ &= \int_0^\infty \epsilon \Psi_{ni} u^{-(\epsilon+1)} e^{-\Psi u^{-\epsilon}} du.\end{aligned} \tag{C.10}$$

Note that:

$$\frac{d}{du}\left[\frac{1}{\Psi} e^{-\Psi u^{-\epsilon}}\right] = \epsilon u^{-(\epsilon+1)} e^{-\Psi u^{-\epsilon}}. \tag{C.11}$$

Using this result to evaluate the integral above, the probability that the worker chooses to live in location n and work in location i is:

$$\lambda_{ni} = \frac{L_{ni}}{\bar{L}} = \frac{\Psi_{ni}}{\Psi} = \frac{(B_n w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{C.12}$$

where L_{ni} is the measure of commuters from residence n to workplace i; $\bar{L}$ is the overall measure of workers that choose to live in the city.

Summing across workplaces i in equation (C.12), we obtain the probability that a worker ichooses to live in residence n ($R_n/\bar{L}$):

$$\lambda_n^R = \frac{R_n}{\bar{L}} = \frac{\sum_{i \in N} (B_n w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{C.13}$$

Similarly, summing across residences n in equation (C.12), we obtain the probability that a worker chooses workplace i ($L_i/\bar{L}$):

$$\lambda_i^L = \frac{L_i}{\bar{L}} = \frac{\sum_{n \in N} (B_n w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{C.14}$$

For the measure of workers in location i (L_i), we can evaluate the conditional probability that they commute from location n (conditional on having chosen to work in location i):

$$\begin{aligned}\lambda_{ni|i}^L = \frac{\lambda_{ni}}{\lambda_i^L} &= \Pr\left[u_{ni} \geq \max\{u_{ri}\}; \forall r\right], \\ &= \int_0^\infty \prod_{r \neq n} G_{ri}(u) g_{ni}(u) du, \\ &= \int_0^\infty e^{-\Psi_i^L u^{-\epsilon}} \epsilon \Psi_{ni} u^{-(\epsilon+1)} du.\end{aligned} \tag{C.15}$$

where

$$\Psi_i^L \equiv \sum_{k \in N} (B_k w_i)^{\epsilon} \left(\kappa_{ki} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}. \tag{C.16}$$

Using the result (C.11) to evaluate the integral in equation (C.15), the probability that a worker commutes from residence n to workplace i conditional on having chosen to work in location i is:

$$\lambda_{ni|i}^L = \frac{\lambda_{ni}}{\lambda_i^L} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} (B_k w_i)^{\epsilon} \left(\kappa_{ki} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{C.17}$$

which further simplifies to:

$$\lambda_{ni|i}^L = \frac{B_n^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} B_k^{\epsilon} \left(\kappa_{ki} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{C.18}$$

For the measure of residents of location n (R_n), we can evaluate the conditional probability that they commute to location i (conditional on having chosen to live in location n):

$$\begin{aligned} \lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} &= \Pr\left[u_{ni} \geq \max\{u_{n\ell}\}; \forall \ell\right], \\ &= \int_0^{\infty} \prod_{\ell \neq i} G_{n\ell}(u) g_{ni}(u) du, \\ &= \int_0^{\infty} e^{-\Psi_n^R u^{-\epsilon}} \epsilon \Psi_{ni} u^{-(\epsilon+1)} du, \end{aligned} \tag{C.19}$$

where

$$\Psi_n^R \equiv \sum_{\ell \in N} (B_n w_\ell)^{\epsilon} \left(\kappa_{n\ell} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}. \tag{C.20}$$

Using the result (C.11) to evaluate the integral in equation (C.19), the probability that a worker commutes to location i conditional on having chosen to live in location n is:

$$\lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{\ell \in N} (B_n w_\ell)^{\epsilon} \left(\kappa_{n\ell} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}, \tag{C.21}$$

which further simplifies to:

$$\lambda_{ni|n}^R = \frac{(w_i/\kappa_{ni})^{\epsilon}}{\sum_{\ell \in N} (w_\ell/\kappa_{n\ell})^{\epsilon}}. \tag{C.22}$$

Commuter market clearing requires that the measure of workers employed in each location i (L_i) equals the sum across all locations n of their measures of residents (R_n) times their conditional probabilities of commuting to i ($\lambda_{ni|n}^R$):

$$\begin{aligned} L_i &= \sum_{n \in N} \lambda_{ni|n}^R R_n \\ &= \sum_{n \in N} \frac{(w_i/\kappa_{ni})^{\epsilon}}{\sum_{\ell \in N} (w_\ell/\kappa_{n\ell})^{\epsilon}} R_n, \end{aligned} \tag{C.23}$$

where, since there is a continuous measure of workers residing in each location, there is no uncertainty in the supply of workers to each employment location.

