Part

Online Appendix: Neoclassical Growth in an Interdependent World*

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A Introduction

This Online Appendix reports the theoretical derivations for all results in the paper and additional empirical evidence. Section B characterizes the allocation of wealth across countries within each time period. Section C characterizes optimal consumption-saving decisions. Section D confirms that the balance of payments accounting identities hold in our framework. Section E summarizes the system of equations for general equilibrium, and establishes some key relationships between capital and labor incomes and the trade and capital share matrices.

Section F analyzes the steady-state equilibrium of the full nonlinear model. Section G characterizes its transition dynamics. Section H shows how the full nonlinear model can be inverted to recover the unobserved location fundamentals that rationalize the observed data as an equilibrium outcome.

Section I linearizes the model and derives a closed-form solution for the evolution of the state variables, in terms of an impact matrix that captures the initial impact of shocks, and a transition matrix that governs the updating of the state variables. Section J analyzes the relationship between goods and capital market integration and the speed of convergence to steady state.

Section K contains additional empirical results that supplement those reported in the paper. Section L provides further information on the data sources and definitions.

B Wealth Allocation

In this section of the Online Appendix, we characterize the allocation of wealth within each time period. In the next section of this Online Appendix, we characterize the intertemporal consumption-saving decision.

B.1 Gravity Equation for Wealth Allocations

We have the following monotonic relationship between the return (v) and efficiency units (φ) for a unit of wealth invested from investor n in producer i:

$$\frac{\kappa_{nit} v}{r_{it}} = \varphi.$$

Using this monotonic relationship in the distribution of efficiency units (6), we obtain the following distribution for bilateral returns:

$$F_{nit}(v) = e^{-\Psi_{nit} v^{-\epsilon}}, \qquad \Psi_{nit} \equiv \left(\eta_i r_{it} / \kappa_{nit}\right)^{\epsilon}.$$

Each unit of wealth is allocated to the country with the highest return. Therefore the distribution of returns across all producers i for investor n is

$$F_{nt}(v) = \prod_{i=1}^{N} e^{-\Psi_{nit} v^{-\epsilon}},$$

which can be written as

$$F_{nt}(v) = e^{-\Psi_{nt} v^{-\epsilon}}, \qquad \Psi_{nt} \equiv \sum_{i=1}^{N} \Psi_{nit}.$$

Using these distributions of returns, the probability that a unit of wealth from investor n is allocated to producer i is

$$\begin{aligned}
b_{nit} &= \text{Prob}\left[v_{nit} \geq \max\left\{v_{nkt}\right\} \forall k\right], \\
&= \int_0^\infty \prod_{k \neq i} F_{nkt}(v) f_{nit}(v) dv, \\
&= \int_0^\infty \prod_{k \neq i} e^{-\Psi_{nkt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-(\epsilon+1)} e^{-\Psi_{nit} v^{-\epsilon}} dv, \\
&= \int_0^\infty \prod_{k} e^{-\Psi_{nkt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-(\epsilon+1)} dv, \\
&= \int_0^\infty e^{-\Psi_{nt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-(\epsilon+1)} dv.
\end{aligned}$$

Note that

$$\frac{d}{dv}\left[\frac{1}{\Psi_{nt}} e^{-\Psi_{nt} v^{-\epsilon}}\right] = \epsilon v^{-(\epsilon+1)} e^{-\Psi_{nt} v^{-\epsilon}}.$$

Using this result to evaluate the integral above, the probability that a unit of wealth from investor n is allocated to producer i is

$$b_{nit} = \frac{\Psi_{nit}}{\Psi_{nt}} = \frac{\left(\eta_{it} r_{it} / \kappa_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}}. \tag{B.1}$$

B.2 Derivation of Capital Income Rate (v_{nt})

Using the above Fréchet distributions of returns to a unit of wealth, the capital income rate from investor n across all producers i (v_{nt}) is

$$v_{nt} = \int_0^\infty \epsilon \Psi_{nt} v^{-\epsilon} e^{-\Psi_{nt} v^{-\epsilon}} dv.$$

Now define the following change of variables:

$$y = \Psi_{nt} v^{-\epsilon}, \qquad\qquad dy = -\epsilon \Psi_{nt} v^{-(\epsilon+1)} dv.$$

Using this change of variables, the capital income rate from investor n can be written as

$$v_{nt} = \int_0^{\infty} \Psi_{nt}^{1/\epsilon} y^{-1/\epsilon} e^{-y} dy,$$

which can be in turn written as

$$v_{nt} = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right) \left[\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}\right]^{\frac{1}{\epsilon}}. \tag{B.2}$$

We now show that the distribution of returns for investor n from each producer i conditional on allocating wealth to that producer country is the same across all producers i and equal to the overall distribution of returns for investor n. The distribution of returns for investor n from each producer i conditional on allocating wealth to that producer country is

$$\begin{aligned}
=&\frac{1}{b_{nit}} \int_0^{v} \prod_{k \neq i} F_{nkt}(v)\, v f_{nit}(v)\, dv, \\
=&\frac{1}{b_{nit}} \int_0^{v} \prod_{k \neq i} e^{-\Psi_{nkt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-\epsilon} e^{-\Psi_{nit} v^{-\epsilon}} dv, \\
=&\frac{1}{b_{nit}} \int_0^{v} \prod_{i} e^{-\Psi_{nit} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-\epsilon} dv, \\
=&\frac{1}{b_{nit}} \int_0^{v} e^{-\Psi_{nt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-\epsilon} dv, \\
=&\frac{\Psi_{nt}}{\Psi_{nit}} \int_0^{v} e^{-\Psi_{nt} v^{-\epsilon}} \epsilon \Psi_{nit} v^{-\epsilon} dv, \\
=&e^{-\Psi_{nt} v^{-\epsilon}},
\end{aligned}$$

which is the same as the overall distribution of returns for investor n above. Therefore the capital income rate for investor n for each producer i conditional on allocating wealth to that producer country (v_{nit}) is equal to the overall capital income rate from investor n across all producer countries above (v_{nt}).

B.3 Derivation of Average Efficiency Units

Recall that the distribution of returns for investor n from producer i conditional on allocating wealth to that producer country is

$$F_{nt}(v) = e^{-\Psi_{nt} v^{-\epsilon}}.$$

Recall that we have the following relationship between returns and efficiency units:

$$v = \frac{r_{it}\varphi}{\kappa_{nit}}.$$

Therefore, the distribution of efficiency units for investments from investor n conditional on investing in producer i is

$$\begin{aligned} F_{nt}(\varphi) &= e^{-\Psi_{nt}(r_{it}/\kappa_{nit})^{-\epsilon}\varphi^{-\epsilon}}, \\ &= e^{-(\Psi_{nt}/\Psi_{nit})\alpha^{-\epsilon}}. \end{aligned}$$

Expected efficiency units for investments from investor n conditional on investing in producer i are:

$$\overline{\alpha}_{nit} = \int_0^\infty \epsilon\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon} e^{-\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon}} d\varphi.$$

Now define the following change of variables:

$$y = \left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon},$$

$$dy = -\epsilon\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-(\epsilon+1)} d\varphi,$$

and hence:

$$\varphi = \left(\frac{y}{\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)}\right)^{-\frac{1}{\epsilon}},$$

$$d\varphi = -\frac{dy}{\epsilon\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-(\epsilon+1)}}.$$

Using these relationships, we have:

$$\overline{\varphi}_{nit} = \int_0^\infty \epsilon\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon} e^{-\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon}} \frac{dy}{\epsilon\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-(\epsilon+1)}},$$

$$\overline{\varphi}_{nit} = \int_0^\infty \varphi^{-\epsilon} e^{-\left(\eta_{it}^{\epsilon}\Psi_{nt}/\Psi_{nit}\right)\varphi^{-\epsilon}} \frac{dy}{\varphi^{-(\epsilon+1)}},$$

$$\overline{\varphi}_{nit} = \int_0^\infty \varphi^{-\epsilon} e^{-y} \frac{dy}{\varphi^{-(\epsilon+1)}},$$

$$\overline{\varphi}_{nit} = \int_0^\infty \varphi e^{-y} dy,$$

$$\overline{\varphi}_{nit} = \int_0^\infty \left(\frac{y}{(\eta_{it}^\epsilon \Psi_{nt}/\Psi_{nit})} \right)^{-\frac{1}{\epsilon}} e^{-y} dy,$$

$$\overline{\varphi}_{nit} = \int_0^\infty \left((\eta_{it}^\epsilon \Psi_{nt}/\Psi_{nit}) \right)^{\frac{1}{\epsilon}} y^{-1/\epsilon} e^{-y} dy,$$

$$\overline{\varphi}_{nit} = \int_0^\infty \left(\frac{\eta_{it}^\epsilon}{b_{nit}} \right)^{\frac{1}{\epsilon}} y^{-1/\epsilon} e^{-y} dy,$$

$$\overline{\varphi}_{nit} = \Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left(\frac{\eta_{it}^\epsilon}{b_{nit}} \right)^{\frac{1}{\epsilon}}, \tag{B.3}$$

where $\Gamma(\cdot)$ is the Gamma function. As a check on this derivation of average efficiency units, note that we have two equivalent ways of writing the expected capital payment:

$$v_{nit} a_{nit} = \frac{r_{it} \overline{\varphi}_{nit} a_{nit}}{\kappa_{nit}}.$$

Using our result for $v_{nit} = v_{nt}$ from equation (B.2) above, this becomes:

$$\Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left[\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right]^{\frac{1}{\epsilon}} a_{nit} = \frac{r_{it} \overline{\varphi}_{nit} a_{nit}}{\kappa_{nit}}.$$

Using our result for $\overline{\varphi}_{nit}$ from equation (B.3) above, this becomes:

$$\Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left[\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right]^{\frac{1}{\epsilon}} a_{nit} = \frac{r_{it} a_{nit}}{\kappa_{nit}} \Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left(\frac{\eta_{it}^\epsilon}{b_{nit}} \right)^{\frac{1}{\epsilon}}.$$

Using our result for b_{nit} from equation (B.1) above, this becomes:

$$\Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left[\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right]^{\frac{1}{\epsilon}} a_{nit} = \frac{r_{it} a_{nit}}{\kappa_{nit}} \Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left(\frac{\eta_{it}^\epsilon \left(\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right)}{\left(\eta_{it} r_{it} / \kappa_{nit} \right)^\epsilon} \right)^{\frac{1}{\epsilon}},$$

which simplifies to:

$$\Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left[\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right]^{\frac{1}{\epsilon}} a_{nit} = \Gamma\left(\frac{\epsilon - 1}{\epsilon} \right) \left[\sum_{h=1}^N \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^\epsilon \right]^{\frac{1}{\epsilon}} a_{nit},$$

and confirms our derivation of average efficiency units above.

C Optimal Consumption-Savings Decisions

In Subsection C.1, we characterize optimal consumption-savings decisions in the general model with a representative agent and CRRA preferences. In Subsection C.2, we characterize optimal consumption-savings decisions for a special case of the model with (i) a separation between workers (who live hand to mouth) and capitalists (who can save) and (ii) log utility.

C.1 Representative Agent and CRRA Preferences

We begin with the general model with a representative agent and CRRA preferences. This representative agent chooses consumption and saving to maximize intertemporal utility subject to their intertemporal budget constraint.

$$\max_{\{c_{nt+s},a_{nt+s+1}\}} \sum_{s=0}^{\infty} \beta^{t+s} \frac{(c_{nt+s})^{1-1/\psi}}{1-1/\psi}, \tag{C.1}$$

$$\text{subject to} \qquad p_{nt}c_{nt} + p_{nt}\left(a_{nt+1} - (1-\delta)\, a_{nt}\right) = v_{nt}a_{nt} + w_{nt}\ell_{nt}.$$

Following Angeletos (2007), we can rewrite this consumption-saving problem recursively as the following value function:

$$v\left(a_{nt};t\right) = \max_{c_{nt},a_{nt+1}} \frac{c_{nt}^{1-1/\psi}}{1-1/\psi} + \beta v\left(a_{nt+1};t+1\right), \tag{C.2}$$

$$\text{s.t. } c_{nt} + a_{nt+1} = \mathcal{R}_{nt}a_{nt} + \frac{w_{nt}\ell_{nt}}{p_{nt}},$$

where we have defined $\mathcal{R}_{nt} \equiv \mathcal{R}_{nt}^{nom} / \left(p_{nt}/p_{nt-1}\right)$ and $\mathcal{R}_{nt}^{nom} \equiv \left[p_{nt}\left(1-\delta\right) + v_{nt}\right]/p_{nt-1}$. We begin by defining $h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s}\ell_{nt+s}/p_{nt+s}}{\prod_{u=1}^{s}\mathcal{R}_{nt+u}}$ as the present-discounted value of future wage income. We now establish the following results:

$$c_{nt} = \varsigma_{nt}\left(\mathcal{R}_{nt}a_{nt} + w_{nt}\ell_{nt} + h_{nt}\right), \tag{C.3}$$

where ς_{nt} is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi}\mathcal{R}_{nt+1}^{\psi-1}\varsigma_{nt+1}^{-1}. \tag{C.4}$$

Proof Conjecture: $v\left(k_{nt};t\right) = \frac{\left(d_{nt}(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt})\right)^{1-1/\psi}}{1-1/\psi}$, $c_{nt} = \varsigma_{nt}\left(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt}\right)$. We setup the Lagrangian as

$$\mathcal{L}_{nt} = \frac{c_{nt}^{1-1/\psi}}{1-1/\psi} + \beta v\left(a_{nt+1};t+1\right) + \xi_{nt}\left[\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}-c_{nt}-a_{nt+1}\right].$$

Taking first-order conditions:

$$\{c_{nt}\} \qquad c_{nt}^{-1/\psi} = \xi_{nt},$$

$$\begin{aligned}\{a_{nt+1}\} \qquad \xi_{nt} &= \beta v'\left(a_{nt+1};t+1\right), \\ &= \beta d_{nt+1}^{1-1/\psi}\mathcal{R}_{nt+1}\left(\mathcal{R}_{nt+1}a_{nt+1}+w_{nt+1}\ell_{nt+1}+h_{nt+1}\right)^{-1/\psi}.\end{aligned}$$

Hence

$$c_{nt}^{-1/\psi} = \beta d_{nt+1}^{1-1/\psi}\mathcal{R}_{nt+1}\left(\mathcal{R}_{nt+1}a_{nt+1}+w_{nt+1}\ell_{nt+1}+h_{nt+1}\right)^{-1/\psi}. \tag{C.5}$$

The envelope condition implies

$$d_{nt}^{1-1/\psi}\left(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt}\right)^{-1/\psi} = c_{nt}^{-1/\psi}.$$

Using our conjecture that $c_{nt} = \varsigma_{nt}(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt})$, we have:

$$d_{nt}^{1-\psi} = \varsigma_{nt}.$$

Substituting $a_{nt+1} = (1-\varsigma_{nt})\left(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt}\right)-h_{nt}$ and $d_{nt}^{1-\psi} = \varsigma_{nt}$ into equation (C.5), we obtain:

$$\begin{aligned}c_{nt} &= \beta^{-\psi}d_{nt+1}^{1-\psi}\mathcal{R}_{nt+1}^{-\psi}\left(\mathcal{R}_{nt+1}a_{nt+1}+w_{nt+1}\ell_{nt+1}+h_{nt+1}\right), \\ &= \beta^{-\psi}\varsigma_{nt+1}\mathcal{R}_{nt+1}^{-\psi}\left(\mathcal{R}_{nt+1}\left(1-\varsigma_{nt}\right)\left(\mathcal{R}_{nt}a_{nt}+w_{nt}\ell_{nt}+h_{nt}\right)\underbrace{-\mathcal{R}_{nt+1}h_{nt}+w_{nt+1}\ell_{nt+1}+h_{nt+1}}_{\equiv 0}\right), \\ &= \beta^{-\psi}\varsigma_{nt+1}\mathcal{R}_{nt+1}^{1-\psi}\left(1-\varsigma_{nt}\right)c_{nt}\varsigma_{nt}^{-1}, \\ &= \beta^{-\psi}\varsigma_{nt+1}\mathcal{R}_{nt+1}^{1-\psi}\left(\varsigma_{nt}^{-1}-1\right)c_{nt}.\end{aligned}$$

Hence:

$$\begin{aligned}1 &= \beta^{-\psi}\varsigma_{nt+1}\mathcal{R}_{nt+1}^{1-\psi}\left(\varsigma_{nt}^{-1}-1\right), \\ \varsigma_{nt}^{-1} &= 1+\beta^{\psi}\mathcal{R}_{nt+1}^{\psi-1}\varsigma_{nt+1}^{-1}.\end{aligned} \tag{C.6}$$

Therefore, we have established that consumption and saving are linear functions of current-period wealth: ς_{nt} and $1-\varsigma_{nt}$, respectively.

C.2 Capitalists-Workers and Logarithmic Preferences

We now characterize optimal consumption-saving decisions for a special case of the model with (i) a separation between workers (who live hand to mouth) and capitalists (who can save) and (ii) log utility. Capitalists choose consumption and investment to maximize intertemporal utility subject to their intertemporal budget constraint:

$$\max_{\{c_{nt},k_{nt+1}\}} \sum_{t=0}^{\infty} \beta^t \ln c_{nt}, \tag{C.7}$$

$$\text{subject to} \qquad p_{nt}c_{nt} + p_{nt}\left(a_{nt+1} - (1-\delta)\, a_{nt}\right) = v_{nt}a_{nt}.$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c_{nt} - \mu_{nt}\left[p_{nt}c_{nt} + p_{nt}\left(a_{nt+1} - (1-\delta)\, a_{nt}\right) - v_{nt}a_{nt}\right].$$

The first-order conditions are:

$$\{c_{nt}\} \qquad \frac{\beta^t}{c_{nt}} - p_{nt}\mu_{nt} = 0,$$

$$\{a_{nt+1}\} \qquad \left[v_{nt+1} + p_{nt+1}\left(1-\delta\right)\right]\mu_{nt+1} - p_{nt}\mu_{nt} = 0.$$

Together these first-order conditions imply:

$$\frac{c_{nt+1}}{c_{nt}} = \beta \frac{p_{nt}\mu_{nt}}{p_{nt+1}\mu_{nt+1}} = \beta\left[v_{nt+1}/p_{nt+1} + (1-\delta)\right], \tag{C.8}$$

where the transversality condition implies:

$$\lim_{t\to\infty} \beta^t \frac{a_{nt+1}}{c_{nt}} = 0. \tag{C.9}$$

With logarithmic utility, capitalists optimal consumption-saving decision involves a constant saving rate. We conjecture the following policy functions:

$$p_{nt}c_{nt} = (1-\beta)\left(v_{nt} + p_{nt}\left(1-\delta\right)\right)a_{nt}, \tag{C.10}$$

$$a_{nt+1} = \beta\left(v_{nt}/p_{nt} + (1-\delta)\right)a_{nt}. \tag{C.11}$$

Substituting the consumption policy function (C.10) into the Euler equation (C.8), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice

$$\begin{aligned}\frac{c_{nt+1}}{c_{nt}} &= \frac{\left(v_{nt+1}/p_{nt+1} + (1-\delta)\right) a_{nt+1}}{\left(v_{nt}/p_{nt} + (1-\delta)\right) a_{nt}}, \\ &= \beta \left(v_{nt+1}/p_{nt+1} + (1-\delta)\right).\end{aligned}$$

D Balance of Payments

We now use our framework to illustrate the conventional balance of payments accounting identities. The financial account (FA_{it}) is defined as the increase in foreign assets in country i minus the increase in country i's assets abroad:

$$FA_{it} = \underbrace{\left(\sum_{n=1}^{N} p_{nt} a_{nit+1} - \sum_{n=1}^{N} p_{nt-1} a_{nit}\right)}_{\text{Increase in foreign assets in country } i} - \underbrace{\left(p_{it} a_{it+1} - p_{it-1} a_{it}\right)}_{\text{Increase in country } i\text{'s assets abroad}}. \tag{D.1}$$

Trade balance (TB_{it}) corresponds to the difference between the value of goods produced in a country and the value of goods used in that country:

$$TB_{it} = \underbrace{w_{it}\ell_i + \sum_{n=1}^{N} v_{nt} a_{nit}}_{\text{Value of goods produced}} - \underbrace{\left(p_{it} c_{it} + \sum_{n=1}^{N} p_{nt} a_{nit+1} - (1-\delta) \sum_{n=1}^{N} p_{nt} a_{nit}\right)}_{\text{Value of goods used in the country}}. \tag{D.2}$$

Net investment income (NII_{it}) is the difference between income receipts from assets owned by country i minus income payments on foreign-owned assets used in country i:

$$\text{NII}_{it} = \underbrace{\left(\mathcal{R}_{it}^{Nom} - 1\right) p_{it-1} a_{it}}_{\text{Income receipts from assets owned}} - \underbrace{\sum_{n=1}^{N} \left(\mathcal{R}_{nt}^{Nom} - 1\right) p_{nt-1} a_{nit}}_{\text{Income payments to foreign-owned assets}}. \tag{D.3}$$

Combining these definitions in equations (D.1)-(D.3), we confirm that the conventional balance of payments accounting identity holds:

$$CA_{it} = TB_{it} + NII_{it} = -FA_{it}. \tag{D.4}$$

In the special case of our model with capital autarky and open goods markets, the financial account and the current account of the balance of payments in equation (D.4) are necessarily

equal to zero ($-FA_{it} = CA_{it} = 0$), but there can be a trade imbalance in equation (D.4) that is offset by net investment income from domestic assets ($TB_{it} + NII_{it} = 0$).

