Online Appendix for "Trade and Geography" (Not for Publication)

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A Introduction

This online appendix reports additional technical derivations for the main paper. Section B reports these additional results for the Helpman (1998) model in Section 3 of the paper. Section C establishes a number of isomorphisms between this new economic geography model with imperfect competition and internal increasing returns to scale and neoclassical models of perfect competition and external increasing returns to scale. Section D derives the relationship between firm and consumer market access reported in the paper.

B Modeling Economic Activity Between Cities and Regions

We consider an economy consisting of a set N of regions indexed by n. Each region is endowed with an exogenous supply of floor space (H_i). The economy as a whole is endowed with a measure $\bar{L}$ of workers, where each worker has one unit of labor that is supplied inelastically with zero disutility. Workers are perfectly geographically mobile and hence in equilibrium real wages are equalized across all populated regions. Regions are connected by a bilateral transport network that can be used to ship goods subject to symmetric iceberg trade costs, such that $d_{ni} = d_{in} > 1$ units must be shipped from region i in order for one unit to arrive in region $n \neq i$, where $d_{nn} = 1$.

B1 Consumer Preferences

Preferences are defined over goods consumption and residential floor space use. We assume that these preferences take the Cobb-Douglas form, such that indirect utility for a worker in location n is given by:[1]

$$U_n = \frac{B_n v_n}{P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1, \tag{B.1}$$

where B_n denotes amenities; v_n is worker income; P_n is the consumption goods price index; Q_n is the price of floor space. The consumption goods price index is assumed to take the constant elasticity of substitution (CES) form:

$$P_n = \left[\sum_{i \in N} \int_0^{M_i} p_{ni}(\psi)^{1-\sigma} \, d\psi \right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{B.2}$$

where M_i is the endogenous measure of varieties produced in each location; $p_{ni}(\psi)$ is the cost to a consumer in location n of a variety ψ from location i; and we assume that varieties are substitutes.

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[1] For empirical evidence using U.S. data in support of the constant housing expenditure share implied by the Cobb-Douglas functional form, see Davis and Ortalo-Magné (2011).

B2 Production

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of F units of labor and a constant variable cost in terms of labor that depends on a location's productivity A_i. Therefore the total amount of labor ($l_i(\psi)$) required to produce $x_i(\psi)$ units of a variety ψ in location i is

$$l_i(\psi) = F + \frac{x_i(\psi)}{A_i}. \tag{B.3}$$

The producer of each variety chooses its prices to maximize its profits subject to the downward-sloping demand curve for its variety:

$$\max_{p_i(\psi)} \left\{ p_i(\psi)x_i(\psi) - w_i \left(F + \frac{x_i(\psi)}{A_i} \right) \right\}. \tag{B.4}$$

Profit maximization implies that equilibrium prices are a constant markup over the marginal cost of supplying a variety to a market,

$$p_{ni}(\psi) = p_{ni} = \left(\frac{\sigma}{\sigma - 1} \right) d_{ni} \frac{w_i}{A_i}. \tag{B.5}$$

Using this equilibrium pricing rule (B.5) in the definition of profits (B.4), free entry and zero profits imply that equilibrium output of each variety is equal to a constant that depends on location productivity, namely,

$$x_i(\psi) = \bar{x}_i = A_i(\sigma - 1)F, \tag{B.6}$$

which implies that equilibrium employment for each variety is the same for all locations, so

$$l_i(\psi) = \bar{l} = \sigma F. \tag{B.7}$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total measure of varieties supplied by each location is proportional to the endogenous supply of workers choosing to locate there:

$$M_i = \frac{L_i}{\sigma F}. \tag{B.8}$$

B3 Price Indices and Expenditure Shares

Using the symmetry of equilibrium pricing, the consumption price index in equation (B.2) can be written as follows:

$$P_n = \left[\sum_{i \in N} M_i p_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{B.9}$$

Using equilibrium prices (B.5) and labor market clearing (B.8), this consumption goods price index can be re-written as follows:

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{1}{\sigma F} \right)^{\frac{1}{1-\sigma}} \left[\sum_{i \in N} L_i \left(d_{ni} w_i / A_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{B.10}$$

Using the CES expenditure function, equilibrium prices (B.5) and labor market clearing (B.8), the share of location n's expenditure on goods produced in location i is

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in N} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(d_{ni} w_i / A_i \right)^{1-\sigma}}{\sum_{k \in N} L_k \left(d_{nk} w_k / A_k \right)^{1-\sigma}}. \tag{B.11}$$

The model therefore implies a "gravity equation" for goods trade, where the bilateral trade between locations n and i depends on both "bilateral resistance" (bilateral trade costs d_{ni}) and "multilateral resistance" (trade costs to all other locations k d_{nk}), as in Anderson and van Wincoop (2003). Together (8) and (9) imply that each location's price index can be again written in terms of its trade share with itself, so

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{L_n}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}. \tag{B.12}$$

