Online Supplement for "Dynamic Spatial General Equilibrium" (Replication Material)*

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December 2022

*The latest version of the paper can be downloaded from here. The latest version of the Online Appendix can be downloaded from here. The latest version of this Online Supplement containing further theoretical extensions, additional empirical results and the data appendix can be downloaded from here. A toolkit illustrating our spectral analysis for a model economy can be downloaded from here.

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S.1 Introduction

This Online Supplement contains additional derivations for our baseline model, theoretical extensions and generalizations, supplementary empirical results, and the data appendix. In Section S.2, we report additional derivations for our baseline model with a single traded sector from Section 2 of the paper, including workers' migration choice probabilities and their expected value of living in each location. In Section S.3, we establish a number of isomorphisms, in which we show that our results hold throughout the class of trade models with a constant trade elasticity.

In Section S.4, we introduce a number of extension of our baseline specification, as discussed in Section 4 of the paper. Subsection S.4.1 shows that our framework naturally accommodates shocks to trade and migration costs. Subsection S.4.2 allows for agglomeration forces in production and residence and provides a characterization of the steady-state equilibrium in the presence of these agglomeration forces.

Subsection S.4.3 introduces multiple final goods sectors with region-specific capital. Section S.4.4 incorporates multiple final goods sectors with region-sector-specific capital. Section S.4.5 further generalizes the analysis to allow for multiple final goods sectors with region-sector-specific capital and input-output linkages. Subsection S.4.6 incorporates trade deficits following the conventional approach of the quantitative international trade literature in treating these deficits as exogenous. Section S.4.7 allows capital to be used residentially (for housing) as well as commercially (in production). Section S.4.8 reports an extension to allow landlords to invest in other locations. Section S.4.9 discusses an extension to incorporate an endogenous labor participation decision.

In Section S.5, we present the derivations for the extension of our baseline model with a single traded sector and single non-traded sector used for our baseline quantitative analysis in Section 5 of the paper.

Section S.6 reports additional empirical results that are discussed in the paper. Subsection S.6.1 shows that individual U.S. states differ substantially in terms of the dynamics of their capital-labor ratios, highlighting the empirical relevance of capital accumulation for income convergence. Subsection S.6.2 provides evidence of substantial net migration between U.S. states, highlighting the empirical salience of migration for the population dynamics of U.S. states. Subsection S.6.3 show that the model's gravity equation predictions provide a good approximation to the observed data on trade and migration flows.

Subsection S.6.4 examines the evolution of the real interest rate in terms of the local consumption price index along the transition path to steady-state. Subsection S.6.5 reports additional evidence on the predictive power of convergence to the initial steady-state for the observed population growth of U.S. states. Subsection S.6.6 reports additional empirical results for our spectral analysis in Section 5.4 of the paper. Subsection S.6.7 provides further information about the implied fundamentals from inverting the non-linear model. Subsection S.6.8 reports additional empirical results for our multi-sector extension that is discussed in Section 5.5 of the paper.

Section S.7 reports further details about the data sources and definitions.

S.2 Baseline Dynamic Spatial Model

In Subsection S.2.1, we show how our baseline model can be inverted to recover the unobserved locational fundamentals implied by the observed data. Subsection S.2.2 provides further details about the solution algorithm used to solve for the economy's transition path in the non-linear model. In Subsection S.2.3, we provide the closed-form solution for the economy's transition path for any convergent sequence of future shocks to productivities and amenities under perfect foresight, as discussed in Section 3.4 of the paper . In Subsection S.2.4, we provide the closed-form solution for the economy's transition path for the case in which agents observe an initial shock to fundamentals and form rational expectations about future shocks based on a known stochastic process for fundamentals, as discussed in Section 3.4 of the paper. In Subsection S.2.5, we characterize the distributional consequences of shocks to fundamentals. In Subsection S.2.6, we report the derivations for the expression for expected utility in the paper. In Subsection S.2.7, we provide the derivations for the expression for the migration choice probabilities in the paper.

S.2.1 Model Inversion

In this section of the Online Supplement, we show how our generalization of dynamic exact-hat algebra to incorporate forward-looking capital investments in Proposition 2 of the paper can be used to invert the model and recover the unobserved changes in fundamentals (z_{it}, b_{it}, τ_{nit}, κ_{git}) implied by the observed data. We solve for the unobserved changes in these fundamentals from the general equilibrium conditions of the model and the observed data on bilateral trade and migration flows, population, capital stock and labor income per capita under the assumption of perfect foresight. We recover these unobserved fundamentals, without making assumptions about where the economy lies on the transition path to steady-state or the specific trajectory of fundamentals, because the observed changes in migration flows and the capital stock capture agents' expectations about this sequence of future fundamentals. We show that this model inversion has a sequential structure, such that we can recover unobserved fundamentals in a sequence of steps, where we make the minimal set of assumptions in each step, before adding further assumptions in the next step to recover additional fundamentals.

We use our baseline values for the model's parameters from Section 5.1 of the paper based on central values from the existing empirical literature. In a first step, we recover bilateral trade frictions (τ_{nit}) from observed bilateral trade shares (S_{nit}). Assuming that bilateral trade frictions are symmetric ($\tau_{nit} = \tau_{int}$), and normalizing own trade frictions to one ($\tau_{nnt} = \tau_{iit} = 1$), the model's gravity equation predictions for goods trade in equation (13) in the paper imply:

$$\frac{S_{nit} S_{int}}{S_{nnt} S_{iit}} = \left(\frac{\tau_{nit} \tau_{int}}{\tau_{nnt} \tau_{iit}} \right)^{-\theta} = \left(\tau_{nit} \right)^{-2\theta}. \tag{S.2.1}$$

In second step, we solve for productivity (z_{it}) using observed population (ℓ_{it}), labor income per capita (w_{it}), and the capital stock (k_{it}) and our solutions for bilateral trade frictions (τ_{nit}) from the previous step. From the model's goods market clearing condition in equation (12) in the paper, we have:

$$w_{it} \ell_{it} = \sum_{n=1}^{N} \frac{\left(w_{it} \left(\ell_{it} / k_{it} \right)^{1-\mu} \tau_{nit} / z_{it} \right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt} \left(\ell_{mt} / k_{mt} \right)^{1-\mu} \tau_{nmt} / z_{mt} \right)^{-\theta}} w_{nt} \ell_{nt}, \tag{S.2.2}$$

which uniquely determines productivity (z_{it}) up to normalization (or a choice of units). Since we normalize own trade frictions to one ($\tau_{nnt} = \tau_{iit} = 1$), a change in trade costs with all trade partners (including oneself) is captured in productivity (z_{it}). As these solutions for bilateral trade frictions (τ_{nit}) and productivity (z_{it}) only use the predictions of the static Armington trade model and condition on the observed capital stock and population, they hold regardless of what assumptions are made about capital accumulation and migration.

In a third step, we recover bilateral migration frictions (κ_{git}) from observed bilateral migration flows (D_{igt}). Assuming that bilateral migration frictions are symmetric ($\kappa_{git} = \kappa_{igt}$), and normalizing own migration frictions to one ($\kappa_{ggt} = \kappa_{iit} = 1$), the model's gravity equation predictions for migration in equation (16) in the paper imply:

$$\frac{D_{igt} D_{git}}{D_{ggt} D_{iit}} = \left(\frac{\kappa_{git} \kappa_{igt}}{\kappa_{ggt} \kappa_{iit}} \right)^{-1/\rho} = \left(\kappa_{git} \right)^{-2/\rho}. \tag{S.2.3}$$

In a fourth step, we solve for the expected value of living in each location (v_{gt+1}^{w}) from observed population (ℓ_{it}) and our solutions for bilateral migration frictions (κ_{git}) from the previous step. From the model's population flow condition in equation (15) in the paper and the assumption of perfect foresight, we have:

$$\ell_{gt+1} = \sum_{i=1}^{N} \frac{\left(\exp\left(\beta v_{gt+1}^{w} \right) / \kappa_{git} \right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta v_{mt+1}^{w} \right) / \kappa_{mit} \right)^{1/\rho}} \ell_{it}, \tag{S.2.4}$$

which uniquely determines the expected values (v_{gt+1}^{w}) up to a normalization (or choice of units). Since we normalize own migration frictions to one ($\kappa_{ggt} = \kappa_{iit} = 1$), a change in migration frictions with all locations (including oneself) is captured in the expected value (v^w_{gt+1}) and hence in amenities (b_{gt}) in the next step. As these solutions for bilateral migration frictions (κ_{git}) and expected values (v^w_{gt+1}) use only the predictions of the migration model, they hold regardless of what assumptions are made about patterns of trade in goods.

In a fifth and final step, we recover amenities in each location (b_{it}) from observed goods trade (S_{nit}), observed migration flows (D_{git}), and our solutions for productivity (z_{it}) and expected values (v^w_{it+1}) from the previous steps. Using the model's value function (14) for a pair of time periods and the assumption of perfect foresight, we have:

$$\ln b_{it} = \left(v^w_{it} - v^w_{it+1}\right) + (1-\beta)\, v^w_{it+1} - \ln \frac{S_{iit}^{-\frac{1}{\theta}}}{\left(D_{iit}\right)^{\rho}} - \ln z_{it} - (1-\mu) \ln \left(\frac{k_{it}}{\ell_{it}}\right), \tag{S.2.5}$$

which uniquely determines amenities (b_{it}) up to our choices of units for productivity and expected values. In this final step, we use the predictions of both the migration and trade blocs of the model. Note that this final step for amenities in equation (S.2.5) requires expected values for both periods t and $t+1$, and hence requires migration flows for both periods from equations (S.2.3) and (S.2.4).

We thus obtain values for the unobserved fundamentals (z_{it}, b_{it}, τ_{nit}, κ_{nit}) implied by the observed values of the endogenous variables under the assumption of perfect foresight, without making assumptions about where on the transition path to steady-state the economy lies or about the particular expected future trajectory of fundamentals. Note that these fundamentals are derived under the assumption of symmetric trade and migration costs, which need not necessarily be satisfied in the data. Therefore, these fundamentals do not exactly rationalize the observed expenditure shares (S_{nit}) and outmigration probabilities (D_{igt}), although we find that the model's predictions under this symmetry assumption are strongly correlated with the observed data.

As general equilibrium allocations in the model are homogenous of degree zero in productivity and amenities, multiplying these fundamentals by scalars leaves allocations unchanged. Therefore, without loss of generality, we focus on shocks to relative productivity and amenities, which are invariant to the units in which these variables are measured.

S.2.2 Non-Linear Model Solution Algorithm

In Section 5.3 of the paper, we use our spectral analysis to provide an analytical characterization of the speed of convergence to steady-state and the interaction between the capital and labor adjustment margins. Although this spectral analysis uses a linearization of the model, we show that this linearization provides a good approximation to the transition path of the full non-linear model. In this section of the Online Supplement, we provide further details on the solution algorithm used to solve for the economy's transition path in the non-linear model.

Solving for the Sequential Competitive Equilibrium Consider an economy on a transition path to some unknown steady-state starting from an initial allocation $\left(\{l_{i0}\}_{i=1}^{N}, \{k_{i0}\}_{i=1}^{N}, \{k_{i1}\}_{i=1}^{N}, \{S_{ni0}\}_{n,i=1}^{N}, \{D_{ni,-1}\}_{n,i=1}^{N}\right)$, given an anticipated sequence of changes in fundamentals, $\left\{\{\dot{z}_{it}\}_{i=1}^{N}, \left\{\dot{b}_{it}\right\}_{i=1}^{N}, \{\dot{\tau}_{ijt}\}_{i,j=1}^{N}, \{\dot{\kappa}_{ijt}\}_{i,j=1}^{N}\right\}_{t=1}^{\infty}$. The strategy to solve the sequential competitive equilibrium is as follows:

1. Initiate the algorithm at $t=0$: guess the path of relative changes in transformed expected utility $\left\{\dot{\boldsymbol{u}}_t^{(0)}\right\}_{t=1}^{T+1}$, where we define $u_{it} \equiv \exp\left(\frac{\beta}{\rho} v_{it}^w\right)$,[1] and the path of landlord consumption rates, $\left\{\boldsymbol{\varsigma}_t^{(0)}\right\}_{t=1}^{T+1}$, for a sufficient large T. The path should converge by period $T+1$, i.e. $\dot{u}_{iT+1}^{(0)} = 1$.

2. Set the rental rates in period $t=1$ in accordance to the guessed consumption rates and the observed allocation

$$R_{i1} = \left(\frac{\chi_{i1}\ell_{i1}}{\chi_{i0}\ell_{i0}}\right)\left(1-\varsigma_{i1}^{(0)}\right)^{-1} \quad \forall i.$$

3. Use the path of transformed expected utility $\left\{\dot{\boldsymbol{u}}_t^{(0)}\right\}_{t=1}^{T+1}$ to get migration rates $\{\boldsymbol{D_t}\}_{t=1}^{T+1}$:

$$\dot{D}_{igt+1} = \frac{\dot{u}_{gt+2}^{(0)} / \left(\dot{\kappa}_{git+1}\right)^{1/\rho}}{\sum_{m=1}^{N} D_{imt}\dot{u}_{mt+2}^{(0)} / \left(\dot{\kappa}_{mit+1}\right)^{1/\rho}}.$$

4. Use the migration rates to get employment levels in all periods $t>2$:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt}\ell_{it}$$

5. For each period $t>0$:

   (a) Use $\boldsymbol{\ell}_t$, $\boldsymbol{\ell}_{t-1}$, $\boldsymbol{\chi}_t$, $\boldsymbol{\chi}_{t-1}$ and $\boldsymbol{S}_{t-1}$ to solve for the relative changes in wages $\dot{\boldsymbol{w}}_{t+1}$ and the new expenditure shares $\boldsymbol{S}_{t+1}$, by solving the system of non-linear equations

$$\dot{w}_{it+1}\dot{\ell}_{it+1} = \sum_{n=1}^{N} \frac{S_{nit+1}w_{nt}\ell_{nt}}{\sum_{k=1}^{N} S_{kit}w_{kt}\ell_{kt}} \dot{w}_{nt+1}\dot{\ell}_{nt+1}$$

   and

$$\dot{S}_{nit+1} \equiv \frac{\left(\dot{\tau}_{nit+1}\dot{w}_{it+1}\left(\dot{\chi}_{it+1}\right)^{\mu-1}/\dot{z}_{it+1}\right)^{-\theta}}{\sum_{k=1}^{N} S_{nkt}\left(\dot{\tau}_{nkt+1}\dot{w}_{kt+1}\left(\dot{\chi}_{kt+1}\right)^{\mu-1}/\dot{z}_{kt+1}\right)^{-\theta}}.$$

   where $\dot{\chi}_{it+1} \equiv \dot{k}_{it+1}/\dot{\ell}_{it+1}$. Note that the initial level of wages $\boldsymbol{w}_t$ can be recovered from the expenditure share matrix $\boldsymbol{S}_t$ and the population levels $\boldsymbol{\ell}_t$. Also note that this system can be solved through an iterative procedure after guessing a vector of relative changes in wages $\dot{\boldsymbol{w}}_{t+1}$.

   (b) Solve for the implied relative changes in price indices $\dot{\boldsymbol{p}}_{t+1}$ from

$$\dot{p}_{it+1} = \left(\sum_{m=1}^{N} S_{imt}\left(\dot{\tau}_{imt+1}\dot{w}_{mt+1}\left(\dot{\chi}_{mt+1}\right)^{\mu-1}/\dot{z}_{mt+1}\right)^{-\theta}\right)^{-1/\theta}.$$

[1] Note that we express the set of equilibrium conditions in terms of transformed workers utility $u_{it} \equiv \exp\left(\frac{\beta}{\rho} v_{it}^w\right)$, whereas in Caliendo et al. (2018), the equilibrium conditions are expressed in terms of $\exp\left(v_{it}^w\right)$.

(c) Solve for the new rental rates $\dot{\boldsymbol{R}}_{t+1}$ using

$$R_{it+1} = (1-\delta) + \frac{\dot{w}_{it+1}}{\dot{p}_{it+1}\dot{\chi}_{it+1}} \left(R_{it} - (1-\delta)\right).$$

(d) Finally, update the capital labor ratios $\dot{\boldsymbol{\chi}}_{t+2}$ if $t < T$ using

$$\chi_{it+2}\ell_{it+2} = \left(1 - \varsigma_{it+1}\right) R_{it+1}\chi_{it+1}\ell_{it+1},$$

where recall that we have already solved for the path of population levels in all periods.

6. For each t, solve backwards for $\left\{\dot{\boldsymbol{u}}_t^{(1)}\right\}_{t=1}^{T+1}$ using

$$\dot{u}_{it+1} = \left(\dot{b}_{it+1}\frac{\dot{w}_{it+1}}{\dot{p}_{it+1}}\right)^{\frac{\beta}{\rho}} \left(\sum_{g=1}^{N} D_{igt}\dot{u}_{gt+2}/\left(\dot{\kappa}_{git+1}\right)^{\frac{1}{\rho}}\right)^{\beta}.$$

7. For each t, solve backwards for $\left\{\boldsymbol{\varsigma}_t^{(1)}\right\}_{t=1}^{T+1}$ using

$$\varsigma_{it} = \frac{\varsigma_{it+1}}{\varsigma_{it+1} + \beta^{\psi} R_{it+1}^{\psi-1}},$$

imposing $R_{T+1} = 1/\beta$.

8. Take the new paths for $\left\{\dot{\boldsymbol{u}}_t^{(1)}\right\}_{t=1}^{T+1}$ and $\left\{\boldsymbol{\varsigma}_t^{(1)}\right\}_{t=1}^{T+1}$ as the new initial conditions, and return to step 2.

9. Continue until convergence of $\left\{\dot{\boldsymbol{u}}_t^{(1)}\right\}_{t=1}^{T+1}$ and $\left\{\boldsymbol{\varsigma}_t^{(1)}\right\}_{t=1}^{T+1}$.

S.2.3 Convergent Sequence of Shocks Under Perfect Foresight

We now generalize our analysis of the model's transition dynamics to any convergent sequence of future shocks to productivities and amenities under perfect foresight. In particular, we consider an economy that is somewhere on a convergence path towards an initial steady-state with constant fundamentals at time $t = 0$, when agents learn about a convergent sequence of future shocks to productivity and amenities $\left\{\widetilde{\boldsymbol{f}}_s\right\}_{s\geq 1}$ from time $t = 1$ onwards, where $\widetilde{\boldsymbol{f}}_s$ is a vector of log differences in fundamentals between times s and 0 for each location.

Proposition S.1. ***Sequence of Shocks Under Perfect Foresight.*** *Consider an economy that is somewhere on a convergence path towards steady-state at time $t = 0$, when agents learn about a convergent sequence of future shocks to productivity and amenities $\left\{\widetilde{\boldsymbol{f}}_{\boldsymbol{s}}\right\}_{s\geq 1} = \left\{\begin{bmatrix}\widetilde{\boldsymbol{z}}_{\boldsymbol{s}}\\ \widetilde{\boldsymbol{b}}_{\boldsymbol{s}}\end{bmatrix}\right\}_{s\geq 1}$ from time $t = 1$ onwards. There exists a $2N \times 2N$ transition matrix ($\boldsymbol{P}$) and a $2N \times 2N$ impact* *matrix ($\boldsymbol{R}$) such that the second-order difference equation system in equation (22) in the paper has a closed-form solution of the form:*

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=t+1}^{\infty} \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)} \boldsymbol{R}\left(\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right) + \boldsymbol{R}\widetilde{\boldsymbol{f}}_t + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} \qquad \textit{for all } t \geq 1, \tag{S.2.6}$$

with initial condition $\widetilde{\boldsymbol{x}}_0 = \mathbf{0}$ and where $\boldsymbol{\Psi}, \boldsymbol{\Gamma}$ are matrices from the second-order difference equation (22) in the paper and are derived in Online Appendix B.4.7.

Proof. We start by proving the case with a single fundamental shock $\widetilde{\boldsymbol{f}}_s$ at future time s, with $\widetilde{\boldsymbol{f}}_t = 0$ for all $t \neq s$. We then exploit the linear structure and consider a sequence of shocks. Given that the shock takes place at time s, we know from Proposition 3 in the paper that:

$$\widetilde{\boldsymbol{x}}_s = \boldsymbol{R}\widetilde{\boldsymbol{f}}_s + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{s-1}. \tag{S.2.7}$$

Following our derivations for (B.71) in the Online Appendix, we know the state variables follow a system of second-order difference equations:

$$\boldsymbol{\Psi}\widetilde{\boldsymbol{x}}_{t+2} = \begin{cases} \boldsymbol{\Gamma}\widetilde{\boldsymbol{x}}_{t+1} + \boldsymbol{\Theta}\widetilde{\boldsymbol{x}}_t + \boldsymbol{\Pi}\widetilde{\boldsymbol{f}} & t \geq s-1, \\ \boldsymbol{\Gamma}\widetilde{\boldsymbol{x}}_{t+1} + \boldsymbol{\Theta}\widetilde{\boldsymbol{x}}_t & 0 \leq t < s-1. \end{cases} \tag{S.2.8}$$

We now solve the second-order difference equation (S.2.8) backwards for $0 \leq t < s-1$. Starting from $t = s-2$, we have:

$$\boldsymbol{\Gamma}\widetilde{\boldsymbol{x}}_{s-1} = \boldsymbol{\Psi}\widetilde{\boldsymbol{x}}_s - \boldsymbol{\Theta}\widetilde{\boldsymbol{x}}_{s-2}.$$

Substitute using (S.2.7) and equation (B.72) in the Online Appendix, we obtain:

$$\widetilde{\boldsymbol{x}}_{s-1} = \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-1} \boldsymbol{R}\widetilde{\boldsymbol{f}} + \boldsymbol{P}x_{s-2}.$$

We can show by induction that, for all $t \geq 1$:

$$\widetilde{\boldsymbol{x}}_t = \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)} \boldsymbol{R}\widetilde{\boldsymbol{f}} + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1}.$$

Hence, with a single shock $\widetilde{\boldsymbol{f}}_s$ at time $s > 0$, the law of motion of the state variables follows:

$$\widetilde{\boldsymbol{x}}_t = \begin{cases} \boldsymbol{R}\widetilde{\boldsymbol{f}}_s + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} & t \geq s \\ \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)} \boldsymbol{R}\widetilde{\boldsymbol{f}}_s + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} & 1 \leq t < s. \end{cases}$$

Given linearity, the law of motion with a sequence of convergent fundamentals follows:

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=t+1}^{\infty} \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)} \boldsymbol{R}\left(\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right) + \boldsymbol{R}\widetilde{\boldsymbol{f}}_t + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} \qquad \text{for all } t \geq 1,$$

where $\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}$ is the change in fundamental in period s and $\widetilde{\boldsymbol{f}}_t$ is the cumulative change in fundamental at time t relative to time 0. That the sequence of fundamentals converges ($\lim_{s\to\infty}\left(\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right) \to \mathbf{0}$) ensures the summation is well defined. □

Therefore, even though we consider a general convergent sequence of shocks to productivity and amenities in a setting with many locations connected by a rich geography, and with multiple sources of dynamics from investment and migration, we are again able to obtain a closed-form solution for the transition path of the spatial distribution of economic activity. Both the transition matrix $\boldsymbol{P}$ and impact matrix $\boldsymbol{R}$ remain the same same as those in the previous subsection, and can be recovered from our observed trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$ and the structural parameters of the model $\{\psi, \theta, \beta, \rho, \mu, \delta\}$.

S.2.4 Stochastic Location Characteristics and Rational Expectations

Most previous research on dynamic spatial models has focused on perfect foresight, because of the challenges of solving non-linear dynamic models in the presence of expectational errors. We now show that our linearization of the economy's transition path also accommodates the case in which agents observe an initial shock to fundamentals and form rational expectations about future shocks based on a known stochastic process for fundamentals.

In particular, we assume the following AR(1) process for fundamentals, which allows shocks to productivity and amenities to have permanent effects on the level of these variables, and hence to affect the steady-state equilibrium:

$$\begin{aligned} \Delta \ln z_{it+1} &= \rho^z \Delta \ln z_{it} + \varpi_{it}^z, & \left|\rho^z\right| < 1, \\ \Delta \ln b_{it+1} &= \rho^b \Delta \ln b_{it} + \varpi_{it}^b, & \left|\rho^b\right| < 1, \end{aligned} \tag{S.2.9}$$

where we use $\Delta \ln$ to denote log changes between two periods, such that $\Delta \ln z_{it} \equiv \ln z_{it} - \ln z_{it-1}$; $\rho^z = \rho^b = 0$ corresponds to the special case of a random walk; and ϖ_{it}^z and ϖ_{it}^b are mean zero and independently and identically distributed innovations. Given this assumed AR(1) process, we can write the expected values of these future fundamental shocks as:

$$\mathbb{E}_t\left[\Delta \ln \boldsymbol{f}_{t+s}\right] = \boldsymbol{N}^s \Delta \ln \boldsymbol{f}_t, \qquad \boldsymbol{N} \equiv \begin{bmatrix} \rho^z \cdot \boldsymbol{I}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \rho^b \cdot \boldsymbol{I}_{N\times N} \end{bmatrix}, \tag{S.2.10}$$

where $\mathbb{E}_t\left[\cdot\right]$ is the expectation conditional on the realizations of shocks up to time t.

Proposition S.2. ***Stochastic Fundamentals and Rational Expectations.*** *Suppose that productivity and amenities evolve stochastically according to the AR(1) process (S.2.9) and agents have rational expectations. There exists a $2N \times 2N$ transition matrix ($\boldsymbol{P}$) and a $2N \times 2N$ impact matrix ($\boldsymbol{R}$) such that the evolution of the economy's state variables (x_t) has the following closed-form solution:*

$$\Delta \ln \boldsymbol{x}_{t+1} = \boldsymbol{P}\Delta \ln \boldsymbol{x}_t + \boldsymbol{R}\Delta \ln \boldsymbol{f}_t + \sum_{s=0}^{\infty} \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-s} \boldsymbol{R}\boldsymbol{N}^{s+1}\left(\Delta \ln \boldsymbol{f}_t - \Delta \ln \boldsymbol{f}_{t-1}\right). \tag{S.2.11}$$

Proof. The state variables at time $t+1$ are chosen by agents as functions of past state variables and fundamental shocks realized up to time t. Under rational expectation, agents at each time t expect a sequence of future fundamental shocks according to (S.2.10). Thus, from Proposition S.1, we know

$$\left(\ln x_{t+1} - \ln x_t^*\right) - P\left(\ln x_t - \ln x_t^*\right) = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\Psi^{-1}\Gamma - P\right)^{-s} R\Delta \ln f_{t+s+1},$$

$$\left(\ln x_t - \ln x_{t-1}^*\right) - P\left(\ln x_{t-1} - \ln x_{t-1}^*\right) = \mathbb{E}_{t-1}\sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} R\Delta\ln f_{t+s},$$

where x_t^* is the steady-state implied by fundamentals at time t. Taking the difference between the two equations, we get

$$\begin{aligned} & \left(\ln x_{t+1} - \ln x_t^*\right) - P\left(\ln x_t - \ln x_t^*\right) - \left(\ln x_t - \ln x_{t-1}^*\right) + P\left(\ln x_{t-1} - \ln x_{t-1}^*\right) \\ = \; & \mathbb{E}_t\sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} R\Delta\ln f_{t+s+1} - \mathbb{E}_{t-1}\sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} R\Delta\ln f_{t+s}. \end{aligned}$$

The left-hand side (LHS) of the above equation can be written as:

$$LHS \;=\; \Delta\ln x_{t+1} - P\Delta\ln x_t - \left(I - P\right)\Delta\ln x_t^*.$$

We know $\mathbb{E}_t\hat{f}_{t+s} = N^s\hat{f}_t$, and hence the right-hand side (RHS) of the above equation can be written as:

$$\begin{aligned} RHS \;=\; & \sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} R\mathbb{E}_t\Delta\ln f_{t+s+1} - \sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} R\mathbb{E}_{t-1}\Delta\ln f_{t+s}, \\ =\; & \sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} RN^{s+1}\Delta\ln f_t - \sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} RN^{s+1}\Delta\ln f_{t-1}, \\ =\; & \sum_{s=0}^{\infty}\left(\Psi^{-1}\Gamma - P\right)^{-s} RN^{s+1}\left(\Delta\ln f_t - \Delta\ln f_{t-1}\right). \end{aligned}$$

We also know:

$$\Delta\ln x_t^* = \left(I - P\right)^{-1} R\Delta\ln f_t.$$

We obtain the Proposition by setting the LHS to be equal to the RHS. □

In this case, the innovations in fundamental shocks at time t not only affect the current-period state variables ($\boldsymbol{\ell}_t$, $\boldsymbol{k}_t$), but also affect the entire expected sequence of future fundamental shocks, because of serial correlation in fundamental shocks (ρ^z and ρ^b not equal to zero).

S.2.5 Distributional Consequences

The presence of gradual adjustment in the model from migration frictions and capital accumulation has two important implications for the welfare effects of shocks to productivity and amenities. First, these welfare effects depend not only on the change in steady-state, but also on the transition dynamics. Second, there is a distribution of these welfare effects, both across landlords because they are geographically immobile, and across workers because of migration frictions, which imply that a worker's initial location matters for the welfare impact of these shocks.

As our approach provides sufficient statistics for the economy's transition path in response to shocks to fundamentals, it also provides sufficient statistics for the welfare effects of these shocks. In the remainder of this subsection, we illustrate these sufficient statistics for welfare, using changes in migration flows to reveal information about continuation values. In particular, we suppose that the economy starts from steady-state at time $t = 0$, at which point agents become aware of a permanent change in fundamentals ($\widetilde{\boldsymbol{f}}$) at time $t=1$. Since fundamentals change from time $t=1$ onwards, the change in workers' welfare at time $t=0$ is completely determined by the change in the continuation value from their optimal location choice:

$$\widetilde{\boldsymbol{v}}_0 = \beta \boldsymbol{D} \widetilde{\boldsymbol{v}}_1, \tag{S.2.12}$$

where this change in continuation value ($\beta \boldsymbol{D} \widetilde{\boldsymbol{v}}_1$) depends on workers' initial location at time $t=0$, because of migration frictions, as captured by the outmigration matrix ($\boldsymbol{D}$).

We now show that the expression for population dynamics in equation (20) in the paper can be used to infer relative changes in continuation values in response to shocks to fundamentals from these population movements:

$$\widetilde{\boldsymbol{\ell}}_1 = \boldsymbol{E} \widetilde{\boldsymbol{\ell}}_0 + \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} \right) \left(\widetilde{\boldsymbol{v}}_1 + \varsigma \right),$$

where the first term ($\boldsymbol{E} \widetilde{\boldsymbol{\ell}}_0$) is equal to zero, because of our assumption that the economy starts from an initial steady state at time $t=0$ ($\widetilde{\boldsymbol{\ell}}_0 = \ln \boldsymbol{\ell}_{\mathbf{0}} - \ln \boldsymbol{\ell}^* = 0$); the presence of the constant ς reflects the fact that migration decisions depend on relative expected values across locations, and hence are invariant to a common change in expected values across all locations.

To compute the impact on the overall level of welfare, we set this constant equal to the average change in expected values across all locations weighted by population shares ($\boldsymbol{\ell}^{*\prime} \cdot \widetilde{\boldsymbol{v}}_1$), where we stack the $\boldsymbol{\ell}^{*\prime}$ vector N times into an $N \times N$ matrix $\boldsymbol{L} \equiv [\boldsymbol{\ell}^{*\prime}, \ldots, \boldsymbol{\ell}^{*\prime}]$, such that $\varsigma = -\boldsymbol{L} \widetilde{\boldsymbol{v}}_1$. This convenient choice has two simplifying properties: (i) $\boldsymbol{L}^2 = \boldsymbol{L}$; (ii) $\boldsymbol{L}\boldsymbol{D} = \boldsymbol{L}$, because $\boldsymbol{\ell}^{*\prime}$ is the Perron-eigenvector of $\boldsymbol{D}$.[2] Using these properties, we can re-write the above population dynamics equation as follows:[3]

$$\left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_1 = \frac{\rho}{\beta} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} + \boldsymbol{L} \right)^{-1} \widetilde{\boldsymbol{\ell}}_1.$$

Combining this result with equation (S.2.12), we obtain the following key implication that population movements at time $t=1$ in response to these shocks to fundamentals are sufficient statistics for their impact on relative expected values for workers in different locations at time $t=0$:[4]

$$\left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_0 = \rho \boldsymbol{D} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} + \boldsymbol{L} \right)^{-1} \widetilde{\boldsymbol{\ell}}_1,$$

where $\boldsymbol{L} \widetilde{\boldsymbol{v}}_0$ is again a constant vector that represents the average change in expected values across all locations weighted by initial population shares, and the right-hand side captures relative changes in expected values across locations, as revealed by the first-period population movements.

Finally, we can connect these first-period population movements ($\widetilde{\boldsymbol{\ell}}_1$) to the productivity ($\widetilde{\boldsymbol{z}}$) and amenity ($\widetilde{\boldsymbol{b}}$) shocks using our closed-form solution for the economy's transition path (26), which yields our sufficient statistic for workers' welfare exposure to these shocks.

[2]Since $\boldsymbol{\ell}^{*\prime}$ is the Perron-eigenvector of $\boldsymbol{D}$ and $\boldsymbol{E}$, we have $\boldsymbol{L}\boldsymbol{D} = \boldsymbol{D}\boldsymbol{L} = \boldsymbol{L}\boldsymbol{E} = \boldsymbol{E}\boldsymbol{L} = \boldsymbol{L}$. Since population share sum to one, $\boldsymbol{L} \times \widetilde{\boldsymbol{l}}_1 = \mathbf{0}$.

[3]In particular, we use $\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} \right) \left(\widetilde{\boldsymbol{v}}_1 - \boldsymbol{L} \widetilde{\boldsymbol{v}}_1 \right) = \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} + \boldsymbol{L} \right) \left(\widetilde{\boldsymbol{v}}_1 - \boldsymbol{L} \widetilde{\boldsymbol{v}}_1 \right)$, because $\boldsymbol{L}^2 = \boldsymbol{L}$.

[4]We pre-multiply both sides of equation (S.2.12) by $\left(\boldsymbol{I} - \boldsymbol{L} \right)$ and use $\left(\boldsymbol{I} - \boldsymbol{L} \right) \boldsymbol{D} \widetilde{\boldsymbol{v}}_1 = \boldsymbol{D} \left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_1$.

Proposition S.3. *Consider an economy that is initially in steady-state at time $t=0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}}=\begin{bmatrix}\widetilde{\boldsymbol{z}}\\ \widetilde{\boldsymbol{b}}\end{bmatrix}$) from time $t=1$ onwards.*

(i) *The relative welfare impact for agents initially in each location at time 0 is*

$$\widetilde{\boldsymbol{v}}_0-\boldsymbol{L}\widetilde{\boldsymbol{v}}_0=\rho\boldsymbol{D}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}+\boldsymbol{L}\right)^{-1}\boldsymbol{R}^{\ell}\widetilde{\boldsymbol{f}},$$

where $\boldsymbol{R}^{\ell}$ is the matrix representing the first N rows of $\boldsymbol{R}$.

(ii) *The average welfare impact on all agents, weighted by initial population shares, is*

$$\boldsymbol{L}\widetilde{\boldsymbol{v}}_0=\frac{\beta}{1-\beta}\boldsymbol{L}\left(\underbrace{\begin{bmatrix}\boldsymbol{C} & \boldsymbol{I}\end{bmatrix}\widetilde{\boldsymbol{f}}}_{\substack{\text{direct effects from}\\ \text{changes in fundamentals}}}+\underbrace{\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\left(\boldsymbol{I}-(1-\beta)\boldsymbol{P}\left(\boldsymbol{I}-\beta\boldsymbol{P}\right)^{-1}\right)\left(\boldsymbol{I}-\boldsymbol{P}\right)^{-1}\boldsymbol{R}\widetilde{\boldsymbol{f}}}_{\substack{\text{indirect effects from}\\ \text{changes in state variables}}}\right),$$

where $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}$ are matrices from equation (B.61) of the Online Appendix.

Proof. We start from the migration equation (20) in the paper:

$$\begin{aligned}\widetilde{\boldsymbol{\ell}}_1 &= \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}\right)\left(\widetilde{\boldsymbol{v}}_1-\boldsymbol{L}\widetilde{\boldsymbol{v}}_1\right),\\ &= \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}+\boldsymbol{L}\right)\left(\widetilde{\boldsymbol{v}}_1-\boldsymbol{L}\widetilde{\boldsymbol{v}}_1\right),\end{aligned}$$

where the second equality follows from $\boldsymbol{L}$=$\boldsymbol{L}^2$. Hence

$$\begin{aligned}\left(\boldsymbol{I}-\boldsymbol{L}\right)\widetilde{\boldsymbol{v}}_1 &= \frac{\rho}{\beta}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}+\boldsymbol{L}\right)^{-1}\widetilde{\boldsymbol{l}}_1,\\ &= \frac{\rho}{\beta}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}+\boldsymbol{L}\right)^{-1}\boldsymbol{R}^{\ell}\widetilde{\boldsymbol{f}},\end{aligned}$$

and the changes in welfare at $t=0$ follow

$$\begin{aligned}\widetilde{\boldsymbol{v}}_0 &= \beta\boldsymbol{D}\widetilde{\boldsymbol{v}}_1,\\ &= \beta\boldsymbol{L}\widetilde{\boldsymbol{v}}_1+\beta\boldsymbol{D}\left(\boldsymbol{I}-\boldsymbol{L}\right)\widetilde{\boldsymbol{v}}_1,\\ &= \boldsymbol{L}\widetilde{\boldsymbol{v}}_0+\rho\boldsymbol{D}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}+\boldsymbol{L}\right)^{-1}\boldsymbol{R}^{\ell}\widetilde{\boldsymbol{f}},\end{aligned}$$

where the third equality follows from $\boldsymbol{L}\boldsymbol{D}=\boldsymbol{L}$; this completes the proof of the first part of the Proposition.

To prove the second part, note

$$\begin{aligned}\boldsymbol{L}\tilde{\boldsymbol{v}}_0 &= \boldsymbol{L}\beta\boldsymbol{D}\tilde{\boldsymbol{v}}_1,\\ &= \boldsymbol{L}\sum_{s=1}^{\infty}\left(\beta\boldsymbol{D}\right)^s\left(\boldsymbol{C}\tilde{\boldsymbol{z}}+\widetilde{\boldsymbol{b}}+\boldsymbol{A}\tilde{\boldsymbol{\ell}}_s+\boldsymbol{B}\tilde{\boldsymbol{\chi}}_s\right),\\ &= \frac{1}{1-\beta}\boldsymbol{L}\begin{bmatrix}\boldsymbol{C} & \boldsymbol{I}\end{bmatrix}\widetilde{\boldsymbol{f}}+\boldsymbol{L}\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\sum_{s=1}^{\infty}\beta^s\left(\boldsymbol{I}-\boldsymbol{P}^s\right)\left(\boldsymbol{I}-\boldsymbol{P}\right)^{-1}\boldsymbol{R}\widetilde{\boldsymbol{f}},\\ &= \frac{\beta}{1-\beta}\boldsymbol{L}\left(\begin{bmatrix}\boldsymbol{C} & \boldsymbol{I}\end{bmatrix}\widetilde{\boldsymbol{f}}+\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\left(\boldsymbol{I}-(1-\beta)\boldsymbol{P}\left(\boldsymbol{I}-\beta\boldsymbol{P}\right)^{-1}\right)\left(\boldsymbol{I}-\boldsymbol{P}\right)^{-1}\boldsymbol{R}\widetilde{\boldsymbol{f}}\right),\end{aligned}$$

where the third equality follows from Proposition 4 and the fact $\boldsymbol{L}\boldsymbol{D}=\boldsymbol{L}$. □

S.2.6 Derivation of Expected Utility

In this subsection, we derive the expected value for a worker of living in location i at time t (v_{it}^{w}) in equation (14) in the paper. Recall that idiosyncratic mobility shocks are drawn from an extreme value distribution with the following cumulative distribution function:

$$F(\epsilon) = e^{-e^{(-\epsilon-\bar{\gamma})}},$$

and corresponding probability density function:

$$f(\epsilon) = e^{(-\epsilon-\bar{\gamma})}e^{-e^{(-\epsilon-\bar{\gamma})}}.$$

Using this extreme value distribution, note that:

$$\text{Prob}\left[\beta\mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\epsilon_{gt} \geq \beta\mathbb{E}_t v_{mt+1}^{w} - \kappa_{mi} + \rho\epsilon_{mt}\right], \qquad \forall\, m \neq g,$$

$$\text{Prob}\left[\beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - (\kappa_{gi} - \kappa_{mi}) + \rho\epsilon_{it} \geq \rho\epsilon_{mt}\right],$$

$$\text{Prob}\left[\rho\epsilon_{mt} \leq \beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - (\kappa_{gi} - \kappa_{mi}) + \rho\epsilon_{gt}\right],$$

$$\text{Prob}\left[\rho\epsilon_{mt} \leq \rho\bar{\epsilon}_{igmt} + \rho\epsilon_{gt}\right],$$

$$\bar{\epsilon}_{igmt} \equiv \frac{\beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - (\kappa_{gi} - \kappa_{mi})}{\rho},$$

$$\text{Prob}\left[\epsilon_{kt} \leq \bar{\epsilon}_{igmt} + \epsilon_{gt}\right].$$

Now define the expected continuation value for an agent in location i at time t:

$$\Phi_{it} = \max_{\{g\}_1^N}\left\{\beta\mathbb{E}_t\mathbb{E}_\epsilon\left[\mathbb{V}_{gt+1}^{w}\right] - \kappa_{gi} + \rho\epsilon_{gt}\right\}$$

$$\Phi_{it} = \sum_{g=1}^{N}\int_{-\infty}^{\infty}\left(\beta\mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\epsilon_{gt}\right) f(\epsilon_{gt})\prod_{m\neq g} F\left(\bar{\epsilon}_{igmt} + \epsilon_{gt}\right) d\epsilon_{gt}.$$

Using our assumed functional form, we have:

$$\Phi_{it} = \sum_{g=1}^{N}\int_{-\infty}^{\infty}\left(\beta\mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\epsilon_{gt}\right) e^{(-\epsilon_{gt}-\bar{\gamma})}e^{-e^{(-\epsilon_{gt}-\bar{\gamma})}}e^{-\sum_{m\neq g} e^{\left(-\bar{\epsilon}_{igmt}-\epsilon_{gt}-\bar{\gamma}\right)}} d\epsilon_{gt},$$

$$\Phi_{it} = \sum_{g=1}^{N}\int_{-\infty}^{\infty}\left(\beta\mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\epsilon_{gt}\right) e^{(-\epsilon_{gt}-\bar{\gamma})}e^{-\sum_{m=1}^{N} e^{\left(-\bar{\epsilon}_{igmt}-\epsilon_{gt}-\bar{\gamma}\right)}} d\epsilon_{gt},$$

since $\bar{\epsilon}_{immt} = 0$.

$$\Phi_{it} = \sum_{g=1}^{N}\int_{-\infty}^{\infty}\left(\beta\mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\epsilon_{gt}\right) e^{(-\epsilon_{gt}-\bar{\gamma})}e^{-e^{(-\epsilon_{gt}-\bar{\gamma})}\sum_{m=1}^{N} e^{\left(-\bar{\epsilon}_{igmt}\right)}} d\epsilon_{gt}.$$

Define:

$$\lambda_{igt} \equiv \log\sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}},$$

$$e^{\lambda_{igt}} = \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}},$$

$$\zeta_{gt} \equiv \epsilon_{gt} + \bar{\gamma},$$

Using these definitions:

$$\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\zeta_{gt} - \bar{\gamma}\right)\right) e^{(-\zeta_{gt})} e^{-e^{(-\zeta_{gt})} \sum_{m=1}^{N} e^{\left(-\bar{\epsilon}_{igmt}\right)}} d\zeta_{gt},$$

$$\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\zeta_{gt} - \bar{\gamma}\right)\right) e^{(-\zeta_{gt})} e^{-e^{(-\zeta_{gt})} e^{\lambda_{igt}}} d\zeta_{gt},$$

$$\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\zeta_{gt} - \bar{\gamma}\right)\right) e^{(-\zeta_{gt})} e^{-e^{\left(-\left(\zeta_{gt} - \lambda_{igt}\right)\right)}} d\zeta_{gt}.$$

Now define another change of variables:

$$\tilde{y}_{igt} \equiv \zeta_{gt} - \lambda_{igt}.$$

Using this definition:

$$\Phi_{it} = \sum_{g=1}^{N} \int_{-\infty}^{\infty} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\tilde{y}_{igt} + \lambda_{igt} - \bar{\gamma}\right)\right) e^{\left(-(\tilde{y}_{igt} + \lambda_{igt}) - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt}.$$

$$\Phi_{it} = \sum_{g=1}^{N} \left(\begin{array}{c} \int_{-\infty}^{\infty} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\lambda_{igt} - \bar{\gamma}\right)\right) e^{\left(-(\tilde{y}_{igt} + \lambda_{igt}) - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} \\ + \rho \int_{-\infty}^{\infty} \tilde{y}_{igt} e^{\left(-(\tilde{y}_{igt} + \lambda_{igt}) - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} \end{array} \right).$$

$$\Phi_{it} = \sum_{i=1}^{N} e^{-\lambda_{igt}} \left(\begin{array}{c} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\lambda_{igt} - \bar{\gamma}\right)\right) \int_{-\infty}^{\infty} e^{\left(-\tilde{y}_{igt} - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} \\ + \rho \int_{-\infty}^{\infty} \tilde{y}_{igt} e^{\left(-(\tilde{y}_{igt} + \lambda_{igt}) - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} \end{array} \right).$$

Now note that:

$$\frac{d}{dy} \left[e^{-e^{-y}} \right] = e^{-y - e^{-y}},$$

$$\int_{-\infty}^{\infty} e^{\left(-\tilde{y}_{igt} - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} = \left[e^{-e^{-\tilde{y}_{igt}}} \right]_{-\infty}^{\infty} = [1 - 0],$$

which implies:

$$\Phi_{it} = \sum_{g=1}^{N} e^{-\lambda_{igt}} \left(\begin{array}{c} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \left(\lambda_{igt} - \bar{\gamma}\right)\right) \\ + \rho \int_{-\infty}^{\infty} \tilde{y}_{igt} e^{\left(-(\tilde{y}_{igt} + \lambda_{igt}) - e^{\left(-\tilde{y}_{igt}\right)}\right)} d\epsilon_{gt} \end{array} \right).$$

Now note also that:

$$\rho\bar{\gamma} = \rho \int_{-\infty}^{\infty} \tilde{y}_{igt} e^{\left(-(\tilde{y}_{igt}+\lambda_{igt})-e^{\left(-\tilde{y}_{igt}\right)}\right)} d\epsilon_{gt},$$

Therefore:

$$\Phi_{it} = \sum_{g=1}^{N} e^{-\lambda_{igt}} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho\lambda_{igt}\right).$$

Using the definition of λ_{igt}, we have:

$$\Phi_{it} = \sum_{g=1}^{N} e^{-\log\sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}} \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}\right).$$

Recall that

$$\bar{\epsilon}_{igmt} \equiv \frac{\beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - (\kappa_{gi} - \kappa_{mi})}{\rho}.$$

Therefore

$$\begin{aligned}
\left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}\right) &= \left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi} + \rho \log \sum_{k=1}^{N} e^{-\frac{\beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - \left(\kappa_{gi}-\kappa_{mi}\right)}{\nu}}\right) \\
&= \rho \log\left(\sum_{m=1}^{N} e^{\frac{\beta \mathbb{E}_t v_{mt+1}^{w} - \kappa_{mi}}{\nu}}\right) \\
&= \rho \log\left(\sum_{m=1}^{N} e^{\left(\beta \mathbb{E}_t v_{mt+1}^{w} - \kappa_{mi}\right)^{1/\rho}}\right),
\end{aligned}$$

and

$$\begin{aligned}
\sum_{g=1}^{N} e^{-\log\sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}} &= \sum_{g=1}^{N} e^{-\log\sum_{m=1}^{N} e^{-\frac{\beta\left(\mathbb{E}_t v_{gt+1}^{w} - \mathbb{E}_t v_{mt+1}^{w}\right) - \left(\kappa_{gi}-\kappa_{mi}\right)}{\nu}}}, \\
&= \sum_{g=1}^{N} e^{-\log\left[e^{-\left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi}\right)^{1/\rho}} \sum_{m=1}^{N} e^{\left(\beta \mathbb{E}_t v_{mt+1}^{w} - \kappa_{mi}\right)^{1/\rho}}\right]} \\
&= \sum_{g=1}^{N} e^{\left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi}\right)^{1/\rho}} \sum_{m=1}^{N} e^{-\left(\beta \mathbb{E}_t v_{mt+1}^{w} - \kappa_{mi}\right)^{1/\rho}} \\
&= 1.
\end{aligned}$$

Therefore, we have:

$$\Phi_{it} = \max_{\{g\}_1^N} \left\{\beta \mathbb{E}_t \mathbb{E}_\epsilon \left[\mathbb{V}_{gt+1}^{w}\right] - \kappa_{gi} + \rho\epsilon_{gt}\right\} = \rho \log\left(\sum_{g=1}^{N} e^{\left(\beta \mathbb{E}_t v_{gt+1}^{w} - \kappa_{gi}\right)^{1/\rho}}\right).$$

Using this result, we obtain the expression for expected utility in equation (14) in the paper:

$$v_{it}^{w} = \ln\left(\frac{w_{it}}{p_{it}}\right) + \ln b_{it} + \rho \log \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{w}\right)/\kappa_{gi}\right)^{1/\rho}.$$

S.2.7 Derivation of Outmigration Probabilities

In this subsection, we derive the outmigration probabilities (D_{igt}) in equation (16) in the paper. The probability that a worker migrates from location i to location g at the end of period t is given by:

$$D_{igt} = \text{Prob}\left[\frac{\beta \mathbb{E}_t v^w_{gt+1} - \kappa_{gi}}{\rho} + \epsilon_{gt} \geq \max_{m \neq g}\left\{\frac{\beta \mathbb{E}_t v^w_{mt+1} - \kappa_{mi}}{\rho} + \epsilon_{mt}\right\}\right],$$

$$D_{igt} = \text{Prob}\left[\frac{\beta\left(\mathbb{E}_t v^w_{gt+1} - \mathbb{E}_t v^w_{mt+1}\right) - \left(\kappa_{gi} - \kappa_{mi}\right)}{\rho} + \epsilon_{gt} \geq \max_{m \neq g}\left\{\epsilon_{mt}\right\}\right].$$

Therefore this outmigration probability can be written as:

$$D_{igt} = \int_{-\infty}^{\infty} f\left(\epsilon_{gt}\right) \prod_{m \neq g} F\left(\frac{\beta\left(\mathbb{E}_t v^w_{gt+1} - \mathbb{E}_t v^w_{mt+1}\right) - \left(\kappa_{gi} - \kappa_{mi}\right)}{\rho} + \epsilon_{gt}\right) d\epsilon_{gt}.$$

Using our extreme value distributional assumption and the definition of $\bar{\epsilon}_{igmt}$ in the previous subsection, we can write this as:

$$D_{igt} = \int_{-\infty}^{\infty} e^{\left(-\epsilon_{gt} - \bar{y}\right)} e^{-e^{\left(-\epsilon_{gt} - \bar{\gamma}\right)} \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}} d\epsilon_{gt},$$

Recall from the previous subsection the following definitions:

$$\lambda_{igt} \equiv \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}},$$

$$e^{\lambda_{igt}} = \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}},$$

$$\zeta_{gt} \equiv \epsilon_{gt} + \bar{\gamma}.$$

Using these definitions, our outmigration probability can be written as follows:

$$D_{igt} = \int_{-\infty}^{\infty} e^{-\zeta_{gt}} e^{-e^{-\zeta_{gt}} e^{\lambda_{igt}}} d\zeta_{gt},$$

Now recall the following additional definition from the previous subsection:

$$\tilde{y}_{igt} \equiv \zeta_{gt} - \lambda_{igt}.$$

$$D_{igt} = \int_{-\infty}^{\infty} e^{-\left(\tilde{y}_{igt} + \lambda_{igt}\right)} e^{-e^{-\left(\tilde{y}_{igt} + \lambda_{igt}\right)} e^{\lambda_{igt}}} d\tilde{y}_{igt},$$

$$D_{igt} = e^{-\lambda_{igt}} \int_{-\infty}^{\infty} e^{-\tilde{y}_{igt}} e^{-e^{-\left(\tilde{y}_{igt}\right)}} d\tilde{y}_{igt},$$

$$D_{igt} = e^{-\lambda_{igt}} \int_{-\infty}^{\infty} e^{-\tilde{y}_{igt} - e^{-\left(\tilde{y}_{igt}\right)}} d\tilde{y}_{igt},$$

Recall that:

$$\int_{-\infty}^{\infty} e^{\left(-\tilde{y}_{igt}-e^{\left(-\tilde{y}_{igt}\right)}\right)} d\tilde{y}_{igt} = \left[e^{-e^{-\tilde{y}_{igt}}}\right]_{-\infty}^{\infty} = [1-0].$$

Therefore we have

$$D_{igt} = e^{-\lambda_{igt}}.$$

Recall

$$\lambda_{igt} \equiv \log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}},$$

Therefore

$$D_{igt} = e^{-\left[\log \sum_{m=1}^{N} e^{-\bar{\epsilon}_{igmt}}\right]},$$

Recall

$$\bar{\epsilon}_{igmt} \equiv \frac{\beta\left(\mathbb{E}_t v_{gt+1} - \mathbb{E}_t v_{mt+1}\right) - \left(\kappa_{gi} - \kappa_{mi}\right)}{\rho}.$$

Therefore

$$D_{igt} = e^{-\log\left[\sum_{m=1}^{N} e^{-\frac{\beta\left(\mathbb{E}_t v_{gt+1} - \mathbb{E}_t v_{mt+1}\right) - \left(\kappa_{gi} - \kappa_{mi}\right)}{\rho}}\right]},$$

$$D_{igt} = e^{-\log\left[e^{-\left(\beta \mathbb{E}_t v_{gt+1} - \kappa_{gi}\right)^{1/\rho}} \sum_{m=1}^{N} e^{\left(\beta \mathbb{E}_t v_{mt+1} - \kappa_{mi}\right)^{1/\rho}}\right]},$$

$$D_{igt} = e^{\log\left[e^{\left(\beta \mathbb{E}_t v_{gt+1} - \kappa_{gi}\right)^{1/\rho}} \sum_{m=1}^{N} e^{-\left(\beta \mathbb{E}_t v_{mt+1} - \kappa_{mi}\right)^{1/\rho}}\right]},$$

$$D_{igt} = e^{\log\left[\frac{e^{\left(\beta \mathbb{E}_t v_{gt+1} - \kappa_{gi}\right)^{1/\rho}}}{\sum_{m=1}^{N} e^{\left(\beta \mathbb{E}_t v_{mt+1} - \kappa_{mi}\right)^{1/\rho}}}\right]},$$

$$D_{igt} = \frac{e^{\left(\beta \mathbb{E}_t v_{gt+1} - \kappa_{gi}\right)^{1/\rho}}}{\sum_{m=1}^{N} e^{\left(\beta \mathbb{E}_t v_{mt+1} - \kappa_{mi}\right)^{1/\rho}}},$$

which yields equation (16) in the paper:

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}\right)/\kappa_{gi}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}\right)/\kappa_{mi}\right)^{1/\rho}}.$$

S.3 Isomorphisms

In Section 2 of the paper, we derive our baseline results using the Armington (1969) model of trade, in which goods are differentiated by location. In this section of the Online Supplement, we show that our results also hold in the class of trade models with a constant trade elasticity considered by Arkolakis et al. (2012), henceforth ACR.

In Section S.3.1, we consider the Ricardian model of trade based on technology differences of Eaton and Kortum (2002), in which markets are perfectly competitive and production technologies are constant returns to scale. In Section S.3.2 we consider the new trade theory model of Krugman (1980), in which markets are monopolistically competitive and production technologies are increasing return to scale. Although for simplicity we assume a representative firm in Section S.3.2, analogous results also hold in the heterogeneous firm model of Melitz (2003) with a Pareto productivity distribution.

In Section S.3.1, the goods market clearing condition in the Eaton and Kortum (2002) model takes exactly the same form as in our Armington (1969) model in Section 2 of the paper. Combining this goods market clearing condition with our specifications of migration decisions and capital accumulation, we obtain the same system of equations for general equilibrium as in Section 2 of the paper. The only difference is that the trade elasticity (θ) in the Eaton Kortum (2002) specification depends on the shape parameter of the Fréchet productivity distribution rather than the elasticity of substitution between varieties.

In Section S.3.2, the presence of love of variety, increasing returns and transport costs in the Krugman (1980) model gives rise to agglomeration forces. As a result, the goods market clearing condition takes a similar form as in the extension of our baseline Armington (1969) model to incorporate agglomeration forces. Combining this goods market clearing condition with our specifications of migration decisions and capital accumulation, we obtain a similar system of equations for general equilibrium as in the extension of our baseline Armington (1969) model to incorporate agglomeration forces.

S.3.1 Ricardian Technology Differences

We consider a version of Eaton and Kortum (2002) with labor and capital as the two factors of production. Migration and capital accumulation are modeled in the same way as in Section 2 of the paper. The only difference from our baseline Armington model in that section of the paper is the specification of preferences and production.

S.3.1.1 Preferences

Workers' indirect utility function in location n at time t is assumed to take the following form:

$$\ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \tag{S.3.1}$$

where b_{nt} are amenities; w_{nt} is the wage; and p_{nt} is the consumption goods price index. Landowners' indirect utility function takes the same form, but their income depends on the rental rate for capital (r_{nt}) rather than the wage (w_{nt}). The consumption goods price index (p_{nt}) is defined over consumption of a fixed continuum of goods according to the constant elasticity of substitution (CES) functional form:

$$p_{nt} = \left[\int_0^1 p_{nt}\left(\vartheta\right)^{1-\sigma} d\vartheta\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{S.3.2}$$

where $p_{nt}(\vartheta)$ denotes the price of good ϑ in location n.

