Online Appendix for "Quantitative Urban Models" (Not for Publication)

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A Introduction

Section B reports additional derivations for our baseline quantitative model in Section 4 of the paper. Section C shows that the same predictions for workplace and residence decisions in the commuter market hold in an entire class of quantitative urban models that make different assumptions about preferences, trade costs and market structure in the goods market. Section D reports additional derivations for the extensions and generalizations in Section 6 of the paper.

B Baseline Quantitative Urban Model

In this Section of the Online Appendix, we report additional derivations for the baseline quantitative urban model introduced in Section 4 of the paper. Locations can differ in terms of productivity, amenities, the density of development (the ratio of floor space to land area), and access to transport infrastructure. Both productivity and amenities are influenced by natural advantages and agglomeration forces. Congestion forces take the form of an inelastic supply of land and commuting costs that are increasing in travel time, where travel time depends on the transport network.

We consider a city that is embedded in a wider economy. The city consists of a discrete set of locations (city blocks) indexed by $n, i \in \mathbb{N}$. Time is discrete and is indexed by t. There are

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two types of agents: workers and landlords. Workers are mobile across locations within the city. We consider two different assumptions about worker mobility with the wider economy: (i) A "closed-city" specification, with an exogenous measure of worker ($L_{\mathbb{N}t} = L_{\mathbb{N}}$), in which worker utility is endogenous; (ii) An "open-city" specification, in which the measure of workers ($L_{\mathbb{N}t}$) is endogenously determined by population mobility with a wider economy that provides a reservation level of utility $\overline{U}_t$. In the baseline version of the model, we assume a continuous measure of workers $L_{\mathbb{N}t}$, which ensures that the realized value of variables equals their expected values, and abstracts from any issues of granularity or small sample variation. Firms produce a single final good under conditions of perfect competition and constant returns to scale. This final good is costlessly traded and chosen as the numeraire ($P_{nt} = 1$).

The baseline version of the model is static, but productivity, amenities, the supply of floor space and the transport network are allowed to evolve over time. We discuss extensions and generalization of this baseline specification, including dynamics, in Section 6 of the paper.

B.1 Workplace-Residence Choices

Worker preferences are defined over the final consumption good and residential floor space. We assume that these preferences take the Cobb-Douglas form, such that the indirect utility for a worker ω residing in n and working in i is:

$$u_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1, \tag{B.1}$$

where we suppress the time subscript, except where important; P_n is the price of the final consumption good; Q_n is the price of residential floor space; w_i is the wage; κ_{ni} is an iceberg commuting cost; we model this iceberg commuting cost as depending on bilateral travel time (τ_{ni}) using the transport network: $\kappa_{ni} = e^{\kappa \tau_{ni}} \in [1, \infty)$, where $\kappa > 0$ parameterizes the magnitude of commuting costs; B_n captures residential amenities that are common across all workers and could be endogenous to the surrounding concentration of economic activity through agglomeration forces; and $b_{ni}(\omega)$ is an idiosyncratic amenity draw that captures all the idiosyncratic factors that can cause an individual to live and work in particular locations within the city.

Residential amenities (B_n) are assumed to depend on residential fundamentals ($\overline{B}_n$) and residential externalities ($\mathbb{B}_n$). Residential fundamentals capture features of physical geography that make a location a more of less attractive place to live independently of the surrounding concentration of economic activity (e.g., green areas). Residential externalities capture the interactions between residents within the city (e.g., positive externalities through local public goods and negative externalities through crime):

$$B_n = \overline{B}_n \mathbb{B}_n^{\eta^B}, \qquad \mathbb{B}_n \equiv \sum_{i \in \mathbb{N}} e^{-\delta^B \tau_{ni}} R_i, \tag{B.2}$$

where η^B governs the magnitude of these residential externalities and δ^B parameterizes their spatial decay with travel time.

Idiosyncratic amenities ($b_{ni}(\omega)$) are drawn from an independent extreme value (Fréchet) distribution for each residence-workplace pair and worker:

$$G(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{B.3}$$

where we normalize the Fréchet scale parameter in equation (B.3) to one, because it enters the worker choice probabilities isomorphically to common amenities B_n in equation (B.1); the smaller the Fréchet shape parameter ϵ, the greater the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables.

These idiosyncratic preference shocks make solving the model's commuter market clearing condition tractable by ensuring that each residence-workplace pair faces an upward-sloping supply function for commuters. As shown in Section B.8 of this Online Appendix, the probability that a worker chooses to reside in n and work in i is:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k\in\mathbb{N}} \sum_{\ell\in\mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.4}$$

where L_{ni} is the measure of commuters from n to i and $L_{\mathbb{N}}$ is the measure of workers in the city.

A key implication of equation (B.4) is that bilateral commuting flows satisfy a gravity equation, consistent with the empirical evidence presented in Section 2.4 of the paper. As in the structural gravity equation literature in international trade, bilateral commuting flows depend not only "bilateral resistance" (κ_{ni}) between a pair of locations n and i, but also on "multilateral resistance" ($\kappa_{k\ell}$ for all $k, \ell \in \mathbb{N}$). Each residence-workplace pair must offer a higher real wage adjusted for amenities ($B_n w_i / \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)$) in order to attractive additional commuters with elasticity ϵ. Although individual workers have idiosyncratic amenity shocks for each residence-workplace pair, because there is a continuous measure of these workers, there is no uncertainty in the supply of commuters for each residence-workplace pair.

Summing across workplaces in equation (B.4), we obtain the probability of residing in each location ($\lambda_n^R = \sum_{\ell\in\mathbb{N}} \lambda_{n\ell}$):

$$\lambda_n^R = \frac{R_n}{L_{\mathbb{N}}} = \frac{(B_n)^{\epsilon}\, \Phi_n^R \left(P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k\in\mathbb{N}} \sum_{\ell\in\mathbb{N}} (B_k)^{\epsilon}\, \Phi_k^R \left(P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}} \qquad \Phi_n^R \equiv \sum_{\ell\in\mathbb{N}} \left(w_\ell / \kappa_{n\ell}\right)^{\epsilon}, \tag{B.5}$$

where R_n is employment by residence, or "residents" for brevity; and Φ_n^R is a measure of residents' commuting market access; which depends on commuting costs and the wages offered in each workplace.

Summing across residences in equation (B.4), we obtain the probability of being employed in each location ($\lambda_i^L = \sum_{k \in \mathbb{N}} L_{ki}$):

$$\lambda_i^L = \frac{L_i}{L_{\mathbb{N}}} = \frac{(w_i)^{\epsilon} \Phi_i^L}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (w_\ell)^{\epsilon} \Phi_\ell^L}, \qquad \Phi_i^L \equiv \sum_{k \in \mathbb{N}} B_k \left(\kappa_{ki} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}, \tag{B.6}$$

where L_i is employment by workplace, or "employment" for brevity; and Φ_i^L is a measure of workplace commuting market access, which depends on commuting costs and the amenity-adjusted cost of living in each residence.

As shown in Section B.8 of this Online Appendix, expected utility conditional on choosing a residence-workplace pair is the same across all residence and workplace pairs within the city:

$$U = \mathbb{E}[u] = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}, \tag{B.7}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\delta \equiv \Gamma((\epsilon - 1)/\epsilon)$; and $\Gamma(\cdot)$ is the Gamma function.

B.2 Production

Markets are assumed to be competitive. The single final good is produced using labor and commercial floor space according to the following constant returns to scale Cobb-Douglas technology:

$$Y_n = A_n \left(\frac{L_n}{\beta}\right)^{\beta} \left(\frac{H_n^L}{1-\beta}\right)^{1-\beta}, \qquad 0 < \beta < 1, \tag{B.8}$$

where A_n is productivity and H_n^L is the supply of commercial floor space.

We allow productivity (A_n) to depend on production fundamentals ($\overline{A}_n$) and production externalities ($\mathbb{A}_n$). Production fundamentals capture features of physical geography that make a location a more of less attractive place to produce independently of the surrounding concentration of economic activity (e.g., access to natural water). Production externalities capture the interactions between workers within the city (e.g., knowledge spillovers):

$$A_n = \overline{A}_n \mathbb{A}_n^{\eta^A}, \qquad \mathbb{A}_n \equiv \sum_{i \in \mathbb{N}} e^{-\delta^A \tau_{ni}} L_i \tag{B.9}$$

where η^A governs the magnitude of these production externalities and δ^A parameterizes their spatial decay with travel time.

From the first-order conditions for profit maximization, equilibrium employment and use of commercial floor space satisfy:

$$w_n = A_n \left(\frac{\beta}{1-\beta} \frac{H_n^L}{L_n}\right)^{1-\beta},$$

$$q_n = A_n \left(\frac{1-\beta}{\beta} \frac{L_n}{H_n^L} \right)^{\beta}.$$

To determine the equilibrium price of commercial floor space (q_n), we use the requirement that profits are zero if the final good is produced:

$$Y_n = A_n \left(\frac{L_n}{\beta} \right)^{\beta} \left(\frac{H_n^L}{1-\beta} \right)^{1-\beta} - w_n L_n - q_n H_n^L = 0,$$

which together with the first-order conditions for profit maximization conditions above yields the following requirement for zero-profits that price equals unit cost:

$$1 = \frac{1}{A_n} w_n^{\beta} q_n^{1-\beta}.$$

Intuitively, blocks that have higher productivity (A_n) or lower wages (w_n) are more attractive production locations, and hence must be characterized by higher commercial floor prices in an equilibrium in which firms make zero profits in all locations with positive production.

B.3 Commuter Market Clearing

Commuter market clearing implies that employment in each workplace equals the sum of residents commuting to that workplace:

$$L_i = \sum_{n \in \mathbb{N}} \lambda_{ni|n}^R R_n, \qquad \lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{(w_i/\kappa_{ni})^{\epsilon}}{\sum_{\ell \in \mathbb{N}} (w_\ell/\kappa_{n\ell})^{\epsilon}}, \tag{B.10}$$

and average income in each residence is equal to the wage in each workplace multiplied by the probability of commuting to that workplace:

$$v_n = \sum_{i \in \mathbb{N}} \lambda_{ni|n} w_i. \tag{B.11}$$

B.4 Land Market Clearing

Market clearing for residential floor space implies that income from the ownership of residential floor space equals payments for its use:

$$Q_n H_n^R = (1-\alpha) v_n R_n, \tag{B.12}$$

where H_n^R is the supply of residential floor space. Similarly, market clearing for commercial floor space implies that income from the ownership of commercial floor space equals payments for its use:

$$q_n H_n^L = \frac{1-\beta}{\beta} w_n L_n, \tag{B.13}$$

where H_n^L is the supply of commercial floor space.

These supplies of residential and commercial floor space in each location reflect geographical land area, the ratio of floor space to geographical land area (as reflected in the height of buildings), and the shares of floor space allocated to each use. In our baseline model here, we assume exogenous supplies of residential floor space (H_n^R) and commercial floor space (H_n^L) in each location. In reality, both the overall supply of floor space and its allocation to each use are influenced by market forces, and we discuss extensions to allow for both an endogenous supply of floor space and an endogenous allocation to each use in Section 6 of the paper. Floor space is assumed to be owned by absentee landlords.[1]

B.5 Existence and Uniqueness

The general equilibrium spatial distribution of economic activity within the city is determined by the model parameters (α, β, κ, ϵ, η^B, δ^B, η^A, δ^A) and the following exogenous location characteristics: residential fundamentals ($\overline{B}_n$), production fundamentals ($\overline{A}_n$), the supplies of residential and commercial floor space (H_n^R, H_n^L), and the transport network (τ_{ni}). Given these parameters and exogenous location characteristics, the closed-city general equilibrium of the model is referenced by residents (R_n), employment (L_n), wages (w_n), average residential income (v_n), the prices of residential and commercial floor space (Q_r, q_r), and expected utility (U), given exogenous total city population ($L_{\mathbb{N}}$). The open-city general equilibrium is analogous, except that total city population is endogenously determined by population mobility with the wider economy and its exogenous reservation level of utility ($\overline{U}$). Given these equilibrium objects, all the other endogenous variables of the model can be determined.

We now provide a sufficient condition for the existence of a unique equilibrium. We combine the general equilibrium conditions of the model to obtain a system of equations that takes the required form to apply Theorem 1 from Allen, Arkolakis and Li (2024):

$$x_{nh} = f_{nih}(x_i) = \sum_{i \in \mathbb{N}} \mathcal{K}_{nih} \prod_{h' \in \mathbb{H}} x_{ih'}^{\gamma_{nhh'}}. \tag{B.14}$$

where $n, i \in \mathbb{N}$ denote locations and $h \in \mathbb{H}$ denote economic interactions, which here include residents, employment, and the prices of residential and commercial floor space. We begin by rewriting each of the general equilibrium conditions in the form required to apply this theorem.

[1] Instead of absentee landlords, one can assume that landlords consume only the final good. Since this final good is costlessly traded with the wider economy and chosen as the numeraire, this alternative assumption leaves all equilibrium conditions in the model unchanged.

Population Mobility As a preliminary step, note that expected utility (B.7) can be re-written as:

$$\left(\frac{U}{\vartheta}\right)^{\epsilon^o} = \left[\sum_{n\in\mathbb{N}}\sum_{i\in\mathbb{N}} (B_n w_i)^{\epsilon} \left(\kappa_{ni} Q_n^{1-\alpha}\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}, \tag{B.15}$$

where we have used our choice of numeraire ($P_n = 1$). We now use this expression for expected utility (U) to rewrite the other general equilibrium conditions of the model.

