Online Appendix to "Accounting for Trade Patterns" (Not for Publication)*

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A.1 Introduction

This Online Appendix contains additional theoretical derivations and supplementary empirical results for the main paper. In Section A.2, we report additional derivations for our theoretical framework from Section 2 of the paper. In Section A.3, we provide further detail on our structural estimation approach from Section 3 of the paper. In Section A.4, we report further information on the data sources and definitions for our U.S. and Chilean data from Section 4 of the paper.

In Section A.5, we report additional empirical results using our U.S. data that supplement those from Section 5 of the paper. In Section A.6, we replicate all of our empirical results from Section 5 of the paper, but using Chilean data instead of U.S. data. In Section A.7, we show that our theoretical approach allows for unobserved differences in product composition within observed product categories.

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A.2 Theoretical Framework Derivations

This section of the Online Appendix reports additional derivations for Section 2 of the paper. Each subsection has the same name as the corresponding subsection in Section 2 of the paper.

A.2.1 Demand

No further derivations required for Section 2.1 of the paper.

A.2.2 Non-traded Sectors

No further derivations required for Section 2.2 of the paper.

A.2.3 Domestic Versus Foreign Varieties Within Tradable Sectors

No further derivations required for Section 2.3 of the paper.

A.2.4 Exporter Price Indexes

No further derivations required for Section 2.4 of the paper.

A.2.5 Expenditure Shares

This section of the Online Appendix reports additional derivations for Section 2.5 of the paper. Corresponding to the firm expenditure share (S_{ut}^{U} in equation (12) in the paper), we can define the share of an individual foreign firm in expenditure on foreign imports within a sector ($\mathbb{S}_{ft}^{F}$) as:

$$\mathbb{S}_{ft}^{F} = \frac{\left(P_{ft}^{F} / \varphi_{ft}^{F}\right)^{1-\sigma_{g}^{F}}}{\sum_{i \in \Omega_{jgt}^{E}} \sum_{m \in \Omega_{jigt}^{F}} \left(P_{mt}^{F} / \varphi_{mt}^{F}\right)^{1-\sigma_{g}^{F}}}, \tag{A.2.1}$$

where we use "blackboard" font $\mathbb{S}_{ft}^{F}$ for the firm expenditure share to emphasize that this variable is defined as a share of expenditure on *foreign* firms (since $\Omega_{jgt}^{E} \equiv \left\{\Omega_{jgt}^{I} : i \neq j\right\}$ in the denominator of equation (A.2.1)). Similarly, we can define the share of an individual tradable sector in all expenditure on tradable sectors ($\mathbb{S}_{jgt}^{T}$)

$$\mathbb{S}_{jgt}^{T} = \frac{\left(P_{jgt}^{G} / \varphi_{jgt}^{G}\right)^{1-\sigma^{G}}}{\sum_{k \in \Omega^{T}} \left(P_{jkt}^{G} / \varphi_{jkt}^{G}\right)^{1-\sigma^{G}}}, \tag{A.2.2}$$

where we use the blackboard font $\mathbb{S}_{jgt}^{T}$ and superscript T for the sector expenditure share to signal that this variable is defined across *tradable* sectors (since $\Omega^{T} \subseteq \Omega^{G}$ in the denominator of equation (A.2.2)).

A.2.6 Model Inversion

In this section of the Online Appendix, we report additional derivations for Section 2.6 of the paper. In particular, given the observed data on prices and expenditures for each product $\{P_{ut}^U, X_{ut}^U\}$ and the substitution parameters $\{\sigma_g^U, \sigma_g^F, \sigma^G\}$, the model is invertible, such that unique values of appeal can be recovered from the observed data (up to a normalization or choice of units). We start with the solution for product appeal in equation (13) in the paper, reproduced below:

$$\frac{\varphi_{ut}^U}{\mathbb{M}_{ft}^{U*}\left[\varphi_{ut}^U\right]} = \frac{P_{ut}^U}{\mathbb{M}_{ft}^{U*}\left[P_{ut}^U\right]}\left(\frac{S_{ut}^U}{\mathbb{M}_{ft}^{U*}\left[S_{ut}^U\right]}\right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.2.3}$$

where we choose units in which to measure product appeal such that its geometric mean across common products within each firm is equal to one:

$$\mathbb{M}_{ft}^{U*}\left[\varphi_{ut}^U\right] \equiv \left(\prod_{u\in\Omega_{ft,t-1}^U}\varphi_{ut}^U\right)^{\frac{1}{N_{t,t-1}^U}} = 1. \tag{A.2.4}$$