S.3.1.2 Production

Goods are produced with labor and capital according to a constant returns to scale production technology. These goods can be traded between locations subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from location i to location n in order for one unit to arrive (where $\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). Therefore, the price for consumers in location n of purchasing a good ϑ from location i is:

$$p_{nit}(\vartheta) = \frac{\tau_{nit} w_{it}^{\mu} r_{it}^{1-\mu}}{z_{it} a_i(\vartheta)}, \qquad 0 < \mu < 1, \tag{S.3.3}$$

where z_{it} captures common determinants of productivity across goods in location i and $a_i(\vartheta)$ captures idiosyncratic determinants of productivity for each good ϑ within that location. Productivity for each good ϑ in each location i is drawn independently from the following Fréchet distribution:

$$F_i(a) = \exp\left(-a^{-\theta}\right), \qquad \theta > 1,$$

where we normalize the Fréchet scale parameter to one, because it enters the model isomorphically to z_{it}. Using the properties of this Fréchet distribution, location n's share of expenditure on goods produced in location i is:

$$s_{nit} = \frac{\left(\tau_{nit} w_{it}^{\mu} r_{it}^{1-\mu} / z_{it}\right)^{-\theta}}{\sum_{m=1}^{N} \left(\tau_{nmt} w_{mt}^{\mu} r_{mt}^{1-\mu} / z_{mt}\right)^{-\theta}}, \tag{S.3.4}$$

and location n's price index can be expressed as:

$$p_{nt} = \left[\sum_{m=1}^{N} \left(\tau_{nmt} w_{mt}^{\mu} r_{mt}^{1-\mu} / z_{mt}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{S.3.5}$$

S.3.1.3 Market Clearing

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

$$(w_{it}\ell_{it} + r_{it}k_{it}) = \sum_{n=1}^{N} S_{nit}\left(w_{nt}\ell_{nt} + r_{nt}k_{nt}\right). \tag{S.3.6}$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using profit maximization and zero profits, this capital market clearing condition can be expressed as follows:

$$r_{it}k_{it} = \frac{1-\mu}{\mu} w_{it}\ell_{it}. \tag{S.3.7}$$

S.3.1.4 General Equilibrium

Given the state variables $\{\ell_{i0}, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables. We now show that the system of equations for general equilibrium in this version of the Eaton and Kortum (2002) model takes exactly the same form as in our baseline Armington model.

Capital Accumulation: Using capital market clearing (S.3.7), the price index (S.3.5) and the analogous derivations for landlords' consumption-investment decision as in our baseline Armington model, the capital accumulation equation can be expressed as:

$$k_{it+1} = \beta \frac{1-\mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1-\delta) k_{it}, \tag{S.3.8}$$

$$p_{nt} = \left[\sum_{i=1}^{N} \left(w_{it} \left(\frac{1-\mu}{\mu} \right)^{1-\mu} (\ell_{it}/k_{it})^{1-\mu} \tau_{ni}/z_i \right)^{-\theta} \right]^{-1/\theta}, \tag{S.3.9}$$

where for simplicity we assume logarithmic intertemporal utility.

Goods Market Clearing: Using the expenditure share (S.3.4) and capital market clearing (S.3.7) in the goods market clearing condition (S.3.6), we obtain:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}, \tag{S.3.10}$$

$$S_{nit} = \frac{\left(w_{it} (\ell_{it}/k_{it})^{1-\mu} \tau_{ni}/z_i \right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt} (\ell_{mt}/k_{mt})^{1-\mu} \tau_{nm}/z_m \right)^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it}\ell_{it}}, \tag{S.3.11}$$

where S_{nit} is the expenditure share of importer n on exporter i at time t; we have defined T_{int} as the corresponding income share of exporter i from importer n at time t; and note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{S.3.12}$$

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v^w_{gt+1} \right) / \kappa_{git} \right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v^w_{mt+1} \right) / \kappa_{mit} \right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \tag{S.3.13}$$

where D_{igt} is the outmigration probability from location i to location g between time t and $t+1$; we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$; and again note that the order of subscripts switches between the outmigration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location n at time t can be written as:

$$v_{nt}^{w} = \ln b_{nt} + \ln\left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{w}\right) / \kappa_{gnt}\right)^{1/\rho}. \tag{S.3.14}$$

S.3.2 New Trade Model

We consider a version of Krugman (1980) with labor and capital as the two factors of production. Migration and capital accumulation are modeled in the same way as in Section 2 of the paper. The only difference from our baseline Armington model in that section of the paper is the specification of preferences and production. We now show that the system of equations for general equilibrium in this version of the Krugman (1980) model takes a similar form as in the extension of our baseline Armington model with agglomeration economies.

S.3.2.1 Preferences

Workers' indirect period utility function in location n at time t is assumed to take the following form:

$$\ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \tag{S.3.15}$$

where b_{nt} are amenities; w_{nt} is the wage; and p_{nt} is the consumption goods price index. Landowners' indirect utility function takes the same form, but their income depends on the rental rate for capital (r_{nt}) rather than the wage (w_{nt}). The consumption goods price index (p_{nt}) is defined over the consumption of a mass of varieties (M_{it}) from each location i according to the constant elasticity of substitution (CES) functional form:

$$p_{nt} = \left[\sum_{i=1}^{N} \int_{0}^{M_{it}} p_{nit}(j)^{1-\sigma} \, dj\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{S.3.16}$$

where $p_{nit}(j)$ is the price in country n of a variety j produced in country i at time t; the mass of varieties (M_{it}) is endogenously determined by free entry; and varieties are substitutes ($\sigma > 1$).

S.3.2.2 Production

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost (F) and a constant marginal cost that depends on a location's productivity (z_{it}). The production technology is assumed to be homothetic, such that the fixed and marginal cost use labor and capital with the same intensity. In particular, the total cost of producing $x_i(j)$ units of variety j in location i is given by:

$$\varpi_i(j) = \left(F + \frac{x_{it}(j)}{z_{it}}\right) w_{it}^{\mu} r_{it}^{1-\mu}, \qquad 0 < \mu < 1. \tag{S.3.17}$$

Varieties can be traded between countries subject to iceberg variable costs of trade, such that $\tau_{ni} \geq 1$ units must be shipped from country i to country n in order for one unit to arrive (where

$\tau_{ni} > 1$ for $n \neq i$ and $\tau_{nn} = 1$). The cost to the consumer in location n of sourcing a variety from location i is thus:

$$p_{nit}(j) = \tau_{nit} p_{iit}(j), \tag{S.3.18}$$

where $p_{iit}(j)$ is the "free on board" price before transport costs. Profit maximization and zero profits imply that this free on board price is a constant markup over marginal cost:

$$p_{iit}(j) = \overline{p}_{iit} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{w_{it}^{\mu} r_{it}^{1-\mu}}{z_{it}}, \tag{S.3.19}$$

and equilibrium variety output is equal to a constant that depends on location productivity:

$$x_{it}(j) = \overline{x}_{it} = z_{it}(\sigma - 1) F. \tag{S.3.20}$$

Multiplying equilibrium prices and output, variety revenue is given by:

$$\overline{y}_{it} = \overline{p}_{iit} \overline{x}_{it} = \sigma F w_{it}^{\mu} r_{it}^{1-\mu}.$$

Additionally, cost minimization implies that capital payments are a constant multiple of labor payments:

$$r_{it} k_{it} = \frac{1-\mu}{\mu} w_{it} \ell_{it}. \tag{S.3.21}$$

Using this implication of cost minimization, variety revenue can be re-written as:

$$\overline{y}_{it} = \sigma F w_{it} \left(\frac{1-\mu}{\mu}\right)^{1-\mu} \left(\frac{1}{\chi_{it}}\right)^{1-\mu}. \tag{S.3.22}$$

The mass of varieties in each location equals aggregate revenue divided by variety variety:

$$M_{it} = \frac{r_{it} k_{it} + w_{it} \ell_{it}}{\overline{y}_{it}}.$$

Using the constant relationship between capital payments and labor payments (S.3.21) and the expression for variety revenue (S.3.22), the mass of varieties can be expressed as:

$$M_{it} = \frac{\ell_{it} (\chi_{it})^{1-\mu}}{\sigma F \lambda \left(\frac{1-\mu}{\mu}\right)^{1-\mu}}. \tag{S.3.23}$$

Using the properties of CES demand, country n's share of expenditure on goods produced in country i is:

$$s_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{m=1}^{N} M_m p_{nm}^{1-\sigma}}.$$

Using equilibrium prices in equations (S.3.18) and (S.3.19) and the mass of varieties (S.3.23), we can re-write this expenditure share as:

$$s_{ni} = \frac{\ell_{it} (\chi_{it})^{1-\mu} \left(\tau_{nit} w_{it}^{\mu} r_{it}^{1-\mu} / z_{it}\right)^{1-\sigma}}{\sum_{m=1}^{N} \ell_{mt} (\chi_{mt})^{1-\mu} \left(\tau_{nmt} w_{mt}^{\mu} r_{mt}^{1-\mu} / z_{mt}\right)^{1-\sigma}}, \tag{S.3.24}$$

and the price index (S.3.16) as:

$$p_{nt} = \left[\sum_{m=1}^{N} \frac{\ell_{mt} \left(\chi_{mt}\right)^{1-\mu}}{\sigma F \lambda \left(\frac{1-\mu}{\mu}\right)^{1-\mu}} \left(\frac{\sigma}{\sigma - 1} \tau_{nmt} w_{mt}^{\mu} r_{mt}^{1-\mu} / z_{mt} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{S.3.25}$$

Using the relationship between capital and labor payments (S.3.21), we can further re-write the expenditure share (S.3.24) as:

$$s_{ni} = \frac{\ell_{it} \left(\chi_{it}\right)^{\sigma(1-\mu)} \left(\tau_{nit} w_{it} / z_{it}\right)^{1-\sigma}}{\sum_{m=1}^{N} \ell_{mt} \left(\chi_{mt}\right)^{\sigma(1-\mu)} \left(\tau_{nmt} w_{mt} / z_{mt}\right)^{1-\sigma}}, \tag{S.3.26}$$

and the price index (S.3.25) as:

$$p_{nt} = \left[\sum_{m=1}^{N} \frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma F \lambda \left(\frac{1-\mu}{\mu}\right)^{1-\mu}} \ell_{mt} \left(\chi_{mt}\right)^{\sigma(1-\mu)} \left(\tau_{nmt} w_{mt} \left(\frac{1-\mu}{\mu} \right)^{1-\mu} / z_{mt} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{S.3.27}$$

S.3.2.3 Market Clearing

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

$$\left(w_{it} \ell_{it} + r_{it} k_{it}\right) = \sum_{n=1}^{N} S_{nit} \left(w_{nt} \ell_{nt} + r_{nt} k_{nt}\right). \tag{S.3.28}$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using profit maximization and zero profits, this capital market clearing condition is given by equation (S.3.21) above.

S.3.2.4 General Equilibrium

Given the state variables $\{\ell_{i0}, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables. We now show that the system of equations for general equilibrium in this version of the Eaton and Kortum (2002) model takes a similar form as in the extension of our baseline Armington model with agglomeration economies.

Capital Accumulation: Using capital market clearing (S.3.21), the price index (S.3.27) and the analogous derivations for landlords' consumption-investment decisions as in our baseline Armington model, the capital accumulation equation can be expressed as:

$$k_{it+1} = \beta \frac{1-\mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1-\delta) k_{it}, \tag{S.3.29}$$

$$p_{nt} = \left[\sum_{m=1}^{N} \frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}{\sigma F \lambda \left(\frac{1-\mu}{\mu}\right)^{1-\mu}} \ell_{mt} \left(\chi_{mt}\right)^{\sigma(1-\mu)} \left(\tau_{nmt} w_{mt} \left(\frac{1-\mu}{\mu} \right)^{1-\mu} / z_{mt} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{S.3.30}$$

where for simplicity we assume logarithmic intertemporal utility.

Goods Market Clearing: Using the expenditure share (S.3.26) and capital market clearing (S.3.21) in the goods market clearing condition (S.3.28), we obtain:

$$w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}, \tag{S.3.31}$$

$$S_{nit} = \frac{\ell_{it} \left(\chi_{it}\right)^{\sigma(1-\mu)} \left(\tau_{nit} w_{it} / z_{it}\right)^{1-\sigma}}{\sum_{m=1}^{N} \ell_{mt} \left(\chi_{mt}\right)^{\sigma(1-\mu)} \left(\tau_{nmt} w_{mt} / z_{mt}\right)^{1-\sigma}}, \qquad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}}, \tag{S.3.32}$$

where S_{nit} is the expenditure share of importer n on exporter i at time t; we have defined T_{int} as the corresponding income share of exporter i from importer n at time t; and note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{S.3.33}$$

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{w}\right) / \kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^{w}\right) / \kappa_{mit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \tag{S.3.34}$$

where D_{igt} is the outmigration probability from location i to location g between time t and $t+1$; we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$; and again note that the order of subscripts switches between the outmigration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location n at time t can be written as:

$$v_{nt}^{w} = \ln b_{nt} + \ln\left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{w}\right) / \kappa_{gnt}\right)^{1/\rho}. \tag{S.3.35}$$

S.4 Extensions

In this section of the Online Supplement, we consider a number of extensions of our baseline specification from Section 2 of the paper. In Subsection S.4.1, we show that our results naturally generalize to accommodate shocks to trade and migration frictions, in addition to shocks to productivity and amenities.

In Subsection S.4.2, we allow for agglomeration and dispersion forces, such that both productivity and amenities are endogenous to the surrounding concentration of economic activity. In Subsection S.4.3 we introduce multiple final goods sectors with region-specific capital. In Section S.4.4, we incorporate multiple final goods sectors with region-sector-specific capital. In Section S.4.5, we further generalizes the analysis to allow for multiple final goods sectors with region-sector-specific capital and input-output linkages.

In Subsection S.4.6, we generalize our baseline specification to allow for trade deficits, following the standard approach in the quantitative international trade literature of treating these trade deficits as exogenous. In Subsection S.4.7, we allow capital to be used residentially (housing) as well as commercially.

In Subsection S.4.8, we report an extension to allow landlords to invest in other locations. Finally, in Section S.4.9, we discuss an extension to incorporate an endogenous labor participation decision.

S.4.1 Shocks to Trade and Migration Costs

In this Subsection, we derive sufficient statistics for changes in steady-states and the transition path, allowing for shocks to trade and migration costs, as well as to productivity and amenities, as discussed in Section 4 of the paper. In the interests of brevity, we focus on the case in which the economy starts from a steady-state, for which we observe the trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$). For simplicity, we also assume logarithmic intertemporal utility. We derive sufficient statistics for changes in steady-states and the transition path in response to small changes in productivities ($\mathrm{d}\ln \boldsymbol{z}$), amenities ($\mathrm{d}\ln \boldsymbol{b}$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau}$) and migration costs ($\mathrm{d}\ln \kappa$), using the observed trade and migration matrices from the initial steady-state.

S.4.1.1 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1} = k_{it} = k_i^*$, $\ell_{it+1} = \ell_{it} = \ell_i^*$, $w_{it+1}^* = w_{it}^* = w_i^*$ and $v_{it+1}^* = v_{it}^* = v_i^*$, where we use an asterisk to denote a steady-state value.

Capital Accumulation. From the capital accumulation equation (11) in the paper, the steady-state stock of capital solves:

$$(1-\beta(1-\delta))\,\chi_i^* = (1-\beta(1-\delta))\,\frac{k_i^*}{\ell_i^*} = \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln\left(\frac{w_i^*}{p_i^*}\right).$$

Using the total derivative of real income in equation (B.24) in the Online Appendix, this becomes:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln w_i^* - \sum_{m=1}^{N} S_{im}^*\left[\mathrm{d}\ln\tau_{im} + \mathrm{d}\ln w_m^* - (1-\mu)\,\mathrm{d}\ln\chi_m^* - \mathrm{d}\ln z_m\right],$$

which can be re-written as:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln w_i^* - \sum_{m=1}^{N} S_{im}^*\left[\mathrm{d}\ln w_m^* - (1-\mu)\,\mathrm{d}\ln\chi_m^* - \mathrm{d}\ln z_m\right] - \mathrm{d}\ln\tau_i^{\text{in}},$$

where $\mathrm{d}\ln\tau_i^{\text{in}}$ is a measure of weighted-average incoming trade costs defined as:

$$\mathrm{d}\ln\tau_i^{\text{in}} \equiv \sum_{m=1}^{N} S_{im}^*\,\mathrm{d}\ln\tau_{im}.$$

This relationship has the matrix representation:

$$\mathrm{d}\ln\boldsymbol{\chi}^* = \mathrm{d}\ln\boldsymbol{w}^* - \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{w}^* + (1-\mu)\,\boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{\chi}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z} - \mathrm{d}\ln\boldsymbol{\tau}^{\text{in}},$$

$$(\boldsymbol{I} - (1-\mu)\,\boldsymbol{S})\,\mathrm{d}\ln\boldsymbol{\chi}^* = (\boldsymbol{I} - \boldsymbol{S})\,\mathrm{d}\ln\boldsymbol{w}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z} - \mathrm{d}\ln\boldsymbol{\tau}^{\text{in}}. \tag{S.4.1}$$

Goods Market Clearing. The total derivative of the goods market clearing condition in equation (B.26) in the paper can be re-written as:

$$\left[\begin{array}{c}\mathrm{d}\ln w_{it}\\ +\,\mathrm{d}\ln\ell_{it}\end{array}\right] = \left[\begin{array}{c}\sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln w_{nt} + \mathrm{d}\ln\ell_{nt}\right)\\ +\theta\sum_{n=1}^{N}\sum_{m=1}^{N} T_{int}S_{nmt}\left(\mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln\chi_{mt} - \mathrm{d}\ln z_{mt}\right)\\ -\theta\sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln w_{it} - (1-\mu)\,\mathrm{d}\ln\chi_{it} - \mathrm{d}\ln z_{it}\right)\\ +\theta\sum_{n=1}^{N} T_{int}\,\mathrm{d}\ln\tau_{nt}^{\text{in}} - \theta\,\mathrm{d}\ln\tau_{it}^{\text{out}}\end{array}\right],$$

where $\mathrm{d}\ln\tau_{it}^{\text{in}}$ is defined above and $\mathrm{d}\ln\tau_{it}^{\text{out}}$ is defined as:

$$\mathrm{d}\ln\tau_{it}^{\text{out}} \equiv \sum_{n=1}^{N} T_{int}\,\mathrm{d}\ln\tau_{nit}.$$

This relationship above has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{w_t} + \mathrm{d}\ln \boldsymbol{\ell_t} = \boldsymbol{T}\left(\mathrm{d}\ln \boldsymbol{w_t} + \mathrm{d}\ln \boldsymbol{\ell_t}\right) + \theta \begin{bmatrix} \left(\boldsymbol{TS}-\boldsymbol{I}\right)\left(\mathrm{d}\ln \boldsymbol{w_t} - (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi_t} - \mathrm{d}\ln \boldsymbol{z_t}\right) \\ +\boldsymbol{T}\,\mathrm{d}\ln \boldsymbol{\tau}_{\boldsymbol{t}}^{\text{in}} - \mathrm{d}\ln \boldsymbol{\tau}_{\boldsymbol{t}}^{\text{out}} \end{bmatrix}.$$

We can re-write this relationship as:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln \boldsymbol{w_t} = \begin{bmatrix} -\left(\boldsymbol{I}-\boldsymbol{T}\right)\,\mathrm{d}\ln \boldsymbol{\ell_t} + \theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln \boldsymbol{z_t} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi_t}\right) \\ +\theta\left[\boldsymbol{T}\,\mathrm{d}\ln \boldsymbol{\tau}_{\boldsymbol{t}}^{\text{in}} - \mathrm{d}\ln \boldsymbol{\tau}_{\boldsymbol{t}}^{\text{out}}\right] \end{bmatrix}.$$

In steady-state we have:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln \boldsymbol{w}^{*} = \begin{bmatrix} -\left(\boldsymbol{I}-\boldsymbol{T}\right)\,\mathrm{d}\ln \boldsymbol{\ell}^{*} + \theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^{*}\right) \\ +\theta\left[\boldsymbol{T}\,\mathrm{d}\ln \boldsymbol{\tau}^{\text{in}} - \mathrm{d}\ln \boldsymbol{\tau}^{\text{out}}\right] \end{bmatrix}. \tag{S.4.2}$$

Population Flow. The total derivative of the population flow condition in equation (15) in the paper can be re-written as:

$$\begin{aligned} \mathrm{d}\ln \ell_{gt+1} = {} & \sum_{i=1}^{N} E_{git}\,\mathrm{d}\ln \ell_{it} + \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1} - \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\,\mathrm{d}\ln \kappa_{git} \\ & - \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\sum_{m=1}^{N} D_{imt}\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1} + \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\sum_{m=1}^{N} D_{imt}\,\mathrm{d}\ln \kappa_{mit}, \end{aligned}$$

and hence:

$$\begin{aligned} \mathrm{d}\ln \ell_{gt+1} = {} & \sum_{i=1}^{N} E_{git}\,\mathrm{d}\ln \ell_{it} + \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1} - \frac{1}{\rho}\,\mathrm{d}\ln \kappa_{gt}^{\text{in}} \\ & - \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\sum_{m=1}^{N} D_{imt}\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1} + \frac{1}{\rho}\sum_{i=1}^{N} E_{git}\,\mathrm{d}\ln \kappa_{it}^{\text{out}}, \end{aligned}$$

where we have defined:

$$\mathrm{d}\ln \kappa_{gt}^{\text{in}} \equiv \sum_{i=1}^{N} E_{git}\,\mathrm{d}\ln \kappa_{git},$$

$$\mathrm{d}\ln \kappa_{it}^{\text{out}} \equiv \sum_{m=1}^{N} D_{imt}\,\mathrm{d}\ln \kappa_{mit}.$$

This total derivative has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}} - \frac{1}{\rho}\left(\mathrm{d}\ln \boldsymbol{\kappa}_{\boldsymbol{t}}^{\text{in}} - \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\kappa}_{\boldsymbol{t}}^{\text{out}}\right).$$

In steady-state, we have:

$$\mathrm{d}\ln \boldsymbol{\ell}^{*} = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell}^{*} + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\,\mathrm{d}\boldsymbol{v}^{*} - \frac{1}{\rho}\left(\mathrm{d}\ln \boldsymbol{\kappa}^{\text{in}} - \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\kappa}^{\text{out}}\right). \tag{S.4.3}$$

Value function. The total derivative of the value function in equation (14) in the paper can be re-written as:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \sum_{m=1}^{N} S_{imt}\left(\mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln \chi_{mt} - \mathrm{d}\ln z_{mt}\right) - \mathrm{d}\ln \tau_{it}^{\text{in}} \\ + \mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt}\left(\beta \mathbb{E}_t\, \mathrm{d}v_{mt+1}\right) - \mathrm{d}\ln \kappa_{it}^{\text{out}} \end{array} \right],$$

where $\mathrm{d}\ln \tau_{it}^{\text{in}}$ and $\mathrm{d}\ln \kappa_{it}^{\text{out}}$ are defined above. The above relationship has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w_t} + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z_t} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi_t}\right) - \mathrm{d}\ln \boldsymbol{\tau_t}^{\text{in}} \\ + \mathrm{d}\ln \boldsymbol{b_t} - \mathrm{d}\ln \boldsymbol{\kappa_t}^{\text{out}} + \beta \boldsymbol{D}\mathbb{E}_t\, \mathrm{d}\boldsymbol{v_{t+1}} \end{array} \right].$$

In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^* = \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) - \mathrm{d}\ln \boldsymbol{\tau}^{\text{in}} \\ + \mathrm{d}\ln \boldsymbol{b} - \mathrm{d}\ln \boldsymbol{\kappa}^{\text{out}} + \beta \boldsymbol{D}\,\mathrm{d}\boldsymbol{v}^* \end{array} \right]. \tag{S.4.4}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left[\boldsymbol{I} - (1-\mu)\,\boldsymbol{S}\right]^{-1}\left[(\boldsymbol{I} - \boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z} - \mathrm{d}\ln \boldsymbol{\tau}^{\text{in}}\right]. \tag{S.4.5}$$

$$\mathrm{d}\ln \boldsymbol{w}^* = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\,(\boldsymbol{I} - \boldsymbol{TS})\right]^{-1} \left[\begin{array}{c} \left(-(\boldsymbol{I} - \boldsymbol{T})\,\mathrm{d}\ln \boldsymbol{\ell}^* + (\boldsymbol{I} - \boldsymbol{TS})\,\theta\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right)\right) \\ +\theta\left[\boldsymbol{T}\,\mathrm{d}\ln \boldsymbol{\tau}^{\text{in}} - \mathrm{d}\ln \boldsymbol{\tau}^{\text{out}}\right] \end{array} \right]. \tag{S.4.6}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = (\boldsymbol{I} - \boldsymbol{E})^{-1}\left[\frac{\beta}{\rho}(\boldsymbol{I} - \boldsymbol{ED})\,\mathrm{d}\boldsymbol{v}^* - \frac{1}{\rho}\left(\mathrm{d}\ln \boldsymbol{\kappa}^{\text{in}} - \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\kappa}^{\text{out}}\right)\right]. \tag{S.4.7}$$

$$\mathrm{d}\boldsymbol{v}^* = \left[\boldsymbol{I} - \beta \boldsymbol{D}\right]^{-1} \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) - \mathrm{d}\ln \boldsymbol{\tau}^{\text{in}} \\ + \mathrm{d}\ln \boldsymbol{b} - \mathrm{d}\ln \boldsymbol{\kappa}^{\text{out}} \end{array} \right]. \tag{S.4.8}$$

As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^{N} w_i^* \ell_i^* = \sum_{i=1}^{N} q_i^* = \overline{q} = 1$), which implies that the log changes in incomes satisfy $\boldsymbol{q}^*\,\mathrm{d}\ln \boldsymbol{q}^* = \sum_{i=1}^{N} q_i^*\,\mathrm{d}\ln q_i^* = \sum_{i=1}^{N} q_i^* \frac{\mathrm{d}q_i^*}{q_i^*} = \sum_{i=1}^{N} \mathrm{d}q_i^* = 0$, where $\boldsymbol{q}^*$ is a row vector of the steady-state income of each location. Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_i = \overline{\ell} = 1$, which implies $\boldsymbol{\ell}^*\,\mathrm{d}\ln \boldsymbol{\ell}^* = \sum_{i=1}^{N} \ell_i^*\,\mathrm{d}\ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{\mathrm{d}\ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} \mathrm{d}\ell_i^* = 0$, where $\boldsymbol{\ell}^*$ is a row vector of the steady-state population of each location.

S.4.1.2 Sufficient Statistics for Transition Dynamics Starting from Steady-State

We suppose that the economy starts from an initial steady-state distribution of economic activity $\{k_i^*, \ell_i^*, w_i^*, v_i^*\}$. We consider small shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$), amenities ($\mathrm{d}\ln \boldsymbol{b}$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau}$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa}$), holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \overline{\ell} = 0$). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\widetilde{\chi}_{it} = \ln \chi_{it} - \ln \chi_i^*$, for all variables except for the worker value function v_{it}; with a slight abuse of notation we use $\widetilde{v}_{it} \equiv v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (11) in the paper, we have:

$$
\begin{aligned}
k_{it+1} &= \beta\left(1-\delta\right)k_{it} + \beta\frac{1-\mu}{\mu}\frac{w_{it}}{p_{it}}\ell_{it},\\
\frac{k_{it+1}}{\ell_{it+1}}\frac{\ell_{it+1}}{\ell_{it}} &= \beta\left(1-\delta\right)\frac{k_{it}}{\ell_{it}} + \beta\frac{1-\mu}{\mu}\frac{w_{it}}{p_{it}},\\
\chi_{it+1}\frac{\ell_{it+1}}{\ell_{it}} &= \beta\left(1-\delta\right)\chi_{it} + \beta\frac{1-\mu}{\mu}\frac{w_{it}}{p_{it}}, \qquad \text{(S.4.9)}
\end{aligned}
$$

while in steady-state we have:

$$
\begin{aligned}
\frac{k_i^*}{\ell_i^*} &= \beta\left(1-\delta\right)\frac{k_i^*}{\ell_i^*} + \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*},\\
\chi_i^* &= \beta\left(1-\delta\right)\chi_i^* + \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}.\\
\chi_i^* &= \frac{\beta}{\left(1-\beta\left(1-\delta\right)\right)}\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}. \qquad \text{(S.4.10)}
\end{aligned}
$$

Dividing both sides of equation (S.4.9) by χ_i^*, we have:

$$
\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \frac{\beta}{\chi_i^*}\frac{1-\mu}{\mu}\frac{w_{it}}{p_{it}},
$$

which using (S.4.10) can be re-written as:

$$
\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \left(1-\beta\left(1-\delta\right)\right)\frac{w_{it}/w_i^*}{p_{it}/p_i^*},
$$

which can be further re-written as:

$$
\begin{aligned}
\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 &= \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \left(1-\beta\left(1-\delta\right)\right)\frac{w_{it}/w_i^*}{p_{it}/p_i^*} - 1,\\
\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 &= \beta\left(1-\delta\right)\left(\frac{\chi_{it}}{\chi_i^*} - 1\right) + \left(1-\beta\left(1-\delta\right)\right)\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*} - 1\right).
\end{aligned}
$$

Noting that:

$$
\begin{aligned}
\frac{x_{it}}{x_i^*} - 1 &\simeq \ln\left(\frac{x_{it}}{x_i^*}\right),\\
\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 &\simeq \ln\left(\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}}\right),
\end{aligned}
$$

we have:

$$
\begin{aligned}
\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right) + \ln\left(\frac{\ell_{it+1}}{\ell_{it}}\right) &= \beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),\\
\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right) + \ln\left(\frac{\ell_{it+1}/\ell_i^*}{\ell_{it}/\ell_i^*}\right) &= \beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),
\end{aligned}
$$

which can be re-written as follows:

$$\widetilde{\chi}_{it+1} = \beta(1-\delta)\widetilde{\chi}_{it} + (1-\beta(1-\delta))(\widetilde{w}_{it} - \widetilde{p}_{it}) - \widetilde{\ell}_{it+1} + \widetilde{\ell}_{it},$$

We can re-write the above relationship for the log deviation of the capital-labor ratio from the initial steady-state as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \beta(1-\delta)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + (1-\beta(1-\delta))(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}}) - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}. \tag{S.4.11}$$

Taking the total derivative of real income relative to the initial steady-state, we have:

$$\widetilde{w}_{it} - \widetilde{p}_{it} = \widetilde{w}_{it} - \sum_{m=1}^{N} S_{imt}\left[\widetilde{\tau}_{imt} + \widetilde{w}_{mt} - (1-\mu)\widetilde{\chi}_{mt} - \widetilde{z}_m\right],$$

$$\widetilde{w}_{it} - \widetilde{p}_{it} = \widetilde{w}_{it} - \sum_{m=1}^{N} S_{imt}\left[\widetilde{w}_{mt} - (1-\mu)\widetilde{\chi}_{mt} - \widetilde{z}_m\right] - \widetilde{\tau}_{imt}^{\text{in}},$$

where

$$\widetilde{\tau}_{imt}^{\text{in}} \equiv \sum_{m=1}^{N} S_{imt}\widetilde{\tau}_{imt}.$$

We can re-write this relationship in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}} = (\boldsymbol{I} - \boldsymbol{S})\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + (1-\mu)\boldsymbol{S}\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}} - \widetilde{\boldsymbol{\tau}}^{\text{in}}.$$

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \begin{bmatrix} \left[\beta(1-\delta)\boldsymbol{I} + (1-\beta(1-\delta))(1-\mu)\boldsymbol{S}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ +(1-\beta(1-\delta))(\boldsymbol{I}-\boldsymbol{S})\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + (1-\beta(1-\delta))\boldsymbol{S}\widetilde{\boldsymbol{z}} \\ -(1-\beta(1-\delta))\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{bmatrix}. \tag{S.4.12}$$

Goods Market Clearing. The total derivative of the goods market clearing condition in equation (12) in the paper relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} = \begin{bmatrix} \boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \theta(\boldsymbol{TS} - \boldsymbol{I})(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - (1-\mu)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{z}}) \\ +\theta\left[\boldsymbol{T}\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\tau}}^{\text{out}}\right] \end{bmatrix},$$

where

$$\widetilde{\tau}_{it}^{\text{out}} \equiv \sum_{n=1}^{N} T_{int}\widetilde{\tau}_{nit}.$$

We can re-write this relationship as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta(\boldsymbol{I} - \boldsymbol{TS})\right]^{-1}\begin{bmatrix} -(\boldsymbol{I}-\boldsymbol{T})\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta(\boldsymbol{I}-\boldsymbol{TS})(\widetilde{\boldsymbol{z}} + (1-\mu)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}) \\ +\theta\left[\boldsymbol{T}\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\tau}}^{\text{out}}\right] \end{bmatrix}. \tag{S.4.13}$$

Population Flow. The total derivative of the population flow condition in equation (15) in the paper relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} - \frac{1}{\rho}\left(\widetilde{\boldsymbol{\kappa}}^{\text{in}} - \boldsymbol{E}\widetilde{\boldsymbol{\kappa}}^{\text{out}}\right), \tag{S.4.14}$$

where

$$\widetilde{\kappa}_{gt}^{\text{in}} \equiv \sum_{i=1}^{N} E_{git}\widetilde{\kappa}_{git},$$

$$\widetilde{\kappa}_{git}^{\text{out}} \equiv \sum_{m=1}^{N} D_{imt}\widetilde{\kappa}_{mit}.$$

Value Function. The total derivative of the value function in equation (14) in the paper relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{c} \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}} + \left(1-\mu\right)\boldsymbol{S}\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\tau}}^{\text{in}} \\ +\widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{\kappa}}^{\text{out}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array}\right]. \tag{S.4.15}$$

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (S.4.12), the goods market clearing condition (S.4.13), the population flow condition (S.4.14), and the value function (S.4.15), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\mu\right)\boldsymbol{S}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ +\left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} \\ -\left(1-\beta\left(1-\delta\right)\right)\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.4.16}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[\begin{array}{c} -\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left(\widetilde{\boldsymbol{z}} + \left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\right) \\ +\theta\left[\boldsymbol{T}\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\tau}}^{\text{out}}\right] \end{array}\right]. \tag{S.4.17}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} - \frac{1}{\rho}\left(\widetilde{\boldsymbol{\kappa}}^{\text{in}} - \boldsymbol{E}\widetilde{\boldsymbol{\kappa}}^{\text{out}}\right). \tag{S.4.18}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{c} \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}} + \left(1-\mu\right)\boldsymbol{S}\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\tau}}^{\text{in}} \\ +\widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{\kappa}}^{\text{out}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array}\right]. \tag{S.4.19}$$

S.4.2 Agglomeration Forces

In this section of the Online Supplement, we generalize our baseline specification from Section 2 of the paper to introduce agglomeration forces. We allow productivity and amenities to both have an exogenous component, which captures locational fundamentals such as climate and access to natural water, and an endogenous component, which captures agglomeration forces, and depends on the surrounding concentration of economic activity.

S.4.2.1 Productivity and Amenities

We follow the standard approach in the spatial economics literature of modelling these agglomeration forces as a power function of a location's own population: $z_{it} = \bar{z}_{it}\ell_{it}^{\eta^z}$ and $b_{it} = \bar{b}_{it}\ell_{it}^{\eta^b}$, where $\eta^z > 0$ and $\eta^b > 0$ parameterize the strength of agglomeration forces in productivity and amenities respectively.[5] In this extension, the general equilibrium conditions of the model remain as in Section 2.6 of the paper, except that the price index (10) and the expenditure share (13) are modified to incorporate agglomeration forces in production ($z_{it} = \bar{z}_{it}\ell_{it}^{\eta^z}$), and the value function (14) is adjusted to include agglomeration forces in amenities ($b_{it} = \bar{b}_{it}\ell_{it}^{\eta^b}$).

S.4.2.2 General Equilibrium

Given the state variables $\{\ell_{i0}, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing in equation (9) in the paper, the price index in equation (4) in the paper and the equilibrium pricing rule in equation (2) in the paper, the law of motion for capital is:

$$k_{it+1} = \beta\frac{1-\mu}{\mu}\frac{w_{it}}{p_{it}}\ell_{it} + \beta(1-\delta)k_{it}, \tag{S.4.20}$$

$$p_{nt} = \left[\sum_{i=1}^{N}\left(w_{it}\left(\frac{1-\mu}{\mu}\right)^{1-\mu}(\ell_{it}/k_{it})^{1-\mu}\tau_{ni}/\left(\bar{z}_{it}\ell_{it}^{\eta^z}\right)\right)^{-\theta}\right]^{-1/\theta}, \tag{S.4.21}$$

where for simplicity we assume logarithmic intertemporal utility. The presence of agglomeration forces implies that the term in $\ell_{it}^{\eta^z}$ appears in the expression for the price index (p_{nt}) in equation (S.4.21).

Goods Market Clearing: Using the equilibrium pricing rule in equation (2) in the paper, the CES expenditure share, and capital market clearing in equation (9) in the paper, together with the goods market clearing condition in equation (8) in the paper, we obtain:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit}w_{nt}\ell_{nt}, \tag{S.4.22}$$

[5]While for simplicity we assume that agglomeration and dispersion forces only depend on a location's own population, it is straightforward to also introduce spillovers across locations, as in Ahlfeldt, Redding, Sturm and Wolf (2015) and Allen, Arkolakis and Li (2020). Dispersion forces in productivity and amenities can be introduced through $\eta^z < 0$ and $\eta^b < 0$ respectively.

$$S_{nit} = \frac{\left(w_{it}\left(\ell_{it}/k_{it}\right)^{1-\mu}\tau_{ni}/\left(\bar{z}_{it}\ell_{it}^{\eta^z}\right)\right)^{-\theta}}{\sum_{m=1}^{N}\left(w_{mt}\left(\ell_{mt}/k_{mt}\right)^{1-\mu}\tau_{nm}/\left(\bar{z}_{mt}\ell_{mt}^{\eta^z}\right)\right)^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit}w_{nt}\ell_{nt}}{w_{it}\ell_{it}}, \tag{S.4.23}$$

where S_{nit} is the expenditure share of importer n on exporter i at time t; we have defined T_{int} as the corresponding income share of exporter i from importer n at time t; and the only difference from our baseline specification in the paper is the terms in $\ell_{it}^{\eta^z}$ in the expenditure share (S_{nit}) in equation (S.4.23). Note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the outmigration probabilities, the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt}\ell_{it}, \tag{S.4.24}$$

$$D_{igt} = \frac{\left(\exp\left(\beta\mathbb{E}_t v_{gt+1}^w\right)/\kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N}\left(\exp\left(\beta\mathbb{E}_t v_{mt+1}^w\right)/\kappa_{mit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it}D_{igt}}{\ell_{gt+1}}, \tag{S.4.25}$$

where D_{igt} is the outmigration probability from location i to location g between time t and $t+1$, and we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$. Note that the order of subscripts switches between the outmigration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker value function: Using the worker indirect utility function in equation (3) in the paper in the value function, the expected value from living in location n at time t can be written as:

$$v_{nt}^w = \ln\left(\frac{\bar{b}_{nt}\ell_{nt}^{\eta^b}w_{nt}}{p_{nt}}\right) + \rho\ln\sum_{g=1}^{N}\left(\exp\left(\beta\mathbb{E}_t v_{gt+1}^w\right)/\kappa_{gnt}\right)^{1/\rho}, \tag{S.4.26}$$

where the only difference from our baseline specification in the paper is the term in $l_{nt}^{\eta^b}$ in the value function in equation (S.4.26).

S.4.2.3 Existence and Uniqueness (Proof of Proposition 6 in the Paper)

We now use the system of equations for general equilibrium (S.4.20)-(S.4.26) to characterize the existence and uniqueness of a deterministic steady-state equilibrium with time-invariant fundamentals $\{\bar{z}_i,\ \bar{b}_i,\ \tau_{ni},\ \kappa_{ni}\}$ and endogenous variables $\{v_i^*, w_i^*, \ell_i^*, k_i^*\}$. Given these time-invariant fundamentals, we can drop the expectation over future fundamentals, such that $\mathbb{E}_t v_{gt+1}^w = v_{gt+1}^w$.

Capital-Labor Ratio In steady-state, $k_{it+1} = k_{it} = k_i^*$, and we can use the capital accumulation condition (S.4.20) to solve for the steady-state capital-labor ratio:

$$k_i^* = \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}\ell_i^* + \beta\left(1-\delta\right)k_i^*,$$

$$\frac{k_i^*}{\ell_i^*} = \frac{\beta}{1-\beta\left(1-\delta\right)} \frac{1-\mu}{\mu} \frac{w_i^*}{p_i^*}. \tag{S.4.27}$$

Price Index Using this result for the steady-state capital-labor ratio, we can re-write the price index in equation (S.4.21) as follows:

$$\left(p_n^*\right)^{-\theta} = \sum_{i=1}^{N} \left(w_i^* \left(\frac{1-\mu}{\mu}\right)^{1-\mu} \left(\ell_i^*/k_i^*\right)^{1-\mu} \tau_{ni} / \left(\bar{z}_i \left(\ell_i^*\right)^{\eta^z}\right) \right)^{-\theta},$$

$$\left(p_n^*\right)^{-\theta} = \sum_{i=1}^{N} \left(\frac{1-\beta\left(1-\delta\right)}{\beta}\right)^{-\theta(1-\mu)} \left(w_i^*\right)^{-\theta\mu} \left(p_i^*\right)^{-\theta(1-\mu)} \left(\ell_i^*\right)^{\eta^z\theta} \left(\tau_{ni}/\bar{z}_i\right)^{-\theta},$$

$$\left(p_n^*\right)^{-\theta} = \sum_{i=1}^{N} \psi \widetilde{\tau}_{ni} \left(w_i^*\right)^{-\theta\mu} \left(p_i^*\right)^{-\theta(1-\mu)} \left(\ell_i^*\right)^{\eta^z\theta}, \tag{S.4.28}$$

$$\psi \equiv \left(\frac{1-\beta\left(1-\delta\right)}{\beta}\right)^{-\theta(1-\mu)}, \qquad \widetilde{\tau}_{ni} \equiv \left(\tau_{ni}/z_i\right)^{-\theta}.$$

Goods Market Clearing Condition Using this result for the steady-state capital-labor ratio, we can also re-write the goods market clearing condition (S.4.22) as follows:

$$w_i^*\ell_i^* = \sum_{n=1}^{N} \frac{\left(w_i^* \left(\ell_i^*/k_i^*\right)^{1-\mu} \tau_{ni}/\left(\bar{z}_i\left(\ell_i^*\right)^{\eta^z}\right)\right)^{-\theta}}{\sum_{m=1}^{N}\left(w_m^* \left(\ell_m^*/k_m^*\right)^{1-\mu} \tau_{nm}/\left(\bar{z}_m\left(\ell_m^*\right)^{\eta^z}\right)\right)^{-\theta}} w_n^*\ell_n^*,$$

$$w_i^*\ell_i^* = \sum_{n=1}^{N} \frac{\left(w_i^* \left(\frac{1-\mu}{\mu}\right)^{1-\mu} \left(\ell_i^*/k_i^*\right)^{1-\mu} \tau_{ni}/\left(\bar{z}_i\left(\ell_i^*\right)^{\eta^z}\right)\right)^{-\theta}}{\left(p_n^*\right)^{-\theta}} w_n^*\ell_n^*,$$

$$w_i^*\ell_i^* = \sum_{n=1}^{N} \frac{\left(w_i^*\right)^{-\theta\mu} \left(\frac{1-\beta(1-\delta)}{\beta}\right)^{-\theta(1-\mu)} \left(p_i^*\right)^{-\theta(1-\mu)} \left(\ell_i^*\right)^{\eta^z\theta} \left(\tau_{ni}/\bar{z}_i\right)^{-\theta}}{\left(p_n^*\right)^{-\theta}} w_n^*\ell_n^*,$$

$$\left(\ell_i^*\right)^{1-\eta^z\theta} \left(w_i^*\right)^{1+\theta\mu} \left(p_i^*\right)^{\theta(1-\mu)} = \sum_{n=1}^{N} \left(\frac{1-\beta\left(1-\delta\right)}{\beta}\right)^{-\theta(1-\mu)} \left(p_n^*\right)^{\theta} \left(\tau_{ni}/\bar{z}_i\right)^{-\theta} w_n^*\ell_n^*,$$

$$\left(\ell_i^*\right)^{1-\eta^z\theta} \left(w_i^*\right)^{1+\theta\mu} \left(p_i^*\right)^{\theta(1-\mu)} = \sum_{n=1}^{N} \psi \widetilde{\tau}_{ni} \left(p_n^*\right)^{\theta} w_n^*\ell_n^*. \tag{S.4.29}$$

Value Function We now show how the value function (S.4.26) can be re-written using our change of variables:

$$v_n^{w*} = \ln\left(\frac{\bar{b}_n \left(\ell_n^*\right)^{\eta^b} w_n^*}{p_n^*}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta v_g^{w*}\right)/\kappa_{gn}\right)^{1/\rho},$$

$$\exp\left(v_n^{w*}\right) = \left(\frac{\bar{b}_n \left(\ell_n^*\right)^{\eta^b} w_n^*}{p_n^*}\right) \left[\sum_{g=1}^{N} \left(\exp\left(\beta v_g^{w*}\right)/\kappa_{gn}\right)^{1/\rho}\right]^{\rho},$$

$$\exp\left(\frac{\beta}{\rho}v_n^{w*}\right) = \left(\ell_n^*\right)^{\beta\eta^b/\rho} \left(\frac{w_n^*}{p_n^*}\right)^{\beta/\rho} \left[\sum_{g=1}^{N} \left(\kappa_{gn}/\bar{b}_n^{\beta}\right)^{-1/\rho} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right)\right]^{\beta},$$

$$\exp\left(\frac{\beta}{\rho}v_n^{w*}\right) = \left(\ell_n^*\right)^{\beta\eta^b/\rho} \left(\frac{w_n^*}{p_n^*}\right)^{\beta/\rho} \left[\sum_{g=1}^{N} \widetilde{\kappa}_{gn} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right)\right]^{\beta}, \qquad \widetilde{\kappa}_{gn} \equiv \left(\kappa_{gn}/\bar{b}_n^{\beta}\right)^{-1/\rho},$$

$$\exp\left(\frac{\beta}{\rho}v_n^{w*}\right) = \left(\ell_n^*\right)^{\beta\eta^b/\rho} \left(\frac{w_n^*}{p_n^*}\right)^{\beta/\rho} \phi_n^{\beta}, \qquad \phi_n \equiv \sum_{g=1}^{N} \widetilde{\kappa}_{gn} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right). \tag{S.4.30}$$

Using this solution in the definition of ϕ_n immediately above, we have:

$$\phi_n = \sum_{g=1}^{N} \widetilde{\kappa}_{gn} \left(\ell_g^*\right)^{\beta\eta^b/\rho} \left(p_g^*\right)^{-\beta/\rho} \left(w_g^*\right)^{\beta/\rho} \phi_g^{\beta}. \tag{S.4.31}$$

Population Flow Condition We now show how the population flow condition (S.4.24) can be re-written using our change of variables:

$$\ell_g^* = \sum_{i=1}^{N} \frac{\left(\exp\left(\beta v_g^{w*}\right)/\kappa_{gi}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta v_m^{w*}\right)/\kappa_{mi}\right)^{1/\rho}} \ell_i^*,$$

$$\ell_g^* = \sum_{i=1}^{N} \frac{\kappa_{gi}^{-1/\rho} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right)}{\sum_{m=1}^{N} \kappa_{mi}^{-1/\rho} \exp\left(\frac{\beta}{\rho}v_m^{w*}\right)} \ell_i^*,$$

$$\ell_g^* = \sum_{i=1}^{N} \kappa_{gi}^{-1/\rho} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right) \left[\sum_{m=1}^{N} \kappa_{mi}^{-1/\rho} \exp\left(\frac{\beta}{\rho}v_m^{w*}\right)\right]^{-1} \ell_i^*,$$

$$\ell_g^* = \sum_{i=1}^{N} \widetilde{\kappa}_{gi} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right) \left[\sum_{m=1}^{N} \widetilde{\kappa}_{mi} \exp\left(\frac{\beta}{\rho}v_m^{w*}\right)\right]^{-1} \ell_i^*, \qquad \widetilde{\kappa}_{gi} \equiv \left(\kappa_{gi}/\bar{b}_i^{\beta}\right)^{-1/\rho},$$

$$\ell_g^* = \sum_{i=1}^{N} \widetilde{\kappa}_{gi} \exp\left(\frac{\beta}{\rho}v_g^{w*}\right) \phi_i^{-1} \ell_i^*, \qquad \phi_i \equiv \sum_{m=1}^{N} \widetilde{\kappa}_{mi} \exp\left(\frac{\beta}{\rho}v_m^{w*}\right).$$

Now using the value function result (S.4.30) above, we have:

$$\ell_g^* = \sum_{i=1}^{N} \widetilde{\kappa}_{gi} \left(\ell_g^*\right)^{\beta\eta^b/\rho} \left(\frac{w_g^*}{p_g^*}\right)^{\beta/\rho} \phi_g^{\beta} \phi_i^{-1} \ell_i^*,$$

$$\left(p_g^*\right)^{\beta/\rho} \left(w_g^*\right)^{-\beta/\rho} \left(\ell_g^*\right)^{1-\beta\eta^b/\rho} \phi_g^{-\beta} = \sum_{i=1}^{N} \widetilde{\kappa}_{gi} \ell_i^* \phi_i^{-1}. \tag{S.4.32}$$

S.4.2.4 System of Equations

Collecting together these results, the steady-state equilibrium of the model $\{p_i^*, w_i^*, \ell_i^*, \phi_i^*\}$ can be expressed as the solution to the following system of equations:

$$\left(p_i^*\right)^{-\theta} = \sum_{n=1}^{N} \psi \widetilde{\tau}_{in} \left(p_n^*\right)^{-\theta(1-\mu)} \left(w_n^*\right)^{-\theta\mu} \left(\ell_n^*\right)^{\eta^z \theta}, \tag{S.4.33}$$

$$\left(p_i^*\right)^{\theta(1-\mu)} \left(w_i^*\right)^{1+\theta\mu} \left(\ell_i^*\right)^{1-\eta^z \theta} = \sum_{n=1}^{N} \psi \widetilde{\tau}_{ni} \left(p_n^*\right)^{\theta} w_n^* \ell_n^*, \tag{S.4.34}$$

$$\left(p_i^*\right)^{\beta/\rho} \left(w_i^*\right)^{-\beta/\rho} \left(\ell_i^*\right)^{1-\beta\eta^b/\rho} \left(\phi_i^*\right)^{-\beta} = \sum_{n=1}^{N} \widetilde{\kappa}_{in} \ell_n^* \left(\phi_n^*\right)^{-1}, \tag{S.4.35}$$

$$\phi_i^* = \sum_{n=1}^{N} \widetilde{\kappa}_{ni} \left(p_n^*\right)^{-\beta/\rho} \left(w_n^*\right)^{\beta/\rho} \left(\ell_n^*\right)^{\beta\eta^b/\rho} \left(\phi_n^*\right)^{\beta}, \tag{S.4.36}$$

where we have the following definitions:

$$\psi \equiv \left(\frac{1-\beta(1-\delta)}{\beta}\right)^{-\theta(1-\mu)}, \qquad \widetilde{\tau}_{ni} \equiv \left(\tau_{ni}/z_i\right)^{-\theta},$$

$$\phi_i^* \equiv \sum_{n=1}^{N} \widetilde{\kappa}_{ni} \exp\left(\frac{\beta}{\rho} v_n^{w*}\right), \qquad \widetilde{\kappa}_{in} \equiv \left(\kappa_{in}/\overline{b}_n^{\beta}\right)^{-1/\rho}.$$

We now provide a sufficient condition for the existence of a unique steady-state equilibrium in terms of the properties of a coefficient matrix ($\boldsymbol{A}$) of model parameters $\{\psi, \theta, \beta, \rho, \mu, \delta, \eta^z, \eta^b\}$ following the approach of Allen, Arkolakis and Li (2020).