B4 Market Clearing

Expenditure on floor space in each location is assumed to be redistributed lump sum to the workers residing in that location. Noting that Cobb-Douglas utility implies that expenditure on floor space is a constant share of income, it follows that income per capita in each location (v_n) equals labor income (w_n) plus expenditure on residential floor space per capita ($(1-\alpha)v_n$):

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n = \frac{w_n L_n}{\alpha}. \tag{B.13}$$

Goods market clearing implies that revenue in each location equals expenditure on goods produced in that location. We assume that trade in balanced, such that expenditure equals income in each location. Using zero profits, which implies that revenue equals labor income, and utility maximization, which implies that expenditure on goods is a constant share of income, this goods market clearing condition can be written as:

$$w_i L_i = \sum_{n \in N} \alpha \pi_{ni} v_n L_n = \sum_{n \in N} \pi_{ni} w_n L_n. \tag{B.14}$$

Land market clearing implies that the supply of floor space equals the demand for floor space. Using utility maximization in this land market clearing condition, the price of floor space (Q_n) is given by:

$$Q_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}. \tag{B.15}$$

B5 Population Mobility

Population mobility implies that workers receive the same real income in all populated locations, hence

$$U_n = \frac{v_n}{P_n^{\alpha} Q_n^{1-\alpha}} = \bar{U}. \tag{B.16}$$

Using the price index (B.12), the relationship between income and wages (B.13), and land market clearing (B.15), we can rewrite this population mobility condition as follows:

$$\bar{U} = \frac{A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} L_n^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}}{\alpha \left(\frac{\sigma}{\sigma-1}\right)^{\alpha} \left(\frac{1}{\sigma F}\right)^{\frac{\alpha}{1-\sigma}} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}. \tag{B.17}$$

B6 Gains from Trade and Population Shares

From population mobility (B.17), all regions experience the same welfare gains from trade, and the welfare gain for each region can be expressed in terms of the change in its domestic trade share and the change in its population:

$$\frac{\bar{V}^T}{\bar{V}^A} = \frac{V_n^T}{V_n^A} = \left(\pi_{nn}^T\right)^{-\frac{\alpha}{\sigma-1}} \left(\frac{L_n^T}{L_n^A}\right)^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}. \tag{B.18}$$

Rearranging the population mobility condition (B.17) to obtain an expression for L_n, and dividing by total labor supply $\bar{L} = \sum_{n \in N} L_n$, the population share of each location ($\lambda_n \equiv L_n/\bar{L}$) depends on its productivity (A_n), amenities (B_n), supply of floor space (H_n) and domestic trade share (π_{nn}) relative to those of all other locations,

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}{\sum_{k\in N}\left[A_k^{\alpha} B_k H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\sigma-1)}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}, \tag{B.19}$$

where each location's domestic trade share (π_{nn}) summarizes its market access to other locations.

B7 Existence and Uniqueness

The general equilibrium of the model can be characterized using two systems of equations for wages and population across locations, one from the population mobility condition, and the other from the gravity of trade flows. We begin with the population mobility condition, which can be re-written as:

$$P_n^{\alpha} = \frac{B_n v_n}{\bar{U} Q_n^{1-\alpha}}. \tag{B.20}$$

Using the relationship between income and wages $v_n = w_n/\alpha$, and land market clearing ($Q_n = \frac{1-\alpha}{\alpha}\frac{w_n L_n}{H_n}$), we can further re-write this population mobility condition as:

$$P_n = \frac{B_n^{\frac{1}{\alpha}} w_n}{\bar{W}} \left(\frac{H_n}{L_n}\right)^{\frac{1-\alpha}{\alpha}}, \qquad \bar{W} \equiv \left[\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{U}\right]^{\frac{1}{\alpha}}. \tag{B.21}$$

Now recall from equation (B.10) that the consumption goods price index can be written as:

$$P_n = \frac{\sigma}{\sigma-1}\left(\frac{1}{\sigma F}\right)^{\frac{1}{1-\sigma}}\left[\sum_{i\in N} L_i\left(d_{ni}\frac{w_i}{A_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{B.22}$$

Combining equations (B.21) and (B.22), we obtain our first system of equations for wages and population across locations from population mobility:

$$\bar{W}^{1-\sigma}\frac{1}{\sigma F}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = \frac{B_i^{\frac{1-\sigma}{\alpha}} w_i^{1-\sigma}\left(\frac{H_i}{L_i}\right)^{(1-\sigma)\frac{1-\alpha}{\alpha}}}{\sum_{n\in N} L_n\left(d_{in}\frac{w_n}{A_n}\right)^{1-\sigma}}. \tag{B.23}$$

We now turn to our second system of equations for wages and population across locations from the gravity of trade flows. Combining the gravity equation for trade and the equality of income and expenditure, we obtain:

$$w_i L_i = \sum_{n\in N}\frac{\frac{L_i}{\sigma F}\left(\frac{\sigma}{\sigma-1} d_{ni}\frac{w_i}{A_i}\right)^{1-\sigma}}{P_n^{1-\sigma}} w_n L_n. \tag{B.24}$$