Having solved for product appeal (φ_{ut}^U) using equations (A.2.3) and (A.2.4), we use equation (3) in the paper to compute the firm price index, as reproduced below:

$$P_{ft}^F = \left[\sum_{u\in\Omega_{ft}^U}\left(P_{ut}^U/\varphi_{ut}^U\right)^{1-\sigma_g^U}\right]^{\frac{1}{1-\sigma_g^U}}. \tag{A.2.5}$$

Using this solution for the firm price index (P_{ft}^F) from equation (A.2.5), we divide the share of a foreign firm in sectoral imports in equation (A.2.1) by its geometric mean across common foreign firms within that sector to obtain the following solution for appeal for each foreign firm:

$$\frac{\varphi_{ft}^F}{\mathbb{M}_{jgt}^{F*}\left[\varphi_{ft}^F\right]} = \frac{P_{ft}^F}{\mathbb{M}_{jgt}^{F*}\left[P_{ft}^F\right]}\left(\frac{\mathbb{S}_{ft}^F}{\mathbb{M}_{jgt}^{F*}\left[\mathbb{S}_{ft}^F\right]}\right)^{\frac{1}{\sigma_g^F-1}}, \tag{A.2.6}$$

where we choose units in which to measure firm appeal such that its geometric mean across common foreign firms within each sector is equal to one:

$$\mathbb{M}_{jgt}^{F*}\left[\varphi_{ft}^F\right] \equiv \left(\prod_{i\in\Omega_{jgt,t-1}^E}\prod_{f\in\Omega_{jigt,t-1}^F}\varphi_{ft}^F\right)^{\frac{1}{N_{jgt,t-1}^F}} = 1. \tag{A.2.7}$$

Having solved for firm appeal (φ_{ft}^F) for each foreign firm using equations (A.2.6) and (A.2.7), we use equations (7) and (9) in the paper to compute the sector price index, as reproduced below:

$$P_{jgt}^G = \left(\mu_{jgt}^G\right)^{\frac{1}{\sigma_g^F-1}}\left[\sum_{i\in\Omega_{jgt}^E}\sum_{f\in\Omega_{jigt}^F}\left(P_{ft}^F/\varphi_{ft}^F\right)^{1-\sigma_g^F}\right]^{\frac{1}{1-\sigma_g^F}}, \tag{A.2.8}$$

where recall that μ^{G}_{jgt} is the observed share of expenditure on foreign varieties within each sector.

Using this solution for the sector price index (P^{G}_{jgt}) from equation (A.2.8), we divide the share of an individual tradable sector in all expenditure on tradable sectors in equation (A.2.2) by its geometric mean across these tradable sectors to obtain the following solution for sector appeal for each tradable sector:

$$\frac{\varphi^{G}_{jgt}}{\mathbb{M}^{T}_{jt}\left[\varphi^{G}_{jgt}\right]} = \frac{P^{G}_{jgt}}{\mathbb{M}^{T}_{jt}\left[P^{G}_{jgt}\right]}\left(\frac{\mathbb{S}^{T}_{jgt}}{\mathbb{M}^{T}_{jt}\left[\mathbb{S}^{T}_{jgt}\right]}\right)^{\frac{1}{\sigma^{G}-1}}, \tag{A.2.9}$$

where we choose units in which to measure sector appeal such that its geometric mean across tradable sectors is equal to one:

$$\mathbb{M}^{T}_{jt}\left[\varphi^{G}_{jgt}\right] \equiv \left(\prod_{g\in\Omega^{T}}\varphi^{G}_{jgt}\right)^{\frac{1}{N^{T}}} = 1. \tag{A.2.10}$$