Proposition. ***(Proposition 6 in the paper)*** *A sufficient condition for the existence of a unique steady-state spatial distribution of economic activity $\{\ell_i^*, k_i^*, w_i^*, R_i^*, v_i^*\}$ (up to a choice of units) given time-invariant locational fundamentals $\{\overline{z}_i^*, \overline{b}_i^*, \tau_{ni}^*, \kappa_{ni}^*\}$ is that the spectral radius of a coefficient matrix ($\boldsymbol{A}^{\text{Agg}}$) of model parameters $\{\psi, \theta, \beta, \rho, \mu, \delta, \eta^z, \eta^b\}$ is less than or equal to one.*

Proof. The exponents on the variables on the left-hand side of the system of equations (S.4.33)-(S.4.36) can be represented as the following matrix:

$$\boldsymbol{\Lambda}^{\text{Agg}} = \begin{bmatrix} -\theta & 0 & 0 & 0 \\ \theta(1-\mu) & (1+\theta\mu) & (1-\eta^z\theta) & 0 \\ \beta/\rho & -\beta/\rho & \left(1-\beta\eta^b/\rho\right) & -\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The exponents on the variables on the right-hand side of the system of equations (S.4.33)-(S.4.36) can be represented as the following matrix:

$$\boldsymbol{\Gamma}^{\text{Agg}} = \begin{bmatrix} -\theta(1-\mu) & -\theta\mu & \eta^z\theta & 0 \\ \theta & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -\beta/\rho & \beta/\rho & \beta\eta^b/\rho & \beta \end{bmatrix}.$$

Let $\boldsymbol{A}^{\text{Agg}} \equiv \left| \boldsymbol{\Gamma}^{\text{Agg}} \left(\boldsymbol{\Lambda}^{\text{Agg}} \right)^{-1} \right|$ and denote the spectral radius (eigenvalue with the largest absolute value) of this matrix by $\rho \left(\boldsymbol{A}^{\text{Agg}} \right)$. From Theorem 1 in Allen, Arkolakis and Li (2020), a sufficient condition for the existence of a unique equilibrium (up to a choice of units) is $\rho \left(\boldsymbol{A}^{\text{Agg}} \right) \leq 1$.
We next derive a sharper sufficient for the case of quasi-symmetric trade and migration costs: $\tau_{in} = \widetilde{\tau}_{in} \widetilde{\tau}_i^a \widetilde{\tau}_n^b$ and $\kappa_{in} = \widetilde{\kappa}_{in} \widetilde{\kappa}_i^c \widetilde{\kappa}_n^d$, where $\widetilde{\tau}_{in} = \widetilde{\tau}_{ni}$ and $\widetilde{\kappa}_{in} = \widetilde{\kappa}_{ni}$, as assumed in our empirical application. In this case of quasi-symmetric trade and migration costs, we can re-write the system of equations (S.4.33)-(S.4.36) as follows:

$$p_i^{-\theta} \left(\widetilde{\tau}_i^a \right)^{-1} = \sum_{n=1}^{N} \widetilde{\tau}_{in} \widetilde{\tau}_n^b p_n^{-\theta} q_n^{-\theta\mu} \ell_n^{\eta^z \theta}, \tag{S.4.37}$$

$$p_i^{\theta-1} q_i^{1+\theta\mu} \ell_i^{1-\eta^z \theta} \left(\widetilde{\tau}_i^b \right)^{-1} = \sum_{n=1}^{N} \widetilde{\tau}_{in} \widetilde{\tau}_n^a p_n^{\theta-1} q_n \ell_n, \tag{S.4.38}$$

$$q_i^{-\beta/\rho} \ell_i^{1-\beta\eta^b/\rho} \phi_i^{-\beta} \left(\widetilde{\kappa}_i^c \right)^{-1} = \sum_{n=1}^{N} \widetilde{\kappa}_{in} \widetilde{\kappa}_n^d \ell_n \phi_n^{-1}, \tag{S.4.39}$$

$$\phi_i \left(\widetilde{\kappa}_i^d \right)^{-1} = \sum_{n=1}^{N} \widetilde{\kappa}_{in} \widetilde{\kappa}_n^c q_n^{\beta/\rho} \ell_n^{\beta\eta^b/\rho} \phi_n^{\beta}. \tag{S.4.40}$$

From equation (S.4.40), we know:

$$1 = \sum_{n=1}^{N} \frac{\widetilde{\kappa}_{in} \widetilde{\kappa}_n^c q_n^{\beta/\rho} \ell_n^{\beta\eta^b/\rho} \phi_n^{\beta}}{\phi_i \left(\widetilde{\kappa}_i^d \right)^{-1}}.$$

Multiply the left-hand side of equation (S.4.39) by $\sum_{n=1}^{N} \frac{\widetilde{\kappa}_{in} \widetilde{\kappa}_n^c q_n^{\beta/\rho} \ell_n^{\beta\eta^b/\rho} \phi_n^{\beta}}{\phi_i \left(\widetilde{\kappa}_i^d \right)^{-1}}$, and move $\left(\widetilde{\kappa}_i^c \right)^{-1} \phi_i^{-\beta} q_i^{-\beta/\rho} \ell_i^{-\beta\eta^b/\rho}$ to the right-hand side:

$$\sum_{n=1}^{N} \frac{\widetilde{\kappa}_{in} \widetilde{\kappa}_n^c q_n^{\beta/\rho} \ell_n^{\beta\eta^b/\rho} \phi_n^{\beta}}{\phi_i \left(\widetilde{\kappa}_i^d \right)^{-1}} \ell_i = \sum_{n=1}^{N} \frac{\widetilde{\kappa}_{in} \phi_i^{\beta} \widetilde{\kappa}_i^c q_i^{\beta/\rho} \ell_i^{\beta\eta^b/\rho}}{\phi_n \left(\widetilde{\kappa}_n^d \right)^{-1}} \ell_n,$$

$$\iff \frac{\ell_i \widetilde{\kappa}_i^d / \phi_i}{\sum_{n=1}^{N} \widetilde{\kappa}_{in} \ell_n \widetilde{\kappa}_n^d / \phi_n} = \frac{\widetilde{\kappa}_i^c \phi_i^{\beta} q_i^{\beta/\rho} \ell_i^{\beta\eta^b/\rho}}{\sum_{n=1}^{N} \widetilde{\kappa}_{in} \widetilde{\kappa}_n^c q_n^{\beta/\rho} \ell_n^{\beta\eta^b/\rho} \phi_n^{\beta}}.$$

By the Perron-Frobenius theorem, $\ell_i \widetilde{\kappa}_i^d / \phi_i = x \widetilde{\kappa}_i^c \phi_i^{\beta} q_i^{\beta/\rho} \ell_i^{\beta\eta^b/\rho}$ for some constant x. Since the scale of ℓ_i is not pinned down by the system of equations—if $\{\ell_i\}$ is part of a solution to the system of equations, so is $\{2\ell_i\}$—we can without loss of generality set $x = 1$. Hence:

$$\ell_i^{1-\beta\eta^b/\rho} = \widetilde{\kappa}_i^c \left(\widetilde{\kappa}_i^d \right)^{-1} \phi_i^{1+\beta} q_i^{\beta/\rho},$$

$$\ell_i = \left(c_i \left(\widetilde{\kappa}_i^d \right)^{-1} \phi_i^{1+\beta} q_i^{\beta/\rho} \right)^{\frac{1}{1-\beta\eta^b/\rho}}. \tag{S.4.41}$$

Now we use the same strategy to reduce equations (S.4.37) and (S.4.38) down to one. Re-write equation (S.4.37) as:

$$1=\sum_{n=1}^{N}\frac{\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}p_{n}^{-\theta}q_{n}^{-\theta\mu}\ell_{n}^{\eta^{z}\theta}}{p_{i}^{-\theta}\left(\widetilde{\tau}_{i}^{a}\right)^{-1}}.$$

Multiplying the left-hand side of equation (S.4.38) by $\sum_{n=1}^{N}\frac{\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}p_{n}^{-\theta}q_{n}^{-\theta\mu}\ell_{n}^{\eta^{z}\theta}}{p_{i}^{-\theta}\left(\widetilde{\tau}_{i}^{a}\right)^{-1}}$:

$$\sum_{n=1}^{N}\frac{\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}p_{n}^{-\theta}q_{n}^{-\theta\mu}\ell_{n}^{\eta^{z}\theta}}{p_{i}^{-\theta}\left(\widetilde{\tau}_{i}^{a}\right)^{-1}}p_{i}^{\theta-1}q_{i}^{1+\theta\mu}\ell_{i}^{1-\eta^{z}\theta}\left(\widetilde{\tau}_{i}^{b}\right)^{-1}=\sum_{n=1}^{N}\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{a}p_{n}^{\theta-1}q_{n}\ell_{n},$$

$$\iff\frac{\widetilde{\tau}_{i}^{a}p_{i}^{\theta-1}q_{i}\ell_{i}}{\sum_{n=1}^{N}\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{a}p_{n}^{\theta-1}q_{n}\ell_{n}}=\frac{\widetilde{\tau}_{i}^{b}p_{i}^{-\theta}q_{i}^{-\theta\mu}\ell_{i}^{\eta^{z}\theta}}{\sum_{n=1}^{N}\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}p_{n}^{-\theta}q_{n}^{-\theta\mu}\ell_{n}^{\eta^{z}\theta}}.$$

Again by the Perron-Frobenius theorem, $\widetilde{\tau}_{i}^{a}p_{i}^{\theta-1}q_{i}\ell_{i}=y\widetilde{\tau}_{i}^{b}p_{i}^{-\theta}q_{i}^{-\theta\mu}\ell_{i}^{\eta^{z}\theta}$ for some constant y. Since p_i is a nominal variable, we can without loss of generality set $y=1$; hence

$$\widetilde{\tau}_{i}^{a}p_{i}^{\theta-1}q_{i}\ell_{i}=\widetilde{\tau}_{i}^{b}p_{i}^{-\theta}q_{i}^{-\theta\mu}\ell_{i}^{\eta^{z}\theta},$$

$$\begin{aligned}
p_{i}^{-\theta} &= e_{i}q_{i}^{-\theta\frac{1+\theta\mu}{1-2\theta}}\ell_{i}^{-\theta\frac{1-\eta^{z}\theta}{1-2\theta}},\\
&= e_{i}q_{i}^{-\theta\frac{1+\theta\mu}{1-2\theta}}\left(\phi_{i}^{1+\beta}q_{i}^{\beta/\rho}\right)^{\frac{-\theta}{1-2\theta}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}},\\
&= e_{i}q_{i}^{-\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right)}\phi_{i}^{\frac{-\theta}{1-2\theta}(1+\beta)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}},
\end{aligned}\tag{S.4.42}$$

where $e_{i}=\widetilde{\tau}_{i}^{b}\left(\widetilde{\tau}_{i}^{a}\right)^{-1}$. Now substitute (S.4.41) and (S.4.42) into (S.4.37) and (S.4.40):

$$\begin{aligned}
&e_{i}q_{i}^{-\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right)}\phi_{i}^{\frac{-\theta}{1-2\theta}(1+\beta)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}}\left(\widetilde{\tau}_{i}^{a}\right)^{-1},\\
=&\sum_{n=1}^{N}\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}e_{n}q_{n}^{-\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right)}\phi_{n}^{\frac{-\theta}{1-2\theta}(1+\beta)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}}q_{n}^{-\theta\mu}\left(\widetilde{\kappa}_{n}^{c}\left(\widetilde{\kappa}_{n}^{d}\right)^{-1}\phi_{n}^{1+\beta}q_{n}^{\beta/\rho}\right)^{\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho}},\\
=&\sum_{n=1}^{N}\widetilde{\tau}_{in}\widetilde{\tau}_{n}^{b}e_{n}\left(\widetilde{\kappa}_{n}^{c}\left(\widetilde{\kappa}_{n}^{d}\right)^{-1}\right)^{\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho}}q_{n}^{-\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right)-\theta\mu-\frac{\beta}{\rho}\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho}}\phi_{n}^{\frac{-\theta}{1-2\theta}(1+\beta)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}+(1+\beta)\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho}},
\end{aligned}$$

$$\begin{aligned}
\phi_{i}\left(\widetilde{\kappa}_{i}^{d}\right)^{-1} &= \sum_{n=1}^{N}\widetilde{\kappa}_{in}\widetilde{\kappa}_{n}^{c}q_{n}^{\beta/\rho}\left(\widetilde{\kappa}_{n}^{c}\left(\widetilde{\kappa}_{n}^{d}\right)^{-1}\phi_{n}^{1+\beta}q_{n}^{\beta/\rho}\right)^{\frac{\beta\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}}\phi_{n}^{\beta},\\
&= \sum_{n=1}^{N}\widetilde{\kappa}_{in}\left(\widetilde{\kappa}_{n}^{c}\left(\widetilde{\kappa}_{n}^{d}\right)^{-1}\right)^{\frac{\beta\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}}\widetilde{\kappa}_{n}^{c}q_{n}^{\frac{\beta}{\rho}\left(1+\frac{\beta\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}\right)}\phi_{n}^{\beta\left(1+(1+\beta)\frac{\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}\right)}.
\end{aligned}$$

We now have two sets of equations in two sets of endogenous variables q_i,ϕ_i. We now again apply Theorem 1 in Allen, Arkolakis and Li (2020) for this system of two equations. The matrix of coefficients on the left and right-hand side of this system of equations are respectively:

$$\mathbf{\Lambda}^{\text{Agg}}=\begin{bmatrix}-\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right) & \frac{-\theta}{1-2\theta}\left(1+\beta\right)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\\ 0 & 1\end{bmatrix}$$

$$\mathbf{\Gamma}^{\mathrm{Agg}} = \begin{bmatrix} -\frac{\theta}{1-2\theta}\left(1+\theta\mu+\frac{\beta}{\rho}\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}\right)-\theta\mu-\frac{\beta}{\rho}\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho} & \frac{-\theta}{1-2\theta}\left(1+\beta\right)\frac{1-\eta^{z}/\theta}{1-\beta\eta^{b}/\rho}+\left(1+\beta\right)\frac{\eta^{z}\theta}{1-\beta\eta^{b}/\rho} \\ \frac{\beta}{\rho}\left(1+\frac{\beta\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}\right) & \beta\left(1+\left(1+\beta\right)\frac{\eta^{b}/\rho}{1-\beta\eta^{b}/\rho}\right) \end{bmatrix}.$$

A sufficient condition for a unique equilibrium is again that the spectral radius of $\boldsymbol{A}^{\mathrm{Agg}} \equiv \left|\mathbf{\Gamma}^{\mathrm{Agg}}\left(\mathbf{\Lambda}^{\mathrm{Agg}}\right)^{-1}\right|$ is less than or equal to one: $\rho\left(\boldsymbol{A}^{\mathrm{Agg}}\right) \leq 1$. As $\eta^{z}, \eta^{b} \to 0$, the coefficient matrices $\mathbf{\Lambda}^{\mathrm{Agg}}$ and $\mathbf{\Gamma}^{\mathrm{Agg}}$ reduce to those in our baseline specification without agglomeration forces, as characterized in Proposition 1 in the paper. $\square$

As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a choice of units or numeraire in order to solve for wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^{N} w_{it}\ell_{it} = \sum_{i=1}^{N} q_{it} = \overline{q}_{t} = 1$). Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_{i} = \overline{\ell} = 1$, which implies $\sum_{i=1}^{N} \ell_{i}^{*} \,\mathrm{d}\ln \ell_{i}^{*} = \sum_{i=1}^{N} \ell_{i}^{*} \frac{\mathrm{d}\ell_{i}^{*}}{\ell_{i}^{*}} = \sum_{i=1}^{N} \mathrm{d}\ell_{i}^{*} = 0$.

S.4.2.5 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path. In the interests of brevity, we focus on differences from our baseline specification without agglomeration economies.

Goods Market Clearing Totally differentiating the goods market clearing condition, we have:

$$\begin{bmatrix} \mathrm{d}\ln w_{it} \\ +\mathrm{d}\ln \ell_{it} \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln w_{nt}+\mathrm{d}\ln \ell_{nt}\right) \\ +\theta\sum_{n=1}^{N}\sum_{m=1}^{N} T_{int}S_{nmt}\left(\mathrm{d}\ln \tau_{nmt}+\mathrm{d}\ln w_{mt}-\left(1-\mu\right)\mathrm{d}\ln \chi_{mt}-\eta^{z}\,\mathrm{d}\ln \ell_{mt}-\mathrm{d}\ln z_{mt}\right) \\ -\theta\sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln \tau_{nit}+\mathrm{d}\ln w_{it}-\left(1-\mu\right)\mathrm{d}\ln \chi_{it}-\eta^{z}\,\mathrm{d}\ln \ell_{it}-\mathrm{d}\ln z_{it}\right) \end{bmatrix}. \tag{S.4.43}$$

Value Function. Totally differentiating the value function, we have:

$$\mathrm{d}v_{it} = \begin{bmatrix} \mathrm{d}\ln w_{it}-\sum_{m=1}^{N} S_{imt}\left(\mathrm{d}\ln \tau_{nmt}+\mathrm{d}\ln w_{mt}-\left(1-\mu\right)\mathrm{d}\ln \chi_{mt}-\eta^{z}\,\mathrm{d}\ln \ell_{mt}-\mathrm{d}\ln z_{mt}\right) \\ +\mathrm{d}\ln b_{it}+\eta^{b}\,\mathrm{d}\ln \ell_{mt}+\sum_{m=1}^{N} D_{imt}\left(\beta\mathbb{E}_{t}\,\mathrm{d}v_{mt+1}-\mathrm{d}\ln \kappa_{mit}\right) \end{bmatrix}. \tag{S.4.44}$$

S.4.2.6 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1} = k_{it} = k_{i}^{*}$, $\ell_{it+1} = \ell_{it} = \ell_{i}^{*}$, $w_{it+1}^{*} = w_{it}^{*} = w_{i}^{*}$ and $v_{it+1}^{*} = v_{it}^{*} = v_{i}^{*}$, where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \overline{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$).

Capital Accumulation. From the capital accumulation equation (S.4.20), the steady-state stock of capital solves:

$$\left(1-\beta\left(1-\delta\right)\right)\chi_i^* = \left(1-\beta\left(1-\delta\right)\right)\frac{k_i^*}{\ell_i^*} = \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln\left(\frac{w_i^*}{p_i^*}\right).$$

Totally differentiating real income, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln w_i^* - \sum_{m=1}^{N} S_{im}^*\left[\mathrm{d}\ln w_m^* - \left(1-\mu\right)\mathrm{d}\ln\chi_m^* - \eta^z\,\mathrm{d}\ln\ell_m^* - \mathrm{d}\ln z_m\right],$$

where we have used and $\mathrm{d}\ln\tau_{nm} = 0$. This relationship has the matrix representation:

$$\begin{aligned}
\mathrm{d}\ln\boldsymbol{\chi}^* &= \mathrm{d}\ln\boldsymbol{w}^* - \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{w}^* + \left(1-\mu\right)\boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{\chi}^* + \eta^z\boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{\ell}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z},\\
\left(\boldsymbol{I}-\left(1-\mu\right)\boldsymbol{S}\right)\mathrm{d}\ln\boldsymbol{\chi}^* &= \left(\boldsymbol{I}-\boldsymbol{S}\right)\mathrm{d}\ln\boldsymbol{w}^* + \eta^z\boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{\ell}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z}. \qquad \text{(S.4.45)}
\end{aligned}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.43) has the following matrix representation:

$$\mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t} = \begin{bmatrix} \boldsymbol{T}\left(\mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t}\right) \\ +\theta\left(\boldsymbol{TS}-\boldsymbol{I}\right)\left(\mathrm{d}\ln\boldsymbol{w_t} - \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi_t} - \eta^z\,\mathrm{d}\ln\boldsymbol{\ell_t} - \mathrm{d}\ln\boldsymbol{z}\right) \end{bmatrix},$$

where we have used $\mathrm{d}\ln\boldsymbol{\tau} = 0$. We can re-write this relationship as:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln\boldsymbol{w_t} = \begin{bmatrix} -\left(\boldsymbol{I}-\boldsymbol{T}-\theta\eta^z\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right)\mathrm{d}\ln\boldsymbol{\ell_t} \\ +\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi_t}\right) \end{bmatrix}.$$

In steady-state we have:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln\boldsymbol{w}^* = \begin{bmatrix} -\left(\boldsymbol{I}-\boldsymbol{T}-\theta\eta^z\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right)\mathrm{d}\ln\boldsymbol{\ell}^* \\ +\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi}^*\right) \end{bmatrix}. \qquad \text{(S.4.46)}$$

Population Flow. The total derivative of the population flow condition has the same matrix representation as in our baseline model:

$$\mathrm{d}\ln\boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln\boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}}.$$

In steady-state, we have:

$$\mathrm{d}\ln\boldsymbol{\ell}^* = \boldsymbol{E}\,\mathrm{d}\ln\boldsymbol{\ell}^* + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathrm{d}\boldsymbol{v}^*. \qquad \text{(S.4.47)}$$

Value function. The total derivative of the value function (S.4.44) has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left[\begin{array}{c} (\boldsymbol{I}-\boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w_t} + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi_t}\right) \\ +\,\mathrm{d}\ln \boldsymbol{b} + \left(\eta^z \boldsymbol{S} + \boldsymbol{\eta^b}\right)\ \mathrm{d}\ln \boldsymbol{\ell} + \beta \boldsymbol{D}\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}} \end{array} \right],$$

where we have used $\mathrm{d}\ln \boldsymbol{\tau} = \mathrm{d}\ln \boldsymbol{\kappa} = 0$ and $\boldsymbol{\eta^b}$ is a $N \times N$ diagonal matrix with the parameter η^b along its diagonal. In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^* = \left[\begin{array}{c} (\boldsymbol{I}-\boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) \\ +\,\mathrm{d}\ln \boldsymbol{b} + \left(\eta^z \boldsymbol{S} + \boldsymbol{\eta^b}\right)\ \mathrm{d}\ln \boldsymbol{\ell}^* + \beta \boldsymbol{D}\,\mathrm{d}\boldsymbol{v}^* \end{array} \right]. \tag{S.4.48}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left(\boldsymbol{I} - (1-\mu)\,\boldsymbol{S}\right)^{-1}\left((\boldsymbol{I}-\boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w}^* + \eta^z \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{\ell}^* + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z}\right). \tag{S.4.49}$$

$$\mathrm{d}\ln \boldsymbol{w}^* = \left(\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right)^{-1} \left[\begin{array}{c} -\left(\boldsymbol{I} - \boldsymbol{T} - \theta\eta^z\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right)\ \mathrm{d}\ln \boldsymbol{\ell}^* \\ +\left(\boldsymbol{I} - \boldsymbol{TS}\right)\theta\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) \end{array} \right]. \tag{S.4.50}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\ \mathrm{d}\boldsymbol{v}^*. \tag{S.4.51}$$

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta\boldsymbol{D}\right)^{-1}\left\{ \begin{array}{c} \mathrm{d}\ln \boldsymbol{w}^* - \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{w}^* - \mathrm{d}\ln \boldsymbol{z} - (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) \\ +\,\mathrm{d}\ln \boldsymbol{b} + \left(\eta^z \boldsymbol{S} + \boldsymbol{\eta^b}\right)\ \mathrm{d}\ln \boldsymbol{\ell}^* \end{array} \right\}. \tag{S.4.52}$$

As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^{N} w_i^* \ell_i^* = \sum_{i=1}^{N} q_i^* = \overline{q} = 1$), which implies that the log changes in incomes satisfy $\boldsymbol{q}^*\,\mathrm{d}\ln \boldsymbol{q}^* = \sum_{i=1}^{N} q_i^*\,\mathrm{d}\ln q_i^* = \sum_{i=1}^{N} q_i^* \frac{\mathrm{d}q_i^*}{q_i^*} = \sum_{i=1}^{N} \mathrm{d}q_i^* = 0$, where $\boldsymbol{q}^*$ is a row vector of the steady-state income of each location. Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_i = \overline{\ell} = 1$, which implies $\boldsymbol{\ell}^*\,\mathrm{d}\ln \boldsymbol{\ell}^* = \sum_{i=1}^{N} \ell_i^*\,\mathrm{d}\ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{\mathrm{d}\ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} \mathrm{d}\ell_i^* = 0$, where $\boldsymbol{\ell}^*$ is a row vector of the steady-state population of each location.

S.4.2.7 Sufficient Statistics for Transition Dynamics Starting from Steady-State

We suppose that the economy starts from an initial steady-state distribution of economic activity $\{k_i^*,\ \ell_i^*,\ w_i^*,\ v_i^*\}$. We consider small shocks to productivity $(\mathrm{d}\ln \boldsymbol{z})$ and amenities $(\mathrm{d}\ln \boldsymbol{b})$ in each location, holding constant the economy's aggregate labor endowment $(\mathrm{d}\ln \overline{\ell})$, trade costs $(\mathrm{d}\ln \boldsymbol{\tau} = 0)$ and commuting costs $(\mathrm{d}\ln \boldsymbol{\kappa} = 0)$. We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\widetilde{\chi}_{it} = \ln \chi_{it} - \ln \chi_i^*$, for all variables except for the worker value function v_{it}; with a slight abuse of notation we use $\widetilde{v}_{it} \equiv v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (S.4.20), and following the same line of argument as in the baseline model without agglomeration economies, we can derive the following expression for the deviation of the capital-ratio from its steady-state value:

$$\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right)+\ln\left(\frac{\ell_{it+1}/\ell_i^*}{\ell_{it}/\ell_i^*}\right)=\beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right)+\left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),$$

which can be re-written as follows:

$$\widetilde{\chi}_{it+1}=\beta\left(1-\delta\right)\widetilde{\chi}_{it}+\left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{w}_{it}-\widetilde{p}_{it}\right)-\widetilde{\ell}_{it+1}+\widetilde{\ell}_{it},$$

We can re-write the above relationship for the log deviation of the capital-labor ratio from the initial steady-state as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}}=\beta\left(1-\delta\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}+\left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{p}}_{\boldsymbol{t}}\right)-\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}. \tag{S.4.53}$$

Taking the total derivative of real income relative to the initial steady-state, we have:

$$\widetilde{w}_{it}-\widetilde{p}_{it}=\widetilde{w}_{it}-\sum_{m=1}^{N}S_{imt}\left[\widetilde{w}_{mt}-\left(1-\mu\right)\widetilde{\chi}_{mt}-\widetilde{z}_m-\eta^z\widetilde{\ell}_m\right],$$

where we have used $\mathrm{d}\ln\tau_{nm}=0$. We can re-write this relationship in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{p}}_{\boldsymbol{t}}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\left(1-\mu\right)\boldsymbol{S}\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}+\boldsymbol{S}\widetilde{\boldsymbol{z}}+\eta^z\boldsymbol{S}\widetilde{\boldsymbol{\ell}}.$$

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}}=\left[\begin{array}{c}\left[\beta\left(1-\delta\right)\boldsymbol{I}+\left(1-\beta\left(1-\delta\right)\right)\left(1-\mu\right)\boldsymbol{S}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\\+\left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}}\\-\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}+\left[1+\eta^z\left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\right]\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\end{array}\right]. \tag{S.4.54}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.43) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}=\boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right)+\theta\left(\boldsymbol{TS}-\boldsymbol{I}\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{z}}-\eta^z\widetilde{\boldsymbol{\ell}}\right),$$

where we have used $\mathrm{d}\ln\boldsymbol{\tau}=0$. We can re-write this relationship as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}=\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]^{-1}\left[\begin{array}{c}-\left(\boldsymbol{I}-\boldsymbol{T}-\theta\eta^z\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\\+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\widetilde{\boldsymbol{z}}+\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\right)\end{array}\right]. \tag{S.4.55}$$

Population Flow. The total derivative of the population flow condition relative to the initial steady-state has the same matrix representation as in the baseline model without agglomeration economies:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}=\boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}+\frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.56}$$

Value Function. The total derivative of the value function (S.4.44) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{S}) \, \widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S} \widetilde{\boldsymbol{z}} + (1 - \mu) \, \boldsymbol{S} \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ + \left(\eta^z \boldsymbol{S} + \boldsymbol{\eta}^{\boldsymbol{b}} \right) \, \mathrm{d} \ln \boldsymbol{\ell}^* + \widetilde{\boldsymbol{b}} + \beta \boldsymbol{D} \mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array} \right], \tag{S.4.57}$$

where we have used $\mathrm{d} \ln \boldsymbol{\tau} = \mathrm{d} \ln \boldsymbol{\kappa} = 0$.

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (S.4.54), the goods market clearing condition (S.4.55), the population flow condition (S.4.56), and the value function (S.4.57), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta \left(1 - \delta \right) \boldsymbol{I} + \left(1 - \beta \left(1 - \delta \right) \right) \left(1 - \mu \right) \boldsymbol{S} \right] \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ + \left(1 - \beta \left(1 - \delta \right) \right) \left(\boldsymbol{I} - \boldsymbol{S} \right) \widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left(1 - \beta \left(1 - \delta \right) \right) \boldsymbol{S} \widetilde{\boldsymbol{z}} \\ - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \left[1 + \eta^z \left(1 - \beta \left(1 - \delta \right) \right) \boldsymbol{S} \right] \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array} \right]. \tag{S.4.58}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta \left(\boldsymbol{I} - \boldsymbol{T} \boldsymbol{S} \right) \right]^{-1} \left[\begin{array}{c} - \left(\boldsymbol{I} - \boldsymbol{T} - \theta \eta^z \left(\boldsymbol{I} - \boldsymbol{T} \boldsymbol{S} \right) \right) \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \\ + \theta \left(\boldsymbol{I} - \boldsymbol{T} \boldsymbol{S} \right) \left(\widetilde{\boldsymbol{z}} + \left(1 - \mu \right) \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \right) \end{array} \right]. \tag{S.4.59}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E} \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} \right) \mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.60}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{S}) \, \widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S} \widetilde{\boldsymbol{z}} + (1 - \mu) \, \boldsymbol{S} \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ + \left(\eta^z \boldsymbol{S} + \boldsymbol{\eta}^{\boldsymbol{b}} \right) \, \mathrm{d} \ln \widetilde{\boldsymbol{\ell}}^* + \widetilde{\boldsymbol{b}} + \beta \boldsymbol{D} \mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array} \right]. \tag{S.4.61}$$

S.4.3 Multiple Sectors (Region-Specific Capital)

We consider an economy that consists of many locations indexed by $i \in \{1, \ldots, N\}$ and many sectors indexed by $j \in \{1, \ldots, J\}$. Time is discrete and is indexed by t. The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate δ.

S.4.3.1 Worker Migration Decisions

At the beginning of period t, the economy inherits a mass of workers in each location i and sector j (ℓ_{it}^j), with the total labor endowment of the economy given by $\bar{\ell} = \sum_{i=1}^N \sum_{j=1}^J \ell_{it}^j$. Workers first produce and consume in their location and sector in period t, before observing mobility shocks $\{\epsilon_{gt}^h\}$ for all possible locations $g \in \{1, \ldots, N\}$ and sectors $h \in \{1, \ldots, J\}$ and deciding where to move for period $t+1$. Workers face bilateral migration costs $\{\kappa_{git}^{hj}\}$, which vary by both location and sector. The value function for a worker in location i and sector j at time t ($\mathbb{V}_{it}^{j,w}$) is equal to the current flow of utility in that location and sector plus the expected continuation value next period from the optimal choice of location and sector:

$$\mathbb{V}_{it}^{j,w} = \ln u_{it}^{j,w} + \max_{\{g\}_1^N \{h\}_1^J} \left\{ \beta \mathbb{E}_t \left[\mathbb{V}_{gt+1}^{h,w} \right] - \kappa_{git}^{hj} + \rho \epsilon_{gt}^{h} \right\}, \tag{S.4.62}$$

where we use the superscript w to denote workers; we assume logarithmic flow utility ($\ln u_{it}^{j,w}$); β is the discount rate; $\mathbb{E}[\cdot]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; ρ captures the dispersion of idiosyncratic mobility shocks; and we assume $\kappa_{iit}^{jj} = 1$ and $\kappa_{git}^{hj} > 1$ for $g \neq i$ and $h \neq j$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

$$F(\epsilon) = e^{-e^{(-\epsilon - \bar{\gamma})}}, \tag{S.4.63}$$

where $\overline{\gamma}$ is the Euler-Mascheroni constant.

Under this assumption, the expected value for a worker of living in location i at time t ($v_{it}^{j,w}$) can be re-written in the following form:

$$v_{it}^{j,w} = \ln u_{it}^{j,w} + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{J} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho}. \tag{S.4.64}$$

The corresponding probability of migrating from location-sector ij to location-sector gh satisfies a gravity equation:

$$D_{igt}^{jh} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^{o,w} \right) / \kappa_{mit}^{oj} \right)^{1/\rho}}. \tag{S.4.65}$$

S.4.3.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in their location and sector that period. Worker flow indirect utility in location n and sector j depends on local amenities (b_{nt}^{j}), the wage (w_{nt}^{j}), and the consumption goods price index (p_{it}):

$$\ln u_{nt}^{j,w} = \ln b_{nt}^{j} + \ln w_{nt}^{j} - \ln p_{nt}, \tag{S.4.66}$$

where amenities (b_{nt}^{j}) capture characteristics of a location and sector that make it a more attractive place to live and work regardless of the wage and cost of consumption goods (e.g., climate and rewarding work). In this section of the Online Supplement, we assume that amenities are exogenous.

The consumption goods price index (p_{nt}) in location n depends on the consumption goods price index for each sector h in that location (p_{nt}^{h}):

$$p_{nt} = \prod_{h=1}^{J} \left(p_{nt}^{h} \right)^{\psi^h}, \qquad 0 < \psi^h < 1, \qquad \sum_{h=1}^{J} \psi^h = 1, \tag{S.4.67}$$

where the consumption goods price index for each sector h in location n depends on the price of the variety sourced from each location i within that sector h (p^h_{nit}):

$$p^h_{nt} = \left[\sum_{i=1}^{N} \left(p^h_{nit}\right)^{-\theta}\right]^{-1/\theta}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{S.4.68}$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties; $\theta = \sigma - 1$ is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector ($p^h_{nt}c^h_{nt}$) is a constant share of overall goods consumption expenditure ($p_{nt}c_{nt}$) in each location:

$$p^h_{nt}c^h_{nt} = \psi^h p_{nt}c_{nt} = \psi^h \sum_{j=1}^{N} w^j_{nt}\ell^j_{nt}. \tag{S.4.69}$$

Using constant elasticity of substitution (CES) demand for individual varieties of goods, the share of location n's expenditure within sector h on the goods produced by location i is:

$$S^h_{nit} \equiv \frac{\left(p^h_{nit}\right)^{-\theta}}{\sum_{m=1}^{N} \left(p^h_{nmt}\right)^{-\theta}}. \tag{S.4.70}$$

S.4.3.3 Production

Producers in each location i and sector j use labor (ℓ^j_{it}) and capital (k^j_{it}) to produce output (y^j_{it}) of the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$y^j_{it} = z^j_{it} \left(\frac{\ell^j_{it}}{\mu^j}\right)^{\mu^j} \left(\frac{k^j_{it}}{1-\mu^j}\right)^{1-\mu^j}, \qquad 0 < \mu^j < 1, \tag{S.4.71}$$

where z^j_{it} denotes productivity in location i in sector j at time t. As for amenities above, we assume in this section of the Online Supplement that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau^j_{nit} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n, where $\tau^j_{nit} > 1$ for $n \neq i$ and $\tau^j_{iit} = 1$. From profit maximization, the cost to a consumer in location n of sourcing the good produced by location i in sector j depends on iceberg trade costs and constant marginal costs:

$$p^j_{nit} = \tau^j_{nit}p^j_{iit} = \frac{\tau^j_{nit}\left(w^j_{it}\right)^{\mu^j}\left(r_{it}\right)^{1-\mu^j}}{z^j_{it}}, \tag{S.4.72}$$

where p^j_{iit} is the "free on board" price of the good supplied by location i before transport costs.

From profit maximization and zero profits, total payments to each factor of production are a constant share of total revenue:

$$w^j_{it}\ell^j_{it} = \mu^j p^j_{iit}y^j_{it}, \tag{S.4.73}$$

$$r_{it}k^j_{it} = \left(1-\mu^j\right)p^j_{iit}y^j_{it}, \tag{S.4.74}$$

where capital mobility across sectors within regions ensures the same return to capital across sectors within regions ($r^j_{it} = r_{it}$).

S.4.3.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^k = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^k, \tag{S.4.75}$$

where we use the superscript k to denote landlords; c_{it}^k is the consumption goods index for landlords; and β is the discount rate. Since landlords are geographically immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords (c_{it}^k) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index (p_{nt}) takes the same form as in equation (S.4.67), and the consumption goods price index for each sector (p_{nt}^j) takes the same form as in equation (S.4.68). Under these assumptions, the landlords' utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords' optimal consumption-saving decision, we assume that the investment technology for capital in each location uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location can produce one unit of capital in that location using one unit of the consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate δ. The intertemporal budget constraints for landlords in each location requires that total income from the existing stock of capital ($\sum_{j=1}^{J} r_{it} k_{it}^j$) equals the total value of goods consumption ($p_{it} c_{it}^k$) and net investment ($p_{it}\left(k_{it+1} - (1-\delta) k_{it}\right)$):

$$r_{it} k_{it} = \sum_{j=1}^{J} r_{it} k_{it}^j = p_{it} c_{it}^k + r_{it} k_{it} + p_{it}\left(k_{it+1} - (1-\delta) k_{it}\right). \tag{S.4.76}$$

Combining landlords' intertemporal utility (S.4.75) and budget constraint (S.4.76), the landlords' intertemporal optimization problem is:

$$\max_{\left\{c_t, k_{t+1}^k\right\}} \sum_{t=0}^{\infty} \beta^t \ln c_{it}^k, \tag{S.4.77}$$

$$\text{subject to} \qquad p_{it} c_{it}^k + p_{it}\left(k_{it+1} - (1-\delta) k_{it}\right) = r_{it} k_{it}.$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^k - \xi_t \left[p_{it} c_{it}^k + p_{it}\left(k_{it+1} - (1-\delta) k_{it}\right) - r_{it} k_{it}\right]. \tag{S.4.78}$$

The first-order conditions are:

$$\{c_{it}\} \qquad \frac{\beta^t}{c_{it}} - p_{it}\xi_t = 0,$$

$$\{k_{it+1}\} \qquad \left(r_{it+1} + p_{it+1}\left(1-\delta\right)\right)\xi_{t+1} - p_{it}\xi_t = 0,$$

Together these first-order conditions imply:

$$\frac{c_{it+1}}{c_{it}} = \beta\frac{p_{it}\mu_t}{p_{it+1}\mu_{t+1}} = \beta\left(r_{it+1}/p_{it+1} + \left(1-\delta\right)\right), \tag{S.4.79}$$

where the transversality condition implies:

$$\lim_{t\to\infty}\beta^t\frac{k_{it+1}}{c_{it}} = 0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords' optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it}c_{it}^k = \left(1-\beta\right)\left(r_{it} + p_{it}\left(1-\delta\right)\right)k_{it}, \tag{S.4.80}$$

$$k_{it+1} = \beta\left(r_{it}/p_{it} + \left(1-\delta\right)\right)k_{it}. \tag{S.4.81}$$

Substituting the consumption policy function (S.4.80) into the Euler equation (S.4.79), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\begin{aligned}\frac{c_{it+1}^k}{c_{it}^k} &= \frac{\left(r_{it+1}/p_{it+1} + \left(1-\delta\right)\right)k_{it+1}}{\left(r_{it}/p_{it} + \left(1-\delta\right)\right)k_{it}},\\ &= \beta\left(r_{it+1}/p_{it+1} + \left(1-\delta\right)\right).\end{aligned}$$

Given this optimal consumption-saving decision in equations (S.4.80)-(S.4.81), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (S.4.69). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location n allocate the same share of expenditure on location i within sector j, as for workers in equation (S.4.70).

S.4.3.5 Market Clearing

Goods market clearing implies that revenue in each location in each sector equals expenditure on the goods produced by that location and sector:

$$p_{it}^j y_{it}^j = \psi^j \sum_{n=1}^{N}\sum_{h=1}^{J} S_{nit}^j\left(w_{nt}^h \ell_{nt}^h + r_{nt}k_{nt}^h\right),$$

$$w_{it}^j \ell_{it}^j + r_{nt}k_{nt}^j = \psi^j \sum_{n=1}^{N}\sum_{h=1}^{J} S_{nit}^j\left(w_{nt}^h \ell_{nt}^h + r_{nt}k_{nt}^h\right),$$

$$w_{it}^{j}\ell_{it}^{j}+\frac{1-\mu^{j}}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}=\psi^{j}\sum_{n=1}^{N}\sum_{h=1}^{J}S_{nit}^{j}\left(w_{nt}^{h}\ell_{nt}^{h}+\frac{1-\mu^{h}}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}\right),$$

$$\frac{1}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}=\psi^{j}\sum_{n=1}^{N}\sum_{h=1}^{J}S_{nit}^{j}\frac{1}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}. \tag{S.4.82}$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (S.4.73) and (S.4.74), we can write payments for capital in each sector as:

$$r_{it}k_{it}^{j}=\frac{1-\mu^{j}}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}. \tag{S.4.83}$$

Therefore capital market clearing implies:

$$k_{it}^{j}=\frac{\frac{1-\mu^{j}}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}}{\sum_{o=1}^{J}\frac{1-\mu^{o}}{\mu^{o}}w_{it}^{o}\ell_{it}^{o}}k_{it}. \tag{S.4.84}$$

Re-arranging the relationship between sector-level payments to capital and labor in equation (S.4.83), the equilibrium rental rate for capital is given by:

$$r_{it}=\frac{1-\mu^{j}}{\mu^{j}}\frac{w_{it}^{j}\ell_{it}^{j}}{k_{it}^{j}}.$$

Using this result in capital market clearing (S.4.84), we can re-write this capital market clearing condition as:

$$r_{it}=\left(\sum_{o=1}^{J}\frac{1-\mu^{o}}{\mu^{o}}w_{it}^{o}\ell_{it}^{o}\right)/k_{it}. \tag{S.4.85}$$

S.4.3.6 General Equilibrium

Given the state variables $\{\ell_{it}^{j},\,k_{it}^{j}\}$ for each sector j and location i, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\left\{\ell_{it}^{j},k_{it}^{j},w_{it}^{j},v_{it}^{j}\right\}_{t}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing (S.4.85), the price index (S.4.67) and the equilibrium pricing rule (S.4.72), the capital accumulation equation (S.4.81) becomes:

$$k_{it+1}=\beta\left(1-\delta\right)k_{it}+\beta\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}, \tag{S.4.86}$$

$$\vartheta_{it}^{o}=\frac{1-\mu^{o}}{\mu^{o}}\frac{w_{it}^{o}}{p_{it}}, \qquad k_{it}^{j}=\frac{\vartheta_{it}^{j}\ell_{it}^{j}}{\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}}k_{it}.$$

$$p_{nt}=\prod_{j=1}^{J}\left[\sum_{i=1}^{N}\left(w_{it}^{j}\left(\frac{1-\mu^{j}}{\mu^{j}}\right)^{1-\mu^{j}}\left(\ell_{it}^{j}/k_{it}^{j}\right)^{1-\mu^{j}}\tau_{nit}^{j}/z_{it}^{j}\right)^{-\theta}\right]^{-\psi^{j}/\theta}. \tag{S.4.87}$$

Goods Market Clearing: From the goods market clearing condition (S.4.82), we have:

$$\frac{1}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}=\psi^{j}\sum_{n=1}^{N}\sum_{h=1}^{J}S_{nit}^{j}\frac{1}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}. \tag{S.4.88}$$

$$S_{nit}^{j}=\frac{\left(\tau_{nit}^{j}\left(w_{it}^{j}\right)^{\mu^{j}}\left(r_{it}\right)^{1-\mu^{j}}/z_{it}^{j}\right)^{-\theta}}{\sum_{m=1}^{N}\left(\tau_{nmt}^{j}\left(w_{mt}^{j}\right)^{\mu^{j}}\left(r_{mt}\right)^{1-\mu^{j}}/z_{mt}^{j}\right)^{-\theta}}, \qquad T_{int}^{jh}\equiv\frac{\psi^{j}S_{nit}^{j}\frac{1}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}}{\frac{1}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}},$$

where S_{nit}^{j} is the expenditure share of importer n on exporter i in sector j at time t, and we have defined T_{int}^{jh} as the corresponding income share of exporter i from importer n at time t. Note that the order of subscripts switches between the expenditure share (S_{nit}^{j}) and the income share (T_{int}^{jh}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the out-migration probabilities (S.4.65), the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1}^{h}=\sum_{i=1}^{N}\sum_{j=1}^{J}D_{igt}^{jh}\ell_{it}^{j}, \tag{S.4.89}$$

$$D_{igt}^{jh}=\frac{\left(\exp\left(\beta\mathbb{E}_{t}v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho}}{\sum_{m=1}^{N}\sum_{o=1}^{J}\left(\exp\left(\beta\mathbb{E}_{t}v_{mt+1}^{o,w}\right)/\kappa_{mit}^{oj}\right)^{1/\rho}}, \qquad E_{git}^{hj}\equiv\frac{\ell_{it}^{j}D_{igt}^{jh}}{\ell_{gt+1}^{h}},$$

where D_{igt}^{jh} is the outmigration probability from sector j in location i to sector h in location g at time t, and we have defined E_{git}^{hj} as the corresponding inmigration probability to sector h in location g from sector j in location i at time t. Note that the order of subscripts switches between the outmigration probability (D_{igt}^{jh}) and the inmigration probability (E_{git}^{hj}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (S.4.66) in the value function (S.4.62), the expected utility from working in sector j in location n at time t can be written as:

$$v_{nt}^{j,w}=\ln b_{nt}^{j}+\ln\left(\frac{w_{nt}^{j}}{p_{nt}}\right)+\rho\log\sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta\mathbb{E}_{t}v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho}. \tag{S.4.90}$$

S.4.3.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

Prices. Using the relationship between capital and labor payments (S.4.84), the pricing rule (S.4.72) can be re-written as follows:

$$p_{nit}^{j} = \frac{\tau_{nit}^{j}\left(w_{it}^{j}\right)\left(\frac{1-\mu^{j}}{\mu^{j}}\right)^{1-\mu^{j}}\left(\frac{1}{\chi_{it}^{j}}\right)^{1-\mu^{j}}}{z_{it}^{j}}, \tag{S.4.91}$$

where χ_{it}^{j} is the capital-labor ratio in sector j:

$$\chi_{it}^{j} \equiv \frac{k_{it}^{j}}{\ell_{it}^{j}}.$$

Totally differentiating this pricing rule, we have:

$$\mathrm{d}\ln p_{nit}^{j} = \left[\mathrm{d}\ln \tau_{nit}^{j} + \mathrm{d}\ln w_{it}^{j} - \left(1-\mu^{j}\right)\mathrm{d}\ln \chi_{it}^{j} - \mathrm{d}\ln z_{it}^{j}\right]. \tag{S.4.92}$$

Expenditure Shares. Totally differentiating this expenditure share equation (S.4.70), we get:

$$\mathrm{d}\ln S_{nit}^{j} = \theta\left(\sum_{h=1}^{N} S_{nht}^{j}\,\mathrm{d}\ln p_{nht}^{j} - \mathrm{d}\ln p_{nit}^{j}\right). \tag{S.4.93}$$

Price Indices. Totally differentiating the consumption goods price index in equation (S.4.68), we have:

$$\mathrm{d}\ln p_{nt}^{j} = \sum_{m=1}^{N} S_{nmt}^{j}\,\mathrm{d}\ln p_{nmt}^{j}. \tag{S.4.94}$$

Migration Shares. Totally differentiating the outmigration share in equation (S.4.65), we get:

$$\mathrm{d}\ln D_{igt}^{jh} = \frac{1}{\rho}\left[\left(\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1}^{h,w} - \mathrm{d}\ln\kappa_{git}^{hj}\right) - \sum_{m=1}^{N}\sum_{o=1}^{J} D_{imt}^{jo}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln\kappa_{mit}^{oj}\right)\right]. \tag{S.4.95}$$

Real Income. Totally differentiating real income we have:

$$\mathrm{d}\ln\vartheta_{it}^{j} = \mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J}\psi^{h}\,\mathrm{d}\ln p_{it}^{h},$$

$$\mathrm{d}\ln\vartheta_{it}^{j} = \mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J}\psi^{h}\sum_{m=1}^{N} S_{nmt}^{h}\,\mathrm{d}\ln p_{imt}^{h},$$

$$\mathrm{d}\ln\vartheta_{it}^{j} = \mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J}\psi^{h}\sum_{m=1}^{N} S_{imt}^{h}\left[\begin{array}{c}\mathrm{d}\ln\tau_{imt}^{h} + \mathrm{d}\ln w_{mt}^{h}\\ -\left(1-\mu^{h}\right)\mathrm{d}\ln\chi_{mt}^{h} - \mathrm{d}\ln z_{mt}^{h}\end{array}\right], \tag{S.4.96}$$

Goods Market Clearing. Totally differentiating the goods market clearing condition (S.4.82), we have:

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}}+\frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}}=\sum_{n=1}^{N}\sum_{h=1}^{J}\psi^{h}\frac{S_{nit}^{h}\frac{1}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}}{\frac{1}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}}\left(\frac{\mathrm{d}w_{nt}^{h}}{w_{nt}^{h}}+\frac{\mathrm{d}\ell_{nt}^{h}}{\ell_{nt}^{h}}+\frac{\mathrm{d}S_{nit}^{h}}{S_{nit}^{h}}\right).$$

Using our result for the derivative of expenditure shares in equation (S.4.93) above, we can rewrite this as:

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}}+\frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}}=\sum_{n=1}^{N}\sum_{h=1}^{J}T_{int}^{jh}\left(\frac{\mathrm{d}w_{nt}^{h}}{w_{nt}^{h}}+\frac{\mathrm{d}\ell_{nt}^{h}}{\ell_{nt}^{h}}+\theta\left(\sum_{m=1}^{N}S_{nmt}^{h}\frac{\mathrm{d}p_{nmt}^{h}}{p_{nmt}^{h}}-\frac{\mathrm{d}p_{nit}^{h}}{p_{nit}^{h}}\right)\right),$$

$$T_{int}^{jh}\equiv\psi^{h}\frac{S_{nit}^{h}\frac{1}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}}{\frac{1}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}},$$

$$\begin{bmatrix}\mathrm{d}\ln w_{it}^{j}\\+\mathrm{d}\ln\ell_{it}^{j}\end{bmatrix}=\begin{bmatrix}\sum_{n=1}^{N}\sum_{h=1}^{J}T_{int}^{jh}\left(\mathrm{d}\ln w_{nt}^{h}+\mathrm{d}\ln\ell_{nt}^{h}\right)\\+\theta\sum_{n=1}^{N}\sum_{h=1}^{J}\sum_{m=1}^{N}T_{int}^{jh}S_{nmt}^{h}\left(\mathrm{d}\ln\tau_{nmt}^{h}+\mathrm{d}\ln w_{mt}^{h}-\left(1-\mu^{h}\right)\mathrm{d}\ln\chi_{mt}^{h}-\mathrm{d}\ln z_{mt}\right)\\-\theta\sum_{n=1}^{N}\sum_{h=1}^{J}T_{int}^{jh}\left(\mathrm{d}\ln\tau_{nit}^{h}+\mathrm{d}\ln w_{it}^{h}-\left(1-\mu^{h}\right)\mathrm{d}\ln\chi_{it}^{h}-\mathrm{d}\ln z_{it}\right)\end{bmatrix}. \tag{S.4.97}$$

Population Flow. Totally differentiating the population flow condition (S.4.89) we have:

$$\frac{\mathrm{d}\ell_{gt+1}^{h}}{\ell_{gt+1}^{h}}=\sum_{i=1}^{N}\sum_{j=1}^{J}E_{git}^{hj}\left[\frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}}+\frac{\mathrm{d}D_{igt}^{jh}}{D_{igt}^{jh}}\right],$$

$$\mathrm{d}\ln\ell_{gt+1}^{h}=\sum_{i=1}^{N}\sum_{j=1}^{J}E_{git}^{hj}\left[\mathrm{d}\ln\ell_{it}^{j}+\frac{1}{\rho}\left[\left(\beta\mathbb{E}_{t}\,\mathrm{d}v_{gt+1}^{h,w}-\mathrm{d}\ln\kappa_{git}^{hj}\right)-\sum_{m=1}^{N}\sum_{o=1}^{J}D_{imt}^{jo}\left(\beta\mathbb{E}_{t}\,\mathrm{d}v_{mt+1}^{o,w}-\mathrm{d}\ln\kappa_{mit}^{oj}\right)\right]\right]. \tag{S.4.98}$$

Value Function. Note that the value function is:

$$v_{it}^{j,w}=\ln\left(\frac{w_{it}^{j}}{\prod_{h=1}^{J}\left[\sum_{m=1}^{N}\left(p_{imt}^{h}\right)^{-\theta}\right]^{-\psi^{h}/\theta}}\right)+\ln b_{it}^{j}+\rho\ln\sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta\mathbb{E}_{t}v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho},$$

$$\left[\sum_{m=1}^{N}\left(p_{imt}^{h}\right)^{-\theta}\right]^{-1/\theta}=\left(\frac{\left(p_{iit}^{h}\right)^{-\theta}}{S_{iit}^{h}}\right)^{-1/\theta},\qquad\tau_{iit}^{h}=1,$$

$$\prod_{h=1}^{J}\left[\sum_{m=1}^{N}\left(p_{imt}^{h}\right)^{-\theta}\right]^{-\psi^{h}/\theta}=\prod_{h=1}^{J}\left(\frac{\left(p_{iit}^{h}\right)^{-\theta}}{S_{iit}^{h}}\right)^{-\psi^{h}/\theta},$$

$$\sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta\mathbb{E}_{t}v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho}=\frac{\left(\exp\left(\beta\mathbb{E}_{t}v_{it+1}^{j,w}\right)/\kappa_{iit}^{jj}\right)^{1/\rho}}{D_{iit}^{jj}},\qquad\kappa_{iit}^{jj}=1,$$

$$v_{it}^{j,w}=\ln w_{it}^{j}-\sum_{h=1}^{J}\psi^{h}\left(\frac{1}{\theta}\ln S_{iit}^{h}+\ln p_{iit}^{h}\right)+\ln b_{it}^{j}+\beta\mathbb{E}_{t}v_{it+1}^{j,w}-\rho\ln D_{iit}^{jj}. \tag{S.4.99}$$

Note that the value function can be equivalently written as:

$$v_{it}^{j,w} = \ln \xi^j + \ln \vartheta_{it}^{i} + \ln b_{it}^{j} + \beta \mathbb{E}_t v_{it+1}^{j,w} - \rho \ln D_{iit}^{jj},$$

where $\xi^j = \frac{\mu^j}{1-\mu^j}$. Totally differentiating the value function (S.4.99) we have:

$$v_{it}^{j,w} = \ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \left(\frac{1}{\theta} \ln S_{iit}^{h} + \ln p_{iit}^{h} \right) + \ln b_{it}^{j} + \beta \mathbb{E}_t v_{it+1}^{j,w} - \rho \ln D_{iit}^{jj}. \tag{S.4.100}$$

$$\mathrm{d}v_{it}^{j,w} = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \left(\frac{1}{\theta} \mathrm{d}\ln S_{iit}^{h} + \mathrm{d}\ln p_{iit}^{h} \right) + \mathrm{d}\ln b_{it}^{j} + \beta \mathbb{E}_t \,\mathrm{d}v_{it+1}^{j,w} - \rho \,\mathrm{d}\ln D_{iit}^{jj},$$

$$\mathrm{d}\ln S_{iit}^{h} = -\theta \,\mathrm{d}\ln p_{iit}^{h} + \theta \left[\sum_{m=1}^{N} S_{imt}^{h} \,\mathrm{d}\ln p_{imt}^{h} \right],$$

$$\mathrm{d}\ln D_{iit}^{jj} = \frac{1}{\rho} \left[\beta \mathbb{E}_t \,\mathrm{d}v_{it+1}^{j,w} - \mathrm{d}\ln \kappa_{iit}^{jj} - \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln \kappa_{mit}^{oj} \right) \right].$$

Using these results in the derivative of the value function, we have:

$$\mathrm{d}v_{it}^{j} = \left[\begin{array}{c} \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \left(\frac{1}{\theta} \mathrm{d}\ln S_{iit}^{h} + \mathrm{d}\ln p_{iit}^{h} \right) \\ + \mathrm{d}\ln b_{it}^{j} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln \kappa_{mit}^{oj} \right) \end{array} \right],$$

where we have used $\mathrm{d}\ln \kappa_{iit}^{jj} = 0$. Using the total derivative of the pricing rule (S.4.92), we can re-write this derivative of the value function as follows:

$$\mathrm{d}v_{it}^{j} = \left[\begin{array}{c} \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J} \psi^h \sum_{m=1}^{N} S_{imt}^{h} \left(\mathrm{d}\ln \tau_{nmt}^{h} + \mathrm{d}\ln w_{mt}^{h} - \left(1-\mu^h\right) \mathrm{d}\ln \chi_{mt}^{h} - \mathrm{d}\ln z_{mt} \right) \\ + \mathrm{d}\ln b_{it}^{j} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln \kappa_{mit}^{oj} \right) \end{array} \right], \tag{S.4.101}$$

which can be equivalently written as:

$$\mathrm{d}v_{it}^{j} = \left[\mathrm{d}\ln \vartheta_{it}^{j} + \mathrm{d}\ln b_{it}^{j} + \sum_{m=1}^{N} \sum_{o=1}^{J} D_{imt}^{jo} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln \kappa_{mit}^{oj} \right) \right].$$

S.4.3.8 Steady-state Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1}^{j} = k_{it}^{j} = k_i^{j*}$, $\ell_{it+1}^{j} = \ell_{it}^{j} = \ell_i^{j*}$, $w_{it+1}^{j} = w_{it}^{j} = w_i^{j*}$ and $v_{it+1}^{j} = v_{it}^{j} = v_i^{j*}$, where we use an asterisk to denote a steady-state value. We consider a small common shock to productivity across all sectors ($\mathrm{d}\ln \boldsymbol{z}$) and amenities across all sectors ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$).

Capital Accumulation. From the capital accumulation equation (S.4.86), the steady-state stock of capital solves:

$$k_i^* = \beta\left(1-\delta\right)k_i^* + \beta\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*},$$

$$\left(1-\beta\left(1-\delta\right)\right)k_i^* = \beta\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}.$$

But from capital market clearing, we also have:

$$k_i^{j*} = \frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}k_i^*.$$

and hence:

$$\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*} = \vartheta_i^{j*}\ell_i^{j*}\frac{k_i^*}{k_i^{j*}}.$$

Using this result in the capital accumulation equation, we have:

$$\left(1-\beta\left(1-\delta\right)\right)k_i^{j*} = \beta\vartheta_i^{j*}\ell_i^{j*},$$

and hence:

$$\vartheta_i^{j*} = \frac{\left(1-\beta\left(1-\delta\right)\right)}{\beta}\frac{k_i^{j*}}{\ell_i^{j*}} = \frac{\left(1-\beta\left(1-\delta\right)\right)}{\beta}\chi_i^{j*}.$$

Totally differentiating, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln\vartheta_i^{j*},$$

$$\mathrm{d}\ln\vartheta_i^{j*} = \mathrm{d}\ln w_i^* - \mathrm{d}\ln p_i^*.$$

From the total derivative of real income (S.4.96) above, this becomes:

$$\mathrm{d}\ln\vartheta_i^{j*} = \mathrm{d}\ln w_i^* - \sum_{m=1}^{N}\sum_{h=1}^{J}\psi^h S_{im}^h\left[\mathrm{d}\ln w_m^* - \left(1-\mu^h\right)\mathrm{d}\ln\vartheta_i^{j*} - \mathrm{d}\ln z_m^h\right],$$

where we have used $\mathrm{d}\ln\tau_{imt}^h = 0$. This relationship has the following matrix representation:

$$\mathrm{d}\ln\boldsymbol{\vartheta}^* = \mathrm{d}\ln\boldsymbol{w}^* - \boldsymbol{S}\left(\mathrm{d}\ln\boldsymbol{w} - \left(\boldsymbol{I}-\boldsymbol{\mu}\right)\mathrm{d}\ln\boldsymbol{\vartheta}^* - \mathrm{d}\ln\boldsymbol{z}\right), \tag{S.4.102}$$

where $\mathrm{d}\ln\boldsymbol{\vartheta}^*$ and $\mathrm{d}\ln\boldsymbol{w}^*$ are $NJ\times 1$ vectors; $\boldsymbol{S}$ is a $NJ\times NJ$ matrix with elements:

$$S_{nit} = S_{nit}^j = \sum_{h=1}^{J}\psi^h S_{nit},$$

and $\boldsymbol{\mu}$ is $NJ\times NJ$ diagonal matrix whose (ij)-th element on the diagonal is μ^j. Note that the evolution of the regional capital stock is given by:

$$k_{it} = \sum_{j=1}^{J}k_{it}^j,$$

$$\mathrm{d}k_{it} = \sum_{j=1}^{J} \mathrm{d}k_{it}^{j},$$

$$\frac{\mathrm{d}k_{it}}{k_{it}} = \sum_{j=1}^{J} \frac{k_{it}^{j}}{k_{it}} \frac{\mathrm{d}k_{it}^{j}}{k_{it}^{j}},$$

$$\mathrm{d}\ln k_{it} = \sum_{j=1}^{J} \frac{k_{it}^{j}}{k_{it}} \mathrm{d}\ln k_{it}^{j},$$

$$\mathrm{d}\ln k_{it} = \sum_{j=1}^{J} \frac{k_{it}^{j}}{k_{it}} \left(\mathrm{d}\ln \vartheta_{it}^{j} + \mathrm{d}\ln \ell_{it}^{j} \right),$$

which can be written as:

$$\mathrm{d}\ln \boldsymbol{k_t^{reg}} = \mathcal{K} \left(\mathrm{d}\ln \boldsymbol{\vartheta_t} + \mathrm{d}\ln \boldsymbol{\ell_t} \right),$$

where k_t^{reg} is a $N \times 1$ vector of regional capital stocks; $\mathcal{K}$ is the $N \times NJ$ matrix whose (i, jo) element is the steady-state share of capital in location i employed in location j sector o; $\boldsymbol{\vartheta_t}$ and $\boldsymbol{\ell_t}$ are $NJ \times 1$ vectors.