Recall from equation (B.12) that the price index can be expressed as:

$$P_n^{1-\sigma} = \frac{\frac{L_n}{\sigma F}\left(\frac{\sigma}{\sigma-1}\frac{w_n}{A_n}\right)^{1-\sigma}}{\pi_{nn}}. \tag{B.25}$$

Combining equations (B.24) and (B.25), we obtain the following system of equations for wages in each location:

$$w_i^{\sigma} A_i^{1-\sigma} = \sum_{n\in N}\pi_{nn} d_{ni}^{1-\sigma} w_n^{\sigma} A_n^{1-\sigma}. \tag{B.26}$$

We now derive an expression for the domestic trade share (π_{nn}) in this system of equations. To do so, we combine the expression for the price index from equation (B.25) above with the following expression for the price index from the population mobility condition:

$$P_n = \frac{B_n^{\frac{1}{\alpha}} w_n}{\bar{W}} \left(\frac{H_n}{L_n} \right)^{\frac{1-\alpha}{\alpha}}, \qquad \bar{W} \equiv \left[\alpha \left(\frac{1-\alpha}{\alpha} \right)^{1-\alpha} \bar{U} \right]^{\frac{1}{\alpha}}. \tag{B.27}$$

Equating these two expressions for the price index, we obtain the following solution for the domestic trade share

$$\pi_{nn} = \bar{W}^{1-\sigma} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} L_n^{1-(\sigma-1)\frac{1-\alpha}{\alpha}} H_n^{(\sigma-1)\frac{1-\alpha}{\alpha}} A_n^{\sigma-1} B_n^{\frac{\sigma-1}{\alpha}}. \tag{B.28}$$

Using this solution for the domestic trade share (B.28) in the system of equations for wages in each location (B.26), we obtain our second system of equations for wages and population in each location:

$$\bar{W}^{1-\sigma} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} = \frac{w_i^{\sigma} A_i^{1-\sigma}}{\sum_{n \in N} d_{ni}^{1-\sigma} L_n^{1-(\sigma-1)\frac{1-\alpha}{\alpha}} H_n^{(\sigma-1)\frac{1-\alpha}{\alpha}} B_n^{\frac{\sigma-1}{\alpha}} w_n^{\sigma}}. \tag{B.29}$$

Under the assumption of symmetric trade costs ($d_{in} = d_{ni}$), the following eigenvector solves the two systems of equations (B.23) and (B.29):

$$w_n^{1-2\sigma} A_n^{\sigma-1} L_n^{(\sigma-1)\frac{1-\alpha}{\alpha}} H_n^{-(\sigma-1)\frac{1-\alpha}{\alpha}} B_n^{-\frac{\sigma-1}{\alpha}} = \xi. \tag{B.30}$$

where ξ is the corresponding eigenvalue. Using this closed-form solution (B.30) in either equation (B.23) or (B.29), we obtain a single system of equations that determines equilibrium population in equation location:

$$L_n^{\tilde{\sigma}\gamma_1} A_n^{-\tilde{\sigma}(\sigma-1)} B_n^{-\frac{\tilde{\sigma}\sigma}{\alpha}} H_n^{-\tilde{\sigma}\sigma\frac{1-\alpha}{\alpha}} = \bar{W}^{1-\sigma} \sum_{i \in N} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma-1} d_{ni} \right)^{1-\sigma} \left(L_i^{\tilde{\sigma}\gamma_1} \right)^{\frac{\gamma_2}{\gamma_1}} A_i^{\tilde{\sigma}\sigma} B_i^{\frac{\tilde{\sigma}(\sigma-1)}{\alpha}} H_i^{\tilde{\sigma}(\sigma-1)\frac{1-\alpha}{\alpha}}, \tag{B.31}$$

$$\tilde{\sigma} \equiv \frac{\sigma - 1}{2\sigma - 1}, \tag{B.32}$$

$$\gamma_1 \equiv \sigma \frac{1-\alpha}{\alpha}, \tag{B.33}$$

$$\gamma_2 \equiv 1 + \frac{\sigma}{\sigma - 1} - (\sigma - 1)\frac{1-\alpha}{\alpha}. \tag{B.34}$$

We are thus in a position to establish the following proposition in the main text.

Proposition 2 *Assume $\sigma(1-\alpha) > 1$. Given the floor space, productivity and amenity parameters $\{H_n, A_n, B_n\}$ and symmetric bilateral trade frictions $\{d_{ni}\}$ for all locations $n, i \in N$, there exist unique equilibrium populations (L_n^*) that solve this system of equations.*

Proof. The proof follows Allen and Arkolakis (2014). Assume $\sigma(1-\alpha) > 1$. Given the floor space and productivity for each location $\{H_n, A_n\}$ and bilateral trade frictions $\{d_{ni}\}$, there exists a unique fixed point in this system because $\gamma_2/\gamma_1 < 1$ (Fujimoto and Krause 1985). ■

The intuition for this parameter restriction $\sigma(1-\alpha) > 1$ is that the existence of a unique equilibrium requires that the model's agglomeration forces are sufficiently weak relative to its dispersion forces. A higher value of $(1-\alpha)$ implies that floor space accounts for a larger share of expenditure and a higher elasticity of substitution (σ) implies that varieties are closer substitutes, both of which weaken agglomeration forces relative to dispersion forces.