In the special case of our model with trade autarky and open capital markets, imports, exports and the trade balance in equation (D.4) are all equal zero ($TB_{it} = 0$), but there can be imbalances in the current and financial accounts in equation (D.4), which are offset by net investment income from wealth allocated at home and abroad ($CA_{it} = NII_{it} = -FA_{it} \neq 0$).

In our baseline model with open goods and capital markets, trade can be imbalanced ($TB_{it} \neq 0$) in equation (D.4), and there can be offsetting imbalances in the current and financial accounts ($CA_{it} = -FA_{it} \neq 0$) in equation (D.4), which can be attributed to either trade imbalance ($TB_{it} \neq 0$) or net investment income ($NII_{it} \neq 0$).

E Conditions for General Equilibrium

In this section of the Online Appendix, we summarize the system of equations for general equilibrium, and establish some relationships between capital and labor incomes and the trade and capital share matrices.

E.1 General Equilibrium System

Given state variables $\{k_{nt}\}_{n=1}^{N}$, the equilibrium objects $\{w_{nt}, r_{nt}, s_{nt}, v_{nt}, b_{nt}\}_{n=1}^{N}$ can be characterized by the following system of equations:

$$s_{nit} = \frac{\left(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it}\right)^{-\theta}}{\sum_{h=1}^{N} \left(\tau_{nht} w_{ht}^{\mu_h} r_{ht}^{1-\mu_h} / z_{ht}\right)^{-\theta}}, \tag{E.1}$$

$$w_{it}\ell_i = \mu_i \sum_{n=1}^{N} s_{nit} \left(v_{nt} a_{nt} + w_{nt}\ell_n\right), \tag{E.2}$$

$$b_{nit} = \frac{\left(\eta_{it} r_{it} / \kappa_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}}, \tag{E.3}$$

$$v_{nt} = \gamma \left[\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}\right]^{1/\epsilon}, \tag{E.4}$$

$$\sum_{n=1}^{N} v_{nt} b_{nit} a_{nt} = \frac{1-\mu_i}{\mu_i} w_{it}\ell_i, \tag{E.5}$$

along with the normalization:

$$\sum_{i=1}^{N} \frac{1}{\mu_i} w_{it} \ell_i = 1, \tag{E.6}$$

which also implies:

$$\sum_{n=1}^{N} \frac{v_{nt} a_{nt}}{1-\mu_i} = 1. \tag{E.7}$$

The evolution of state variables follows:

$$a_{nt+1} = (1-\varsigma_{nt}) \left(\mathcal{R}_{nt} a_{nt} + \frac{w_{nt} \ell_n}{p_{nt}} + h_{nt} \right) - h_{nt}, \tag{E.8}$$

$$h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s} \ell_{nt+s} / p_{nt+s}}{\prod_{u=1}^{s} \mathcal{R}_{nt+u}}, \tag{E.9}$$

$$p_{nt} \equiv \left[\sum_{i} \left(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it} \right)^{-\theta} \right]^{-1/\theta}, \tag{E.10}$$

where

$$\mathcal{R}_{nt} = 1 - \delta + v_{nt}/p_{nt}, \tag{E.11}$$

and ς_{nt} is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi} \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}. \tag{E.12}$$

E.2 Incomes and Trade and Capital Shares

We now establish some key relationships between labor and capital incomes and the trade and capital share matrices. Throughout the following, we suppress the time subscript to simplify notation. Let $q_n \equiv w_n \ell_n$ denote labor income in country n. Let $\zeta_n \equiv v_n a_n$ denote capital income in country n. Let $S_{nh} \equiv [s_{nh}^*]$ denote the share of country n's expenditure on country h. Let $T_{in} \equiv \frac{S_{ni}(v_n a_n + w_n \ell_n)}{\sum_{h=1}^{N} S_{hi}(v_h a_h + w_h \ell_h)}$ denote the share of country i's income from market n. Let $B_{ni} \equiv [b_{ni}^*]$ denote the share of country n's wealth allocated to country i. Let $X_{in} \equiv \frac{v_n b_{ni} a_n}{\sum_{h=1}^{N} v_h b_{hi} a_h}$ denote the share of capital payments in country i made to country n. Using these definitions, we have the

following relationships:

$$\begin{cases} w_i \ell_i & = \mu_i \sum_{n=1}^{N} S_{ni} \left(v_n a_n + w_n \ell_n \right), \\ \frac{1}{\mu_i} w_i \ell_i T_{in} & = S_{ni} \left(v_n a_n + w_n \ell_n \right), \\ \sum_{n=1}^{N} v_n b_{ni} a_n & = \frac{1-\mu_i}{\mu_i} w_i \ell_i, \\ X_{in} & \equiv \frac{v_n b_{ni} a_n}{\sum_{h=1}^{N} v_h b_{hi} a_h} = \frac{v_n b_{ni} a_n}{\frac{1-\mu_i}{\mu_i} w_i \ell_i}. \end{cases}$$

Assuming the same labor share across countries for the remainder of this subsection ($\mu_i = \mu$), we have:[1]

$$\begin{cases} \boldsymbol{q}' & = \mu \left(\boldsymbol{\zeta} + \boldsymbol{q} \right)' \boldsymbol{S}, \\ \boldsymbol{q}' \boldsymbol{T} & = \mu \left(\boldsymbol{q} + \boldsymbol{\zeta} \right)', \\ \boldsymbol{\zeta}' \boldsymbol{B} & = \frac{1-\mu}{\mu} \boldsymbol{q}', \\ \boldsymbol{\zeta}' & = \frac{1-\mu}{\mu} \boldsymbol{q}' \boldsymbol{X}, \end{cases} \tag{E.13}$$

Hence, we can recover $\boldsymbol{q}$, $\boldsymbol{\zeta}$, $\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$ if we know either one of $\boldsymbol{S}$, $\boldsymbol{T}$ and either one of $\boldsymbol{B}$, $\boldsymbol{X}$. For instance, if we know $\boldsymbol{S}$ and $\boldsymbol{B}$, we can recover $\boldsymbol{\zeta}$ and $\boldsymbol{q}$ as eigenvectors:

$$\boldsymbol{\zeta}' = \boldsymbol{\zeta}' \left(1 - \mu \right) \boldsymbol{S} \left(\boldsymbol{I} - \mu \boldsymbol{S} \right)^{-1} \boldsymbol{B}^{-1},$$

$$\boldsymbol{q}' = \boldsymbol{q}' \left(\left(1 - \mu \right) \boldsymbol{X} + \mu \boldsymbol{I} \right) \boldsymbol{S}.$$

We also have the following results:

$$\left(\boldsymbol{q} + \boldsymbol{\zeta} \right)' \boldsymbol{S} \boldsymbol{T} = \left(\boldsymbol{q} + \boldsymbol{\zeta} \right)',$$

$$\boldsymbol{q}' = \boldsymbol{q}' \boldsymbol{T} \boldsymbol{S},$$

$$\boldsymbol{\zeta}' = \boldsymbol{\zeta}' \boldsymbol{B} \boldsymbol{X},$$

$$\boldsymbol{q}' = \boldsymbol{q}' \boldsymbol{X} \boldsymbol{B},$$

such that $\boldsymbol{q'}$ is an eigenvector of $\boldsymbol{TS}$; $\boldsymbol{\zeta'}$ is an eigenvector of $\boldsymbol{BX}$; and $\boldsymbol{q'}$ is an eigenvector of $\boldsymbol{XB}$.

[1] If the labor share differs across countries, analogous results hold replacing μ and $(1-\mu)/\mu$ with the diagonal matrices $D(\boldsymbol{\mu})$and $D(\boldsymbol{\mu})^{-1}(I - D(\boldsymbol{\mu}))$, respectively, where $\boldsymbol{\mu}$ is the vector of countries' labor shares.

F Steady State

In this section of the Online Appendix, we characterize the steady-state equilibrium of the model. We use an asterisk to denote the steady-state equilibrium values of variables.

F.1 Steady-State Equilibrium Conditions

The steady-state equilibrium is characterized by the following system of equations:

$$s_{ni}^{*}=\frac{\left(\tau_{ni}\left(w_{i}^{*}\right)^{\mu_{i}}\left(r_{i}^{*}\right)^{1-\mu_{i}}/z_{i}\right)^{-\theta}}{\sum_{h=1}^{N}\left(\tau_{nh}\left(w_{h}^{*}\right)^{\mu_{h}}\left(r_{h}^{*}\right)^{1-\mu_{h}}/z_{h}\right)^{-\theta}}, \tag{F.1}$$

$$w_{i}^{*}\ell_{i}=\mu_{i}\sum_{n=1}^{N}s_{ni}^{*}\left(v_{n}^{*}k_{n}^{*}+w_{n}^{*}\ell_{n}\right), \tag{F.2}$$

$$b_{ni}^{*}=\frac{\left(\eta_{i}r_{i}^{*}/\kappa_{ni}\right)^{\epsilon}}{\sum_{h=1}^{N}\left(\eta_{h}r_{h}^{*}/\kappa_{nh}\right)^{\epsilon}}, \tag{F.3}$$

$$v_{n}^{*}=\gamma\left[\sum_{h=1}^{N}\left(\eta_{h}r_{h}^{*}/\kappa_{nh}\right)^{\epsilon}\right]^{1/\epsilon}, \tag{F.4}$$

$$\sum_{n=1}^{N}v_{n}^{*}b_{ni}^{*}a_{n}^{*}=\frac{1-\mu_{i}}{\mu_{i}}w_{i}^{*}\ell_{i}, \tag{F.5}$$

$$\sum_{i=1}^{N}\frac{1}{\mu_{i}}w_{i}^{*}\ell_{i}=1, \tag{F.6}$$

$$a_{n}^{*}=\left(1-\varsigma_{n}^{*}\right)\left(R_{n}^{*}a_{n}^{*}+\frac{w_{n}^{*}\ell_{n}}{p_{n}^{*}}+h_{n}^{*}\right)-h_{n}^{*}, \tag{F.7}$$

$$h_{n}^{*}\equiv\frac{w_{n}^{*}\ell_{n}}{p_{n}^{*}}\sum_{s=1}^{\infty}\left(\mathcal{R}_{n}^{*}\right)^{-s}=\frac{w_{n}^{*}\ell_{n}}{p_{n}^{*}}\frac{1}{\mathcal{R}_{n}^{*}-1} \tag{F.8}$$

$$p_{n}^{*}\equiv\left[\sum_{i=1}^{N}\left(\tau_{ni}\left(w_{i}^{*}\right)^{\mu_{i}}\left(r_{i}^{*}\right)^{1-\mu_{i}}/z_{i}\right)^{-\theta}\right]^{-1/\theta} \tag{F.9}$$

$$\varsigma_{n}^{*}=1-\beta^{\psi}\left(\mathcal{R}_{n}^{*}\right)^{\psi-1} \tag{F.10}$$

$$\left(1-\varsigma_{n}^{*}\right)\left(\mathcal{R}_{n}^{*}a_{n}^{*}+\frac{w_{n}^{*}\ell_{n}}{p_{n}^{*}}\right)-\varsigma_{n}^{*}h_{n}^{*}=\delta a_{n}^{*} \tag{F.11}$$

F.2 Steady-State Equilibrium Values

Let $\xi_n^* \equiv \frac{a_n^*}{a_n^*+h_n^*}$ denote steady-state share of tangible wealth (as opposed to human capital wealth). We know that:

$$\xi_n^* = \frac{a_n^*}{a_n^* + h_n^*} = \frac{\mathcal{R}_n^* a_n^*}{\mathcal{R}_n^* a_n^* + \mathcal{R}_n^* h_n^*} = \frac{\mathcal{R}_n^* a_n^*}{\mathcal{R}_n^* a_n^* + \frac{w_n^* \ell_n}{p_n^*} + h_n^*}.$$

Hence the steady-state equation for capital (F.7) implies:

$$\underbrace{\xi_n^* \left(a_n^* + h_n^*\right)}_{=a_n^*} = \left(1 - \varsigma_n^*\right) \underbrace{\xi_n^* \left(\mathcal{R}_n^* a_n^* + \frac{w_n^* \ell_n}{p_n^*} + h_n^*\right)}_{=R_n^* a_n^*}.$$

$$\implies 1 = \left(1 - \varsigma_n^*\right) \mathcal{R}_n^*.$$

Substituting this result into the steady-state equation for the saving rate, we get:

$$\mathcal{R}_n^* \left(1 - \varsigma_n^*\right) = \beta^\psi \left(\mathcal{R}_n^*\right)^\psi,$$

$$\implies \beta \mathcal{R}_n^* = 1,$$

$$\varsigma_n^* = 1 - \beta.$$

Hence, the steady-state real return on wealth is equalized across countries and is equal to the inverse of the discount rate, and

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta.$$

Collecting equations, the steady-state equilibrium $\{w_i^*, r_i^*, v_i^*, k_i\}$ is characterized by:

$$w_i^* \ell_i = \mu_i \sum_{n=1}^N \frac{\left(\tau_{ni} \left(w_i^*\right)^{\mu_i} \left(r_i^*\right)^{1-\mu_i} / z_i\right)^\theta}{\sum_{h=1}^N \left(\tau_{nh} \left(w_h^*\right)^{\mu_i} \left(r_h^*\right)^{1-\mu_i} / z_h\right)^\theta} \left(v_n^* a_n^* + w_n^* \ell_n\right),$$

$$v_n^* = \gamma \left[\sum_{h=1}^N \left(\eta_h r_h^* / \kappa_{nh}\right)^\epsilon\right]^{1/\epsilon},$$

$$\sum_{n=1}^N v_n^* b_{ni}^* a_n^* = \frac{1-\mu_i}{\mu_i} w_i^* \ell_i,$$

$$v_n^* = \left(\beta^{-1} - 1 + \delta\right) \left[\sum_{i=1}^N \left(\tau_{ni} \left(w_i^*\right)^{\mu_i} \left(r_i^*\right)^{1-\mu_i} / z_i\right)^{-\theta}\right]^{-1/\theta},$$

$$\sum_{i=1}^{N} \frac{1}{\mu_i} w_i^* \ell_i = 1.$$

which provides a system of $4N + 1$ equations and $4N$ unknowns, with 1 equation being the normalization.

F.3 Steady-State Incomes and Trade and Capital Shares

Let $\chi_n^* \equiv \frac{v_n^* a_n^*}{w_n^* \ell_n + v_n^* a_n^*}$ denote the steady-state capital income share of country n. Recall that $\xi_n^* \equiv \frac{a_n^*}{a_n^* + h_n^*}$ is the share of tangible wealth. In steady state, we have:

$$\frac{a_n^*}{h_n^*} = \frac{v_n^* a_n^* / v_n^*}{\frac{\beta w_n^* \ell_n}{(1-\beta) p_n^*}} = \frac{(1-\beta)\, v_n^* a_n^*}{\beta w_n^* \ell_n v_n^* / p_n^*} = \frac{(1-\beta)\, v_n^* a_n^*}{(1-\beta+\delta\beta)\, w_n^* \ell_n}$$

Hence:

$$\begin{aligned} \xi_n^* &= \frac{(1-\beta)\, \chi_n^*}{(1-\beta)\, \chi_n^* + (1-\beta+\delta\beta)\,(1-\chi_n^*)} \\ &= \frac{(1-\beta)\, \chi_n^*}{1-\beta+(1-\chi_n^*)\,\delta\beta} \end{aligned}$$

From Section E and the results above, we can recover χ_n^* and ξ_n^* once we know either one of $\boldsymbol{S}$, $\boldsymbol{T}$ and either one of $\boldsymbol{B}$, $\boldsymbol{X}$.

G Dynamic Exact-hat Algebra (Proof of Proposition 1)

In this section of the Online Appendix, we show that we can solve for the transition path of the economy using dynamic exact-hat algebra techniques: Given observed initial populations $\{\ell_{i0}\}_{i=1}^N$, an initial observed allocation of the economy, ($\{a_{i0}\}_{i=1}^N$, $\{a_{i1}\}_{i=1}^N$, $\{S_{ni0}\}_{n,i=1}^N$, $\{T_{ni0}\}_{n,i=1}^N$, $\{B_{ni0}\}_{n,i=1}^N$, $\{X_{ni0}\}_{n,i=1}^N$), and a convergent sequence of future changes in fundamentals under perfect foresight:

$$\left\{ \{\dot{z}_{it}\}_{i=1}^N, \{\dot{\eta}_{it}\}_{i=1}^N, \{\dot{\tau}_{it}\}_{i,j=1}^N, \{\dot{\kappa}_{it}\}_{i,j=1}^N \right\}_{t=1}^{\infty},$$

the solution for the sequence of changes in the model's endogenous variables does not require information on the level of fundamentals:

$$\left\{ \{z_{it}\}_{i=1}^N, \{\eta_{it}\}_{i=1}^N, \{\tau_{it}\}_{i,j=1}^N, \{\kappa_{it}\}_{i,j=1}^N \right\}_{t=1}^{\infty}.$$

We solve for this transition path using dynamic exact-hat algebra in the following steps:

1. Guess a path for wealth holdings $\left\{\{a_{nt}^0\}_{n=1}^N\right\}_{t=2}^T$, and set wealth holdings for periods 0 and 1 according to their values in the data.