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.97) has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{w_t} + \mathrm{d}\ln \boldsymbol{\ell_t} = \boldsymbol{T} \left(\mathrm{d}\ln \boldsymbol{w_t} + \mathrm{d}\ln \boldsymbol{\ell_t} \right) + \theta \left(\boldsymbol{TS} - \boldsymbol{I} \right) \left(\mathrm{d}\ln \boldsymbol{w_t} - \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d}\ln \boldsymbol{\chi_t} - \mathrm{d}\ln \boldsymbol{z} \right),$$

where these matrices have $NJ \times NJ$ elements and we have used $\mathrm{d}\ln \boldsymbol{\tau} = 0$. In steady-state we have:

$$\mathrm{d}\ln \boldsymbol{w}^{*} + \mathrm{d}\ln \boldsymbol{\ell}^{*} = \boldsymbol{T} \left(\mathrm{d}\ln \boldsymbol{w}^{*} + \mathrm{d}\ln \boldsymbol{\ell}^{*} \right) + \theta \left(\boldsymbol{TS} - \boldsymbol{I} \right) \left(\mathrm{d}\ln \boldsymbol{w}^{*} - \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d}\ln \boldsymbol{\vartheta}^{*} - \mathrm{d}\ln \boldsymbol{z} \right). \tag{S.4.103}$$

Population Flow. The total derivative of the population flow condition (S.4.98) has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{\ell_{t+1}} = \boldsymbol{E} \,\mathrm{d}\ln \boldsymbol{\ell_t} + \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{ED} \right) \mathbb{E}_t \, \mathrm{d}\boldsymbol{v_{t+1}},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\mathrm{d}\ln \boldsymbol{\ell}^{*} = \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E} \right)^{-1} \left(\boldsymbol{I} - \boldsymbol{ED} \right) \mathrm{d}\boldsymbol{v}^{*}. \tag{S.4.104}$$

Value function. The total derivative of the value function has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \mathrm{d}\ln \boldsymbol{\vartheta_t} + \mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D} \mathbb{E}_t \, \mathrm{d}\boldsymbol{v_{t+1}},$$

where these matrices again have $NJ \times NJ$ elements and we have used $\mathrm{d}\ln \boldsymbol{\kappa} = 0$. In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^{*} = \left(\boldsymbol{I} - \beta \boldsymbol{D} \right)^{-1} \left[\mathrm{d}\ln \boldsymbol{\vartheta}^{*} + \mathrm{d}\ln \boldsymbol{b} \right]. \tag{S.4.105}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\vartheta}^* = \mathrm{d}\ln \boldsymbol{w}^* - \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{w} - (\boldsymbol{I}-\boldsymbol{\mu})\,\mathrm{d}\ln \boldsymbol{\vartheta}^* - \mathrm{d}\ln \boldsymbol{z}\right). \tag{S.4.106}$$

$$\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^* = \boldsymbol{T}\left(\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^*\right) + \theta\left(\boldsymbol{T}\boldsymbol{S}-\boldsymbol{I}\right)\left(\mathrm{d}\ln \boldsymbol{w}^* - (\boldsymbol{I}-\boldsymbol{\mu})\,\mathrm{d}\ln \boldsymbol{\vartheta}^* - \mathrm{d}\ln \boldsymbol{z}\right). \tag{S.4.107}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{E}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}\right)\mathrm{d}\boldsymbol{v}^*. \tag{S.4.108}$$

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I}-\beta\boldsymbol{D}\right)^{-1}\left[\mathrm{d}\ln \boldsymbol{\vartheta}^* + \mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.109}$$

$$\mathrm{d}\ln k^{reg*} = \mathcal{K}\left(\mathrm{d}\ln \boldsymbol{\vartheta}_{\boldsymbol{i}}^* + \mathrm{d}\ln \boldsymbol{\ell}_{\boldsymbol{i}}^*\right). \tag{S.4.110}$$

S.4.3.9 Sufficient Statistics for Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$) in each sector and location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$). We use a tilde above a variable to denote a log-deviation from the initial steady-state, such that $\widetilde{\ell}_{it} = \ln \ell_{it} - \ln \ell_i^*$, for all variables except for the worker value function v_{it}; with a slight abuse of notation we use $\widetilde{v}_{it} \equiv v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (S.4.86), we have:

$$k_{it+1} = \beta\left(1-\delta\right)k_{it} + \beta\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}.$$

Dividing by the steady-state capital stock we have:

$$\frac{k_{it+1}}{k_i^*} = \beta\left(1-\delta\right)\frac{k_{it}}{k_i^*} + \beta\sum_{o=1}^{J}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{k_i^*}.$$

We know from above that the steady-state capital stock is given by:

$$k_i^* = \frac{\beta}{1-\beta\left(1-\delta\right)}\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}.$$

Therefore we can re-write the capital accumulation equation as:

$$\frac{k_{it+1}}{k_i^*} = \beta\left(1-\delta\right)\frac{k_{it}}{k_i^*} + \left(1-\beta\left(1-\delta\right)\right)\sum_{o=1}^{J}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}.$$

$$\frac{k_{it+1}}{k_i^*} = \beta\left(1-\delta\right)\frac{k_{it}}{k_i^*} + \left(1-\beta\left(1-\delta\right)\right)\sum_{o=1}^{J}\frac{\vartheta_i^{o*}\ell_i^{o*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{o*}\ell_i^{o*}},$$

which can be further re-written as:

$$\frac{k_{it+1}}{k_i^*} - 1 = \beta\left(1-\delta\right)\left(\frac{k_{it}}{k_i^*} - 1\right) + \left(1-\beta\left(1-\delta\right)\right)\sum_{o=1}^{J}\frac{\vartheta_i^{o*}\ell_i^{o*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\left(\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{o*}\ell_i^{o*}} - 1\right),$$

Noting that

$$\frac{k_{it}}{k_i^*} - 1 \simeq \ln\left(\frac{k_{it}}{k_i^*}\right),$$

we have:

$$\ln\left(\frac{k_{it+1}}{k_i^*}\right) = \beta\left(1-\delta\right)\ln\left(\frac{k_{it}}{k_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\sum_{o=1}^{J}\frac{\vartheta_i^{o*}\ell_i^{o*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\ln\left(\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{o*}\ell_i^{o*}}\right),$$

which can be re-written in matrix form as:

$$\widetilde{\boldsymbol{k}}_{\boldsymbol{t+1}}^{\boldsymbol{reg}} = \beta\left(1-\delta\right)\widetilde{\boldsymbol{k}}_{\boldsymbol{t}}^{\boldsymbol{reg}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{\mathcal{K}}\left(\widetilde{\boldsymbol{\vartheta}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right), \tag{S.4.111}$$

where $\boldsymbol{k}_{\boldsymbol{t+1}}^{\boldsymbol{reg}}$ and $\boldsymbol{k}_{\boldsymbol{t}}^{\boldsymbol{reg}}$ are $N \times 1$ vectors; $\mathcal{K}$ is the $N \times NJ$ matrix whose (i, jo) element is the steady-state share of capital in location i employed in location j sector o; $\boldsymbol{\vartheta}_{\boldsymbol{t}}$ and $\boldsymbol{\ell}_{\boldsymbol{t}}$ are $NJ \times 1$ vectors.

To derive the cross-industry allocation, note:

$$\frac{k_{it}^{j}}{k_i^{j*}} = \frac{\vartheta_{it}^{j}\ell_{it}^{j}}{\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}}\frac{k_{it}}{k_i^{j*}},$$

$$\frac{k_{it}^{j}}{k_i^{j*}} = \frac{\vartheta_{it}^{j}\ell_{it}^{j}}{\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}}\frac{k_{it}}{\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}k_i^*},$$

$$\frac{k_{it}^{j}}{k_i^{j*}} = \frac{\vartheta_{it}^{j}\ell_{it}^{j}/\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_{it}^{o}\ell_{it}^{o}/\left(\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}\right)}\frac{k_{it}}{k_i^*},$$

$$\frac{k_{it}^{j}}{k_i^{j*}} = \frac{\vartheta_{it}^{j}\ell_{it}^{j}/\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}}\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}}\frac{k_{it}}{k_i^*},$$

$$\ln\left(\frac{k_{it}^{j}}{k_i^{j*}}\right) = \ln\left(\frac{\vartheta_{it}^{j}\ell_{it}^{j}}{\vartheta_i^{j*}\ell_i^{j*}}\right) - \ln\left(\sum_{o=1}^{J}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}}\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\right) + \ln\left(\frac{k_{it}}{k_i^*}\right).$$

Note that:

$$\ln\left(\sum_{o=1}^{J}\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}}\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\right) \simeq \sum_{o=1}^{J}\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\ln\left(\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}}\right), \qquad \frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}} \simeq 1.$$

Using this result above, we have:

$$\ln\left(\frac{k_{it}^{j}}{k_i^{j*}}\right) = \ln\left(\frac{\vartheta_{it}^{j}\ell_{it}^{j}}{\vartheta_i^{j*}\ell_i^{j*}}\right) - \sum_{o=1}^{J}\frac{\vartheta_i^{j*}\ell_i^{j*}}{\sum_{o=1}^{J}\vartheta_i^{o*}\ell_i^{o*}}\ln\left(\frac{\vartheta_{it}^{o}\ell_{it}^{o}}{\vartheta_i^{j*}\ell_i^{j*}}\right) + \ln\left(\frac{k_{it}}{k_i^*}\right).$$

In matrix representation, we have:

$$\widetilde{\boldsymbol{k}}_{\boldsymbol{t}}^{\boldsymbol{j}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}^{\boldsymbol{j}} = \widetilde{\boldsymbol{\vartheta}}_{\boldsymbol{t}}^{\boldsymbol{j}} - \underbrace{1}_{N\times 1}\otimes\left(\mathcal{K}\left(\widetilde{\vartheta}_t + \widetilde{\ell}_t\right) + \boldsymbol{k}_{\boldsymbol{t}}^{\boldsymbol{reg}}\right). \tag{S.4.112}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.97) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{t}+\widetilde{\boldsymbol{\ell}}_{t}=\left[\begin{array}{c}\boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{t}+\widetilde{\boldsymbol{\ell}}_{t}\right)\\ +\theta\left(\boldsymbol{T}\boldsymbol{S}-\boldsymbol{I}\right)\left(\widetilde{\boldsymbol{w}}_{t}-\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}_{t}^{j}}-\widetilde{\boldsymbol{\ell}}_{t}\right)-\widetilde{\boldsymbol{z}}\right)\end{array}\right],$$

where these matrices have $NJ\times NJ$ elements; we have used $\mathrm{d}\ln\boldsymbol{\tau}=0$; and we again use the superscript j for capital ($\boldsymbol{k}_{t}^{j}$) to distinguish sector-location capital from aggregate location capital ($\boldsymbol{k}_{t}^{reg}$). We can re-write this matrix representation as:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\right]\widetilde{\boldsymbol{w}}_{t}=-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{t}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}_{t}^{j}}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right],$$

$$\widetilde{\boldsymbol{w}}_{t}=\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{t}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}_{t}^{j}}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.113}$$

Population Flow. The total derivative of the population flow condition (S.4.98) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{t+1}=\boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{t}+\frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_{t}\widetilde{\boldsymbol{v}}_{t+1}, \tag{S.4.114}$$

where again these matrices have $NJ\times NJ$ elements.

Value Function. The total derivative of the value function relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{t}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{t}+\boldsymbol{S}\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}}_{t}^{j}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right]+\widetilde{\boldsymbol{b}}+\beta\boldsymbol{D}\mathbb{E}_{t}\widetilde{\boldsymbol{v}}_{t+1}, \tag{S.4.115}$$

where again these matrices have $NJ\times NJ$ elements; we have used $\mathrm{d}\ln\boldsymbol{\tau}=\mathrm{d}\ln\boldsymbol{\kappa}=0$; and we use the superscript j for capital ($\boldsymbol{k}_{t}^{j}$) to distinguish sector-location capital from aggregate location capital ($\boldsymbol{k}_{t}^{reg}$).

Real Income. The total derivative of real income relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\vartheta}}_{t}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{t}+\boldsymbol{S}\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}}_{t}^{j}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right], \tag{S.4.116}$$

where again these matrices have $NJ\times NJ$ elements and we have used $\mathrm{d}\ln\boldsymbol{\tau}=0$.

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

$$\widetilde{\boldsymbol{\vartheta}}_{t}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{t}+\boldsymbol{S}\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}}_{t}^{j}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right], \tag{S.4.117}$$

$$\widetilde{\boldsymbol{w}}_{t}=\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{t}+\theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}_{t}^{j}}-\widetilde{\boldsymbol{\ell}}_{t}\right)+\widetilde{\boldsymbol{z}}\right]\right], \tag{S.4.118}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{S.4.119}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}}^{\boldsymbol{j}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{S.4.120}$$

$$\widetilde{\boldsymbol{k}}_{\boldsymbol{t+1}}^{\boldsymbol{reg}} = \beta\left(1-\delta\right)\widetilde{\boldsymbol{k}}_{\boldsymbol{t}}^{\boldsymbol{reg}} + \left(1 - \beta\left(1-\delta\right)\right)\boldsymbol{\mathcal{K}}\left(\widetilde{\boldsymbol{\vartheta}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right), \tag{S.4.121}$$

$$\widetilde{\boldsymbol{k}}_{\boldsymbol{t}}^{\boldsymbol{j}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}^{\boldsymbol{j}} = \widetilde{\boldsymbol{\vartheta}}_{\boldsymbol{t}}^{\boldsymbol{j}} - \underbrace{1}_{N\times 1} \otimes \left(\mathcal{K}\left(\widetilde{\vartheta}_t + \widetilde{\ell}_t\right) + \boldsymbol{k}_{\boldsymbol{t}}^{\boldsymbol{reg}}\right). \tag{S.4.122}$$

S.4.4 Multiple Sector-Regions (Sector-Location Specific Capital)

We consider an economy that consists of many locations indexed by $i \in \{1,\ldots,N\}$ and many sectors indexed by $j \in \{1,\ldots,J\}$. Time is discrete and is indexed by t. The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across sectors and locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate δ.

S.4.4.1 Worker Migration Decisions

At the beginning of each period t, the economy inherits a mass of workers in each sector j and location i (ℓ_{it}^{j}), with the total labor endowment of the economy given by $\overline{\ell} = \sum_{i=1}^{N}\sum_{j=1}^{J}\ell_{it}$. Workers first produce and consume in their sector and location in period t, before observing mobility shocks $\{\epsilon_{gt}^{h}\}$ for all possible sectors $h \in \{1,\ldots,J\}$ and locations $g \in \{1,\ldots,N\}$ and deciding where to move for period $t+1$. Workers face bilateral migration costs that vary by sector and location, where κ_{git}^{hj} denotes the cost of moving from sector j in location i to sector h in location g. The value function for a worker in sector j and location i at time t ($\mathbb{V}_{it}^{j,w}$) is equal to the current flow of utility in that sector and location plus the expected continuation value next period from the optimal choice of sector and location:

$$\mathbb{V}_{it}^{j,w} = \ln u_{it}^{j,w} + \max_{\{g\}_1^N\{h\}_1^J}\left\{\beta\mathbb{E}_t\left[\mathbb{V}_{gt+1}^{h,w}\right] - \kappa_{git}^{hj} + \rho\epsilon_{gt}^{h}\right\}, \tag{S.4.123}$$

where we use the superscript w to denote workers; we assume logarithmic flow utility ($\ln u_{it}^{j,w}$); β denotes the discount rate; $\mathbb{E}\left[\cdot\right]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; ρ captures the dispersion of idiosyncratic mobility shocks; and we assume $\kappa_{iit}^{jj} = 1$ and $\kappa_{git}^{hj} > 1$ for $g \neq i$ and $h \neq j$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

$$F\left(\epsilon\right) = e^{-e^{\left(-\epsilon-\overline{\gamma}\right)}}, \tag{S.4.124}$$

where $\overline{\gamma}$ is the Euler-Mascheroni constant.

Under this assumption, the expected value for a worker of living in location i at time t ($v_{it}^{j,w}$) can be re-written in the following form:

$$v_{it}^{j,w} = \ln u_{it}^{j,w} + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{K} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w}\right) / \kappa_{git}^{hj} \right)^{1/\rho}. \tag{S.4.125}$$

The corresponding probability of migrating from location-sector ij to location-sector gh satisfies a gravity equation:

$$D_{igt}^{jh} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w}\right) / \kappa_{git}^{hj} \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^{o,w}\right) / \kappa_{kit}^{oj} \right)^{1/\rho}}. \tag{S.4.126}$$

S.4.4.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in their location and sector that period. Worker flow indirect utility in location n and sector j depends on local amenities (b_{nt}^{j}), the wage (w_{nt}^{j}), and the consumption goods price index (p_{it}):

$$\ln u_{nt}^{j,w} = \ln b_{nt}^{j} + \ln w_{nt}^{j} - \ln p_{nt}, \tag{S.4.127}$$

where amenities (b_{nt}) capture characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g., climate and scenic views). In this section of the Online Supplement, we assume that amenities are exogenous.

The consumption goods price index (p_{nt}) in location n depends on the consumption goods price index for each sector h in that location (p_{nt}^{h}):

$$p_{nt} = \prod_{h=1}^{J} \left(p_{nt}^{h}\right)^{\psi^h}, \qquad 0 < \psi^h < 1, \qquad \sum_{h=1}^{J} \psi^h, \tag{S.4.128}$$

where the consumption goods price index for each sector h in location n depends on the price of the variety sourced from each location i within that sector h (p_{nit}^{h}):

$$p_{nt}^{h} = \left[\sum_{i=1}^{N} \left(p_{nit}^{h}\right)^{-\theta} \right]^{-1/\theta}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{S.4.129}$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties; $\theta = \sigma - 1$ is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector ($p_{nt}^{h} c_{nt}^{h}$) is a constant share of overall goods consumption expenditure ($p_{nt} c_{nt}$) in each location:

$$p_{nt}^{h} c_{nt}^{h} = \psi^h p_{nt} c_{nt}. \tag{S.4.130}$$

Using constant elasticity of substitution (CES) demand for individuals varieties of goods, the share location n's expenditure within sector h on the goods produced by location i is:

$$S_{nit}^{h} \equiv \frac{\left(p_{nit}^{h}\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nmt}^{h}\right)^{-\theta}}. \tag{S.4.131}$$

S.4.4.3 Production

Producers in each location i and sector j use labor (ℓ_{it}^{j}) and capital (k_{it}^{j}) to produce output (y_{it}^{j}) of the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$y_{it}^{j} = z_{it}^{j} \left(\frac{\ell_{it}^{j}}{\mu^{j}} \right)^{\mu^{j}} \left(\frac{k_{it}^{j}}{1-\mu^{j}} \right)^{1-\mu^{j}}, \qquad 0 < \mu^{j} < 1, \tag{S.4.132}$$

where z_{it}^{j} denotes productivity in location i in sector j at time t. As for amenities above, we assume in this section of the Online Supplement that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit}^{j} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n, where $\tau_{nit}^{j} > 1$ for $n \neq i$ and $\tau_{iit}^{j} = 1$. From profit maximization, the cost to a consumer in location n of sourcing the good produced by location i within sector j is:

$$p_{nit}^{j} = \tau_{nit}^{j} p_{iit}^{j} = \frac{\tau_{nit}^{j} \left(w_{it}^{j} \right)^{\mu^{j}} \left(r_{it}^{j} \right)^{1-\mu^{j}}}{z_{it}^{j}}, \tag{S.4.133}$$

where p_{iit}^{j} is the "free on board" price of the good supplied by location i before transport costs; r_{it}^{j} is the rate of return to capital, which now varies across both sectors j and locations i, because capital is specific to both a sector and location.

From profit maximization problem and zero profits, payments for labor and building capital are constant shares of revenue:

$$w_{it}^{j} \ell_{it}^{j} = \mu^{j} p_{iit}^{j} y_{it}^{j}, \tag{S.4.134}$$

$$r_{it}^{j} k_{it}^{j} = \left(1-\mu^{j}\right) p_{iit}^{j} y_{it}^{j}, \tag{S.4.135}$$

where the immobility of capital across sectors once installed implies that the rate of return on capital need not be equalized across sectors and locations out of steady-state ($r_{it}^{j} \neq r_{nt}^{h}$).

S.4.4.4 Landlord Consumption

Landlords in each location and sector choose their consumption and investment in capital to maximize their intertemporal utility subject to the intertemporal budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^{j,k} = \sum_{t=0}^{\infty} \beta^{t} \ln c_{it}^{j,k}, \tag{S.4.136}$$

where we use the superscript k to denote landlords; $c_{it}^{j,k}$ is the consumption index for landlords in location i and sector j; and β denotes the discount rate. Since landlords are immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords ($c_{it}^{j,k}$) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index (p_{nt}) takes the same form as in equation (S.4.128), and the consumption goods price index for each sector (p^j_{nt}) takes the same form as in equation (S.4.129). Under these assumptions, the landlords' utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords' optimal consumption-saving, we assume that the investment technology for capital in each location and sector uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location and sector can produce one unit of capital for that sector and location using one unit of the consumption index for that sector and location. We assume that capital is geographically immobile once installed and depreciates at a constant rate δ. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r^j_{it}k^j_{it}$) equals the total value of goods consumption ($p_{it}c^{j,k}_{it}$) and net investment ($p_{it}\left(k^j_{it+1}-(1-\delta^j)\,k^j_{it}\right)$):

$$r^j_{it}k^j_{it} = p_{it}c^{j,k}_{it} + p_{it}\left(k^j_{it+1} - \left(1-\delta^j\right)k^j_{it}\right). \tag{S.4.137}$$

Combining landlords' intertemporal utility (S.4.136) and budget constraint (S.4.137), their intertemporal optimization problem is:

$$\max_{\left\{c^{j,k}_t,k^{j,k}_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^t \ln c^{j,k}_{it}, \tag{S.4.138}$$

$$\text{subject to} \qquad p_{it}c^{j,k}_{it} + p_{it}\left(k^j_{it+1} - \left(1-\delta^j\right)k^j_{it}\right) = r^j_{it}k^j_{it}.$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c^{j,k}_{it} - \xi^j_t\left[p_{it}c^{j,k}_{it} + p_{it}\left(k^j_{it+1} - \left(1-\delta^j\right)k^j_{it}\right) - r^j_{it}k^j_{it}\right]. \tag{S.4.139}$$

The first-order conditions are:

$$\left\{c^{j,k}_{it}\right\} \qquad \frac{\beta^t}{c^{j,k}_{it}} - p_{it}\xi^j_t = 0,$$

$$\left\{k^j_{it+1}\right\} \qquad \left(r^j_{it+1} + p_{it+1}\left(1-\delta^j\right)\right)\xi^j_{t+1} - p_{it}\xi^j_t = 0.$$

Together these first-order conditions imply:

$$\frac{c^{j,k}_{it+1}}{c^{j,k}_{it}} = \beta\frac{p_{it}\mu^j_t}{p_{it+1}\mu^j_{t+1}} = \beta\left(r^j_{it+1}/p_{it+1} + \left(1-\delta^j\right)\right), \tag{S.4.140}$$

where the transversality condition implies:

$$\lim_{t\to\infty} \beta^t \frac{k^j_{it+1}}{c^{j,k}_{it}} = 0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords' optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it}c_{it}^{j,k} = (1-\beta)\left(r_{it}^{j} + p_{it}\left(1-\delta^{j}\right)\right)k_{it}^{j}, \tag{S.4.141}$$

$$k_{it+1}^{j} = \beta\left(r_{it}^{j}/p_{it} + \left(1-\delta^{j}\right)\right)k_{it}^{j}. \tag{S.4.142}$$

Substituting the consumption policy function (S.4.141) into the Euler equation (S.4.140), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\begin{aligned}\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} &= \frac{\left(r_{it+1}^{j}/p_{it+1} + (1-\delta^{j})\right)k_{it+1}^{j}}{\left(r_{it}^{j}/p_{it} + (1-\delta^{j})\right)k_{it}^{j}}, \\ &= \beta\left(r_{it+1}^{j}/p_{it+1} + \left(1-\delta^{j}\right)\right).\end{aligned}$$

Given this optimal consumption-saving decision in equations (S.4.141)-(S.4.142), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (S.4.130). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location n allocate the same share of expenditure in location i within sector j as for workers in equation (S.4.131).

S.4.4.5 Market Clearing

Goods market clearing implies that revenue in each region-sector equals expenditure on the goods produced by that region-sector:

$$p_{iit}^{j}y_{it}^{j} = \sum_{n=1}^{N}\psi^{j}S_{nit}^{j}\sum_{h=1}^{J}\left(w_{nt}^{h}\ell_{nt}^{h} + r_{nt}^{h}k_{nt}^{h}\right),$$

$$w_{it}^{j}\ell_{it}^{j} + r_{nt}^{j}k_{nt}^{j} = \sum_{n=1}^{N}\psi^{j}S_{nit}^{j}\sum_{h=1}^{J}\left(w_{nt}^{h}\ell_{nt}^{h} + r_{nt}^{h}k_{nt}^{h}\right),$$

$$w_{it}^{j}\ell_{it}^{j} + \frac{1-\mu^{j}}{\mu^{j}}w_{it}^{j}\ell_{it}^{j} = \sum_{n=1}^{N}\psi^{j}S_{nit}^{j}\sum_{h=1}^{J}\left(w_{nt}^{h}\ell_{nt}^{h} + \frac{1-\mu^{h}}{\mu^{h}}w_{nt}^{h}\ell_{nt}^{h}\right),$$

$$\left(\frac{1}{\mu^{j}}\right)w_{it}^{j}\ell_{it}^{j} = \sum_{n=1}^{N}\sum_{h=1}^{J}\psi^{j}S_{nit}^{j}\left(\frac{1}{\mu^{h}}\right)w_{nt}^{h}\ell_{nt}^{h}. \tag{S.4.143}$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (S.4.134) and (S.4.135), we can write payments for capital in each sector as:

$$r_{it}^{j}k_{it}^{j} = \frac{1-\mu^{j}}{\mu^{j}}w_{it}^{j}\ell_{it}^{j}. \tag{S.4.144}$$

S.4.4.6 General Equilibrium

Given the state variables $\{\ell_{i0}^{j},\ k_{i0}^{j}\}$ for each sector j and location i, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\left\{\ell_{it}^{j}, k_{it}^{j}, w_{it}^{j}, v_{it}^{j}\right\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing (S.4.144), the price index (S.4.128) and the equilibrium pricing rule (S.4.133), the capital accumulation equation (S.4.142) can be rewritten as:

$$k_{it+1}^{j} = \beta \frac{1-\mu^{j}}{\mu^{j}} \frac{w_{it}^{j}}{p_{it}} \ell_{it}^{j} + \beta \left(1-\delta^{j}\right) k_{it}^{j}, \tag{S.4.145}$$

$$p_{nt} = \prod_{h=1}^{J} \left[\sum_{i=1}^{N} \left(w_{it}^{j} \left(\frac{1-\mu^{j}}{\mu^{j}} \right)^{1-\mu^{j}} \left(\ell_{it}^{j} / k_{it}^{j} \right)^{1-\mu^{j}} \tau_{nit}^{j} / z_{it}^{j} \right)^{-\theta} \right]^{-\psi^{h}/\theta}. \tag{S.4.146}$$

Goods Market Clearing: Using the equilibrium pricing rule (S.4.133), the expenditure share (S.4.131) and capital market clearing (S.4.144) in the goods market clearing condition (S.4.143), we obtain:

$$\left(\frac{1}{\mu^{j}} \right) w_{it}^{j} \ell_{it}^{j} = \sum_{n=1}^{N} \sum_{h=1}^{J} \psi^{j} S_{nit}^{j} \left(\frac{1}{\mu^{h}} \right) w_{nt}^{h} \ell_{nt}^{h}, \tag{S.4.147}$$

$$S_{nit}^{h} \equiv \frac{\left(w_{it}^{j} \left(\ell_{it}^{j} / k_{it}^{j} \right)^{1-\mu^{j}} \tau_{nit}^{j} / z_{it}^{j} \right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt}^{j} \left(\ell_{mt}^{j} / k_{mt}^{j} \right)^{1-\mu^{j}} \tau_{nmt}^{j} / z_{mt}^{j} \right)^{-\theta}}, \qquad T_{int}^{jh} \equiv \frac{\psi^{j} S_{nit}^{j} \left(1/\mu^{h}\right) w_{nt}^{h} \ell_{nt}^{h}}{\left(1/\mu^{j}\right) w_{it}^{j} \ell_{it}^{j}}, \tag{S.4.148}$$

where S_{nit}^{h} is the expenditure share of importer n on each exporter i at time t, and we have defined T_{int}^{jh} as the corresponding income share of exporter i from each importer n at time t. Note that the order of subscripts switches between the expenditure share (S_{nit}^{h}) and the income share (T_{int}^{jh}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the outmigration probabilities (S.4.126), the population flow condition for the evolution of the employment distribution over time is given by:

$$\ell_{gt+1}^{h} = \sum_{i=1}^{N} \sum_{j=1}^{J} D_{igt}^{jh} \ell_{it}^{j}, \tag{S.4.149}$$

$$D_{igt}^{jh} = \frac{\left(\exp\left(\beta \mathbb{E}_{t} v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left(\exp\left(\beta \mathbb{E}_{t} v_{mt+1}^{o,w} \right) / \kappa_{kit}^{oj} \right)^{1/\rho}}, \qquad E_{git}^{hj} \equiv \frac{\ell_{it}^{j} D_{igt}^{jh}}{\ell_{gt+1}^{h}}, \tag{S.4.150}$$

where D_{igt}^{jh} is the outmigration probability from sector j in location i to sector h in location g between time t and $t+1$, and we have defined E_{git}^{hj} as the corresponding inmigration probability to sector h in location g from sector j in location i between time t and $t+1$. Note that the order of subscripts switches between the outmigration probability (D_{igt}^{jh}) and the inmigration probability (E_{git}^{hj}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (S.4.127) in the value function (S.4.125), the expected value from living in location n at time t can be written as:

$$v_{it}^{j,w} = \ln\left[\frac{b_{nt}^{j} w_{nt}^{j}}{p_{nt}}\right] + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{K} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w}\right) / \kappa_{git}^{hj}\right)^{1/\rho}. \tag{S.4.151}$$

S.4.4.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

Prices. Using the relationship between capital and labor payments (S.4.144), the pricing rule (S.4.133) can be re-written as follows:

$$p_{nit}^{j} = \frac{\tau_{nit}^{j} w_{it}^{j} \left(\frac{1-\mu^j}{\mu^j}\right)^{1-\mu^j} \left(\frac{1}{\chi_{it}^{j}}\right)^{1-\mu^j}}{z_{it}^{j}}, \tag{S.4.152}$$

where χ_{it}^{j} is the capital-labor ratio in sector j in region i:

$$\chi_{it}^{j} \equiv \frac{k_{it}^{j}}{\ell_{it}^{j}}.$$

Totally differentiating this pricing rule, we have:

$$\mathrm{d}\ln p_{nit}^{j} = \mathrm{d}\ln \tau_{nit}^{j} + \mathrm{d}\ln w_{it}^{j} - \left(1-\mu^j\right) \mathrm{d}\ln \chi_{it}^{j} - \mathrm{d}\ln z_{it}^{j}. \tag{S.4.153}$$

Expenditure Shares. Totally differentiating the expenditure share equation (S.4.131), we get:

$$\mathrm{d}\ln S_{nit}^{j} = \theta \left(\sum_{h=1}^{N} S_{nht}^{j} \,\mathrm{d}\ln p_{nht}^{j} - \mathrm{d}\ln p_{nit}^{j}\right). \tag{S.4.154}$$

Price Indices. Totally differentiating the industry consumption goods price index in equation (S.4.128), we have:

$$\mathrm{d}\ln p_{nt}^{j} = \sum_{m=1}^{N} S_{nmt}^{j} \,\mathrm{d}\ln p_{nmt}^{j}. \tag{S.4.155}$$

Migration Shares. Totally differentiating the outmigration share equation (S.4.126), we get:

$$\mathrm{d}\ln D_{igt}^{jh} = \frac{1}{\rho}\left[\left(\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1}^{h,w} - \mathrm{d}\ln\kappa_{git}^{hj}\right) - \sum_{m=1}^{N}\sum_{o=1}^{J} D_{imt}^{jo}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln\kappa_{mit}^{oj}\right)\right]. \quad \text{(S.4.156)}$$

Real Income. Totally differentiating real income we have:

$$\mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \mathrm{d}\ln p_{it},$$

$$\mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J}\psi^{h}\sum_{m=1}^{N} S_{imt}^{h}\,\mathrm{d}\ln p_{imt}^{h},$$

$$\mathrm{d}\ln\left(\frac{w_{it}^{j}}{p_{it}}\right) = \mathrm{d}\ln w_{it}^{j} - \sum_{h=1}^{J}\psi^{h}\sum_{m=1}^{N} S_{imt}^{h}\left[\mathrm{d}\ln\tau_{imt}^{h} + \mathrm{d}\ln w_{mt}^{h} - \left(1-\mu^{h}\right)\mathrm{d}\ln\chi_{mt}^{h} - \mathrm{d}\ln z_{mt}^{h}\right]. \quad \text{(S.4.157)}$$

Goods Market Clearing. Totally differentiating the goods market clearing condition (S.4.143), we have:

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}} + \frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}} = \sum_{n=1}^{N}\sum_{h=1}^{J}\frac{\psi^{j} S_{nit}^{j}\left(1/\mu^{h}\right) w_{nt}^{h}\ell_{nt}^{h}}{\left(1/\mu^{j}\right) w_{it}^{j}\ell_{it}^{j}}\left(\frac{\mathrm{d}w_{nt}^{h}}{w_{nt}^{h}} + \frac{\mathrm{d}\ell_{nt}^{h}}{\ell_{nt}^{h}} + \frac{\mathrm{d}S_{nit}^{j}}{S_{nit}^{j}}\right).$$

Using our result for the derivative of expenditure shares in equation (S.4.154) above, we can rewrite this as:

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}} + \frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}} = \sum_{n=1}^{N}\sum_{h=1}^{J} T_{int}^{jh}\left(\frac{\mathrm{d}w_{nt}^{h}}{w_{nt}^{h}} + \frac{\mathrm{d}\ell_{nt}^{h}}{\ell_{nt}^{h}} + \theta\left(\sum_{m=1}^{N} S_{nmt}^{j}\frac{\mathrm{d}p_{nmt}^{j}}{p_{nmt}^{j}} - \frac{\mathrm{d}p_{nit}^{j}}{p_{nit}^{j}}\right)\right),$$

$$T_{int}^{jh} \equiv \frac{\psi^{j} S_{nit}^{j}\left(1/\mu^{h}\right) w_{nt}^{h}\ell_{nt}^{h}}{\left(1/\mu^{j}\right) w_{it}^{j}\ell_{it}^{j}}.$$

$$\begin{bmatrix} \mathrm{d}\ln w_{it}^{j} \\ +\mathrm{d}\ln\ell_{it}^{j} \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N}\sum_{h=1}^{J} T_{int}^{jh}\left(\mathrm{d}\ln w_{nt}^{h} + \mathrm{d}\ln\ell_{nt}^{h}\right) \\ +\theta\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{h=1}^{J} T_{int}^{jh} S_{nmt}^{j}\left(\mathrm{d}\ln\tau_{nmt}^{j} + \mathrm{d}\ln w_{mt}^{j} - \left(1-\mu^{j}\right)\mathrm{d}\ln\chi_{mt}^{j} - \mathrm{d}\ln z_{mt}^{j}\right) \\ -\theta\left(\mathrm{d}\ln\tau_{nit}^{j} + \mathrm{d}\ln w_{it}^{j} - \left(1-\mu^{j}\right)\mathrm{d}\ln\chi_{it}^{j} - \mathrm{d}\ln z_{it}^{j}\right) \end{bmatrix}. \quad \text{(S.4.158)}$$

Population Flow. Totally differentiating the population flow condition (S.4.149) we have:

$$\frac{\mathrm{d}\ell_{gt+1}^{h}}{\ell_{gt+1}^{h}} = \sum_{i=1}^{N}\sum_{j=1}^{J} E_{git}^{hj}\left[\frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}} + \frac{\mathrm{d}D_{igt}^{jh}}{D_{igt}^{jh}}\right],$$

$$\mathrm{d}\ln\ell_{gt+1}^{h} = \sum_{i=1}^{N}\sum_{j=1}^{J} E_{git}^{hj}\left[\mathrm{d}\ln\ell_{it}^{j} + \frac{1}{\rho}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1}^{h} - \mathrm{d}\ln\kappa_{gi}^{hj} - \sum_{m=1}^{N}\sum_{o=1}^{J} D_{imt}^{jo}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{o} - \mathrm{d}\ln\kappa_{mit}^{oj}\right)\right)\right]. \quad \text{(S.4.159)}$$

Value Function. Note that the value function can be re-written using the following results:

$$v_{it}^{j,w} = \ln \frac{w_{it}^{j}}{\prod_{o=1}^{J}\left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-\psi^o/\theta}} + \ln b_{it}^{j} + \rho \ln \sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho},$$

$$\prod_{o=1}^{J}\left[\sum_{m=1}^{N}\left(p_{imt}^{o}\right)^{-\theta}\right]^{-\psi^o/\theta} = \prod_{o=1}^{J}\left(\frac{\left(p_{iit}^{o}\right)^{-\theta}}{S_{iit}^{o}}\right)^{-\psi^o/\theta}, \qquad \tau_{iit}^{o} = 1,$$

$$\sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{it+1}^{j,w}\right)/\kappa_{iit}^{jj}\right)^{1/\rho}}{D_{iit}^{jj}}, \qquad \kappa_{iit}^{jj} = 1,$$

$$v_{it}^{j,w} = \ln w_{it}^{j} + \sum_{o=1}^{J}\psi^o\left[-\frac{1}{\theta}\ln S_{iit}^{o} - \ln p_{iit}^{o}\right] + \ln b_{it}^{j} + \beta \mathbb{E}_t v_{it+1}^{j,w} - \rho \ln D_{iit}^{jj}. \tag{S.4.160}$$

Totally differentiating the value function (S.4.160) we have:

$$\mathrm{d}v_{it}^{j,w} = \mathrm{d}\ln w_{it}^{j} + \sum_{o=1}^{J}\psi^o\left[-\frac{1}{\theta}\mathrm{d}\ln S_{iit}^{o} - \mathrm{d}\ln p_{iit}^{o}\right] + \mathrm{d}\ln b_{it}^{j} + \beta \mathbb{E}_t\, \mathrm{d}v_{it+1}^{j,w} - \rho\, \mathrm{d}\ln D_{iit}^{jj},$$

$$\mathrm{d}\ln S_{iit}^{o} = -\theta\, \mathrm{d}\ln p_{iit}^{o} + \theta\left[\sum_{m=1}^{N} S_{imt}^{o}\, \mathrm{d}\ln p_{imt}^{o}\right],$$

$$\mathrm{d}\ln D_{iit}^{jj} = \frac{1}{\rho}\left[\beta \mathbb{E}_t\, \mathrm{d}v_{it+1}^{j} - \mathrm{d}\ln \kappa_{iit}^{jj} - \sum_{m=1}^{N}\sum_{h=1}^{J} D_{imt}^{jh}\left(\beta \mathbb{E}_t v_{mt+1}^{h} - \mathrm{d}\ln \kappa_{mit}^{hj}\right)\right].$$

Using these results in the derivative of the value function, we have:

$$\mathrm{d}v_{it}^{j,w} = \left[\begin{array}{c} \mathrm{d}\ln w_{it}^{j} - \sum_{o=1}^{J}\psi^o \sum_{m=1}^{N} S_{imt}^{o}\, \mathrm{d}\ln p_{imt}^{o} \\ + \mathrm{d}\ln b_{it}^{j} + \sum_{m=1}^{N}\sum_{h=1}^{J} D_{imt}^{jh}\left(\beta \mathbb{E}_t\, \mathrm{d}v_{mt+1}^{h} - \mathrm{d}\ln \kappa_{mit}^{hj}\right) \end{array}\right],$$

where we have used $\mathrm{d}\ln \kappa_{iit}^{jj} = 0$. Using the total derivative of the pricing rule (S.4.153), we can re-write this derivative of the value function as follows:

$$\mathrm{d}v_{it}^{j,w} = \left[\begin{array}{c} \mathrm{d}\ln w_{it}^{j} - \sum_{o=1}^{J}\psi^o \sum_{m=1}^{N} S_{imt}^{o}\left(\mathrm{d}\ln \tau_{nmt}^{o} + \mathrm{d}\ln w_{mt}^{o} - (1-\mu^o)\, \mathrm{d}\ln \chi_{mt}^{o} - \mathrm{d}\ln z_{mt}^{o}\right) \\ + \mathrm{d}\ln b_{it}^{j} + \sum_{m=1}^{N}\sum_{h=1}^{J} D_{imt}^{jh}\left(\beta \mathbb{E}_t\, \mathrm{d}v_{mt+1}^{h} - \mathrm{d}\ln \kappa_{mit}^{hj}\right) \end{array}\right]. \tag{S.4.161}$$

S.4.4.8 Steady-state

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1}^{j} = k_{it}^{j} = k_{i}^{j*}$, $\ell_{it+1}^{j} = \ell_{it}^{j} = \ell_{i}^{j*}$, $w_{it+1}^{j*} = w_{it}^{j*} = w_{i}^{j*}$ and $v_{it+1}^{j*} = v_{it}^{j*} = v_{i}^{j*}$, where we use an asterisk to denote a steady-state value. We consider small common shocks to productivities across all sectors ($\mathrm{d}\ln \boldsymbol{z}$) and to amenities across all sectors ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \overline{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$).

Capital Accumulation. From the capital accumulation equation (S.4.145), the steady-state stock of building capital solves:

$$k_i^{j*} = \beta \left[\frac{r_i^{j*}}{p_i^*} + \left(1 - \delta^j\right) \right] k_i^{j*}.$$

$$\left(1 - \beta \left(1 - \delta^j\right)\right) k_i^{j*} = \beta \frac{r_i^j}{p_i} k_i^{j*}.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}^j}{p_{it}} k_{it}^j = \frac{1 - \mu^j}{\mu^j} \frac{w_{it}^j \ell_{it}^j}{p_{it}}.$$

Using this result in the expression for the steady-state capital stock above, we have:

$$\left(1 - \beta \left(1 - \delta^j\right)\right) k_i^{j*} = \beta \frac{1 - \mu^j}{\mu^j} \frac{w_i^{j*} \ell_i^{j*}}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d} \ln \chi_i^{j*} = \mathrm{d} \ln \left(\frac{w_i^{j*}}{p_i^{j*}} \right).$$

From the total derivative of real income (S.4.157) above, this becomes:

$$\mathrm{d} \ln \chi_i^{j*} = \mathrm{d} \ln w_{it}^{j*} - \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S_{imt}^h \left[\mathrm{d} \ln w_{mt}^{h*} - \left(1 - \mu^h\right) \mathrm{d} \ln \chi_{mt}^{h*} - \mathrm{d} \ln z_{mt} \right].$$

which has the matrix representation:

$$\mathrm{d} \ln \boldsymbol{\chi}^* = \mathrm{d} \ln \boldsymbol{w}^* - \boldsymbol{S} \left(\mathrm{d} \ln \boldsymbol{w}^* - (\boldsymbol{I} - \boldsymbol{\mu}) \, \mathrm{d} \ln \boldsymbol{\chi}^* - \mathrm{d} \ln \boldsymbol{z} \right), \tag{S.4.162}$$

where $\mathrm{d} \ln \boldsymbol{\chi}^*$ and $\mathrm{d} \ln \boldsymbol{w}^*$ are $NJ \times 1$ vectors; $\boldsymbol{S}$ is a $NJ \times NJ$ matrix with elements:

$$S_{nit} = S_{nit}^j = \sum_{h=1}^{J} \psi^h S_{imt}^h.$$

and $\boldsymbol{\mu}$ is $NJ \times NJ$ diagonal matrix whose (ij)-th element on the diagonal is μ^j.

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.158) has the following matrix representation:

$$\mathrm{d} \ln \boldsymbol{w_t} + \mathrm{d} \ln \boldsymbol{\ell_t} = \boldsymbol{T} \left(\mathrm{d} \ln \boldsymbol{w_t} + \mathrm{d} \ln \boldsymbol{\ell_t} \right) + \theta \left(\boldsymbol{TS} - \boldsymbol{I} \right) \left(\mathrm{d} \ln \boldsymbol{w} - (\boldsymbol{I} - \boldsymbol{\mu}) \, \mathrm{d} \ln \boldsymbol{\chi_t} - \mathrm{d} \ln \boldsymbol{z} \right),$$

where these matrices have $NJ \times NJ$ elements. In steady-state we have:

$$\mathrm{d} \ln \boldsymbol{w}^* + \mathrm{d} \ln \boldsymbol{\ell}^* = \boldsymbol{T} \left(\mathrm{d} \ln \boldsymbol{w}^* + \mathrm{d} \ln \boldsymbol{\ell}^* \right) + \theta \left(\boldsymbol{TS} - \boldsymbol{I} \right) \left(\mathrm{d} \ln \boldsymbol{w}^* - (\boldsymbol{I} - \boldsymbol{\mu}) \, \mathrm{d} \ln \boldsymbol{\chi}^* - \mathrm{d} \ln \boldsymbol{z} \right). \tag{S.4.163}$$

Population Flow. The total derivative of the population flow condition (S.4.159) has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\,\mathrm{d}\boldsymbol{v}^*. \tag{S.4.164}$$

Value function. The total derivative of the value function has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w_t} + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi_t}\right) + \,\mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D}\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta\boldsymbol{D}\right)^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + \left(\boldsymbol{I} - \boldsymbol{\mu}\right)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \,\mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.165}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\,\mathrm{d}\ln \boldsymbol{\chi}^* + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z}. \tag{S.4.166}$$

$$\mathrm{d}\ln \boldsymbol{w}^* + \,\mathrm{d}\ln \boldsymbol{\ell}^* = \boldsymbol{T}\left(\,\mathrm{d}\ln \boldsymbol{w}^* + \,\mathrm{d}\ln \boldsymbol{\ell}^*\right) + \theta\left(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}\right)\left(\,\mathrm{d}\ln \boldsymbol{w}^* - \left(\boldsymbol{I} - \boldsymbol{\mu}\right)\,\mathrm{d}\ln \boldsymbol{\chi}^* - \,\mathrm{d}\ln \boldsymbol{z}\right). \tag{S.4.167}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\,\mathrm{d}\boldsymbol{v}^*. \tag{S.4.168}$$

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta\boldsymbol{D}\right)^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + \left(\boldsymbol{I} - \boldsymbol{\mu}\right)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \,\mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.169}$$

S.4.4.9 Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity ($\,\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\,\mathrm{d}\ln \boldsymbol{b}$) in each sector and location, holding constant the economy's aggregate labor endowment ($\,\mathrm{d}\ln \overline{\ell} = 0$), trade costs ($\,\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\,\mathrm{d}\ln \boldsymbol{\kappa} = 0$). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\widetilde{\ell}_{it} = \ell_{it} - \ell_i^*$, for all variables except for the worker value function v_{it}, where with a slight abuse of notation we use $\widetilde{v}_{it} = v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (S.4.145), we have:

$$k_{it+1}^j = \beta \frac{r_{it}^j}{p_{it}} k_{it}^j + \beta\left(1 - \delta^j\right) k_{it}^j.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}^j}{p_{it}} k_{it}^j = \frac{1-\mu^j}{\mu^j}\frac{w_{it}^j \ell_{it}^j}{p_{it}}.$$

Using this result in the capital accumulation equation above, we have:

$$k^j_{it+1} = \beta \left(1 - \delta^j\right) k^j_{it} + \beta \frac{1-\mu^j}{\mu^j} \frac{w^j_{it} \ell^j_{it}}{p_{it}},$$

$$\frac{k^j_{it+1}}{\ell^j_{it+1}} \frac{\ell^j_{it+1}}{\ell^j_{it}} = \beta \left(1 - \delta^j\right) \frac{k^j_{it}}{\ell^j_{it}} + \beta \frac{1-\mu^j}{\mu^j} \frac{w^j_{it}}{p_{it}},$$

$$\chi^j_{it+1} \frac{\ell^j_{it+1}}{\ell^j_{it}} = \beta \left(1 - \delta^j\right) \chi^j_{it} + \beta \frac{1-\mu^j}{\mu^j} \frac{w^j_{it}}{p_{it}}, \tag{S.4.170}$$

while in steady-state we have:

$$\frac{k^{j*}_i}{\ell^{j*}_i} = \beta \left(1 - \delta^j\right) \frac{k^{j*}_i}{\ell^{j*}_i} + \beta \frac{1-\mu^j}{\mu^j} \frac{w^{j*}_i}{p^*_i},$$

$$\chi^{j*}_i = \beta \left(1 - \delta^j\right) \chi^{j*}_i + \beta \frac{1-\mu^j}{\mu^j} \frac{w^{j*}_i}{p^*_i}.$$

$$\chi^{j*}_i = \frac{\beta}{\left(1 - \beta \left(1 - \delta^j\right)\right)} \frac{1-\mu^j}{\mu^j} \frac{w^{j*}_i}{p^*_i}. \tag{S.4.171}$$

Dividing both sides of equation (S.4.170) by χ^{j*}_i, we have:

$$\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}} = \beta \left(1 - \delta^j\right) \frac{\chi^j_{it}}{\chi^{j*}_i} + \frac{\beta}{\chi^{j*}_i} \frac{1-\mu^j}{\mu^j} \frac{w^j_{it}}{p^j_{it}},$$

which using (S.4.171) can be re-written as:

$$\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}} = \beta \left(1 - \delta^j\right) \frac{\chi^j_{it}}{\chi^{j*}_i} + \left(1 - \beta \left(1 - \delta^j\right)\right) \frac{w^j_{it}/w^{j*}_i}{p_{it}/p^*_i},$$

which can be further re-written as:

$$\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}} - 1 = \beta \left(1 - \delta^j\right) \frac{\chi^j_{it}}{\chi^{j*}_i} + \left(1 - \beta \left(1 - \delta^j\right)\right) \frac{w^j_{it}/w^{j*}_i}{p_{it}/p^*_i} - 1,$$

$$\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}} - 1 = \beta \left(1 - \delta^j\right) \left(\frac{\chi^j_{it}}{\chi^{j*}_i} - 1\right) + \left(1 - \beta \left(1 - \delta^j\right)\right) \left(\frac{w^j_{it}/w^{j*}_i}{p_{it}/p^*_i} - 1\right).$$

Noting that:

$$\frac{x_{it}}{x^*_i} - 1 \simeq \ln \left(\frac{x_{it}}{x^*_i}\right),$$

$$\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}} - 1 \simeq \ln \left(\frac{\chi^j_{it+1}}{\chi^{j*}_i} \frac{\ell^j_{it+1}}{\ell^j_{it}}\right),$$

we have:

$$\ln\left(\frac{\chi^j_{it+1}}{\chi^{j*}_i}\right)+\ln\left(\frac{\ell^j_{it+1}}{\ell^j_{it}}\right)=\beta\left(1-\delta^j\right)\ln\left(\frac{\chi^j_{it}}{\chi^{j*}_i}\right)+\left(1-\beta\left(1-\delta^j\right)\right)\ln\left(\frac{w^j_{it}/w^{j*}_i}{p_{it}/p^*_i}\right),$$

$$\ln\left(\frac{\chi^j_{it+1}}{\chi^{j*}_i}\right)+\ln\left(\frac{\ell^j_{it+1}/\ell^{j*}_i}{\ell^j_{it}/\ell^{j*}_i}\right)=\beta\left(1-\delta^j\right)\ln\left(\frac{\chi^j_{it}}{\chi^{j*}_i}\right)+\left(1-\beta\left(1-\delta^j\right)\right)\ln\left(\frac{w^j_{it}/w^{j*}_i}{p_{it}/p^*_i}\right),$$

which can be re-written as follows:

$$\widetilde{\chi}^j_{it+1}=\beta\left(1-\delta^j\right)\widetilde{\chi}^j_{it}+\left(1-\beta\left(1-\delta^j\right)\right)\left(\widetilde{w}^j_{it}-\widetilde{p}_{it}\right)-\widetilde{\ell}^j_{it+1}+\widetilde{\ell}^j_{it}.$$

We can rewrite this relationship in matrix form as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}}=\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}+\left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{p}}_{\boldsymbol{t}}\right)-\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}},$$

where these matrices have $NJ\times NJ$ elements. Now, from the total derivative of real income (S.4.157), we have :

$$\widetilde{w}^j_{it}-\widetilde{p}_{it}=\widetilde{w}^j_{it}-\sum_{m=1}^{N}S^j_{nmt}\left[\widetilde{w}^j_{mt}-\left(1-\mu^j\right)\widetilde{\chi}^j_{mt}+\widetilde{z}^j_m\right],$$

where we have used $\mathrm{d}\ln\tau^j_{im}=0$. We can re-write this relationship in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{p}}_{\boldsymbol{t}}=\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\boldsymbol{S}\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}+\boldsymbol{S}\widetilde{\boldsymbol{z}}.$$

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}}=\left[\begin{array}{c}\left[\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)+\left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\boldsymbol{S}\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\\+\left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}\\\left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}}-\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\end{array}\right].\qquad\text{(S.4.172)}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.158) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}=\boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}+\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right)+\theta\left(\boldsymbol{TS}-\boldsymbol{I}\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}-\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}-\widetilde{\boldsymbol{z}}\right),$$

where these matrices have $NJ\times NJ$ elements and we have used $\mathrm{d}\ln\boldsymbol{\tau}=0$. This expression can be re-written as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}}=\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]^{-1}\left[-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}+\widetilde{\boldsymbol{z}}\right]\right].\qquad\text{(S.4.173)}$$

Population Flow. The total derivative of the population flow condition (S.4.159) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}}=\boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}+\frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}},\qquad\text{(S.4.174)}$$

where again these matrices have $NJ\times NJ$ elements.

Value function. The total derivative of the value function relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = (\boldsymbol{I} - \boldsymbol{S})\,\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[(\boldsymbol{I} - \boldsymbol{\mu})\,\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta \boldsymbol{D}\mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{S.4.175}$$

where again these matrices have $NJ \times NJ$ elements and we have used $\mathrm{d}\ln \boldsymbol{\tau} = 0$ and $\mathrm{d}\ln \kappa = 0$.

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\,(\boldsymbol{I} - \boldsymbol{\delta})\,\boldsymbol{I} + (\boldsymbol{I} - \beta\,(\boldsymbol{I} - \boldsymbol{\delta}))\,\boldsymbol{S}\right]\left[(\boldsymbol{I} - \boldsymbol{\mu})\,\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] \\ + (\boldsymbol{I} - \beta\,(\boldsymbol{I} - \boldsymbol{\delta}))\,(\boldsymbol{I} - \boldsymbol{S})\,\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array} \right]. \tag{S.4.176}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\,(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S})\right]^{-1}\left[-\,(\boldsymbol{I} - \boldsymbol{T})\,\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\,(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S})\left[(\boldsymbol{I} - \boldsymbol{\mu})\,\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.177}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\,(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D})\,\mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.178}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = (\boldsymbol{I} - \boldsymbol{S})\,\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[(\boldsymbol{I} - \boldsymbol{\mu})\,\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta \boldsymbol{D}\mathbb{E}_t \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.179}$$

S.4.5 Input-Output Linkages (Sector-Location Specific Capital)

We consider an economy that consists of many locations indexed by $i \in \{1, \ldots, N\}$ and many sectors indexed by $j \in \{1, \ldots, J\}$. Time is discrete and is indexed by t. The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across sectors and locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate δ.

S.4.5.1 Worker Migration Decisions

At the beginning of each period t, the economy inherits a mass of workers in each sector j and location i (ℓ_{it}^{j}), with the total labor endowment of the economy given by $\overline{\ell} = \sum_{i=1}^{N}\sum_{j=1}^{J}\ell_{it}$. Workers first produce and consume in their sector and location in period t, before observing mobility shocks $\{\epsilon_{gt}^{h}\}$ for all possible sectors $h \in \{1, \ldots, J\}$ and locations $g \in \{1, \ldots, N\}$ and deciding where to move for period $t+1$. Workers face bilateral migration costs that vary by sector and location, where κ_{git}^{hj} denotes the cost of moving from sector j in location i to sector h in location g. The value function for a worker in sector j and location i at time t ($\mathbb{V}_{it}^{j,w}$) is equal to the current flow of utility in that sector and location plus the expected continuation value next period from the optimal choice of sector and location:

$$\mathbb{V}_{it}^{j,w} = \ln u_{it}^{j,w} + \max_{\{g\}_1^N \{h\}_1^J} \left\{\beta \mathbb{E}_t\left[\mathbb{V}_{gt+1}^{h,w}\right] - \kappa_{git}^{hj} + \rho \epsilon_{gt}^{h}\right\}, \tag{S.4.180}$$

where we use the superscript w to denote workers; we assume logarithmic flow utility ($\ln u_{it}^{j,w}$); β denotes the discount rate; $\mathbb{E}[\cdot]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; ρ captures the dispersion of idiosyncratic mobility shocks; and we assume $\kappa_{iit}^{jj} = 1$ and $\kappa_{git}^{hj} > 1$ for $g \neq i$ and $h \neq j$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution:

$$F(\epsilon) = e^{-e^{(-\epsilon-\bar{\gamma})}}, \tag{S.4.181}$$

where $\overline{\gamma}$ is the Euler-Mascheroni constant.

Under this assumption, the expected value for a worker of living in location i at time t ($v_{it}^{j,w}$) can be re-written in the following form:

$$v_{it}^{j,w} = \ln u_{it}^{j,w} + \rho \log \sum_{g=1}^{N} \sum_{h=1}^{K} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho}. \tag{S.4.182}$$

The corresponding probability of migrating from location-sector ij to location-sector gh satisfies a gravity equation:

$$D_{igt}^{jh} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho}}{\sum_{m=1}^{N} \sum_{o=1}^{J} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^{o,w} \right) / \kappa_{kit}^{oj} \right)^{1/\rho}}. \tag{S.4.183}$$

The population flow condition implies:

$$\ell_{gt+1}^{h} = \sum_{i=1}^{N} \sum_{j=1}^{J} D_{igt}^{jh} \ell_{it}^{j}. \tag{S.4.184}$$

We also define a corresponding *inmigration* probability E_{git}^{hj}, which captures the share of workers in destination g and sector h at time $t+1$ that inmigrated from origin i and sector j at time t:

$$E_{git}^{hj} \equiv \frac{\ell_{it}^{j} D_{igt}^{jh}}{\ell_{gt+1}^{h}}. \tag{S.4.185}$$

Note that the order of subscripts switches between the outmigration probability and the inmigration probability, because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

S.4.5.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade. As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in their location and sector that period. Worker flow indirect utility in location n and sector j depends on local amenities (b_{nt}^{j}), the wage (w_{nt}^{j}), and the consumption goods price index (p_{nt}):

$$\ln u_{nt}^{j,w} = \ln b_{nt}^{j} + \ln w_{nt}^{j} - \ln p_{nt}, \tag{S.4.186}$$

where amenities (b_{nt}) capture characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g., climate and scenic views). In this section of the Online Supplement, we assume that amenities are exogenous.

The consumption goods price index (p_{nt}) in location n depends on the consumption goods price index for each sector h in that location (p_{nt}^h):

$$p_{nt} = \prod_{h=1}^{J} \left(p_{nt}^h\right)^{\psi^h}, \qquad 0 < \psi^h < 1, \qquad \sum_{h=1}^{J} \psi^h, \tag{S.4.187}$$

where the consumption goods price index for each sector h in location n depends on the price of the variety sourced from each location i within that sector h (p_{nit}^h):

$$p_{nt}^h = \left[\sum_{i=1}^{N} \left(p_{nit}^h\right)^{-\theta}\right]^{-1/\theta}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{S.4.188}$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties; $\theta = \sigma - 1$ is the trade elasticity; and for simplicity, we assume a common elasticity of substitution and trade elasticity across all sectors.

Utility maximization implies that goods consumption expenditure on each sector ($p_{nt}^h c_{nt}^h$) is a constant share of overall goods consumption expenditure ($p_{nt} c_{nt}$) in each location:

$$p_{nt}^h c_{nt}^h = \psi^h p_{nt} c_{nt}. \tag{S.4.189}$$

Using constant elasticity of substitution (CES) demand for individual varieties of goods, the share of location n's expenditure within sector h on the goods produced by location i is:

$$S_{nit}^h \equiv \frac{\left(p_{nit}^h\right)^{-\theta}}{\sum_{m=1}^{N} \left(p_{nmt}^h\right)^{-\theta}}. \tag{S.4.190}$$

S.4.5.3 Production

Producers in each location i and sector j use labor, capital and intermediate inputs to produce the variety supplied by that location in that sector. Production is assumed to occur under conditions of perfect competition and subject to the following unit cost function:

$$\mathbb{C}_{it}^j = \left[\left(\frac{w_{it}^j}{z_{it}^j}\right)^{\mu^j} (r_{it})^{1-\mu^j}\right]^{\gamma^j} \prod_{h=1}^{J} \left(p_{it}^h\right)^{\gamma^{j,h}}, \qquad \sum_{h=1}^{J} \gamma^{j,h} = 1 - \gamma^j, \tag{S.4.191}$$

where $(1 - \gamma^j)$ is the share of intermediates in production costs; $\gamma^{j,h}$ is the share of materials from sector h used in sector j; z_{it}^j denotes labor-augmenting productivity in location i in sector j at time t. As for amenities above, we assume in this section of the Online Supplement that productivity is exogenous.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit}^j \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n, where $\tau_{nit}^j > 1$ for $n \neq i$ and $\tau_{iit}^j = 1$. From profit maximization, the cost to a consumer in location n of sourcing the good produced by location i within sector j is:

$$p_{nit}^j = \tau_{nit}^j p_{iit}^j = \tau_{nit}^j \left[\left(\frac{w_{it}^j}{z_{it}^j}\right)^{\mu^j} (r_{it})^{1-\mu^j}\right]^{\gamma^j} \prod_{h=1}^{J} \left(p_{it}^h\right)^{\gamma^{j,h}}, \tag{S.4.192}$$

where p_{iit}^j is the "free on board" price of the good supplied by location i before transport costs.