Residential Choice Probabilities Using expected utility (B.7), the residential choice probabilities (B.79) for each location can be re-written as:

$$R_n = \xi \left(B_n \left(\Phi_n^R\right)^{\frac{1}{\epsilon^o}} Q_n^{\alpha-1}\right)^{\epsilon}, \tag{B.16}$$

where ξ is an endogenous scalar:

$$\xi \equiv L_{\mathbb{N}} \left(\frac{U}{\vartheta}\right)^{-\epsilon^o}, \tag{B.17}$$

and Φ_n^R our measure of residential commuting market access that is a commuting cost weighted average of wages in each workplace:

$$\Phi_n^R \equiv \sum_{i\in\mathbb{N}} \kappa_{ni}^{-\epsilon} w_i^{\epsilon}. \tag{B.18}$$

Workplace Choice Probabilities Using expected utility (B.7), , the workplace choice probabilities (B.6) for each location can be re-written as:

$$L_i = \xi \left(w_i \left(\Phi_t^L\right)^{\frac{1}{\epsilon}}\right)^{\epsilon}, \tag{B.19}$$

where we defined the endogenous scalar ξ in equation (B.17) above and Φ_i^L is a measure of workplace commuting market access that is a commuting cost weighted average and amenities and the price of residential floor space in each residence:

$$\Phi_i^L \equiv \sum_{n\in\mathbb{N}} \kappa_{ni}^{-\epsilon} \left(B_n^o Q_n^{\alpha-1}\right)^{\epsilon}. \tag{B.20}$$

Residential Commuting Market Access From the workplace choice probabilities (B.19), we have the following relationship:

$$(w_i)^{\epsilon} = \frac{1}{\xi}\frac{L_i}{\Phi_i^L}.$$

Using this relationship, we can re-write residential commuting market access (Φ_n^R) in equation (B.18) as follows:

$$\Phi_n^R = \frac{1}{\xi}\sum_{i\in\mathbb{N}} \kappa_{ni}^{-\epsilon}\frac{L_i}{\Phi_i^L}. \tag{B.21}$$

Workplace Commuting Market Access From the residential choice probabilities (B.16), we have the following relationship:

$$\left(B_n Q_{nt}^{\alpha-1}\right)^{\epsilon} = \frac{1}{\xi}\frac{R_n}{\Phi_n^R}$$

Using this relationship, we can re-write workplace commuting market access (Φ_i^E) in equation (B.20) as follows:

$$\Phi_i^L = \frac{1}{\xi}\sum_{n\in\mathbb{N}} \kappa_{ni}^{-\epsilon}\frac{R_n}{\Phi_n^R}. \tag{B.22}$$

Output From the Cobb-Douglas production technology, output of the final good implies (Y_i) is:

$$Y_i = A_i \left(\frac{L_i}{\beta}\right)^{\beta}\left(\frac{H_i^L}{1-\beta}\right)^{1-\beta}. \tag{B.23}$$

Residential Income Using the definition of residential commuting market access from equation (B.18) in equation (B.90), average residential income (v_n) can be written as:

$$\Phi_n^R v_n = \sum_{i\in\mathbb{N}} \kappa_{ni}^{-\epsilon}\left(w_i\right)^{\epsilon+1}. \tag{B.24}$$

Wages From the zero-profit condition for the final good to be produced:

$$w_n = A_n^{\frac{1}{\beta}} q_n^{-\left(\frac{1-\beta}{\beta}\right)}, \tag{B.25}$$

where we have again used our choice of numeraire ($P_n = 1$).

Residential and Commercial Floor Space Prices The prices of residential (Q_n) and commercial (q_n) floor space are determined by the market clearing conditions for residential and commercial floor space in equations (B.12) and (B.13), respectively, given the suppliers of residential and commercial floor space (H_n^R, H_n^L).

Amenities and Productivity Amenities (B_n) and productivity (A_n) are determined by fundamentals and externalities according to equations (B.2) and (B.9), respectively.

System of General Equilibrium Conditions Using equations (B.18), (B.20), (B.21), (B.22), (B.23), (B.24), (B.25), (B.12), (B.13), (B.2) and (B.9), the system of general equilibrium conditions of the model can be written in the form of equation (B.14) as follows:

$$R_n = \xi \left(B_n \left(\Phi_n^R \right)^{\frac{1}{\epsilon}} Q_n^{\alpha-1} \right)^{\epsilon}, \tag{B.26}$$

$$L_n = \xi \left(w_n \left(\Phi_n^L \right)^{\frac{1}{\epsilon}} \right)^{\epsilon}, \tag{B.27}$$

$$\Phi_n^R = \frac{1}{\xi} \sum_{i \in \mathbb{N}} e^{-\kappa\epsilon\tau_{ni}} L_i \left(\Phi_i^L \right)^{-1}, \tag{B.28}$$

$$\Phi_n^L = \frac{1}{\xi} \sum_{i \in \mathbb{N}} e^{-\kappa\epsilon\tau_{ni}} R_i \left(\Phi_i^R \right)^{-1}, \tag{B.29}$$

$$w_n = A_n^{\frac{1}{\beta}} q_n^{-\left(\frac{1-\beta}{\beta}\right)}, \tag{B.30}$$

$$\Phi_n^R v_n = \sum_{i \in \mathbb{N}} e^{-\kappa\epsilon\tau_{ni}} \left(w_i \right)^{\epsilon+1}, \tag{B.31}$$

$$Q_n = (1-\alpha) v_n R_n \left(H_n^R \right)^{-1}, \tag{B.32}$$

$$q_n = A_n \left(\frac{L_n}{\beta} \right)^{\beta} \left(\frac{H_n^L}{1-\beta} \right)^{-\beta}, \tag{B.33}$$

$$B_n = \overline{B}_n \left(\sum_{i \in \mathbb{N}} e^{-\delta^B \tau_{ni}} R_i \right)^{\eta^B}, \tag{B.34}$$

$$A_n = \overline{A}_n \left(\sum_{i \in \mathbb{N}} e^{-\delta^A \tau_{ni}} L_i \right)^{\psi}, \tag{B.35}$$

where we have used our parameterization of commuting costs as $\kappa_{ni}^{-\epsilon} = e^{-\kappa\epsilon\tau_{ni}}$; and the supplies of residential floor space (H_{nt}^R) and commercial floor space (H_{nt}^E) are exogenous in this baseline specification.

The exponents on the variables on the left-hand side of this system of equations can be represented as the following matrix:

$$\mathbf{\Lambda} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1H} \\ \Lambda_{21} & \Lambda_{22} & \dots & \Lambda_{2H} \\ \vdots & \vdots & \vdots & \vdots \\ \Lambda_{H1} & \Lambda_{H2} & \dots & \Lambda_{HH} \end{bmatrix}.$$

The exponents on the variables on the right-hand side of this system of equations can be represented as the following matrix:

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \dots & \Gamma_{1H} \\ \Gamma_{21} & \Gamma_{22} & \dots & \Gamma_{2H} \\ \vdots & \vdots & \vdots & \vdots \\ \Gamma_{H1} & \Gamma_{H2} & \dots & \Gamma_{HH} \end{bmatrix}$$

Let $\boldsymbol{\Upsilon} \equiv |\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-1}|$ and denote the spectral radius (eigenvalue with the largest absolute value) of this matrix by $\rho(\boldsymbol{\Upsilon})$. From Theorem 1 in Allen, Arkolakis and Li (2024), a sufficient condition for the existence of a unique equilibrium (up to scale) is $\rho(\boldsymbol{\Upsilon}) \leq 1$.

Special Case One special case that is particularly tractable is the case with no residential floor space use ($\alpha = 1$) and no commercial floor space use ($\beta = 1$), as analyzed in Allen, Arkolakis and Li (2024). In this special case, the commuting probability (B.4) can be written as:

$$L_{ni} = \xi \left(\frac{B_n w_i}{\kappa_{ni}} \right)^{\epsilon}, \tag{B.36}$$

where $\xi \equiv L_{\mathbb{N}} \left(U/\vartheta\right)^{-\epsilon}$; $U = \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(\frac{B_k w_\ell}{\kappa_{k\ell}} \right)^{\epsilon} \right]^{1/\epsilon}$; and the wage is now pinned down by productivity alone:

$$w_i = A_i. \tag{B.37}$$

Commuter market clearing requires:

$$L_n = \sum_{i \in \mathbb{N}} L_{in}, \tag{B.38}$$

$$R_n = \sum_{i \in \mathbb{N}} L_{ni}. \tag{B.39}$$

Substituting the commuting probability (B.36) into these commuter market clearing conditions, and using equations (B.2) and (B.9) for amenities (B_n) and productivity (A_n), the system of general equilibrium conditions can be written as:

$$\begin{aligned}
L_n A_n^{-\epsilon} &= \xi \sum_{i \in \mathbb{N}} e^{-\kappa \epsilon \tau_{ni}} B_i^{\epsilon}, \\
R_n B_n^{-\epsilon} &= \xi \sum_{i \in \mathbb{N}} e^{-\kappa \epsilon \tau_{ni}} A_i^{\epsilon}, \\
A_n^{\frac{1}{\eta^A}} &= \overline{A}_n^{\frac{1}{\eta^A}} \sum_{i \in \mathbb{N}} e^{-\delta^A \tau_{ni}} L_i, \\
B^{\frac{1}{\eta^B}} &= \overline{B}_n^{\frac{1}{\eta^B}} \sum_{i \in \mathbb{N}} e^{-\delta^B \tau_{ni}} R_i,
\end{aligned}$$

where we have used our parameterization of commuting costs as $\kappa_{ni}^{-\epsilon} = e^{-\kappa \epsilon \tau_{ni}}$. The exponents on the variables on the left-hand side of this system of equations can be represented as the following matrix:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & -\epsilon & 0 \\ 0 & 1 & 0 & -\epsilon \\ 0 & 0 & \frac{1}{\eta^A} & 0 \\ 0 & 0 & 0 & \frac{1}{\eta^B} \end{bmatrix}.$$

The exponents on the variables on the right-hand side of this system of equations can be represented as the following matrix:

$$\mathbf{\Gamma} = \begin{bmatrix} 0 & 0 & 0 & \epsilon \\ 0 & 0 & \epsilon & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

We thus have:

$$\mathbf{\Upsilon} \equiv \left|\mathbf{\Gamma}\mathbf{\Lambda}^{-1}\right| = \begin{bmatrix} 0 & 0 & 0 & \left|\eta^B \epsilon\right| \\ 0 & 0 & \left|\eta^A \epsilon\right| & 0 \\ 1 & 0 & \left|\eta^A \epsilon\right| & 0 \\ 0 & 1 & 0 & \left|\eta^B \epsilon\right| \end{bmatrix}.$$

From the Collatz-Wielandt Formula, a sufficient condition for uniqueness of the equilibrium is hence $\left|\eta^A\right| \epsilon \leq \frac{1}{2}$ and $\left|\eta^B\right| \epsilon \leq \frac{1}{2}$, which corresponds to the case in which both the agglomeration forces in the model (as parameterized by production externalities (η^A) and residential externalities (η^B)) are sufficiently weak relative to the dispersion forces from the heterogeneity in idiosyncratic amenities (as parameterized by (ϵ)).

B.6 Market Access Representation

For the special case of the model in which productivity and amenities are exogenous ($\eta^B = \eta^A = 0$), we now show that the impact of a change in commuting costs ($\kappa_{ni}^{-\epsilon}$) on the spatial distribution of economic activity within the city can be represented solely in terms of changes in commuting market access.