2. For each period t, use $\{a_{nt}^0\}_{n=1}^N$, $\{a_{nt-1}^0\}_{n=1}^N$, and the trade and capital share matrices from period $t-1$, $\boldsymbol{S}_{t-1}, \boldsymbol{T}_{t-1}, \boldsymbol{B}_{t-1}, \boldsymbol{X}_{t-1}$, to solve for the contemporary equilibrium in relative changes$\{\hat{w}_{nt}, \hat{r}_{nt}, \hat{p}_{nt}, \hat{v}_{nt}\}_{n=1}^N$:

   (a) Make an initial guess for $\{\hat{w}_{nt}, \hat{r}_{nt}, \hat{p}_{nt}, \hat{v}_{nt}\}_{n=1}^N$.

   (b) Update the price indices $\hat{p}_{nt}$ from:

$$\hat{p}_{nt} = \left[\sum_{i=1}^N s_{nit-1}\left(\frac{\hat{\tau}_{nit}\hat{w}_{it}^{\mu_i}\hat{r}_{it}^{1-\mu_i}}{\hat{z}_{it}}\right)^{-\theta}\right]^{-1/\theta}.$$

   (c) Update the return to wealth $\hat{v}_{nt}$ from:

$$\hat{v}_{nt} = \left[\sum_{h=1}^N b_{nht-1}\left(\frac{\hat{\eta}_{ht}\hat{r}_{ht}}{\hat{\kappa}_{nht}}\right)^{\epsilon}\right]^{1/\epsilon}.$$

   (d) Update wage levels $\hat{w_{it}}$ from:

$$\hat{w}_{it}\hat{\ell}_{it} = \sum_{n=1}^N t_{nit}\hat{s}_{nit}\left(\chi_{nt-1}\hat{v}_{nt}\hat{a}_{nt}^0 + (1-\chi_{nt-1})\,\hat{w}_{nt}\hat{\ell}_{nt}\right),$$

   where

$$\hat{s}_{nit} = \left(\frac{\hat{\tau}_{nit}\hat{w}_{it}^{\mu_i}\hat{r}_{it}^{1-\mu_i}}{\hat{z}_{it}}\frac{1}{\hat{p}_{nt}}\right)^{-\theta},$$

   and we define $\chi_{nt-1} \equiv \frac{v_{nt-1}a_{nt-1}^0}{v_{nt-1}a_{nt-1}^0 + w_{nt-1}\ell_{nt-1}}$.

   (e) Update the return to capital $\hat{r}_{it}$ from:

$$\sum_{n=1}^N x_{nit-1}\hat{v}_{nt}\hat{a}_{nt}^0\hat{b}_{nit} = \hat{w}_{it}\hat{\ell}_{it}, \quad \hat{b}_{nit} = \frac{\left(\hat{\eta}_{it}\hat{r}_{it}/\hat{\kappa}_{nit}\right)^{\epsilon}}{\hat{v}_{nt}^{\epsilon}}.$$

   (f) Return to step (2.b) and repeat until convergence of the temporary equilibrium.

   (g) Update the values of the matrices $\boldsymbol{S}_t, \boldsymbol{T}_t, \boldsymbol{B}_t, \boldsymbol{X}_t$ at period t using their values at period $t-1$, $\boldsymbol{S}_{t-1}, \boldsymbol{T}_{t-1}, \boldsymbol{B}_{t-1}, \boldsymbol{X}_{t-1}$, and the relative changes in the trade and capital flow shares:

$$\hat{s}_{nit} = \left(\hat{\tau}_{nit}\hat{w}_{it}^{\mu_i}\hat{r}_{it}^{1-\mu_i}/\left(\hat{z}_{it}\hat{p}_{nt}\right)\right)^{-\theta},$$

$$\hat{t}_{int} \equiv \hat{s}_{nit}\left(\chi_{nt}\hat{v}_{nt}\hat{a}^0_{nt} + (1-\chi_{nt})\,\hat{w}_{nt}\hat{\ell}_{nt}\right) / \left(\hat{w}_{it}\hat{\ell}_{it}\right),$$

$$\hat{b}_{nit} = \left(\hat{\eta}_{it}\hat{r}_{it} / \left(\hat{\kappa}_{nit}\hat{v}_{nt}\right)\right)^{\epsilon},$$

$$\hat{x}_{int} = \hat{b}_{nit}\hat{v}_{nt}\hat{a}^0_{nt} / \left(\hat{w}_{it}\hat{\ell}_{it}\right).$$

3. Compute the path for the gross rental rates $\left\{\{\mathcal{R}_{nt}\}_{n=1}^{N}\right\}_{t=1}^{T}$ by solving backwards:

$$\mathcal{R}_{nt} = 1 - \delta + \left(\mathcal{R}_{nt+1} - (1-\delta)\right) / \frac{\hat{v}_{nt+1}}{\hat{p}_{nt+1}},$$

   where recall that $\mathcal{R}_{nT} = 1/\beta$.

4. Compute the path for consumption rates $\left\{\{\varsigma_{nt}\}_{n=1}^{N}\right\}_{t=0}^{T}$ by solving backwards:

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi}\mathcal{R}_{nt+1}^{\psi-1}\varsigma_{nt+1}^{-1},$$

   where recall that $\varsigma_{nT} = 1 - \beta$.

5. Compute the path for human capital $\left\{\{h_{nt}\}_{n=1}^{N}\right\}_{t=0}^{T}$ by solving backwards:

$$h_{nt} = \frac{1}{\mathcal{R}_{nt+1}}\left(\frac{1-\chi_{nt+1}}{\chi_{nt+1}}\left(\mathcal{R}_{nt+1} - (1-\delta)\right)a^0_{nt+1} + h_{nt+1}\right),$$

   where human capital in steady state is given by $h_{nT} = \frac{\beta}{1-\beta}\left(\frac{1}{\beta} - (1-\delta)\right)\frac{1-\chi_{nT}}{\chi_{nT}}a^0_{nT}$.

6. Compute the implied price index in period 0 for each location using the relationship

$$p_{n0} = \frac{\frac{1-\chi_{n0}}{\chi_{n0}}v_{n0}a^0_{n0}}{\frac{a^0_{n1}+h_{n0}}{1-\varsigma_{n0}} - \mathcal{R}_{n0}a^0_{n0} - h_{n0}}.$$

7. Update the path for wealth levels $\left\{\{a^1_{nt}\}_{n=1}^{N}\right\}_{t=2}^{T}$ from

$$a^1_{nt+1} = (1-\varsigma_{nt})\left(\mathcal{R}_{nt}a^0_{nt} + \frac{1-\chi_{nt}}{\chi_{nt}}\frac{v_{nt}}{p_{nt}}a^0_{nt} + h_{nt}\right) - h_{nt}$$

   where $\frac{v_{nt}}{p_{nt}}$ is obtained by updating $\frac{v_{n0}}{p_{n0}}$ with the relative changes from the contemporary equilibrium $\frac{\hat{v}_{nt}}{\hat{p}_{nt}}$; v_{n0} is taken as given in the data from the ratio of capital income to a_{n0}; and p_{n0} is recovered in step (6).

8. Normalize all nominal variables to ensure that the sum of global GDP in each period is equal to 1: $\sum_{i=1}^{N} \frac{w_{it}\ell_{it}}{\mu_i} = 1$.

9. Return to step (2) and repeat until convergence of the path for wealth levels $\left\{ \{a_{nt}\}_{n=1}^{N} \right\}_{t=2}^{T}$.

H Model Inversion

We now show how we can invert the model to recover fundamentals: trade frictions (τ_{nit}), financial frictions (κ_{nit}), goods productivity (z_{nt}), and capital use efficiency (η_{nt}). As part of this model inversion, we show how to solve for unobserved endogenous variables, such as the rental rate for capital (r_{nt}), the consumption price index (p_{nt}), and total income from wealth ($v_{nt}a_{nt}$).

In our quantitative analysis, we only use this model inversion to recover total income from wealth ($v_{nt}a_{nt}$) in a model-consistent way from the observed national accounts, trade and capital holdings data. When we undertake counterfactuals, we use either dynamic exact-hat algebra or our linearization, neither of which requires us to solve for fundamentals (κ_{nit}, τ_{nit}, z_{nt}, η_{nt}).

H.1 Capital Frictions (κ_{nit})

We can recover capital frictions from the portfolio allocation shares:

$$b_{nit} = \frac{\left(\eta_{it} r_{it} / \kappa_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}}. \tag{H.1}$$

We can represent these portfolio shares as the following gravity equation:

$$b_{nit} = \omega_{nt} \vartheta_{it} \kappa_{nit}^{-\epsilon},$$

where we can estimate $\kappa_{nit}^{-\epsilon}$ as the residual from a Poisson Pseudo Maximum Likelihood (PPML) regression of portfolio shares on investor fixed effects and producer fixed effects. Any common components of capital frictions across countries are absorbed into the fixed effects here and will be captured in our solutions for capital productivities below.

H.2 Trade Frictions (τ_{nit})

From the trade shares:

$$s_{nit} = \frac{\left(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it}\right)^{-\theta}}{\sum_{h=1}^{N} \left(\tau_{nht} w_{ht}^{\mu_i} r_{ht}^{1-\mu_i} / z_{ht}\right)^{-\theta}}. \tag{H.2}$$

We can represent these trade shares as the following gravity equation:

$$s_{nit} = \omega_{nt}\vartheta_{it}\tau_{nit}^{-\theta},$$

where we can estimate $\tau_{nit}^{-\theta}$ as the residual from a Poisson Pseudo Maximum Likelihood (PPML) regression of trade shares on importer fixed effects and exporter fixed effects. Any common components of trade frictions across countries are absorbed into the fixed effects here and will be captured in our solutions for productivities below.

H.3 Real Rental Rate (r_{it}/p_{it})

Suppose that we observe GDP ($GDP_{it} = w_{it}\ell_{it} + r_{it}k_{it}$) and labor compensation ($w_{it}\ell_{it}$). We can recover capital payments from:

$$r_{it}k_{it} = GDP_{it} - w_{it}\ell_{it} = \frac{1}{\mu_i}w_{it}\ell_{it}, \tag{H.3}$$

where for internal consistency in the model, we allow the labor share (μ_i) to vary across countries. Given data on nominal capital stocks ($p_{it}k_{it}$) we recover the real rental rate ($\bar{r}_{it} = r_{it}/p_{it}$) using:

$$\bar{r}_{it} \equiv \frac{r_{it}}{p_{it}} = \frac{r_{it}k_{it}}{p_{it}k_{it}} = \frac{GDP_{it} - w_{it}\ell_{it}}{p_{it}k_{it}}.$$

H.4 Capital Income ($v_{it}a_{it}$)

In the model, GDP in each country i must equal to the sum of expenditure by all other countries on the output of country i:

$$w_{it}\ell_{it}/\mu_{it} = \sum_{n=1}^{N} s_{nit}\left(w_{nt}\ell_{nt} + v_{nt}a_{nt}\right). \tag{H.4}$$

Using data on GDP ($w_{it}\ell_{it}/\mu_{it}$), labor compensation ($w_{it}\ell_{it}$) and expenditure shares (s_{nit}), we use the above equation to recover capital income $v_{nt}a_{nt}$ for each country.

H.5 Goods Productivity (z_{it}) and Capital Use Efficiency (η_{it})

Using these solutions for the real rental rate ($\bar{r}_{it}$), capital income ($v_{it}a_{it}$), trade frictions (τ_{nit}), and capital frictions (κ_{nit}), we jointly recover goods productivity (z_{it}), capital use efficiency (η_{it}), and the consumption price index (p_{it}) by iterating over the definition of the price index and the

market clearing conditions for goods and capital:

$$p_{nt} = \left[\sum_{i=1}^{N} \left(\frac{\tau_{nit} w_{it}^{\mu_i} \left(\bar{r}_{it} p_{it} \right)^{1-\mu_i}}{z_{it}} \right)^{-\theta} \right]^{-1/\theta}, \tag{H.5}$$

$$\frac{1-\mu_i}{\mu_i} w_{it} \ell_{it} = \sum_{n=1}^{N} \frac{\left(\eta_{it} \bar{r}_{it} p_{it} / \kappa_{nit} \right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} \bar{r}_{ht} p_{ht} / \kappa_{nht} \right)^{\epsilon}} v_{nt} a_{nt}, \tag{H.6}$$

$$w_{it} \ell_{it} / \mu_i = \sum_{n=1}^{N} \frac{\left(\tau_{nit} w_{it}^{\mu_i} \left(\bar{r}_{it} p_{it} \right)^{1-\mu_i} / z_{it} \right)^{-\theta}}{\sum_{h=1}^{N} \left(\tau_{nht} w_{ht}^{\mu_h} \left(\bar{r}_{ht} p_{ht} \right)^{1-\mu_h} / z_{ht} \right)^{-\theta}} \left(w_{nt} \ell_{nt} + v_{nt} a_{nt} \right), \tag{H.7}$$

where we can solve for unique vectors of capital productivities (η_{it}), goods productivities (z_{it}), and price indices (p_{it}), in each country up to a normalization. We impose the normalization that the geometric mean of the capital productivities is equal to one. We impose a similar normalization for goods productivities. Note that the price indices (p_{it}) are recovered as auxiliary variables, and we do not directly use them in our quantitative analysis.

I Linearization

In this section of the Online Appendix, we linearize the model, and derive a closed-form solution for the evolution of the state variables along the transition path towards steady state. In Subsection I.1, we linearize the general equilibrium conditions of the model. In Subsection I.2, we represent the linearized general equilibrium conditions in matrix form.

I.1 Linearized Equilibrium Conditions

Let $\widetilde{x}_{nt} \equiv \ln x_{nt} - \ln x_n^*$ denote log deviation relative to the initial steady state. We begin by totally differentiating the conditions for general equilibrium in equations (E.1)-(E.12) with respect to shocks to fundamentals in the form of goods productivity ($\widetilde{z}_{it}$), capital use efficiency ($\widetilde{\eta}_{it}$), trade frictions ($\widetilde{\tau}_{ni}$), and capital market frictions ($\widetilde{\kappa}_{ni}$). We thus obtain the following system of linearized equilibrium conditions:

$$-\frac{1}{\theta} \widetilde{S}_{nit} = \widetilde{\tau}_{nit} + \mu_i \widetilde{w}_{it} + (1-\mu_i) \widetilde{r}_{it} - \widetilde{z}_{it} - \sum_{h} S_{nh} \left(\widetilde{\tau}_{nht} + \mu_i \widetilde{w}_{ht} + (1-\mu_i) \widetilde{r}_{ht} - \widetilde{z}_{ht} \right). \tag{I.1}$$

$$\widetilde{w}_{it} = \sum_{n=1}^{N} T_{in} \left(\widetilde{S}_{nit} + \chi_n \left(\widetilde{v}_{nt} + \widetilde{a}_{nt} \right) + (1-\chi_n) \widetilde{w}_{nt} \right). \tag{I.2}$$

$$\frac{1}{\epsilon}\widetilde{B}_{nit} = \widetilde{\eta}_{it} + \widetilde{r}_{it} - \widetilde{\kappa}_{nit} - \sum_h B_{nh}\left(\widetilde{\eta}_{ht} + \widetilde{r}_{ht} - \widetilde{\kappa}_{nht}\right). \tag{I.3}$$

$$\widetilde{v}_{nt} = \sum_h B_{nh}\left(\widetilde{\eta}_{ht} + \widetilde{r}_{ht} - \widetilde{\kappa}_{nht}\right). \tag{I.4}$$

$$\widetilde{r}_{it} + \sum_n X_{in}\left(\widetilde{\eta}_{it} + (1-1/\epsilon)\,\widetilde{B}_{nit} + \widetilde{a}_{nt} - \widetilde{\kappa}_{nit}\right) = \widetilde{w}_{it}. \tag{I.5}$$

$$\sum_i \frac{1}{\mu_i} q_i \widetilde{w}_{it} = 0. \tag{I.6}$$

$$\widetilde{p}_{nt} \equiv \sum_i S_{ni}\left(\widetilde{\tau}_{nit} + \mu_i \widetilde{w}_{it} + (1-\mu_i)\,\widetilde{r}_{it} - \widetilde{z}_{it}\right). \tag{I.7}$$

$$\begin{aligned}
\widetilde{h}_{nt} &\equiv \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s} - \sum_{u=1}^{s}\widetilde{\mathcal{R}}_{nt+u}\right), \qquad (I.8)\\
&= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\sum_{u=1}^{s}\widetilde{\mathcal{R}}_{nt+u},\\
&= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \frac{1-\beta}{\beta}\left(\sum_{s=1}^{\infty}\beta^s\widetilde{\mathcal{R}}_{nt+1} + \beta\sum_{s=1}^{\infty}\beta^s\widetilde{\mathcal{R}}_{nt+2} + \dots\right),\\
&= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \frac{1-\beta}{\beta}\left(\frac{\widetilde{\mathcal{R}}_{nt+1}}{1-\beta} + \beta\frac{\widetilde{\mathcal{R}}_{nt+2}}{1-\beta} + \dots\right),\\
&= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \left(\widetilde{\mathcal{R}}_{nt+1} + \beta\widetilde{\mathcal{R}}_{nt+2} + \dots\right),\\
&= \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \frac{1}{\beta}\sum_{s=1}^{\infty}\beta^s\widetilde{\mathcal{R}}_{nt+s}.
\end{aligned}$$

$$\widetilde{\mathcal{R}}_{nt} = (1-\beta+\beta\delta)\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right). \tag{I.9}$$

$$\begin{aligned}
\xi_n\widetilde{a}_{nt+1} &= -\frac{1-\beta}{\beta}\widetilde{\varsigma}_n + \xi_n\left(\widetilde{\mathcal{R}}_{nt} + \widetilde{a}_{nt}\right) \qquad (I.10)\\
&\quad + (1-\xi_n)(1-\beta)\left(\widetilde{w}_{nt} - \widetilde{p}_{nt} - \widetilde{h}_{nt}\right),
\end{aligned}$$

$$-\widetilde{\varsigma}_{nt} = \beta\left((\psi-1)\,\widetilde{\mathcal{R}}_{nt+1} - \widetilde{\varsigma}_{nt+1}\right). \tag{I.11}$$

In this system of linear equations, there are no terms in the change in the trade and capital share matrices, because these terms are second-order in the underlying Taylor-series expansion,

involving interactions between the shocks to fundamentals and the resulting changes in trade and capital share matrices. As we consider first-order changes in fundamentals, these second-order, nonlinear terms drop out of the linearization. Therefore, we can write the trade and capital share matrices with no time subscript (S_{ni}, T_{in}, B_{ni}, X_{in}) for first-order changes in fundamentals.

I.2 Matrix Representation

We now represent this system of linearized equations in matrix form. We proceed in two steps. First, we derive prices and allocation in the static block as a function of the current-period state variable $\widetilde{a}_t$. Second, we then derive the law of motion for the state variable as a second-order difference equation. To simplify the exposition, we assume a common labor share across countries ($\mu_i = \mu$) throughout this section of the Online Appendix.