From profit maximization problem and zero profits, payments for labor and capital in each sector are constant shares of revenue in that sector:

$$w_{it}^j \ell_{it}^j = \gamma^j \mu^j y_{it}^j, \tag{S.4.193}$$

$$r_{it}^j k_{it}^j = \gamma^j \left(1 - \mu^j\right) y_{it}^j, \tag{S.4.194}$$

where ℓ_{it}^j is labor input; k_{it}^j is capital input; and y_{it}^j denotes revenue; the immobility of capital across sectors and locations once installed implies that the rate of return on capital need not be equalized across sectors and locations out of steady-state ($r_{it}^j \neq r_{nt}^h$).

S.4.5.4 Landlord Consumption

Landlords in each location and sector choose their consumption and investment in capital to maximize their intertemporal utility subject to the intertemporal budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^{j,k} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k}, \tag{S.4.195}$$

where we use the superscript k to denote landlords; $c_{it}^{j,k}$ is the consumption index for landlords in location i and sector j; and β denotes the discount rate. Since landlords are immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

The consumption goods index for landlords ($c_{it}^{j,k}$) takes exactly the same form as for workers and is a Cobb-Douglas aggregate of consumption indexes for each sector, where these consumption indexes for each sector are constant elasticity of substitution (CES) functions of the consumption of varieties from each location. Therefore, the consumption goods price index (p_{nt}) takes the same form as in equation (S.4.187), and the consumption goods price index for each sector (p_{nt}^j) takes the same form as in equation (S.4.188). Under these assumptions, landlords' utility maximization problem is weakly separable. First, we solve for the optimal consumption-savings decision across time periods for overall goods consumption. Second, we solve for the optimal allocation of consumption across sectors within each time period. Third, we solve for the optimal allocation of consumption across location varieties within each sector.

Beginning with landlords' optimal consumption-saving decision, we assume that the investment technology for capital in each location and sector uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location and sector can produce one unit of capital in that location and sector using one unit of the consumption index for that location and sector. We assume that capital is geographically immobile once installed and depreciates at a constant rate δ^j. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r_{it}^j k_{it}^j$) equals the total value of goods consumption ($p_{it} c_{it}^{j,k}$) and net investment ($p_{it}\left(k_{it+1}^j - \left(1 - \delta^j\right) k_{it}^j\right)$):

$$r_{it}^j k_{it}^j = p_{it} c_{it}^{j,k} + p_{it}\left(k_{it+1}^j - \left(1 - \delta^j\right) k_{it}^j\right). \tag{S.4.196}$$

Combining landlords' intertemporal utility (S.4.195) and budget constraint (S.4.196), their intertemporal optimization problem is:

$$\max_{\left\{c_t^{j,k},k_{t+1}^{j,k}\right\}} \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k}, \tag{S.4.197}$$

$$\text{subject to} \qquad p_{it} c_{it}^{j,k} + p_{it}\left(k_{it+1}^j - \left(1-\delta^j\right) k_{it}^j\right) = r_{it}^j k_{it}^j.$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^{j,k} - \xi_t^j \left[p_{it} c_{it}^{j,k} + p_{it}\left(k_{it+1}^j - \left(1-\delta^j\right) k_{it}^j\right) - r_{it}^j k_{it}^j\right]. \tag{S.4.198}$$

The first-order conditions are:

$$\left\{c_{it}^{j,k}\right\} \qquad \frac{\beta^t}{c_{it}^{j,k}} - p_{it}\xi_t^j = 0,$$

$$\left\{k_{it+1}^j\right\} \qquad \left(r_{it+1}^j + p_{it+1}\left(1-\delta^j\right)\right)\xi_{t+1}^j - p_{it}\xi_t^j = 0,$$

Together these first-order conditions imply:

$$\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} = \beta \frac{p_{it}\mu_t^j}{p_{it+1}\mu_{t+1}^j} = \beta\left(r_{it+1}^j / p_{it+1} + \left(1-\delta^j\right)\right), \tag{S.4.199}$$

where the transversality condition implies:

$$\lim_{t\to\infty} \beta^t \frac{k_{it+1}^j}{c_{it}^{j,k}} = 0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords' optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it} c_{it}^{j,k} = (1-\beta)\left(r_{it}^j + p_{it}\left(1-\delta^j\right)\right) k_{it}^j, \tag{S.4.200}$$

$$k_{it+1}^j = \beta\left(r_{it}^j / p_{it} + \left(1-\delta^j\right)\right) k_{it}^j. \tag{S.4.201}$$

Substituting the consumption policy function (S.4.200) into the Euler equation (S.4.199), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\begin{aligned}\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}} &= \frac{\left(r_{it+1}^j / p_{it+1} + (1-\delta^j)\right) k_{it+1}^j}{\left(r_{it}^j / p_{it} + (1-\delta^j)\right) k_{it}^j}, \\ &= \beta\left(r_{it+1}^j / p_{it+1} + \left(1-\delta^j\right)\right).\end{aligned}$$

Given this optimal consumption-saving decision in equations (S.4.200)-(S.4.201), our assumption of Cobb-Douglas preferences across sectors implies that landlords allocate constant shares of consumption expenditure across sectors within time periods, as for workers in equation (S.4.189). Similarly, our assumption of constant elasticity of substitution (CES) preferences across locations within sectors implies that landlords in location n allocate the same share of expenditure on location i within sector j, as for workers in equation (S.4.190).

S.4.5.5 Goods Market Clearing

Goods market clearing implies that income in each location and sector equals expenditure on the goods produced in that location and sector:

$$y_{it}^{j} = \sum_{n=1}^{N} S_{nit}^{j} x_{n}^{j}, \tag{S.4.202}$$

where y_{it}^{j} is total income in sector j in location i and x_{n}^{j} is total expenditure on industry j in region n at time t. Total expenditure is the sum of final consumption and intermediate goods expenditure and is given by:

$$x_{nt}^{j} = \psi^{j} \sum_{h=1}^{J} \left(w_{nt}^{h} \ell_{nt}^{h} + r_{nt}^{h} k_{nt}^{h} \right) + \sum_{h=1}^{J} \gamma^{h,j} y_{nt}^{h}.$$

Combining these two relationships, we have:

$$y_{it}^{j} = \sum_{n=1}^{N} S_{nit}^{j} \left[\psi^{j} \sum_{h=1}^{J} \left(w_{nt}^{h} \ell_{nt}^{h} + r_{nt}^{h} k_{nt}^{h} \right) + \sum_{h=1}^{J} \gamma^{h,j} y_{nt}^{h} \right],$$

which can be re-written as:

$$\begin{aligned} y_{it}^{j} &= \sum_{n=1}^{N} S_{nit}^{j} \left[\psi^{j} \sum_{h=1}^{J} \gamma^{h} y_{nt}^{h} + \sum_{h=1}^{J} \gamma^{h,j} y_{nt}^{h} \right], \qquad \text{(S.4.203)} \\ &= \sum_{n=1}^{N} \sum_{h=1}^{J} S_{nit}^{j} \left[\psi^{j} \gamma^{h} + \gamma^{h,j} \right] y_{nt}^{h}. \end{aligned}$$

S.4.5.6 Capital Market Clearing

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using the property that payments to capital and labor are constant shares of total revenue in equations (S.4.193) and (S.4.194), we can write payments for capital in each sector as:

$$r_{it}^{j} k_{it}^{j} = \frac{1 - \mu^{j}}{\mu^{j}} w_{it}^{j} \ell_{it}^{j}. \tag{S.4.204}$$

S.4.5.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

Industry Price Indices. Totally differentiating the industry consumption goods price index in equation (S.4.187), we have:

$$\frac{\mathrm{d}p_{nt}^{j}}{p_{nt}^{j}} = \sum_{m=1}^{N} S_{nmt}^{j} \frac{\mathrm{d}p_{nmt}^{j}}{p_{nmt}^{j}},$$

$$\mathrm{d}\ln p_{nt}^{j} = \sum_{m=1}^{N} S_{nmt}^{j} \,\mathrm{d}\ln p_{nmt}^{j}. \tag{S.4.205}$$

Prices. Using the relationship between capital and labor payments (S.4.204), the pricing rule (S.4.192) can be re-written as follows:

$$p_{nit}^{j} = \tau_{nit}^{j} \left(\frac{w_{it}^{j}}{z_{it}^{j}}\right)^{\gamma^{j}} \left(\frac{1-\mu^{j}}{\mu^{j}}\right)^{\left(1-\mu^{j}\right)\gamma^{j}} \left(\frac{1}{\chi_{it}^{j}}\right)^{\left(1-\mu^{j}\right)\gamma^{j}} \prod_{h=1}^{J} \left(p_{it}^{h}\right)^{\gamma^{j,h}}, \tag{S.4.206}$$

where χ_{it}^{j} is the capital-labor ratio in sector j in region i:

$$\chi_{it}^{j} \equiv \frac{k_{it}^{j}}{\ell_{it}^{j}}.$$

Totally differentiating this pricing rule, we have:

$$\mathrm{d}\ln p_{nit}^{j} = \left[\begin{array}{c} \mathrm{d}\ln \tau_{nit}^{j} + \gamma^{j}\,\mathrm{d}\ln w_{it}^{j} - \left(1-\mu^{j}\right)\gamma^{j}\,\mathrm{d}\ln \chi_{it}^{j} \\ -\gamma^{j}\,\mathrm{d}\ln z_{it}^{j} + \sum_{h=1}^{J} \gamma^{j,h}\,\mathrm{d}\ln p_{it}^{h} \end{array}\right]. \tag{S.4.207}$$

Combining the total derivatives of the the price index (S.4.205) and prices (S.4.209), we have:

$$\mathrm{d}\ln p_{nit}^{j} = \left[\begin{array}{c} \mathrm{d}\ln \tau_{nit}^{j} + \gamma^{j}\,\mathrm{d}\ln w_{it}^{j} - \left(1-\mu^{j}\right)\gamma^{j}\,\mathrm{d}\ln \chi_{it}^{j} \\ -\gamma^{j}\,\mathrm{d}\ln z_{it}^{j} + \sum_{h=1}^{J}\sum_{m=1}^{N} \gamma^{j,h} S_{imt}^{h}\,\mathrm{d}\ln p_{imt}^{h} \end{array}\right],$$

which can be re-written as:

$$\mathrm{d}\ln p_{nit}^{j} = \left[\begin{array}{c} \mathrm{d}\ln \tau_{nit}^{j} + \gamma^{j}\,\mathrm{d}\ln w_{it}^{j} - \left(1-\mu^{j}\right)\gamma^{j}\,\mathrm{d}\ln \chi_{it}^{j} \\ -\gamma^{j}\,\mathrm{d}\ln z_{it}^{j} + \sum_{h=1}^{J}\sum_{m=1}^{N} \Sigma_{im}^{jh}\,\mathrm{d}\ln p_{imt}^{h} \end{array}\right],$$

$$\text{where} \qquad \Sigma_{im}^{jh} \equiv \gamma^{j,h} S_{imt}^{h}.$$

Define $\boldsymbol{\Gamma} \equiv \left[\boldsymbol{I} - \boldsymbol{\Sigma}\right]^{-1}$ as the Leontief inverse of the shares Σ_{im}^{jh}, we can write this relationship as:

$$\mathrm{d}\ln p_{nit}^{j} = \sum_{m=1}^{N}\sum_{o=1}^{J} \Gamma_{im}^{jo}\left[\mathrm{d}\ln \tau_{nmt}^{o} + \gamma^{o}\,\mathrm{d}\ln w_{mt}^{o} - \left(1-\mu^{o}\right)\gamma^{o}\,\mathrm{d}\ln \chi_{mt}^{o} - \gamma^{o}\,\mathrm{d}\ln z_{mt}^{o}\right]. \tag{S.4.208}$$

Expenditure Shares. Totally differentiating the expenditure share equation (S.4.190), we get:

$$\mathrm{d}\ln S_{nit}^{j} = \theta\left(\sum_{h=1}^{N} S_{nht}^{j}\,\mathrm{d}\ln p_{nht}^{j} - \,\mathrm{d}\ln p_{nit}^{j}\right). \tag{S.4.209}$$

Using the total derivatives of prices above (S.4.208), this total derivative of the expenditure shares can be written as:

$$\mathrm{d}\ln S_{nit}^{j} = \theta\left[\sum_{h=1}^{N} S_{nht}^{j}\sum_{m=1}^{N}\sum_{o=1}^{J}\Gamma_{hm}^{jo} - \sum_{m=1}^{N}\sum_{o=1}^{J}\Gamma_{im}^{jo}\right]\left[\begin{array}{c} \mathrm{d}\ln \tau_{nmt}^{o} + \gamma^{o}\,\mathrm{d}\ln w_{mt}^{o} \\ -\left(1-\mu^{o}\right)\gamma^{o}\,\mathrm{d}\ln \chi_{mt}^{o} - \gamma^{o}\,\mathrm{d}\ln z_{mt}^{o} \end{array}\right]. \tag{S.4.210}$$

Migration Shares. Totally differentiating this expenditure share equation (S.4.183), we get:

$$\mathrm{d}\ln D_{igt}^{jh} = \frac{1}{\rho}\left[\left(\beta\mathbb{E}_t\,\mathrm{d}v_{gt+1}^{h,w} - \mathrm{d}\ln\kappa_{git}^{hj}\right) - \sum_{m=1}^{N}\sum_{o=1}^{J} D_{imt}^{jo}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{o,w} - \mathrm{d}\ln\kappa_{mit}^{oj}\right)\right]. \quad \text{(S.4.211)}$$

Labor Payments. Totally differentiating labor payments (S.4.193), we have:

$$w_{it}^{j}\ell_{it}^{j} = \gamma^{j}\mu^{j}y_{it}^{j},$$

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}}w_{it}^{j}\ell_{it}^{j} + \frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}}w_{it}^{j}\ell_{it}^{j} = \gamma^{j}\mu^{j}y_{it}^{j}\frac{\mathrm{d}y_{it}^{j}}{y_{it}^{j}},$$

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}} + \frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}} = \frac{\gamma^{j}\mu^{j}y_{it}^{j}}{w_{it}^{j}\ell_{it}^{j}}\frac{\mathrm{d}y_{it}^{j}}{y_{it}^{j}},$$

$$\frac{\mathrm{d}w_{it}^{j}}{w_{it}^{j}} + \frac{\mathrm{d}\ell_{it}^{j}}{\ell_{it}^{j}} = \xi_i^{j}\frac{\mathrm{d}y_{it}^{j}}{y_{it}^{j}},$$

$$\mathrm{d}\ln w_{it}^{j} + \mathrm{d}\ln\ell_{it}^{j} = \xi_i^{j}\,\mathrm{d}\ln y_{it}^{j}, \quad \text{(S.4.212)}$$

$$\xi_i^{j} \equiv \frac{\gamma^{j}\mu^{j}y_{it}^{j}}{w_{it}^{j}\ell_{it}^{j}}.$$

Goods Market Clearing. Totally differentiating the goods market clearing condition (S.4.203), we have:

$$y_{it}^{j} = \sum_{n=1}^{N}\sum_{h=1}^{J} S_{nit}^{j}\left[\psi^{j}\gamma^{h} + \gamma^{h,j}\right]y_{nt}^{h},$$

$$\frac{\mathrm{d}y_{it}^{j}}{y_{it}^{j}}y_{it}^{j} = \sum_{n=1}^{N}\sum_{h=1}^{J} S_{nit}^{j}\left[\psi^{j}\gamma^{h} + \gamma^{h,j}\right]y_{nt}^{h}\frac{\mathrm{d}S_{nit}^{j}}{S_{nit}^{j}} + \sum_{n=1}^{N}\sum_{h=1}^{J} S_{nit}^{j}\left[\psi^{j}\gamma^{h} + \gamma^{h,j}\right]y_{nt}^{h}\frac{\mathrm{d}y_{nt}^{h}}{y_{nt}^{h}},$$

which can be re-written as:

$$\mathrm{d}\ln y_{it}^{j} = \left[\begin{array}{c} \sum_{n=1}^{N}\vartheta_{in}^{j}\,\mathrm{d}\ln S_{nit}^{j} + \sum_{n=1}^{N}\sum_{h=1}^{J}\Theta_{in}^{jh}\,\mathrm{d}\ln S_{nit}^{j} \\ +\sum_{n=1}^{N}\vartheta_{in}^{j}\,\mathrm{d}\ln y_{nt}^{h} + \sum_{n=1}^{N}\sum_{h=1}^{J}\Theta_{in}^{jh}\,\mathrm{d}\ln y_{nt}^{h} \end{array}\right],$$

$$\text{where} \qquad \vartheta_{in}^{j} \equiv \frac{S_{nit}^{j}\psi^{j}\sum_{h=1}^{J}\gamma^{h}y_{nt}^{h}}{y_{it}^{j}}, \qquad \Theta_{in}^{jh} \equiv \frac{S_{nit}^{j}\gamma^{h,j}y_{nt}^{h}}{y_{it}^{j}}.$$

Using equation (S.4.212), we can re-write this relationship as:

$$\mathrm{d}\ln y_{it}^{j} = \left[\begin{array}{c} \sum_{n=1}^{N}\vartheta_{in}^{j}\,\mathrm{d}\ln S_{nit}^{j} + \sum_{n=1}^{N}\sum_{h=1}^{J}\Theta_{in}^{jh}\,\mathrm{d}\ln S_{nit}^{j} \\ +\sum_{n=1}^{N}\sum_{h=1}^{J}\frac{S_{nit}^{j}\psi^{j}\gamma^{h}y_{nt}^{h}}{y_{it}^{j}}\frac{1}{\xi_i^{h}}\left(\mathrm{d}\ln w_{nt}^{h} + \mathrm{d}\ln\ell_{nt}^{h}\right) + \sum_{n=1}^{N}\sum_{h=1}^{J}\Theta_{in}^{jh}\,\mathrm{d}\ln y_{nt}^{h} \end{array}\right].$$

Using the definition of $\xi_i^j \equiv \frac{\gamma^j \mu^j y_{it}^j}{w_{it}^j \ell_{it}^j}$, we have:

$$\mathrm{d}\ln y_{it}^j = \left[\begin{array}{c} \sum_{n=1}^N \vartheta_{in}^j \,\mathrm{d}\ln S_{nit}^j + \sum_{n=1}^N \sum_{h=1}^J \Theta_{in}^{jh} \,\mathrm{d}\ln S_{nit}^j \\ + \sum_{n=1}^N \sum_{h=1}^J \frac{S_{nit}^j \psi^j \gamma^h y_{nt}^h}{y_{it}^j} \frac{w_{it}^h \ell_{it}^h}{\gamma^h \mu^h y_{it}^h} \left(\mathrm{d}\ln w_{nt}^h + \mathrm{d}\ln \ell_{nt}^h \right) + \sum_{n=1}^N \sum_{h=1}^J \Theta_{in}^{jh} \,\mathrm{d}\ln y_{nt}^h \end{array} \right].$$

Using $w_{it}^j \ell_{it}^j = \gamma^j \mu^j y_{it}^j$, we can re-write this further as:

$$\mathrm{d}\ln y_{it}^j = \left[\begin{array}{c} \sum_{n=1}^N \vartheta_{in}^j \,\mathrm{d}\ln S_{nit}^j + \sum_{n=1}^N \sum_{h=1}^J \Theta_{in}^{jh} \,\mathrm{d}\ln S_{nit}^j \\ + \sum_{n=1}^N \sum_{h=1}^J \vartheta_{in}^{jh} \left(\mathrm{d}\ln w_{nt}^h + \mathrm{d}\ln \ell_{nt}^h \right) + \sum_{n=1}^N \sum_{h=1}^J \Theta_{in}^{jh} \,\mathrm{d}\ln y_{nt}^h \end{array} \right],$$

$$\text{where} \qquad \vartheta_{in}^{jh} \equiv \frac{S_{nit}^j \psi^j \gamma^h y_{nt}^h}{y_{it}^j}.$$

$$\mathrm{d}\ln y_{it}^j \left[1 - \sum_{n=1}^N \sum_{h=1}^J \Theta_{in}^{jh} \,\mathrm{d}\ln y_{nt}^h \right] = \left[\begin{array}{c} \sum_{n=1}^N \sum_{h=1}^J \vartheta_{in}^{jh} \left(\mathrm{d}\ln w_{nt}^h + \mathrm{d}\ln \ell_{nt}^h \right) \\ + \sum_{n=1}^N \left[\vartheta_{in}^j + \sum_{h=1}^J \Theta_{in}^{jh} \right] \mathrm{d}\ln S_{nit}^j \end{array} \right].$$

Taking the Leontief inverse of Θ_{in}^{jh}, we have:

$$\mathrm{d}\ln y_{it}^j = \sum_{m=1}^N \sum_{o=1}^J \Delta_{im}^{jo} \left[\begin{array}{c} \sum_{n=1}^N \sum_{h=1}^J \vartheta_{mn}^{oh} \left(\mathrm{d}\ln w_{nt}^h + \mathrm{d}\ln \ell_{nt}^h \right) \\ + \sum_{n=1}^N \left[\vartheta_{mn}^o + \sum_{h=1}^J \Theta_{mn}^{oh} \right] \mathrm{d}\ln S_{nmt}^o \end{array} \right].$$

Using equation (S.4.212), this becomes:

$$\mathrm{d}\ln w_{it}^j + \mathrm{d}\ln \ell_{it}^j = \xi_i^j \sum_{m=1}^N \sum_{o=1}^J \Delta_{im}^{jo} \left[\begin{array}{c} \sum_{n=1}^N \sum_{h=1}^J \vartheta_{mn}^{oh} \left(\mathrm{d}\ln w_{nt}^h + \mathrm{d}\ln \ell_{nt}^h \right) \\ + \sum_{n=1}^N \left[\vartheta_{mn}^o + \sum_{h=1}^J \Theta_{mn}^{oh} \right] \mathrm{d}\ln S_{nmt}^o \end{array} \right]. \tag{S.4.213}$$

Population Flow. Totally differentiating the population flow condition (S.4.184) we have:

$$\mathrm{d}\ln \ell_{gt+1}^h = \sum_{i=1}^N \sum_{j=1}^J E_{git}^{hj} \left[\mathrm{d}\ln \ell_{it}^j + \mathrm{d}\ln D_{igt}^{jh} \right],$$

$$\mathrm{d}\ln \ell_{gt+1}^h = \sum_{i=1}^N \sum_{j=1}^J E_{git}^{hj} \left[\mathrm{d}\ln \ell_{it}^j + \frac{1}{\rho} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{gt+1}^h - \mathrm{d}\ln \kappa_{gi}^{hj} - \sum_{m=1}^N \sum_{o=1}^J D_{imt}^{jo} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1}^o - \mathrm{d}\ln \kappa_{mit}^{oj} \right) \right) \right]. \tag{S.4.214}$$

Value Function. Note that the value function can be re-written using the following results:

$$v_{it}^{j,w} = \ln \frac{w_{it}^j}{\prod_{o=1}^J \left[\sum_{m=1}^N p_{imt}^{-\theta} \right]^{-\psi^o/\theta}} + \ln b_{it}^j + \rho \ln \sum_{g=1}^N \sum_{h=1}^J \left(\exp \left(\beta \mathbb{E}_t v_{gt+1}^{h,w} \right) / \kappa_{git}^{hj} \right)^{1/\rho},$$

$$\prod_{o=1}^J \left[\sum_{m=1}^N \left(p_{imt}^o \right)^{-\theta} \right]^{-\psi^o/\theta} = \prod_{o=1}^J \left(\frac{\left(p_{iit}^o \right)^{-\theta}}{S_{iit}^o} \right)^{-\psi^o/\theta}, \qquad \tau_{iit}^o = 1,$$

$$\sum_{g=1}^{N}\sum_{h=1}^{J}\left(\exp\left(\beta\mathbb{E}_t v_{gt+1}^{h,w}\right)/\kappa_{git}^{hj}\right)^{1/\rho}=\frac{\left(\exp\left(\beta\mathbb{E}_t v_{it+1}^{j,w}\right)/\kappa_{iit}^{jj}\right)^{1/\rho}}{D_{iit}^{jj}},\qquad \kappa_{iit}^{jj}=1,$$

$$v_{it}^{j,w}=\ln w_{it}^{j}+\sum_{o=1}^{J}\psi^{o}\left[-\frac{1}{\theta}\ln S_{iit}^{o}-\ln p_{iit}^{o}\right]+\ln b_{it}^{j}+\beta\mathbb{E}_t v_{it+1}^{j,w}-\rho\ln D_{iit}^{jj}. \tag{S.4.215}$$

Totally differentiating the value function (S.4.215) we have:

$$\mathrm{d}v_{it}^{j,w}=\mathrm{d}\ln w_{it}^{j}+\sum_{o=1}^{J}\psi^{o}\left[-\frac{1}{\theta}\,\mathrm{d}\ln S_{iit}^{o}-\mathrm{d}\ln p_{iit}^{o}\right]+\mathrm{d}\ln b_{it}^{j}+\beta\mathbb{E}_t\,\mathrm{d}v_{it+1}^{j,w}-\rho\,\mathrm{d}\ln D_{iit}^{jj},$$

$$\mathrm{d}\ln S_{iit}^{o}=-\theta\,\mathrm{d}\ln p_{iit}^{o}+\theta\left[\sum_{m=1}^{N}S_{imt}^{o}\,\mathrm{d}\ln p_{imt}^{o}\right],$$

$$\mathrm{d}\ln D_{iit}^{jj}=\frac{1}{\rho}\left[\beta\mathbb{E}_t\,\mathrm{d}v_{it+1}^{j}-\mathrm{d}\ln\kappa_{iit}^{jj}-\sum_{m=1}^{N}\sum_{h=1}^{J}D_{imt}^{jh}\left(\beta\mathbb{E}_t v_{mt+1}^{h}-\mathrm{d}\ln\kappa_{mit}^{hj}\right)\right].$$

Using these results in the derivative of the value function, we have:

$$\mathrm{d}v_{it}^{j,w}=\left[\begin{array}{c}\mathrm{d}\ln w_{it}^{j}-\sum_{o=1}^{J}\psi^{o}\sum_{m=1}^{N}S_{imt}^{o}\,\mathrm{d}\ln p_{imt}^{o}\\+\mathrm{d}\ln b_{it}^{j}+\sum_{m=1}^{N}\sum_{h=1}^{J}D_{imt}^{jh}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{h}-\mathrm{d}\ln\kappa_{mit}^{hj}\right)\end{array}\right],$$

where we have used $\mathrm{d}\ln\kappa_{iit}^{jj}=0$. From the total derivative of prices in equation (S.4.208), we have:

$$\mathrm{d}\ln p_{imt}^{o}=\sum_{n=1}^{N}\sum_{h=1}^{J}\Gamma_{mz}^{oh}\left[\mathrm{d}\ln\tau_{int}^{h}+\gamma^{h}\,\mathrm{d}\ln w_{nt}^{h}-\left(1-\mu^{h}\right)\gamma^{h}\,\mathrm{d}\ln\chi_{nt}^{h}-\gamma^{h}\,\mathrm{d}\ln z_{nt}^{h}\right].$$

Using this result in the value function above, we obtain:

$$\mathrm{d}v_{it}^{j,w}=\left[\begin{array}{c}\mathrm{d}\ln w_{it}^{j}-\sum_{o=1}^{J}\sum_{m=1}^{N}\psi^{o}S_{imt}^{o}\sum_{n=1}^{N}\sum_{h=1}^{J}\Gamma_{mz}^{oh}\left[\begin{array}{c}\mathrm{d}\ln\tau_{int}^{h}+\gamma^{h}\,\mathrm{d}\ln w_{nt}^{h}\\-\left(1-\mu^{h}\right)\gamma^{h}\,\mathrm{d}\ln\chi_{nt}^{h}-\gamma^{h}\,\mathrm{d}\ln z_{nt}^{h}\end{array}\right]\\+\mathrm{d}\ln b_{it}^{j}+\sum_{m=1}^{N}\sum_{h=1}^{J}D_{imt}^{jh}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1}^{h}-\mathrm{d}\ln\kappa_{mit}^{hj}\right)\end{array}\right]. \tag{S.4.216}$$

S.4.5.8 Steady-state

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1}^{j}=k_{it}^{j}=k_{i}^{j*}$, $\ell_{it+1}^{j}=\ell_{it}^{j}=\ell_{i}^{j*}$, $w_{it+1}^{j*}=w_{it}^{j*}=w_{i}^{j*}$ and $v_{it+1}^{j*}=v_{it}^{j*}=v_{i}^{j*}$, where we use an asterisk to denote a steady-state value. We consider small common shocks to productivities across all sectors ($\mathrm{d}\ln\boldsymbol{z}$) and to amenities across all sectors ($\mathrm{d}\ln\boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln\overline{\ell}=0$), trade costs ($\mathrm{d}\ln\boldsymbol{\tau}=0$) and commuting costs ($\mathrm{d}\ln\boldsymbol{\kappa}=0$).

Capital Accumulation. From the capital accumulation equation (S.4.201), the steady-state stock of building capital solves:

$$k_i^{j*} = \beta \left[\frac{r_i^j}{p_i} + \left(1 - \delta^j\right) \right] k_i^{j*}.$$

$$\left(1 - \beta \left(1 - \delta^j\right)\right) k_i^{j*} = \beta \frac{r_i^j}{p_i} k_i^{j*}.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}^j}{p_{it}} k_{it}^j = \frac{1 - \mu^j}{\mu^j} \frac{w_{it}^j \ell_{it}^j}{p_{it}},$$

Using this result in the expression for the steady-state capital stock above, we have:

$$\left(1 - \beta \left(1 - \delta^j\right)\right) k_i^{j*} = \beta \frac{1 - \mu^j}{\mu^j} \frac{w_i^{j*} \ell_i^{j*}}{p_i^*}. \tag{S.4.217}$$

We now derive an expression for the total derivative of real income:

$$\mathrm{d} \ln \left(\frac{w_i^{j*}}{p_i^*} \right) = \mathrm{d} \ln w_i^{j*} - \mathrm{d} \ln p_i^*.$$

The total derivative of the aggregate price index is given by:

$$\mathrm{d} \ln p_i^* = \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S_{im}^{h*} \, \mathrm{d} \ln p_{im}^{h*}.$$

Using our expression for the total derivative of prices above (S.4.208), we can re-write this total derivative of the aggregate price index as:

$$\mathrm{d} \ln p_i^* = \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S_{im}^{h*} \sum_{n=1}^{N} \sum_{o=1}^{J} \Gamma_{mn}^{ho} \left[\gamma^o \, \mathrm{d} \ln w_{nt}^{o*} - \left(1 - \mu^o\right) \gamma^o \, \mathrm{d} \ln \chi_{nt}^{o*} - \gamma^o \, \mathrm{d} \ln z_{nt}^o \right].$$

where have used $\mathrm{d} \ln \tau_{nit}^j = 0$. Using these results in equation (S.4.217), the change the steady-state capital labor ratio is given by:

$$\mathrm{d} \ln \chi_i^{j*} = \mathrm{d} \ln w_{it}^{j*} - \sum_{m=1}^{N} \sum_{h=1}^{J} \psi^h S_{im}^{h*} \sum_{n=1}^{N} \sum_{o=1}^{J} \Gamma_{mn}^{ho} \left[\gamma^o \, \mathrm{d} \ln w_{nt}^{o*} - \left(1 - \mu^o\right) \gamma^o \, \mathrm{d} \ln \chi_{nt}^{o*} - \gamma^o \, \mathrm{d} \ln z_{nt}^o \right],$$

which has the matrix representation:

$$\mathrm{d} \ln \boldsymbol{\chi}^* = \mathrm{d} \ln \boldsymbol{w}^* - \boldsymbol{S} \left(\mathrm{d} \ln \boldsymbol{w}^* - \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d} \ln \boldsymbol{\chi}^* - \mathrm{d} \ln \boldsymbol{z} \right), \tag{S.4.218}$$

where $\mathrm{d} \ln \boldsymbol{\chi}^*$ and $\mathrm{d} \ln \boldsymbol{w}^*$ are $NJ \times NJ$ matrices; $\boldsymbol{\lambda}$ is a $NJ \times NJ$ diagonal matrix whose (ij)-th element on the diagonal is λ^j; and $\boldsymbol{S}$ is a $NJ \times NJ$ matrix with elements:

$$S_{nit}^j = \sum_{n=1}^{N} \sum_{o=1}^{J} \sum_{m=1}^{N} \sum_{h=1}^{J} \Gamma_{mn}^{ho} \gamma^o.$$

Goods Market Clearing. Recall that the total derivative of the expenditure share in equation (S.4.210) is:

$$\mathrm{d}\ln S_{nmt}^{j}=\theta\left[\sum_{h=1}^{N}S_{nht}^{j}\sum_{g=1}^{N}\sum_{o=1}^{J}\Gamma_{hg}^{jo}-\sum_{g=1}^{N}\sum_{o=1}^{J}\Gamma_{mg}^{jo}\right]\left[\gamma^{o}\,\mathrm{d}\ln w_{gt}^{o}-\left(1-\mu^{o}\right)\gamma^{o}\,\mathrm{d}\ln\chi_{gt}^{o}-\gamma^{o}\,\mathrm{d}\ln z_{gt}^{o}\right],$$

where we have used $\mathrm{d}\ln\tau_{nit}^{j}=0$. We can re-write this total derivative of the expenditure share as:

$$\mathrm{d}\ln S_{nmt}^{j}=\theta\left[\sum_{h=1}^{N}S_{nht}^{j}\sum_{g=1}^{N}\sum_{o=1}^{J}\Lambda_{hg}^{jo}-\sum_{g=1}^{N}\sum_{o=1}^{J}\Lambda_{mg}^{jo}\right]\left[\mathrm{d}\ln w_{gt}^{o}-\left(1-\mu^{o}\right)\,\mathrm{d}\ln\chi_{gt}^{o}-\,\mathrm{d}\ln z_{gt}^{o}\right],$$

$$\text{where}\qquad \Lambda_{hg}^{jo}\equiv\gamma^{o}\Gamma_{hg}^{jo}.$$

Recall that the total derivative of the goods market clearing condition is:

$$\mathrm{d}\ln w_{it}^{j}+\,\mathrm{d}\ln\ell_{it}^{j}=\xi_{i}^{j}\sum_{m=1}^{N}\sum_{o=1}^{J}\Delta_{im}^{jo}\left[\begin{array}{c}\sum_{n=1}^{N}\sum_{h=1}^{J}\vartheta_{mn}^{oh}\left(\,\mathrm{d}\ln w_{nt}^{h}+\,\mathrm{d}\ln\ell_{nt}^{h}\right)\\ +\sum_{n=1}^{N}\left[\vartheta_{mn}^{o}+\sum_{h=1}^{J}\Theta_{mn}^{oh}\right]\,\mathrm{d}\ln S_{nmt}^{o}\end{array}\right].$$

Using this expression for the total derivative of the expenditure share in the total derivative of the goods market clearing condition in equation (S.4.213), we obtain:

$$\mathrm{d}\ln w_{it}^{j}+\,\mathrm{d}\ln\ell_{it}^{j}=\left[\begin{array}{c}\xi_{i}^{j}\sum_{m=1}^{N}\sum_{o=1}^{J}\Delta_{im}^{jo}\sum_{n=1}^{N}\sum_{h=1}^{J}\vartheta_{mn}^{oh}\left(\,\mathrm{d}\ln w_{nt}^{h}+\,\mathrm{d}\ln\ell_{nt}^{h}\right)\\ +\left\{\begin{array}{c}\xi_{i}^{j}\sum_{m=1}^{N}\sum_{o=1}^{J}\Delta_{im}^{jo}\theta\sum_{n=1}^{N}\left[\vartheta_{mn}^{o}+\sum_{h=1}^{J}\Theta_{mn}^{oh}\right]\\ \times\sum_{g=1}^{N}\left[\sum_{h=1}^{N}S_{nht}^{j}\sum_{o=1}^{J}\Lambda_{hg}^{jo}-\sum_{o=1}^{J}\Lambda_{mg}^{jo}\right]\\ \times\left[\begin{array}{c}\mathrm{d}\ln w_{gt}^{o}-\left(1-\mu^{o}\right)\,\mathrm{d}\ln\chi_{gt}^{o}\\ -\,\mathrm{d}\ln z_{gt}^{o}\end{array}\right]\end{array}\right\}\end{array}\right].$$

To simplify notation, we define $\Pi_{im}^{jo}\equiv\xi_{i}^{j}\Delta_{im}^{jo}$ as the network-adjusted share of income in sector j in location i derived from selling to sector o in location m. We also define $\Upsilon_{nmg}^{j}\equiv\sum_{h=1}^{N}S_{nht}^{j}\sum_{o=1}^{J}\Lambda_{hg}^{jo}-\sum_{o=1}^{J}\Lambda_{mg}^{jo}$ as the elasticity of location n's expenditure in sector j on goods from location i with respect to the price of goods in that sector from location m. Using this notation, we can re-write the above goods market clearing condition as:

$$\mathrm{d}\ln w_{it}^{j}+\,\mathrm{d}\ln\ell_{it}^{j}=\left[\begin{array}{c}\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{o=1}^{J}\sum_{h=1}^{J}\Pi_{im}^{jo}\vartheta_{mn}^{oh}\left(\,\mathrm{d}\ln w_{nt}^{h}+\,\mathrm{d}\ln\ell_{nt}^{h}\right)\\ +\left\{\begin{array}{c}\theta\sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{o=1}^{J}\sum_{g=1}^{N}\Pi_{im}^{jo}\left[\vartheta_{mn}^{o}+\sum_{h=1}^{J}\Theta_{mn}^{oh}\right]\\ \times\Upsilon_{nmg}^{j}\left[\,\mathrm{d}\ln w_{gt}^{o}-\left(1-\mu^{o}\right)\,\mathrm{d}\ln\chi_{gt}^{o}-\,\mathrm{d}\ln z_{gt}^{o}\right]\end{array}\right\}\end{array}\right].$$

We can write this goods market clearing condition in matrix form as:

$$\mathrm{d}\ln\boldsymbol{w_t}+\,\mathrm{d}\ln\boldsymbol{\ell_t}=\boldsymbol{T}\left(\,\mathrm{d}\ln\boldsymbol{w_t}+\,\mathrm{d}\ln\boldsymbol{\ell_t}\right)+\theta\boldsymbol{M}\left(\,\mathrm{d}\ln\boldsymbol{w_t}-\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\,\mathrm{d}\ln\boldsymbol{\chi_t}-\,\mathrm{d}\ln\boldsymbol{z}\right),$$

where these matrices have $NJ\times NJ$ elements. In particular, $\boldsymbol{T}$ is a $NJ\times NJ$ matrix with elements:

$$T_{in}=\sum_{m=1}^{N}\sum_{o=1}^{J}\sum_{h=1}^{J}\Pi_{im}^{jo}\vartheta_{mn}^{oh},$$

and $\boldsymbol{M}$ is a $NJ \times NJ$ matrix with elements:

$$M_{in} = \sum_{m=1}^{N} \sum_{o=1}^{J} \sum_{g=1}^{N} \Pi_{im}^{jo} \left[\vartheta_{mn}^{o} + \sum_{h=1}^{J} \Theta_{mn}^{oh} \right] \Upsilon_{nmg}^{j}.$$

In steady-state we have:

$$\mathrm{d}\ln \boldsymbol{w}^{*} + \mathrm{d}\ln \boldsymbol{\ell}^{*} = \boldsymbol{T} \left(\mathrm{d}\ln \boldsymbol{w}^{*} + \mathrm{d}\ln \boldsymbol{\ell}^{*} \right) + \theta \boldsymbol{M} \left(\mathrm{d}\ln \boldsymbol{w}^{*} - \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d}\ln \boldsymbol{\chi}^{*} - \mathrm{d}\ln \boldsymbol{z} \right). \tag{S.4.219}$$

Population Flow. The total derivative of the population flow condition (S.4.214) has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{\ell}_{\boldsymbol{t+1}} = \boldsymbol{E}\, \mathrm{d}\ln \boldsymbol{\ell}_{\boldsymbol{t}} + \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} \right) \mathbb{E}_{t}\, \mathrm{d}\boldsymbol{v}_{\boldsymbol{t+1}},$$

where these matrices again have $NJ \times NJ$ elements. In steady-state, we have:

$$\mathrm{d}\ln \boldsymbol{\ell}^{*} = \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E} \right)^{-1} \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} \right) \mathrm{d}\boldsymbol{v}^{*}. \tag{S.4.220}$$

Value function. Recall from equation (S.4.216) that the total derivative of the value function is given by:

$$\mathrm{d}v_{it}^{j,w} = \left[\begin{array}{c} \mathrm{d}\ln w_{it}^{j} - \sum_{n=1}^{N} \sum_{h=1}^{J} \sum_{m=1}^{N} \sum_{o=1}^{J} \psi^{o} S_{imt}^{o} \Gamma_{mz}^{oh} \left[\begin{array}{c} \gamma^{h}\, \mathrm{d}\ln w_{nt}^{h} \\ - \left(1 - \mu^{h}\right) \gamma^{h}\, \mathrm{d}\ln \chi_{nt}^{h} - \gamma^{h}\, \mathrm{d}\ln z_{nt}^{h} \end{array} \right] \\ + \mathrm{d}\ln b_{it}^{j} + \sum_{n=1}^{N} \sum_{h=1}^{J} D_{int}^{jh} \beta \mathbb{E}_{t}\, \mathrm{d}v_{nt+1}^{h} \end{array} \right],$$

where we have used $\mathrm{d}\ln \tau_{nit}^{j} = 0$ and $\mathrm{d}\ln \kappa_{mit}^{hj} = 0$. This total derivative of the value function has the following matrix representation:

$$\mathrm{d}\boldsymbol{v}_{\boldsymbol{t}} = \mathrm{d}\ln \boldsymbol{w}_{\boldsymbol{t}} - \boldsymbol{S} \left[\mathrm{d}\ln \boldsymbol{w}_{\boldsymbol{t}} - \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d}\ln \boldsymbol{\chi}_{\boldsymbol{t}} - \mathrm{d}\ln \boldsymbol{z} \right] + \mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D} \mathbb{E}_{t}\, \mathrm{d}\boldsymbol{v}_{\boldsymbol{t+1}},$$

where these matrices again have $NJ \times NJ$ elements. Recall that the matrix $\boldsymbol{S}$ has elements S_{int}^{j} given by:

$$S_{int}^{j} = \sum_{h=1}^{J} \sum_{m=1}^{N} \sum_{o=1}^{J} \psi^{o} S_{im}^{o*} \Gamma_{mn}^{oh} \gamma^{h}.$$

The matrix $\boldsymbol{D}$ has elements D_{nit}^{j} given by:

$$D_{int}^{j} = \sum_{h=1}^{J} D_{int}^{jh}.$$

In steady-state, this total derivative of the value function becomes:

$$\mathrm{d}\boldsymbol{v}^{*} = \left(\boldsymbol{I} - \beta \boldsymbol{D} \right)^{-1} \left[\left(\boldsymbol{I} - \boldsymbol{S} \right) \mathrm{d}\ln \boldsymbol{w}^{*} + \boldsymbol{S} \left(\mathrm{d}\ln \boldsymbol{z} + \left(\boldsymbol{I} - \boldsymbol{\mu} \right) \mathrm{d}\ln \boldsymbol{\chi}^{*} \right) + \mathrm{d}\ln \boldsymbol{b} \right]. \tag{S.4.221}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = (\boldsymbol{I}-\boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}(\boldsymbol{I}-\boldsymbol{\mu})\,\mathrm{d}\ln \boldsymbol{\chi}^* + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z}. \tag{S.4.222}$$

$$\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^* = \boldsymbol{T}\left(\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^*\right) + \theta \boldsymbol{M}\left(\mathrm{d}\ln \boldsymbol{w}^* - (\boldsymbol{I}-\boldsymbol{\mu})\,\mathrm{d}\ln \boldsymbol{\chi}^* - \mathrm{d}\ln \boldsymbol{z}\right). \tag{S.4.223}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}(\boldsymbol{I}-\boldsymbol{E})^{-1}(\boldsymbol{I}-\boldsymbol{E}\boldsymbol{D})\,\mathrm{d}\boldsymbol{v}^*. \tag{S.4.224}$$

$$\mathrm{d}\boldsymbol{v}^* = (\boldsymbol{I}-\beta \boldsymbol{D})^{-1}\left[(\boldsymbol{I}-\boldsymbol{S})\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (\boldsymbol{I}-\boldsymbol{\mu})\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.225}$$

S.4.5.9 Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small common shock to productivity across sectors ($\mathrm{d}\ln \boldsymbol{z}$) and amenities across sectors ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\widetilde{\ell}_{it} = \ell_{it} - \ell_i^*$, for all variables except for the worker value function v_{it}, where with a slight abuse of notation we use $\widetilde{v}_{it} = v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (S.4.201), we have:

$$k_{it+1}^j = \beta \frac{r_{it}^j}{p_{it}} k_{it}^j + \beta\left(1-\delta^j\right) k_{it}^j.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}^j}{p_{it}} k_{it}^j = \frac{1-\mu^j}{\mu^j}\frac{w_{it}^j \ell_{it}^j}{p_{it}}.$$

Using this result in the capital accumulation equation above, we have:

$$k_{it+1}^j = \beta\left(1-\delta^j\right) k_{it} + \beta \frac{1-\mu^j}{\mu^j}\frac{w_{it}^j \ell_{it}^j}{p_{it}},$$

$$\frac{k_{it+1}^j}{\ell_{it+1}^j}\frac{\ell_{it+1}^j}{\ell_{it}^j} = \beta\left(1-\delta^j\right)\frac{k_{it}^j}{\ell_{it}^j} + \beta \frac{1-\mu^j}{\mu^j}\frac{w_{it}^j}{p_{it}},$$

$$\chi_{it+1}^j \frac{\ell_{it+1}^j}{\ell_{it}^j} = \beta\left(1-\delta^j\right)\chi_{it}^j + \beta \frac{1-\mu^j}{\mu^j}\frac{w_{it}^j}{p_{it}}, \tag{S.4.226}$$

while in steady-state we have:

$$\frac{k_i^{j*}}{\ell_i^{j*}} = \beta\left(1-\delta^j\right)\frac{k_i^{j*}}{\ell_i^{j*}} + \beta \frac{1-\mu^j}{\mu^j}\frac{w_i^{j*}}{p_i^*},$$

$$\chi_i^{j*} = \beta\left(1-\delta^j\right)\chi_i^{j*} + \beta \frac{1-\mu^j}{\mu^j}\frac{w_i^{j*}}{p_i^*}.$$

$$\chi_i^{j*} = \frac{\beta}{\left(1-\beta\left(1-\delta^j\right)\right)} \frac{1-\mu^j}{\mu^j} \frac{w_i^{j*}}{p_i^*}. \tag{S.4.227}$$

Dividing both sides of equation (S.4.226) by χ_i^{j*}, we have:

$$\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j} = \beta\left(1-\delta^j\right) \frac{\chi_{it}^j}{\chi_i^{j*}} + \frac{\beta}{\chi_i^{j*}} \frac{1-\mu^j}{\mu^j} \frac{w_{it}^j}{p_{it}^j},$$

which using (S.4.227) can be re-written as:

$$\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j} = \beta\left(1-\delta^j\right) \frac{\chi_{it}^j}{\chi_i^{j*}} + \left(1-\beta\left(1-\delta^j\right)\right) \frac{w_{it}^j / w_i^{j*}}{p_{it}/p_i^*},$$

which can be further re-written as:

$$\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j} - 1 = \beta\left(1-\delta^j\right) \frac{\chi_{it}^j}{\chi_i^{j*}} + \left(1-\beta\left(1-\delta^j\right)\right) \frac{w_{it}^j / w_i^{j*}}{p_{it}/p_i^*} - 1,$$

$$\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j} - 1 = \beta\left(1-\delta^j\right) \left(\frac{\chi_{it}^j}{\chi_i^{j*}} - 1\right) + \left(1-\beta\left(1-\delta^j\right)\right) \left(\frac{w_{it}^j / w_i^{j*}}{p_{it}/p_i^*} - 1\right).$$

Noting that:

$$\frac{x_{it}}{x_i^*} - 1 \simeq \ln\left(\frac{x_{it}}{x_i^*}\right),$$

$$\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j} - 1 \simeq \ln\left(\frac{\chi_{it+1}^j}{\chi_i^{j*}} \frac{\ell_{it+1}^j}{\ell_{it}^j}\right),$$

we have:

$$\ln\left(\frac{\chi_{it+1}^j}{\chi_i^{j*}}\right) + \ln\left(\frac{\ell_{it+1}^j}{\ell_{it}^j}\right) = \beta\left(1-\delta^j\right) \ln\left(\frac{\chi_{it}^j}{\chi_i^{j*}}\right) + \left(1-\beta\left(1-\delta^j\right)\right) \ln\left(\frac{w_{it}^j / w_i^{j*}}{p_{it}/p_i^*}\right),$$

$$\ln\left(\frac{\chi_{it+1}^j}{\chi_i^{j*}}\right) + \ln\left(\frac{\ell_{it+1}^j / \ell_i^{j*}}{\ell_{it}^j / \ell_i^{j*}}\right) = \beta\left(1-\delta^j\right) \ln\left(\frac{\chi_{it}^j}{\chi_i^{j*}}\right) + \left(1-\beta\left(1-\delta^j\right)\right) \ln\left(\frac{w_{it}^j / w_i^{j*}}{p_{it}/p_i^*}\right),$$

which can be re-written as follows:

$$\widetilde{\chi}_{it+1}^j = \beta\left(1-\delta^j\right) \widetilde{\chi}_{it}^j + \left(1-\beta\left(1-\delta^j\right)\right) \left(\widetilde{w}_{it}^j - \widetilde{p}_{it}\right) - \widetilde{\ell}_{it+1}^j + \widetilde{\ell}_{it}^j.$$

We can rewrite this relationship in matrix form as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right) \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right) \left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}}\right) - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}},$$

where these matrices have $NJ \times NJ$ elements. Following an analogous analysis as for steady-state above, the total derivative of real income relative to the initial steady-state can be written in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}} = \left(\boldsymbol{I}-\boldsymbol{S}\right) \widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left(\boldsymbol{I}-\boldsymbol{\mu}\right) \widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}},$$

where we have used $\mathrm{d}\ln\widetilde{\boldsymbol{\tau}} = 0$. Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right) + \left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\boldsymbol{S}\right]\left[\left(\boldsymbol{I}-\boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}}\right] \\ + \left(\boldsymbol{I}-\beta\left(\boldsymbol{I}-\boldsymbol{\delta}\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.4.228}$$

Goods Market Clearing. Following an analogous analysis as for steady-state above, the total derivative of the goods market clearing condition can be written in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} = \boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \theta \boldsymbol{M}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - (\boldsymbol{I} - \boldsymbol{\mu})\,\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{z}}\right),$$

where these matrices have $NJ \times NJ$ elements and we have used $\mathrm{d}\ln\widetilde{\boldsymbol{\tau}} = 0$. This expression can be re-written as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} - \theta\boldsymbol{M}\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} - \theta\boldsymbol{M}\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.229}$$

Population Flow. The total derivative of the population flow condition (S.4.214) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{S.4.230}$$

where again these matrices have $NJ \times NJ$ elements.

Value function. Following an analogous analysis as for steady-state above, the total derivative of the value function relative to the initial steady-state can be written in matrix form as:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{S.4.231}$$

where again these matrices have $NJ \times NJ$ elements and we have used $\mathrm{d}\ln\boldsymbol{\tau} = 0$ and $\mathrm{d}\ln\boldsymbol{\kappa} = 0$.

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(\boldsymbol{I} - \boldsymbol{\delta}\right)\boldsymbol{I} + \left(\boldsymbol{I} - \beta\left(\boldsymbol{I} - \boldsymbol{\delta}\right)\right)\boldsymbol{S}\right]\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] \\ + \left(\boldsymbol{I} - \beta\left(\boldsymbol{I} - \boldsymbol{\delta}\right)\right)\left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.4.232}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} - \theta\boldsymbol{M}\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} - \theta\boldsymbol{M}\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.233}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.234}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(\boldsymbol{I} - \boldsymbol{\mu}\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.235}$$

S.4.6 Trade Deficits

In this section of the Online Supplement, we consider an extension of our baseline model in Section 2 of the paper to allow for trade deficits. As the model does not generate predictions for how trade imbalances respond to shocks, we follow the standard approach in the quantitative international trade literature of treating these imbalances as exogenous. We apportion these trade deficits fully to worker income, assuming that expenditure equals income for landlords. In particular, we allow the ratio of per capita expenditure to per capita income (d_{nt}) for workers to differ exogenously across locations and over time. When workers choose whether to move to a location, they take into account not only the labor income in that region but also this exogenous ratio of expenditure to income, which corresponds to a transfer to workers by location.

S.4.6.1 General Equilibrium

Given the state variables $\{\ell_{i0},\ k_{i0}\}$ and a path for the ratio of expenditure to income $\{d_{it}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables. The conditions for general equilibrium take a similar form as in our baseline model in Section 2 of the paper.

Capital Accumulation: Using capital market clearing from equation (11) in the paper, the price index from equation in the paper (4) and the equilibrium pricing rule from equation (2) in the paper, and assuming logarithmic intertemporal utility for simplicity, the capital accumulation equation becomes:

$$k_{it+1} = \beta \frac{1-\mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta \left(1-\delta\right) k_{it}. \tag{S.4.236}$$

$$p_{nt} = \left[\sum_{i=1}^{N} \left(w_{it} \left(\frac{1-\mu}{\mu} \right)^{1-\mu} \left(\ell_{it}/k_{it}\right)^{1-\mu} \tau_{ni}/z_i \right)^{-\theta} \right]^{-1/\theta}. \tag{S.4.237}$$

Goods Market Clearing: Using the equilibrium pricing rule (2) in the paper, the expenditure share (13) in the paper and the relationship between factor payments from equation (11) in the paper, the goods market clearing condition with trade deficits can be written as:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} d_{nt} w_{nt} \ell_{nt}, \tag{S.4.238}$$

$$S_{nit} = \frac{\left(w_{it} \left(\ell_{it}/k_{it}\right)^{1-\mu} \tau_{ni}/z_i \right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt} \left(\ell_{mt}/k_{mt}\right)^{1-\mu} \tau_{nm}/z_m \right)^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},$$

where S_{nit} is the expenditure share of importer n on exporter i at time t, and we have defined T_{int} as the corresponding income share of exporter i from importer n at time t. Note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the out-migration probabilities, the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{S.4.239}$$

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v^w_{gt+1}\right) / \kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v^w_{mt+1}\right) / \kappa_{mit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}},$$

where D_{igt} is the out-migration probability from location i to location g between time t and $t+1$, and we have defined E_{git} as the corresponding in-migration probability to location g from location i between time t and $t+1$. Note that the order of subscripts switches between the out-migration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function from equation (4) in the paper in the value function from equation (7), the expected value from living in location n at time t can be written as:

$$v^w_{nt} = \ln b_{nt} + \ln\left(\frac{d_{nt} w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v^w_{gt+1}\right) / \kappa_{gnt}\right)^{1/\rho}. \tag{S.4.240}$$

S.4.6.2 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics. In the interests of brevity, we focus on relationships that differ from the baseline model in Section 2 of the paper.