We start with the conditions for general equilibrium in the model. Substituting the wage (B.30) into employment (B.27), we obtain:

$$L_n = \xi A_n^{\frac{\epsilon}{\beta}} q_n^{-\epsilon\left(\frac{1-\beta}{\beta}\right)} \Phi_n^L.$$

Substituting the commercial floor price (B.33) into the above equation, we get:

$$L_n = \xi A_n^{\epsilon} \left(\frac{L_n}{\beta}\right)^{-\epsilon(1-\beta)} \left(\frac{H_n^L}{1-\beta}\right)^{\epsilon(1-\beta)} \Phi_n^L,$$

and hence:

$$L_n = \xi^{\frac{1}{1+\epsilon(1-\beta)}} \beta^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} (1-\beta)^{-\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} A_n^{\frac{\epsilon}{1+\epsilon(1-\beta)}} \left(H_n^L\right)^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{1}{1+\epsilon(1-\beta)}}, \tag{B.40}$$

$$\frac{L_n}{\beta} = \xi^{\frac{1}{1+\epsilon(1-\beta)}} \beta^{-\frac{1}{1+\epsilon(1-\beta)}} (1-\beta)^{-\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} A_n^{\frac{\epsilon}{1+\epsilon(1-\beta)}} \left(H_n^L\right)^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{1}{1+\epsilon(1-\beta)}}.$$

Substituting this result into the commercial floor price (B.33):

$$q_n = A_n \xi^{\frac{\beta}{1+\epsilon(1-\beta)}} \beta^{-\frac{\beta}{1+\epsilon(1-\beta)}} A_n^{\frac{\beta\epsilon}{1+\epsilon(1-\beta)}} \left(\frac{H_n^L}{1-\beta}\right)^{\frac{\beta\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{\beta}{1+\epsilon(1-\beta)}} \left(\frac{H_n^L}{1-\beta}\right)^{-\beta},$$

and hence:

$$q_n = \xi^{\frac{\beta}{1+\epsilon(1-\beta)}} \beta^{-\frac{\beta}{1+\epsilon(1-\beta)}} A_n^{\frac{1+\epsilon}{1+\epsilon(1-\beta)}} \left(\frac{H_n^L}{1-\beta} \right)^{-\frac{\beta}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{\beta}{1+\epsilon(1-\beta)}} . \tag{B.41}$$

Substituting the residential floor price (B.32) into residents (B.26), we obtain:

$$R_n = \xi B_n^{\epsilon} (1-\alpha)^{\epsilon(\alpha-1)} v_n^{\epsilon(\alpha-1)} R_n^{\epsilon(\alpha-1)} \left(H_n^R\right)^{-\epsilon(\alpha-1)} \Phi_n^R ,$$

and hence:

$$R_n = \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} v_n^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{1}{1+\epsilon(1-\alpha)}} .$$

In an equilibrium in which commuting costs between locations are prohibitive ($\kappa_{ni}^{-\epsilon} \approx 0$ for $n \neq i$), $v_n \approx w_n \approx \left(\Phi_n^R\right)^{\frac{1}{\epsilon}}$ and we can further rewrite this equation as:

$$R_n \approx \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{\alpha}{1+\epsilon(1-\alpha)}} . \tag{B.42}$$

Substituting this result into the residential floor price (B.32), we have:

$$Q_n \approx (1-\alpha) v_n \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{1+2\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{\alpha}{1+\epsilon(1-\alpha)}} .$$

Again approximating around an equilibrium in which commuting costs between locations are prohibitive ($\kappa_{ni}^{-\epsilon} \approx 0$ for $n \neq i$), $v_n = w_n = \left(\Phi_n^R\right)^{\frac{1}{\epsilon}}$ and we can further rewrite this equation as:

$$Q_n \approx (1-\alpha) \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{1+2\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{1+\epsilon}{\epsilon(1+\epsilon(1-\alpha))}} . \tag{B.43}$$

Collecting together equations (B.40), (B.41), (B.42) and (B.43), we have obtain the following system of equations for employment (L_n), the price of commercial floor space (q_n), residents (R_n) and the price of residential floor space (Q_n) in terms of commuting market access (Φ_n^L, Φ_n^R), the structural residuals (A_n, B_n, H_n^R, H_n^L), and parameters:

$$\begin{aligned}
L_n &= \xi^{\frac{1}{1+\epsilon(1-\beta)}} \beta^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} (1-\beta)^{-\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} A_n^{\frac{\epsilon}{1+\epsilon(1-\beta)}} \left(H_n^L\right)^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{1}{1+\epsilon(1-\beta)}} , \\
q_n &= \xi^{\frac{\beta}{1+\epsilon(1-\beta)}} \beta^{-\frac{\beta}{1+\epsilon(1-\beta)}} A_n^{\frac{1+\epsilon}{1+\epsilon(1-\beta)}} \left(\frac{H_n^L}{1-\beta} \right)^{-\frac{\beta}{1+\epsilon(1-\beta)}} \left(\Phi_n^L\right)^{\frac{\beta}{1+\epsilon(1-\beta)}} , \\
R_n &\approx \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{\alpha}{1+\epsilon(1-\alpha)}} , \\
Q_n &\approx (1-\alpha) \xi^{\frac{1}{1+\epsilon(1-\alpha)}} B_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} (1-\alpha)^{\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(H_n^R\right)^{-\frac{1+2\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \left(\Phi_n^R\right)^{\frac{1+\epsilon}{\epsilon(1+\epsilon(1-\alpha))}} .
\end{aligned}$$

Taking log differences between a counterfactual and an initial equilibrium, we can represent relative changes in employment ($\widehat{L}_n$), the price of commercial floor space ($\widehat{q}_n$), residents ($\widehat{R}_n$) and the price of residential floor space ($\widehat{Q}_n$) as the following system of equations:

$$\log \widehat{L}_n = \frac{1}{1+\epsilon(1-\beta)} \log \widehat{\Phi}_n^L + \log \left(\widehat{A}_n^{\frac{\epsilon}{1+\epsilon(1-\beta)}} \left(\widehat{H}_n^L \right)^{\frac{\epsilon(1-\beta)}{1+\epsilon(1-\beta)}} \right),$$

$$\log \widehat{q}_n = \frac{\beta}{1+\epsilon(1-\beta)} \log \widehat{\Phi}_n^L + \log \left(\widehat{A}_n^{\frac{1+\epsilon}{1+\epsilon(1-\beta)}} \left(\widehat{H}_n^L \right)^{-\frac{\beta}{1+\epsilon(1-\beta)}} \right),$$

$$\log \widehat{R}_n \approx \frac{\alpha}{1+\epsilon(1-\alpha)} \log \widehat{\Phi}_n^R + \log \left(\widehat{B}_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} \left(\widehat{H}_n^R \right)^{-\frac{\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \right),$$

$$\log \widehat{Q}_n \approx \frac{1+\epsilon}{\epsilon(1+\epsilon(1-\alpha))} \log \widehat{\Phi}_n^R + \log \left(\widehat{B}_n^{\frac{\epsilon}{1+\epsilon(1-\alpha)}} \left(\widehat{H}_n^R \right)^{-\frac{1+2\epsilon(\alpha-1)}{1+\epsilon(1-\alpha)}} \right),$$

where recall that a hat above a variable denotes a relative change between the counterfactual (prime) and initial (no prime) equilibria: $\widehat{x} = x'/x$.

This system of equations can be represented in the same form as equation (26) in the paper:

$$\begin{aligned} \ln \widehat{y}_n^R &\approx \rho^R \ln \widehat{\Phi}_n^R + e_n^R, \\ \ln \widehat{y}_n^L &= \rho^L \ln \widehat{\Phi}_n^L + e_n^R, \end{aligned} \tag{B.44}$$

where the our come variables $\widehat{y}_n^R = \left[\widehat{R}_n, \widehat{Q}_n\right]$ and $y_n^L = \left[\widehat{L}_n, \widehat{q}_n\right]$ are changes in residents, residential floor prices, employment, and commercial floor prices; the reduced-form coefficients (ρ^R, ρ^L) depend on structural parameters of the model; the residuals (e_n^R, e_n^L) capture changes in location characteristics (productivity, amenities, and the supplies of residential and commercial floor space); in the specification model considered here, in which workers idiosyncratic shocks are to preferences (rather than productivity), the first equation involves an approximation around an equilibrium with prohibitive commuting costs between locations (such that $v_n \approx \left(\Phi_n^R\right)^{1/\epsilon}$).

B.7 Counterfactuals

In this subsection of the Online Appendix, we show that the counterfactual equilibrium conditions of the model can be written in the exact-hat algebra form in equations (18)-(B.66) in the paper.

We denote the value of a variable in the counterfactual equilibrium by a prime (x_n'), the value of variable in the initial equilibrium without a prime (x_n), and the relative change in a variable by a hat ($\widehat{x}_n = x_n'/x_n$). We hold residential fundamentals ($\overline{B}_n$), production fundamentals ($\overline{A}_n$), the supply of residential floor space (H_n^R) and the supply of commercial floor space (H_n^L) constant at their values in the initial equilibrium. We consider an exogenous change in travel times, such that travel times and commuting costs differ from their values in the initial equilibrium: $\tau'_{ni} \neq \tau_{ni}$ and hence $\kappa'_{ni} \neq \kappa_{ni}$. The system of equations for a counterfactual equilibrium can be written as follows:

$$\lambda_n^{R\prime} = \frac{\sum_{\ell \in \mathbb{N}} \left(B'_n w'_\ell\right)^\epsilon \left(e^{-\phi \tau'_{n\ell}}\right) \left(\left(Q'_n\right)^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B'_k w'_\ell\right)^\epsilon \left(e^{-\phi \tau'_{k\ell}}\right) \left(\left(Q'_k\right)^{1-\alpha}\right)^{-\epsilon}} \tag{B.45}$$

$$\lambda_n^{L\prime} = \frac{\sum_{k \in \mathbb{N}} \left(B'_k w'_n\right)^\epsilon \left(e^{-\phi \tau'_{kn}}\right) \left(\left(Q'_k\right)^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \lambda_{k\ell} \left(B'_k w'_\ell\right)^\epsilon \left(e^{-\phi \tau'_{k\ell}}\right) \left(\left(Q'_k\right)^{1-\alpha}\right)^{-\epsilon}} \tag{B.46}$$

$$\lambda_{ni|n}^{R\prime} = \frac{\left(w'_i\right)^\epsilon \left(e^{-\phi \tau'_{ni}}\right)}{\sum_{\ell \in \mathbb{N}} \left(w'_\ell\right)^\epsilon \left(e^{-\phi \tau'_{n\ell}}\right)}, \tag{B.47}$$

$$R'_n = \lambda_n^{R\prime} L_{\mathbb{N}}, \tag{B.48}$$

$$L'_n = \lambda_n^{L\prime} L_{\mathbb{N}}, \tag{B.49}$$

$$q'_n = \left(A'_n\right)^{\frac{1}{1-\beta}} \left(w'_n\right)^{-\frac{\beta}{1-\beta}}, \tag{B.50}$$

$$v'_n = \sum_{i \in \mathbb{N}} \lambda'_{ni|n} w'_i, \tag{B.51}$$

$$B'_n = \overline{B}_n \left(\sum_{i \in \mathbb{N}} e^{-\delta^B \tau'_{ni}} R'_i\right)^{\eta^B}, \tag{B.52}$$

$$A'_n = \overline{A}_n \left(\sum_{i \in \mathbb{N}} e^{-\delta^A \tau'_{ni}} L'_i\right)^{\eta^A}, \tag{B.53}$$

$$w'_n L'_n = \frac{\beta}{1-\beta} q'_n H_n^L, \tag{B.54}$$

$$Q'_n H_n^R = (1-\alpha) v'_n R'_n, \tag{B.55}$$

where we have used $\left(\kappa'_{ni}\right)^{-\epsilon} = e^{-\kappa \epsilon \tau'_{ni}} = e^{-\phi \tau'_{ni}}$ and $\overline{B}'_n = \overline{B}_n$, $\overline{A}'_n = \overline{A}_n$, $H_n^{R\prime} = H_n^R$ and $H_n^{L\prime} = H_n^L$.

We now rewrite this system of equations for a counterfactual equilibrium in terms of the relative changes of the endogenous variables and the values of these endogenous variables in the initial equilibrium. Given the exogenous change in travel times and hence commuting costs, and initial guesses for changes in wages ($\widehat{w}_n^0$) and the price of residential floor space ($\widehat{Q}_n^0$), we can re-write the system of equations as follows:

$$\widehat{\lambda}_n^R \lambda_n^R = \frac{\sum_{\ell \in \mathbb{N}} \lambda_{n\ell} \left(\widehat{B}_n \widehat{w}_\ell\right)^\epsilon \left(e^{-\phi \widehat{\tau}_{n\ell}}\right) \left(\widehat{Q}_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \lambda_{k\ell} \left(\widehat{B}_k \widehat{w}_\ell\right)^\epsilon \left(e^{-\phi \widehat{\tau}_{k\ell}}\right) \left(\widehat{\kappa}_{k\ell} \widehat{Q}_k^{1-\alpha}\right)^{-\epsilon}} \tag{B.56}$$

$$\widehat{\lambda}_n^L \lambda_n^L = \frac{\sum_{k\in\mathbb{N}} \lambda_{kn} \left(\widehat{B}_k \widehat{w}_n\right)^{\epsilon} \left(e^{-\phi\widehat{\tau}_{kn}}\right) \left(\widehat{Q}_k^{1-\alpha}\right)^{-\epsilon}}{\sum_{k\in\mathbb{N}} \sum_{\ell\in\mathbb{N}} \lambda_{k\ell} \left(\widehat{B}_k \widehat{w}_\ell\right)^{\epsilon} \left(\widehat{\kappa}_{k\ell} \widehat{Q}_k^{1-\alpha}\right)^{-\epsilon}} \tag{B.57}$$

$$\widehat{\lambda}_{ni|n}^R \lambda_{ni|n}^R = \frac{\lambda_{ni|n}^R \widehat{w}_i^{\epsilon} \left(e^{-\phi\widehat{\tau}_{ni}}\right)}{\sum_{\ell\in\mathbb{N}} \lambda_{n\ell|n}^R \widehat{w}_\ell^{\epsilon} \left(e^{-\phi\widehat{\tau}_{n\ell}}\right)}, \tag{B.58}$$

$$\widehat{R}_n = \widehat{\lambda}_n^R \lambda_n^R L_{\mathbb{N}}, \tag{B.59}$$

$$\widehat{L}_n = \widehat{\lambda}_n^L \lambda_n^L L_{\mathbb{N}}, \tag{B.60}$$

$$\widehat{q}_n = \left(\widehat{A}_n\right)^{\frac{1}{1-\beta}} \left(\widehat{w}_n\right)^{-\frac{\beta}{1-\beta}}, \tag{B.61}$$

$$\widehat{v}_n v_n = \sum_{i\in\mathbb{N}} \widehat{\lambda}_{ni|n} \lambda_{ni|n} \widehat{w}_i w_i, \tag{B.62}$$

$$\widehat{B}_n = \left(\frac{\sum_{i\in\mathbb{N}} e^{-\delta^B \widehat{\tau}_{ni} + \tau_{ni}} \widehat{R}_i R_i}{\sum_{i\in\mathbb{N}} e^{-\delta^B \tau_{ni}} R_i}\right)^{\eta^B}, \tag{B.63}$$

$$\widehat{A}_n = \left(\frac{\sum_{i\in\mathbb{N}} e^{-\delta^A \widehat{\tau}_{ni} + \tau_{ni}} \widehat{L}_i L_i}{\sum_{i\in\mathbb{N}} e^{-\delta^A \tau_{ni}} L_i}\right)^{\eta^A}, \tag{B.64}$$

where, with a slight abuse of notation, we denote $\widehat{\tau}_{ni} = \tau'_{ni} - \tau_{ni}$; and where we have used $\widehat{\overline{A}}_n = \widehat{\overline{B}}_n = \widehat{H}_n^R = \widehat{H}_n^L = 1$. From this system of equations, we obtain implied changes in labor demand ($\widehat{L}_n^D$) and labor supply ($\widehat{L}_n^S$) in each location:

$$\widehat{L}_n^D = \frac{\widehat{q}_n}{\widehat{w}_n^0}, \qquad \widehat{L}_n^S = \widehat{\lambda}_n^L, \tag{B.65}$$

and the implied changes in income from residential floor space (Ω_n^S) and expenditure (Ω_n^D) on residential floor space:

$$\widehat{\Omega}_n^S = \widehat{Q}_n^0, \qquad \widehat{\Omega}_n^D = \widehat{v}_n \widehat{R}_n. \tag{B.66}$$