I.2.1 Static Bloc of Equations

We begin by defining the following measures of incoming and outgoing changes in trade and capital market frictions: $\widetilde{\tau}_{nt}^{in} \equiv \sum_i S_{nit}\widetilde{\tau}_{nit}$, $\widetilde{\tau}_{it}^{out} \equiv \sum_n T_{int}\widetilde{\tau}_{nit}$, $\widetilde{\kappa}_{nt}^{out} \equiv \sum_i B_{nit}\widetilde{\kappa}_{nit}$, $\widetilde{\kappa}_{it}^{in} \equiv \sum_n X_{int}\widetilde{\kappa}_{nit}$. Let $\boldsymbol{D}(\boldsymbol{x})$ denote the diagonal matrix with the vector $\boldsymbol{x}$ on the diagonal. Using this notation, we can represent the system of linearized equations from Subsection I.1 in the following form:

$$\begin{aligned}\widetilde{\boldsymbol{w}}_t &= \boldsymbol{T}\left[\boldsymbol{D}(\boldsymbol{\chi})\left(\boldsymbol{B}(\widetilde{\boldsymbol{\eta}}_t + \widetilde{\boldsymbol{r}}_t) - \widetilde{\boldsymbol{\kappa}}_t^{out} + \widetilde{\boldsymbol{a}}_t\right) + (\boldsymbol{I} - \boldsymbol{D}(\boldsymbol{\chi}))\,\widetilde{\boldsymbol{w}}_t\right] \\ &\quad +\theta\left\{\boldsymbol{T}\widetilde{\boldsymbol{\tau}}_t^{in} - \widetilde{\boldsymbol{\tau}}_t^{out} + (\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I})\left(\mu\widetilde{\boldsymbol{w}}_t + (1-\mu)\,\widetilde{\boldsymbol{r}}_t - \widetilde{\boldsymbol{z}}_t\right)\right\}. \end{aligned} \tag{I.12}$$

$$\widetilde{\boldsymbol{r}}_t + \widetilde{\boldsymbol{\eta}}_t + (\epsilon - 1)\left((\boldsymbol{I} - \boldsymbol{X}\boldsymbol{B})(\widetilde{\boldsymbol{\eta}}_t + \widetilde{\boldsymbol{r}}_t) + \boldsymbol{X}\widetilde{\boldsymbol{\kappa}}_t^{out} - \widetilde{\boldsymbol{\kappa}}_t^{in}\right) + \boldsymbol{X}\widetilde{\boldsymbol{a}}_t - \widetilde{\boldsymbol{\kappa}}_t^{in} = \widetilde{\boldsymbol{w}}_t. \tag{I.13}$$

$$\frac{1}{\mu}\boldsymbol{q}'\widetilde{\boldsymbol{w}}_t = \widetilde{\boldsymbol{w}}_t. \tag{I.14}$$

The first two equations and the normalization equation can be solved to express changes in wages ($\widetilde{\boldsymbol{w}}_t$) and changes in rental rates ($\widetilde{\boldsymbol{r}}_t$) as functions of changes in the state variable $\widetilde{\boldsymbol{a}}_t$ and funda-

mental shocks ($\widetilde{\boldsymbol{z}}_t$, $\widetilde{\boldsymbol{\eta}}_t$, $\widetilde{\boldsymbol{\kappa}}_t^{in}$, $\widetilde{\boldsymbol{\kappa}}_t^{out}$, $\widetilde{\boldsymbol{\tau}}_t^{in}$, $\widetilde{\boldsymbol{\tau}}_t^{out}$):

$$\begin{bmatrix} \widetilde{\boldsymbol{w}}_t \\ \widetilde{\boldsymbol{r}}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}^w \\ \boldsymbol{L}^r \end{bmatrix} \widetilde{\boldsymbol{a}}_t + \begin{bmatrix} \boldsymbol{M}^w \\ \boldsymbol{M}^r \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{z}}_t \\ \widetilde{\boldsymbol{\eta}}_t \\ \widetilde{\boldsymbol{\kappa}}_t^{in} \\ \widetilde{\boldsymbol{\kappa}}_t^{out} \\ \widetilde{\boldsymbol{\tau}}_t^{in} \\ \widetilde{\boldsymbol{\tau}}_t^{out} \end{bmatrix}. \tag{I.15}$$

Note that our choice of numeraire implies the following normalization equations: $\frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\widetilde{\boldsymbol{w}}_t = \mathbf{0}$ and

$$\frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\boldsymbol{B}\widetilde{\boldsymbol{r}}_t = -\frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\widetilde{\boldsymbol{a}} + \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}' \underbrace{\begin{bmatrix} \mathbf{0} & -\boldsymbol{B} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\equiv \boldsymbol{E}} \widetilde{\boldsymbol{f}}.$$

Derivation of normalization equations We begin by deriving these normalization equation from our choice of numeraire. Given our choice of world GDP as numeraire, we know $\boldsymbol{q}'\widetilde{\boldsymbol{w}}_t = 0$, and hence:

$$\left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)\widetilde{\boldsymbol{w}}_t = \mathbf{0}.$$

Likewise, our choice of numeraire implies:

$$\frac{\sum_{n=1}^N v_{nt}a_{nt}}{1-\mu} = 1$$

$$\frac{1}{1-\mu}\sum_{n=1}^N \zeta_n\left(\widetilde{v}_{nt} + \widetilde{a}_{nt}\right) = 0$$

$$\frac{1}{1-\mu}\sum_{n=1}^N \zeta_n\left(\sum_{h=1}^N B_{nh}\left(\widetilde{\eta}_{ht} + \widetilde{r}_{ht} - \widetilde{\kappa}_{nht}\right) + \widetilde{a}_{nt}\right) = 0$$

$$\frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\left[\boldsymbol{B}\left(\widetilde{\boldsymbol{\eta}} + \widetilde{\boldsymbol{r}}_t\right) - \widetilde{\boldsymbol{\kappa}}^{out} + \widetilde{\boldsymbol{a}}_t\right] = \mathbf{0}$$

$$\frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\boldsymbol{B}\widetilde{\boldsymbol{r}}_t = -\frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\widetilde{\boldsymbol{k}} + \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}' \underbrace{\begin{bmatrix} \mathbf{0} & -\boldsymbol{B} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\equiv \boldsymbol{E}} \widetilde{\boldsymbol{f}}$$

Matrix definitions We now provide the definitions of the matrices ($\boldsymbol{L}^r$, $\boldsymbol{L}^w$, $\boldsymbol{M}^r$, $\boldsymbol{M}^w$) in equation (I.15):

$$\boldsymbol{L}^r = -\left(\boldsymbol{I} - \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\boldsymbol{B}\right)\left(\boldsymbol{A}_1\left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)\boldsymbol{B}_2 + \boldsymbol{B}_1\right)^{-1}\left(\boldsymbol{A}_1\left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)\boldsymbol{C}_2 + \boldsymbol{C}_1\right) - \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'$$

$$\boldsymbol{L}^{w} = \left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)(\boldsymbol{C}_2 + \boldsymbol{B}_2\boldsymbol{L}^{r})$$

$$\boldsymbol{M}^{r} = -\left(\boldsymbol{I} - \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\boldsymbol{B}\right)\left(\boldsymbol{A}_1\left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)\boldsymbol{B}_2 + \boldsymbol{B}_1\right)^{-1}\left(\boldsymbol{A}_1\left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)\boldsymbol{D}_2 + \boldsymbol{D}_1\right) + \frac{1}{1-\mu}\mathbf{1}\boldsymbol{\zeta}'\boldsymbol{E}$$

$$\boldsymbol{M}^{w} = \left(\boldsymbol{I} - \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'\right)(\boldsymbol{D}_2 + \boldsymbol{B}_2\boldsymbol{M}^{r})$$

$$\boldsymbol{A}_1 = (\boldsymbol{T}(\boldsymbol{I} - \boldsymbol{D}(\boldsymbol{\chi})) + \theta(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I})\mu - \boldsymbol{I})$$

$$\boldsymbol{B}_1 = (\boldsymbol{T}\boldsymbol{D}(\boldsymbol{\chi})\boldsymbol{B} + \theta(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I})(1-\mu))$$

$$\boldsymbol{C}_1 = \boldsymbol{T}\boldsymbol{D}(\boldsymbol{\chi})$$

$$\boldsymbol{B}_2 = ((\epsilon - 1)(\boldsymbol{I} - \boldsymbol{X}\boldsymbol{B}) + \boldsymbol{I})$$

$$\boldsymbol{C}_2 = \boldsymbol{X}$$

$$\boldsymbol{D}_1 = \left[\begin{array}{ccccccc} -\theta(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}), & \boldsymbol{T}\boldsymbol{D}(\boldsymbol{\chi})\boldsymbol{B}, & \mathbf{0}, & -\boldsymbol{T}\boldsymbol{D}(\boldsymbol{\chi}), & \theta\boldsymbol{T}, & -\theta\boldsymbol{I} \end{array}\right]$$

$$\boldsymbol{D}_2 = \left[\begin{array}{ccccccc} \mathbf{0}, & \boldsymbol{I} + (\epsilon - 1)(\boldsymbol{I} - \boldsymbol{X}\boldsymbol{B}), & -\epsilon\boldsymbol{I}, & (\epsilon - 1)\boldsymbol{X}, & \mathbf{0}, & \mathbf{0} \end{array}\right]$$

$$\boldsymbol{E} = \left[\begin{array}{cccccc} \mathbf{0} & -\boldsymbol{B} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} \end{array}\right]$$

I.2.2 Dynamic Bloc of Equations

We now show that the evolution of the wealth state variables can be written as a second-order difference equation. We start with the dynamic bloc of linearized equations for general equilibrium from Section I.1 above. We have the following relationships:

$$\widetilde{\boldsymbol{p}}_t \equiv \widetilde{\boldsymbol{\tau}}_t^{in} + \boldsymbol{S}(\mu\widetilde{\boldsymbol{w}}_t + (1-\mu)\widetilde{\boldsymbol{r}}_t - \widetilde{\boldsymbol{z}}_t)$$

$$\widetilde{\boldsymbol{h}}_t \equiv \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s(\widetilde{\boldsymbol{w}}_{t+s} - \widetilde{\boldsymbol{p}}_{t+s}) - \frac{1}{\beta}\sum_{s=1}^{\infty}\beta^s\widetilde{\boldsymbol{R}}_{t+s}$$

$$\widetilde{\boldsymbol{\mathcal{R}}}_t = (1 - \beta + \beta\delta)\left(\boldsymbol{B}(\widetilde{\boldsymbol{\eta}}_t + \widetilde{\boldsymbol{r}}_t) - \widetilde{\boldsymbol{\kappa}}_t^{out} - \widetilde{\boldsymbol{p}}_t\right)$$

$$-\widetilde{\boldsymbol{\varsigma}}_t = \beta\left((\psi - 1)\widetilde{\boldsymbol{\mathcal{R}}}_{t+1} - \widetilde{\boldsymbol{\varsigma}}_{t+1}\right)$$

$$\begin{aligned}\boldsymbol{D}(\boldsymbol{\xi})\widetilde{\boldsymbol{a}}_{t+1} &= -\frac{1-\beta}{\beta}\widetilde{\boldsymbol{\varsigma}}_t + \boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{\mathcal{R}}}_t + \widetilde{\boldsymbol{a}}_t\right) \\ &\quad + (\boldsymbol{I} - \boldsymbol{D}(\boldsymbol{\xi}))(1-\beta)\left(\widetilde{\boldsymbol{w}}_t - \widetilde{\boldsymbol{p}}_t - \widetilde{\boldsymbol{h}}_t\right)\end{aligned}$$

$$\begin{aligned}\beta \boldsymbol{D}(\boldsymbol{\xi})\widetilde{\boldsymbol{a}}_{t+2} &= -\beta\frac{1-\beta}{\beta}\widetilde{\boldsymbol{\varsigma}}_{t+1}+\beta \boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}+\widetilde{\boldsymbol{a}}_{t+1}\right)\\ &\quad +\beta(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi}))(1-\beta)\left(\widetilde{\boldsymbol{w}}_{t+1}-\widetilde{\boldsymbol{p}}_{t+1}-\widetilde{\boldsymbol{h}}_{t+1}\right)\end{aligned}$$

$$\begin{aligned}\boldsymbol{D}(\boldsymbol{\xi})(\widetilde{\boldsymbol{a}}_{t+1}-\beta\widetilde{\boldsymbol{a}}_{t+2}) &= -\frac{1-\beta}{\beta}(\widetilde{\boldsymbol{\varsigma}}_t-\beta\widetilde{\boldsymbol{\varsigma}}_{t+1})+\boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{\mathcal{R}}}_t+\widetilde{\boldsymbol{a}}_t-\beta\left(\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}+\widetilde{\boldsymbol{a}}_{t+1}\right)\right)\\ &\quad +(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi}))(1-\beta)\left(\left(\widetilde{\boldsymbol{w}}_t-\widetilde{\boldsymbol{p}}_t-\widetilde{\boldsymbol{h}}_t\right)-\beta\left(\widetilde{\boldsymbol{w}}_{t+1}-\widetilde{\boldsymbol{p}}_{t+1}-\widetilde{\boldsymbol{h}}_{t+1}\right)\right)\end{aligned}$$

$$\widetilde{\boldsymbol{h}}_t \equiv \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s(\widetilde{\boldsymbol{w}}_{t+s}-\widetilde{\boldsymbol{p}}_{t+s})-\frac{1}{\beta}\sum_{s=1}^{\infty}\beta^s\widetilde{\boldsymbol{\mathcal{R}}}_{t+s}$$

$$\widetilde{\boldsymbol{h}}_{t+1} \equiv \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s(\widetilde{\boldsymbol{w}}_{t+s+1}-\widetilde{\boldsymbol{p}}_{t+s+1})-\frac{1}{\beta}\sum_{s=1}^{\infty}\beta^s\widetilde{\boldsymbol{\mathcal{R}}}_{t+s+1}$$

$$\widetilde{\boldsymbol{h}}_t-\beta\widetilde{\boldsymbol{h}}_{t+1}=(1-\beta)(\widetilde{\boldsymbol{w}}_{t+1}-\widetilde{\boldsymbol{p}}_{t+1})-\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}$$

$$\begin{aligned}\boldsymbol{D}(\boldsymbol{\xi})(\widetilde{\boldsymbol{a}}_{t+1}-\beta\widetilde{\boldsymbol{a}}_{t+2}) &= (1-\beta)\left((\psi-1)\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\right)\\ &\quad +\boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{\mathcal{R}}}_t+\widetilde{\boldsymbol{a}}_t-\beta\left(\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}+\widetilde{\boldsymbol{a}}_{t+1}\right)\right)\\ &\quad +(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi}))(1-\beta)\left((\widetilde{\boldsymbol{w}}_t-\widetilde{\boldsymbol{p}}_t)-(\widetilde{\boldsymbol{w}}_{t+1}-\widetilde{\boldsymbol{p}}_{t+1})+\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\right)\end{aligned}$$

$$\widetilde{\mathcal{R}}_t=(1-\beta+\beta\delta)(\widetilde{\boldsymbol{v}}_t-\widetilde{\boldsymbol{p}}_t)$$

Assume fundamentals are constant along the transition (with a possible initial shock), so

$$\begin{aligned}\boldsymbol{D}(\boldsymbol{\xi})(\widetilde{\boldsymbol{a}}_{t+1}-\beta\widetilde{\boldsymbol{a}}_{t+2}) &= \boldsymbol{D}(\boldsymbol{\xi})(\widetilde{\boldsymbol{a}}_t-\beta\widetilde{\boldsymbol{a}}_{t+1})\\ &\quad +(1-\beta)\psi\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\\ &\quad +\boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{\mathcal{R}}}_t-\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\right)\\ &\quad +(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi}))(1-\beta)((\boldsymbol{I}-\mu\boldsymbol{S})(\widetilde{\boldsymbol{w}}_t-\widetilde{\boldsymbol{w}}_{t+1})-\boldsymbol{S}(1-\mu)(\widetilde{\boldsymbol{r}}_t-\widetilde{\boldsymbol{r}}_{t+1}))\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{a}}_{t+1}-\beta\widetilde{\boldsymbol{a}}_{t+2}\right) &= \boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{a}}_{t}-\beta\widetilde{\boldsymbol{a}}_{t+1}\right)\\
&\quad+(1-\beta)\,\psi\,(1-\beta+\beta\delta)\left(\boldsymbol{B}\left(\widetilde{\boldsymbol{\eta}}+\widetilde{\boldsymbol{r}}_{t+1}\right)-\widetilde{\boldsymbol{\kappa}}^{out}-\widetilde{\boldsymbol{\tau}}^{in}-\boldsymbol{S}\left(\mu\widetilde{\boldsymbol{w}}_{t+1}+(1-\mu)\,\widetilde{\boldsymbol{r}}_{t+1}-\widetilde{\boldsymbol{z}}\right)\right)\\
&\quad+\boldsymbol{D}(\boldsymbol{\xi})\,(1-\beta+\beta\delta)\left(\boldsymbol{B}\left(\widetilde{\boldsymbol{r}}_{t}-\widetilde{\boldsymbol{r}}_{t+1}\right)-\boldsymbol{S}\left(\mu\left(\widetilde{\boldsymbol{w}}_{t}-\widetilde{\boldsymbol{w}}_{t+1}\right)+(1-\mu)\left(\widetilde{\boldsymbol{r}}_{t}-\widetilde{\boldsymbol{r}}_{t+1}\right)\right)\right)\\
&\quad+\left(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi})\right)(1-\beta)\left(\left(\boldsymbol{I}-\mu\boldsymbol{S}\right)\left(\widetilde{\boldsymbol{w}}_{t}-\widetilde{\boldsymbol{w}}_{t+1}\right)-\boldsymbol{S}\,(1-\mu)\left(\widetilde{\boldsymbol{r}}_{t}-\widetilde{\boldsymbol{r}}_{t+1}\right)\right)\\
&= \boldsymbol{D}(\boldsymbol{\xi})\left(\widetilde{\boldsymbol{a}}_{t}-\beta\widetilde{\boldsymbol{a}}_{t+1}\right)\\
&\quad+\left[\left(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi})\right)(1-\beta)-\mu\,(1-\beta+\beta\delta\boldsymbol{D}(\boldsymbol{\xi}))\,\boldsymbol{S}\right]\widetilde{\boldsymbol{w}}_{t}\\
&\quad\left[-\left(\boldsymbol{I}-\boldsymbol{D}(\boldsymbol{\xi})\right)(1-\beta)+\mu\,(1-\beta+\beta\delta\boldsymbol{D}(\boldsymbol{\xi}))\,\boldsymbol{S}-\mu\,(1-\beta)\,\psi\,(1-\beta+\beta\delta)\,\boldsymbol{S}\right]\widetilde{\boldsymbol{w}}_{t+1}\\
&\quad\left[+(1-\beta+\beta\delta)\,\boldsymbol{D}(\boldsymbol{\xi})\,\boldsymbol{B}-(1-\beta+\beta\delta\boldsymbol{D}(\boldsymbol{\xi}))\,\boldsymbol{S}\,(1-\mu)\right]\widetilde{\boldsymbol{r}}_{t}\\
&\quad\left[-(1-\beta+\beta\delta)\,\boldsymbol{D}(\boldsymbol{\xi})\,\boldsymbol{B}+(1-\beta+\beta\delta\boldsymbol{D}(\boldsymbol{\xi}))\,\boldsymbol{S}\,(1-\mu)\right]\widetilde{\boldsymbol{r}}_{t+1}\\
&\quad+(1-\beta)\,\psi\,(1-\beta+\beta\delta)\left(\boldsymbol{B}-\boldsymbol{S}\,(1-\mu)\right)\widetilde{\boldsymbol{r}}_{t+1}\\
&\quad+\psi\,(1-\beta)\,(1-\beta+\beta\delta)\left(\boldsymbol{B}\widetilde{\boldsymbol{\eta}}-\widetilde{\boldsymbol{\kappa}}^{out}-\widetilde{\boldsymbol{\tau}}^{in}+\boldsymbol{S}\widetilde{\boldsymbol{z}}\right).
\end{aligned}
$$