Real Expenditure. Totally differentiating real expenditure we have:

$$\mathrm{d}\ln\left(\frac{d_{it} w_{it}}{p_{it}}\right) = \mathrm{d}\ln d_{it} + \mathrm{d}\ln w_{it} - \mathrm{d}\ln p_{it},$$

$$\mathrm{d}\ln\left(\frac{d_{it} w_{it}}{p_{it}}\right) = \left[\begin{array}{c} \mathrm{d}\ln d_{it} + \mathrm{d}\ln w_{it} \\ -\sum_{m=1}^{N} S_{nmt}\left[\mathrm{d}\ln \tau_{nmt} + \mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln \chi_{mt} - \mathrm{d}\ln z_{mt}\right] \end{array}\right]. \tag{S.4.241}$$

Goods Market Clearing Totally differentiating the goods market clearing condition in equation (12) in the paper, we have:

$$\frac{\mathrm{d}w_{it}}{w_{it}} + \frac{\mathrm{d}\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} \frac{S_{nit} d_{nt} w_{nt} \ell_{nt}}{w_{it} \ell_{it}} \left(\frac{\mathrm{d}d_{nt}}{d_{nt}} + \frac{\mathrm{d}w_{nt}}{w_{nt}} + \frac{\mathrm{d}\ell_{nt}}{\ell_{nt}} + \theta\left(\sum_{h=1}^{N} S_{nht} \frac{\mathrm{d}p_{nht}}{p_{nht}} - \frac{\mathrm{d}p_{nit}}{p_{nit}}\right)\right),$$

$$\frac{\mathrm{d}w_{it}}{w_{it}} + \frac{\mathrm{d}\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} T_{int} \left(\frac{\mathrm{d}d_{nt}}{d_{nt}} + \frac{\mathrm{d}w_{nt}}{w_{nt}} + \frac{\mathrm{d}\ell_{nt}}{\ell_{nt}} + \theta\left(\sum_{h \in N} S_{nht} \frac{\mathrm{d}p_{nht}}{p_{nht}} - \frac{\mathrm{d}p_{nit}}{p_{nit}}\right)\right),$$

$$T_{int} = \frac{S_{nit} d_{nt} w_{nt} \ell_{nt}}{w_{it} \ell_{it}},$$

$$\left[\begin{array}{c} \mathrm{d}\ln w_{it} \\ +\mathrm{d}\ln \ell_{it} \end{array}\right] = \left[\begin{array}{c} \sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln d_{nt} + \mathrm{d}\ln w_{nt} + \mathrm{d}\ln \ell_{nt}\right) \\ +\theta \sum_{n=1}^{N} \sum_{m=1}^{N} T_{int} S_{nmt}\left(\mathrm{d}\ln \tau_{nmt} + \mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln \chi_{mt} - \mathrm{d}\ln z_{mt}\right) \\ -\theta \sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln \tau_{nit} + \mathrm{d}\ln w_{it} - (1-\mu)\,\mathrm{d}\ln \chi_{it} - \mathrm{d}\ln z_{it}\right) \end{array}\right]. \tag{S.4.242}$$

Value Function. Note that the value function can be re-written using the following results:

$$v_{it} = \ln\left[\frac{d_{it}w_{it}}{\left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-1/\theta}}\right] + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho},$$

$$\left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-1/\theta} = \left(\frac{p_{iit}^{-\theta}}{S_{iit}}\right)^{-1/\theta}, \qquad \tau_{iit} = 1,$$

$$\sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{it+1}\right)/\kappa_{iit}\right)^{1/\rho}}{D_{iit}}, \qquad \kappa_{iit} = 1,$$

$$v_{it} = -\frac{1}{\theta}\ln S_{iit} + \ln d_{it} + \ln w_{it} - \ln p_{iit} + \ln b_{it} + \beta \mathbb{E}_t v_{it+1} - \rho \ln D_{iit}. \tag{S.4.243}$$

Totally differentiating this expression for the value function, we have:

$$\mathrm{d}v_{it} = -\frac{1}{\theta}\,\mathrm{d}\ln S_{iit} + \mathrm{d}\ln d_{it} + \mathrm{d}\ln w_{it} - \mathrm{d}\ln p_{iit} + \mathrm{d}\ln b_{it} + \beta \mathbb{E}_t\,\mathrm{d}v_{it+1} - \rho\,\mathrm{d}\ln D_{iit},$$

where

$$\mathrm{d}\ln S_{iit} = -\theta\,\mathrm{d}\ln p_{iit} + \theta\left[\sum_{m=1}^{N} S_{imt}\,\mathrm{d}\ln p_{imt}\right],$$

$$\mathrm{d}\ln D_{iit} = \frac{1}{\rho}\left[\beta \mathbb{E}_t\,\mathrm{d}v_{it+1} - \mathrm{d}\ln \kappa_{iit} - \sum_{m=1}^{N} D_{imt}\left(\beta \mathbb{E}_t v_{mt+1} - \mathrm{d}\ln \kappa_{mit}\right)\right].$$

Using these results for $\mathrm{d}\ln S_{iit}$ and $\mathrm{d}\ln D_{iit}$ in the expression for $\mathrm{d}v_{it}$ above, we have:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln d_{it} + \mathrm{d}\ln w_{it} - \sum_{m=1}^{N} S_{imt}\,\mathrm{d}\ln p_{imt} \\ +\,\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt}\left(\beta \mathbb{E}_t\,\mathrm{d}v_{mt+1} - \mathrm{d}\ln \kappa_{mit}\right) \end{array}\right],$$

where we have used $\mathrm{d}\ln \kappa_{iit} = 0$. Using the pricing rule, we can re-write this derivative of the value function as follows:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln d_{it} + \mathrm{d}\ln w_{it} - \sum_{m=1}^{N} S_{imt}\left(\mathrm{d}\ln \tau_{nmt} + \mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln \chi_{mt} - \mathrm{d}\ln z_{mt}\right) \\ +\,\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt}\left(\beta \mathbb{E}_t\,\mathrm{d}v_{mt+1} - \mathrm{d}\ln \kappa_{mit}\right) \end{array}\right]. \tag{S.4.244}$$

S.4.6.3 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant fundamentals $\{z_i, b_i, d_i, \tau_{ni}, \kappa_{ni}\}$ and constant values of the endogenous variables: $k_{it+1}^{j} = k_{it}^{j} = k_i^{j*}$, $\ell_{it+1}^{j} = \ell_{it}^{j} = \ell_i^{j*}$, $w_{it+1}^{j} = w_{it}^{j} = w_i^{j*}$ and $v_{it+1}^{j} = v_{it}^{j} = v_i^{j*}$, where we use an asterisk to denote a steady-state value. We consider a small common shock to productivity across sectors ($\mathrm{d}\ln \boldsymbol{z}$), amenities across sectors ($\mathrm{d}\ln \boldsymbol{b}$) and trade deficits across sectors ($\mathrm{d}\ln \boldsymbol{d}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell}$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau}$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$).

Capital Accumulation. From the capital accumulation equation in equation (11) in the paper, the steady-state stock of capital solves:

$$\left(1-\beta\left(1-\delta\right)\right)\chi_i^* = \left(1-\beta\left(1-\delta\right)\right)\frac{k_i^*}{\ell_i^*} = \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln\left(\frac{w_i^*}{p_i^*}\right).$$

Using the total derivative of real income, this becomes:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln w_i^* - \sum_{m=1}^{N} S_{im}^*\left[\mathrm{d}\ln w_m^* - \left(1-\mu\right)\mathrm{d}\ln\chi_m^* - \mathrm{d}\ln z_m\right],$$

where we have used and $\mathrm{d}\ln\tau_{nm} = 0$. This relationship has the matrix representation:

$$\mathrm{d}\ln\boldsymbol{\chi}^* = \mathrm{d}\ln\boldsymbol{w}^* - \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{w}^* + \left(1-\mu\right)\boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{\chi}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z},$$

$$\left(\boldsymbol{I} - \left(1-\mu\right)\boldsymbol{S}\right)\mathrm{d}\ln\boldsymbol{\chi}^* = \left(\boldsymbol{I}-\boldsymbol{S}\right)\mathrm{d}\ln\boldsymbol{w}^* + \boldsymbol{S}\,\mathrm{d}\ln\boldsymbol{z}. \tag{S.4.245}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.242) has the following matrix representation:

$$\mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t} = \left[\begin{array}{c}\boldsymbol{T}\left(\mathrm{d}\ln\boldsymbol{d} + \mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t}\right)\\ +\theta\left(\boldsymbol{TS}-\boldsymbol{I}\right)\left(\mathrm{d}\ln\boldsymbol{w_t} - \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi_t} - \mathrm{d}\ln\boldsymbol{z}\right)\end{array}\right],$$

where we have used $\mathrm{d}\ln\boldsymbol{\tau} = 0$. We can re-write this relationship as:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln\boldsymbol{w_t} = \left[\begin{array}{c}\boldsymbol{T}\,\mathrm{d}\ln\boldsymbol{d} - \left(\boldsymbol{I}-\boldsymbol{T}\right)\mathrm{d}\ln\boldsymbol{\ell_t}\\ +\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi_t}\right)\end{array}\right].$$

In steady-state we have:

$$\left[\boldsymbol{I}-\boldsymbol{T}+\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\right]\mathrm{d}\ln\boldsymbol{w}^* = \left[\begin{array}{c}\boldsymbol{T}\,\mathrm{d}\ln\boldsymbol{d} - \left(\boldsymbol{I}-\boldsymbol{T}\right)\mathrm{d}\ln\boldsymbol{\ell}^*\\ +\theta\left(\boldsymbol{I}-\boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + \left(1-\mu\right)\mathrm{d}\ln\boldsymbol{\chi}^*\right)\end{array}\right]. \tag{S.4.246}$$

Population Flow. The total derivative of the population flow condition has the same matrix representation as in our baseline model:

$$\mathrm{d}\ln\boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln\boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}}.$$

In steady-state, we have:

$$\mathrm{d}\ln\boldsymbol{\ell}^* = \boldsymbol{E}\,\mathrm{d}\ln\boldsymbol{\ell}^* + \frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathrm{d}\boldsymbol{v}^*. \tag{S.4.247}$$

Value function. The total derivative of the value function (S.4.244) has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left[\begin{array}{c} \mathrm{d}\ln \boldsymbol{d} + (\boldsymbol{I} - \boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w_t} \\ +\boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi_t}\right) + \ \mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D}\mathbb{E}_t\, \mathrm{d}\boldsymbol{v_{t+1}} \end{array} \right],$$

where we have used $\mathrm{d}\ln \boldsymbol{\tau} = \mathrm{d}\ln \boldsymbol{\kappa} = 0$. In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^* = \left[\begin{array}{c} \mathrm{d}\ln \boldsymbol{d} + (\boldsymbol{I} - \boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w}^* \\ +\boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) + \ \mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D}\, \mathrm{d}\boldsymbol{v}^* \end{array} \right]. \tag{S.4.248}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left[\boldsymbol{I} - (1-\mu)\,\boldsymbol{S}\right]^{-1}\left[(\boldsymbol{I} - \boldsymbol{S})\ \mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z}\right]. \tag{S.4.249}$$

$$\mathrm{d}\ln \boldsymbol{w}^* = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\,(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S})\right]^{-1} \left[\begin{array}{c} \boldsymbol{T}\,\mathrm{d}\ln \boldsymbol{d} - (\boldsymbol{I} - \boldsymbol{T})\ \mathrm{d}\ln \boldsymbol{\ell}^* \\ + (\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S})\,\theta\left(\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) \end{array} \right]. \tag{S.4.250}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\,(\boldsymbol{I} - \boldsymbol{E})^{-1}\,(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D})\ \mathrm{d}\boldsymbol{v}^*. \tag{S.4.251}$$

$$\mathrm{d}\boldsymbol{v}^* = \left[\boldsymbol{I} - \beta \boldsymbol{D}\right]^{-1} \left[\begin{array}{c} \mathrm{d}\ln \boldsymbol{d} + \ \mathrm{d}\ln \boldsymbol{w}^* \\ -\boldsymbol{S}\left(\mathrm{d}\ln \boldsymbol{w}^* - \ \mathrm{d}\ln \boldsymbol{z} - (1-\mu)\ \mathrm{d}\ln \boldsymbol{\chi}^*\right) + \ \mathrm{d}\ln \boldsymbol{b} \end{array} \right]. \tag{S.4.252}$$

As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^N w_i^* \ell_i^* = \sum_{i=1}^N q_i^* = \overline{q} = 1$), which implies that the log changes in incomes satisfy $\boldsymbol{q}^*\,\mathrm{d}\ln \boldsymbol{q}^* = \sum_{i=1}^N q_i^*\,\mathrm{d}\ln q_i^* = \sum_{i=1}^N q_i^* \frac{\mathrm{d}q_i^*}{q_i^*} = \sum_{i=1}^N \mathrm{d}q_i^* = 0$, where $\boldsymbol{q}^*$ is a row vector of the steady-state income of each location. Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^N \ell_i = \overline{\ell} = 1$, which implies $\boldsymbol{\ell}^*\,\mathrm{d}\ln \boldsymbol{\ell}^* = \sum_{i=1}^N \ell_i^*\,\mathrm{d}\ln \ell_i^* = \sum_{i=1}^N \ell_i^* \frac{\mathrm{d}\ell_i^*}{\ell_i^*} = \sum_{i=1}^N \mathrm{d}\ell_i^* = 0$, where $\boldsymbol{\ell}^*$ is a row vector of the steady-state population of each location.

In the interest of brevity, we focus above on deriving sufficient statistics for changes in steady-states in the presence of trade deficits. Nevertheless, analogous sufficient statistics results for transition paths can be derived in the presence of trade deficits, as in our baseline model in Section 2 of the paper.

S.4.7 Residential Capital (Housing)

In this section of the Online Supplement, we consider an extension of our baseline model in Section 2 of the paper to allow capital to be used residentially as well as commercially. We consider an economy that consists of many locations indexed by $i \in \{1, \ldots, N\}$. Time is discrete and is indexed by t. The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed and depreciates gradually at a constant rate δ.

S.4.7.1 Worker Migration Decisions

Worker migration decisions are modeled as in our baseline model in Section 2 of the paper. The expected value for a worker of living in location i at time t (v_{it}^{w}) is:

$$v_{it}^{w} = \ln u_{it}^{w} + \rho \log \sum_{g=1}^{N} \left(\exp \left(\beta \mathbb{E}_t v_{gt+1}^{w} \right) / \kappa_{git} \right)^{1/\rho}. \tag{S.4.253}$$

The probability of migrating from location i to location g is:

$$D_{igt} = \frac{\left(\exp \left(\beta \mathbb{E}_t v_{gt+1}^{w} \right) / \kappa_{git} \right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp \left(\beta \mathbb{E}_t v_{mt+1}^{w} \right) / \kappa_{kit} \right)^{1/\rho}}. \tag{S.4.254}$$

S.4.7.2 Worker Consumption

As workers do not have access to an investment technology, they choose their consumption of varieties each period to maximize their flow utility in the location in which they have chosen to live in that period. Worker static utility depends on local amenities (b_{nt}), goods consumption (c_{nt}^{w}) and residential use of capital (k_{nt}^{w}):

$$\ln u_{nt} = \ln b_{nt} + \alpha \ln c_{nt}^{w} + (1-\alpha) \ln k_{nt}^{w}, \qquad 0 < \alpha < 1, \tag{S.4.255}$$

where c_{nt}^{w} is a consumption index for workers in location n defined over the consumption of the variety supplied by each location i (c_{nit}^{w}):

$$c_{nt}^{w} = \left[\sum_{i=1}^{N} \left(c_{ni}^{w} \right)^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{S.4.256}$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties and $\theta = \sigma - 1$ is the trade elasticity. Amenities (b_{nt}) capture exogenous characteristics of a location that make it a more attractive place to live regardless of the wage and cost of consumption goods (e.g., climate and scenic views).

The corresponding worker indirect utility function depends on amenities (b_{nt}), the wage (w_{nt}), the rental rate for capital (r_{nt}) and the consumption goods price index (p_{nt}):

$$\ln u_{nt} = \ln b_{nt} + \ln w_{nt} - \alpha \ln p_{nt} - (1-\alpha) \ln r_{nt}, \tag{S.4.257}$$

where the consumption goods price index (p_{nt}) in location n depends of the price of the variety sourced from each location i (p_{nit}):

$$p_{nt} = \left[\sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}. \tag{S.4.258}$$

From the first-order conditions for worker utility maximization, total worker payments for goods consumption and residential capital use are constant multiples of total worker income:

$$p_{nt}c_{nt} = \alpha w_{nt}\ell_{nt}, \tag{S.4.259}$$

$$r_{nt}k^{w}_{nt} = (1-\alpha)\, w_{nt}\ell_{nt}. \tag{S.4.260}$$

Using constant elasticity of substitution (CES) demand for individual varieties of goods, the share location n's expenditure on the goods produced by location i is:

$$S_{nit} \equiv \frac{p_{nit}^{-\theta}}{\sum_{m=1}^{N} p_{nmt}^{-\theta}}. \tag{S.4.261}$$

S.4.7.3 Production

Producers in each location use labor (ℓ_{it}) and commercial capital (k^{y}_{it}) to produce output (y_{it}) of the variety supplied by that location. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$y_{it} = z_{it}\left(\frac{\ell_{it}}{\mu}\right)^{\mu}\left(\frac{k^{y}_{it}}{1-\mu}\right)^{1-\mu}, \qquad 0<\mu<1, \tag{S.4.262}$$

where z_{it} denotes exogenous productivity in location i at time t.

We assume that trade between locations is subject to iceberg variable costs of trade, such that $\tau_{nit} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n, where $\tau_{nit} > 1$ for $n \neq i$ and $\tau_{iit} = 1$. From profit maximization, the cost to a consumer in location n of sourcing the good produced by location i is:

$$p_{nit} = \tau_{nit}p_{iit} = \frac{\tau_{nit}w_{it}^{\mu}r_{it}^{1-\mu}}{z_{it}}, \tag{S.4.263}$$

where p_{iit} is the "free on board" price of the good supplied by location i before trade costs.

From profit maximization problem and zero profits, total payments to each factor of production are a constant share of total revenue:

$$w_{it}\ell_{it} = \mu p_{it}y_{it}, \tag{S.4.264}$$

$$r_{it}k^{y}_{it} = (1-\mu)\, p_{it}y_{it}. \tag{S.4.265}$$

S.4.7.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v^{k}_{it} = \sum_{t=0}^{\infty}\beta^{t}\left[\alpha \ln c^{k}_{it} + (1-\alpha)\ln k^{k}_{it}\right], \tag{S.4.266}$$

where c^k_{it} is a consumption index defined over the consumption of the good supplied by each location (c^k_{imt}) as in equation (S.4.256); k^k_{nt} denotes is landlords' residential use of capital; and β denotes the discount rate.

We assume that the investment technology for capital in each location uses the varieties from all locations with the same functional form as consumption. In particular, landlords in a given location can produce one unit of capital in that location using one unit of the consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate δ. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r_{it}k_{it}$) equals the total value of goods consumption ($p_{it}c^k_{it}$), residential capital use ($r_{it}k^k_{it}$), and net investment ($p_{it}\left(k_{it+1} - (1-\delta)\,k_{it}\right)$):

$$r_{it}k_{it} = p_{it}c^k_{it} + r_{it}k^k_{it} + p_{it}\left(k_{it+1} - (1-\delta)\,k_{it}\right). \tag{S.4.267}$$

Combining the landlords intertemporal utility (S.4.266) and budget constraint (S.4.267), the landlord's intertemporal optimization problem is:

$$\max_{\left\{c_t, k^k_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^t \left[\alpha \ln c^k_{it} + (1-\alpha)\ln k^k_{it}\right], \tag{S.4.268}$$

$$\text{subject to} \qquad p_{it}c^k_{it} + p_{it}\left(k_{it+1} - (1-\delta)\,k_{it}\right) = r_{it}\left(k_{it} - k^k_{it}\right).$$

We can write this problem as the following Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\alpha \ln c^k_{it} + (1-\alpha)\ln k^k_{it}\right] - \xi_t \left[p_{it}c^k_{it} + p_{it}\left(k_{it+1} - (1-\delta)\,k_{it}\right) - r_{it}\left(k_{it} - k^k_{it}\right)\right]. \tag{S.4.269}$$

The first-order conditions are:

$$\{c_{it}\} \qquad \alpha\frac{\beta^t}{c_{it}} - p_{it}\xi_t = 0,$$

$$\{k_{it+1}\} \qquad \left(r_{it+1} + p_{it+1}\,(1-\delta)\right)\xi_{t+1} - p_{it}\xi_t = 0,$$

$$\left\{k^k_{it}\right\} \qquad (1-\alpha)\,\frac{\beta^t}{k^k_{it}} - r_{it}\xi_t = 0.$$

Together these first-order conditions imply:

$$\frac{c_{it+1}}{c_{it}} = \beta\frac{p_{it}\mu_t}{p_{it+1}\mu_{t+1}} = \beta\left(r_{it+1}/p_{it+1} + (1-\delta)\right), \tag{S.4.270}$$

$$\frac{r_{it}k^k_{it}}{p_{it}c_{it}} = \frac{1-\alpha}{\alpha}, \tag{S.4.271}$$

where the transversality condition implies:

$$\lim_{t\to\infty} \beta^t \frac{k_{it+1}}{c_{it}} = 0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords' optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it} c_{it}^{k} = \alpha \left(1-\beta\right) \left(r_{it} + p_{it} \left(1-\delta\right)\right) k_{it}, \tag{S.4.272}$$

$$r_{it} k_{it}^{k} = \left(1-\alpha\right) \left(1-\beta\right) \left(r_{it} + p_{it} \left(1-\delta\right)\right) k_{it}, \tag{S.4.273}$$

$$k_{it+1} = \beta \left(r_{it}/p_{it} + \left(1-\delta\right)\right) k_{it}. \tag{S.4.274}$$

Substituting the consumption policy function (S.4.272) into the Euler equation (S.4.270), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\begin{aligned} \frac{c_{it+1}^{k}}{c_{it}^{k}} &= \frac{\left(r_{it+1}/p_{it+1} + \left(1-\delta\right)\right) k_{it+1}}{\left(r_{it}/p_{it} + \left(1-\delta\right)\right) k_{it}}, \\ &= \beta \left(r_{it+1}/p_{it+1} + \left(1-\delta\right)\right). \end{aligned}$$

S.4.7.5 Market Clearing

Goods market clearing implies that revenue in each location equals expenditure on the goods produced by that location:

$$p_{it} y_{it} = \alpha \sum_{n=1}^{N} S_{nit} \left(w_{nt} \ell_{nt} + r_{nt} k_{nt}\right),$$

$$w_{it} \ell_{it} + r_{it} k_{it}^{y} = \alpha \sum_{n=1}^{N} S_{nit} \left(w_{nt} \ell_{nt} + r_{nt} k_{nt}\right),$$

$$w_{it} \ell_{it} + \frac{1-\mu}{\mu} w_{it} \ell_{it} = \alpha \sum_{n=1}^{N} S_{nit} \left(w_{nt} \ell_{nt} + r_{nt} k_{nt}\right),$$

$$w_{it} \ell_{it} + \frac{1-\mu}{\mu} w_{it} \ell_{it} = \alpha \sum_{n=1}^{N} S_{nit} \left(w_{nt} \ell_{nt} + \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right) w_{nt} \ell_{nt}\right),$$

$$w_{it} \ell_{it} + \frac{1-\mu}{\mu} w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} \left(\alpha w_{nt} \ell_{nt} + \left(\left(1-\alpha\right) + \frac{1-\mu}{\mu}\right) w_{nt} \ell_{nt}\right),$$

$$\frac{1}{\mu} w_{it} \ell_{it} = \frac{\alpha\mu + \left(1-\alpha\right)\mu + \left(1-\mu\right)}{\mu} \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt},$$

$$\frac{1}{\mu} w_{it} \ell_{it} = \frac{1}{\mu} \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt},$$

$$w_{it} \ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}. \tag{S.4.275}$$

The capital market clearing condition equates the income received by landlords from ownership of capital to payments for the residential and commercial use of capital. Using workers' expenditure on residential capital (S.4.260), payments for labor (S.4.264) and capital (S.4.265) in production, and landlords' expenditure on residential capital (S.4.271), this capital market clearing condition can be expressed as:

$$r_{it}k_{it} = r_{it}k_{it}^{k} + r_{it}k_{it}^{w} + r_{it}k_{it}^{y},$$

$$r_{it}k_{it} = (1-\alpha)\, r_{it}k_{it} + (1-\alpha)\, w_{it}\ell_{it} + \frac{1-\mu}{\mu} w_{it}\ell_{it}$$

$$r_{it}k_{it} = \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right) w_{it}\ell_{it}. \tag{S.4.276}$$

S.4.7.6 General Equilibrium

Given the state variables $\{\ell_{i0}, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing (S.4.276), the price index (S.4.258) and the equilibrium pricing rule (S.4.263), the capital accumulation equation (S.4.274) becomes:

$$k_{it+1} = \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}}{p_{it}}\ell_{it} + \beta\,(1-\delta)\, k_{it}, \tag{S.4.277}$$

$$p_{nt} = \left[\sum_{i=1}^{N}\left(w_{it}\left(\frac{1-\mu}{\mu}\right)^{1-\mu}(\ell_{it}/k_{it})^{1-\mu}\,\tau_{nit}/z_{it}\right)^{-\theta}\right]^{-1/\theta}.$$

Goods Market Clearing: Using the equilibrium pricing rule (S.4.263), the expenditure share (S.4.261) and capital market clearing (S.4.276), the goods market clearing condition (S.4.275) can be written as:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit}w_{nt}\ell_{nt}. \tag{S.4.278}$$

$$S_{nit} \equiv \frac{p_{nit}^{-\theta}}{\sum_{m=1}^{N} p_{nmt}^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit}w_{nt}\ell_{nt}}{w_{it}\ell_{it}},$$

where S_{nit} is the expenditure share of importer n on exporter i at time t, and we have defined T_{int} as the corresponding income share of exporter i from importer n at time t. Note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the outmigration probabilities (S.4.254), the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{S.4.279}$$

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^w\right) / \kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^w\right) / \kappa_{kit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}},$$

where D_{igt} is the outmigration probability from location i to location g between time t and time $t+1$, and we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$. Note that the order of subscripts switches between the outmigration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (S.4.255) in the value function (S.4.253), the expected value from living in location n at time t can be written as:

$$v_{it}^w = \ln b_{nt} + \ln \left(\frac{w_{nt}^{\alpha}}{p_{nt}^{\alpha} \left(\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu} \right) \frac{\ell_{nt}}{k_{nt}} \right)^{1-\alpha}} \right) + \rho \log \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^w\right) / \kappa_{git}\right)^{1/\rho}.$$

S.4.7.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path.

Prices Using the relationship between capital and labor payments (S.4.276), the pricing rule (S.4.263) can be re-written as follows:

$$p_{nit} = \frac{\tau_{nit} w_{it} \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu} \right)^{1-\mu} \left(\frac{1}{\chi_{it}} \right)^{1-\mu}}{z_{it}}, \tag{S.4.280}$$

where χ_{it} is the capital-labor ratio:

$$\chi_{it} \equiv \frac{k_{it}}{\ell_{it}}.$$

Totally differentiating this pricing rule, we have:

$$\mathrm{d}\ln p_{nit} = \mathrm{d}\ln \tau_{nit} + \mathrm{d}\ln w_{it} - (1-\mu)\, \mathrm{d}\ln \chi_{it} - \mathrm{d}\ln z_{it}. \tag{S.4.281}$$

Expenditure Shares Totally differentiating this expenditure share equation (S.4.261), we get:

$$\mathrm{d}\ln S_{nit} = \theta \left(\sum_{h=1}^{N} S_{nht}\, \mathrm{d}\ln p_{nht} - \mathrm{d}\ln p_{nit} \right). \tag{S.4.282}$$

Price Indices Totally differentiating the consumption goods price index in equation (S.4.258), we have:

$$d\ln p_{nt} = \sum_{m=1}^{N} S_{nmt}\, d\ln p_{nmt}. \tag{S.4.283}$$

Migration Shares Totally differentiating the outmigration share in equation (S.4.254), we get:

$$d\ln D_{igt} = \frac{1}{\rho}\left[\left(\beta \mathbb{E}_t\, dv_{gt+1} - d\ln \kappa_{git}\right) - \sum_{h=1}^{N} D_{iht}\left(\beta \mathbb{E}_t\, dv_{ht+1} - d\ln \kappa_{hit}\right) \right]. \tag{S.4.284}$$

Real Income Totally differentiating real income we have:

$$d\ln\left(\frac{w_{it}}{p_{it}}\right) = d\ln w_{it} - d\ln p_{it},$$

$$d\ln\left(\frac{w_{it}}{p_{it}}\right) = d\ln w_{it} - \sum_{m=1}^{N} S_{nmt}\, d\ln p_{nmt},$$

$$d\ln\left(\frac{w_{it}}{p_{it}}\right) = d\ln w_{it} - \sum_{m=1}^{N} S_{nmt}\left[d\ln \tau_{nmt} + d\ln w_{mt} - (1-\mu)\, d\ln \chi_{mt} - d\ln z_{mt}\right]. \tag{S.4.285}$$

Goods Market Clearing Totally differentiating the goods market clearing condition (S.4.275), we have:

$$\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it}\ell_{it}}\left(\frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \frac{dS_{nit}}{S_{nit}}\right).$$

Using our result for the derivative of expenditure shares in equation (S.4.282) above, we can rewrite this as:

$$\frac{dw_{it}}{w_{it}} + \frac{d\ell_{it}}{\ell_{it}} = \sum_{n=1}^{N} T_{int}\left(\frac{dw_{nt}}{w_{nt}} + \frac{d\ell_{nt}}{\ell_{nt}} + \theta\left(\sum_{h\in N} S_{nht}\frac{dp_{nht}}{p_{nht}} - \frac{dp_{nit}}{p_{nit}}\right)\right),$$

$$T_{int} \equiv \frac{S_{nit} w_{nt}\ell_{nt}}{w_{it}\ell_{it}},$$

$$d\ln w_{it} + d\ln \ell_{it} = \left[\begin{array}{c} \sum_{n=1}^{N} T_{int}\left(d\ln w_{nt} + d\ln \ell_{nt}\right) \\ +\theta \sum_{n=1}^{N}\sum_{m=1}^{N} T_{int} S_{nmt}\left(d\ln \tau_{nmt} + d\ln w_{mt} - (1-\mu)\, d\ln \chi_{mt} - d\ln z_{mt}\right) \\ -\theta \sum_{n=1}^{N} T_{int}\left(d\ln \tau_{nit} + d\ln w_{it} - (1-\mu)\, d\ln \chi_{it} - d\ln z_{it}\right) \end{array}\right]. \tag{S.4.286}$$

Population Flow. Totally differentiating the population flow condition (S.4.279) we have:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt}\ell_{it},$$

$$d\ln \ell_{gt+1} = \sum_{i=1}^{N} E_{git}\left[d\ln \ell_{it} + \frac{1}{\rho}\left(\beta \mathbb{E}_t\, dv_{gt+1} - d\ln \kappa_{gi} - \sum_{m=1}^{N} D_{imt}\left(\beta \mathbb{E}_t\, dv_{mt+1} - d\ln \kappa_{mit}\right)\right)\right]. \tag{S.4.287}$$

Value Function. Note that the value function can be re-written using the following results:

$$v_{it} = \ln \frac{w_{it}}{\left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-1/\theta}} + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}\right) / \kappa_{git}\right)^{1/\rho},$$

$$\left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-1/\theta} = \left(\frac{p_{iit}^{-\theta}}{S_{iit}}\right)^{-1/\theta}, \qquad \tau_{iit} = 1,$$

$$\sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}\right) / \kappa_{git}\right)^{1/\rho} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{it+1}\right) / \kappa_{iit}\right)^{1/\rho}}{D_{iit}}, \qquad \kappa_{iit} = 1,$$

$$v_{it} = -\frac{1}{\theta} \ln S_{iit} + \ln w_{it} - \ln p_{iit} + \ln b_{it} + \beta \mathbb{E}_t v_{it+1} - \rho \ln D_{iit}. \tag{S.4.288}$$

Totally differentiating this value function (S.4.288) we have:

$$\mathrm{d}v_{it} = -\frac{1}{\theta} \,\mathrm{d}\ln S_{iit} + \,\mathrm{d}\ln w_{it} - \,\mathrm{d}\ln p_{iit} + \,\mathrm{d}\ln b_{it} + \beta \mathbb{E}_t \,\mathrm{d}v_{it+1} - \rho \,\mathrm{d}\ln D_{iit},$$

$$\mathrm{d}\ln S_{iit} = -\theta \,\mathrm{d}\ln p_{iit} + \theta \left[\sum_{m=1}^{N} S_{imt} \,\mathrm{d}\ln p_{imt}\right],$$

$$\mathrm{d}\ln D_{iit} = \frac{1}{\rho} \left[\beta \mathbb{E}_t \,\mathrm{d}v_{it+1} - \,\mathrm{d}\ln \kappa_{iit} - \sum_{m=1}^{N} D_{imt} \left(\beta \mathbb{E}_t v_{mt+1} - \,\mathrm{d}\ln \kappa_{mit}\right)\right].$$

Using these results in the derivative of the value function, we have:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \sum_{m=1}^{N} S_{imt} \,\mathrm{d}\ln p_{imt} \\ +\,\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1} - \,\mathrm{d}\ln \kappa_{mit}\right) \end{array}\right],$$

where we have used $\mathrm{d}\ln \kappa_{iit} = 0$. Using the total derivative of the pricing rule (S.4.281), we can re-write this derivative of the value function as follows:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \sum_{m=1}^{N} S_{imt} \left(\mathrm{d}\ln \tau_{nmt} + \,\mathrm{d}\ln w_{mt} - (1-\mu) \,\mathrm{d}\ln \chi_{mt} - \,\mathrm{d}\ln z_{mt}\right) \\ +\,\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt} \left(\beta \mathbb{E}_t \,\mathrm{d}v_{mt+1} - \,\mathrm{d}\ln \kappa_{mit}\right) \end{array}\right]. \tag{S.4.289}$$

S.4.7.8 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1} = k_{it} = k_i^*$, $\ell_{it+1} = \ell_{it} = \ell_i^*$, $w_{it+1}^* = w_{it}^* = w_i^*$ and $v_{it+1}^* = v_{it}^* = v_i^*$, where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau}$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa}$).

Capital Accumulation. From the capital accumulation equation (S.4.277), the steady-state stock of capital solves:

$$k_i^* = \beta \left[\frac{r_i}{p_i} + (1-\delta) \right] k_i^*.$$

$$(1 - \beta (1-\delta)) k_i^* = \beta \frac{r_i}{p_i} k_i^*.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}}{p_{it}} k_{it} = \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu} \right) \frac{w_{it}\ell_{it}}{p_{it}},$$

Using this result in the expression for the steady-state capital stock above, we have:

$$(1 - \beta (1-\delta)) k_i^* = \beta \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu} \right) \frac{w_i^*\ell_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d} \ln \chi_i^* = \mathrm{d} \ln \left(\frac{w_i^*}{p_i^*} \right).$$

From the total derivative of real income (S.4.285) above, this becomes:

$$\mathrm{d} \ln \chi_i^* = \mathrm{d} \ln w_i^* - \sum_{m=1}^{N} S_{im} \left[\mathrm{d} \ln w_m^* - (1-\mu) \, \mathrm{d} \ln \chi_m^* - \mathrm{d} \ln z_m \right],$$

where we have used $\mathrm{d} \ln \tau_{im} = 0$. This relationship has the following matrix representation:

$$\mathrm{d} \ln \boldsymbol{\chi}^* = \mathrm{d} \ln \boldsymbol{w}^* - \boldsymbol{S} \, \mathrm{d} \ln \boldsymbol{w}^* + (1-\mu) \, \boldsymbol{S} \, \mathrm{d} \ln \boldsymbol{\chi}^* + \boldsymbol{S} \, \mathrm{d} \ln \boldsymbol{z},$$

$$(\boldsymbol{I} - (1-\mu) \, \boldsymbol{S}) \, \mathrm{d} \ln \boldsymbol{\chi}^* = (\boldsymbol{I} - \boldsymbol{S}) \, \mathrm{d} \ln \boldsymbol{w}^* + \boldsymbol{S} \, \mathrm{d} \ln \boldsymbol{z},$$

which can be written as:

$$\mathrm{d} \ln \boldsymbol{\chi}^* = (\boldsymbol{I} - (1-\mu) \, \boldsymbol{S})^{-1} \left[(\boldsymbol{I} - \boldsymbol{S}) \, \mathrm{d} \ln \boldsymbol{w}^* + \boldsymbol{S} \, \mathrm{d} \ln \boldsymbol{z} \right]. \tag{S.4.290}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.286) has the following matrix representation:

$$\mathrm{d} \ln \boldsymbol{w_t} + \mathrm{d} \ln \boldsymbol{\ell_t} = \boldsymbol{T} \left(\mathrm{d} \ln \boldsymbol{w_t} + \mathrm{d} \ln \boldsymbol{\ell_t} \right) + \theta \left(\boldsymbol{TS} - \boldsymbol{I} \right) \left(\mathrm{d} \ln \boldsymbol{w_t} - (1-\mu) \, \mathrm{d} \ln \boldsymbol{\chi_t} - \mathrm{d} \ln \boldsymbol{z} \right),$$

where we have used $\mathrm{d} \ln \boldsymbol{\tau} = 0$. We can re-write this relationship in steady-state as:

$$\mathrm{d} \ln \boldsymbol{w}^* = \left[\boldsymbol{I} - \boldsymbol{T} + \theta \left(\boldsymbol{I} - \boldsymbol{TS} \right) \right]^{-1} \left[- \left(\boldsymbol{I} - \boldsymbol{T} \right) \mathrm{d} \ln \boldsymbol{\ell}^* + \theta \left(\boldsymbol{I} - \boldsymbol{TS} \right) \left(\mathrm{d} \ln \boldsymbol{z} + (1-\mu) \, \mathrm{d} \ln \boldsymbol{\chi}^* \right) \right]. \tag{S.4.291}$$

Population Flow. The total derivative of the population flow condition (S.4.287) has the following matrix representation:

$$\mathrm{d}\ln \boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}},$$

which can be written in steady-state as:

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\,\mathrm{d}\boldsymbol{v}^*. \tag{S.4.292}$$

Value function. The total derivative of the value function (S.4.289) has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w_t} + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi_t}\right) + \,\mathrm{d}\ln \boldsymbol{b} + \beta\boldsymbol{D}\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}},$$

where we have used $\mathrm{d}\ln \boldsymbol{\tau} = \mathrm{d}\ln \boldsymbol{\kappa} = 0$. We can re-write this relationship in steady-state as:

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta\boldsymbol{D}\right)^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \,\mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.293}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left(\boldsymbol{I} - (1-\mu)\,\boldsymbol{S}\right)^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w} + \boldsymbol{S}\,\mathrm{d}\ln \boldsymbol{z}\right]. \tag{S.4.294}$$

$$\mathrm{d}\ln \boldsymbol{w}^* = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\,\mathrm{d}\ln \boldsymbol{\ell}^* + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right)\right]. \tag{S.4.295}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\,\mathrm{d}\boldsymbol{v}^*. \tag{S.4.296}$$

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta\boldsymbol{D}\right)^{-1}\left[\left(\boldsymbol{I} - \boldsymbol{S}\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \boldsymbol{S}\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \,\mathrm{d}\ln \boldsymbol{b}\right]. \tag{S.4.297}$$

S.4.7.9 Sufficient Statistics for Transition Dynamics

Suppose that the economy starts from an initial steady-state. Consider a small shock to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \overline{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$). We use a tilde above a variable to denote a log-deviation from the initial steady-state, such that $\widetilde{\ell}_{it} = \ln \ell_{it} - \ln \ell_i^*$, for all variables except for the worker value function v_{it}; with a slight abuse of notation we use $\widetilde{v}_{it} \equiv v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. From the capital accumulation equation (S.4.277), we have:

$$k_{it+1} = \beta\frac{r_{it}}{p_{it}}k_{it} + \beta\left(1-\delta\right)k_{it}.$$

From the relationship between labor and capital payments, we have:

$$\frac{r_{it}}{p_{it}}k_{it} = \left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}\ell_{it}}{p_{it}},$$

$$k_{it+1} = \beta\left(1-\delta\right)k_{it} + \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}}{p_{it}}\ell_{it},$$

$$\frac{k_{it+1}}{\ell_{it+1}}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\frac{k_{it}}{\ell_{it}} + \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}}{p_{it}},$$

$$\chi_{it+1}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\chi_{it} + \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}}{p_{it}}, \tag{S.4.298}$$

while in steady-state we have:

$$\frac{k_i^*}{\ell_i^*} = \beta\left(1-\delta\right)\frac{k_i^*}{\ell_i^*} + \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_i^*}{p_i^*},$$

$$\chi_i^* = \beta\left(1-\delta\right)\chi_i^* + \beta\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_i^*}{p_i^*}.$$

$$\chi_i^* = \frac{\beta}{\left(1-\beta\left(1-\delta\right)\right)}\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_i^*}{p_i^*}. \tag{S.4.299}$$

Dividing both sides of equation (S.4.298) by χ_i^*, we have:

$$\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \frac{\beta}{\chi_i^*}\left(\frac{1-\alpha}{\alpha} + \frac{1-\mu}{\alpha\mu}\right)\frac{w_{it}}{p_{it}},$$

which using (S.4.299) can be re-written as:

$$\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} = \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \left(1-\beta\left(1-\delta\right)\right)\frac{w_{it}/w_i^*}{p_{it}/p_i^*},$$

which can be further re-written as:

$$\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 = \beta\left(1-\delta\right)\frac{\chi_{it}}{\chi_i^*} + \left(1-\beta\left(1-\delta\right)\right)\frac{w_{it}/w_i^*}{p_{it}/p_i^*} - 1,$$

$$\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 = \beta\left(1-\delta\right)\left(\frac{\chi_{it}}{\chi_i^*} - 1\right) + \left(1-\beta\left(1-\delta\right)\right)\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*} - 1\right).$$

Noting that:

$$\frac{x_{it}}{x_i^*} - 1 \simeq \ln\left(\frac{x_{it}}{x_i^*}\right),$$

$$\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}} - 1 \simeq \ln\left(\frac{\chi_{it+1}}{\chi_i^*}\frac{\ell_{it+1}}{\ell_{it}}\right),$$

we have:

$$\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right) + \ln\left(\frac{\ell_{it+1}}{\ell_{it}}\right) = \beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),$$

$$\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right) + \ln\left(\frac{\ell_{it+1}/\ell_i^*}{\ell_{it}/\ell_i^*}\right) = \beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),$$

which can be re-written as follows:

$$\widetilde{\chi}_{it+1} = \beta\left(1-\delta\right)\widetilde{\chi}_{it} + \left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{w}_{it} - \widetilde{p}_{it}\right) - \widetilde{\ell}_{it+1} + \widetilde{\ell}_{it},$$

We can rewrite this relationship in matrix form as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \beta\left(1-\delta\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}}\right) - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}.$$

Now note that:

$$\widetilde{w}_{it} - \widetilde{p}_{it} = \widetilde{w}_{it} - \sum_{m=1}^{N} S_{nmt}\left[\widetilde{w}_{mt} - \left(1-\mu\right)\widetilde{\chi}_{mt} - \widetilde{z}_{m}\right],$$

which can be written in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right].$$

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \beta\left(1-\delta\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\left(\left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right) - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}},$$

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\mu\right)\boldsymbol{S}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} \\ + \left(1-\beta\left(1-\delta\right)\right)\left(I-S\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.4.300}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.4.286) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} = \boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \theta\left(\boldsymbol{TS} - \boldsymbol{I}\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{z}}\right),$$

which can be re-written as:

$$\left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = -\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right],$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.301}$$

Population Flow. The total derivative of the population flow condition (S.4.287) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\mathbb{E}_{t}\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.302}$$

Value function. The total derivative of the value function (S.4.289) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_{t}\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.303}$$

System of Equations for Transition Dynamics. Collecting together the system of equations for the transition dynamics, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\mu\right)\boldsymbol{S}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} \\ +\left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array} \right]. \tag{S.4.304}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right]\right]. \tag{S.4.305}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.306}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\left[\left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{z}}\right] + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.4.307}$$

S.4.8 Landlord Investment in Other Locations

In this section of the Online Supplement, we consider an extension of our baseline model in Section 2 of the paper to allow landlords to invest in other locations in addition to their own location. We consider an economy that consists of many locations indexed by $i \in \{1, \ldots, N\}$. Time is discrete and is indexed by t. The economy consists of two types of infinitely-lived agents: workers and landlords. Both workers and landlords have the same flow preferences, which are modeled as in the standard Armington model of international trade. The continuous measure of workers are each endowed with one unit of labor that is supplied inelasticity and are geographically mobile across locations subject to bilateral migration costs. Workers do not have access to an investment technology and live hand to mouth as in Kaplan and Violante (2014). The continuous measure of landlords in each location are geographically immobile and own a stock of capital that can be allocated to any location. Landlords make a forward-looking decision over consumption and investment in this stock of capital, which depreciates gradually at a constant rate δ.

S.4.8.1 Capital Allocation

At the beginning of period t, the landlords in location n inherit an existing stock of capital k_{nt}, and decide where to allocate this existing capital and how much to invest in accumulating additional capital. Once these decisions have been made, production and consumption occur. At the end of period t, new capital is created from the investment decisions made at the beginning of the period, and the depreciation of existing capital occurs. In the remainder of this subsection, we characterize landlords' decisions at the beginning of period t of where to allocate the existing stock of capital. In the next subsection, we characterize landlords' optimal consumption-investment decision.

The stock of existing capital owned by landlords in source location n can be employed in each host location i. The productivity of each unit of capital owned in location n is subject to an idiosyncratic productivity shock for each of the possible locations i to which it can be allocated α_{nit}. This productivity shock determines the number of effective units of capital, and has an interpretation as a Keynesian marginal efficiency of capital draw, which captures all the idiosyncratic factors that affect the productivity of capital invested in a location. Landlords face financial frictions or management costs in allocating capital to other locations, which are assumed to take the iceberg form, such that $\phi_{nit} \geq 1$ units of capital from location n must be allocated to location i in order for one unit to be available for production, where $\phi_{nnt} = 1$ and $\phi_{nit} > 1$ for

$n \neq i$. The realized rate of return to a landlord in location n from allocating one unit of capital to location i is:

$$\mathcal{R}_{nit} = \frac{\alpha_{nit} r_{it}}{\phi_{nit}}, \tag{S.4.308}$$

where the idiosyncratic productivity shock (α_{nit}) corresponds to the number of effective units of capital before financial frictions or management costs (ϕ_{nit}) are incurred; and r_{it} corresponds to the rate of return per effective unit of capital.

We assume that these idiosyncratic shocks to the productivity of capital are drawn independently across source and host locations and units of capital from the following Fréchet distribution:

$$F_{nit}(\alpha) = e^{-(\alpha/a_{it})^{-\epsilon}}, \qquad a_{it} > 0, \qquad \epsilon > 1, \tag{S.4.309}$$

where the Fréchet scale parameter (a_{it}) controls the average productivity of capital allocated to host location i. The Fréchet shape parameter ϵ controls the dispersion of idiosyncratic shocks to the productivity of capital, and regulates the sensitivity of the capital allocation to economic variables such as the rate of return per effective unit of capital.

Using the properties of this Fréchet distribution, the share of capital from location n that is employed in location i satisfies a gravity equation:

$$\zeta_{nit} = \frac{k_{nit}}{k_{nt}} = \frac{\left(a_{it} r_{it}/\phi_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(a_{ht} r_{ht}/\phi_{nht}\right)^{\epsilon}}, \tag{S.4.310}$$

which provides a natural explanation for findings of home bias in capital investments, because financial frictions or management costs abroad are greater than those at home ($\phi_{nit} > \phi_{nnt}$ for $n \neq i$).

Using the properties of this Fréchet distribution, the realized rate of return on capital owned by location n at time t is the same across all host locations i and given by:

$$\mathcal{R}_{nit} = \mathcal{R}_{nt} = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right)\left[\sum_{h=1}^{N} \left(a_{ht} r_{ht}/\phi_{nht}\right)^{\epsilon}\right]^{\frac{1}{\epsilon}}, \tag{S.4.311}$$

where $\Gamma(\cdot)$ is the Gamma function.

Using the properties of the Fréchet distributions for productivity (a_{nit}) and the realized rate of return ($\mathcal{R}_{nt}$), the average productivity of capital from source country n in host country i conditional on capital being allocated to that host country is given by:

$$\overline{a}_{nit} = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right)\left(\frac{a_{it}^{\epsilon}}{\zeta_{nit}}\right)^{\frac{1}{\epsilon}}. \tag{S.4.312}$$

S.4.8.2 Capital Accumulation Across Periods

Landlords in each location choose their consumption and investment to maximize their intertemporal utility subject to their budget constraint. Landlords' intertemporal utility equals the expected present discounted value of their flow utility:

$$v_{nt}^{k} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \frac{\left(c_{nt+s}^{k}\right)^{1-1/\psi}}{1 - 1/\psi}, \tag{S.4.313}$$

where we use the superscript k to denote landlords; c^k_{nt} is the consumption index; β is the discount rate; ψ is the elasticity of intertemporal substitution. Since landlords are geographically immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

We assume that the investment technology in each location uses the varieties from all locations with the same functional form as consumption. Therefore, landlords in each location can produce one unit of capital using one unit of the consumption index in that location, where this unit of capital can be allocated to any location, as characterized in the previous subsection. We interpret capital as buildings and structures, which depreciate at the constant rate δ, and we allow for the possibility of negative investment. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($\mathcal{R}_{nt}k_{nt}$) equals the total value of their consumption ($p_{nt}c^k_{nt}$) plus the total value of net investment ($p_{nt}\left(k_{nt+1} - (1-\delta)\,k_{nt}\right)$):

$$\mathcal{R}_{nt}k_{nt} = p_{nt}\left(c^k_{nt} + k_{nt+1} - (1-\delta)\,k_{nt}\right), \tag{S.4.314}$$

where we have used the property established in the previous subsection that the realized rate of return is the same across all host locations for a given source location ($\mathcal{R}_{nit} = \mathcal{R}_{nt}$). We use $R_{nt} \equiv 1 - \delta + \mathcal{R}_{nt}/p_{nt}$ to denote the gross realized return on capital. Following the same line of argument as in Section 2 of the paper, the optimal consumption of landlords in location n satisfies $c_{nt} = \varsigma_{nt}R_{nt}k_{nt}$, where ς_{nt} is defined recursively as:

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi}\left(\mathbb{E}_t\left[R_{nt+1}^{\frac{\psi-1}{\psi}}\varsigma_{t+1}^{-\frac{1}{\psi}}\right]\right)^{\psi}. \tag{S.4.315}$$

Landlords' corresponding optimal saving and investment decisions satisfy $k_{nt+1} = (1-\varsigma_{nt})\,R_{nt}k_{nt}$. Therefore, the landlords in each location have a linear saving rate $(1-\varsigma_{nt})$ out of current period wealth $R_{nt}k_{nt}$, as in Angeletos (2007) and our baseline specification in the paper. The remainder of our quantitative analysis goes through as in our baseline specification in Section 2 of the paper, modifying the capital market clearing condition to take into account that capital from each location is allocated to all locations, as characterized in the previous subsection.

S.4.9 Labor Participation Decision

In this section of the Online Supplement, we discuss an extension of the model to incorporate a labor participation decision. Following Caliendo et al. (2019), we model this labor participation decision in terms of home production.

At the beginning of period t, the economy inherits a mass of workers (ℓ^j_{nt}) in each location n and sector j, where sector $j = 1$ corresponds to market production (employed) and sector $j = 0$ corresponds to home production (non-employed). An employed worker supplies a unit of labor inelastically and receives a competitive wage (w^j_{nt}). She allocates her income over consumption of goods in the same way as in our baseline specification in Section 2 of the paper. In contrast, a non-employed worker obtains consumption from home production ($h_n > 0$).

Under these assumptions, worker preferences take a similar form as in equation (3) in the paper. The flow utility function of a worker in location n in period t depends on amenities (b_{nt}) and a consumption index ($c^{j,w}_{nt}$):

$$u^{j,w}_{nt} = b_{nt}c^{j,w}_{nt}. \tag{S.4.316}$$

The consumption index now depends on whether the worker is employed or non-employed:

$$c_{nt}^{j,w} = \left\{ \begin{array}{ll} \left[\sum_{i=1}^{N} \left(c_{ni}^{j,w} \right)^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}} & \text{if} \quad j = 1 \\ h_n & \text{if} \quad j = 0 \end{array} \right| , \qquad \theta = \sigma - 1, \qquad \sigma > 1. \tag{S.4.317}$$

Under these assumptions, all of our analysis goes through as in our baseline specification in the paper, except that we now need to keep track of an additional state in each location (non-employment), where flow utility in this state depends on home production.

S.5 Tradable and Non-tradable Sector

We consider an economy with many locations indexed by $i \in \{1, \ldots, N\}$. Time is discrete and is denoted by t. There are two sectors: tradable and non-tradable. There are two types of infinitely-lived agents: workers and landlords. Workers are endowed with one unit of labor that is supplied inelasticity and are geographically mobile subject to migration costs. Workers do not have access to an investment technology and hence live hand to mouth, as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital stock in their location. They make a forward-looking decision over consumption and investment in this local stock of capital. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate δ.

S.5.1 Worker Migration Decisions

Worker migration decisions are modeled in exactly the same way as in our baseline Armington model with a single sector. We assume that workers have idiosyncratic preferences across locations and face bilateral migration costs in moving between locations. We assume perfect labor mobility across sectors within each location, such that there is a common wage across sectors within each location.

S.5.2 Worker Consumption

Worker preferences are defined over both traded and non-traded goods. The traded sector is modeled as in the standard Armington model of trade with constant elasticity of substitution (CES) preferences. The non-traded sector consists of a single local non-traded good. The indirect utility function each period depends on worker's wage (w_{nt}), the cost of living (p_{it}) and amenities (b_{nt}):

$$\ln u_{nt}^{w} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}, \tag{S.5.1}$$

where amenities (b_{nt}) capture characteristics of a location that make it a more attractive place to live regardless of goods consumption (e.g., climate and scenic views). In this section of the Online Supplement, we assume that amenities are exogenous. The cost of living (p_{nt}) in location n depends on the price index for traded goods (p_{it}^{T}) and the price of the non-traded good (p_{it}^{NT}):

$$p_{it} = \left(p_{it}^{T} \right)^{\gamma} \left(p_{it}^{NT} \right)^{1-\gamma}. \tag{S.5.2}$$

The price index for traded goods depends on the price of the variety sourced from each location i (p_{nit}):

$$p_{it}^{T}=\left[\sum_{m=1}^{N}p_{imt}^{-\theta}\right]^{-1/\theta}, \tag{S.5.3}$$

where $\sigma>1$ is the constant elasticity of substitution and $\theta=\sigma-1>0$ is the constant trade elasticity.

Using the properties of these CES preferences, the share of expenditure in importer n on the goods supplied by exporter i in the traded sector takes the standard form:

$$S_{nit}=\frac{\left(p_{nit}\right)^{-\theta}}{\sum_{m=1}^{N}\left(p_{nmt}\right)^{-\theta}}. \tag{S.5.4}$$

S.5.3 Production

Production in each sector uses labor and capital. Production is assumed to occur under conditions of perfect competition and using a constant returns to scale Cobb-Douglas production technology. For simplicity, we assume the same factor intensity and productivity in the traded and non-traded sectors. Profit maximization and zero profits implies the following equilibrium prices in the two sectors:

$$p_{nit}=\frac{\tau_{nit}w_{it}^{\mu}r_{it}^{1-\mu}}{z_{it}}, \tag{S.5.5}$$

$$p_{it}^{NT}=\frac{w_{it}^{\mu}r_{it}^{1-\mu}}{z_{it}}, \tag{S.5.6}$$

where z_{it} denotes productivity in location i at time t. In this section of the Online Supplement, we assume that productivity is exogenous.

S.5.4 Landlord Consumption

Landlords in each location choose their consumption and investment in capital to maximize their intertemporal utility subject to their intertemporal budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which we assume for simplicity takes the same logarithmic form as for workers:

$$v_{it}^{k}=\sum_{t=0}^{\infty}\beta^{t}\ln c_{it}^{k}, \tag{S.5.7}$$

where $c_{it}^{k}=\left(c_{it}^{T,k}\right)^{\gamma}\left(c_{it}^{NT,k}\right)^{1-\gamma}$ is the overall consumption for landlords, which depends on the consumption index for tradables ($c_{it}^{T,k}$) and the consumption index for non-tradables ($c_{it}^{NT,k}$).

Landlords in a given location can produce one unit of capital in that location using one unit of the overall consumption index in that location. We assume that capital is geographically immobile once installed and depreciates at a constant rate δ. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital

($r_{it}k_{it}$) equals the total value of their consumption ($p_{it}c_{it}^k$) plus the total value of net investment ($p_{it}\left(k_{it+1}-(1-\delta)\,k_{it}\right)$):

$$r_{it}k_{it}=p_{it}\left(c_{it}^k+k_{it+1}-(1-\delta)\,k_{it}\right). \tag{S.5.8}$$

Combining the landlords' intertemporal utility (S.5.7) and budget constraint (S.5.8), the landlords' intertemporal optimization problem is:

$$\max_{\left\{c_t^k,k_{t+1}\right\}}\sum_{t=0}^{\infty}\beta^t\ln c_{it}^k, \tag{S.5.9}$$

$$\text{subject to}\qquad p_{it}c_{it}^k+p_{it}\left(k_{it+1}-(1-\delta)\,k_{it}\right)=r_{it}k_{it}.$$

We can write this problem as the following Lagrangian:

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\ln c_{it}^k-\xi_t\left[p_{it}c_{it}^k+p_{it}\left(k_{it+1}-(1-\delta)\,k_{it}\right)-r_{it}k_{it}\right]. \tag{S.5.10}$$

The first-order conditions are:

$$\left\{c_{it}^k\right\}\qquad \frac{\beta^t}{c_{it}^k}-p_{it}\xi_t=0,$$

$$\left\{k_{it+1}\right\}\qquad \left(r_{it+1}+p_{it+1}\left(1-\delta\right)\right)\xi_{t+1}-p_{it}\xi_t=0.$$

Together these first-order conditions imply the familiar Euler equation linking the marginal utility of consumption between any two time periods:

$$\frac{c_{it+1}^k}{c_{it}^k}=\beta\frac{p_{it}\mu_t}{p_{it+1}\mu_{t+1}}=\beta\left(r_{it+1}/p_{it+1}+(1-\delta)\right), \tag{S.5.11}$$

where the transversality condition implies:

$$\lim_{t\to\infty}\beta^t\frac{k_{it+1}}{c_{it}^k}=0.$$

Our assumption of logarithmic flow utility and the property that the intertemporal budget constraint is linear in the stock of capital together imply that landlords optimal consumption-saving decision involves a constant saving rate, as in Moll (2014). We conjecture the following policy functions:

$$p_{it}c_{it}^k=(1-\beta)\left(r_{it}+p_{it}\left(1-\delta\right)\right)k_{it}, \tag{S.5.12}$$

$$k_{it+1}=\beta\left(r_{it}/p_{it}+(1-\delta)\right)k_{it}. \tag{S.5.13}$$

Substituting the consumption policy function (S.5.12) into the Euler equation (S.5.11), we confirm that these conjectured policy functions are indeed the optimal consumption-savings choice:

$$\begin{aligned}\frac{c_{it+1}^{j,k}}{c_{it}^{j,k}}&=\frac{\left(r_{it+1}^j/p_{it+1}+(1-\delta^j)\right)k_{it+1}^j}{\left(r_{it}^j/p_{it}+(1-\delta^j)\right)k_{it}^j},\\&=\beta\left(r_{it+1}^j/p_{it+1}+\left(1-\delta^j\right)\right).\end{aligned}$$

S.5.5 Market Clearing

Income equals expenditure implies that the sum of the income of workers and landlords in each location is equal to expenditure on the goods produced by that location:

$$\left[\begin{array}{c}\left(w_{it}\ell_{it}^{T}+r_{it}k_{it}^{T}\right)\\+\left(w_{it}\ell_{it}^{NT}+r_{it}k_{it}^{NT}\right)\end{array}\right]=\left[\begin{array}{c}\gamma\sum_{n=1}^{N}S_{nit}\left(w_{nt}\ell_{nt}+r_{nt}k_{nt}\right)\\+\left(1-\gamma\right)\left(w_{it}\ell_{it}+r_{it}k_{it}\right)\end{array}\right]. \tag{S.5.14}$$

Non-traded goods market clearing implies that income in the non-traded sector is equal to local expenditure on non-traded goods:

$$\left(w_{it}\ell_{it}^{NT}+r_{it}k_{it}^{NT}\right)=\left(1-\gamma\right)\left[\left(w_{it}\ell_{it}+r_{it}k_{it}\right)\right]. \tag{S.5.15}$$

Using this non-traded goods market clearing condition (S.5.15), our equality between income and expenditure in equation (S.5.14) simplifies to:

$$\left(w_{it}\ell_{it}^{T}+r_{it}k_{it}^{T}\right)=\gamma\sum_{n=1}^{N}S_{nit}\left(w_{nt}\ell_{nt}+r_{nt}k_{nt}\right). \tag{S.5.16}$$

Now note that factor market clearing implies:

$$\left(w_{it}\ell_{it}^{T}+r_{it}k_{it}^{T}\right)+\left(w_{it}\ell_{it}^{NT}+r_{it}k_{it}^{NT}\right)=\left(w_{it}\ell_{it}+r_{it}k_{it}\right). \tag{S.5.17}$$

Combining this factor market clearing condition (S.5.17) with non-traded goods market clearing (S.5.15), total payments for factors of production in the traded sector are also a constant share of total factor payments:

$$\left(w_{it}\ell_{it}^{T}+r_{it}k_{it}^{T}\right)=\gamma\left(w_{nt}\ell_{nt}+r_{nt}k_{nt}\right) \tag{S.5.18}$$

Using this result, the goods market clearing condition (S.5.16) can be re-written as:

$$\left(w_{it}\ell_{it}+r_{it}k_{it}\right)=\sum_{n=1}^{N}S_{nit}\left(w_{nt}\ell_{nt}+r_{nt}k_{nt}\right). \tag{S.5.19}$$

Additionally, from profit maximization and zero-profits, capital payments are the same constant multiple of labor payments in each sector:

$$r_{it}k_{it}^{T}=\frac{1-\mu}{\mu}w_{it}\ell_{it}^{T}, \tag{S.5.20}$$

$$r_{it}k_{it}^{NT}=\frac{1-\mu}{\mu}w_{it}\ell_{it}^{NT}. \tag{S.5.21}$$

Combining these results with factor market clearing, we obtain:

$$r_{it}k_{it}=\frac{1-\mu}{\mu}w_{it}\ell_{it}. \tag{S.5.22}$$

Using this property that capital payments are a constant multiple of labor payments, the goods market clearing condition (S.5.19) simplifies to:

$$w_{it}\ell_{it}=\sum_{n=1}^{N}S_{nit}w_{nt}\ell_{nt}. \tag{S.5.23}$$

Finally, combining the relationships between capital and labor payments in each sector in equations (S.5.20) and (S.5.21), with our earlier results in equations (S.5.15) and (S.5.18) that total factor payments in each sector are a constant multiple of total factor payments, we find that a constant share of each location's labor and capital is allocated to the traded and non-traded sectors:

$$\ell_{it}^{T} = \gamma \ell_{it}, \qquad \ell_{it}^{NT} = (1-\gamma)\,\ell_{it}, \tag{S.5.24}$$

$$k_{it}^{T} = \gamma k_{it}, \qquad k_{it}^{NT} = (1-\gamma)\,k_{it}. \tag{S.5.25}$$

S.5.6 General Equilibrium

Given the state variables $\{\ell_{i0},\, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and saving decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables. In particular, we immediately recover the sectoral allocation of labor and capital from equations (S.5.24) and (S.5.25).