Recall that there is no asterisk in the superscript of the geometric mean operator across tradable sectors, because the set of tradable sectors is constant over time. Having solved for sector appeal (φ^{G}_{jgt}) for each tradable sector using equations (A.2.9) and (A.2.10), we use equations (4) and (6) in the paper to compute the aggregate price index, as reproduced below:

$$P_{jt} = \left(\mu^{T}_{jt}\right)^{\frac{1}{\sigma^{G}-1}}\left[\sum_{g\in\Omega^{T}}\left(P^{G}_{jgt}/\varphi^{G}_{jgt}\right)^{1-\sigma^{G}}\right]^{\frac{1}{1-\sigma^{G}}}, \tag{A.2.11}$$

where recall that μ^{T}_{jt} the observed share of aggregate expenditure on tradable sectors. This completes our inversion of the model to recover the structural residuals for product, firm and sector appeal $\{\varphi^{U}_{ut}, \varphi^{F}_{ft}, \varphi^{G}_{jgt}\}$.

A.2.7 Log-Linear CES Price Index

No further derivations required for Section 2.7 of the paper.

A.2.8 Entry, Exit and the Unified Price Index

In this section of the Online Appendix, we report additional derivations for Section 2.8 of the paper. In particular, we derive the expression for the change in the unified price index over time, taking into account entry and exit. Using the shares of expenditure on common goods in equation (19) in the paper, the change in the firm price index between periods $t-1$ and t (P^{F}_{ft}/P^{F}_{ft-1}) can be re-written as:

$$\frac{P^{F}_{ft}}{P^{F}_{ft-1}} = \left(\frac{\lambda^{U}_{ft}}{\lambda^{U}_{ft-1}}\right)^{\frac{1}{\sigma^{U}_{g}-1}}\left[\frac{\sum_{u\in\Omega^{U}_{ft,t-1}}\left(P^{U}_{ut}/\varphi^{U}_{ut}\right)^{1-\sigma^{U}_{g}}}{\sum_{u\in\Omega^{U}_{ft,t-1}}\left(P^{U}_{ut-1}/\varphi^{U}_{ut-1}\right)^{1-\sigma^{U}_{g}}}\right]^{\frac{1}{1-\sigma^{U}_{g}}} = \left(\frac{\lambda^{U}_{ft}}{\lambda^{U}_{ft-1}}\right)^{\frac{1}{\sigma^{U}_{g}-1}}\frac{P^{F*}_{ft}}{P^{F*}_{ft-1}}, \tag{A.2.12}$$

where the superscript asterisk indicates that a variable is defined for the common set of varieties. We can also define the share of expenditure on an individual common product in expenditure on all common products within the firm as:

$$S_{ut}^{U*} = \frac{\left(P_{ut}^{U}/\varphi_{ut}^{U}\right)^{1-\sigma_g^U}}{\sum_{\ell \in \Omega_{ft,t-1}^{U}} \left(P_{\ell t}^{U}/\varphi_{\ell t}^{U}\right)^{1-\sigma_g^U}} = \frac{\left(P_{ut}^{U}/\varphi_{ut}^{U}\right)^{1-\sigma_g^U}}{\left(P_{ft}^{F*}\right)^{1-\sigma_g^U}}. \tag{A.2.13}$$

Rearranging this common product expenditure share (A.2.13), taking logarithms, and taking means of both sides of the equation, we obtain the following expression for the log of the common goods firm price index (P_{ft}^{F*}):

$$\ln P_{ft}^{*} = \mathbb{E}_{ft}^{U*}\left[P_{ut}^{U}\right] - \mathbb{E}_{ft}^{U*}\left[\varphi_{ut}^{U}\right] + \frac{1}{\sigma_g^U - 1}\mathbb{E}_{ft}^{U*}\left[S_{ut}^{U*}\right] \tag{A.2.14}$$

where $\mathbb{E}_{ft}^{U*}\left[\ln P_{ut}^{U}\right] \equiv \frac{1}{N_{ft,t-1}^{U*}} \sum_{u \in \Omega_{ft,t-1}^{U}} \ln\left(P_{ut}^{U}\right)$; the superscript U^{*} indicates that the mean is taken across common products; and the subscripts f and t indicate that this mean varies across firms and over time. Taking logarithms in equation (A.2.12), and using the expression for the common goods firm price index in equation (A.2.14), we obtain equation (20) in the paper.