Let $\left[\boldsymbol{A}^{w}\boldsymbol{A}^{w+}\boldsymbol{A}^{r}\boldsymbol{A}^{r+}\right]$ be defined such that

$$
\begin{aligned}
\boldsymbol{D}(\boldsymbol{\xi})\left(-\beta\widetilde{\boldsymbol{a}}_{t+2}+(1+\beta)\,\widetilde{\boldsymbol{a}}_{t+1}-\widetilde{\boldsymbol{a}}_{t}\right) &\equiv \left[\begin{array}{cccc}\boldsymbol{A}^{w} & \boldsymbol{A}^{w+} & \boldsymbol{A}^{r} & \boldsymbol{A}^{r+}\end{array}\right]\left[\begin{array}{c}\widetilde{\boldsymbol{w}}_{t}\\ \widetilde{\boldsymbol{w}}_{t+1}\\ \widetilde{\boldsymbol{r}}_{t}\\ \widetilde{\boldsymbol{r}}_{t+1}\end{array}\right]+\boldsymbol{J}\underbrace{\left[\begin{array}{c}\widetilde{\boldsymbol{z}}\\ \widetilde{\boldsymbol{\eta}}\\ \widetilde{\boldsymbol{\kappa}}^{in}\\ \widetilde{\boldsymbol{\kappa}}^{out}\\ \widetilde{\boldsymbol{\tau}}^{in}\\ \widetilde{\boldsymbol{\tau}}^{out}\end{array}\right]}_{\equiv\widetilde{\boldsymbol{f}}}\\
&= \left(\boldsymbol{A}^{w+}\boldsymbol{L}^{w}+\boldsymbol{A}^{r+}\boldsymbol{L}^{r}\right)\widetilde{\boldsymbol{a}}_{t+1}+\left(\boldsymbol{A}^{w}\boldsymbol{L}^{w}+\boldsymbol{A}^{r}\boldsymbol{L}^{r}\right)\widetilde{\boldsymbol{a}}_{t}\\
&\quad+\left[\boldsymbol{J}+\boldsymbol{A}^{w+}\boldsymbol{M}^{w}+\boldsymbol{A}^{r+}\boldsymbol{M}^{r}+\boldsymbol{A}^{w}\boldsymbol{M}^{w}+\boldsymbol{A}^{r}\boldsymbol{M}^{r}\right]\widetilde{\boldsymbol{f}}.
\end{aligned}
$$

Then the law of motion for the wealth state variables is

$$
\boldsymbol{\Psi}\widetilde{\boldsymbol{a}}_{t+2}=\boldsymbol{\Gamma}\widetilde{\boldsymbol{a}}_{t+1}+\boldsymbol{\Theta}\widetilde{\boldsymbol{a}}_{t}+\boldsymbol{\Pi}\widetilde{\boldsymbol{f}} \tag{I.16}
$$

$$
\boldsymbol{\Psi}=-\beta\boldsymbol{D}(\boldsymbol{\xi})
$$

$$
\boldsymbol{\Gamma}=-(1+\beta)\,\boldsymbol{D}(\boldsymbol{\xi})+\left(\boldsymbol{A}^{w+}\boldsymbol{L}^{w}+\boldsymbol{A}^{r+}\boldsymbol{L}^{r}\right)
$$

$$
\boldsymbol{\Theta}=\boldsymbol{D}(\boldsymbol{\xi})+\left(\boldsymbol{A}^{w}\boldsymbol{L}^{w}+\boldsymbol{A}^{r}\boldsymbol{L}^{r}\right)
$$

$$
\boldsymbol{J}\equiv\psi\,(1-\beta)\,(1-\beta+\beta\delta)\left[\begin{array}{ccccc}\boldsymbol{S}, & \boldsymbol{B}, & \boldsymbol{0}, & -\boldsymbol{I}, & -\boldsymbol{I}, & \boldsymbol{0}\end{array}\right]
$$

$$
\boldsymbol{\Pi}=\left(\boldsymbol{A}^{w}+\boldsymbol{A}^{w+}\right)\boldsymbol{M}^{w}+\left(\boldsymbol{A}^{r}+\boldsymbol{A}^{r+}\right)\boldsymbol{M}^{r}+\boldsymbol{J}.
$$

I.3 Proof of Proposition 3 (Closed-Form Transition Path)

Proof. We now solve the second-order difference equation (I.16) using the method of undetermined coefficients following Uhlig (1999). We first conjecture the following linear closed-form solution:

$$\widetilde{\boldsymbol{a}}_t = \boldsymbol{P}\widetilde{\boldsymbol{a}}_{t-1} + \boldsymbol{R}\widetilde{\boldsymbol{f}}. \tag{I.17}$$

We next show that this conjecture is indeed the solution to the second-order difference equation. We being by substituting this conjecture into the second-order difference equation (I.16) to obtain a matrix system of quadratic equations:

$$\left(\boldsymbol{\Psi}\boldsymbol{P}^2 - \boldsymbol{\Gamma}\boldsymbol{P} - \boldsymbol{\Theta}\right)\widetilde{\boldsymbol{a}}_{\boldsymbol{t}} + \left[\left(\boldsymbol{\Psi}\boldsymbol{P} + \boldsymbol{\Psi} - \boldsymbol{\Gamma}\right)\boldsymbol{R} - \boldsymbol{\Pi}\right]\widetilde{\boldsymbol{f}} = 0. \tag{I.18}$$

For the system (I.18) to have a solution for $\widetilde{\boldsymbol{a}}_{\boldsymbol{t}} \neq 0$ and $\widetilde{\boldsymbol{f}} \neq 0$, we require:

$$\boldsymbol{\Psi}\boldsymbol{P}^2 - \boldsymbol{\Gamma}\boldsymbol{P} - \boldsymbol{\Theta} = 0, \tag{I.19}$$

$$\boldsymbol{R} = \left(\boldsymbol{\Psi}\boldsymbol{P} + \boldsymbol{\Psi} - \boldsymbol{\Gamma}\right)^{-1}\boldsymbol{\Pi}. \tag{I.20}$$

Following Uhlig (1999), we can write this first condition (I.19) as the following generalized eigenvector-eigenvalue problem, where e is a generalized eigenvector and ξ is a generalized eigenvalue of $\boldsymbol{\Xi}$ with respect to $\boldsymbol{\Delta}$:

$$\xi\boldsymbol{\Delta}e = \boldsymbol{\Xi}e,$$

where:

$$\boldsymbol{\Xi} \equiv \begin{bmatrix} \boldsymbol{\Gamma} & \boldsymbol{\Theta} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}, \qquad \boldsymbol{\Delta} \equiv \begin{bmatrix} \boldsymbol{\Psi} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}.$$

If e_h is a generalized eigenvector and ξ_h is a generalized eigenvalue of $\boldsymbol{\Xi}$ with respect to $\boldsymbol{\Delta}$, then e_h can be written for some $h \in \Re^N$ as

$$e_h = \begin{bmatrix} \xi_h \bar{e}_h \\ \bar{e}_h \end{bmatrix}.$$

Assuming that the transition matrix has distinct eigenvalues, which we verify empirically, there are $2N$ linearly independent generalized eigenvectors $(e_1, \ldots, e_{2N})$ and corresponding stable eigenvalues $(\xi_1, \ldots, \xi_{2N})$, and the transition matrix $(\boldsymbol{P})$ is given by:

$$\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{-\boldsymbol{1}},$$

where $\mathbf{\Lambda}$ is the diagonal matrix of the $2N$ eigenvalues and $\boldsymbol{U}$ is the matrix stacking the corresponding $2N$ eigenvectors $\{\bar{e}_h\}$. The impact matrix ($\boldsymbol{R}$) in the second condition (I.20) can be recovered using:

$$\boldsymbol{R} = (\mathbf{\Psi}\boldsymbol{P} + \mathbf{\Psi} - \mathbf{\Gamma})^{-1}\,\mathbf{\Pi},$$

and our conjecture (I.17) is satisfied. □

I.4 Proof of Proposition 4 (Spectral Analysis)

Proof. The proposition follows from the eigendecomposition of the transition matrix: $\boldsymbol{P} \equiv \boldsymbol{U}\mathbf{\Lambda}\boldsymbol{V}$, where $\boldsymbol{V} = \boldsymbol{U}^{-1}$, which implies $P^s = \sum_{h=1}^{2N} \lambda_h^s u_h v_h'$, and hence:

$$\widetilde{a}_t \;=\; \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{s=0}^{t-1}\left(\sum_{h=1}^{2N} \lambda_h^s u_h v_h'\right)\boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{h=1}^{2N}\left(\sum_{s=0}^{t-1}\lambda_h^s\right) u_h v_h' \boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{h=1}^{2N}\frac{1-\lambda_h^t}{1-\lambda_h} u_h v_h' \boldsymbol{R}\widetilde{\boldsymbol{f}}.$$

To decompose the impact of any observed shock $\widetilde{\boldsymbol{f}}$ as a linear combination $\boldsymbol{\varrho}$ of the impact of the eigen-shocks $\left\{\widetilde{\boldsymbol{f}}_{(h)}\right\}$, let $\boldsymbol{F}$ denote the matrix whose h-th column is the h-th eigen-shock. Then $\boldsymbol{RF}\boldsymbol{\varrho} = \boldsymbol{R}\widetilde{\boldsymbol{f}} \iff \boldsymbol{\varrho} = \left((\boldsymbol{RF})'(\boldsymbol{RF})\right)^{-1}(\boldsymbol{RF})\,\widetilde{\boldsymbol{f}}$, which implies that $\boldsymbol{\varrho}$ can be recovered as the coefficients from a regression of $\boldsymbol{R}\widetilde{\boldsymbol{f}}$ on the impact of the eigen-shocks. □

I.5 Proof of Proposition 5 (Speed of Convergence)

Proof. If the initial impact of the shock to fundamentals on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) coincides with a real eigenvector ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$), we can rewrite equation (41) in Proposition 4 in the paper as follows:

$$\widetilde{\boldsymbol{a}}_t = \sum_{h=2}^{2N}\left(\frac{\lambda_h^t}{1-\lambda_h}\right)\boldsymbol{u}_h\boldsymbol{v}_h'\boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{j=2}^{2N}\frac{1-\lambda_j^t}{1-\lambda_j}\boldsymbol{u}_j\boldsymbol{v}_j'\boldsymbol{u}_h = \frac{1-\lambda_h^t}{1-\lambda_h}\boldsymbol{u}_h,$$

where we have used $\boldsymbol{v}_i'\boldsymbol{u}_h = 0$ for $i \neq h$ and $\boldsymbol{v}_i'\boldsymbol{u}_h = 1$ for $i = h$. Taking differences between periods $t+1$ and t, we have:

$$\widetilde{\boldsymbol{a}}_{t+1} - \widetilde{\boldsymbol{a}}_t = \frac{1-\lambda_h^{t+1}}{1-\lambda_h}\boldsymbol{u}_h - \frac{1-\lambda_h^t}{1-\lambda_h}\boldsymbol{u}_h,$$

which simplifies to: $(1-\lambda_h)\left(\widetilde{\boldsymbol{a}}_{t+1} - \widetilde{\boldsymbol{a}}_t\right) = (1-\lambda_h)\,\lambda_h^t\boldsymbol{u}_h$. Therefore: $\left(\widetilde{\boldsymbol{a}}_{t+1} - \widetilde{\boldsymbol{a}}_t\right) = \lambda_h^t\boldsymbol{u}_h$. Noting that $\widetilde{\boldsymbol{a}}_t = \ln \boldsymbol{a}_t - \ln \boldsymbol{a}^*_{\text{initial}}$, we have: $\ln \boldsymbol{a}_{t+1} - \ln \boldsymbol{a}_t = \lambda_h^t\boldsymbol{u}_h$. This implies exponential convergence to steady state, such that for each location i: $\frac{a_{it+1}}{a_{it}} = \exp\left(\lambda_h^t u_{ih}\right)$. We measure the speed of convergence to steady state using the conventional measure of the half-life. In particular, we define the half-life of a shock $\widetilde{\boldsymbol{f}}$ for the i-th state variable as the time it takes for that state

variable to converge half of the way to steady state:

$$\arg\max_t \frac{|\widetilde{a}_{it} - \widetilde{a}_{i\infty}|}{\max_s |\widetilde{a}_{is} - \widetilde{a}_{i\infty}|} \geq \frac{1}{2}, \tag{I.21}$$

where $\widetilde{a}_{i\infty} = a^*_{i,\text{new}} - a^*_{i,\text{initial}}$. Using this definition, we can solve for the half-life as

$$\frac{\frac{1-\lambda_h^t}{1-\lambda_h}u_h}{\frac{1}{1-\lambda_h}u_h} = \frac{1}{2}, \quad \Rightarrow \quad \lambda_h^t = \frac{1}{2}, \quad \Rightarrow \quad \ln\frac{1}{2} = t\ln\lambda_h, \quad \Rightarrow \quad t = -\frac{\ln 2}{\ln\lambda_h}.$$

Imposing the requirement that t is an integer, we obtain: $t = -\left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil$, for all state variables $i = 2, \cdots, 2N$, where $\lceil \cdot \rceil$ is the ceiling function. □

J Goods and Capital Market Integration and Convergence

We begin by considering the special case of the model with a separation between (i) workers, who earn wage income and live hand to mouth, and (ii) capitalists, who have log utility and make forward-looking consumption-saving decisions. We later generalize our analysis to a representative agent and CRRA preferences. We assume a common labor share across countries ($\mu_i = \mu$) throughout this section of the Online Appendix.

J.1 Worker-Capitalists with Log Utility

In this special case of a separation between workers and capitalists with log utility, the solution to capitalists' optimal consumption-saving decision is given by equation (C.11) in Subsection C.2, as reproduced below:

$$a_{nt+1} = \beta\left[v_{nt}/p_{nt} + (1-\delta)\right] a_{nt}. \tag{J.1}$$

Dividing both sides of this wealth accumulation equation by a_{nt}, we obtain:

$$\frac{a_{nt+1}}{a_{nt}} = \beta v_{nt}/p_{nt} + \beta\left(1-\delta\right).$$

Now recall from Subsection F that the steady-state real return is given by:

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - (1-\delta) = \frac{1-\beta\left(1-\delta\right)}{\beta}.$$

Therefore, we can rewrite the wealth accumulation equation above as

$$\frac{a_{nt+1}/a_n^*}{a_{nt}/a_n^*} = \beta \frac{1-\beta(1-\delta)}{\beta} \frac{v_{nt}/p_{nt}}{v_n^*/p_n^*} + \beta(1-\delta)$$

$$\frac{a_{nt+1}/a_n^*}{a_{nt}/a_n^*} = (1-\beta(1-\delta)) \frac{v_{nt}/p_{nt}}{v_n^*/p_n^*} + \beta(1-\delta),$$

which can be further written as

$$\frac{a_{nt+1}/a_n^*}{a_{nt}/a_n^*} - 1 = [1-\beta(1-\delta)] \left[\frac{v_{nt}/p_{nt}}{v_n^*/p_n^*} - 1 \right].$$

Now note that:

$$\frac{x_{it}}{x_i^*} - 1 \simeq \log\left(\frac{x_{it}}{x_i^*}\right).$$

Therefore, we can further rewrite the wealth accumulation equation as

$$\log\left(\frac{a_{nt+1}/a_n^*}{a_{nt}/a_n^*}\right) = [1-\beta(1-\delta)] \log\left(\frac{v_{nt}/p_{nt}}{v_n^*/p_n^*}\right),$$

and hence:

$$\widetilde{a}_{nt+1} - \widetilde{a}_{nt} = (1-\beta+\beta\delta)(\widetilde{v}_{nt} - \widetilde{p}_{nt}), \tag{J.2}$$

which corresponds to equation (42) in the paper.

A conventional measure of the speed of convergence to steady state is the ordinary least squares (OLS) regression slope coefficient of the log change in the deviations of the wealth state variables from steady state on the log initial level of the deviation of the wealth state variables from steady state. From the properties of OLS, this regression slope coefficient is given by: $\text{Cov}(\widetilde{a}_{nt+1} - \widetilde{a}_{nt}, \widetilde{a}_{nt}) / \text{Var}(\widetilde{a}_{nt})$.

Using equation (J.2), this regression slope coefficient capturing the speed of convergence depends on the covariance between the log deviation of the real return from steady state and the log deviation of the initial level of wealth from steady state:

$$\frac{\text{Cov}(\widetilde{a}_{nt+1} - \widetilde{a}_{nt}, \widetilde{a}_{nt})}{\text{Var}(\widetilde{a}_{nt})} = [1-\beta(1-\delta)] \frac{\text{Cov}(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt})}{\text{Var}(\widetilde{a}_{nt})}. \tag{J.3}$$

Recall that the nominal return (v_{nt}) is given by:

$$v_{nt} = \gamma \left[\sum_{h=1}^{N} (\eta_{ht} r_{ht} / \kappa_{nht})^{\epsilon} \right]^{\frac{1}{\epsilon}}. \tag{J.4}$$

Note also that cost minimization with our Cobb-Douglas production technology implies:

$$p_{nnt}\frac{(1-\mu)}{z_{nt}}\left(\frac{\ell_n}{\mu k_{nt}}\right)^{\mu}\left(\frac{1}{1-\mu}\right)^{1-\mu}=r_{nt}.$$

Taking log deviations from steady state, we obtain:

$$\widetilde{p}_{nnt}-\mu\widetilde{k}_{nt}=\widetilde{r}_{nt}. \tag{J.5}$$

J.2 Proof of Proposition 6

We now generalize our analysis to a representative agent and CRRA preferences.