Capital Accumulation: Using capital market clearing (S.5.22), the price index (S.5.2), the price index for traded goods (S.5.3), and the equilibrium pricing rules (S.5.5) and (S.5.6), the capital accumulation equation (S.5.13) becomes:

$$k_{it+1} = \beta \frac{1-\mu}{\mu} \frac{w_{it}}{p_{it}} \ell_{it} + \beta\,(1-\delta)\,k_{it}, \tag{S.5.26}$$

$$p_{it} = \left(p_{it}^{T}\right)^{\gamma} \left(p_{it}^{NT}\right)^{1-\gamma}, \tag{S.5.27}$$

$$p_{nt}^{T} = \left[\sum_{i=1}^{N} \left(w_{it} \left(\frac{1-\mu}{\mu}\right)^{1-\mu} \left(\ell_{it}/k_{it}\right)^{1-\mu} \tau_{ni}/z_i \right)^{-\theta}\right]^{-1/\theta}, \tag{S.5.28}$$

$$p_{nt}^{NT} = w_{nt} \left(\frac{1-\mu}{\mu}\right)^{1-\mu} \left(\ell_{nt}/k_{nt}\right)^{1-\mu} / z_{nt}. \tag{S.5.29}$$

Goods Market Clearing: Using the equilibrium pricing rule in the traded sector (S.5.5), the expenditure share (S.5.4) and capital market clearing condition (S.5.22) in the goods market clearing condition (S.5.23), we obtain:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}, \tag{S.5.30}$$

$$S_{nit} = \frac{\left(w_{it} \left(\ell_{it}/k_{it}\right)^{1-\mu} \tau_{ni}/z_i\right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt} \left(\ell_{mt}/k_{mt}\right)^{1-\mu} \tau_{nm}/z_m\right)^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it}\ell_{it}}, \tag{S.5.31}$$

where S_{nit} is the expenditure share of importer n on exporter i at time t; we have defined T_{int} as the corresponding income share of exporter i from importer n at time t; and note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the analogous derivations for migration decisions as in our baseline Armington model, the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{S.5.32}$$

$$D_{igt} = \frac{\left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^w\right) / \kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{mt+1}^w\right) / \kappa_{mit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \tag{S.5.33}$$

where D_{igt} is the outmigration probability from location i to location g between time t and $t+1$; we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$; and again note that the order of subscripts switches between the out-migration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the analogous derivations for migration decisions as in our baseline Armington model, the expected value from living in location n at time t can be written as:

$$v_{nt}^w = \ln b_{nt} + \ln\left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta \mathbb{E}_t v_{gt+1}^w\right) / \kappa_{gnt}\right)^{1/\rho}. \tag{S.5.34}$$

S.5.7 Comparative Statics

We now totally differentiate the conditions for general equilibrium to obtain comparative static expressions that we use in our sufficient statistics for changes in steady-state and the entire transition path. In the interests of brevity, we focus on differences from the specification in our baseline single-sector Armington model.

Prices Totally differentiating the pricing rules in the traded and non-traded sectors, we have:

$$\mathrm{d}\ln p_{nit} = \mathrm{d}\ln \tau_{nit} + \mathrm{d}\ln w_{it} - (1-\mu)\, \mathrm{d}\ln \chi_{it} - \mathrm{d}\ln z_{it}. \tag{S.5.35}$$

$$\mathrm{d}\ln p_{nt}^{NT} = \mathrm{d}\ln \tau_{nit} + \mathrm{d}\ln w_{it} - (1-\mu)\, \mathrm{d}\ln \chi_{it} - \mathrm{d}\ln z_{it} \tag{S.5.36}$$

Price Indices Totally differentiating the consumption goods price index in equation (S.5.2), we have:

$$\mathrm{d}\ln p_{nt} = \gamma\, \mathrm{d}\ln p_{nt}^{T} + (1-\gamma)\, \mathrm{d}\ln p_{nt}^{NT}. \tag{S.5.37}$$

$$\mathrm{d}\ln p_{nt}^{T} = \sum_{m=1}^{N} S_{nmt}\, \mathrm{d}\ln p_{nmt}^{T}.$$

Real Income. Totally differentiating real income we have:

$$\mathrm{d}\ln\left(\frac{w_{it}}{p_{it}}\right) = \mathrm{d}\ln w_{it} - \mathrm{d}\ln p_{it},$$

$$\mathrm{d}\ln\left(\frac{w_{it}}{p_{it}}\right) = \mathrm{d}\ln w_{it} - \gamma\sum_{m=1}^{N} S_{imt}\,\mathrm{d}\ln p_{imt} - (1-\gamma)\ \mathrm{d}\ln p_{it}^{NT},$$

$$\mathrm{d}\ln\left(\frac{w_{it}}{p_{it}}\right) = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \gamma\sum_{m=1}^{N} S_{imt}\left[\mathrm{d}\ln\tau_{imt} + \mathrm{d}\ln w_{mt} - (1-\mu)\ \mathrm{d}\ln\chi_{mt} - \mathrm{d}\ln z_{mt}\right] \\ -(1-\gamma)\left[\mathrm{d}\ln w_{it} - (1-\mu)\ \mathrm{d}\ln\chi_{it} - \mathrm{d}\ln z_{it}\right] \end{array}\right]. \tag{S.5.38}$$

Goods Market Clearing Totally differentiating the goods market clearing condition (S.5.30), we obtain the same expression as in our baseline single-sector Armington model:

$$\left[\begin{array}{c} \mathrm{d}\ln w_{it} \\ +\mathrm{d}\ln\ell_{it} \end{array}\right] = \left[\begin{array}{c} \sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln w_{nt} + \mathrm{d}\ln\ell_{nt}\right) \\ +\theta\sum_{n=1}^{N}\sum_{m=1}^{N} T_{int}S_{nmt}\left(\mathrm{d}\ln\tau_{nmt} + \mathrm{d}\ln w_{mt} - (1-\mu)\ \mathrm{d}\ln\chi_{mt} - \mathrm{d}\ln z_{mt}\right) \\ -\theta\sum_{n=1}^{N} T_{int}\left(\mathrm{d}\ln\tau_{nit} + \mathrm{d}\ln w_{it} - (1-\mu)\ \mathrm{d}\ln\chi_{it} - \mathrm{d}\ln z_{it}\right) \end{array}\right]. \tag{S.5.39}$$

Value Function. Note that the value function (S.5.34) can be re-written using the following results:

$$v_{it} = \ln\left[\frac{w_{it}}{\left(p_{it}^{T}\right)^{\gamma}\left(p_{it}^{NT}\right)^{1-\gamma}}\right] + \ln b_{it} + \rho\ln\sum_{g=1}^{N}\left(\exp\left(\beta\mathbb{E}_t v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho},$$

$$p_{it}^{T} = \left[\sum_{m=1}^{N} p_{imt}^{-\theta}\right]^{-1/\theta} = \left(\frac{p_{iit}^{-\theta}}{S_{iit}}\right)^{-1/\theta}, \qquad \tau_{iit} = 1,$$

$$\sum_{g=1}^{N}\left(\exp\left(\beta\mathbb{E}_t v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho} = \frac{\left(\exp\left(\beta\mathbb{E}_t v_{it+1}\right)/\kappa_{iit}\right)^{1/\rho}}{D_{iit}}, \qquad \kappa_{iit} = 1,$$

$$v_{it} = \ln w_{it} - \frac{\gamma}{\theta}\ln S_{iit} - \gamma\ln p_{iit} - (1-\gamma)\ln p_{it}^{NT} + \ln b_{it} + \beta\mathbb{E}_t v_{it+1} - \rho\ln D_{iit}.$$

Totally differentiating this expression for the value function, we have:

$$\mathrm{d}v_{it} = \mathrm{d}\ln w_{it} - \frac{\gamma}{\theta}\,\mathrm{d}\ln S_{iit} - \gamma\,\mathrm{d}\ln p_{iit} - (1-\gamma)\ \mathrm{d}\ln p_{it}^{NT} + \mathrm{d}\ln b_{it} + \beta\,\mathrm{d}v_{it+1} - \rho\,\mathrm{d}\ln D_{iit},$$

where

$$\mathrm{d}\ln S_{iit} = -\theta\,\mathrm{d}\ln p_{iit} + \theta\left[\sum_{m=1}^{N} S_{imt}\,\mathrm{d}\ln p_{imt}\right],$$

$$\mathrm{d}\ln D_{iit} = \frac{1}{\rho}\left[\beta\mathbb{E}_t\,\mathrm{d}v_{it+1} - \mathrm{d}\ln\kappa_{iit} - \sum_{m=1}^{N} D_{imt}\left(\beta\mathbb{E}_t v_{mt+1} - \mathrm{d}\ln\kappa_{mit}\right)\right].$$

Using these results for $\mathrm{d}\ln S_{iit}$ and $\mathrm{d}\ln D_{iit}$ in the expression for $\mathrm{d}v_{it}$ above, we have:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \gamma\sum_{m=1}^{N} S_{imt}\,\mathrm{d}\ln p_{imt} - (1-\gamma)\ \mathrm{d}\ln p_{it}^{NT} \\ +\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1} - \mathrm{d}\ln\kappa_{mit}\right) \end{array}\right],$$

where we have used $\mathrm{d}\ln\kappa_{iit} = 0$. Using the total derivative of the pricing rules (S.5.35) and (S.5.36), we can re-write this derivative of the value function as follows:

$$\mathrm{d}v_{it} = \left[\begin{array}{c} \mathrm{d}\ln w_{it} - \gamma\sum_{m=1}^{N} S_{imt}\left(\mathrm{d}\ln\tau_{nmt} + \mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln\chi_{mt} - \mathrm{d}\ln z_{mt}\right) \\ (1-\gamma)\left(\mathrm{d}\ln w_{mt} - (1-\mu)\,\mathrm{d}\ln\chi_{mt} - \mathrm{d}\ln z_{mt}\right) \\ +\,\mathrm{d}\ln b_{it} + \sum_{m=1}^{N} D_{imt}\left(\beta\mathbb{E}_t\,\mathrm{d}v_{mt+1} - \mathrm{d}\ln\kappa_{mit}\right) \end{array} \right]. \tag{S.5.40}$$

S.5.8 Steady-State Sufficient Statistics

Suppose that the economy starts from an initial steady-state with constant values of the endogenous variables: $k_{it+1} = k_{it} = k_i^*$, $\ell_{it+1} = \ell_{it} = \ell_i^*$, $w_{it+1}^* = w_{it}^* = w_i^*$ and $v_{it+1}^* = v_{it}^* = v_i^*$, where we use an asterisk to denote a steady-state value, and drop the time subscript for the remainder of this subsection, since we are concerned with steady-states. We consider small shocks to productivity ($\mathrm{d}\ln\boldsymbol{z}$) and amenities ($\mathrm{d}\ln\boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln\bar{\ell} = 0$), trade costs ($\mathrm{d}\ln\boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln\boldsymbol{\kappa} = 0$).

Capital Accumulation. From the capital accumulation equation (S.5.26), the steady-state stock of capital solves:

$$(1-\beta(1-\delta))\,\chi_i^* = (1-\beta(1-\delta))\,\frac{k_i^*}{\ell_i^*} = \beta\frac{1-\mu}{\mu}\frac{w_i^*}{p_i^*}.$$

Totally differentiating, we have:

$$\mathrm{d}\ln\chi_i^* = \mathrm{d}\ln\left(\frac{w_i^*}{p_i^*}\right).$$

Using the total derivative of real income (S.5.38) above, this becomes:

$$\mathrm{d}\ln\chi_i^* = \left[\begin{array}{c} \mathrm{d}\ln w_i^* - \gamma\sum_{m=1}^{N} S_{im}^*\left[\mathrm{d}\ln w_m^* - (1-\mu)\,\mathrm{d}\ln\chi_m^* - \mathrm{d}\ln z_m\right] \\ -\,(1-\gamma)\left[\mathrm{d}\ln w_m^* - (1-\mu)\,\mathrm{d}\ln\chi_m^* - \mathrm{d}\ln z_m\right] \end{array} \right],$$

where we have used and $\mathrm{d}\ln\tau_{nm} = 0$. This relationship has the matrix representation:

$$\mathrm{d}\ln\boldsymbol{\chi}^* = \left[\begin{array}{c} \mathrm{d}\ln\boldsymbol{w}^* - \left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\,\mathrm{d}\ln\boldsymbol{w}^* \\ +\,(1-\mu)\left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\,\mathrm{d}\ln\boldsymbol{\chi}^* + \left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\,\mathrm{d}\ln\boldsymbol{z} \end{array} \right],$$

$$\left(\boldsymbol{I} - (1-\mu)\left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right)\,\mathrm{d}\ln\boldsymbol{\chi}^* = \left[\begin{array}{c} \left(\boldsymbol{I} - \left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right)\,\mathrm{d}\ln\boldsymbol{w}^* \\ +\left[\gamma\boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\mathrm{d}\ln\boldsymbol{z} \end{array} \right]. \tag{S.5.41}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.5.39) has the following matrix representation:

$$\mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t} = \boldsymbol{T}\left(\mathrm{d}\ln\boldsymbol{w_t} + \mathrm{d}\ln\boldsymbol{\ell_t}\right) + \theta\left(\boldsymbol{TS} - \boldsymbol{I}\right)\left(\mathrm{d}\ln\boldsymbol{w_t} - (1-\mu)\,\mathrm{d}\ln\boldsymbol{\chi_t} - \mathrm{d}\ln\boldsymbol{z}\right),$$

where we have used $\mathrm{d}\ln\boldsymbol{\tau} = 0$. We can re-write this relationship as:

$$\left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]\,\mathrm{d}\ln\boldsymbol{w_t} = -\left(\boldsymbol{I} - \boldsymbol{T}\right)\,\mathrm{d}\ln\boldsymbol{\ell_t} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln\boldsymbol{\chi_t}\right).$$

In steady-state we have:

$$\left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]\,\mathrm{d}\ln\boldsymbol{w}^* = \left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\,\mathrm{d}\ln\boldsymbol{\ell}^* + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\left(\mathrm{d}\ln\boldsymbol{z} + (1-\mu)\,\mathrm{d}\ln\boldsymbol{\chi}^*\right)\right]. \tag{S.5.42}$$

Population Flow. The total derivative of the population flow condition has the same matrix representation as in our baseline single-sector Armington model:

$$\mathrm{d}\ln \boldsymbol{\ell_{t+1}} = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell_t} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}}.$$

In steady-state, we have:

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell}^* + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\,\mathrm{d}\boldsymbol{v}^*. \tag{S.5.43}$$

Value function. The total derivative of the value function (S.5.40) has the following matrix representation:

$$\mathrm{d}\boldsymbol{v_t} = \left(\boldsymbol{I} - \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right)\,\mathrm{d}\ln \boldsymbol{w_t} + \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\,\mathrm{d}\ln \boldsymbol{\chi_t}\right) + \,\mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D}\mathbb{E}_t\,\mathrm{d}\boldsymbol{v_{t+1}},$$

where we have used $\,\mathrm{d}\ln \boldsymbol{\tau} = \,\mathrm{d}\ln \boldsymbol{\kappa} = 0$. In steady-state, we have:

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right)\,\mathrm{d}\ln \boldsymbol{w}^* + \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) + \,\mathrm{d}\ln \boldsymbol{b} + \beta \boldsymbol{D}\,\mathrm{d}\boldsymbol{v}^*. \tag{S.5.44}$$

System of Steady-State Equations. Collecting together the system of steady-state equations, we have:

$$\mathrm{d}\ln \boldsymbol{\chi}^* = \left[\boldsymbol{I} - (1-\mu)\left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right]^{-1}\left[\begin{array}{c} \left(\boldsymbol{I} - \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\right)\,\mathrm{d}\ln \boldsymbol{w}^* \\ + \left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\,\mathrm{d}\ln \boldsymbol{z} \end{array}\right]. \tag{S.5.45}$$

$$\mathrm{d}\ln \boldsymbol{w}^* = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\,\mathrm{d}\ln \boldsymbol{\ell}^* + \left(\boldsymbol{I} - \boldsymbol{TS}\right)\theta\left(\,\mathrm{d}\ln \boldsymbol{z} + (1-\mu)\,\,\mathrm{d}\ln \boldsymbol{\chi}^*\right)\right]. \tag{S.5.46}$$

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\,\mathrm{d}\boldsymbol{v}^*\boldsymbol{.} \tag{S.5.47}$$

$$\mathrm{d}\boldsymbol{v}^* = \left(\boldsymbol{I} - \beta \boldsymbol{D}\right)^{-1}\left[\begin{array}{c} \mathrm{d}\ln \boldsymbol{w}^* + \,\mathrm{d}\ln \boldsymbol{b} \\ -\left[\gamma \boldsymbol{S} + (1-\gamma)\,\boldsymbol{I}\right]\left(\,\mathrm{d}\ln \boldsymbol{w}^* - \,\mathrm{d}\ln \boldsymbol{z} - (1-\mu)\,\,\mathrm{d}\ln \boldsymbol{\chi}^*\right) \end{array}\right]. \tag{S.5.48}$$

As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a numeraire in order for solve for changes in wages. We choose the total income of all locations as our numeraire ($\sum_{i=1}^{N} w_i^* \ell_i^* = \sum_{i=1}^{N} q_i^* = \overline{q} = 1$), which implies that the log changes in incomes satisfy $\boldsymbol{q}^*\,\mathrm{d}\ln \boldsymbol{q}^* = \sum_{i=1}^{N} q_i^*\,\mathrm{d}\ln q_i^* = \sum_{i=1}^{N} q_i^* \frac{\mathrm{d}q_i^*}{q_i^*} = \sum_{i=1}^{N} \,\mathrm{d}q_i^* = 0$, where $\boldsymbol{q}^*$ is a row vector of the steady-state income of each location. Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_i = \overline{\ell} = 1$, which implies $\boldsymbol{\ell}^*\,\mathrm{d}\ln \boldsymbol{\ell}^* = \sum_{i=1}^{N} \ell_i^*\,\mathrm{d}\ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{\mathrm{d}\ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} \,\mathrm{d}\ell_i^* = 0$, where $\boldsymbol{\ell}^*$ is a row vector of the steady-state population of each location.

S.5.9 Sufficient Statistics for Transition Dynamics

We suppose that the economy starts from an initial steady-state distribution of economic activity $\{k_i^*, \ell_i^*, w_i^*, v_i^*\}$. We consider small shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$) in each location, holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \bar{\ell}$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$). We use a tilde above a variable to denote a log deviation from the initial steady-state, such that $\widetilde{\chi}_{it} = \ln \chi_{it} - \ln \chi_i^*$, for all variables except for the worker value function v_{it}; with a slight abuse of notation we use $\widetilde{v}_{it} \equiv v_{it} - v_i^*$ to denote the deviation in levels for the worker value function.

Capital Accumulation. Following the same line of argument in our baseline single-sector Armington model, the log deviation of the capital-labor ratio from steady-state can be written as:

$$\ln\left(\frac{\chi_{it+1}}{\chi_i^*}\right) + \ln\left(\frac{\ell_{it+1}/\ell_i^*}{\ell_{it}/\ell_i^*}\right) = \beta\left(1-\delta\right)\ln\left(\frac{\chi_{it}}{\chi_i^*}\right) + \left(1-\beta\left(1-\delta\right)\right)\ln\left(\frac{w_{it}/w_i^*}{p_{it}/p_i^*}\right),$$

which can be re-written as:

$$\widetilde{\chi}_{it+1} = \beta\left(1-\delta\right)\widetilde{\chi}_{it} + \left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{w}_{it} - \widetilde{p}_{it}\right) - \widetilde{\ell}_{it+1} + \widetilde{\ell}_{it}.$$

We can re-write this relationship in matrix form as:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \beta\left(1-\delta\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}}\right) - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}. \tag{S.5.49}$$

Taking the total derivative of real income relative to the initial steady-state, we have:

$$\widetilde{w}_{it} - \widetilde{p}_{it} = \left[\begin{array}{c} \widetilde{w}_{it} - \gamma\sum_{m=1}^{N} S_{imt}\left[\widetilde{w}_{mt} - \left(1-\mu\right)\widetilde{\chi}_{mt} - \widetilde{z}_m\right] \\ -\left(1-\gamma\right)\left[\widetilde{w}_{it} - \left(1-\mu\right)\widetilde{\chi}_{it} - \widetilde{z}_i\right] \end{array}\right],$$

where we have used $\mathrm{d}\ln \tau_{nm} = 0$. We can re-write this relationship in matrix form as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{p}}_{\boldsymbol{t}} = \left(\boldsymbol{I} - \left[\gamma \boldsymbol{S} + \left(1-\gamma\right)\boldsymbol{I}\right]\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left(1-\mu\right)\left[\gamma\boldsymbol{S} + \left(1-\gamma\right)\boldsymbol{I}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}}.$$

Using this result in our expression for the dynamics of the capital-labor ratio above, we have:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\mu\right)\left[\gamma\boldsymbol{S} + \left(1-\gamma\right)\boldsymbol{I}\right]\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ +\left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I} - \left[\gamma\boldsymbol{S} + \left(1-\gamma\right)\boldsymbol{I}\right]\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} \\ +\left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.5.50}$$

Goods Market Clearing. The total derivative of the goods market clearing condition (S.5.39) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} = \boldsymbol{T}\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \theta\left(\boldsymbol{TS} - \boldsymbol{I}\right)\left(\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} - \left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{z}}\right),$$

where we have used $\mathrm{d}\ln \boldsymbol{\tau} = 0$. We can re-write this relationship as:

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I} - \boldsymbol{TS}\right)\left(\widetilde{\boldsymbol{z}} + \left(1-\mu\right)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\right)\right]. \tag{S.5.51}$$

Population Flow. Following the same line of argument in our baseline single-sector Armington model, the total derivative of the population flow condition relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.5.52}$$

Value Function. The total derivative of the value function (S.5.40) relative to the initial steady-state has the following matrix representation:

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{l} \left(\boldsymbol{I} - \left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\widetilde{\boldsymbol{z}} \\ + (1-\mu)\left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array}\right], \tag{S.5.53}$$

where we have used $\mathrm{d}\ln\boldsymbol{\tau} = \mathrm{d}\ln\boldsymbol{\kappa} = 0$.

System of Equations for Transition Dynamics Relative to the Initial Steady-State. Collecting together the capital accumulation equation (S.5.50), the goods market clearing condition (S.5.51), the population flow condition (S.5.52), and the value function (S.5.53), the system of equations for the transition dynamics relative to the initial steady-state takes the following form:

$$\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta(1-\delta)\boldsymbol{I} + (1-\beta(1-\delta))(1-\mu)\left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} \\ + (1-\beta(1-\delta))\left(\boldsymbol{I} - \left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} \\ + (1-\beta(1-\delta))\left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\widetilde{\boldsymbol{z}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right]. \tag{S.5.54}$$

$$\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]^{-1}\left[-\left(\boldsymbol{I} - \boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\left(\widetilde{\boldsymbol{z}} + (1-\mu)\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}}\right)\right]. \tag{S.5.55}$$

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}. \tag{S.5.56}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\begin{array}{l} \left(\boldsymbol{I} - \left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\widetilde{\boldsymbol{z}} \\ + (1-\mu)\left[\gamma\boldsymbol{S} + (1-\gamma)\boldsymbol{I}\right]\widetilde{\boldsymbol{\chi}}_{\boldsymbol{t}} + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\mathbb{E}_t\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} \end{array}\right]. \tag{S.5.57}$$

S.6 Additional Empirical Results

In this section of the Online Supplement, we report additional empirical results that are discussed in the paper. Subsection S.6.1 shows that individual U.S. states differ substantially in terms of the dynamics of their capital-labor ratios, highlighting the empirical relevance of capital accumulation for income convergence. Subsection S.6.2 provides evidence of substantial net migration between U.S. states, highlighting the empirical salience of migration for the population dynamics of U.S. states. Subsection S.6.3 show that the model's gravity equation predictions provide a good approximation to the observed data on trade and migration flows.

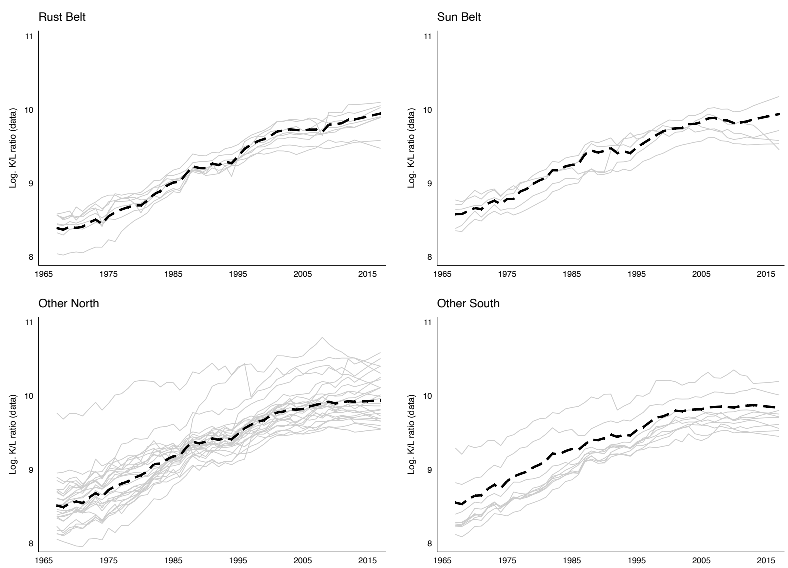
Subsection S.6.4 examines the evolution of the real interest rate in terms of the local consumption price index along the transition path to steady-state. Subsection S.6.5 reports additional evidence on the predictive power of convergence to the initial steady-state for the observed population growth of U.S. states. Subsection S.6.6 reports additional empirical results for our spectral analysis in Section 5.4 of the paper. Subsection S.6.7 provides further information about the implied fundamentals from inverting the non-linear model. Subsection S.6.8 reports additional empirical results for our multi-sector extension that is discussed in Section 5.5 of the paper.

S.6.1 Capital Dynamics

We introduce forward-looking capital accumulation into the dynamic discrete choice model of migration of Caliendo et al. (2019). In this section of the Online Supplement, we show that U.S. states differ substantially in terms of the dynamics of their capital-labor ratios over our sample period, highlighting the empirical relevance of capital accumulation for income convergence.

In Figure S.6.1, we show the capital-labor over time for each U.S. state using the solid gray lines. We also show the population-share weighted average of these capital-labor ratios for our four geographical groupings using the black dashed lines. The capital-labor ratio is measured as the ratio of the real capital stock to population. While all U.S. states experience an increase in the capital-labor ratio, the rate of increase differs substantially across states, implying that capital accumulation plays an important role in regional income convergence. We find substantial heterogeneity in the trends in the capital-labor ratio across states within all four of our geographical groupings, with this heterogeneity greatest for the Other Northern and Other Southern states.

Figure S.6.1: Capital-Labor Ratios for U.S. States over Time



Note: Gray lines show capital-labor ratios for each U.S. state and year; black dashed lines show the population-weighted average of these steady-state gaps for the four geographical regions of the Rust Belt, Sun Belt, Other Northern States and Other Southern States, as defined in the main text; capital-labor ratios measured as the ratio of the real capital stock to population.

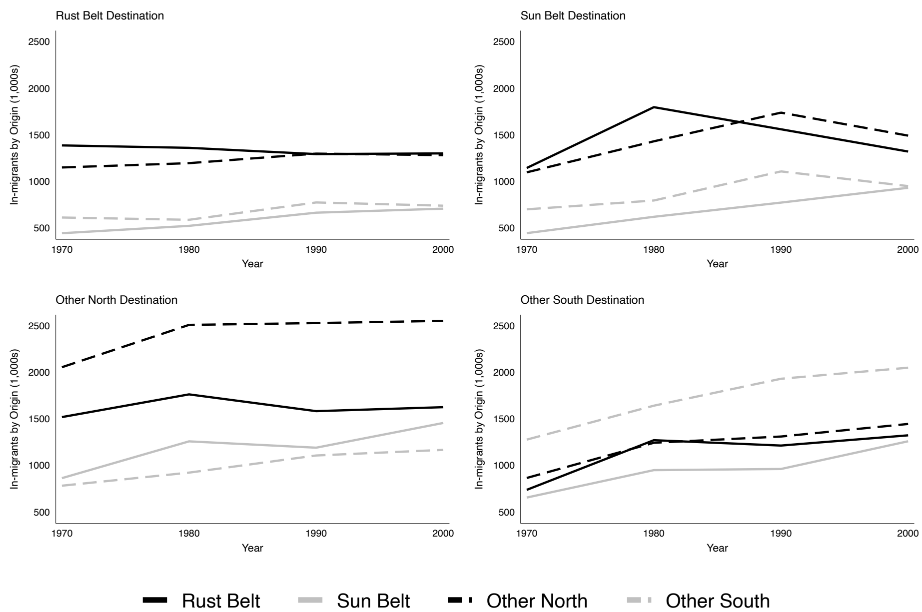
S.6.2 Migration Flows

In this section of the Online Supplement, we provide evidence on the role of internal migration as a source of population changes for the four groups of states. Internal migration is measured as movements of people between states within the United States and excludes international migration. We focus for brevity on in-migrants, measured as inflows of internal migrants (in thousands) into each state, separated out by origin state.

In Figure S.6.2, we show in-migration flows for our four geographical groupings of states. Three features are noteworthy. First, geographical proximity matters for migration flows, such that other Rust Belt states are one of the leading sources of in-migrants for the Rust Belt (top-left panel), consistent with our model's gravity equation predictions. Second, all groups of states receive non-negligible in-migration flows, such that gross migration flows are larger than net migration flows, in line with the idiosyncratic mobility shocks in our model. Third, despite the role for geography, the Rust Belt and Other Northern states are the two largest sources of in-migrants for the Sun Belt, consistent with internal migration contributing to the observed reorientation of population shares.

Finally, although not shown in these figures, we find a modest decline in rates of internal migration between states in the later years of our sample, which is in line the findings of a number of studies, including Kaplan and Schulhofer-Wohl (2017) and Molloy et al. (2011). Consistent with the comparison of several different sources of administrative data in Hyatt et al. (2018), we find that this decline in rates of internal migration between states is smaller in the population census data than in Current Population Survey (CPS) data.

Figure S.6.2: Internal In-migration to Each Destination Region by Source Region from 1960-2000



Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. Other Southern all other former members of the Confederacy. Other Northern all other Union states during the Civil War

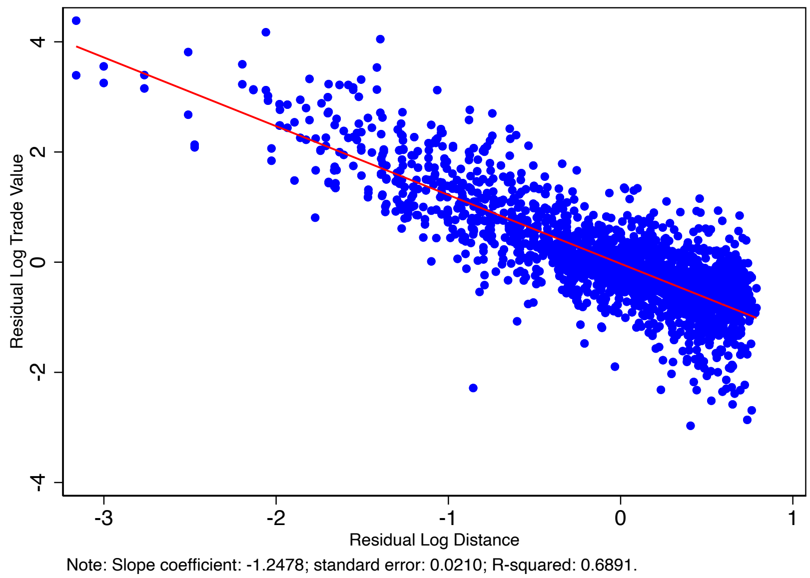
Notes: Internal in-migration to each destination region by source region from 1960-2000; internal migration includes all movements of people between states within the United States and excludes international migration.

S.6.3 Gravity in Trade and Migration

In this section of the Online Supplement, we show that bilateral flows of goods and migrants between U.S. states both exhibit strong gravity equations, as predicted by our theoretical framework.

We begin with the gravity equation for the bilateral value of trade in goods, where we model bilateral trade costs as a constant elasticity function of bilateral geographical distance, measured as the Great Circle distance between the population centers of states. First, we regress the log bilateral value of trade on origin and destination fixed effects, and generate the residuals. Second, we regress log bilateral geographical distance on origin and destination fixed effects, and generate the residuals. Third, we display the two sets of residuals against one another and the linear regression relationship between them, using the Frisch-Waugh-Lovell theorem. As shown in Figure S.6.3, we find a strong, negative and statistically significant and approximately log linear conditional correlation between the bilateral value of trade and bilateral geographical distance, consistent with our model's gravity equation predictions for goods trade.

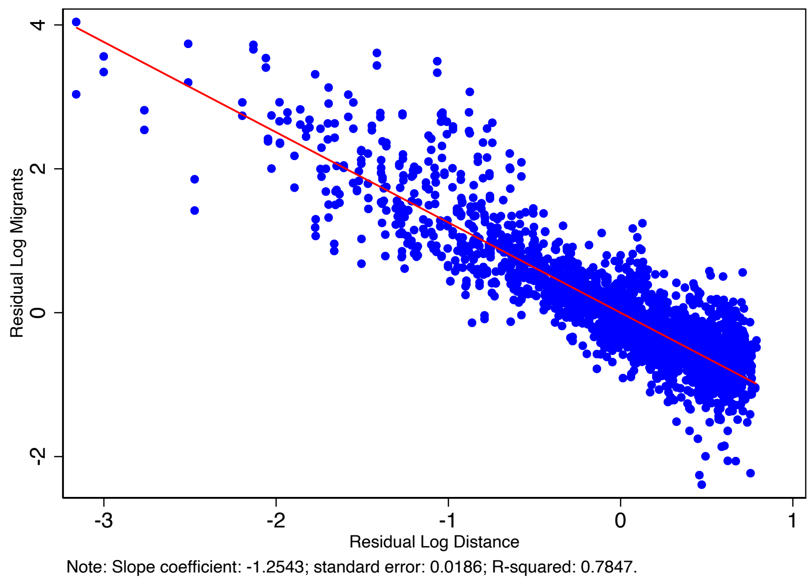
Figure S.6.3: Gravity Equation for the Bilateral Value of Goods Trade in 2017



Notes: Conditional correlation between the log bilateral value of goods trade between U.S. states and the log of bilateral distance between the population centers of U.S. states; residual log trade value and residual log distance from conditioning on origin and destination fixed effects.

We next turn to the gravity equation for bilateral migration flows, where we again model bilateral migration costs as a constant elasticity function of bilateral geographical distance. First, we regress log bilateral migration on origin and destination fixed effects, and generate the residuals. Second, we regress log bilateral geographical distance on origin and destination fixed effects, and generate the residuals. Third, we display the two sets of residuals against one another and the linear regression relationship between them, using the Frisch-Waugh-Lovell theorem. As shown in Figure S.6.4, we find a strong, negative and statistically significant and approximately log linear conditional correlation between bilateral migration and bilateral geographical distance, consistent with our model's gravity equation predictions for migration flows.

Figure S.6.4: Gravity Equation for Bilateral Migration in 2000



Notes: Conditional correlation between log bilateral migration flows between U.S. states and the log of bilateral distance between the population centers of U.S. states; residual log migrants and residual log distance from conditioning on origin and destination fixed effects.

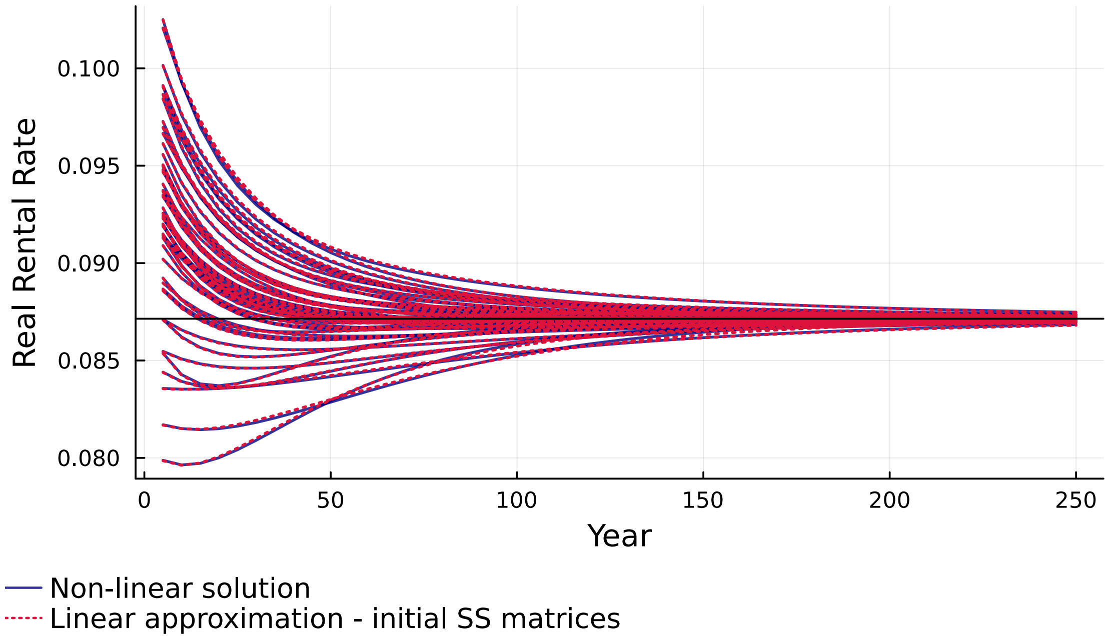
Taken together, these results confirm that the gravity equation is a strong empirical feature of both bilateral goods trade and bilateral migration flows between U.S. states, as predicted by our theoretical framework.

S.6.4 Real Interest Rate

Our baseline specification develops a tractable theoretical framework for incorporating forward-looking investment into a dynamic discrete choice migration model that overcomes the challenge of a high-dimensional state space. We assume that capital is geographically immobile once installed, and that landlords can only invest in their own location, which generates gradual adjustment in local capital because of consumption smoothing. While adjustment costs provide an alternative potential explanation for gradual adjustment in local capital, our approach is analytically tractable, and we show in this section that for standard values of model parameters it implies only small differences across locations in the real rental rate in terms of the consumption good along the transition path to steady-state.

We first use our inversion of the non-linear model from our generalization of dynamic exact-hat algebra in Proposition 2 in the paper to recover the implied empirical distribution of productivities, amenities, trade costs and migration costs, as discussed in Online Supplement S.2.1. We next solve for the steady-state of the non-linear model implied by the 1990 values of these fundamentals $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. Starting from this steady-state, we then undertake counterfactuals for the economy's transition path in response to the empirical distribution of productivity shocks from 1990-2000, which includes substantial changes in relative productivity ranging from around -30 to 30 percent. We solve for this transition path in both the non-linear model using Proposition 2 in the paper and the linearized model using Proposition 3 in the paper.

Figure S.6.5: Real Rental Rate along the Transition Path to Steady-State



Note: We solve for the steady-state of the non-linear model implied by the 1990 values of these fundamentals $\{z_i$, b_i, τ_{ni}, $\kappa_{ni}\}$. Starting from this steady-state, we then undertake counterfactuals for the economy's transition path in response to the empirical distribution of productivity shocks from 1990-2000, which includes substantial changes in relative productivity ranging from around -30 to 30 percent. We solve for this transition path in both the non-linear model using Proposition 2 in the paper and the linearized model using Proposition 3 in the paper. The figure shows the real rental rate in terms of the consumption index (r_{it}/p_{it}) in each U.S. state along the transition path to steady-state.

In Figure S.6.5, we show the real rental rate in terms of the consumption index (r_{it}/p_{it}) in each U.S. state along the transition path to steady-state, where the solid blue line shows the non-linear model solution and the dashed red line shows the linearized solution. Despite the substantial changes in relative productivity for these decadal shocks from 1990-2000, we find relatively small differences in the real rental rate (r_{it}/p_{it}) across locations along the transition path.

In steady-state, there is common real rental rate in terms of the consumption good across all locations: $r_i^*/p_i^* = r^*/p^* = (1 - \beta(1 - \delta))/\beta$. In Online Supplement S.4.8, we develop an extension of this baseline specification, in which we allow landlords to invest in other locations subject to financial frictions, and bilateral investment flows satisfy a gravity equation.

S.6.5 Convergence to Steady-state

In Sections 5.2-5.3 of the paper, we provide evidence on the decline in the rate of income convergence across U.S. states over time. In this section of the Online Supplement, we provide additional evidence on patterns of income convergence over time. In Subsection S.6.5.1, we present evidence on the evolution of steady-state population gaps for each state over time. In Subsection S.6.5.2, we provide further evidence on the relationship between predicted population growth based on initial conditions and actual population growth.

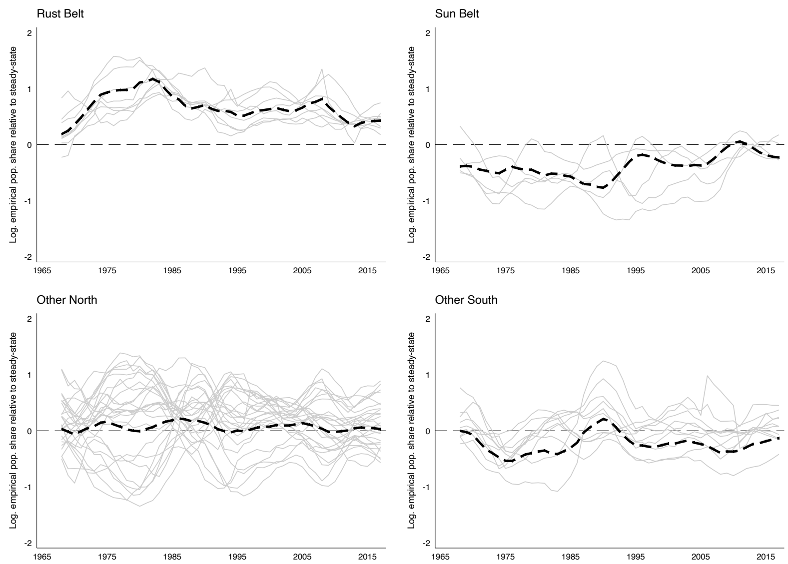
S.6.5.1 Steady-State Gaps in Population Shares

We compute steady-state gaps in population shares using the following two steps. First, we compute the implied steady-state population share for each U.S. state and year, by using our generalization of exact-hat algebra in Proposition 2 in the paper to solve for the economy's transition path in the absence of further changes in fundamentals, given the observed values of the labor and capital state variables in each year. Second, we calculate the implied steady-state gap in population shares for each year separately, measured as the log-ratio of the actual population share to the steady-state population share.

In Figure S.6.6, we display the evolution of these steady-state gaps for each U.S. state using the solid gray lines. We also show the population-share weighted average of these steady-state gaps using the black dashed lines for our four broad geographical regions: The Rust Belt; the Sun Belt; Other Northern States; and Other Southern States. Three main features are apparent. First, the population shares of the Rust Belt states were substantially above steady-state, and the population shares of the Sun Belt states were substantially below steady-state, even at the beginning of our sample period. Second, Rust Belt states move substantially further away from steady-state from the mid-1960s until around 1980, which in the model is driven by shocks to fundamentals, such as productivity and amenities. Third, both individual states and these four geographical regions remain persistently away from steady-state for decades, consistent with slow convergence towards steady-state.

We find that these deviations from steady-state for both Rust and Sun Belt states are substantial. For example, in 1975, the population-weighted average of the gaps of population shares from steady-state for the Rust Belt and Sun Belt groups of states are 99 and -41 percent, respectively. By 2015, the corresponding population-weighted averages of the gaps of population shares from steady-state are 41 and -18 percent, respectively.

Figure S.6.6: Steady-State Gaps of Population Shares for U.S. States over Time



Note: Gray lines show gaps of population shares from steady-state for each U.S. state and year; black dashed lines show the population-weighted average of these steady-state gaps for the four geographical regions of the Rust Belt, Sun Belt, Other Northern States and Other Southern States, as defined in the main text; we compute the steady-state population share for each U.S. state and year by using our generalization of exact-hat algebra in Proposition 2 in the paper to solve for the economy's transition path in the absence of further changes in fundamentals, given the observed values of the labor and capital state variables in each year; gap from steady-state is measured as the log-ratio of actual population share to steady-state population share.

S.6.5.2 Predicted Population Growth Based on Initial Conditions

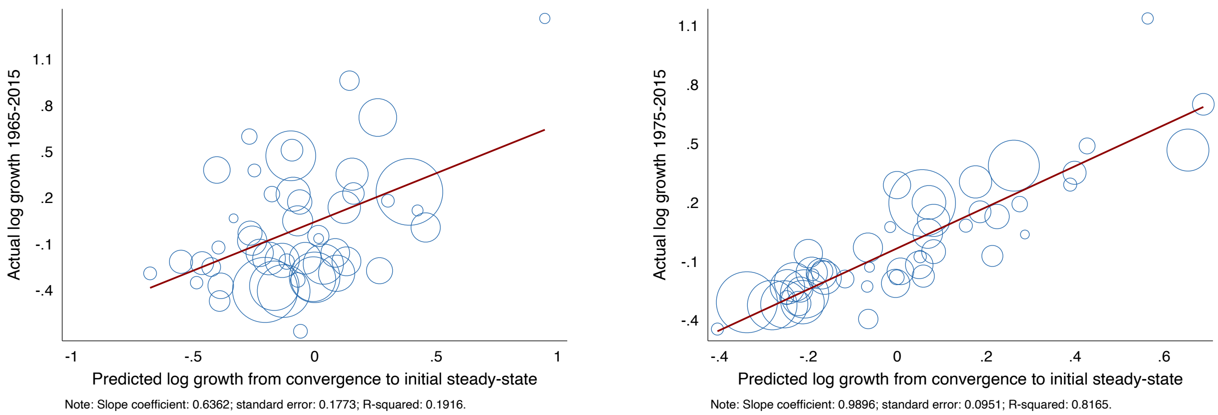
In Section 5.3 of the paper, we provide evidence that much of the observed decline in the rate of income convergence is explained by initial conditions at the beginning of our sample period rather than by any subsequent fundamental shocks. In this section of the Online Supplement, we provide further evidence on the role of initial conditions in explaining subsequent growth, by regressing actual population growth on predicted population growth based on convergence towards an initial steady-state with unchanged fundamentals. Importantly, predicted population growth is calculated using only the initial values of the labor and capital state variables and the initial trade and migration share matrices, and uses no information about subsequent population growth.

In Figure S.6.7a, we display actual population growth from 1965-2015 against predicted population growth based on convergence to an initial steady-state with 1965 fundamentals. The predictions based on convergence to an initial steady-state with unchanged fundamentals use only the 1964 and 1965 values of the state variables (population and the capital stock in each location) and the 1965 values of the trade and migration share matrices. Each circle in the figure corresponds to a different US state and the sizes of the circles are proportional to the initial population size of each state. The red line shows the linear fit between the two variables. We find a strong positive and statistically significant relationship between actual and predicted population growth, with a regression slope (standard error) of 0.64 (0.18) and R-squared of 0.19.

As discussed above, we find the largest contribution from shocks to fundamentals to the evolution of state population shares over time at the beginning of our sample period. From 1975 onwards, we find that predicted population growth based on convergence to an initial steady-state with unchanged fundamentals has even greater predictive power for actual population growth. In Figure S.6.7b, we display actual population growth from 1975-2015 against predicted population growth based on convergence to an initial steady-state with 1975 fundamentals. We find an even stronger positive and statistically significant relationship between actual and predicted population growth, with a regression slope (standard error) of of 0.99 (0.095) and R-squared of 0.82. We find a similar pattern of results for later periods, such as 1985-2015 and 1995-2015.

Figure S.6.7: Actual Growth in Population Shares Versus Predicted Growth in Population Shares Based on Convergence to an Initial Steady-State with Unchanged Fundamentals

(a) 1965-2015 (b) 1975-2015



Note: Vertical axis is actual log population growth; horizontal axis is predicted log population growth based on convergence to the implied initial steady-state assuming no further changes in fundamentals; left-panel shows results for 1965-2015; right-panel shows results for 1975-2015; size of circles for each US state is proportional to initial population size.

In Table S.6.1 below, we show that these results are robust to controlling for the initial level and growth of economic activity. In Column (1), we augment the regression between actual and predicted population growth from 1965-2015 in Figure S.6.7a with the initial log population in 1965, initial log capital stock in 1965 and the initial growth in population from 1965-6. We continue to find a positive and statistically significant relationship between actual and predicted population growth, with the inclusion of these additional control variables having relatively little impact on the estimated coefficient and regression R-squared. In Columns (2)-(4), we show that we find the same pattern of results for 1975-2015, 1985-2015 and 1995-2015, with somewhat larger slope coefficients and R-squared, which reflects the smaller residual contributions from shocks to fundamentals for these later time periods. Therefore, the predictive power of initial convergence towards steady-state does not simply reflect mean reversion, because we find substantial independent information in this variable, even after controlling for initial levels of population and the capital stock and initial population growth.

Table S.6.1: Predictive Power of Convergence Towards Initial Steady-State with Unchanged Fundamentals for Population Growth

Outcome: Base-year - 2015 pop. log growth	(1) 1965	(2) 1975	(3) 1985	(4) 1995
Base-year - 2015 predicted pop. growth	0.521**	0.892***	1.179***	0.735**
	(0.258)	(0.163)	(0.224)	(0.346)
Log base-year population	-0.113**	0.0235	-0.0180	0.00155
	(0.0556)	(0.0204)	(0.0197)	(0.00818)
Log base-year K-L ratio	0.194	-0.108	-0.119**	0.0590**
	(0.165)	(0.0800)	(0.0584)	(0.0281)
Base-year pop. growth rate	10.77	5.573	1.952	4.396
	(6.649)	(5.505)	(3.025)	(3.613)
N	49	49	49	49
R^2	0.341	0.783	0.722	0.826

Note: Dependent variable is actual log population growth between each base year and 2015; base years include 1965, 1975, 1985 and 1995 (columns 1-4, respectively); predicted log population growth is predicted based on convergence to the implied initial steady-state at the beginning of each base year using equation (25) in the paper; log base-year population is log population at each base-year; log base-year K-L ratio is the log capital-labor ratio at each base-year; pop. growth rate is the rate of population growth between each base-year and the subsequent year.

S.6.6 Spectral Analysis

In Section 5.4 of the paper, we use our spectral analysis to provide evidence on the role of capital accumulation and migration dynamics in shaping income convergence and the persistent and heterogeneous impact of local shocks. In this section of the Online Supplement, we report additional empirical results for this spectral analysis.

In Subsection S.6.6.1, we compare half lives of convergence to steady-state computed using transition matrices ($\boldsymbol{P}$) based on steady-state versus observed trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$). In Subsection S.6.6.2, we report additional empirical results on the relationship between the speed of convergence to steady-state and the correlation across locations between the steady-state gaps of the labor and capital state variables.

In Subsection S.6.6.3, we present further empirical results on the relationship between the speed of convergence to steady-state and the correlation across locations between productivity and amenity shocks. In Subsection S.6.6.4, we give additional results for the impulse response of the labor and capital state variables for empirical shocks to the relative productivity of Michigan (as a Rust Belt state) and the relative amenities of Arizona (as a Sun Belt state).

S.6.6.1 Steady-State Versus Observed Transition Matrices

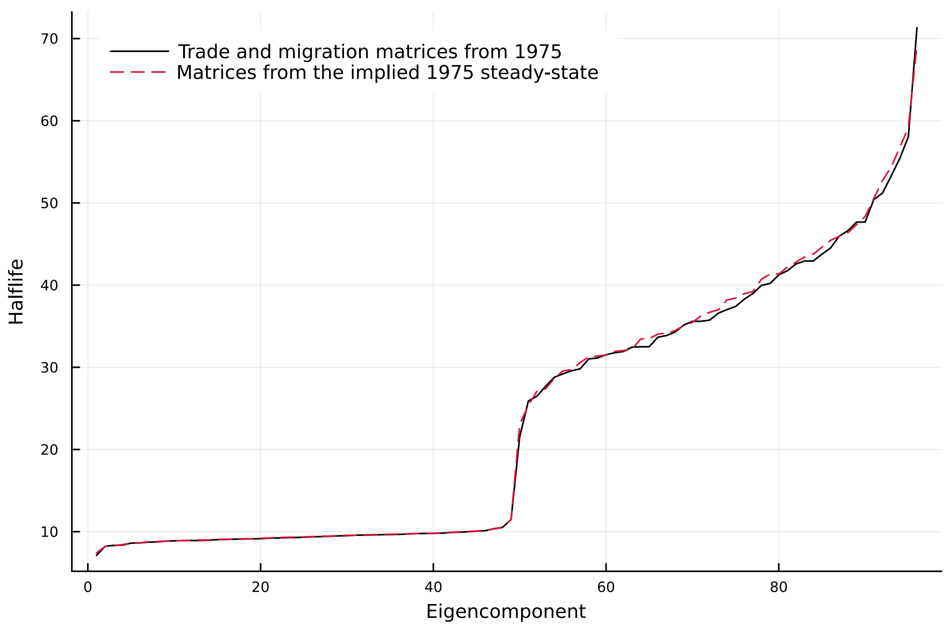
In Section 5.4 of the paper, we use Propositions 3-5 to compute half lives of convergence to steady-state, as determined by eigenvalues of the transition matrix. In Figure 3 in the paper, we show these half lives for the entire spectrum of $2N$ eigenvalues, sorted in terms of increasing half life.

Each eigencomponent with a non-zero eigenvalue corresponds to an eigen-shock for which the initial impact of the shock on the state variables is equal to an eigenvector of the transition matrix ($\boldsymbol{u}_h = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$).

In Figure 3 in the paper, we display results based on the transition matrix ($\boldsymbol{P}$) computed using the 1975 steady-state trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$). We compute these steady-state matrices using using our dynamic exact-hat algebra results from Proposition 2 in the paper. In Figure S.6.8 below, we reproduce these half-lives of convergence to steady-state based on the 1975 steady-state transition matrix, as shown by the red dashed line. As a robustness check, Figure S.6.8 also displays results based on the transition matrix ($\boldsymbol{P}$) computed using the 1975 observed trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$), as shown by the black solid line. We find half lives of convergence to steady-state that are barely distinguishable from one another using these two approaches, which reflects the fact that the steady-state and observed trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$) are strongly correlated with one another. We focus on 1975, because U.S. states are on average furthest from steady-state in this year, but we find a similar pattern of results for other years.

Therefore, we find that our results for the half life of convergence to steady-state are not sensitive to whether we use the steady-state or observed transition matrices, or to the precise year for which we compute these transition matrices.

Figure S.6.8: Half Lives of Convergence to Steady-State using Actual Versus Steady-State Trade and Migration Share Matrices



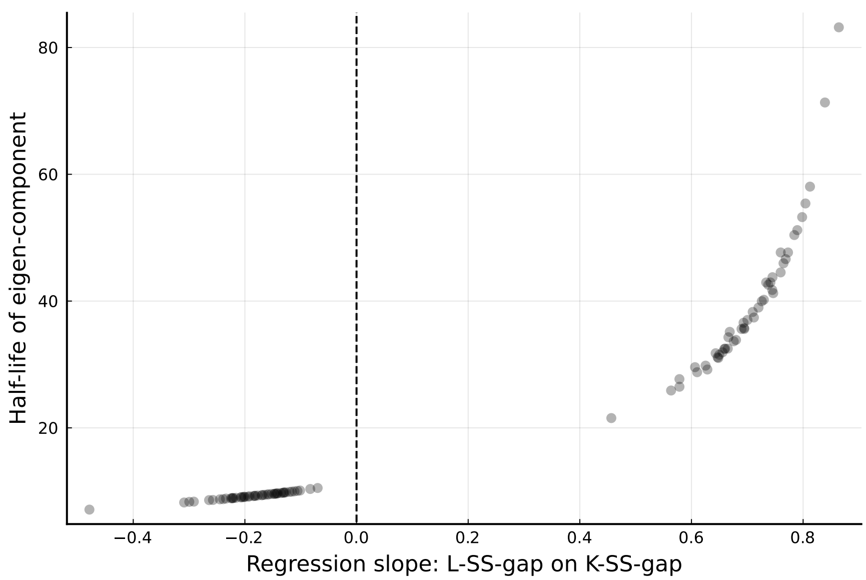
Note: half lives of convergence to steady-state for the full spectrum of eigen-shocks, computed using transition matrices ($\boldsymbol{P}$) based on either the 1975 steady-state trade and migration share matrices (red dashed line), or the 1975 observed trade and migration share matrices (black solid line); half lives of convergence computed using Proposition 5 in the paper.