A.2.9 Exporter Price Movements

In this section of the Online Appendix, we report additional derivations for Section 2.9 of the paper. In particular, we derive the log linear decompositions of the exporter price index ($\mathbb{P}_{jigt}^{E}$) for a given exporter and sector in equations (21) and (22) in the paper. We first use the CES expression for the share an individual foreign firm f in country j's imports from a foreign exporting country $i \neq j$ within a sector g:

$$\mathbb{S}_{ft}^{EF} = \frac{\left(P_{ft}^{F}/\varphi_{ft}^{F}\right)^{1-\sigma_g^F}}{\sum_{k \in \Omega_{jigt}^{F}} \left(P_{kt}^{F}/\varphi_{kt}^{F}\right)^{1-\sigma_g^F}} = \frac{\left(P_{ft}^{F}/\varphi_{ft}^{F}\right)^{1-\sigma_g^F}}{\left(\mathbb{P}_{jigt}^{E}\right)^{1-\sigma_g^F}}, \qquad i \neq j, \tag{A.2.15}$$

where the superscript EF is a mnemonic for exporter and firm, and indicates that this firm expenditure share is computed as a share of imports from a single foreign exporting country.

Re-arranging equation (A.2.15), taking logarithms of both sides, adding and subtracting $\frac{1}{\sigma_g^F - 1} \ln N_{jigt}^{F}$, and taking means across foreign firms from that exporter and sector, we obtain the following expression for the log of the exporter price index:

$$\ln \mathbb{P}_{jigt}^{E} = \mathbb{E}_{jigt}^{F}\left[\ln P_{ft}^{F}\right] - \mathbb{M}_{jigt}^{F}\left[\ln \varphi_{ft}^{F}\right] - \frac{1}{\sigma_g^F - 1}\ln N_{jigt}^{F} + \frac{1}{\sigma_g^F - 1}\mathbb{E}_{jigt}^{F}\left[\mathbb{S}_{ft}^{EF} - \ln \frac{1}{N_{jigt}^{F}}\right], \tag{A.2.16}$$

where $\mathbb{E}_{jigt}^{F}\left[\cdot\right]$ is the mean for importer j across firms from exporter i within sector g at time t, such that $\mathbb{E}_{jigt}^{F}\left[\ln P_{ft}^{F}\right] \equiv \frac{1}{N_{jigt}^{F}} \sum_{f \in \Omega_{jigt}^{F}} \ln P_{ft}^{F}$.

Substituting the firm price index (P_{ft}^{F}) from equation (18) in the paper into equation (A.2.16) above, we obtain our exact log linear decomposition of the exporter price index in equation (21) in the paper, which is reproduced below:

$$\ln \mathbb{P}^{E}_{jigt} = \underbrace{\mathbb{E}^{FU}_{jigt}\left[\ln P^{U}_{ut}\right]}_{\substack{\text{(i) Average log}\\ \text{prices}}} - \underbrace{\left\{\mathbb{E}^{F}_{jigt}\left[\ln \varphi^{F}_{ft}\right] + \mathbb{E}^{FU}_{jigt}\left[\ln \varphi^{U}_{ut}\right]\right\}}_{\substack{\text{(ii) Average log}\\ \text{appeal}}} + \underbrace{\left\{\frac{1}{\sigma^{U}_{g}-1}\mathbb{E}^{FU}_{jigt}\left[\ln S^{U}_{ut} - \ln \frac{1}{N^{U}_{ft}}\right] + \frac{1}{\sigma^{F}_{g}-1}\mathbb{E}^{F}_{jigt}\left[\ln \mathbb{S}^{EF}_{ft} - \ln \frac{1}{N^{F}_{jigt}}\right]\right\}}_{\text{(iii) Dispersion of appeal-adjusted prices}} \tag{A.2.17}$$