B8 Extension to Idiosyncratic Amenities

We now extend the baseline Helpman (1998) model from Section 3 of the paper to incorporate idiosyncratic preferences across locations. We show that the model exhibits similar properties to the baseline specification in the paper, but that the idiosyncratic preferences act as an additional dispersion force. The indirect utility of worker ω residing in location n now depends on goods income (v_n), the consumption goods price index (P_n), the price of floor space (Q_n), amenities that are common for all workers (B_n), and an idiosyncratic amenity shock ($b_n(\vartheta)$):

$$U_n(\vartheta) = \frac{b_n(\vartheta) B_n v_n}{P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1. \tag{B.35}$$

This idiosyncratic amenity shock ($b_n(\vartheta)$) captures the idea that workers have heterogeneous preferences from living in each location (e.g. different preferences for climate, proximity to the coast, etc). We assume that these amenity shocks are drawn independently across locations and workers from the following Fréchet distribution:

$$F(b) = e^{-b^{-\epsilon}}, \qquad \epsilon > 1, \tag{B.36}$$

where we have normalized the Fréchet scale parameter to one, because it enters the model isomorphically to the common amenities parameter (B_n). The Fréchet shape parameter (ϵ) regulates the dispersion of idiosyncratic amenities, which controls the sensitivity of worker location decisions to economic variables (e.g. wages and the cost of living). The smaller the shape parameter (ϵ), the greater the heterogeneity in idiosyncratic amenities, and the less sensitive are worker location decisions to economic variables.

Using the property that the indirect utility function in equation (B.35) is a monotonic function of the idiosyncratic amenity draw, indirect utility also has a Fréchet distribution:

$$F_n(U) = e^{-\psi_n U^{-\epsilon}}, \qquad \psi \equiv \left(B_n v_n / P_n^{\alpha} Q_n^{1-\alpha}\right)^{\epsilon}. \tag{B.37}$$

Using this distribution of indirect utility, the probability that a worker chooses to live in location $n \in N$ is given by:

$$\lambda_n \equiv \frac{L_n}{\bar{L}} = \frac{\left(B_n v_n / P_n^{\alpha} Q_n^{1-\alpha}\right)^{\epsilon}}{\sum_{k \in N} \left(B_k v_k / P_k^{\alpha} Q_k^{1-\alpha}\right)^{\epsilon}}, \tag{B.38}$$

and expected utility is given by:

$$\bar{U} = \delta \left[\sum_{k \in N} \left(B_k v_k / P_k^{\alpha} Q_k^{1-\alpha}\right)^{\epsilon}\right]^{\frac{1}{\epsilon}}, \tag{B.39}$$

where $\delta = \Gamma((\epsilon - 1)/\epsilon)$ and $\Gamma(\cdot)$ is the Gamma function. Using this expression for expected utility (B.39), we can re-write the location choice probabilities (B.38) as follows:

$$\frac{L_n}{\bar{L}} = \frac{\left(B_n v_n / P_n^{\alpha} Q_n^{1-\alpha}\right)^{\epsilon}}{\left(\bar{U}/\delta\right)^{\epsilon}}. \tag{B.40}$$

Now recall the following relationships:

$$v_n = \frac{w_n}{\alpha},$$

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{L_n}{\sigma F \pi_{nn}}\right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}.$$

$$Q_n = \frac{1 - \alpha}{\alpha} \frac{w_n L_n}{H_n}.$$

Using these three relationships in the choice probabilities (B.40), we obtain:

$$\frac{L_n}{\bar{L}} = \frac{\left(\alpha^{-1}\left(\frac{1}{\sigma F}\right)^{\frac{\alpha}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\frac{\alpha}{\sigma-1}} L_n^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}\right)^{\epsilon}}{\left(\bar{U}/\delta\right)^{\epsilon}}. \tag{B.41}$$

Re-arranging this equation to take terms in L_n over to the left-hand side, and dividing by its sum across all locations, we obtain the following expression for the location choice probabilities:

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^{\alpha} B_n H_n^{(1-\alpha)} \pi_{nn}^{-\alpha/(\sigma-1)}\right]^{\frac{1}{(1/\epsilon)+\left[\frac{\sigma(1-\alpha)-1}{\sigma-1}\right]}}}{\sum_{k\in N}\left[A_k^{\alpha} B_k H_k^{(1-\alpha)} \pi_{kk}^{-\alpha/(\sigma-1)}\right]^{\frac{1}{(1/\epsilon)+\left[\frac{\sigma(1-\alpha)-1}{\sigma-1}\right]}}}. \tag{B.42}$$

Taking the limit as $\epsilon \to \infty$, which corresponds to the case of no dispersion in idiosyncratic preferences, we obtain the same expression for the location choice probabilities as in the baseline Helpman (1998) model without idiosyncratic preferences in equation (16) in the paper.