To solve for the counterfactual equilibrium, we update our initial guesses for changes in wages ($\widehat{w}_n^{o0}$) and the price of residential floor space ($\widehat{Q}_n^0$) until the changes in the demand and supply of labor are equal to one another in equation (B.65) and the changes in the demand and supply for residential and commercial floor space are equal to one another in equation (B.66), and all the other equilibrium conditions of the model are satisfied.

B.8 Derivation of Residence and Workplace Choices

B.8.1 Distribution of Utility

From the indirect utility function in equation (1) in the paper, we have the following monotonic relationship between idiosyncratic amenities ($b_{ni}(\omega)$) and utility ($U_{ni}(\omega)$):

$$b_{ni}(\omega) = \frac{U_{ni}(\omega)\, \kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}{B_{ni} w_i}. \tag{B.67}$$

We assume that idiosyncratic amenities ($b_{ni}(\omega)$) are drawn from an independent extreme value (Fréchet) distribution for each residence-workplace pair and each worker:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{B.68}$$

where we normalize the Fréchet scale parameter in equation (B.68) to one, because it enters worker choice probabilities isomorphically to the common bilateral amenities parameter B_{ni}.

Together equations (B.67) and (B.68) imply that the distribution of utility for residence n and workplace i is:

$$G_{ni}(u) = e^{-\Psi_{ni} u^{-\epsilon}}, \qquad \Psi_{ni} \equiv (B_{ni} w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}. \tag{B.69}$$

From all possible pairs of residence and workplace, each worker chooses the bilateral commute that offers the maximum utility. Since the maximum of a sequence of Fréchet distributed random variables is itself Fréchet distributed, the distribution of utility across all possible pairs of residence and workplace is:

$$1 - G(u) = 1 - \prod_{k \in \mathbb{N}} \prod_{\ell \in \mathbb{N}} e^{-\Psi_{k\ell} u^{-\epsilon}},$$

where the left-hand side is the probability that a worker has a utility greater than u, and the right-hand side is one minus the probability that the worker has a utility less than u for all possible pairs of residence and employment locations. Therefore we have:

$$G(u) = e^{-\Psi_{\mathbb{N}} u^{-\epsilon}}, \qquad \Psi_{\mathbb{N}} = \sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \Psi_{k\ell}. \tag{B.70}$$

Given this Fréchet distribution for utility, expected utility is:

$$\mathbb{E}[u] = \int_0^{\infty} \epsilon \Psi_{\mathbb{N}} u^{-\epsilon} e^{-\Psi_{\mathbb{N}} u^{-\epsilon}} du. \tag{B.71}$$

Now define the following change of variables:

$$y = \Psi_{\mathbb{N}} u^{-\epsilon}, \qquad dy = -\epsilon \Psi_{\mathbb{N}} u^{-(\epsilon+1)} du. \tag{B.72}$$

Using this change of variables, expected utility can be written as:

$$\mathbb{E}[u] = \int_0^{\infty} \Psi_{\mathbb{N}}^{1/\epsilon} y^{-1/\epsilon} e^{-y} dy, \tag{B.73}$$

which can be in turn written as:

$$\mathbb{E}[u] = \vartheta \Psi_{\mathbb{N}}^{1/\epsilon}, \qquad \vartheta = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right), \tag{B.74}$$

where $\Gamma(\cdot)$ is the Gamma function. Therefore we obtain the expression in equation (7) in the main text:

$$U = \mathbb{E}[u] = \vartheta \Psi_{\mathbb{N}}^{1/\epsilon} = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_{\ell})^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}\right]^{1/\epsilon}. \tag{B.75}$$

B.8.2 Commuting Probabilities

Using the distribution of utility for pairs of residence and employment locations, the probability that a worker chooses the bilateral commute from n to i out of all possible bilateral commutes is:

$$\begin{aligned}\lambda_{ni} &= \Pr\left[u_{ni} \geq \max\{u_{k\ell}\}; \forall k, \ell\right], \\ &= \int_0^\infty \prod_{\ell \neq i} G_{n\ell}(u) \left[\prod_{k \neq n} \prod_{\ell \in \mathbb{N}} G_{k\ell}(u)\right] g_{ni}(u) du, \\ &= \int_0^\infty \prod_{k \in \mathbb{N}} \prod_{\ell \in \mathbb{N}} \epsilon \Psi_{ni} u^{-(\epsilon+1)} e^{-\Psi_{k\ell} u^{-\epsilon}} du, \\ &= \int_0^\infty \epsilon \Psi_{ni} u^{-(\epsilon+1)} e^{-\Psi_{\mathbb{N}} u^{-\epsilon}} du.\end{aligned} \tag{B.76}$$

Note that:

$$\frac{d}{du}\left[\frac{1}{\Psi_{\mathbb{N}}} e^{-\Psi_{\mathbb{N}} u^{-\epsilon}}\right] = \epsilon u^{-(\epsilon+1)} e^{-\Psi_{\mathbb{N}} u^{-\epsilon}}. \tag{B.77}$$

Using this result to evaluate the integral above, the probability that the worker chooses to live in location n and work in location i is:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{\Psi_{ni}}{\Psi_{\mathbb{N}}} = \frac{(B_{ni} w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.78}$$

where L_{ni} is the measure of commuters from residence n to workplace i; $L_{\mathbb{N}}$ is the total measure of workers in the city; and this expression corresponds to equation (4) in the main text.

Summing across workplaces i in equation (B.78), we obtain the probability that a worker in chooses to live in residence n (λ_n^R):

$$\lambda_n^R = \frac{R_n}{L_{\mathbb{N}}} = \frac{\sum_{i \in \mathbb{N}} (B_{ni} w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.79}$$

where R_n denotes employment by residence in location n; and this corresponds to the expression in equation (5) in the main text.

Similarly, summing across residences n in equation (B.78), we obtain the probability that a worker in Greater London chooses workplace i (λ_i^L):

$$\lambda_i^L = \frac{L_i}{L_{\mathbb{N}}} = \frac{\sum_{n \in \mathbb{N}} (B_{ni} w_i)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_\ell)^\epsilon \left(\kappa_{k\ell} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.80}$$

which L_i denotes employment by workplace in location i; and this corresponds to the expression in equation (6) in the paper.

For the measure of workers in location i (L_i), we can evaluate the conditional probability that they commute from location n (conditional on having chosen to work in location i):

$$\lambda_{ni|i}^{L} = \frac{\lambda_{ni}}{\lambda_i^L} = \Pr\left[u_{ni} \geq \max\{u_{ri}\}; \forall r\right], \tag{B.81}$$
$$= \int_0^\infty \prod_{r \neq n} G_{ri}(u) g_{ni}(u) du,$$
$$= \int_0^\infty e^{-\Psi_i^L u^{-\epsilon}} \epsilon \Psi_{ni} u^{-(\epsilon+1)} du.$$

where

$$\Psi_i^L \equiv \sum_{k \in \mathbb{N}} \left(B_{ki} w_i\right)^\epsilon \left(\kappa_{ki} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}. \tag{B.82}$$

Using the result (B.77) to evaluate the integral in equation (B.81), the probability that a worker commutes from residence n to workplace i conditional on having chosen to work in location i is:

$$\lambda_{ni|i}^{L} = \frac{\lambda_{ni}}{\lambda_i^L} = \frac{\left(B_{ni} w_i\right)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \left(B_{ki} w_i\right)^\epsilon \left(\kappa_{ki} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{B.83}$$

which further simplifies to:

$$\lambda_{ni|i}^{L} = \frac{B_{ni}^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} B_{ki}^\epsilon \left(\kappa_{ki} P_k^\alpha Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{B.84}$$

For the measure of residents of location n (R_n), we can evaluate the conditional probability that they commute to location i (conditional on having chosen to live in location n):

$$\lambda_{ni|n}^{R} = \frac{\lambda_{ni}}{\lambda_n^R} = \Pr\left[u_{ni} \geq \max\{u_{n\ell}\}; \forall \ell\right], \tag{B.85}$$
$$= \int_0^\infty \prod_{\ell \neq i} G_{n\ell}(u) g_{ni}(u) du,$$
$$= \int_0^\infty e^{-\Psi_n^R u^{-\epsilon}} \epsilon \Psi_{ni} u^{-(\epsilon+1)} du,$$

where

$$\Psi_n^R \equiv \sum_{\ell \in \mathbb{N}} \left(B_{n\ell} w_\ell\right)^\epsilon \left(\kappa_{n\ell} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}. \tag{B.86}$$

Using the result (B.77) to evaluate the integral in equation (B.85), the probability that a worker commutes to location i conditional on having chosen to live in location n is:

$$\lambda_{ni|n}^{R} = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{\left(B_{ni} w_i\right)^\epsilon \left(\kappa_{ni} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{\ell \in \mathbb{M}} \left(B_{n\ell} w_\ell\right)^\epsilon \left(\kappa_{n\ell} P_n^\alpha Q_n^{1-\alpha}\right)^{-\epsilon}}, \tag{B.87}$$

which further simplifies to:

$$\lambda_{ni|n}^{R} = \frac{\left(B_{ni}w_i/\kappa_{ni}\right)^{\epsilon}}{\sum_{\ell\in\mathbb{N}}\left(B_{n\ell}w_\ell/\kappa_{n\ell}\right)^{\epsilon}}. \tag{B.88}$$

Commuter market clearing requires that the measure of workers employed in each location i (L_i) equals the sum across all locations n of their measures of residents (R_n) times their conditional probabilities of commuting to i ($\lambda_{ni|n}^{R}$):

$$\begin{aligned} L_i &= \sum_{n\in\mathbb{N}} \lambda_{ni|n}^{R} R_n \\ &= \sum_{n\in\mathbb{N}} \frac{\left(B_{ni}w_i/\kappa_{ni}\right)^{\epsilon}}{\sum_{\ell\in\mathbb{N}}\left(B_{n\ell}w_\ell/\kappa_{n\ell}\right)^{\epsilon}} R_n, \end{aligned} \tag{B.89}$$

where, since there is a continuous measure of workers residing in each location, there is no uncertainty in the supply of workers to each employment location.

Expected worker income conditional on living in location n equals the wages in all possible workplace locations weighted by the probabilities of commuting to those locations conditional on living in n:

$$\begin{aligned} \bar{v}_n &= \mathbb{E}\left[w|n\right] \\ &= \sum_{i\in\mathbb{N}} \lambda_{ni|n}^{R} w_i, \\ &= \sum_{i\in\mathbb{N}} \frac{\left(B_{ni}w_i/\kappa_{ni}\right)^{\epsilon}}{\sum_{\ell\in\mathbb{N}}\left(B_{n\ell}w_\ell/\kappa_{n\ell}\right)^{\epsilon}} w_i, \end{aligned} \tag{B.90}$$

where $\mathbb{E}$ denotes the expectations operator and the expectation is taken over the distribution for idiosyncratic amenities. Intuitively, expected worker income is high in locations that have low commuting costs (low κ_{ni}) to high-wage employment locations.