Proposition. ***Goods and Capital Market Integration (Proposition 6 in the paper).*** *The speed of convergence to steady state is* ***faster*** *than in the CNGM with* ***either*** *(i) frictionless trade and capital autarky* ***or*** *(ii) trade autarky and perfect capital markets. This speed of convergence is* ***slower*** *than in the CNGM with (iii)* ***both*** *frictionless trade and perfect capital markets.*

Proof. We now characterize the speed of convergence to steady state for four limiting cases of goods and capital market integration, assuming steady-state levels of fundamentals. Recall $\boldsymbol{q}$ is the vector of labor income across countries, and $\boldsymbol{Q}\equiv\frac{1}{\mu}\mathbf{1}\boldsymbol{q}'$, where each row of $\boldsymbol{Q}$ is equal to the vector of GDP across countries. We consider the following four limiting cases:

1. Trade Autarky and Capital Autarky (CNGM): $\boldsymbol{T}=\boldsymbol{S}=\boldsymbol{B}=\boldsymbol{X}=\boldsymbol{I}$.
2. Frictionless Trade and Capital Autarky: $\boldsymbol{T}=\boldsymbol{S}=\boldsymbol{Q}$, $\boldsymbol{B}=\boldsymbol{X}=\boldsymbol{I}$.
3. Perfect capital markets and Trade Autarky: $\boldsymbol{B}=\boldsymbol{X}=\boldsymbol{Q}$, $\boldsymbol{T}=\boldsymbol{S}=\boldsymbol{I}$.
4. Frictionless Trade and Perfect capital markets: $\boldsymbol{T}=\boldsymbol{S}=\boldsymbol{B}=\boldsymbol{X}=\boldsymbol{Q}$.

In all of these cases, capital income in each country $\boldsymbol{\zeta}$ is proportional to labor income $\boldsymbol{q}$ ($\boldsymbol{\zeta}=\frac{1-\mu}{\mu}\boldsymbol{q}$), the capital income share ($\chi_n\equiv\frac{v_n a_n}{w_n\ell_n+v_n a_n}$) is $\chi=1-\mu$, and $TS=S$, $XB=B$. Let $\boldsymbol{L}^v$, $\boldsymbol{L}^w$, $\boldsymbol{L}^r$, $\boldsymbol{L}^p$, be defined such that $\widetilde{\boldsymbol{v}}_t=\boldsymbol{L}^v\widetilde{\boldsymbol{a}}_t$, $\widetilde{\boldsymbol{w}}_t=\boldsymbol{L}^w\widetilde{\boldsymbol{a}}_t$, $\widetilde{\boldsymbol{r}}_t=\boldsymbol{L}^r\widetilde{\boldsymbol{a}}_t$, $\widetilde{\boldsymbol{p}}_t=\boldsymbol{L}^p\widetilde{\boldsymbol{a}}_t$. From our linearization of the general equilibrium conditions of the model in Section I.2 of this Online Appendix, we have derived that:

$$\boldsymbol{L}^r=-\left(\boldsymbol{I}-\boldsymbol{Q}\boldsymbol{B}\right)\left(\boldsymbol{A}_1\left(\boldsymbol{I}-\boldsymbol{Q}\right)\boldsymbol{B}_2+\boldsymbol{B}_1\right)^{-1}\left(\boldsymbol{A}_1\left(\boldsymbol{I}-\boldsymbol{Q}\right)\boldsymbol{X}+\left(1-\mu\right)\boldsymbol{S}\right)-\boldsymbol{Q}, \tag{J.6}$$

$$\boldsymbol{L}^w \;=\; \left(\boldsymbol{I}-\boldsymbol{Q}\right)\left(\boldsymbol{X}+\boldsymbol{B}_2\boldsymbol{L}^r\right), \tag{J.7}$$

$$\boldsymbol{A}_1 = \mu\left(\theta+1\right)\boldsymbol{S} - \boldsymbol{I}\left(1+\theta\mu\right),$$

$$\boldsymbol{B}_1 = \left(\left(1-\mu\right)\boldsymbol{S}\boldsymbol{B} + \theta\left(\boldsymbol{S}-\boldsymbol{I}\right)\left(1-\mu\right)\right),$$

$$\boldsymbol{B}_2 = \left(\left(\epsilon-1\right)\left(\boldsymbol{I}-\boldsymbol{B}\right)+\boldsymbol{I}\right).$$

$$\boldsymbol{L}^p = \mu\boldsymbol{S}\boldsymbol{L}^w + \left(1-\mu\right)\boldsymbol{S}\boldsymbol{L}^r, \tag{J.8}$$

$$\boldsymbol{L}^v = \boldsymbol{B}\boldsymbol{L}^r. \tag{J.9}$$

We also know that the share of capital wealth in steady state ξ_n follows

$$\xi_n \equiv \frac{a_n}{a_n+h_n} = \frac{\left(1-\beta\right)\chi}{\left(1-\beta\right)\chi + \left(1-\beta+\delta\beta\right)\left(1-\chi\right)} = \frac{\left(1-\beta\right)\left(1-\mu\right)}{1-\beta+\mu\delta\beta}.$$

From our derivations in Section I.2 of this Online Appendix, the dynamic bloc of the model follows

$$\begin{aligned}
&\boldsymbol{D}\left(\boldsymbol{\xi}\right)\left(\widetilde{\boldsymbol{a}}_{t+1} - \beta\widetilde{\boldsymbol{a}}_{t+2}\right)\\
=\;&\boldsymbol{D}\left(\boldsymbol{\xi}\right)\left(\widetilde{\boldsymbol{a}}_{t} - \beta\widetilde{\boldsymbol{a}}_{t+1}\right)\\
&+\left(1-\beta\right)\psi\widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\\
&+\boldsymbol{D}\left(\boldsymbol{\xi}\right)\left(\widetilde{\boldsymbol{\mathcal{R}}}_{t} - \widetilde{\boldsymbol{\mathcal{R}}}_{t+1}\right)\\
&+\left(\boldsymbol{I}-\boldsymbol{D}\left(\boldsymbol{\xi}\right)\right)\left(1-\beta\right)\left(\left(\widetilde{\boldsymbol{w}}_t - \widetilde{\boldsymbol{w}}_{t+1}\right) - \left(\widetilde{\boldsymbol{p}}_t - \widetilde{\boldsymbol{p}}_{t+1}\right)\right)
\end{aligned}$$

$$\widetilde{\boldsymbol{\mathcal{R}}}_t = \left(1-\beta+\beta\delta\right)\left(BL^r - L^p\right)\widetilde{\boldsymbol{a}}_t$$

$$\begin{aligned}
&\left(1-\mu\right)P\left(I-\beta P\right)a\\
=\;&\left(1-\mu\right)\left(I-\beta P\right)a\\
&+\left(1-\beta+\mu\delta\beta\right)\psi\left(1-\beta+\beta\delta\right)\left(BL^r-L^p\right)P\widetilde{\boldsymbol{a}}\\
&+\left(1-\mu\right)\left(1-\beta+\beta\delta\right)\left(BL^r-L^p\right)\left(I-P\right)\widetilde{\boldsymbol{a}}_t\\
&+\mu\left(1-\beta+\beta\delta\right)\left(L^w-L^p\right)\left(I-P\right)a
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\left(1-\mu\right)\boldsymbol{P}\left(\boldsymbol{I}-\beta\boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t \;=\;&\left(1-\mu\right)\left(\boldsymbol{I}-\beta\boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t\\
&+\left(1-\beta+\mu\delta\beta\right)\psi\left(1-\beta+\beta\delta\right)\left(\boldsymbol{L}^v-\boldsymbol{L}^p\right)\boldsymbol{P}\widetilde{\boldsymbol{a}}_t\\
&+\left(1-\mu\right)\left(1-\beta+\beta\delta\right)\left(\boldsymbol{L}^v-\boldsymbol{L}^p\right)\left(\boldsymbol{I}-\boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t\\
&+\mu\left(1-\beta+\beta\delta\right)\left(\boldsymbol{L}^w-\boldsymbol{L}^p\right)\left(\boldsymbol{I}-\boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t.
\end{aligned} \tag{J.10}$$

Note that in all four limiting cases, capital income must equal to capital expenditure, thus $\widetilde{r}_{it} + \widetilde{k}_{it} = \widetilde{a}_{it} + \widetilde{v}_{it}$. Also note that for each country i, the total payments to labor is always proportional to capital payments, implying $\widetilde{w}_{it} = \widetilde{r}_{it} + \widetilde{k}_{it}$. These facts imply that we can write the deviation of real wages from steady state as

$$\tilde{w}_{it} - \tilde{p}_{it} = \widetilde{r}_{it} + \widetilde{k}_{it} - \tilde{p}_{it} = \widetilde{a}_{it} + \widetilde{v}_{it} - \widetilde{p}_{it}$$

thereby implying $\boldsymbol{L}^w - \boldsymbol{L}^p = \boldsymbol{I} + \boldsymbol{L}^v - \boldsymbol{L}^p$. Equation (J.10) can be written as

$$\begin{aligned} \mathbf{0} &= \left[(1-\mu)\left(\boldsymbol{I} - \beta\boldsymbol{P}\right) + \mu\left(1 - \beta + \beta\delta\right)\boldsymbol{I}\right]\left(\boldsymbol{I} - \boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t \\ &\quad + \left(1 - \beta + \mu\delta\beta\right)\psi\left(1 - \beta + \beta\delta\right)\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)\boldsymbol{P}\widetilde{\boldsymbol{a}}_t \\ &\quad + \left(1 - \beta + \beta\delta\right)\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)\left(\boldsymbol{I} - \boldsymbol{P}\right)\widetilde{\boldsymbol{a}}_t \end{aligned}$$

Recall $\widetilde{\boldsymbol{v}}_t - \widetilde{\boldsymbol{p}}_t = \left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)\widetilde{\boldsymbol{a}}_t$; hence, the object $\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)$, which determines how variations in capital wealth $\widetilde{\boldsymbol{a}}_t$ translates into variations in the real return of capital wealth, is key in pinning down the rate of convergence.

The four special cases (combination of autarky and frictionless trade or capital flow) differ in the object $\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)$. In what follows, we show that in all four cases, $\boldsymbol{L}^v - \boldsymbol{L}^p$ takes the form:

$$\boldsymbol{L}^v - \boldsymbol{L}^p = \alpha_1\boldsymbol{I} + \alpha_2\boldsymbol{Q},$$

for some scalars α_1 and α_2. The constant vector $\mathbf{1}$ is an eigenvector of the transition matrix $\boldsymbol{P}$. Noting that $\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)\mathbf{1} = \alpha_1\boldsymbol{I}\mathbf{1} + \alpha_2\boldsymbol{Q}\mathbf{1} = \left(\alpha_1 + \alpha_2\right)\mathbf{1}$, the associated rate of convergence (the eigenvalue) is the solution λ^{CNGM} to the quadratic equation

$$\begin{aligned} 0 &= \left[(1-\mu)\left(1 - \beta\lambda^{CNGM}\right) + \mu\left(1 - \beta + \beta\delta\right)\right]\left(1 - \lambda^{CNGM}\right) \\ &\quad + \left(1 - \beta + \mu\delta\beta\right)\psi\left(1 - \beta + \beta\delta\right)\left(\alpha_1 + \alpha_2\right)\lambda^{CNGM} \\ &\quad + \left(1 - \beta + \beta\delta\right)\left(\alpha_1 + \alpha_2\right)\left(1 - \lambda^{CNGM}\right) \end{aligned}$$

and it captures the rate of convergence in the CNGM. To derive the eigenvalue λ associated with the rate of convergence when the world total capital wealth is at its steady-state level (but each country's capital wealth may deviate from steady state), note that for any state variable $\widetilde{\boldsymbol{a}}$, $\left(\boldsymbol{I} - \boldsymbol{Q}\right)\widetilde{\boldsymbol{a}}$ captures the deviation from the ($\frac{1}{\mu}\boldsymbol{q}$-weighted) world average capital wealth. Also note that $\boldsymbol{Q}\left(\boldsymbol{I} - \boldsymbol{Q}\right)$ is zero; hence

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}} = \left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)\left(\boldsymbol{I} - \boldsymbol{Q}\right)\widetilde{\boldsymbol{a}}_t = \left(\alpha_1\boldsymbol{I} + \alpha_2\boldsymbol{Q}\right)\left(\boldsymbol{I} - \boldsymbol{Q}\right)\widetilde{\boldsymbol{a}}_t = \alpha_1\widetilde{\boldsymbol{a}}_t.$$

The eigenvalue is thus the solution λ to the quadratic equation

$$\begin{aligned} 0 &= [(1-\mu)(1-\beta\lambda)+\mu(1-\beta+\beta\delta)](1-\lambda) \\ &\quad +(1-\beta+\mu\delta\beta)\psi(1-\beta+\beta\delta)\alpha_1\lambda \\ &\quad +(1-\beta+\beta\delta)\alpha_1(1-\lambda) \end{aligned}$$

We now derive the values of α_1 and α_2 in each of the 4 cases. As we show below, in the case of both trade and capital autarky, $\alpha_2 = 0$, thus λ^{CNGM} coincides with λ and is the only eigenvalue, with algebraic multiplicity N. In the other three cases, $\alpha_2 \neq 0$, and the eigenvalue λ is distinct from λ^{CNGM} and has an algebraic multiplicity of $(N-1)$. We also show that $\alpha_1 < 0$. The quadratic equation implies that λ decreases in the absolute value of α_1 ($\mathrm{d}\lambda/\,\mathrm{d}\alpha_1 > 0$); hence, a more negative α_1 implies a higher rate of convergence when the total capital wealth in the world is initially in steady state.

Case 1. Trade Autarky and Capital Autarky (CNGM) Under capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$) and trade autarky ($\tau_{nit} \to \infty$ for $n \neq i$), we have:

$$\boldsymbol{T} = \boldsymbol{S} = \boldsymbol{B} = \boldsymbol{X} = \boldsymbol{I}.$$

Using these results in (J.6)-(J.9), we get that

$$\boldsymbol{L}^v - \boldsymbol{L}^p = -\mu \boldsymbol{I}.$$

Hence $\alpha_1 = -\mu$, $\alpha_2 = 0$.

Case 2. Frictionless Trade and Capital Autarky Under capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$) and frictionless trade ($\tau_{nit} = 1$ for all n, i), we have:

$$\begin{aligned} \widetilde{k}_{nt} &= \widetilde{a}_{nt}, \\ \widetilde{v}_{nt} &= \widetilde{r}_{nt}, \\ \widetilde{p}_{nt} &= \widetilde{p}_t, \end{aligned}$$

$$\boldsymbol{T} = \boldsymbol{S} = \boldsymbol{Q}, \qquad \boldsymbol{B} = \boldsymbol{X} = \boldsymbol{I}.$$

Using these results in (J.6)-(J.9), we have:

$$\begin{aligned}\boldsymbol{L}^r &= -\frac{1+\theta\mu}{1+\theta}(\boldsymbol{I}-\boldsymbol{Q})-\boldsymbol{Q},\\ &= -\frac{1+\theta\mu}{1+\theta}\boldsymbol{I}-\frac{\theta(1-\mu)}{1+\theta}\boldsymbol{Q}.\end{aligned}$$

$$\begin{aligned}\boldsymbol{L}^w &= (\boldsymbol{I}-\boldsymbol{Q})(\boldsymbol{I}+\boldsymbol{L}^r),\\ &= \frac{\theta(1-\mu)}{1+\theta}(\boldsymbol{I}-\boldsymbol{Q}).\end{aligned}$$

$$\begin{aligned}\boldsymbol{L}^p &= \mu\boldsymbol{Q}\boldsymbol{L}^w+(1-\mu)\boldsymbol{Q}\boldsymbol{L}^r,\\ &= -(1-\mu)\boldsymbol{Q}.\end{aligned}$$

$$\begin{aligned}\boldsymbol{L}^v &= \boldsymbol{B}\boldsymbol{L}^r,\\ &= -\frac{1+\theta\mu}{1+\theta}\boldsymbol{I}-\frac{\theta(1-\mu)}{1+\theta}\boldsymbol{Q}.\end{aligned}$$

$$\boldsymbol{L}^v-\boldsymbol{L}^p = -\frac{1+\theta\mu}{1+\theta}\boldsymbol{I}+\frac{1-\mu}{1+\theta}\boldsymbol{Q},$$

where $\theta=\sigma-1$. Hence, $\alpha_1=-\left(\mu+\frac{1-\mu}{\sigma}\right)$, $\alpha_2=\frac{1-\mu}{\sigma}$.

Case 3. Trade Autarky and Perfect capital markets Under trade autarky ($\tau_{nit}\to\infty$ for $n\neq i$) and perfect capital markets ($\kappa_{nit}=1$ for all n,i), we have

$$\begin{aligned}\widetilde{p}_{nt} &= \widetilde{p}_{nnt},\\ \widetilde{v}_{nt} &= \widetilde{v}_t,\end{aligned}$$

$$\boldsymbol{B}=\boldsymbol{X}=\frac{1}{\mu}\mathbf{1}\boldsymbol{q}', \qquad \boldsymbol{T}=\boldsymbol{S}=\boldsymbol{I}.$$

Using these results in (J.6)-(J.9), we have:

$$\boldsymbol{L}^r = \frac{1}{\epsilon}\boldsymbol{I}-\frac{\epsilon+1}{\epsilon}\boldsymbol{Q}.$$

$$\boldsymbol{L}^w = \boldsymbol{I}-\boldsymbol{Q}.$$

$$\begin{aligned}\boldsymbol{L}^p &= \mu\left(\boldsymbol{I}-\boldsymbol{Q}\right)+\left(1-\mu\right)\left(\frac{1}{\epsilon}\boldsymbol{I}-\frac{\epsilon+1}{\epsilon}\boldsymbol{Q}\right),\\ &= \left(\mu+\frac{1-\mu}{\epsilon}\right)\boldsymbol{I}-\left(1+\frac{1-\mu}{\epsilon}\right)\boldsymbol{Q}.\end{aligned}$$

$$\boldsymbol{L}^v=\boldsymbol{Q}\boldsymbol{L}^r=-\boldsymbol{Q}.$$

$$\boldsymbol{L}^v-\boldsymbol{L}^p=-\left(\mu+\frac{1-\mu}{\epsilon}\right)\boldsymbol{I}+\frac{1-\mu}{\epsilon}\boldsymbol{Q}.$$

Hence, $\alpha_1=-\left(\mu+\frac{1-\mu}{\epsilon}\right)$, $\alpha_2=\frac{1-\mu}{\epsilon}$.

Case 4. Frictionless Trade and Perfect capital markets Under frictionless trade ($\tau_{nit}=1$ for all n,i) and perfect capital markets ($\kappa_{nit}=1$ for all n,i), we have:

$$\widetilde{p}_{nt}=\widetilde{p}_t,$$
$$\widetilde{v}_{nt}=\widetilde{v}_t,$$

$$\boldsymbol{T}=\boldsymbol{S}=\boldsymbol{B}=\boldsymbol{X}=\boldsymbol{Q}.$$

Using these results in (J.6)-(J.9), we have:

$$\boldsymbol{L}^r=-\boldsymbol{Q},$$

$$\boldsymbol{L}^w=\mathbf{1}0',$$

$$\boldsymbol{L}^p=-\left(1-\mu\right)\boldsymbol{Q},$$

$$\boldsymbol{L}^v=-\boldsymbol{Q}.$$

Hence $\alpha_1=0$, $\alpha_2=-\mu$.