C Isomorphisms

In this section of the appendix, we establish a number of isomorphisms to the new economic geography model of Helpman (1998). In subsection C1, we show that the system of equations for the general equilibrium of the model takes the same form as in a version of Eaton and Kortum (2002) with population mobility and external economies of scale, as considered in Redding (2016). In subsection C2, we show that the system of equations for the general equilibrium of the model takes the same form as in a version of the Armington (1969) model with population mobility and external economies of scale, as considered in Allen and Arkolakis (2014).

C1 Eaton and Kortum (2002) Model

We consider an economy consisting of a set N of regions indexed by n. Each region is endowed with an exogenous supply of floor space (H_i). The economy as a whole is endowed with a measure $\bar{L}$ of workers, where each worker has one unit of labor that is supplied inelastically with zero disutility. Workers are perfectly geographically mobile and hence in equilibrium real wages are equalized across all populated regions. Regions are connected by a bilateral transport network that can be used to ship goods subject to symmetric iceberg trade costs, such that $d_{ni} = d_{in} \geq 1$ units must be shipped from region i in order for one unit to arrive in region $n \neq i$, where $d_{ni} > 1$ for $n \neq i$ and $d_{nn} = 1$.

C1.1 Consumer Preferences

Preferences are defined over goods consumption and residential floor space use. We assume that these preferences take the Cobb-Douglas form, such that indirect utility for a worker in location n is given by:

$$U_n = \frac{B_n v_n}{P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1, \tag{C.1}$$

where B_n denotes amenities; v_n is worker income; P_n is the consumption goods price index; Q_n is the price of floor space. The consumption goods price index is defined over a fixed continuum of goods $\psi \in [0,1]$ and is assumed to take the constant elasticity of substitution (CES) form:

$$P_n = \left[\int_0^1 p_n(\psi)^{1-\sigma} d\psi\right]^{\frac{1}{1-\sigma}}, \qquad \sigma > 1, \tag{C.2}$$

where $p_n(\psi)$ denotes the price of good ψ in country n.

C1.2 Production

Each location draws an idiosyncratic productivity $z(\psi)$ for each good ψ. Productivity is independently drawn across goods and locations i from a Fréchet distribution:

$$F_i(z) = e^{-T_i z^{-\theta}}, \qquad T_i = A_i L_i^{\eta}, \qquad \theta > 1, \tag{C.3}$$

where the Fréchet scale parameter T_i determines average productivity for location i; we allow this Fréchet scale parameter, and hence average productivity, to depend on its population through external economies of scale (as parameterized by $\eta \geq 0$); and the Fréchet shape parameter θ controls the dispersion of productivity across goods.

Goods are homogeneous in the sense that one unit of a given good is the same as any other unit of that good. Each good is produced with labor under conditions of perfect competition according to a linear technology. The cost to a consumer in location n of purchasing one unit of good ψ from location i is therefore:

$$p_{ni}(\psi) = p_{ni} = \frac{d_{ni} w_i}{z_i(\psi)}, \tag{C.4}$$

where w_i denotes the wage in location i. The representative consumer in location n sources each good from the lowest-cost supplier to that location:

$$p_n(\psi) = \min\left\{p_{ni}(\psi); i \in N\right\}. \tag{C.5}$$

C1.3 Expenditure Shares and Price Indices

Using equilibrium prices (C.4) and the properties of the Fréchet distribution following Eaton and Kortum (2002), the share of expenditure of location n on goods produced by location i is:

$$\pi_{ni} = \frac{A_i L_i^{\eta} \left(d_{ni} w_i\right)^{-\theta}}{\sum_{k \in N} A_k L_k^{\eta} \left(d_{nk} w_k\right)^{-\theta}}, \tag{C.6}$$

while the consumption goods price index (C.7) can be expressed as:

$$P_n = \gamma \left[\sum_{i \in N} A_i L_i^{\eta} \left(d_{ni} w_i\right)^{-\theta}\right]^{-1/\theta}, \tag{C.7}$$

where $\gamma = \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$ and $\Gamma(\cdot)$ denotes the Gamma function. Using the expenditure share (C.6), the price index (C.7) also can be written as:

$$P_n = \gamma \left(\frac{A_n L_n^{\eta}}{\pi_{nn}}\right)^{-\frac{1}{\theta}} w_n, \tag{C.8}$$

so that locations with higher wages (w_n), lower productivity (A_n), lower population (L_n), and higher domestic trade shares (π_{nn}) have higher consumer goods price indices.