Another implication of the Fréchet distribution of utility is that the distribution of utility conditional on residing in location n and commuting to location i is the same across all bilateral pairs of locations with positive residents and employment, and is equal to the distribution of utility for the economy as a whole. To establish this result, note that the distribution of utility conditional on residing in location n and commuting to location i is:

$$\begin{aligned} &= \frac{1}{\lambda_{ni}} \int_0^u \prod_{s\neq i} G_{ns}(u) \left[\prod_{k\neq n}\prod_{\ell\in\mathbb{N}} G_{k\ell}(u)\right] g_{ni}(u)du, \\ &= \frac{1}{\lambda_{ni}} \int_0^u \left[\prod_{k\in\mathbb{N}}\prod_{\ell\in\mathbb{N}} e^{-\Psi_{k\ell}u^{-\epsilon}}\right] \epsilon\Psi_{ni}u^{-(\epsilon+1)}du, \\ &= \frac{\Psi_{\mathbb{N}}}{\Psi_{ni}} \int_0^u e^{-\Psi_{\mathbb{N}}u^{-\epsilon}} \epsilon\Psi_{ni}u^{-(\epsilon+1)}du, \end{aligned} \tag{B.91}$$

$$= e^{-\Psi_{\mathbb{N}} u^{\epsilon}}.$$

On the one hand, lower land prices in location n or a higher wage in location i raise the utility of a worker with a given realization of idiosyncratic amenities b, and hence increase the expected utility of residing in n and working in i. On the other hand, lower land prices or a higher wage induce workers with lower realizations of idiosyncratic amenities b to reside in n and work in i, which reduces the expected utility of residing in n and working in i. With a Fréchet distribution of utility, these two effects exactly offset one another. Pairs of residence and employment locations with more attractive characteristics attract more commuters on the extensive margin until expected utility is the same across all pairs of residence and employment locations within the economy.

An implication of this result is that expected utility conditional on choosing a residence n and workplace i is the same across all residence-workplace pairs and equal to expected utility in the economy as a whole in equation (B.75):

$$U = \vartheta \Psi_{\mathbb{N}}^{1/\epsilon} = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_{k\ell} w_{\ell} \right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha} \right)^{-\epsilon} \right]^{1/\epsilon}. \tag{B.92}$$

C Isomorphisms

In our baseline quantitative urban model in Section 4 of the paper and Section B of this Online Appendix, we derive predictions for workplace and residence decisions in the canonical urban model with a single tradable final good and zero trade costs. In this section of the online appendix, we show that the same predictions for workplace and residence decisions hold in an entire class of quantitative urban models that make different assumptions about preferences, trade costs and market structure in the goods market.

In Section C.1, we show that these predictions continue to hold in an extension of this canonical urban model that incorporates non-traded goods. In Section C.2, we derive these same predictions in a new economic geography model with monopolistic competition, increasing returns to scale and trade costs, as in Helpman (1998), Redding and Sturm (2008) and Monte, Redding and Rossi-Hansberg (2018). In Section C.3, we show that this new economic geography model is isomorphic to a Ricardian spatial model with perfect competition, constant returns to scale and trade costs, as in Eaton and Kortum (2002) and Redding (2016). In Section C.4, we demonstrate an analogous isomorphism to an Armington spatial model with neoclassical production and trade costs, as in Armington (1969), Allen and Arkolakis (2014) and Allen, Arkolakis and Li (2017). All of these different model structures satisfy a gravity equation for bilateral commuting flows and yield expressions for employment by residence, employment by workplace and commuter market clearing that take the same form as in our baseline quantitative urban model.

C.1 Non-traded Goods Extension of Canonical Urban Model

We consider a city that is embedded in a wider economy. The city consists of a discrete set of locations (city blocks) indexed by $n, i \in \mathbb{N}$. Time is discrete and is indexed by t. There are two types of agents: workers and landlords. Workers are mobile across locations within the city. We consider two different assumptions about worker mobility with the wider economy: (i) A "closed-city" specification, with an exogenous measure of worker ($L_{\mathbb{N}t} = L_{\mathbb{N}}$), in which worker utility is endogenous; (ii) An "open-city" specification, in which the measure of workers ($L_{\mathbb{N}t}$) is endogenously determined by population mobility with a wider economy that provides a reservation level of utility $\overline{U}_t$. In the baseline version of the model, we assume a continuous measure of workers $L_{\mathbb{N}t}$, which ensures that the realized value of variables equals their expected values, and abstracts from any issues of granularity or small sample variation. We allow locations to differ from one another in terms of their attractiveness for production and residence, as determined by productivity in the traded and non-traded sectors, amenities, the supply of floor space, and transport connections, where each of these location characteristics can evolve over time.

C.1.1 Preferences

Worker preferences are defined over consumption of a homogeneous traded good, a homogeneous non-traded good, and residential floor space. The indirect utility function is assumed to take the Cobb-Douglas form such that utility for a worker ω residing in n and working in i is given by:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} \left(P_n^T\right)^{\alpha^T} \left(P_n^N\right)^{\alpha^N} Q_n^{1-\alpha^T-\alpha^N}}, \qquad 0 < \alpha^T, \alpha^N < 1, \qquad 0 < \alpha^T + \alpha^N < 1, \quad \text{(C.1)}$$

where we suppress the time subscript from now onwards, except where important; P_n^T is the price of the traded good; P_n^N is the price of the non-traded good; Q_n is the price of residential floor space; w_i is the wage; κ_{ni} is an iceberg commuting cost; B_n captures residential amenities that are common across all workers and could be endogenous to the surrounding concentration of economic activity through agglomeration forces; and $b_{ni}(\omega)$ is an idiosyncratic amenity draw that captures all the idiosyncratic factors that can cause an individual to live and work in particular locations within the city.

We assume that idiosyncratic amenities ($b_{ni}(\omega)$) are drawn from an independent extreme value (Fréchet) distribution for each residence-workplace pair and each worker:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \quad \text{(C.2)}$$

where we normalize the Fréchet scale parameter in equation (C.2) to one, because it enters worker choice probabilities isomorphically to common bilateral amenities B_n from equation (C.1); the

Fréchet shape parameter ϵ regulates the dispersion of idiosyncratic amenities, which controls the sensitivity of worker location decisions to economic variables (e.g. wages and the cost of living). The smaller the shape parameter ϵ, the greater the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables.

C.1.2 Production

The traded and non-traded goods are produced under conditions of perfect competition and constant returns to scale using labor, machinery capital and commercial floor space, where commercial floor space includes building capital and land. The production technology is assumed to take the Cobb-Douglas form with the following unit costs:

$$P_i^T = \frac{1}{A_i^T} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}, \qquad 0 < \beta^L, \beta^H, \beta^M < 1 \qquad \beta^L + \beta^H + \beta^M = 1, \tag{C.3}$$
$$P_i^N = \frac{1}{A_i^N} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M},$$

where A_i^T and A_i^N are the productivities of traded and non-traded production in location i; q_i is the price of commercial floor space; machinery is assumed to be perfectly mobile across locations with a common price r determined in the wider economy; and, for simplicity, we assume the same factor intensity in both sectors. We allow productivity in each sector (A_i^T, A_i^N) to potentially respond endogenously to changes in the surrounding concentration of economic activity through agglomeration forces. We assume that the traded good is costlessly traded such that:

$$P_i^T = P^T, \qquad \forall i \in \mathbb{N}. \tag{C.4}$$

From profit maximization and zero profits, payments to labor, commercial floor space and machinery capital are constant shares of revenue in each sector:

$$w_i L_i^T = \beta^L X_i^T, \qquad q_i H_i^T = \beta^H X_i^T, \qquad r M_i^T = \beta^M X_i^T \tag{C.5}$$
$$w_i L_i^N = \beta^L X_i^N, \qquad q_i H_i^N = \beta^H X_i^N, \qquad r M_i^N = \beta^M X_i^N$$

where L_n^T and L_n^N denote employment in the traded and non-traded sectors respectively; X_i^T and X_i^N correspond to revenue in the two sectors; H_i^T and H_i^N represent commercial floor space use in the two sectors; and M_i^T and M_i^N are machinery inputs in the two sectors. Therefore, total payments for commercial floor space across the two sectors together are proportional to total workplace income:

$$q_i H_i^L = q_i \left[H_i^T + H_i^N\right] = \frac{\beta^H}{\beta^L} w_i \left[L_i^T + L_i^N\right] = \frac{\beta^H}{\beta^L} w_i L_i. \tag{C.6}$$

Re-arranging equation (C.3), we obtain another key implication of profit maximization and zero profits in each sector for each location with positive production:

$$\begin{aligned} w_i &= \left(P^T A_i^T\right)^{1/\beta^L} q_i^{-\beta^H/\beta^L} r^{-\beta^M/\beta^L}, \\ w_i &= \left(P_i^N A_i^N\right)^{1/\beta^L} q_i^{-\beta^H/\beta^L} r^{-\beta^M/\beta^L}. \end{aligned} \tag{C.7}$$

Intuitively, the maximum wage (w_i) that a location can afford to pay in the traded sector is increasing in its productivity (A_i^T) and the price of the final good (P^T) and decreasing in the price of commercial floor space (q_i) and the common price of machinery capital (r). In equilibrium, each location produces both the traded and non-traded final good, and the wage (w_i) and the price of the non-traded good (P_i^N) adjust, such that zero profits are made in both sectors.

C.1.3 Market Clearing

Land market clearing implies that the total income received by landlords as owners of floor space (which equals rateable value ($\mathbb{Q}_n$) in our data) equals the sum of payments for the use of residential and commercial floor space:

$$\mathbb{Q}_n = Q_n H_n^R + q_n H_n^L = (1-\alpha)\, v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n. \tag{C.8}$$

where H_n^R is residential floor space use; rateable values ($\mathbb{Q}_n$) equal the sum of prices times quantities for both residential and commercial floor space use; $\alpha = \alpha^T + \alpha^N$ is the overall share of expenditure on consumption goods; v_n is the per capita income of location n's residents, as determined below as a function of commuting patterns; and R_n is the measure of these residents.

C.1.4 Workplace and Residence Choices

Using indirect utility (C.1) and the Fréchet distribution of idiosyncratic amenities (C.2), this extension of the canonical urban model exhibits a gravity equation for commuting flows. Following the same analysis as in Section B.8 of this Online Appendix, the probability that a worker chooses to reside in location $n \in \mathbb{N}$ and work in location $i \in \mathbb{N}$ is given by:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} \left(P_n^T\right)^{\alpha^T} \left(P_n^N\right)^{\alpha^N} Q_n^{1-\alpha^T-\alpha^N}\right)^{-\epsilon}}{\sum_{k\in\mathbb{N}} \sum_{\ell\in\mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} \left(P_k^T\right)^{\alpha^T} \left(P_k^N\right)^{\alpha^N} Q_k^{1-\alpha^T-\alpha^N}\right)^{-\epsilon}}, \qquad n, i \in \mathbb{N}, \tag{C.9}$$

which takes the same form as in our baseline quantitative urban model, except that the consumption goods price index is now disaggregated into traded and non-traded goods.

Summing across workplaces $i \in \mathbb{N}$, we obtain the probability that a worker lives in residence $n \in \mathbb{N}$ ($\lambda_n^R = R_n/L_{\mathbb{N}}$). Similarly, summing across residences $n \in \mathbb{N}$, we obtain the probability that a worker is employed in workplace $i \in \mathbb{N}$ ($\lambda_n^L = L_i / L_{\mathbb{N}}$):

$$\lambda_n^R = \frac{\sum_{i \in \mathbb{N}} (B_n w_i)^{\epsilon} \left(\kappa_{ni} \left(P_n^T\right)^{\alpha^T} \left(P_n^N\right)^{\alpha^N} Q_n^{1-\alpha^T-\alpha^N} \right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} \left(P_k^T\right)^{\alpha^T} \left(P_k^N\right)^{\alpha^N} Q_k^{1-\alpha^T-\alpha^N} \right)^{-\epsilon}}, \tag{C.10}$$

$$\lambda_i^L = \frac{\sum_{n \in \mathbb{N}} (B_n w_i)^{\epsilon} \left(\kappa_{ni} \left(P_n^T\right)^{\alpha^T} \left(P_n^N\right)^{\alpha^N} Q_n^{1-\alpha^T-\alpha^N} \right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} \left(P_k^T\right)^{\alpha^T} \left(P_k^N\right)^{\alpha^N} Q_k^{1-\alpha^T-\alpha^N} \right)^{-\epsilon}}. \tag{C.11}$$

Both expressions take the same form as in our baseline quantitative urban model, with the consumption goods price index disaggregated into traded and non-traded goods.

From equations (C.9) and (C.10), the conditional probability that a worker commutes to location i conditional on residing in location n also takes the same form as in our baseline quantitative urban model:

$$\lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{(w_i / \kappa_{ni})^{\epsilon}}{\sum_{\ell \in \mathbb{N}} (w_\ell / \kappa_{n\ell})^{\epsilon}}. \tag{C.12}$$

Using this commuting probability conditional on residence ($\lambda_{ni|n}^R$) from equation (C.12), we obtain an identical expression for per capita income by residence as in our baseline quantitative urban model:

$$v_n = \sum_{i \in \mathbb{N}} \lambda_{ni|n}^R w_i. \tag{C.13}$$

Commuter market clearing implies that employment in each location (L_i) equals the measure of workers choosing to commute to that location. Using the commuting probabilities conditional on residence from equation (C.12), we obtain the same expression for this commuter market clearing condition as in our baseline quantitative urban model:

$$L_i = \sum_{n \in \mathbb{N}} \lambda_{ni|n}^R R_n. \tag{C.14}$$

Finally, using the Fréchet distribution for idiosyncratic amenities (C.2), expected utility conditional on choosing a residence-workplace pair ($\bar{U}$) is equalized across all residence-workplace pairs in the economy and takes the same form as in our baseline quantitative urban model:

$$U = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} \left(P_k^T\right)^{\alpha^T} \left(P_k^N\right)^{\alpha^N} Q_k^{1-\alpha^T-\alpha^N} \right)^{-\epsilon} \right]^{\frac{1}{\epsilon}}, \tag{C.15}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\vartheta \equiv \Gamma((\epsilon - 1)/\epsilon)$; and $\Gamma(\cdot)$ is the Gamma function.