Finally, to see that the solution to the quadratic equation decreases in the absolute value of α_1 ($\mathrm{d}\lambda/\,\mathrm{d}\alpha_1>0$), let $\gamma\equiv-\frac{\lambda}{1-\lambda}\psi\left(1-\beta+\beta\delta\right)\alpha_1$. The quadratic equation can be written as

$$\begin{aligned}0 &= \left[\left(1-\mu\right)\left(1-\beta\lambda\right)+\mu\left(1-\beta+\beta\delta\right)\right]\\ &\quad-\left(1-\beta+\mu\delta\beta\right)\gamma+\left(1-\beta+\beta\delta\right)\alpha_1\end{aligned}$$

Totally differentiating, we note

$$\left(1-\lambda\right)\,\mathrm{d}\gamma\equiv\gamma\,\mathrm{d}\lambda-\lambda\psi\left(1-\beta+\beta\delta\right)\,\mathrm{d}\alpha_1-\psi\left(1-\beta+\beta\delta\right)\alpha_1\,\mathrm{d}\lambda$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\alpha_1} = \frac{(1-\beta+\beta\delta)\left(1-\lambda+(1-\beta+\mu\delta\beta)\lambda\psi\right)}{\left[(1-\lambda)\beta(1-\mu)+(1-\beta+\mu\delta\beta)\left[\gamma-\psi(1-\beta+\beta\delta)\alpha_1\right]\right]}$$

Because $\alpha_1 \leq 0$ in all four special cases, both the numerator and the denominator are positive. Hence $\frac{\mathrm{d}\lambda}{\mathrm{d}\alpha_1} > 0$, as desired.

J.2.1 Marginal Changes in Goods and Capital Market Integration and the Rate of Convergence

We now characterize how a marginal change in trade and capital frictions affect the speed of convergence to steady state, assuming that the initial total stock of world capital wealth is at its steady-state level, but the initial distribution of capital wealth across countries may not be at its steady-state value. Following the main text Section 3.6, we focus on the special case of the model with a separation between (i) workers, who earn wage income and live hand to mouth, and (ii) capitalists, who have log utility and make forward-looking consumption-saving decisions. In this case, the slope coefficient from a regression of the log real returns on log initial levels of capital wealth is a sufficient statistic for the rate of convergence:

$$\frac{\mathrm{Cov}\left(\widetilde{a}_{nt+1}-\widetilde{a}_{nt},\widetilde{a}_{nt}\right)}{\mathrm{Var}\left(\widetilde{a}_{nt}\right)} = (1-\beta+\beta\delta)\frac{\mathrm{Cov}\left(\widetilde{v}_{nt}-\widetilde{p}_{nt},\widetilde{a}_{nt}\right)}{\mathrm{Var}\left(\widetilde{a}_{nt}\right)}.$$

To parametrize intermediate degrees of frictions to trade and capital markets, we focus on the case where all countries in the world have identical labor share μ. Recall $\boldsymbol{q}$ is the vector of labor income across countries, and $\boldsymbol{Q} \equiv \frac{1}{\mu}\mathbf{1}\boldsymbol{q}'$, where each row of $\boldsymbol{Q}$ is equal to the vector of GDP across countries. We assume:

1. Trade income and expenditure share matrices follow

$$\boldsymbol{T} = \boldsymbol{S} = \alpha\boldsymbol{I} + (1-\alpha)\boldsymbol{Q}. \tag{J.11}$$

2. Capital portfolio and payment share matrices follow

$$\boldsymbol{B} = \boldsymbol{X} = \phi\boldsymbol{I} + (1-\phi)\boldsymbol{Q}. \tag{J.12}$$

The parameters $\alpha, \phi \in [0,1]$ respectively parametrize the global degree of frictions to trade and capital markets. A value of $\alpha = 1$ captures the case where all countries are in trade autarky, as the trade income and expenditure share are identity matrices. A value of $\alpha = 0$ captures global frictionless trade, where every country has identical expenditure and income shares across trade partners. Likewise, $\phi = 1$ captures global capital autarky, and $\phi = 0$ captures global perfect

capital markets. For any $\alpha, \phi \in [0,1]$, capital income in each country $\boldsymbol{\zeta}$ is proportional to labor income $\boldsymbol{q}$ ($\boldsymbol{\zeta} = \frac{1-\mu}{\mu}\boldsymbol{q}$), the capital income share ($\chi_n \equiv \frac{v_n a_n}{w_n \ell_n + v_n a_n}$) is $\chi = 1 - \mu$.

Given our parametrization, we have

$$\boldsymbol{TS} = \alpha^2 \boldsymbol{I} + \left(1 - \alpha^2\right) \boldsymbol{Q}$$

$$\boldsymbol{BX} = \phi^2 \boldsymbol{I} + \left(1 - \phi^2\right) \boldsymbol{Q}$$

$$\boldsymbol{SX} = \alpha\phi \boldsymbol{I} + (1 - \alpha\phi) \boldsymbol{Q}$$

$$\boldsymbol{QB} = \boldsymbol{QS} = \boldsymbol{Q}$$

Let $\boldsymbol{L}^v$ and $\boldsymbol{L}^p$ be defined such that $\widetilde{\boldsymbol{v}}_t = \boldsymbol{L}^v \widetilde{\boldsymbol{a}}_t$, $\widetilde{\boldsymbol{p}}_t = \boldsymbol{L}^p \widetilde{\boldsymbol{a}}_t$. Define

$$x \equiv \frac{-\left(\phi\left(\mu\alpha - \theta\mu\left(1-\alpha^2\right) - 1\right) + (1-\mu)\,\alpha\right)}{\left((1-\mu)\left(\alpha\phi - \theta\left(1-\alpha^2\right)\right) + \epsilon\left(\mu\alpha - \theta\mu\left(1-\alpha^2\right) - 1\right)\left(1-\phi^2\right) + \phi^2\left(\mu\alpha - \theta\mu\left(1-\alpha^2\right) - 1\right)\right)} \tag{J.13}$$

$$\zeta \equiv \phi x - \alpha\left[\mu\left(\phi + \left(1 + (\epsilon - 1)\left(1 - \phi^2\right)\right) x\right) + x\,(1-\mu)\right] \tag{J.14}$$

From our linearization of the general equilibrium conditions of the model in Section I.2 of this Online Appendix, and substituting using our parametrization of $\boldsymbol{T}$, $\boldsymbol{S}$, $\boldsymbol{B}$, $\boldsymbol{X}$ from the above, we have:

$$\left(\boldsymbol{L}^v - \boldsymbol{L}^p\right)(\boldsymbol{I} - \boldsymbol{Q})\,\widetilde{\boldsymbol{a}} = \zeta\,(\boldsymbol{I} - \boldsymbol{Q})\,\widetilde{\boldsymbol{a}}$$

Following our discussion in Section 3.6 of the paper, this implies that ζ is the regression coefficient of $(\widetilde{\boldsymbol{a}}_{t+1} - \widetilde{\boldsymbol{a}}_t)$ on $\widetilde{\boldsymbol{a}}_t$ when the cross-country distribution of capital wealth deviates from the steady state at time t (while the total world capital wealth is at the steady state).

The following result generalizes the discussion in Section 3.6 of the paper to marginal changes in goods and capital market integration.

Proposition 7. *Consider a world economy with trade and capital flow matrices parametrized according to (J.11) and (J.12), and assume the initial total stock of world capital wealth is at its steady-state level, but the initial distribution of capital wealth across countries may not be at its steady-state value. In the model with a separation between workers and capitalists with log utility, changing the degrees of trade and capital frictions has the following effects on the rate of convergence:*

1. *Under trade autarky ($\alpha = 1$), reducing capital frictions increases the rate of convergence ($\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}\big|_{\alpha=1} \geq 0$);*

2. *Under capital autarky ($\phi = 1$), reducing trade frictions increases the rate of convergence ($\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}\big|_{\phi=1} \geq 0$);*

3. *Under frictionless trade ($\alpha = 0$), reducing capital frictions reduces the rate of convergence ($\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}|_{\alpha=0} \leq 0$);*

4. *Under perfect capital markets ($\phi = 0$), reducing trade frictions reduces the rate of convergence ($\frac{\mathrm{d}\zeta}{\mathrm{d}\alpha}|_{\phi=0} \leq 0$);*

5. *When trade and capital markets have symmetric frictions ($\alpha = \phi$), reducing both frictions reduces the rate of convergence ($\left(\frac{\mathrm{d}\zeta}{\mathrm{d}\alpha} + \frac{\mathrm{d}\zeta}{\mathrm{d}\phi}\right)|_{\alpha=\phi} \leq 0$).*

Proof. These results can be derived by substituting α, ϕ into ζ (equation J.14) and then taking derivatives. Specifically, we have:

1. Under trade autarky ($\alpha = 1$),

$$x|_{\alpha=1} = \frac{1}{(\epsilon-1)\phi + \epsilon}$$

$$\zeta|_{\alpha=1} = -\mu\phi - (1-\phi)\frac{1+\mu(\epsilon-1)(1+\phi)}{(\epsilon-1)\phi+\epsilon}$$

$$\begin{aligned}\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}|_{\alpha=1} &= -\mu + \frac{1+\mu(\epsilon-1)(1+\phi)}{(\epsilon-1)\phi+\epsilon} + (1-\phi)\frac{(\epsilon-1)(1-\mu)}{((\epsilon-1)\phi+\epsilon)^2} \\ &= \frac{(1-\mu)(2\epsilon-1)}{((\epsilon-1)\phi+\epsilon)^2} \\ &\geq 0\end{aligned}$$

2. Under capital autarky ($\phi = 1$):

$$x|_{\phi=1} = -\frac{\theta\mu(1+\alpha)+1}{\theta(1+\alpha)+1}$$

$$\zeta|_{\phi=1} = -\mu\alpha - (1-\alpha)\frac{\theta\mu(1+\alpha)+1}{\theta(1+\alpha)+1}$$

$$\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}|_{\phi=1} = \frac{1-\mu}{\theta(1+\alpha)+1} + (1-\alpha)\frac{\theta(1-\mu)}{(\theta(1+\alpha)+1)^2} \geq 0$$

3. Under frictionless trade ($\alpha = 0$):

$$x|_{\alpha=0} = -\frac{\phi(1+\theta\mu)}{\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu)}$$

$$\zeta|_{\alpha=0} = -\frac{\phi^2(1+\theta\mu)}{\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu)}$$

$$\begin{aligned}
\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}|_{\alpha=0} &= \frac{2\phi(1+\theta\mu)(1-\epsilon)\phi^2(1+\theta\mu)}{(\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu))^2} \\
&\quad -\frac{2\phi(\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu))(1+\theta\mu)}{(\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu))^2} \\
&= -\frac{2\phi(1+\theta\mu)}{(\theta(1-\mu)+\epsilon(1+\theta\mu)(1-\phi^2)+\phi^2(1+\theta\mu))^2} \\
&\quad \times\left[\epsilon(1+\theta\mu)\phi^2+\theta(1-\mu)+\epsilon(1+\theta\mu)\left(1-\phi^2\right)\right] \\
&\leq 0
\end{aligned}$$

4. Under perfect capital markets ($\phi = 0$):

$$x|_{\phi=0} = \frac{(1-\mu)\alpha}{\theta(1-\alpha^2)(1-\mu)-\epsilon(\mu\alpha-\theta\mu(1-\alpha^2)-1)}$$

$$\zeta|_{\phi=0} = -\frac{(1-\mu)[1+\mu(\epsilon-1)]\alpha^2}{\theta(1-\alpha^2)(1-\mu)-\epsilon(\mu\alpha-\theta\mu(1-\alpha^2)-1)}$$

$$\begin{aligned}
\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}|_{\alpha=0} &= -\frac{[2\alpha\theta(1-\mu)+\mu+2\alpha\theta\mu](1-\mu)[1+\mu(\epsilon-1)]\alpha^2}{(\theta(1-\alpha^2)(1-\mu)-\epsilon(\mu\alpha-\theta\mu(1-\alpha^2)-1))^2} \\
&\quad -\frac{2\alpha[\theta(1-\alpha^2)(1-\mu)+\epsilon\theta\mu(1-\alpha^2)+\epsilon(1-\mu\alpha)](1-\mu)[1+\mu(\epsilon-1)]}{(\theta(1-\alpha^2)(1-\mu)-\epsilon(\mu\alpha-\theta\mu(1-\alpha^2)-1))^2} \\
&\leq 0
\end{aligned}$$

5. When trade and capital markets have symmetric frictions ($\alpha = \phi$):

$$
\begin{aligned}
& -Sign\left(\left(\frac{\mathrm{d}\zeta}{\mathrm{d}\alpha}+\frac{\mathrm{d}\zeta}{\mathrm{d}\phi}\right)|_{\alpha=\phi}\right) \\
= \; & Sign(\alpha^3\epsilon\mu+\alpha^3\mu\theta\left(1-\mu\right)+2\alpha^2\epsilon^2\mu\theta\left(1-\mu\right)+2\alpha^2\epsilon^2\left(1-\mu\right)^2 \\
& +2\alpha^2\epsilon\mu^2\theta+2\alpha^2\epsilon\mu\theta\left(\theta-1\right)+4\alpha^2\epsilon\theta\left(1-\mu\right)+2\alpha^2\theta^2\left(1-\mu\right) \\
& +4\alpha\epsilon^2\mu\theta\left(1-\mu\right)+\left(4\alpha-\alpha^3\mu\right)\epsilon^2\left(1-\mu\right)+5\alpha\epsilon\mu^2\theta+3\alpha\epsilon\mu\theta^2 \\
& +\alpha\epsilon\mu\theta\left(\theta-1\right)+4\epsilon\mu^2\theta+2\epsilon\mu\theta^2+\alpha\epsilon\mu\left(1-\mu\right)+\left(8\alpha-\alpha^3\mu\right)\epsilon\theta\left(1-\mu\right) \\
& +\alpha\left(1-\mu\right)\mu\theta+4\left(1-\mu\right)\alpha\theta^2+2\epsilon^2\mu\theta\left(1-\mu\right)+\left(2-\alpha\mu\right)\epsilon^2\left(1-\mu\right) \\
& +4\epsilon\theta\left(1-\mu\right)+2\mu\theta\left(1-\mu\right)+2\theta^2\left(1-\mu\right)+2\epsilon\mu) \\
\geq \; & 0
\end{aligned}
$$

□

K Additional Empirical Evidence

In this section of the Online Appendix, we present additional empirical results than supplement those reported in the paper. Section K.1 provides further evidence that the gravity equation provides a good approximation to observed data on trade and capital holdings, as considered in Section 4.3 of the paper. Section K.2 estimates gravity equations for subcomponents of capital holdings, as discussed in Section 4.3 of the paper. Section K.3 presents impulse responses to productivity shocks for a country that is large relative to global aggregates, complementing those for a small country in Section 4.4 of the paper. Section K.4 examines how the speed of convergence to steady state varies with model parameters, as discussed in Section 4.5 of the paper.

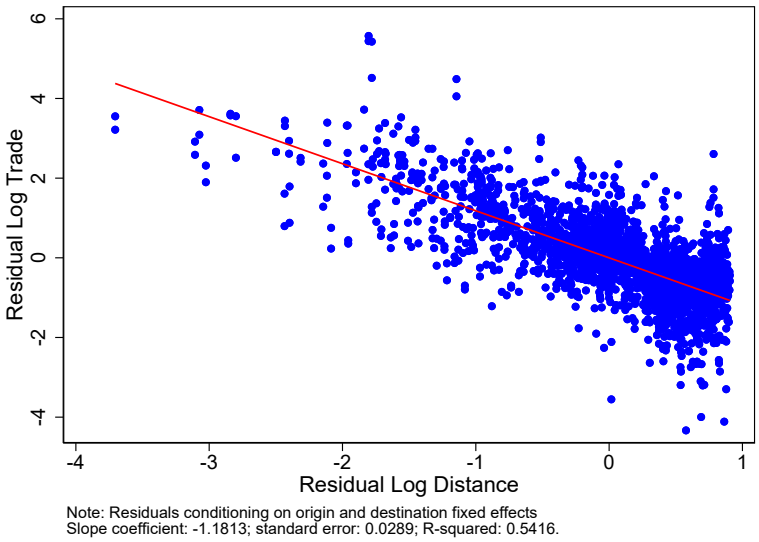
K.1 Gravity in Trade and Capital Holdings

In Table 1 in Section 4.3 of the paper, we report gravity equation estimates for bilateral trade and capital holdings. We show that the gravity equation provides a good approximation for both observed bilateral trade and capital holdings. Although capital holdings are not subject to transportation costs in the way that goods flows are, we find an estimated coefficient on bilateral distance that if anything is larger in absolute value for capital holdings than for trade.

While Table 1 provides evidence on the explanatory power of the gravity equation specification for trade and capital holdings, it does not reveal the relative importance of bilateral distance and the fixed effects for this explanatory power. To separate out the contribution from bilateral distance, we use the Frisch-Waugh-Lovell Theorem. We first run two separate OLS regressions

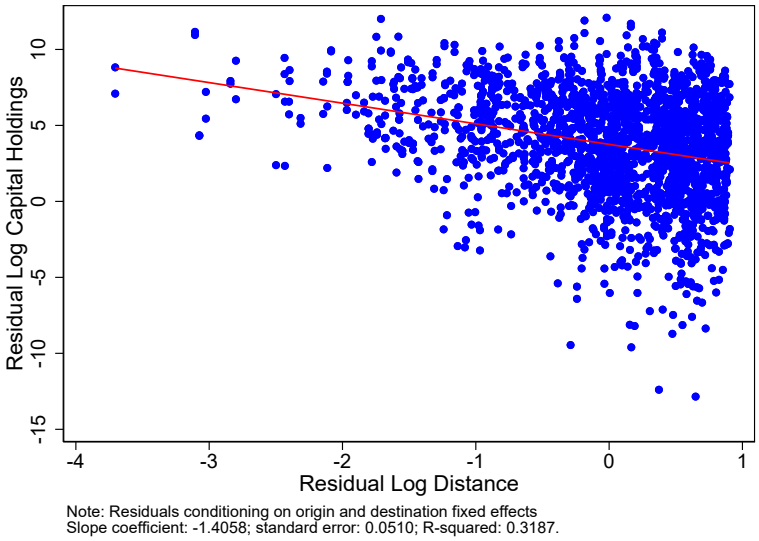
of log values and log distance on origin and destination fixed effects, and generate the two residuals. We next regress these two residuals against one another. In Figures K.1 and K.2, we display these conditional correlations between bilateral log values and log distance, for trade and capital holdings respectively. In both cases, we find negative and highly statistically significant relationships, with a regression R-squared for the conditional correlation of more than 0.25. Therefore, even after removing the origin and destination fixed effects, we find that bilateral distance has a similar explanatory power for capital holdings as for trade.

Figure K.1: Conditional Correlation Between Bilateral Trade and Bilateral Distance



Note: Cross-section of origin and destination countries in 2017; residual log trade and residual log distance are residuals from OLS regressions of log trade and log distance on origin and destination fixed effects, respectively; Blue dots correspond to origin-destination pairs; Red solid line shows the linear fit between these two residuals.

Figure K.2: Conditional Correlation Between Bilateral Capital Holdings and Bilateral Distance



Note: Cross-section of origin and destination countries in 2017; residual log capital holdings and residual log distance are residuals from OLS regressions of log capital holdings and distance on origin and destination fixed effects, respectively; Blue dots correspond to origin-destination pairs; Red solid line shows the linear fit between these two residuals.