Expected worker income conditional on living in location n equals the wages in all possible workplace locations weighted by the probabilities of commuting to those locations conditional on living in n:

$$\begin{aligned}\bar{v}_n &= \mathbb{E}\left[w|n\right] \\ &= \sum_{i\in N} \lambda^R_{ni|n} w_i, \\ &= \sum_{i\in N} \frac{\left(w_i/\kappa_{ni}\right)^{\epsilon}}{\sum_{\ell\in N}\left(w_\ell/\kappa_{n\ell}\right)^{\epsilon}} w_i,\end{aligned} \tag{C.24}$$

where $\mathbb{E}$ denotes the expectations operator and the expectation is taken over the distribution for idiosyncratic amenities. Intuitively, expected worker income is high in locations that have low commuting costs (low κ_{ni}) to high-wage employment locations.

C3 Equalization of Expected Utility

Another implication of the Fréchet distribution of utility is that the distribution of utility conditional on residing in location n and commuting to location i is the same across all bilateral pairs of locations with positive residents and employment, and is equal to the distribution of utility for the economy as a whole. To establish this result, note that the distribution of utility conditional on residing in location n and commuting to location i is:

$$= \frac{1}{\lambda_{ni}} \int_0^u \prod_{s\neq i} G_{ns}(u) \left[\prod_{k\neq n}\prod_{\ell\in N} G_{k\ell}(u)\right] g_{ni}(u)du, \tag{C.25}$$

$$\begin{aligned}&= \frac{1}{\lambda_{ni}} \int_0^u \left[\prod_{k\in N}\prod_{\ell\in N} e^{-\Psi_{k\ell}u^{-\epsilon}}\right] \epsilon\Psi_{ni}u^{-(\epsilon+1)}du, \\ &= \frac{\Psi}{\Psi_{ni}} \int_0^u e^{-\Psi u^{-\epsilon}} \epsilon\Psi_{ni}u^{-(\epsilon+1)}du, \\ &= e^{-\Psi u^{\epsilon}}.\end{aligned}$$

On the one hand, lower land prices in location n or a higher wage in location i raise the utility of a worker with a given realization of idiosyncratic amenities b, and hence increase the expected utility of residing in n and working in i. On the other hand, lower land prices or a higher wage induce workers with lower realizations of idiosyncratic amenities b to reside in n and work in i, which reduces the expected utility of residing in n and working in i. With a Fréchet distribution of utility, these two effects exactly offset one another. Pairs of residence and employment locations with more attractive characteristics attract more commuters on the extensive margin until expected utility is the same across all pairs of residence and employment locations within the economy.

An implication of this result is that expected utility conditional on choosing a residence n and workplace i is the same across all residence-workplace pairs and equal to expected utility in the economy as a whole in equation (C.9):

$$\bar{U} = \vartheta \Psi^{1/\epsilon} = \vartheta \left[\sum_{k \in N} \sum_{\ell \in N} (B_k w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha} \right)^{-\epsilon} \right]^{1/\epsilon}. \tag{C.26}$$

References

Ahlfeldt, G. M., S. J. Redding, D. M. Sturm and N. Wolf (2015) "The Economics of Density: Evidence from the Berlin Wall," *Econometrica*, 83, 2127-89.

Allen, T., C. Arkolakis and X. Li (2021) "On the Equilibrium Properties of Network Models with Heterogeneous Agents," Dartmouth College, mimeograph.

Combes, P., G. Duranton and L. Gobillon (2018) "The Costs of Agglomeration: Land Prices in French Cities," *Review of Economic Studies*, 86(4), 1556-1589.

Davis, M. A. and F. Ortalo-Magne (2011) "Housing Expenditures, Wages, Rents," *Review of Economic Dynamics*, 14(2), 248-61.

Epple, D., B. Gordon and H. Sieg (2010) "A New Approach to Estimating the Production Function for Housing," *American Economic Review*, 100(3), 905-924.

Fortheringham, S. and M. O'Kelly (1989) *Spatial Interaction Models: Formulations and Applications*, Dordrecht: Kluwer.

Grogger, J. and G. Hanson (2011) "Income Maximization and the Selection and Sorting of International Migrants," *Journal of Development Economics*, 95(1), 42-57.

Kennan, J. and J. R. Walker (2011) "The Effect of Expected Income on Individual Migration Decisions," *Econometrica*, 79(1), 211-251.

McDonald, J. and D. McMillan (2010) *Urban Economics and Real Estate: Theory and Policy*, Hoboken, NJ: John Wiley & Sons.

McFadden, D. (1974) "The Measurement of Urban Travel Demand," *Journal of Public Economics*, 3(4), 303-328.