C.2 New Economic Geography Model

We now derive the gravity equation for commuting flows in a new economic geography model following Helpman (1998), Redding and Sturm (2008) and Monte, Redding and Rossi-Hansberg (2018). We consider a city that is embedded in a wider economy. The city consists of a discrete set of locations (city blocks) indexed by $n, i \in \mathbb{N}$. Time is discrete and is indexed by t. There are two types of agents: workers and landlords. Workers are mobile across locations within the city. We consider two different assumptions about worker mobility with the wider economy: (i) A "closed-city" specification, with an exogenous measure of worker ($L_{\mathbb{N}t} = L_{\mathbb{N}}$), in which worker utility is endogenous; (ii) An "open-city" specification, in which the measure of workers ($L_{\mathbb{N}t}$) is endogenously determined by population mobility with a wider economy that provides a reservation level of utility $\overline{U}_t$. In the baseline version of the model, we assume a continuous measure of workers $L_{\mathbb{N}t}$, which ensures that the realized value of variables equals their expected values, and abstracts from any issues of granularity or small sample variation. We allow locations to differ from one another in terms of their attractiveness for production and residence, as determined by productivity, amenities, the supply of floor space, and transport connections, where each of these location characteristics can evolve over time.

C.2.1 Preferences and Endowments

The preferences of a worker ω who lives in location n and works in location i are defined over final goods consumption ($C_n(\omega)$), residential floor space use ($H_n^R(\omega)$), iceberg commuting costs (κ_{ni}), common amenities for all workers (B_n), and an idiosyncratic amenity draw for an individual worker for each residence-workplace pair ($b_{ni}(\omega)$), according to the following Cobb-Douglas functional form:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega)}{\kappa_{ni}} \left(\frac{C_n(\omega)}{\alpha} \right)^{\alpha} \left(\frac{H_n^R(\omega)}{1-\alpha} \right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{C.16}$$

where we suppress the time subscript from now onwards, except where important. The idiosyncratic amenities shock for worker ω for each residence n and workplace i ($b_{ni}(\omega)$) is drawn from an independent Fréchet distribution:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{C.17}$$

where we normalize the Fréchet scale parameter in equation (C.17) to one, because it enters worker choice probabilities isomorphically to common bilateral amenities B_n from equation (C.16); the Fréchet shape parameter $\epsilon > 1$ regulates the dispersion of idiosyncratic amenities, which controls the sensitivity of worker location decisions to economic variables (e.g. wages and the cost of living). The smaller is the shape parameter ϵ, the greater is the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables.

All workers ω residing in location n and working in location i receive the same wage and make the same choices for consumption and residential floor space use. Therefore, we suppress the implicit dependence on ω, except where important.

The goods consumption index in location n takes the constant elasticity of substitution (CES) or Dixit-Stiglitz form and is defined over a continuum of varieties sourced from each location i,

$$C_n = \left[\sum_{i \in \mathbb{N}} \int_0^{\mathcal{M}_i} c_{ni}(j)^{\rho} dj \right]^{\frac{1}{\rho}}, \qquad \sigma = \frac{1}{1-\rho} > 1, \tag{C.18}$$

where $c_{ni}(j)$ is consumption in location n of an individual variety j produced in location i; $\mathcal{M}_i$ is the mass of varieties produced in location i; and ρ is the CES parameter that determines the elasticity of substitution between varieties ($\sigma = 1/(1-\rho) > 1$).

Using the properties of CES preferences (C.18), the equilibrium consumption in location n of each variety j sourced from location i is determined by:

$$c_{ni}(j) = E_n P_n^{\sigma-1} p_{ni}(j)^{-\sigma}, \tag{C.19}$$

where $E_n = P_n C_n$ is total expenditure on consumption goods in location n; P_n is the price index dual to the consumption index (C.18); and $p_{ni}(j)$ is the "cost inclusive of freight" price of variety j produced in location i and consumed in location n.

Goods can be traded between locations subject to iceberg variable trade costs, such that $d_{ni} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n (where $d_{ni} > 1$ for $n \neq i$ and $d_{nn} = 1$). The "cost inclusive of freight" price of a variety in the location of consumption n ($p_{ni}(j)$) is thus a constant multiple of the "free on board" price of that variety in the location of production i ($p_i(j)$), with that multiple determined by these iceberg trade costs:

$$p_{ni}(j) = d_{ni} p_i(j). \tag{C.20}$$

C.2.2 Production

Production is modelled as in the new economic geography literature following Krugman (1991) and Helpman (1998). Varieties are produced under conditions of monopolistic competition using labor, machinery capital, and commercial floor space, where commercial floor space includes both building capital and land. To produce a variety, a firm must incur both a fixed cost and a constant variable cost. We assume that these fixed and variable costs use the three factors of production with the same factor intensity, such that the production technology is homothetic. We allow the variable cost to vary with location productivity A_i, such that the total cost of producing $y_i(j)$

units of a variety j in location i is given by:

$$\Gamma_i(j) = \left(F + \frac{y_i(j)}{A_i}\right) w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}, \qquad 0 < \beta^L, \beta^H, \beta^M < 1, \qquad \beta^L + \beta^H + \beta^M = 1, \tag{C.21}$$

where w_i is the wage; q_i is the price of commercial floor space in location i; and machinery is assumed to be perfectly mobile across locations with a common price r determined in the wider economy. We also allow productivity (A_i) to potentially respond endogenously to changes in the surrounding concentration of economic activity through agglomeration forces. Profit maximization implies that equilibrium variety prices are a constant mark-up over marginal cost:

$$p_{ni}(j) = p_{ni} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}}{A_i}. \tag{C.22}$$

Profit maximization and zero profits imply that the equilibrium output of each variety is the same for all varieties produced in location i:

$$y_i(j) = \bar{y}_i = A_i F \left(\sigma - 1\right). \tag{C.23}$$

Using the equilibrium pricing rule (C.22) and zero profits (C.23), free on board revenue ($x_i(j) = p_i(j) y_i(j)$) for each variety j in location i can be written as:

$$p_i(j) y_i(j) = x_i(j) = \bar{x}_i = \sigma w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} F, \tag{C.24}$$

and the common equilibrium wage bill for each variety j in location i is given by:

$$w_i l_i(j) = w_i \bar{l}_i = \beta^L \bar{x}_i, \tag{C.25}$$

where $l_i(j) = \bar{l}_i$ is workplace employment for variety j in location i.

Aggregating across all varieties produced within location i, profit maximization and zero profits imply that payments for labor, commercial floor space and machinery capital are constant shares of revenue:

$$w_i L_i = \beta^L X_i, \qquad q_i H_i^L = \beta^H X_i \qquad r M_i = \beta^M X_i, \tag{C.26}$$

where L_i is total workplace employment; $X_i = \mathcal{M}_i \bar{x}_i$ is aggregate revenue; H_i^L denotes total commercial floor space use; and M_i is total machinery capital use. Therefore, payments for commercial floor space are proportional to workplace income ($w_i L_i$):

$$q_i H_i^L = \frac{\beta^H}{\beta^L} w_i L_i. \tag{C.27}$$

C.2.3 Trade and Market Clearing

We assume that floor space is owned by landlords, who receive income from residents' and firms' expenditure on floor space, and consume only consumption goods where they live. Total expenditure on consumption goods equals the fraction α of the total income of residents plus the entire income of landlords. This income of landlords equals $(1-\alpha)$ times the total income of residents plus β^H times revenue (which equals β^H/β^L times the total income of workers). Therefore, total expenditure on consumption goods is:

$$E_n = P_n C_n = \alpha v_n R_n + (1-\alpha) v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n = v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n, \tag{C.28}$$

where v_n is the per capita income of location n's residents, as determined below as a function of commuting patterns, and R_n is the measure of these residents.

This new economic geography model implies a gravity equation for bilateral trade in goods between locations. Using CES demand in equation (C.19), and the fact that all varieties supplied from location i to location n charge the same price in equation (C.22), the share of location n's expenditure on goods produced in location i can be written as:

$$\pi_{ni} = \frac{\mathcal{M}_i p_{ni}^{1-\sigma}}{\sum_{k \in \mathbb{N}} \mathcal{M}_k p_{nk}^{1-\sigma}} = \frac{\mathcal{M}_i \left(d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} / A_i \right)^{1-\sigma}}{\sum_{k \in \mathbb{N}} \mathcal{M}_k \left(d_{nk} w_k^{\beta^L} q_k^{\beta^H} r^{\beta^M} / A_k \right)^{1-\sigma}}. \tag{C.29}$$

Therefore trade between locations n and i depends on bilateral trade costs (d_{ni}) in the numerator ("bilateral resistance") and on trade costs to all possible sources of supply k in the denominator ("multilateral resistance"). Goods market clearing and zero profits imply that payments to workers plus payments for commercial floor space use in each location equal expenditure on goods produced in that location:

$$w_i L_i + q_i H_i^L + r M_i = \left[1 + \frac{\beta^H}{\beta^L} + \frac{\beta^M}{\beta^L} \right] w_i L_i = \sum_{n \in \mathbb{N}} \pi_{ni} E_n. \tag{C.30}$$

Using equilibrium prices (C.22), the price index dual to the consumption index (C.18) can be rewritten as:

$$P_n = \left[\sum_{i \in \mathbb{N}} \mathcal{M}_i \left(\frac{\sigma}{\sigma - 1} \frac{d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}}{A_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left(\frac{\mathcal{M}_n}{\pi_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \frac{w_n^{\beta^L} q_n^{\beta^H} r^{\beta^M}}{A_n}, \tag{C.31}$$

where the second equation uses the domestic trade share (π_{nn}) from equation (C.29) and $d_{nn} = 1$.

Labor market clearing implies that total payments to labor in each location equal the mass of varieties times labor payments for each variety. Using this relationship and the Cobb-Douglas production technology, the mass of varieties ($\mathcal{M}_i$) in each location can be written as a function of total labor payments ($w_i L_i$) and firm revenue ($\bar{x}_i$) in that location:

$$\mathcal{M}_i = \frac{w_i L_i}{w_i \bar{l}_i} = \frac{w_i L_i}{\beta^L \bar{x}_i}, \tag{C.32}$$

where L_i is total employment.

Land market clearing implies that the total income received by landlords as owners of floor space (which equals rateable value ($\mathbb{Q}_n$) in our data) equals the sum of payments for the use of residential and commercial floor space:

$$\mathbb{Q}_n = Q_n H_n^R + q_n H_n^L = (1-\alpha) v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n, \tag{C.33}$$

where H_n^R is residential floor space use.

C.2.4 Workplace and Residence Choices

Given the direct utility function (C.16), the corresponding indirect utility function for a worker ω residing in location n and working in location i is:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \tag{C.34}$$

which takes the same form as in our baseline quantitative urban model. The only difference from our baseline model is the underlying determinants of the price index for goods consumption (P_n), as now specified in equation (C.31).

Using indirect utility (C.34) and the Fréchet distribution of idiosyncratic amenities (C.17), this new economic geography model exhibits the same gravity equation predictions for commuting flows as in our baseline quantitative urban model. Following the same analysis as in Section B.8 of this Online Appendix, the probability that a worker chooses to reside in location $n \in \mathbb{N}$ and work in location $i \in \mathbb{N}$ is given by:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{(B_n w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \qquad n, i \in \mathbb{N}, \tag{C.35}$$

which is identical to the specification in our baseline quantitative urban model, except that the price index for goods consumption (P_n) is now determined by equation (C.31).

Summing across workplaces $i \in \mathbb{N}$, we obtain the probability that a worker lives in each residence $n \in \mathbb{N}$ ($\lambda_n^R = R_n / L_{\mathbb{N}}$). Similarly, summing across residences $n \in \mathbb{N}$, we obtain the probability that a worker is employed in each workplace $i \in \mathbb{N}$:

$$\lambda_n^R = \frac{\sum_{i \in \mathbb{N}} (B_{ni} w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \quad \lambda_i^L = \frac{\sum_{n \in \mathbb{N}} (B_{ni} w_i)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_{k\ell} w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{C.36}$$

where both expressions are the same as in our baseline quantitative urban model.

From equations (C.35) and (C.36), the conditional probability that a worker commutes to location i conditional on residing in location n takes the same form as in our baseline quantitative urban model:

$$\lambda^R_{ni|n} = \frac{\lambda_{ni}}{\lambda^R_n} = \frac{(w_i/\kappa_{ni})^{\epsilon}}{\sum_{\ell \in \mathbb{N}} (w_\ell/\kappa_{n\ell})^{\epsilon}}. \tag{C.37}$$

Using this commuting probability conditional on residence ($\lambda^R_{ni|n}$) from equation (C.37), we obtain an identical expression for per capita income conditional on living in location n as in our baseline quantitative urban model:

$$v_n = \sum_{i \in \mathbb{N}} \lambda^R_{ni|n} w_i. \tag{C.38}$$

Commuter market clearing again implies that employment in each location (L_i) equals the measure of workers choosing to commute to that location. Using the commuting probabilities conditional on residence from equation (C.37), we obtain the same expression for this commuter market clearing condition as in our baseline quantitative urban model:

$$L_i = \sum_{n \in \mathbb{N}} \lambda^R_{ni|n} R_n. \tag{C.39}$$

Finally, using the Fréchet distribution for idiosyncratic amenities (C.17), expected utility conditional on choosing a residence-workplace pair ($\bar{U}$) is equalized across all residence-workplace pairs in the economy and takes the same form as in our baseline quantitative urban model:

$$U = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} (B_k w_\ell)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha} \right)^{-\epsilon} \right]^{\frac{1}{\epsilon}}, \tag{C.40}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\vartheta \equiv \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)$; and $\Gamma(\cdot)$ is the Gamma function.