C1.4 Income and Population Mobility

Expenditure on floor space in each location is assumed to be redistributed lump sum to the workers residing in that location. Therefore total income in each location ($v_n L_n$) equals labor income ($w_n L_n$) plus expenditure on residential floor space ($(1-\alpha) v_n L_n$):

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n = \frac{w_n L_n}{\alpha}. \tag{C.9}$$

Goods market clearing implies that revenue in each location equals expenditure on goods produced in that location. We assume that trade in balanced, such that expenditure equals income in each location. Using zero profits, which implies that revenue equals labor income, and utility maximization, which implies that expenditure on goods is a constant share of income, this goods market clearing condition can be written as:

$$w_i L_i = \sum_{n \in N} \pi_{ni} w_n L_n. \tag{C.10}$$

Using land market clearing, we can determine the equilibrium price of floor space (Q_n) as follows:

$$Q_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}. \tag{C.11}$$

Population mobility implies that workers receive the same real income in all populated locations:

$$U_n = \frac{B_n v_n}{P_n^{\alpha} Q_n^{1-\alpha}} = \bar{U}. \tag{C.12}$$

Using land market clearing (C.11), the equality of income and expenditure (C.9) and the price index (C.8), we can rewrite this population mobility conditions as follows:

$$\bar{U} = \frac{A_n^{\frac{\alpha}{\theta}} B_n H_n^{1-\alpha} \pi_{nn}^{-\frac{\alpha}{\theta}} L_n^{-\frac{\theta(1-\alpha)-\eta\alpha}{\theta}}}{\alpha \gamma^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} \tag{C.13}$$

Re-arranging this population mobility condition to take population (L_n) over to the left-hand side, and dividing this expression by its sum across all locations, we obtain the following result that the population share of each location ($\lambda_n = L_n/\bar{L}$) depends on its productivity (A_n), amenities (B_n), supply of floor space (H_n) and domestic trade share (π_{nn}), relative to those of all other locations:

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^{\alpha/\theta} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/\theta}\right]^{\frac{\theta}{\theta(1-\alpha)-\eta\alpha}}}{\sum_{k \in N} \left[A_k^{\alpha/\theta} B_k H_k^{1-\alpha} \pi_{kk}^{-\alpha/\theta}\right]^{\frac{\theta}{\theta(1-\alpha)-\eta\alpha}}}. \tag{C.14}$$

C1.5 General Equilibrium

The general equilibrium of the model can be represented by the share of workers in each location ($\lambda_n = L_n/\bar{L}$), the share of each location's expenditure on goods produced in other locations (π_{ni}) and the wage in each location (w_n). Using labor income (C.10), the trade share (C.6), and population mobility (C.14) this equilibrium triple $\{\lambda_n, \pi_{ni}, w_n\}$ solves the following system of equations for all $i, n \in N$:

$$w_i \lambda_i = \sum_{n \in N} \pi_{ni} w_n \lambda_n, \tag{C.15}$$

$$\pi_{ni} = \frac{L_i^{\eta} \left(d_{ni} w_i / \mathbb{A}_i\right)^{-\theta}}{\sum_{k \in N} L_k^{\eta} \left(d_{nk} w_k / \mathbb{A}_k\right)^{-\theta}}, \tag{C.16}$$

$$\lambda_n = \frac{\left[\mathbb{A}_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/\theta}\right]^{\frac{\theta}{\theta(1-\alpha)-\eta\alpha}}}{\sum_{k \in N} \left[\mathbb{A}_k^{\alpha} B_k H_k^{1-\alpha} \pi_{kk}^{-\alpha/\theta}\right]^{\frac{\theta}{\theta(1-\alpha)-\eta\alpha}}}, \tag{C.17}$$

where $\mathbb{A}_i \equiv A_i^{1/\theta}$. The system of equations (C.15)-(C.17) is isomorphic to the system of equations (17)-(19) in the paper under the following parameter restrictions:

$$\theta = \sigma - 1,$$

$$\eta = 1,$$

$$\mathbb{A}_i = \left(A_i^{\text{EK}}\right)^{1/\theta} = A_i^{\text{Helpman}}.$$

Under these parameter restrictions, both models generate the same equilibrium vector $\{\lambda_n, \pi_{ni}, w_n\}$. As this system of equations (C.15)-(C.17) is isomorphic to the system of equations (17)-(19) in the paper, we obtain the same solutions for locational fundamentals, and the same system of equations for undertaking counterfactuals.

C2 Armington (1969) Model

We consider an economy consisting of a set N of regions indexed by n. Each region is endowed with an exogenous supply of floor space (H_i). The economy as a whole is endowed with a measure $\bar{L}$ of workers, where each worker has one unit of labor that is supplied inelastically with zero disutility. Workers are perfectly geographically mobile and hence in equilibrium real wages are equalized across all populated regions. Regions are connected by a bilateral transport network that can be used to ship goods subject to symmetric iceberg trade costs, such that $d_{ni} = d_{in} \geq 1$ units must be shipped from region i in order for one unit to arrive in region $n \neq i$, where $d_{ni} > 1$ for $n \neq i$ and $d_{nn} = 1$.