S.6.6.2 Speed of Convergence and Steady-State Gaps

In Figure 3 in the paper, we show half lives of convergence to steady-state for the entire spectrum of $2N$ eigenvalues, sorted in terms of increasing half life. Each eigencomponent with a non-zero eigenvalue corresponds to an eigen-shock for which the initial impact of the shock on the state variables is equal to an eigenvector of the transition matrix ($\boldsymbol{u}_h = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$). As in our simple example of two symmetric regions in Section 3.3 in the paper, we find that eigencomponents for which the gaps of the labor and capital state variables from steady-state are positively correlated across locations have slower speeds of convergence to steady-state than eigencomponents for which these steady-state gaps are negatively correlated across locations.

In Figure S.6.9 below, we provide further evidence on this close relationship between the speed of convergence and the correlation between the gaps of the labor and capital state variables from steady-state. On the vertical axis, we display the half-life of convergence to steady-state for each eigen-shock with a non-zero eigenvalue, as determined by this eigenvalue (λ_h). On the horizontal axis, we show the slope coefficients from regressions across locations of the labor gap from steady-state ($\widetilde{\boldsymbol{\ell}}_h$) on the capital gap from steady-state ($\widetilde{\boldsymbol{k}}_h$) for each eigen-shock, as reflected in the eigenvector summarizing the initial impact of the shock on the state variables ($\boldsymbol{u}_h = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$). We display results for the year 2000, but find the same pattern for each year of our sample period. Consistent with the results in Figure 3 in the paper, we find a strong positive relationship between the half-life of convergence to steady-state and the correlation between the gaps from steady-state for the two state variables. We observe low half-lives (fast convergence) for negative correlations and high half-lives (slow convergence) for positive correlations.

Figure S.6.9: Half-lives of Convergence to Steady-State and the Regression Slope Coefficient Between the Labor and Capital Gaps from Steady-State



Note: Half-life on the vertical axis corresponds to the time in years for the state variables to converge half of the way towards steady-state for an eigen-shock in 2000, for which the initial impact of the shock to productivity and amenities on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$) corresponds to an eigenvector ($\boldsymbol{u_h}$) of the transition matrix ($\boldsymbol{P}$), as determined by the corresponding eigenvalue (λ_h); the horizontal axis shows slope coefficients from regressions across locations of the labor gap ($\widetilde{\boldsymbol{\ell}}_h$) from steady-state on the capital gap ($\widetilde{\boldsymbol{k}}_h$) from steady-state for each eigen-shock in 2000 ($\boldsymbol{u_h} = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$); each dot corresponds to a different eigencomponent in 2000.

S.6.6.3 Speed of Convergence and Fundamental Shocks

In Figure S.6.9 in the previous subsection, we examined the relationship between the speed of convergence and the initial impact of the eigen-shock on the state variables (as captured by the eigenvector $\boldsymbol{u}_h = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$). In contrast, we now turn to consider the corresponding relationship between the speed of convergence and the pattern of productivity and amenity shocks captured by each eigen-shock (using $\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{R}^{-1}\boldsymbol{u}_h$).

In the left panel of Figure S.6.10, we again use the vertical axis to display the half-life of convergence to steady-state for each eigen-shock, as determined by the associated eigenvalue (λ_h). On the horizontal axis, we show the slope coefficients from regressions across locations of the productivity shocks ($\widetilde{\boldsymbol{z}}_{(h)}$) on the amenity shocks ($\widetilde{\boldsymbol{b}}_{(h)}$) for each eigen-shock (using $\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{R}^{-1}\boldsymbol{u}_h$). Again we display results for the year 2000, but find the same pattern of results for each year of our sample period. We find a strong, positive relationship between the half-life of convergence to steady-state and the correlation between the productivity and amenity shocks, with low half-lives (fast convergence) for negative correlations, and high half-lives (slow convergence) for positive correlations.

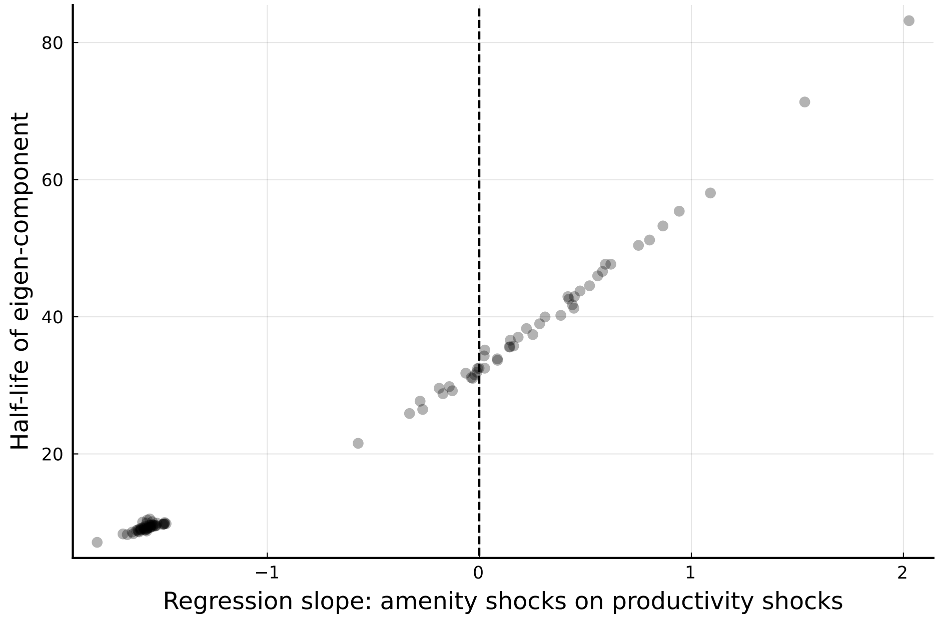
This pattern of results again highlights the interaction between the capital and labor adjustment margins in the model. On the one hand, a positive productivity shock directly raises the marginal productivity of both capital and labor in the production technology, which raises the new steady-state values of labor and capital relative to their initial values. On the other hand, a positive amenity shock directly raises the expected value of living in a location, which increases the new steady-state value of labor relative to its initial value. Therefore, a positive correlation between these two fundamental shocks induces a positive correlation between the labor and capital steady-state gaps, which in turn implies slow convergence to steady-state. In contrast, a strong negative correlation between these shocks is required in order to induce a negative correlation between the labor and capital steady-state gaps, and hence generate fast convergence to steady-state.

S.6.6.4 Impulse Responses

In Section 5.4 of the paper, we provide evidence on the persistent and heterogeneous impact of local shocks by considering individual empirical shocks to productivity and amenities in individual locations. We examine impulse response functions for the labor and capital state variables in each U.S. state following a local shock, starting from the steady-state implied by 1975 fundamentals. Motivated by the observed reallocation of economic activity from the Rust Belt to the Sun Belt, we report results for the empirical shock to relative productivity in Michigan from 1975-2015 (a 15 percent decline) and the empirical shock to relative amenities in Arizona over this same period (a 34 percent rise).

In Figure 6 in Section 5.4 of the paper, we report the impulse response of the population share of each U.S. state for the empirical decline in Michigan's relative productivity from 1975-2015. In Section S.6.6.5 below, we provide the corresponding impulse response of the capital stocks in each U.S. state for this same empirical shock to Michigan's relative productivity. In Section S.6.6.6 below, we report analogous impulse responses of the population share and capital stock in each U.S. state for the empirical increase in Arizona's relative amenities from 1975-2015.

Figure S.6.10: Half-lives of Convergence to Steady-State and the Regression Slope Coefficient Between the Productivity and Amenity Shocks



Note: Half-life on the vertical axis corresponds to the time in years for the state variables to converge half of the way towards steady-state for an eigen-shock in 2000 with a non-zero eigenvalue, for which the initial impact of the shock to productivity and amenities on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$) corresponds to an eigenvector ($\boldsymbol{u_h}$) of the transition matrix ($\boldsymbol{P}$), as determined by the corresponding eigenvalue (λ_h); the horizontal axis shows slope coefficients from regressions across locations of the productivity shocks ($\widetilde{\boldsymbol{z}}_{(h)}$) on the amenity shocks ($\widetilde{\boldsymbol{b}}_{(h)}$) for each eigen-shock in 2000 ($\widetilde{\boldsymbol{f}}_{(h)}$); each dot corresponds to a different eigencomponent in 2000.

S.6.6.5 Michigan Productivity Shock

In this section of the Online Supplement, we examine the dynamic response of the capital state variables in response to the empirical relative decline in Michigan's productivity from 1975-2015. Figure S.6.11 is analogous to Figure 6 in the paper, but shows the impulse responses of the capital stocks (instead of the population shares) of each U.S. state in response to this empirical shock to Michigan's relative productivity.

We find a similar pattern of results for capital stocks as for population shares in the paper. In the top-left panel, we show the log deviation of Michigan's capital stock from the initial steady-state along the transition path to the new steady-state. We find that the decline in Michigan's relative productivity leads to the decumulation of capital, which occurs gradually over time, because of migration frictions and gradual adjustment to capital.

In the top-right panel, we show the corresponding log deviations of capital stocks from the initial steady-state for all other states. We indicate Michigan's neighbors using the blue lines with circle markers and all other states using the gray lines. As for population shares in the paper, we find that the model can generate rich non-monotonic dynamics in capital stocks for individual states. Initially, the decline in Michigan's productivity raises the capital stocks of its neighbors, since workers face lower migration costs in moving to nearby states, and the resulting increase in population induces the accumulation of capital. However, as the economy gradually adjusts towards the new steady-state, the capital stocks in Michigan's neighbors begins to decline, and can even fall below their values in the initial steady-state. Intuitively, workers gradually experience favorable idiosyncratic mobility shocks for states further away from Michigan, which decreases population and reduces the accumulation of capital in neighboring states. Additionally, the decline in Michigan's productivity reduces the size of its market for neighboring locations, which can make those neighboring locations less attractive for workers and reduce the capital stock in the new steady-state. The capital stocks in all other states increase in the new steady-state relative to the initial steady-state.

In the middle panel, we show the log deviations from steady-state for the component of capital stocks attributed to bottom-88 eigencomponents with relatively fast convergence to steady-state. In the middle-left panel, the solid black line shows the overall log deviation of Michigan's capital stock from steady-state (the same as in the top-left left panel), while the dashed black line indicates the component due to the bottom-88 eigencomponents. In the middle-right panel, the solid blue line with circle markers shows the overall log deviation from steady-state of the capital stocks of Michigan's neighbors (same as in the top-right panel); the dashed blue line with circle markers indicates the component of these neighbors' capital stocks due to the bottom-88 eigencomponents; the gray lines represent the capital stocks of all other states (the same as in the top-right panel). Comparing the two sets of blue lines in the middle-right panel, these eigencomponents featuring fast convergence towards steady-state drive the initial rise in the capital stocks of Michigan's neighbors.

In the bottom panel, we show the log deviations from steady-state for the component of capital stocks attributed to the top-10 eigencomponents with relatively slow convergence to steady-state. In the bottom-left panel, the solid black line shows the overall log deviation of Michigan's capital stock from steady-state (the same as in the top-left panel), while the dashed black line indicates the component due to the top-10 eigencomponents. In the bottom-right panel, the solid blue line with circle markers shows the overall log deviation from steady-state of the capital stocks of Michigan's neighbors (same as in the top-right panel); the dashed blue line with circle markers indicates the component of these neighbors' capital stocks due to the top-10 eigencomponents; the gray lines represent the capital stocks of all other states (the same as in the top-right panel). Comparing the two sets of blue lines in the bottom-right panel, these eigencomponents featuring slow convergence towards steady-state drive the ultimate reduction in the capital stocks of Michigan's neighbors. Therefore, the non-monotonic dynamics for Michigan's neighbors in the top-right panel reflect the changing importance over time of the slow and fast-moving components of the economy's adjustment to the productivity shock in the middle-right and bottom-right panels.

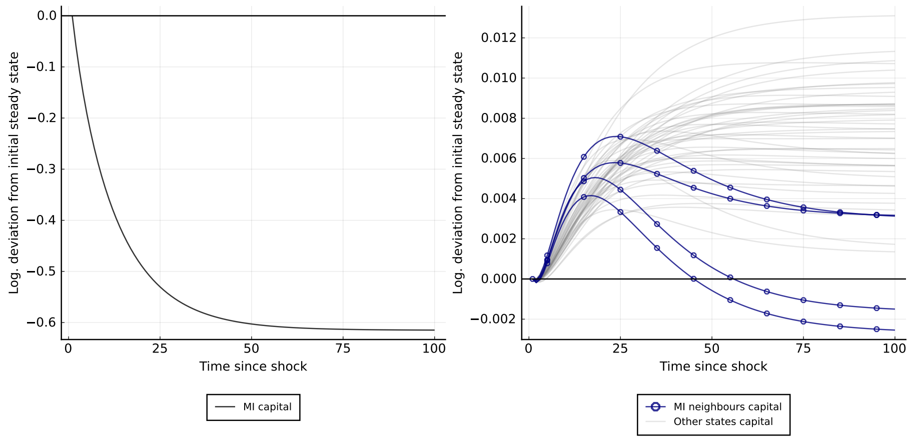
S.6.6.6 Arizona Amenities Shock

In this section of the Online Supplement, we examine the dynamic response of the population and capital state variables in response to the empirical relative increase in Arizona's amenities from 1975-2015.

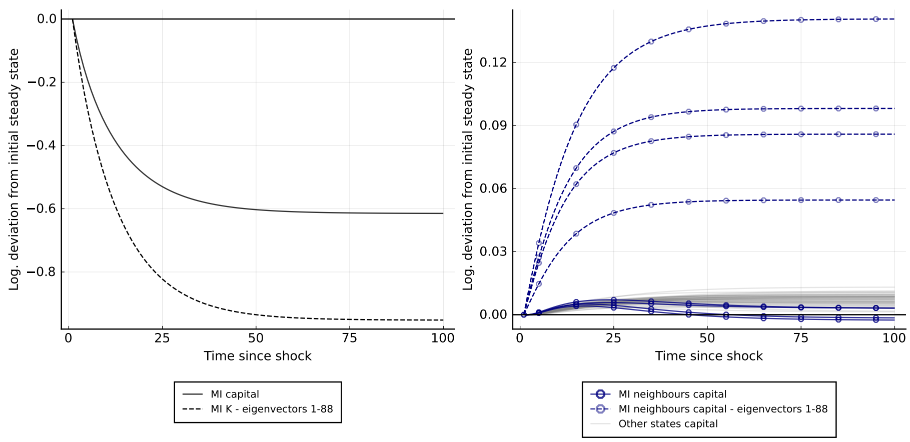
Population Impulse Response Figure S.6.12 is analogous to Figure 6 in the paper, but shows the impulse responses of the population shares of each U.S. state in response to this empirical shock to Arizona's relative amenities (instead of the empirical shock to Michigan's relative productivity). In the top-left panel, we show the log deviation of Arizona's population share from the initial steady-state along the transition path to the new steady-state. We find that the increase

Figure S.6.11: Impulse Response of Capital Stocks for a 15 Percent Decline in Productivity in Michigan

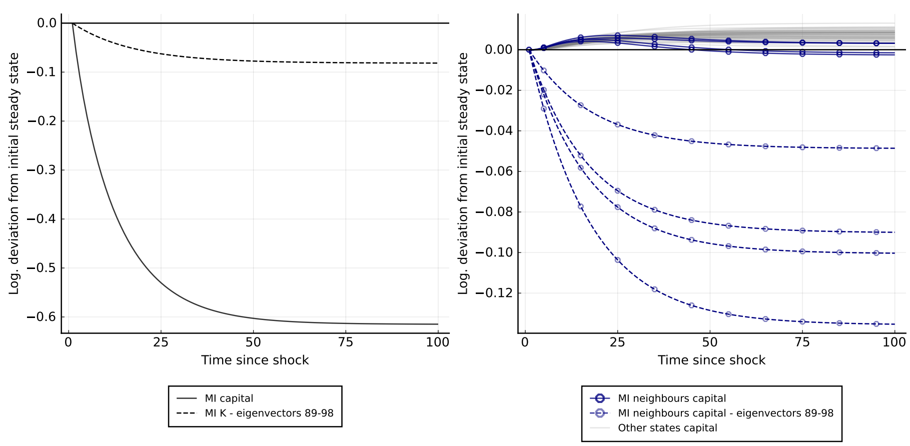
(a) Impulse Response of Overall Capital Stocks



(b) Impulse Response of Capital Stocks for Eigencomponents 1-88



(c) Impulse Response of Capital Stocks for Eigencomponents 88-98



Note: Top-left panel shows overall log deviation of Michigan capital stock from steady-state (vertical axis) against time in years (horizontal axis) for a 15 percent decline in Michigan's productivity (its empirical relative decline in productivity from 1975-2015); Top-right panel shows overall log deviation of other states' capital stocks from steady-state (vertical axis) against time in years (horizontal axis) for this shock to Michigan's productivity; blue lines show Michigan's neighbors; gray lines show other states; Middle and bottom panels decompose this overall impulse response into the contribution of eigencomponents 1-88 and 88-98, respectively.

in Arizona's relative amenities leads to a population inflow, which occurs gradually over time, because of migration frictions and gradual adjustment to capital.

In the top-right panel, we show the corresponding log deviations of population shares from the initial steady-state for all other states. We indicate Arizona's neighbors using the blue lines with circle markers and all other states using the gray lines. Initially, the increase in Arizona's relative amenities leads a stronger decline in population for neighboring states than for other states further away, because the migration frictions between Arizona and its neighboring states are lower. However, as the economy gradually adjusts towards the new steady-state, the decline in the population share of Arizona's neighbors is ultimately smaller than for other states further away. Intuitively, workers in other states further away gradually experience favorable idiosyncratic mobility shocks for Arizona, which decreases the population share of these other states further away. Additionally, the rise in Arizona's population increases the size of its market for neighboring locations by more than for other states further away.

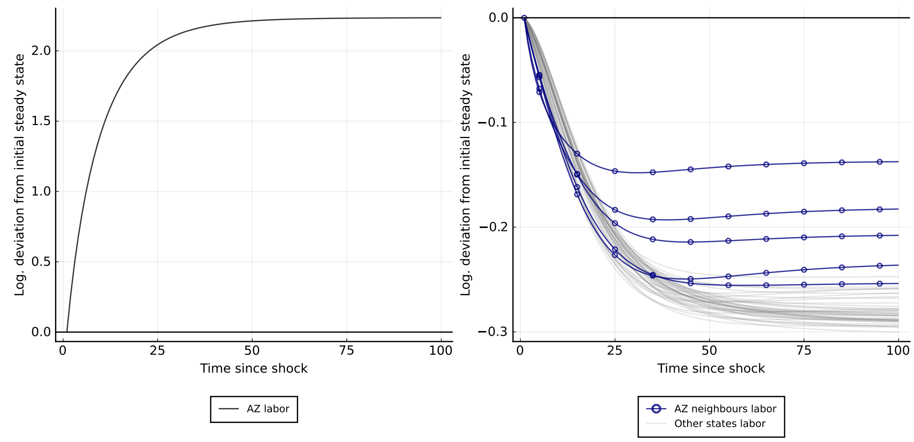
In the middle panel, we show the log deviations from steady-state for the component of population shares attributed to bottom-88 eigencomponents with relatively fast convergence to steady-state. In the middle-left panel, the solid black line shows the overall log deviation of Arizona's population share from steady-state (the same as in the top-left left panel), while the dashed black line indicates the component due to the bottom-88 eigencomponents. In the middle-right panel, the solid blue line with circle markers shows the overall log deviation from steady-state of the population shares of Arizona's neighbors (same as in the top-right panel); the dashed blue line with circle markers indicates the component of these neighbors' population shares due to the bottom-88 eigencomponents; the gray lines represent the population shares of all other states (the same as in the top-right panel). Comparing the two sets of blue lines in the middle-right panel, these eigencomponents featuring fast convergence drive the initially stronger decline in population shares in Arizona's neighbors than in other states further away.

In the bottom panel, we show the log deviations from steady-state for the component of population shares attributed to the top-10 eigencomponents with relatively slow convergence to steady-state. In the bottom-left panel, the solid black line shows the overall log deviation of Arizona's population share from steady-state (the same as in the top-left panel), while the dashed black line indicates the component due to the top-10 eigencomponents. In the bottom-right panel, the solid blue line with circle markers shows the overall log deviation from steady-state of the population shares of Arizona's neighbors (same as in the top-right panel); the dashed blue line with circle markers indicates the component of these neighbors' population shares due to the top-10 eigencomponents; the gray lines represent the population shares of all other states (the same as in the top-right panel). Comparing the two sets of blue lines in the bottom-right panel, these eigencomponents featuring slow convergence towards steady-state drive the ultimately smaller decline in population shares in Arizona's neighbors than in other states further away.

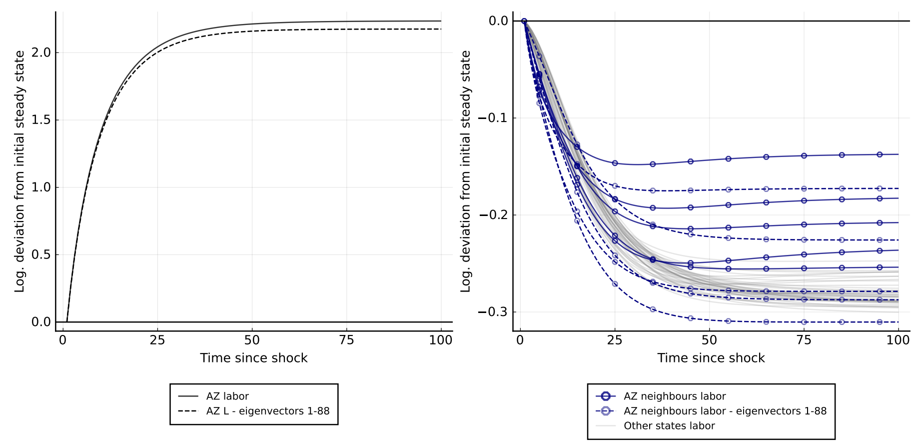
Therefore, the rich overall dynamics in population shares in the top-right panel again reflect the changing importance over time of the slow and fast-moving components of the economy's adjustment to this shock to relative amenities in the middle-right and bottom-right panels, although we find less evidence of non-monotonic dynamics for individual states than for the empirical shock to Michigan's relative productivity above.

Figure S.6.12: Impulse Response of Population Shares for a 34 Percent Increase in Amenities in Arizona

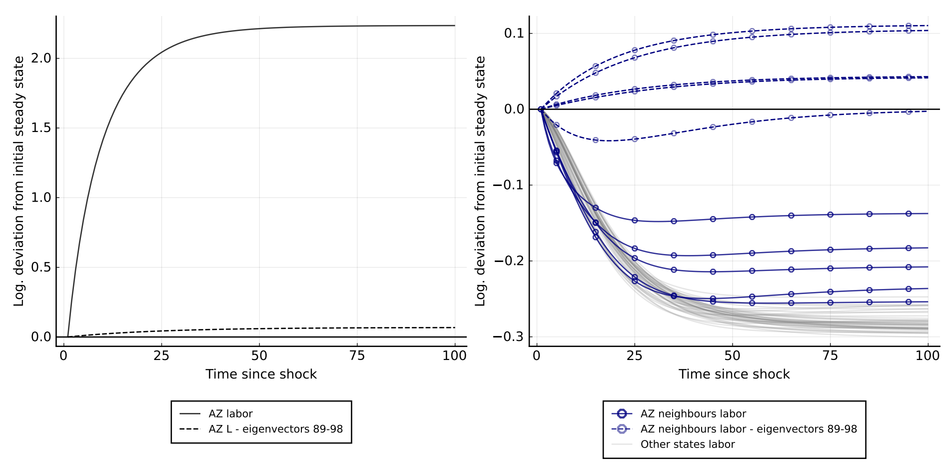
(a) Impulse Response of Overall Population Shares



(b) Impulse Response of Population Shares for Eigencomponents 1-88



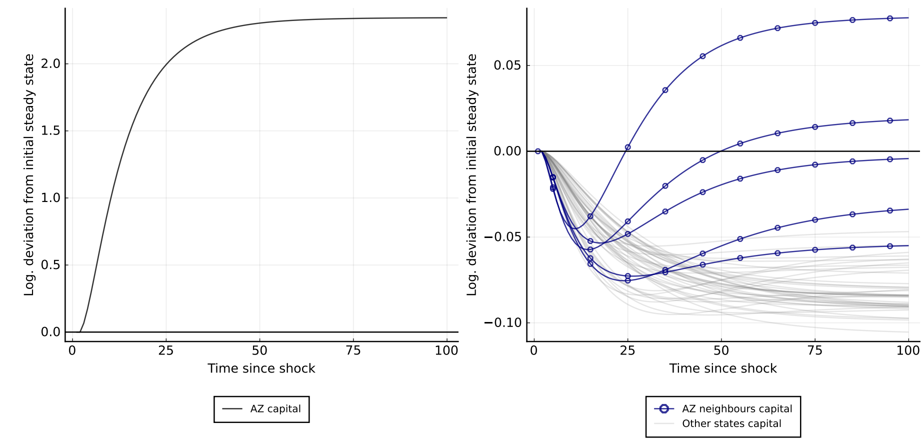
(c) Impulse Response of Population Shares for Eigencomponents 88-98



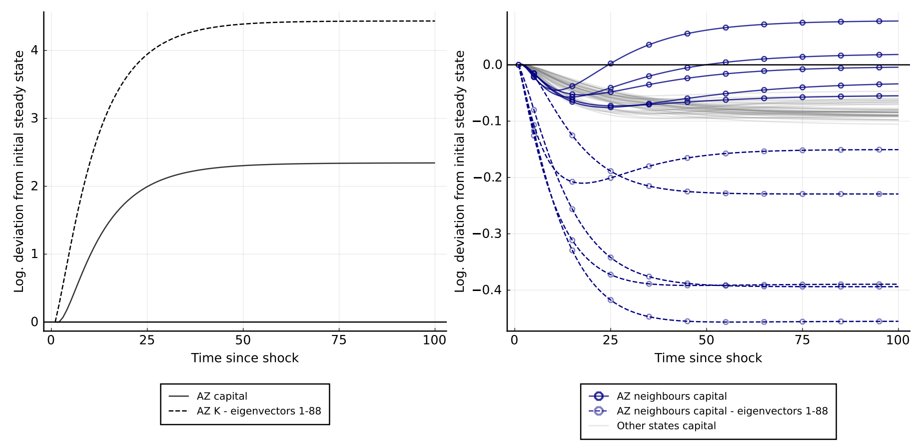
Note: Top left panel shows overall log deviation of Arizona's population share from steady-state (vertical axis) against time in years (horizontal axis) for a 34 percent increase in amenities in Arizona (its empirical relative increase in amenities from 1975-2015); Top right panel shows overall log deviation of other states' population shares from steady-state (vertical axis) against time in years (horizontal axis) for this shock to Arizona's amenities; blue lines show Arizona's neighbors; gray lines show other states; Middle panel decomposes the overall impulse response into the contribution of eigencomponents 1-88; Bottom panel decomposes the overall impulse response into the contribution of eigencomponents 88-98.

Figure S.6.13: Impulse Response of Capital Stocks for a 34 Percent Increase in Amenities in Arizona

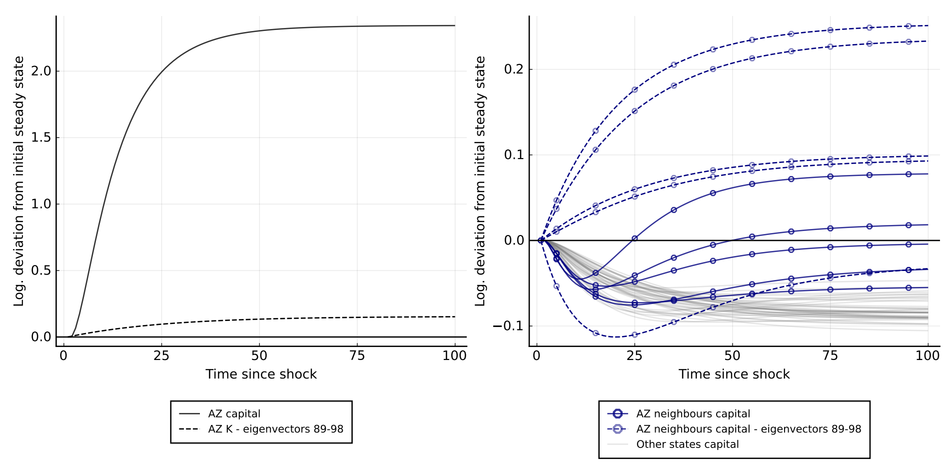
(a) Impulse Response of Overall Capital Stocks



(b) Impulse Response of Capital Stocks for Eigencomponents 1-88



(c) Impulse Response of Capital Stocks for Eigencomponents 88-98



Note: Top left panel shows overall log deviation of Arizona's capital stock from steady-state (vertical axis) against time in years (horizontal axis) for a 34 percent increase in amenities in Arizona (its empirical relative increase in amenities from 1975-2015); Top right panel shows overall log deviation of other states' capital stocks from steady-state (vertical axis) against time in years (horizontal axis) for this shock to Arizona's amenities; blue lines show Arizona's neighbors; gray lines show other states; Middle panel decomposes the overall impulse response into the contribution of eigencomponents 1-88; Bottom panel decomposes the overall impulse response into the contribution of eigencomponents 88-98.

Capital Stock Impulse Response Figure S.6.13 is analogous to Figure S.6.11 in this Online Supplement, but shows the impulse responses of the capital stocks of each U.S. state in response to this empirical shock to Arizona's relative amenities (instead of the empirical shock to Michigan's relative productivity). We again find persistent and heterogeneous effects of the shock across states. Consistent with the results for population shares above, we find that individual states can experience rich dynamics, because of the changing importance over time of the slow and fast-moving components of the economy's adjustment to the shock.

S.6.6.7 Comparing the Linearized and Non-Linear Models

We have so far used our spectral analysis to provide an analytical characterization of the speed of convergence to steady-state and the interaction between the capital and labor adjustment margins. While a caveat is that these analytical results are based on a linearization that is only exact for small shocks (up to first-order), we now show that this linearization provides a good approximation to the transition path of the non-linear model for empirically-reasonable shocks, such as decadal changes in relative productivity.

We first use our inversion of the non-linear model from our generalization of dynamic exact-hat algebra in Proposition 2 in the paper to recover the implied empirical distribution of productivities, amenities, trade costs and migration costs, as discussed in Online Supplement S.2.1. We next solve for the steady-state of the non-linear model implied by the 1990 values of these fundamentals $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. Starting from this steady-state, we then undertake counterfactuals for the economy's transition path in response to the empirical distribution of productivity shocks from 1990-2000, which includes substantial changes in relative productivity ranging from around -30 to 30 percent. We solve for this transition path in both the non-linear model using Proposition 2 in the paper and the linearized model using Proposition 3 in the paper.

In Figure S.6.14, we show the economy's transition path for population shares (left panel) and population relative to the initial steady-state (right panel) for each US state. In both panels, the solid blue line denotes the non-linear model, and the red dashed-line corresponds to the linearized model. We find that the two sets of predictions track one another relatively closely along the transition path of more than one hundred years, consistent with the linearized model providing a good approximation to the solution of the non-linear model. This approximation is somewhat better for population shares (left panel) than for population relative to the initial steady-state (right panel), but remains close in both cases. We find a similar pattern of results for the capital stock and for the response of both state variables to amenity shocks.[6]

These results are consistent with those in Baqaee and Farhi (2019, 2020). In the closed economy, the first-order approximation for the impact of productivity shocks is exact for a static economy with Cobb-Douglas preferences and production technologies. An implication is that this first-order approximation is also exact for the integrated world equilibrium of an economy with Cobb-Douglas preferences and production technologies, in which both goods and factors are perfectly mobile across countries (since the world is a closed economy). We depart from such an economy in three main respects: (i) Although we assume a Cobb-Douglas production technology, we assume constant elasticity of substitution (CES) preferences; (ii) We allow trade and migration

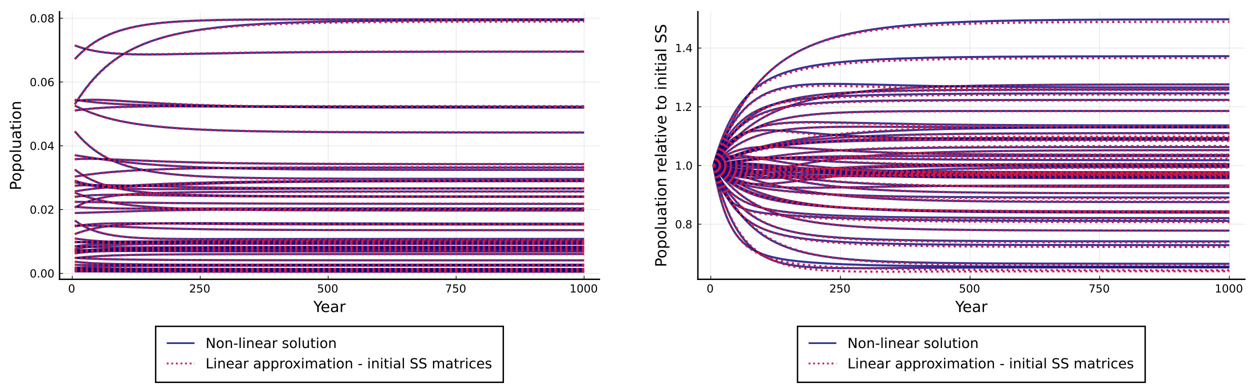
[6] While omitted in the interests of brevity, we also find a close relationship between the predictions of our linearization and the non-linear model solution for changes in steady-states, with a regression slope coefficient of 1.003 and a coefficient of correlation of 0.999. Following the approach of Kleinman et al. (2020) for a static trade model, we can also derive analytical bounds for the quality of the approximation for changes in steady-state.

frictions between locations; (iii) We incorporate capital accumulation and migration dynamics. Nonetheless, in practice, we find that the first-order approximation for the impact of productivity and amenity shocks provides a good approximation to the full non-linear model solution, even for empirically relevant shocks such as decadal changes in the relative productivity of locations. To show large second-order terms, Baqaee and Farhi (2019, 2020) consider an economy with a nested constant elasticity of substitution (CES) network structure, with elasticities of substitution that differ from the Cobb-Douglas case of a unitary elasticity of substitution. Therefore, our results are consistent with the findings that the second-order terms can be large in such a nested CES economy, since we assume a different (Cobb-Douglas) production structure.

Figure S.6.14: Transition Path Predictions of Our Linearization and the Full Non-Linear Model Solution for Counterfactual Changes in Productivity

(a) Transition Path for Population Shares

(b) Transition Path for Population Relative to Initial Steady-State Levels



Note: We first our generalization of dynamic exact-hat algebra in Proposition 2 in the paper to recover the empirical distribution of productivities, amenities, trade costs and migration costs. We next solve for the steady-state in the full non-linear model implied by the 1990 values of these fundamentals. Starting from this steady-state, we then undertake counterfactuals for the economy's transition path in response to the empirical distribution of productivity shocks from 1990-2000. We solve for the economy's transition path in both the non-linear model using Proposition 2 in the paper and the linearized model using Proposition 3 in the paper.

S.6.7 Implied Fundamentals

In this section of the Online Supplement, we provide further evidence on the implied location fundamentals (z_{it}, b_{it}, τ_{nit}, κ_{git}) from inverting the full non-linear model .

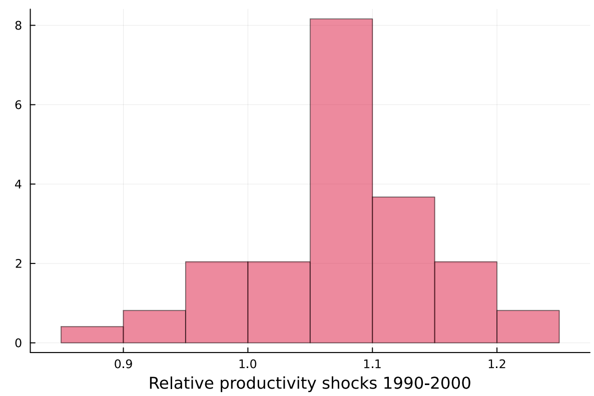
Empirical Distribution of Fundamental Shocks We make the conventional assumption of perfect foresight and use our extension of existing dynamic exact-hat algebra approaches to incorporate forward-looking capital investments from Proposition 2 in the paper. We solve for the unobserved changes in fundamentals from the general equilibrium conditions of the model, using the observed data on bilateral trade and migration flows, population, capital stock and labor income per capita, as discussed in Section S.2.1 above. We recover these unobserved fundamentals, without making assumptions about where the economy lies on the transition path or the specific trajectory of future fundamentals.

In the left and right panels of Figure S.6.15, we show the recovered empirical distributions of relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) across

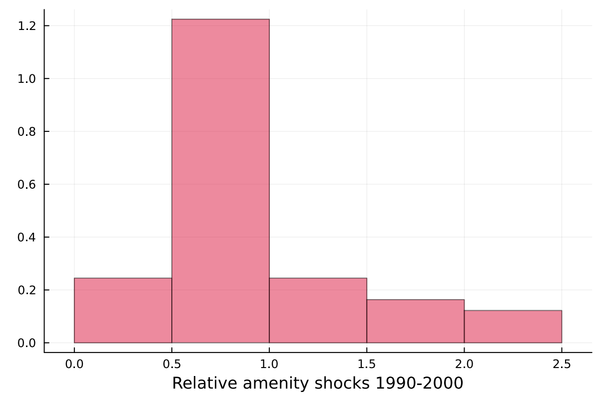
U.S. states from 1990-2000, where both variables are normalized to have a geometric mean of one. We find that relative changes in productivity and amenities are clustered around their geometric mean of one, but individual states can experience substantial decadal changes in relative productivity and amenities from around -30 to 30 percent.

Figure S.6.15: Relative Productivity and Amenity Shocks from 1990-2000 from our Model Inversion

(a) Productivity Shocks ($\widehat{z}_i = z_{i2000}/z_{i1990}$)



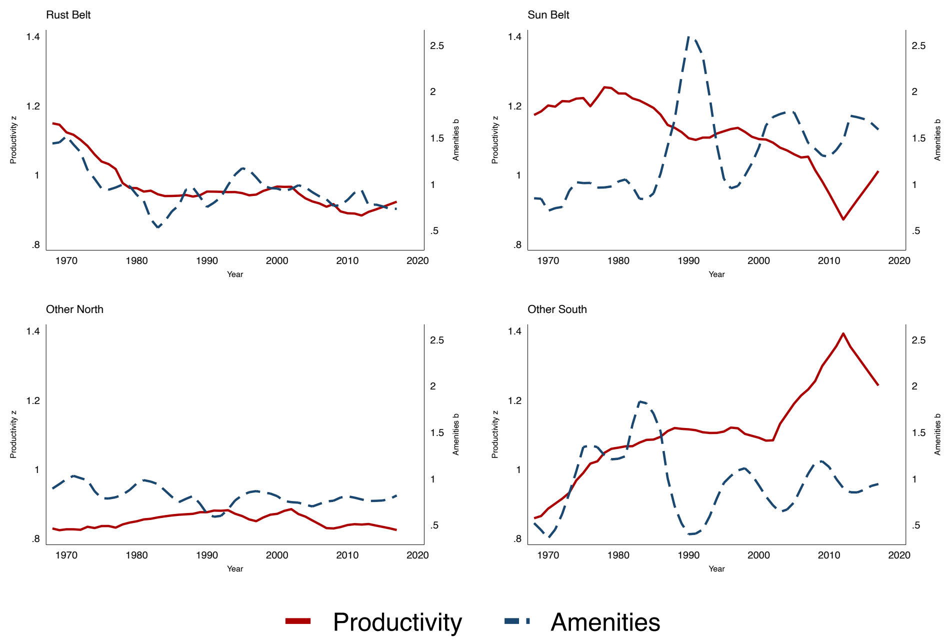
(b) Amenity Shocks ($\widehat{b}_i = b_{i2000}/b_{i1990}$)



Note: Histograms of the distributions of relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) from 1990-2000 from our model inversion, as discussed in Online Supplement S.2.1. Relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) are both normalized to have a geometric mean of one.

In Figure S.6.16, we display relative productivity and amenities for our four geographical groupings of states, where the values for each group are population-weighted averages of those for each state within that region. We find substantial changes in both relative productivity and amenities over time. From the top-left panel, the Rust Belt experiences a substantial decline in its relative productivity in the 1960s and 1970s, consistent with the argument in Holmes (1988) and Alder, Legakos and Ohanian (2019) that high unionization in these states during this time period could have retarded productivity growth relative for example to “right to work” states in Other Southern States. From the top-right panel, the rise in the population and income shares of the Sun Belt in previous figures is largely driven by an increase in its relative amenities. In contrast, relative productivity in the Sun Belt falls over time. From the bottom-right panel, the Other Southern States experience the largest increases in relative productivity over time, consistent with technological catch-up as well as potentially with more pro-business policy environments. Finally, from the bottom-left panel, both relative productivity and amenities are comparatively flat in the Other Northern States.

Figure S.6.16: Relative Productivity and Amenities over Time by Region



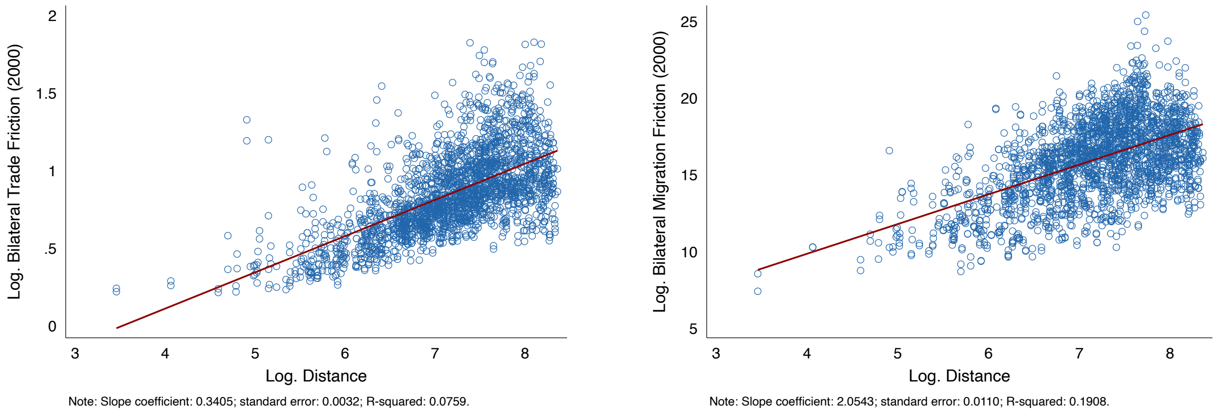
Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. North and South definitions based on Federal and Confederacy states

Notes: Productivity and amenities for each state are recovered from the inversion of the full non-linear model in Online Supplement S.2.1; productivity and amenities are measured in relative terms and are normalized to have a geometric mean of one across U.S. states in each year; productivity and amenities for each group of states are the population-weighted average of their values for each state within that group.

In Figure S.6.17, we show the relationship between our solutions for bilateral trade and migration frictions and bilateral geographical distance. We find a strong positive, statistically significant and approximately log linear relationship between these variables, consistent with the model's gravity equation predictions. In the interests of brevity, we show results for the year 2000, but we find the same pattern of results for all years of our sample period.

Figure S.6.17: Recovered Trade and Migration Frictions

(a) Bilateral Trade Frictions Versus Distance (b) Bilateral Migration Frictions Versus Distance



Note: Bilateral trade and migration frictions recovered from the inversion of the full non-linear model in Online Supplement S.2.1; distance corresponds to the geographical (Great Circle) distance between the centroids of bilateral pairs of US states.

S.6.8 Multi-Sector Extension

In this section of the Online Supplement, we provide further details on our multi-sector extension that is discussed in Section 5.5 of the paper.

S.6.8.1 Data and Parameterization

For our multi-sector extension from 1999-2015, we construct data for the 48 contiguous U.S. states, 43 foreign countries and 23 economic sectors, yielding a total of 2,093 region-sector combinations, where a region is either a U.S. state or a foreign country. We allow for trade across all region-sectors, and for migration across all U.S. states and sectors. We obtain sector-level data on value added, employment and the capital stock for each U.S. state from the national economic accounts of the Bureau of Economic Analysis (BEA).

We construct migration flows between U.S. states in each sector by combining data from the U.S. population census, American Community Survey (ACS), and Current Population Survey (CPS), as discussed in Online Supplement S.7. We use the value of bilateral shipments between U.S. states in each sector from the Commodity Flow Survey (CFS), interpolating between reporting years. We measure foreign trade for each U.S. state and sector using the data on foreign exports by origin of movement (OM) and foreign imports by state of destination (SD) from the U.S. Census Bureau.[7] For each foreign country and sector, we obtain data on value added, employment and the capital stock from the World Input-Output Tables (WIOT).

[7] The Census Bureau constructs these data from U.S. customs transactions, aiming to measure the origin of the movement of each export shipment and the destination of each import shipment. Therefore, these data differ from measures of exports and imports constructed from port of exit/entry, and from the data on the exports of manufacturing enterprises (EME) from the Annual Survey of Manufactures (ASM). See https://www.census.gov/foreign-trade/aip/elom.html and Cassey (2009).

S.6.8.2 Quantitative Results

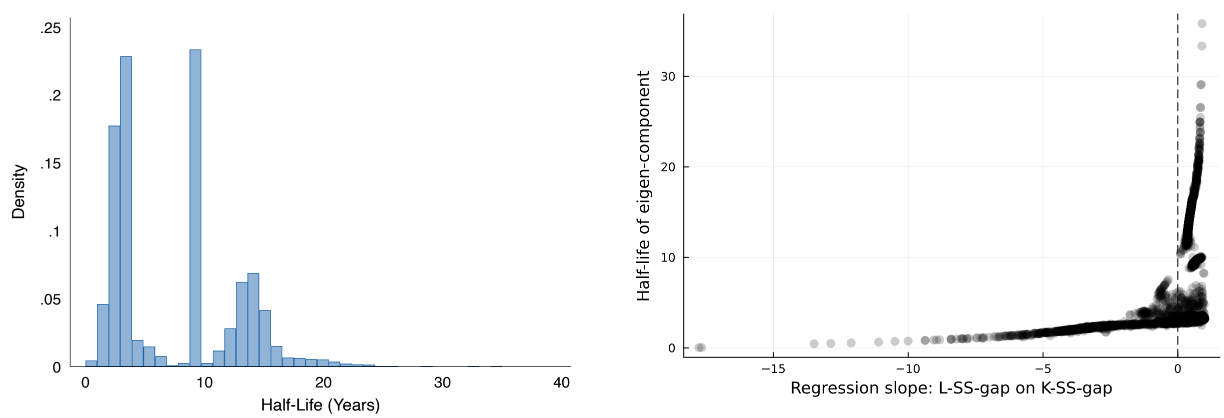
In a final empirical exercise, we implement our multi-sector extension with region-sector specific capital from Section S.4.4 above, using our region-sector data from 1999-2015. We again use our spectral analysis to provide an analytical characterization of the economy's transition path.

We begin by using our generalization of Proposition 5 in the paper for the multi-sector model to compute half-lives of convergence towards steady-state for shocks to productivity or amenities for which the initial impact on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_k}$) of the transition matrix ($\boldsymbol{P}$). In Figure S.6.18, we display the distribution of these half-lives across eigenvectors of the transition matrix the year 2000. We find more rapid convergence to steady-state in our multi-sector extension, with an average half-life of 7 years and a maximum half-life of 35 years (compared to around 20 and 85 years in our baseline single-sector model). This finding is driven by the property of the region-sector migration matrices that flows of people between sectors within states are larger than those between states. A key implication is that the persistence of local labor market shocks depends on whether they induce reallocation across industries within the same location or reallocation across different locations.

Figure S.6.18: Half-lives of Convergence Towards Steady-State in the Multi-Sector

(a) Histogram of Half-lives for Shocks to Productivity and Amenities that Correspond to Eigenvectors of the Transition Matrix ($\boldsymbol{P}$) in 2000

(b) Relationship Between Half-Lives of Convergence Towards Steady-State and the Correlation between the Initial Gaps of the Labor and Capital State Variables from Steady-State in 2000



Note: Half-life corresponds to the time in years for the state variables to converge half of the way towards steady-state for an eigen-shock, for which the initial impact of the shock to productivity and amenities on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_h}$) of the transition matrix ($\boldsymbol{P}$) in the multi-sector model; left panel shows the distribution of half-lives across eigencomponents of the transition matrix in 2000 in the multi-sector model; right panel plots these half-lives of convergence to steady-state for the year 2000 in the multi-sector model against the slope coefficients from regressions across location-sectors of the labor gap ($\widetilde{\boldsymbol{\ell}}_h$) from steady-state on the capital gap ($\widetilde{\boldsymbol{k}}_h$) from steady-state for each nontrivial eigen-shock in 2000 ($\boldsymbol{u_h} = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$).

In the right panel of Figure S.6.18, we show that capital and labor dynamics again interact with one another to shape the speed of convergence towards steady-state in the multi-sector model. On the vertical axis, we display the half-life of convergence to steady-state (in years) for each eigen-shock. On the horizontal axis, we display the regression slope coefficient between the gaps from steady-state for labor and capital for each eigen-shock. As for the single-sector model above, we find a strong, positive and non-linear relationship between the half-life of convergence to steady-state and the correlation between the gaps from steady-state for the two state variables. Although for brevity we display results for the year 2000, we again find the same pattern of results for each year of our sample period.[8]

Therefore, we find a similar pattern of results for the multi-sector model as for the single-sector model in our baseline specification, with slow convergence to steady-state, and an important interaction between capital accumulation and migration dynamics in shaping the persistent and heterogeneous impact of local shocks.

S.7 Data Appendix

In this section of the Online Supplement, we report further details about the data sources and definitions. In Section S.7.1, we discuss the data used for the quantitative analysis of our baseline single-sector model. In Section S.7.2, we discuss the data used for the quantitative analysis of our multi-sector extension.

S.7.1 State Data

In the single-sector version of the model, we consider the 48 contiguous U.S. states (excluding Alaska and Hawaii) plus Washington DC.

State-to-State Migration Data. The decennial population censuses for 1960, 1970, 1980, 1990 and 2000 ask respondents their current state of residence and their state of residence five years ago. From the reported responses, we obtain bilateral five-year migration flows between U.S. states for 1960, 1970, 1980, 1990 and 2000. We construct an analogous bilateral five-year migration flows for 2010 using the American Community Survey (ACS) data for the years 2008-2012. The ACS provides data only on annual migration flows, so we take the 5th power of the annual outmigration shares matrix. The state-to-state migration data are reported for the population over 5 years in age. We construct own-state-to-own-state migration flows as total population over 5 years in age minus total inmigrants from other states. We interpolate between years to obtain five-year bilateral migration flows for each sample year from 1965-2015. We use these bilateral migration flows to construct our outmigration matrix ($\boldsymbol{D}$) and our inmigration matrix ($\boldsymbol{E}$). To take account of international migration to each state and fertility/mortality differences across states, we adjust these migration flows by a scalar for each origin and destination state, such that origin population in year t pre-multiplied by the migration matrix equals destination population in year $t+1$, as required for internal consistency.

State-to-State Trade Data. The Commodity Flow Survey (CFS) reports the value of state-to-state shipments for the years 1993, 1997, 2002, 2007, 2012 and 2017. The CFS covers business establishments in mining, manufacturing, wholesale trade, and selected retail and services trade industries. The survey also covers selected auxiliary establishments (e.g., warehouses) of in-scope, multi-unit, and retail companies. Industries not covered by the CFS include transportation,

[8] Under our assumption of no international migration, the deviation of labor from steady-state is zero for foreign countries in our multi-sector model. Therefore, they adjust to fundamental shocks through capital accumulation alone, which is responsible for the mass of eigen-shocks with intermediate half-lives in both panels of Figure S.6.18.

construction, most retail and services industries, farms, fisheries, foreign establishments, and most government-owned establishments. The CFS collects data on shipments originating from within-scope industries, including exports. Imports are not included until the point that they leave the importer's initial domestic location for shipment to another location. The survey does not cover shipments originating from business establishments located in Puerto Rico and other U.S. possessions and territories.

The predecessor of the CFS was the Commodity Transportation Survey (CTS), which covers the manufacturing sector alone. The 1977 CTS reports the value of shipments from each state of origin to each census division of destination: New England, Middle Atlantic, East North Central, West North Central, South Atlantic, East South Central, West South Central, Mountain, and Pacific. We allocate the value of shipments across destination states within these destination census divisions according to their shares of the value of shipments in the CFS in 1993. We interpolate the value of shipments between years to obtain annual data on the value of shipments for each year of our sample from 1977-2015. We estimate the value of shipments between states for years before 1977 by assuming the following gravity equation:

$$X_{nis} = X_{is} X_{ns} \tau_{nis}, \qquad s \leq t = 1977,$$

where X_{nis} is the value of bilateral shipments from exporter i to importer n in year s; X_{is} is exporter gross domestic product (GDP); X_{ns} is importer GDP; and τ_{nis} captures observed and unobserved bilateral trade costs. Assuming that bilateral trade costs remain constant, the value of bilateral shipments in any previous year $s < t$ can be expressed in the following exact-hat algebra form:

$$X_{nis} = \widehat{X}_{is} \widehat{X}_{ns} X_{nit}, \qquad s \leq t = 1977,$$

where a hat above a variable denotes a relative change between years s and t, such that $\widehat{X}_{is} = X_{is}/X_{it}$. We observe these relative changes in exporter and importer GDP for each year back to the beginning of our sample period in 1965.

We thus obtain the bilateral value of shipments between states for each year of our sample from 1965-2015. We use these bilateral shipments data to construct our expenditure share matrix ($\boldsymbol{S}$) and income share matrix ($\boldsymbol{T}$). For our baseline quantitative analysis, we abstract from direct shipments to and from foreign countries, because of the relatively low level of U.S. trade openness, particularly towards the beginning of our sample period.

Gross Domestic Product, Population and Capital Stock. The Regional Economic Accounts of the Bureau of Economic Analysis (BEA) report population and state gross domestic product (GDP) for each state and year of our sample from 1965-2015. Estimates of real GDP at the state level are available starting from 1977. For the years 1965-1977, we deflate nominal GDP by splicing the national GDP deflator from the years 1965-1977 to the state-level deflator from 1977 onwards. To obtain state-level real capital stocks, we first compute state-industry level nominal gross operating surplus by subtracting labor compensation from GDP. We then deflate each observation by a corresponding estimate of the national capital income deflator, taken from the USA World KLEMS Database. Finally, we sum across industries to obtain a state-level measure of the real capital stock.

Geographical Groupings We consider four main geographical groupings of states. Following Alder, Legakos and Ohanian (2019), we define the Rust Belt as the states of Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin, and the Sun Belt as the states of Arizona, California, Florida, New Mexico and Nevada. We group the remaining states into two categories that capture longstanding differences between the North and South: Other Southern States, which includes all former members of the Confederacy, except those in the Sun Belt; and Other Northern States, which comprises all the Union states from the U.S. Civil War, except those in the Rust Belt or Sun Belt. Therefore, "Other Southern" includes Alabama, Arkansas, Georgia, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas, and Virginia. "Other Northern" includes Colorado, Connecticut, Delaware, District of Columbia, Idaho, Iowa, Kansas, Kentucky, Maine, Maryland, Massachusetts, Minnesota, Missouri, Montana, Nebraska, New Hampshire, New Jersey, North Dakota, Oklahoma, Oregon, Rhode Island, South Dakota, Utah, Vermont, Washington and Wyoming.

S.7.2 Region-Sector Data

In the multi-sector, multi-country version of the model, we consider the 48 contiguous US states, 43 other countries and 23 economic sectors, yielding a total of 2,093 region-sector combinations, where a region is either a US state or a foreign country. We allow for trade across all region-sectors, and for migration across all states and sectors within the US.

State-Sector Migration Data. To implement our multi-sector application, we require a state-sector to state-sector migration matrix for the year 2000. To this end, we first recover state to state worker flows from the 2001 American Community Survey, which includes questions on current state of residence and the state of residence one year ago. We then use data from the Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC), which includes questions on current and past industry of employment, to divide these flows across origin and destination sectors, pooling together data from 1998-2002 to enlarge the sample size. Specifically, we first compute sector to sector transition rates for each state separately, and then assume that these transition probabilities are constant across destination states conditional on the state of origin. We multiply these sectoral transition probabilities by the state to state transition rates to get our matrix of state-sector to state-sector transition rates. Note that due to the small sample size of the CPS, we cannot use it to directly compute transition probabilities across all state-sector combinations, amounting to close to 1,000,000 cells in the migration matrix.

Region-Sector Production, Employment and Capital Stock. For each country-sector combination, we take employment data and nominal value-added, gross-output and capital stocks from the Socio-Economic Accounts of the World Input-Output Tables (WIOT) 2016 release. To obtain real capital stocks, we take country-level estimates from the International Monetary Fund (IMF) Investment and Capital Stock Dataset, which provides private capital stocks in 2005 international dollars for most countries in the WIOT data over the period 1960-2013. We allocate national real capital stocks across sectors according to country-level shares of nominal capital stocks from WIOT. We allocate US aggregates across individual states using states shares in national GDP, gross output and capital using the BEA's Regional Accounts.

State-Sector Foreign Imports and Exports. We use data on exports by state of the origin of movement (OM) and imports by state of destination (SD) from the Foreign Trade Division of the U.S. Census Bureau. The origin of movement (OM) export data are based on the state from which the shipment starts its journey to the port of export. Therefore, the data reflect the transportation origin of an export shipment, which need not correspond to the state in which the good was produced. The state of destination (SD) import data are based on the U.S. state, U.S. territory or U.S. possession where the merchandise is destined, as known at the time of customs filing at the port of entry. If the contents of the shipment are destined to more than one state, territory, or possession, or if the entry summary represents a consolidated shipment, the state of destination is reported as the state with the greatest aggregate value. If in either case, this information is unknown, the state of the ultimate consignee, or the state where the customs entry is filed are reported, in that order. However, before either of those alternatives is used, a good faith effort is required of the customs filer to ascertain the state where the imported merchandise will be delivered.

Therefore, these export origin of movement (OM) and import state of destination (SD) data differ from measures of exports and imports by port of exit and entry. The export data also differ from the exports of manufacturing enterprises (EME) data from the Annual Survey of Manufactures (ASM), which are restricted to manufacturing and based on the state of production. In contrast, our export and import data cover all traded sectors, and are collected by origin of movement (for exports) and destination of shipment (for imports).

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