$$- \underbrace{\left\{\frac{1}{\sigma^{U}_{g}-1}\mathbb{E}^{F}_{jigt}\left[\ln N^{U}_{ft}\right] + \frac{1}{\sigma^{F}_{g}-1}\ln N^{F}_{jigt}\right\}}_{\text{(iv) Variety}},$$

where $\mathbb{E}^{FU}_{jigt}\left[\cdot\right]$ is the mean for importer j across firms and products from exporter i within sector g at time t, such that $\mathbb{E}^{FU}_{jigt}\left[\ln P^{U}_{ut}\right] \equiv \frac{1}{N^{F}_{jigt}}\sum_{f\in\Omega^{F}_{jigt}}\frac{1}{N^{U}_{ft}}\sum_{u\in\Omega^{U}_{ft}}\ln P^{U}_{ut}$.

We next incorporate the entry and exit of varieties. The log change in the exact CES price index for an importer j sourcing goods in sector g from an exporter i between periods $t-1$ and t is:

$$\frac{\mathbb{P}^{E}_{jigt}}{\mathbb{P}^{E}_{jigt-1}} = \left[\frac{\sum_{f\in\Omega^{F}_{jigt}}\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}{\sum_{f\in\Omega^{F}_{jigt-1}}\left(P^{F}_{ft-1}/\varphi^{F}_{ft-1}\right)^{1-\sigma^{F}_{g}}}\right]^{\frac{1}{1-\sigma^{F}_{g}}}, \tag{A.2.18}$$

where the entry and exit of firms over time implies that $\Omega^{F}_{jigt} \neq \Omega^{F}_{jigt-1}$. We define the share of expenditure on common firms $f \in \Omega^{F}_{jigt,t-1}$ within an exporter and sector in periods t and $t-1$ as:

$$\lambda^{F}_{jigt} \equiv \frac{\sum_{f\in\Omega^{F}_{jigt,t-1}}\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}{\sum_{f\in\Omega^{F}_{jigt}}\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}, \qquad \lambda^{F}_{jigt-1} \equiv \frac{\sum_{f\in\Omega^{F}_{jigt,t-1}}\left(P^{F}_{ft-1}/\varphi^{F}_{ft-1}\right)^{1-\sigma^{F}_{g}}}{\sum_{f\in\Omega^{F}_{jigt-1}}\left(P^{F}_{ft-1}/\varphi^{F}_{ft-1}\right)^{1-\sigma^{F}_{g}}}. \tag{A.2.19}$$

Using these definitions from equation (A.2.19), the change in the exporter price index in equation (A.2.18) can be re-written in the following form:

$$\frac{\mathbb{P}^{E}_{jigt}}{\mathbb{P}^{E}_{jigt-1}} = \left(\frac{\lambda^{F}_{jigt}}{\lambda^{F}_{jigt-1}}\right)^{\frac{1}{\sigma^{F}_{g}-1}}\left[\frac{\sum_{f\in\Omega^{F}_{jigt,t-1}}\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}{\sum_{f\in\Omega^{F}_{jigt,t-1}}\left(P^{F}_{ft-1}/\varphi^{F}_{ft-1}\right)^{1-\sigma^{F}_{g}}}\right]^{\frac{1}{1-\sigma^{F}_{g}}} = \left(\frac{\lambda^{F}_{jigt}}{\lambda^{F}_{jigt-1}}\right)^{\frac{1}{\sigma^{F}_{g}-1}}\frac{\mathbb{P}^{E*}_{jigt}}{\mathbb{P}^{E*}_{jigt-1}}, \tag{A.2.20}$$