C.3 Ricardian Spatial Model

We now derive the gravity equation for commuting flows in a Ricardian spatial model following Eaton and Kortum (2002) and Redding (2016). We consider a city that is embedded in a wider economy. The city consists of a discrete set of locations (city blocks) indexed by $n, i \in \mathbb{N}$. Time is discrete and is indexed by t. There are two types of agents: workers and landlords. Workers are mobile across locations within the city. We consider two different assumptions about worker mobility with the wider economy: (i) A "closed-city" specification, with an exogenous measure of worker ($L_{\mathbb{N}t} = L_{\mathbb{N}}$), in which worker utility is endogenous; (ii) An "open-city" specification, in which the measure of workers ($L_{\mathbb{N}t}$) is endogenously determined by population mobility with a wider economy that provides a reservation level of utility $\overline{U}_t$. In the baseline version of the model, we assume a continuous measure of workers $L_{\mathbb{N}t}$, which ensures that the realized value of variables equals their expected values, and abstracts from any issues of granularity or small sample variation. We allow locations to differ from one another in terms of their attractiveness for production and residence, as determined by productivity, amenities, the supply of floor space, and transport connections, where each of these location characteristics can evolve over time.

C.3.1 Preferences and Endowments

The preferences of a worker ω who lives in location n and works in location i are defined over final goods consumption ($C_n(\omega)$), residential floor space use ($H_n^R(\omega)$), iceberg commuting costs (κ_{ni}), common amenities for all workers (B_n), and an idiosyncratic amenity draw for an individual worker for each residence-workplace pair ($b_{ni}(\omega)$), according to the following Cobb-Douglas functional form:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega)}{\kappa_{ni}} \left(\frac{C_n(\omega)}{\alpha} \right)^{\alpha} \left(\frac{H_n^R(\omega)}{1-\alpha} \right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{C.41}$$

where we suppress the time subscript from now onwards, except where important. The idiosyncratic amenities shock for worker ω for each residence n and workplace i ($b_{ni}(\omega)$) is drawn from an independent Fréchet distribution:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{C.42}$$

where we normalize the Fréchet scale parameter in equation (C.42) to one, because it enters worker choice probabilities isomorphically to common bilateral amenities B_{ni} from equation (C.41); the Fréchet shape parameter $\epsilon > 1$ regulates the dispersion of idiosyncratic amenities, which controls the sensitivity of worker location decisions to economic variables (e.g. wages and the cost of living). The smaller is the shape parameter ϵ, the greater is the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables. All workers ω residing in location n and working in location i receive the same wage and make the same choices for consumption and residential floor space use. Therefore, we suppress the implicit dependence on ω, except where important.

The goods consumption index for location n takes the constant elasticity of substitution (CES) form and is defined over a fixed continuum of goods $j \in [0, 1]$:

$$C_n = \left[\int_0^1 c_n(j)^{\rho} dj \right]^{\frac{1}{\rho}}, \tag{C.43}$$

where $c_n(j)$ is consumption of good j in country n; the CES parameter (ρ) determines the elasticity of substitution between goods ($\sigma = 1/(1-\rho) > 1$). The corresponding dual price index for goods consumption (P_n) is:

$$P_n = \left[\int_0^1 p_n(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}, \qquad \sigma = \frac{1}{1-\rho} > 1, \tag{C.44}$$

where $p_n(j)$ is the price of good j in country n.

C.3.2 Production

Each good j can be produced in each location i under conditions of perfect competition using labor, machinery capital and commercial floor space, where commercial floor space includes both building capital and land. The production technology is assumed to take the Cobb-Douglas form. If a good is produced by a location, the requirement of zero profits implies that the good's "free on board" price must equal its constant unit cost of production:

$$p_i(j) = \frac{w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}}{z_i(j)}, \qquad 0 < \beta^L, \beta^H, \beta^M < 1, \qquad \beta^L + \beta^H + \beta^M = 1, \tag{C.45}$$

where w_i denotes the wage; q_i represents the price of commercial floor space in location i; machinery is assumed to be perfectly mobile across locations with a common price r determined in the wider economy; $z_i(j)$ is productivity; and to focus on Ricardian reasons for trade, we assume that factor intensity is the same for all goods, as controlled by (β^L, β^H, β^M).

Each location i draws an idiosyncratic productivity $z_i(j)$ for each good j from an independent Fréchet distribution:

$$F_i(z) = e^{-A_i z^{-\theta}}, \qquad A_i > 0, \quad \theta > 1, \tag{C.46}$$

where the scale parameter A_i determines average productivity for location i and the shape parameter θ controls the dispersion of productivity across goods. We allow this scale parameter that determines each location's average productivity to potentially respond endogenously to changes in the surrounding concentration of economic activity through agglomeration forces.

Goods can be traded between locations subject to iceberg variable trade costs, such that $d_{ni} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n (where $d_{ni} > 1$ for $n \neq i$ and $d_{nn} = 1$). The "cost inclusive of freight" price of a good in the location of consumption n ($p_{ni}(j)$) is thus a constant multiple of the "free on board" price of that good in the location of production i ($p_i(j)$) with that multiple determined by the iceberg trade costs:

$$p_{ni}(j) = d_{ni} p_i(j). \tag{C.47}$$

Combining equations (C.45) and (C.47), the cost to a consumer in location n of purchasing one unit of good j from location i is given by:

$$p_{ni}(j) = \frac{d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M}}{z_i(j)}. \tag{C.48}$$

From profit maximization and zero profits, payments to labor, commercial floor space and machinery capital are constant shares of revenue:

$$w_i L_i = \beta^L X_i, \qquad q_i H_i^L = \beta^H X_i, \qquad r M_i = \beta^M X_i, \tag{C.49}$$

where L_i is workplace employment; X_i denotes revenue; H_i^L represents commercial use of floor space; and M_i is machinery capital use. Therefore, payments for commercial floor space are proportional to workplace income:

$$q_i H_i^L = \frac{\beta^H}{\beta^L} w_i L_i. \tag{C.50}$$

C.3.3 Trade and Market Clearing

We assume that floor space is owned by landlords, who receive income from residents' expenditure on floor space, and consume only consumption goods where they live. Total expenditure on consumption goods equals the fraction α of the total income of residents plus the entire income of landlords. This income of landlords equals $(1-\alpha)$ times the total income of residents plus β^H times revenue (which equals β^H/β^L times the total income of workers). Therefore total expenditure on consumption goods is:

$$E_n = P_n C_n = \alpha v_n R_n + (1-\alpha) v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n = v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n, \tag{C.51}$$

where v_n is the average income of location n's residents, as determined below as a function of commuting patterns, and R_n is the measure of these residents.

This Ricardian spatial model also implies a gravity equation for bilateral trade in goods between locations. Goods are homogeneous in the sense that one unit of a given good is the same as any other unit of that good. Therefore, the representative consumer in a given location sources each good from the lowest-cost supplier to that location. Using equilibrium prices (C.48) and the properties of the Fréchet distribution following Eaton and Kortum (2002), the share of the expenditure of location n on goods produced by location i is:

$$\pi_{ni} = \frac{A_i \left(d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} \right)^{-\theta}}{\sum_{k \in \mathbb{N}} A_k \left(d_{nk} w_k^{\beta^L} q_k^{\beta^H} r^{\beta^M} \right)^{-\theta}}, \tag{C.52}$$

where the elasticity of trade flows to trade costs is determined by the Fréchet shape parameter for productivity θ.

Goods market clearing and zero profits imply that payments to workers plus payments for commercial floor space use plus payments for machinery use in each location equal expenditure on goods produced in that location:

$$w_i L_i + q_i H_i^L + r M_i = \sum_{n \in \mathbb{N}} \pi_{ni} E_n. \tag{C.53}$$

Using equilibrium prices (C.48) and the properties of the Fréchet distribution, the consumption goods price index in equation (C.44) can be rewritten as:

$$P_n = \gamma \left[\sum_{i \in \mathbb{N}} A_i \left(d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{C.54}$$

where $\gamma \equiv \left[\Gamma \left(\frac{\theta - (\sigma - 1)}{\theta} \right) \right]^{\frac{1}{1-\sigma}}$; $\Gamma(\cdot)$ denotes the Gamma function; and we require $\theta > \sigma - 1$ to ensure a finite value for the price index.

Using the trade share (C.52), and noting that $d_{nn} = 1$, the consumption goods price index in equation (C.54) can be further rewritten solely in terms of the domestic trade share (π_{nn}), wages (w_n), the price of commercial floor space (q_n), the common price of machinery capital (r) and parameters:

$$P_n = \gamma \left(\frac{A_n}{\pi_{nn}} \right)^{-\frac{1}{\theta}} \left(w_n^{\beta^L} q_n^{\beta^H} r^{\beta^M} \right). \tag{C.55}$$

Land market clearing implies that the total income received by landlords as owners of floor space (which equals rateable value ($\mathbb{Q}_n$) in our data) equals the sum of payments for the use of residential and commercial floor space:

$$\mathbb{Q}_n = Q_n H_n^R + q_n H_n^L = (1 - \alpha) v_n R_n + \left(\frac{\beta^H}{\beta^L} \right) w_n L_n, \tag{C.56}$$

where H_n^R denotes residential floor space use.

C.3.4 Workplace and Residence Choices

Given the direct utility function (C.41), the corresponding indirect utility function for a worker ω residing in location n and working in location i is:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \tag{C.57}$$

which takes the same form as in our baseline quantitative urban model. The only difference from the canonical urban model is in the underlying determinants of the price index for goods consumption (P_n), as now specified in equation (C.54).

Using indirect utility (C.57) and the Fréchet distribution of idiosyncratic amenities (C.42), this Ricardian spatial model exhibits the same gravity equation predictions for commuting flows as in our baseline quantitative urban model. Following the same analysis as in Section B.8 of this online appendix, the probability that a worker chooses to reside in location $n \in \mathbb{N}$ and work in location $i \in \mathbb{N}$ conditional on choosing a residence-workplace pair in Greater London (λ_{ni}) is given by:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{\left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \qquad n, i \in \mathbb{N}, \tag{C.58}$$

which is the same as in our baseline quantitative urban model, except that the price index for goods consumption (P_n) is now determined by equation (C.54).

Summing across workplaces $i \in \mathbb{N}$, we obtain the probability that a worker lives in each residence $n \in \mathbb{N}$ ($\lambda_n^R = R_n / L_{\mathbb{N}}$). Similarly, summing across residences $n \in \mathbb{N}$, we obtain the probability that a worker is employed in each workplace $i \in \mathbb{N}$ ($\lambda_i^L = L_i / L_{\mathbb{N}}$):

$$\lambda_n^R = \frac{\sum_{i \in \mathbb{N}} \left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \quad \lambda_i^L = \frac{\sum_{n \in \mathbb{N}} \left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}. \tag{C.59}$$

From equations (C.58) and (C.59), the conditional probability that a worker commutes to location i conditional on residing in location n takes the same form as in our baseline quantitative urban model:

$$\lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{\left(w_i / \kappa_{ni}\right)^{\epsilon}}{\sum_{\ell \in \mathbb{N}} \left(w_\ell / \kappa_{n\ell}\right)^{\epsilon}}. \tag{C.60}$$

Using this commuting probability conditional on residence ($\lambda_{ni|n}^R$) from equation (C.60), we obtain an identical expression for per capita income conditional on living in location n as in our baseline quantitative urban model:

$$v_n = \sum_{i \in \mathbb{N}} \lambda_{ni|n}^R w_i. \tag{C.61}$$

Commuter market clearing again implies that employment in each location (L_i) equals the measure of workers choosing to commute to that location. Using the commuting probabilities conditional on residence from equation (C.60), we obtain the same expression for this commuter market clearing condition as in our baseline quantitative urban model:

$$L_i = \sum_{n \in \mathbb{N}} \lambda_{ni|n}^R R_n. \tag{C.62}$$

Finally, using the Fréchet distribution for idiosyncratic amenities (C.42), expected utility conditional on choosing a residence-workplace pair ($\bar{U}$) is equalized across all residence-workplace pairs in the economy and takes the same form as in our baseline quantitative urban model:

$$U = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon} \right]^{\frac{1}{\epsilon}}, \tag{C.63}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\vartheta \equiv \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)$; and $\Gamma(\cdot)$ is the Gamma function.