C2.1 Consumer Preferences

Preferences are defined over goods consumption and residential floor space use. We assume that these preferences takes the Cobb-Douglas form, such that indirect utility for a worker in location n is given by:

$$U_n = \frac{B_n v_n}{P_n^{\alpha} Q_n^{1-\alpha}}, \qquad 0 < \alpha < 1, \tag{C.18}$$

where B_n denotes amenities; v_n is worker income; P_n is the consumption goods price index; Q_n is the price of floor space. The consumption goods price index is defined over the varieties produced by each location and is assumed to take the constant elasticity of substitution (CES) form:

$$P_n = \left[\sum_{i \in N} p_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{C.19}$$

where p_{ni} is the cost to a consumer in location n of the variety produced by location i.

C2.2 Production

Goods are homogeneous in the sense that one unit of a given location's good is the same as any other unit of that location's good. Each location's good is produced with labor under conditions of perfect competition according to a linear technology. The cost to a consumer in location n of purchasing one unit of the good from location i is therefore:

$$p_{ni}(j) = \frac{d_{ni} w_i}{A_i L_i^{\eta}}, \qquad \eta \geq 0, \tag{C.20}$$

where w_i denotes the wage; A_i reflects exogenous determinants of productivity; and L_i^{η} captures external economies of scale in production.

C2.3 Expenditure Shares and Price Indices

Using the properties of CES demand and the equilibrium pricing rule (C.20), the share of expenditure of location n on goods produced by location i is:

$$\pi_{ni} = \frac{L_i^{\eta(\sigma-1)} \left(d_{ni} w_i / A_i\right)^{1-\sigma}}{\sum_{k \in N} L_k^{\eta(\sigma-1)} \left(d_{nk} w_k / A_k\right)^{1-\sigma}}, \tag{C.21}$$

Using the expenditure share (C.21), the price index (C.19) also can be written as:

$$P_n = \left(\frac{1}{\pi_{nn}}\right)^{-\frac{1}{\sigma-1}} \frac{w_n}{A_n L_n^{\eta}}, \tag{C.22}$$

so that locations with higher wages (w_n), lower productivity (A_n), lower population (L_n), and higher domestic trade shares (π_{nn}) have higher consumer goods price indices.

C2.4 Income and Population Mobility

Expenditure on floor space in each location is assumed to be redistributed lump sum to the workers residing in that location. Therefore total income in each location ($v_n L_n$) equals labor income ($w_n L_n$) plus expenditure on residential floor space ($(1-\alpha) v_n L_n$):

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n = \frac{w_n L_n}{\alpha}. \tag{C.23}$$

Goods market clearing implies that revenue in each location equals expenditure on goods produced in that location. We assume that trade in balanced, such that expenditure equals income in each location. Using zero profits, which implies that revenue equals labor income, and utility maximization, which implies that expenditure on goods is a constant share of income, this goods market clearing condition can be written as:

$$w_i L_i = \sum_{n \in N} \pi_{ni} w_n L_n. \tag{C.24}$$

Using land market clearing, the equilibrium price of floor space (Q_n) can be determined as follows:

$$Q_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}. \tag{C.25}$$

Population mobility implies that workers receive the same real income in all populated locations:

$$U_n = \frac{B_n v_n}{P_n^{\alpha} r_n^{1-\alpha}} = \bar{U}. \tag{C.26}$$

Using land market clearing (C.25), the equality of income and expenditure (C.23) and the price index (C.22), we can rewrite this population mobility conditions as follows:

$$\bar{U} = \frac{A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} L_n^{\eta\alpha-(1-\alpha)}}{\alpha \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}. \tag{C.27}$$

Re-arranging this population mobility condition to take population (L_n) over to the left-hand side, and dividing this expression by its sum across all locations, we obtain the following result that the population share of each location ($\lambda_n = L_n/\bar{L}$) depends on its productivity (A_n), amenities (B_n), supply of floor space (H_n) and domestic trade share (π_{nn}), relative to those of all other locations:

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)}\right]^{-\frac{1}{\eta\alpha-(1-\alpha)}}}{\sum_{k \in N} \left[A_k^{\alpha} B_k H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\sigma-1)}\right]^{-\frac{1}{\eta\alpha-(1-\alpha)}}}. \tag{C.28}$$