where the first term ($\left(\lambda^{F}_{jigt}/\lambda^{F}_{jigt-1}\right)^{\frac{1}{\sigma^{F}_{g}-1}}$) corrects for the entry and exit of firms; the second term ($\mathbb{P}^{E*}_{jigt}/\mathbb{P}^{E*}_{jigt-1}$) is the change in the exporter price index for common firms; and we again use the superscript asterisk to denote a variable for common varieties. Using this notation, we can also define the share of expenditure on an individual common firm in overall expenditure on common firms for an exporter and sector:

$$\mathbb{S}^{EF*}_{ft} = \frac{\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}{\sum_{m\in\Omega^{F}_{jigt,t-1}}\left(P^{F}_{mt}/\varphi^{F}_{mt}\right)^{1-\sigma^{F}_{g}}} = \frac{\left(P^{F}_{ft}/\varphi^{F}_{ft}\right)^{1-\sigma^{F}_{g}}}{\left(\mathbb{P}^{E*}_{jigt}\right)^{1-\sigma^{F}_{g}}}. \tag{A.2.21}$$

Rearranging equation (A.2.21) so that the exporter price index for common firms ($\mathbb{P}_{jigt}^{E*}$) is on the left-hand side, taking logarithms, and taking means across the set of common firms within an exporter and sector, we obtain:

$$\ln \mathbb{P}_{jigt}^{E*} = \mathbb{E}_{jigt}^{F*}\left[\ln P_{ft}^{F}\right] - \mathbb{E}_{jigt}^{F*}\left[\ln \varphi_{ft}^{F}\right] + \frac{1}{\sigma_g^F - 1}\mathbb{E}_{jigt}^{F*}\left[\ln S_{ft}^{F}\right], \tag{A.2.22}$$

$\mathbb{E}_{jigt}^{F*}[\cdot]$ is the mean across the common set of firms (superscript F^*) for a given importer (subscript j), exporter (subscript i), sector (subscript g) and time (subscript t) such that:

$$\mathbb{E}_{jigt}^{F*}\left[\ln P_{ft}^{F}\right] = \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \ln P_{ft}^{F}. \tag{A.2.23}$$

Taking differences over time in equation (A.2.22), we obtain the following expression for the log change in the common goods exporter price index:

$$\ln\left(\frac{\mathbb{P}_{jigt}^{E*}}{\mathbb{P}_{jigt-1}^{E*}}\right) = \mathbb{E}_{jigt}^{F*}\left[\ln\left(\frac{P_{ft}^{F}}{P_{ft-1}^{F}}\right)\right] - \mathbb{E}_{jigt}^{F*}\left[\ln\left(\frac{\varphi_{ft}^{F}}{\varphi_{ft-1}^{F}}\right)\right] + \frac{1}{\sigma_g^F - 1}\mathbb{E}_{jigt}^{F*}\left[\ln\left(\frac{\mathbb{S}_{ft}^{EF*}}{\mathbb{S}_{ft-1}^{EF*}}\right)\right]. \tag{A.2.24}$$

We now take logarithms in equation (A.2.20) and use equation (A.2.24) to substitute for $\mathbb{P}_{jigt}^{E*}/\mathbb{P}_{jigt-1}^{E*}$ and arrive at the following expression for the log change in the overall exporter price index:

$$\ln\left(\frac{\mathbb{P}_{jigt}^{E}}{\mathbb{P}_{jigt-1}^{E}}\right) = \frac{1}{\sigma_g^F - 1}\ln\left(\frac{\lambda_{jigt}^{F}}{\lambda_{jigt-1}^{F}}\right) + \mathbb{E}_{jigt}^{F*}\left[\ln\left(\frac{P_{ft}^{F}}{P_{ft-1}^{F}}\right)\right] - \mathbb{E}_{jigt}^{F*}\ln\left[\left(\frac{\varphi_{ft}^{F}}{\varphi_{ft-1}^{F}}\right)\right] + \frac{1}{\sigma_g^F - 1}\mathbb{E}_{jigt}^{F*}\left[\ln\left(\frac{\mathbb{S}_{ft}^{EF*}}{\mathbb{S}_{ft-1}^{EF*}}\right)\right]. \tag{A.2.25}$$