C.4 Armington Model

Finally, we derive our predictions for the impact of the removal of the railway network in an Armington spatial model following Armington (1969), Allen and Arkolakis (2014) and Allen, Arkolakis and Li (2017). We consider a city that is embedded in a wider economy. The city consists of a discrete set of locations (city blocks) indexed by $n, i \in \mathbb{N}$. Time is discrete and is indexed by t. There are two types of agents: workers and landlords. Workers are mobile across locations within the city. We consider two different assumptions about worker mobility with the wider economy: (i) A "closed-city" specification, with an exogenous measure of worker ($L_{\mathbb{N}t} = L_{\mathbb{N}}$), in which worker utility is endogenous; (ii) An "open-city" specification, in which the measure of workers ($L_{\mathbb{N}t}$) is endogenously determined by population mobility with a wider economy that provides a reservation level of utility $\overline{U}_t$. In the baseline version of the model, we assume a continuous measure of workers $L_{\mathbb{N}t}$, which ensures that the realized value of variables equals their expected values, and abstracts from any issues of granularity or small sample variation. We allow locations to differ from one another in terms of their attractiveness for production and residence, as determined by productivity, amenities, the supply of floor space, and transport connections, where each of these location characteristics can evolve over time.

C.4.1 Preferences and Endowments

The preferences of a worker ω who lives in location n and works in location i are defined over final goods consumption ($C_n(\omega)$), residential floor space use ($H_n^R(\omega)$), iceberg commuting costs (κ_{ni}), common bilateral amenities for all workers (B_n), and an idiosyncratic amenity draw for an individual worker for each residence-workplace pair ($b_{ni}(\omega)$), according to the Cobb-Douglas functional form:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega)}{\kappa_{ni}} \left(\frac{C_n(\omega)}{\alpha}\right)^{\alpha} \left(\frac{H_n^R(\omega)}{1-\alpha}\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{C.64}$$

where we suppress the time subscript from now onwards, except where important. The idiosyncratic amenities shock for worker ω for each residence n and workplace i ($b_{ni}(\omega)$) is drawn from an independent Fréchet distribution:

$$G_{ni}(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{C.65}$$

where we normalize the Fréchet scale parameter in equation (C.65) to one, because it enters worker choice probabilities isomorphically to common bilateral amenities (B_{ni}) from equation

(C.64); the Fréchet shape parameter $\epsilon > 1$ regulates the dispersion of idiosyncratic amenities, which controls the sensitivity of worker location decisions to economic variables (e.g. wages and the cost of living). The smaller is the shape parameter ϵ, the greater is the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables. All workers ω residing in location n and working in location i receive the same wage and make the same choices for consumption and residential floor space use. Therefore, we suppress the implicit dependence on ω from now onwards, except where important.

Consumption goods are assumed to be differentiated by location of origin according to the constant elasticity of substitution (CES) functional form. Therefore the consumption index in location n is:

$$C_n = \left[\sum_{i \in \mathbb{N}} c_{ni}^{\rho} \right]^{\frac{1}{\rho}}, \tag{C.66}$$

where c_{ni} denotes consumption in location n of the good produced by location i; and the CES parameter (ρ) determines the elasticity of substitution between the goods produced by each location ($\sigma = 1/(1-\rho) > 1$).

In this specification with differentiation by location of origin, the CES functional form implies that the marginal utility of consuming a location's good approaches infinity as consumption of that good converges to zero. Therefore, in equilibrium, each location consumes the goods produced by all locations. Using the properties of the CES functional form, the corresponding dual price index for goods consumption (P_n) is:

$$P_n = \left[\sum_{i \in \mathbb{N}} p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad \sigma = \frac{1}{1-\rho} > 1, \tag{C.67}$$

where p_{ni} denotes the price in country n of the good produced by country i.

C.4.2 Production

Goods from each location of origin are produced under conditions of perfect competition using labor, machinery capital and commercial floor space, where commercial floor space includes both building capital and land. We assume that the production technology takes the Cobb-Douglas form. Using zero profits and the fact that the goods of all locations are consumed and produced in equilibrium, the "free on board" price of each location's good equals its constant unit cost of production:

$$p_i = w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} / A_i, \qquad 0 < \beta^L, \beta^H, \beta^M < 1, \qquad \beta^L + \beta^H + \beta^M = 1, \tag{C.68}$$

where A_i denotes productivity; w_i is the wage; q_i corresponds to the price of commercial floor space; and machinery is assumed to be perfectly mobile across locations with a common price r

determined in the wider economy. We allow productivity A_i in each location to respond endogenously to changes in the surrounding concentration of economic activity through agglomeration forces.

Goods can be traded between locations subject to iceberg variable trade costs, such that $d_{ni} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n (where $d_{ni} > 1$ for $n \neq i$ and $d_{nn} = 1$). The "cost inclusive of freight" price of a good in the location of consumption n (p_{ni}) is thus a constant multiple of the "free on board" price of that good in the location of production i (p_i) with that multiple determined by the iceberg trade costs:

$$p_{ni} = d_{ni} p_i. \tag{C.69}$$

Combining equations (C.68) and (C.69), the cost to the consumer in location n of purchasing the good produced by location i is:

$$p_{ni} = d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} / A_i. \tag{C.70}$$

From profit maximization and zero profits, payments to labor, commercial floor space and machinery capital are constant shares of revenue:

$$w_i L_i = \beta^L X_i, \qquad q_i H_i^L = \beta^H X_i, \qquad r M_i = \beta^M X_i, \tag{C.71}$$

where L_i is employment; X_i is revenue; H_i^L denotes commercial floor space use; and M_i corresponds to machinery capital use. Therefore, payments for commercial floor space are proportional to workplace income ($w_i L_i$):

$$q_i H_i^L = \frac{\beta^H}{\beta^L} w_i L_i. \tag{C.72}$$

C.4.3 Trade and Market Clearing

We assume that floor space is owned by landlords, who receive income from residents' expenditure on floor space, and consume only consumption goods where they live. Total expenditure on consumption goods equals the fraction α of the total income of residents plus the entire income of landlords. This income of landlords equals $(1 - \alpha)$ times the total income of residents plus β^H times revenue (which equals β^H / β^L times the total income of workers). Therefore, total expenditure on consumption goods is:

$$E_n = P_n C_n = \alpha v_n R_n + (1 - \alpha) v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n = v_n R_n + \frac{\beta^H}{\beta^L} w_n L_n, \tag{C.73}$$

where v_n is the average income of location n's residents, as determined below as a function of commuting patterns, and R_n is the measure of these residents.

This Armington model also implies a gravity equation for bilateral trade in goods between locations. Using again the properties of CES preferences, the share of expenditure of location n on goods produced by location i is:

$$\pi_{ni} = \frac{\left(d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} / A_i\right)^{1-\sigma}}{\sum_{k \in \mathbb{N}} \left(d_{nk} w_k^{\beta^L} q_k^{\beta^H} r^{\beta^M} / A_k\right)^{1-\sigma}}, \tag{C.74}$$

where the elasticity of trade to trade costs $(1-\sigma)$ is now determined by the elasticity of substitution between the goods produced by each location.

Goods market clearing and zero profits imply that payments to workers plus payments for commercial floor space use plus payments for machinery capital use in each location equal expenditure on goods produced in that location:

$$w_i L_i + q_i H_i^L + r M_i = \sum_{n \in \mathbb{N}} \pi_{ni} X_n. \tag{C.75}$$

We now use the expression for the equilibrium price of each location's good in equation (C.70) to rewrite the consumption goods price index in equation (C.67) as follows:

$$P_n = \left[\sum_{i \in \mathbb{N}} \left(d_{ni} w_i^{\beta^L} q_i^{\beta^H} r^{\beta^M} / A_i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{C.76}$$

Using the trade share (C.74), and noting that $d_{nn} = 1$, the consumption goods price index in equation (C.76) can be further rewritten solely in terms of the domestic trade share (π_{nn}), wages (w_n), the price of commercial floor space (q_i), the common price of machinery (r), and parameters:

$$P_n = \left(\frac{1}{\pi_{nn}}\right)^{\frac{1}{1-\sigma}} \left(\frac{w_n^{\beta^L} q_n^{\beta^H} r^{\beta^M}}{A_n}\right). \tag{C.77}$$

Land market clearing implies that the total income received by landlords as owners of floor space (which equals rateable value ($\mathbb{Q}_n$) in our data) equals the sum of payments for the use of residential and commercial floor space:

$$\mathbb{Q}_n = Q_n H_n^R + q_n H_n^L = (1-\alpha) v_n R_n + \left(\frac{\beta^H}{\beta^L}\right) w_n L_n, \tag{C.78}$$

where H_n^R denotes residential floor space use.

C.4.4 Workplace and Residence Choices

Given the direct utility function (C.64), the corresponding indirect utility function for a worker ω residing in location n and working in location i is:

$$U_{ni}(\omega) = \frac{B_n b_{ni}(\omega) w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}, \tag{C.79}$$

which takes the same form as in our baseline quantitative urban model. The only difference from our baseline quantitative urban model is in the underlying determinants of the price index for goods consumption (P_n), as now determined by equation (C.76).

Using indirect utility (C.79) and the Fréchet distribution of idiosyncratic amenities (C.65), this Armington spatial model exhibits the same gravity equation predictions for commuting flows as in our baseline quantitative urban model. Following the same analysis as in Section B.8 of this Online Appendix, the probability that a worker chooses to reside in location $n \in \mathbb{N}$ and work in location $i \in \mathbb{N}$ is given by:

$$\lambda_{ni} = \frac{L_{ni}}{L_{\mathbb{N}}} = \frac{\left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \qquad n, i \in \mathbb{N}, \tag{C.80}$$

which is identical to the specification in our baseline quantitative urban model, except that the price index for goods consumption (P_n) is now determined by equation (C.76).

Summing across workplaces $i \in \mathbb{N}$, we obtain the probability that a worker lives in each residence $n \in \mathbb{N}$ ($\lambda_n^R = R_n / L_{\mathbb{N}}$). Similarly, summing across residences $n \in \mathbb{N}$, we obtain the probability that a worker is employed in each workplace $i \in \mathbb{N}$ ($\lambda_n^L = L_i / L_{\mathbb{N}}$):

$$\lambda_n^R = \frac{\sum_{i \in \mathbb{N}} \left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \quad \lambda_i^L = \frac{\sum_{n \in \mathbb{N}} \left(B_n w_i\right)^{\epsilon} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell\right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha}\right)^{-\epsilon}}, \tag{C.81}$$

where both expressions are the same as in our baseline quantitative urban model.

From equations (C.80) and (C.81), the conditional probability that a worker commutes to location i conditional on residing in location n takes the same form as in our baseline quantitative urban model:

$$\lambda_{ni|n}^R = \frac{\lambda_{ni}}{\lambda_n^R} = \frac{\left(w_i / \kappa_{ni}\right)^{\epsilon}}{\sum_{\ell \in \mathbb{N}} \left(w_\ell / \kappa_{n\ell}\right)^{\epsilon}}. \tag{C.82}$$

Using this commuting probability conditional on residence ($\lambda_{ni|n}^R$) from equation (C.82), we obtain an identical expression for per capita income conditional on living in location n as in our baseline quantitative urban model:

$$v_n = \sum_{i \in \mathbb{N}} \lambda_{ni|n}^R w_i. \tag{C.83}$$

Commuter market clearing again implies that employment in each location (L_i) equals the measure of workers choosing to commute to that location. Using the commuting probabilities conditional on residence from equation (C.82), we obtain the same expression for this commuter market clearing condition as in our baseline quantitative urban model:

$$L_i = \sum_{n \in \mathbb{N}} \lambda_{ni|n}^R R_n. \tag{C.84}$$

Finally, using the Fréchet distribution for idiosyncratic amenities (C.65), expected utility conditional on choosing a residence-workplace pair ($\bar{U}$) is equalized across all residence-workplace pairs in the economy and takes the same form as in our baseline quantitative urban model:

$$U = \vartheta \left[\sum_{k \in \mathbb{N}} \sum_{\ell \in \mathbb{N}} \left(B_k w_\ell \right)^{\epsilon} \left(\kappa_{k\ell} P_k^{\alpha} Q_k^{1-\alpha} \right)^{-\epsilon} \right]^{\frac{1}{\epsilon}}, \tag{C.85}$$

where the expectation is taken over the distribution for idiosyncratic amenities; $\vartheta \equiv \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)$; and $\Gamma(\cdot)$ is the Gamma function.

D Extensions and Generalizations

In this section of the Online Appendix, we report additional derivations for the extensions and generalizations in Section 6 of the paper.

D.1 Floor Space Supply Elasticities

We assume a competitive construction sector that produces floor space (H_n) with land (G_n) and capital (K_n) according to the following Cobb-Douglas production technology:

$$H_n = K_n^{\mu} G_n^{1-\mu}, \qquad 0 < \mu < 1, \tag{D.1}$$

where capital is assumed to be in perfect elastic supply from the wider economy at a constant price p_K.

From cost minimization and zero profits in construction, equilibrium payments for capital are a constant share of payments for residential floor space ($Q_n H_n$):

$$p_K K_n = \mu Q_n H_n. \tag{D.2}$$

Using this equilibrium condition (D.2) to substitute for capital (K_n) in the construction technology (D.1), we obtain a constant elasticity supply function for residential floor space:

$$H_n = \left(\frac{\mu Q_n H_n}{p_K} \right)^{\mu} G_n^{1-\mu},$$

$$H_n^{1-\mu} = \left(\frac{\mu Q_n}{p_K} \right)^{\mu} G_n^{1-\mu},$$

$$H_n = \left(\frac{\mu}{p_K} \right)^{\frac{\mu}{1-\mu}} Q_n^{\frac{\mu}{1-\mu}} G_n. \tag{D.3}$$

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