K.2 Gravity for Subcomponents of Capital Holdings

To stay as close to the CNGM as possible, we aggregate all components of capital holdings (e.g., debt, equity, portfolio and direct) together in our baseline specification. In this section of the Online Appendix, we report a robustness test in which we estimate separate gravity equations for both overall and portfolio capital holdings, and show that we find similar elasticities of capital holdings with respect to distance in both specifications.

We estimate the same gravity equation specification between countries for a single year as in Section 4.3 of the paper:

$$Y_{ni} = \vartheta_i^O \vartheta_n^D \text{dist}_{ni}^{\delta} u_{ni}, \tag{K.1}$$

where H_{ni} is the capital holdings of investor n in producer i; ϑ_i^O is an origin fixed effect; ϑ_n^D is a destination fixed effect; dist_{ni} is bilateral distance; and u_{ni} is a stochastic error. We report standard errors clustered by origin and destination.

In Column (1) of Table K.1, report the results of taking logs in equation (K.1) and estimating this gravity equation for our baseline measure of capital holdings that includes both direct and portfolio investment using ordinary least squares (OLS) with origin and destination fixed effects (Column (3) in Table 1 in the paper). In Column (2) of Table K.1, we demonstrate the same pattern of results if we estimate this gravity equation in levels using the Poisson Pseudo Maximum Likelihood (PPML) estimator (Column (4) in Table 1 in the paper). In Columns (3) and (4) of Table

K.1, we report our robustness test in which we reestimate these specifications using a measure of capital holdings based only on portfolio investment (excluding direct investment). We find similar distance elasticities for portfolio capital holdings as for overall capital holdings, consistent with our aggregating all types of capital holdings together in our baseline specification.

Table K.1: Gravity Equation Regressions For Subcomponents of Capital Holdings

	(1) Overall	(2) Portfolio	(3) Overall	(4) Portfolio
Log Distance	-1.409	-0.627	-1.242	-0.581
	(0.0466)	(0.0512)	(0.0463)	(0.0511)
Estimation	OLS	PPML	OLS	PPML
Origin and Destination Fixed Effects	Yes	Yes	Yes	Yes
Observations	2042	2070	2019	2070
R-squared	0.821		0.831	
Pseudo R-squared		0.917		0.924

Note: Cross-section of origin and destination countries in 2017; all columns include origin and destination fixed effects (FEs); Columns (1)-(2) show results for our baseline measure of bilateral capital holdings; Columns (3) and (4) reestimate the specifications in Columns (1) and (2) using a measure of capital holdings that includes only portfolio investment (excluding direct investment). Columns (1) and (3) estimated in logs using ordinary least squares (OLS); Columns (2) and (4) estimated using the Poisson Pseudo Maximum Likelihood (PPML) estimator; standard errors two-way clustered by origin and destination.

K.3 Impulse Responses to Productivity Shocks

In Section 4.4 of the paper, we provide evidence on the impulse responses of the model's endogenous variables to productivity shocks in a country that is small relative to global aggregates. In this Section of the Online Appendix, we provide evidence on these impulse responses to productivity shocks for a country that is large relative to global aggregates, in which case the productivity shocks in the large country have non-negligible effects on wealth and capital accumulation in other countries.

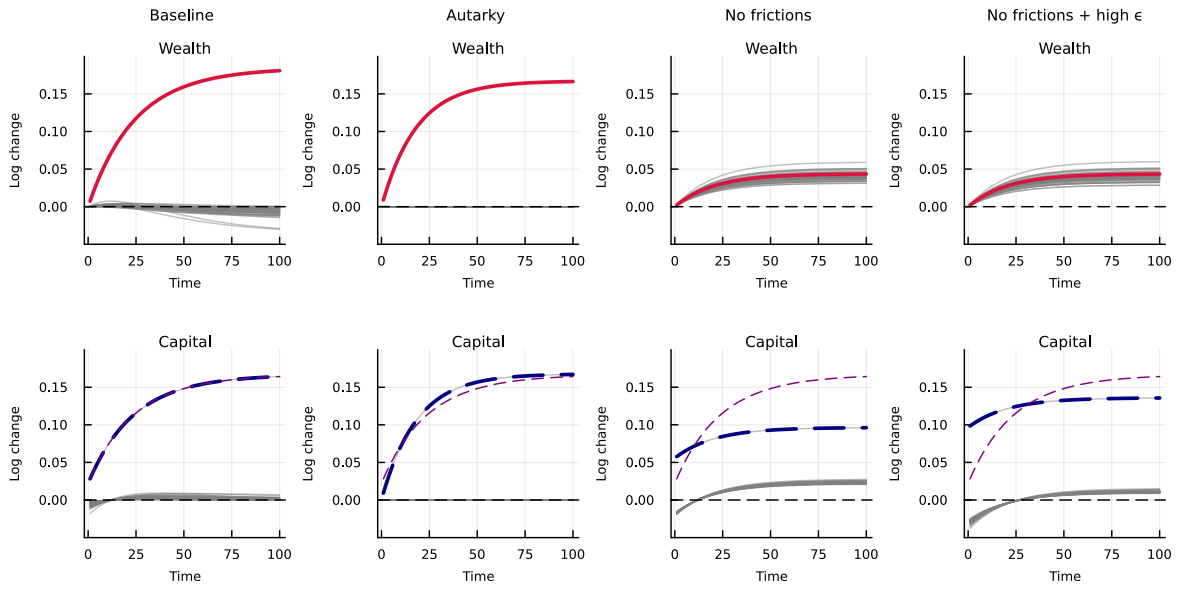
Figure K.3 shows impulse responses for wealth (top row) and capital (bottom row) with respect to a 10 percent productivity shock in a large country (China). Again we display these impulse responses for our baseline model of trade and capital market frictions and imperfect substitutability of goods and capital across countries (far-left panel); the CNGM (middle-left panel); the special case of our model with no goods or capital market frictions and imperfect capital substitutability: $\tau_{ni} = \tau_{nn} = \kappa_{ni} = \kappa_{nn} = 1$ and $\epsilon = 3.15$ (middle-right panel); and the special case of our model with no goods or capital market frictions and perfect capital substitutability: $\tau_{ni} = \tau_{nn} = \kappa_{ni} = \kappa_{nn} = 1$ and $\epsilon \to \infty$ (far-right panel).

The solid red line shows the transition path for the large country's wealth (top row), while the dashed blue line shows the transition path for the large country's capital stock (bottom row). The transition paths for wealth and capital for all other countries are shown by gray lines. For

ease of comparison, we reproduce the transition path for the large country's capital stock in our baseline model (purple dashed line) across each of the panels.

In the CNGM (middle-left panel), the positive productivity shock increases the large country's steady-state levels of wealth and capital. However, with autarky in goods and capital markets, this productivity shock in the large country has no effect on steady-state levels of wealth and capital in other countries. With autarky in capital markets, domestic capital in the large country only can be accumulated through domestic wealth accumulation. Therefore, consumption smoothing implies a gradual accumulation of domestic capital and wealth in the large country along the transition path to steady state, at a rate determined by diminishing marginal returns to capital in the production technology (as controlled by the labor share).

Figure K.3: Large Country Productivity Shock



Note: Impulse responses to a 10 percent productivity shock in a large country (China) for our baseline parameter values; first panel from the left shows impulse responses in our baseline model with trade frictions and imperfect capital markets ($\tau_{ni} > 1$ and $\kappa_{ni} > 1$ for $n \neq i$) and imperfect substitutability ($0 < \sigma < \infty$ and $0 < \epsilon < \infty$); second panel from the left shows impulse responses in the CNGM; third panel from the left shows impulse responses with no trade and capital market frictions ($\tau_{ni} = \kappa_{ni} = 1$) and imperfect substitutability ($0 < \sigma < \infty$ and $0 < \epsilon < \infty$); fourth panel from the left shows impulse responses with no trade and capital market frictions ($\tau_{ni} = \kappa_{ni} = 1$) and imperfect substitutability in goods markets ($0 < \sigma < \infty$) but a perfectly elastic supply of capital ($\epsilon \to \infty$); the red line in the top panel shows impulse responses for wealth in China; the dashed blue line in the bottom panel shows impulse responses for capital in China; the purple dashed line in the bottom panel reproduces the impulse responses for capital in China for our baseline model (first panel from the left) in all other panels for ease of comparison.

In the special case of our framework with no goods and capital market frictions and imperfect capital substitutability (middle-right panel), the positive productivity shock leads to an immediate international capital reallocation, which increases the capital stock in the large country and reduces the capital stock in other countries. With no goods and capital market frictions, the representative agent in each country holds the same global capital portfolio. The increase in the

large country's productivity increases the real return for this global capital portfolio, such that all countries accumulate wealth and capital along the transition path to the new steady state. Although all countries hold the same global capital portfolio, they differ in terms of the relative importance of capital and labor in overall wealth, which generates the observed differences in the impact of the productivity shock on wealth and capital across other countries, both along the transition path and in steady state.

In the special case of our framework with both no goods and capital market frictions and perfect substitutability of capital (far-right panel), we obtain the limiting case of the conventional open-economy neoclassical growth model. Following the positive productivity shock, there is again an immediate international capital reallocation, which is now larger in absolute magnitude because of the greater substitutability of capital. The increase in the large country's productivity again increases the real return for this global capital portfolio, such that all countries accumulate wealth and capital along the transition path to the new steady state. Since countries' differ in terms of the relative importance of capital and labor wealth, the increase in productivity in the large countries has heterogeneous effects on wealth and capital in other countries. With a greater substitutability of capital, the positive productivity shock not only leads to a larger initial capital reallocation but also has a greater impact on the steady-state capital stock in the large country.

In our baseline model with trade and capital market frictions and imperfect substitutability (far-left panel), the predicted impacts of the productivity shock lie in between the extremes of the closed and the open-economy neoclassical growth models. The increase in the large country's productivity leads to an immediate international capital reallocation and affects the real return in other countries. With frictions in goods and capital market, both the initial outflow of capital and the impact on the real return in other countries are heterogeneous. On the one hand, the increase in the large country's productivity makes it more competitive in goods markets, which reduces the demand for other countries' goods through a negative cross-substitution effect, and hence reduces the marginal product of capital and the real return in other countries. On the other hand, the increase in the large country's productivity increases its income, which increases the demand for other countries' through a positive market size effect, and hence reduces the marginal product of capital and the real return in other countries. Depending on the balance between these cross-substitution and market size effects, other countries' can accumulate or decumulate capital and wealth along the transition path to steady state.

Therefore, our framework implies that adjustment to domestic productivity shocks occurs through a combination of the international capital reallocation and domestic capital accumulation. We avoid the extreme predictions of the CNGM (in which all adjustment occurs through domestic wealth accumulation) and the open-economy neoclassical growth model with perfect capital substitutability (in which all adjustment occurs through international reallocations of cap-

ital). Our model provides a natural rationalization for two key features of the data: international capital holdings to take advantage of differences in investment opportunities combined with a strong positive correlation between domestic saving and investment. When a country is large relative to global aggregates shocks to its productivity are transmitted through goods and capital markets to affect capital and wealth accumulation along the transition path in other countries and the steady-state stocks of capital and wealth in other countries.

K.4 Comparative Statics of Speed of Convergence

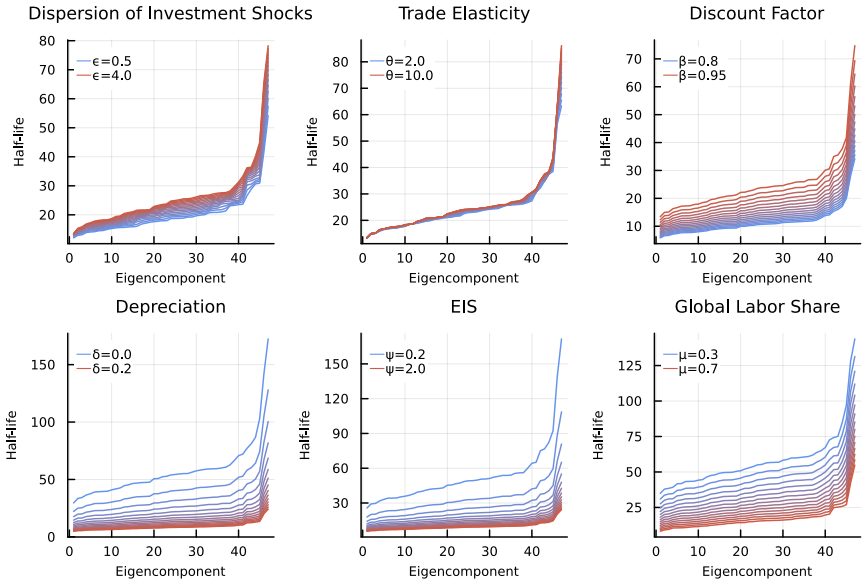
We now examine how the speed of convergence to steady state in our model varies with model parameters. For each alternative value of the model's parameters, we use our eigendecomposition of the transition matrix ($\boldsymbol{P}$) to recover the full spectrum of eigenvalues and eigenvectors (and their associated eigen-shocks). Since the eigen-shocks span all possible fundamental shocks, understanding how parameters affect the entire spectrum of half-lives translates into an analytically sharp understanding of how convergence rates are affected by model parameters.

In Figure K.4, we display the half-lives of convergence to steady state across the entire spectrum of eigen-shocks for different values of model parameters, based on the observed trade and capital shares for the year 2017. Each panel varies the noted parameter, holding constant the other parameters at their baseline values. On the vertical axis, we display the half-life of convergence to steady state. On the horizontal axis, we rank the eigen-shocks in terms of increasing half-lives of convergence to steady state for our baseline parameter values.

In the top-left panel, a higher capital elasticity (ϵ) implies a longer half-life (slower convergence), because greater substitutability of capital across countries reduces the absolute value of the covariance between the real return and the initial level of wealth. In the top-middle panel, a higher trade elasticity (θ) also implies a longer half-life (slower convergence), because greater substitutability of goods across countries also reduces the absolute value of the covariance between the real return and the initial level of wealth. In the top-right panel, a higher discount factor (β) implies a longer half-life (slower convergence), because the representative agent has a higher saving rate, which implies a greater role for wealth accumulation, thereby magnifying the impact of fundamental shocks, and implying a longer length of time for adjustment to occur in response to these shocks.

In the bottom-right panel, we solve the model for alternative values of a common labor share ($\mu_i = \mu$) across countries. A lower labor share (μ) implies a longer half-life (slower convergence), because it implies a greater role for wealth accumulation, which again magnifies the impact of fundamental shocks, and hence requires a greater length of time for adjustment to occur. In the bottom-middle panel, a lower elasticity of intertemporal substitution (ψ) implies a longer half-

Figure K.4: Half Lives of Convergence to Steady State for Alternative Parameter Values



Note: Half lives of convergence to steady state for each eigen-shock for alternative parameter values for our baseline model with trade and capital market frictions and imperfect substitutability between countries for the year 2017; vertical axis shows half-life in years; horizontal axis shows the rank of the eigen-shocks in terms of increasing half lives; each panel varies the noted parameter, holding the other parameters at their baseline value; the blue and red solid lines denote the lower and upper range of the parameter values considered, respectively; each of the other lines in between varies the parameters uniformly in the stated range.

life (slower convergence), because consumption becomes less substitutable across time, which reduces the willingness of the representative agent to respond to investment opportunities. Finally, in the bottom-left panel, a smaller depreciation rate (δ) implies a longer half-life (slower convergence), because it takes longer for investments to depreciate, implying a longer length of time for the distribution of wealth to adjust in response to shocks.

L Data on Bilateral Investment

We construct the data on bilateral investment as follows.

1. We construct the total amounts outstanding by producer country and asset class (i.e., debt securities, equity securities, and fund shares). For the OECD countries, the data are from the OECD (2013–2017). For the non-OECD countries, the data are from the Bank for International Settlements (2013–2017) for debt securities and the market capitalization of listed domestic companies (World Bank, 2013–2017) for equity securities. We restate debt and equity securities from the issuer's residency to nationality, using the issuance-based restatement matrices of the Global Capital Allocation Project (Coppola et al., 2021). We do not restate fund shares, for which the restatement matrices are not available, following

Coppola et al. (2021).

2. Based on the Coordinated Portfolio Investment Survey (International Monetary Fund, 2013–2017), we construct bilateral portfolio investment by asset class. We drop observations for which the investor country is an offshore financial center to avoid double counting of pass-through investment. We define offshore financial centers as eight countries whose ratio of portfolio assets to GDP is above five (Zoromé, 2007): Bermuda, the Cayman Islands, Guernsey, Ireland, the Isle of Man, Jersey, Luxembourg, and the Netherlands Antilles. We split common equity and fund shares, based on the estimates from the Capital Allocation Project. We then restate debt and equity securities (but not fund shares) from the issuer's residency to nationality, using the restatement matrices of the Global Capital Allocation Project. In the order of availability, we use the restatement matrices based on enhanced fund holdings, fund holdings, and issuance.

3. For each producer country and asset class, we construct domestic portfolio investment as the amount outstanding from step 1 minus the sum of foreign portfolio investment from step 2. We then aggregate across asset classes to construct a matrix of bilateral portfolio investment, where each column adds up to the amount outstanding by producer country. We divide each element by the column sum to compute portfolio ownership shares by producer country.

4. We construct the sum of bilateral portfolio investment from step 3 and bilateral direct investment, restated from residency to nationality by Damgaard, Elkjaer, and Johannesen (2019). The data on bilateral direct investment cover only cross-country investment. Therefore, we compute portfolio and direct ownership shares by producer country, excluding domestic investment.

5. We combine steps 3 and 4 to construct a matrix of bilateral ownership shares that sum to one for each producer country. The diagonal elements of the matrix are the domestic portfolio ownership shares from step 3. The off-diagonal elements are the cross-country portfolio and direct ownership shares from step 4, scaled by one minus the domestic portfolio ownership share. The working assumption is that the unobservable domestic portfolio and direct ownership share is equal to the observable domestic portfolio ownership share. That is, the degree of home bias for direct investment is similar to that for portfolio investment.

6. We multiply the capital payment $r_{it}k_{it}$ by the bilateral ownership shares from step 5 to construct the capital income $a_{nit}v_{nit}$ earned by investor n in producer i. We then construct the capital income earned by investor n in the rest of the world (ROW) as the residual:

$a_{n,ROW,t}v_{n,ROW,t} = a_{nt}v_{nt} - \sum_{i \neq ROW} a_{nit}v_{nit}$. This step addresses potential measurement error in the bilateral portfolio investment data outside our sample of 46 countries and ensures that the capital income earned by each investor country satisfies the budget constraint.

7. For a small number of cases, step 6 implies negative capital income earned in the rest of the world. Therefore, we rescale each row of the bilateral ownership shares from step 5 by a positive constant such that step 6 implies positive capital income earned in the rest of the world and the adjusted bilateral capital income $a_{nit}v_{nit}$ sums to total capital income $a_{nt}v_{nt}$ by investor country and total capital payment $r_{it}k_{it}$ by producer country.

8. We construct the bilateral investment shares of investor n as $B_{nit} = a_{nit}v_{nit} / \sum_{h=1}^{N} a_{nht}v_{nht}$. We construct the bilateral capital payment shares of producer i as $X_{int} = a_{nit}v_{nit} / \sum_{h=1}^{N} a_{hit}v_{hit}$.

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