C2.5 General Equilibrium

The general equilibrium of the model can be represented by the share of workers in each location ($\lambda_n = L_n/\bar{L}$), the share of each location's expenditure on goods produced in other locations (π_{ni}) and the wage in each location (w_n). Using goods market clearing (C.24), the trade share (C.21), population mobility (C.28), this equilibrium triple $\{\lambda_n, \pi_{ni}, w_n\}$ solves the following system of equations for all $i, n \in N$:

$$w_i \lambda_i = \sum_{n \in N} \pi_{ni} w_n \lambda_n, \tag{C.29}$$

$$\pi_{ni} = \frac{L_i^{\eta(\sigma-1)} \left(d_{ni} w_i / A_i\right)^{1-\sigma}}{\sum_{k \in N} L_k^{\eta(\sigma-1)} \left(d_{nk} w_k / A_k\right)^{1-\sigma}}, \tag{C.30}$$

$$\lambda_n = \frac{\left[A_n^{\alpha} B_n H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)}\right]^{-\frac{1}{\eta\alpha-(1-\alpha)}}}{\sum_{k \in N} \left[A_k^{\alpha} B_k H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\sigma-1)}\right]^{-\frac{1}{\eta\alpha-(1-\alpha)}}}. \tag{C.31}$$

The system of equations (C.29)-(C.31) is isomorphic to the system of equations (17)-(19) in the paper under the following parameter restriction:

$$\eta = \sigma - 1.$$

Under this parameter restriction, both models generate the same equilibrium vector $\{\lambda_n, \pi_{ni}, w_n\}$. As this system of equations (C.29)-(C.31) is isomorphic to the system of equations (17)-(19) in the paper, we obtain the same solutions for locational fundamentals, and the same system of equations for undertaking counterfactuals.

D Firm and Consumer Market Access

In this section of the online appendix, we derive the relationship between firm and consumer market access reported in the paper. If bilateral trade costs are symmetric ($d_{ni} = d_{in}$), firm and consumer market access are proportional to one another in the class of quantitative spatial models discussed in the paper, as shown in Donaldson and Hornbeck (2016). To demonstrate this result, we start by using the definitions of firm market access in equation (32) in the paper and consumer market access in equation (34) in the paper to obtain a first relationship between these variables:

$$\text{FMA}_i \equiv \sum_{n \in N} d_{ni}^{1-\sigma} \left(w_n L_n\right) \text{CMA}_n^{-1}. \tag{D.1}$$

We next derive a second relationship between these variables using the gravity structure of trade and market clearing. First, from the price index in equation (8) in the paper and the definition of consumer market access in equation (34) in the paper, we have:

$$\text{CMA}_n \equiv \sum_{i \in N} \frac{L_i}{\sigma F} \left(\frac{\sigma}{\sigma-1} \frac{w_i}{A_i} d_{ni}\right)^{1-\sigma}. \tag{D.2}$$

Second, the gravity equation in equation (9) in the paper implies that aggregate bilateral trade between locations n and i (X_{ni}) is:

$$X_{ni} = \frac{L_i}{\sigma F} \left(\frac{\sigma}{\sigma-1} \frac{w_i}{A_i}\right)^{1-\sigma} d_{ni}^{1-\sigma} \left(w_n L_n\right) \text{CMA}_n^{-1}, \tag{D.3}$$

where we have used the fact that total expenditure on consumption goods (X_n) is a constant fraction of total income: $X_n = \alpha v_n L_n = w_n L_n$. Summing across destinations and using the requirement of balanced trade that total income equals total expenditure ($X_i = \sum_{n \in N} X_{ni} = w_i L_i$), we have:

$$w_i L_i = \frac{L_i}{\sigma F} \left(\frac{\sigma}{\sigma - 1} \frac{w_i}{A_i} \right)^{1-\sigma} \sum_{n \in N} d_{ni}^{1-\sigma} \left(w_n L_n \right) \mathrm{CMA}_n^{-1}. \tag{D.4}$$

Using the definition of firm market access in equation (32) in the paper, we can rewrite this expression for total income (D.4) as:

$$\frac{L_i}{\sigma F} \left(\frac{\sigma}{\sigma - 1} \frac{w_i}{A_i} \right)^{1-\sigma} = \frac{w_i L_i}{\mathrm{FMA}_i}. \tag{D.5}$$

Using this result in our expression for consumer market access (D.2) above, we obtain a second relationship between firm and consumer market access:

$$\mathrm{CMA}_n \equiv \sum_{i \in N} d_{ni}^{1-\sigma} \left(w_i L_i \right) \mathrm{FMA}_i^{-1}. \tag{D.6}$$

Under the assumption of symmetric trade costs ($d_{ni} = d_{in}$), the eigenvector that solves this system of two equations (D.1) and (D.6) satisfies: $\mathrm{FMA}_i = \psi \mathrm{CMA}_i$, where ψ is a scalar. We thus obtain the following recursive solution for firm market access:

$$\mathrm{FMA}_i \equiv \sum_{n \in N} d_{ni}^{1-\sigma} \left(w_n L_n \right) \psi \mathrm{FMA}_n^{-1}. \tag{D.7}$$

Therefore, assuming symmetric bilateral trade costs ($d_{ni} = d_{in}$), firm and consumer market access are perfectly correlated with one another and can be recovered (up to a scalar or normalization) from the system of equations (D.7).

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