Substituting the expression the change in the firm price index from equation (20) in the paper into equation (A.2.25), we obtain equation (22) in the paper, which is reproduced below:

$$\begin{aligned}\Delta \ln \mathbb{P}_{jigt}^{E} = {} & \underbrace{\mathbb{E}_{jigt}^{FU*}\left[\Delta \ln P_{ut}^{U}\right]}_{\text{(i) Average log prices}} - \underbrace{\left\{\mathbb{E}_{jigt}^{F*}\left[\Delta \ln \varphi_{ft}^{F}\right] + \mathbb{E}_{jigt}^{FU*}\left[\Delta \ln \varphi_{ut}^{U}\right]\right\}}_{\text{(ii) Average log appeal}} \\ & + \underbrace{\left\{\frac{1}{\sigma_g^U - 1}\mathbb{E}_{jigt}^{FU*}\left[\Delta \ln S_{ut}^{U*}\right] + \frac{1}{\sigma_g^F - 1}\mathbb{E}_{jigt}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF}\right]\right\}}_{\text{(iii) Dispersion of appeal-adjusted prices}} \\ & + \underbrace{\left\{\frac{1}{\sigma_g^U - 1}\mathbb{E}_{jigt}^{F*}\left[\Delta \ln \lambda_{ft}^{U}\right] + \frac{1}{\sigma_g^F - 1}\Delta \ln \lambda_{jigt}^{F}\right\}}_{\text{(iv) Variety}},\end{aligned} \tag{A.2.26}$$

where Δ is the difference operator such that $\Delta \ln \mathbb{P}_{jigt}^{E} \equiv \ln\left(\mathbb{P}_{jigt}^{E}/\mathbb{P}_{jigt-1}^{E}\right)$; $\mathbb{E}_{jigt}^{FU*}[\cdot]$ is a mean, first across common products within firms and then across common firms (superscript FU^*), for a given importer (subscript j), exporter (subscript i), sector (subscript g) and time period (subscript t) such that:

$$\mathbb{E}_{jigt}^{FU*}\left[\Delta \ln P_{ut}^{U}\right] = \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \frac{1}{N_{ft,t-1}^{U}} \sum_{u \in \Omega_{ft,t-1}^{U}} \Delta \ln P_{ut}^{U}. \tag{A.2.27}$$

Recall that our normalization of product appeal in equation (A.2.4) implies $\mathbb{E}_{jigt}^{FU*}\left[\Delta \ln \varphi_{ut}^{U}\right] = 0$. Therefore the log change in the exporter price index in equation (A.2.26) simplifies to:

$$\Delta \ln \mathbb{P}^{E}_{jigt} = \underbrace{\mathbb{E}^{FU*}_{jigt}\left[\Delta \ln P^{U}_{ut}\right]}_{\text{(i) Average log prices}} - \underbrace{\left\{\mathbb{E}^{F*}_{jigt}\left[\Delta \ln \varphi^{F}_{ft}\right]\right\}}_{\text{(ii) Average log appeal}} \tag{A.2.28}$$
$$+ \underbrace{\left\{\frac{1}{\sigma^{U}_{g}-1}\mathbb{E}^{FU*}_{jigt}\left[\Delta \ln S^{U*}_{ut}\right] + \frac{1}{\sigma^{F}_{g}-1}\mathbb{E}^{F*}_{jigt}\left[\Delta \ln \mathbb{S}^{EF}_{ft}\right]\right\}}_{\text{(iii) Dispersion of appeal-adjusted prices}}$$
$$+ \underbrace{\left\{\frac{1}{\sigma^{U}_{g}-1}\mathbb{E}^{F*}_{jigt}\left[\Delta \ln \lambda^{U}_{ft}\right] + \frac{1}{\sigma^{F}_{g}-1}\Delta \ln \lambda^{F}_{jigt}\right\}}_{\text{(iv) Variety}}.$$

A.2.10 Patterns of Trade Across Sectors and Countries

A.2.10.1 Revealed Comparative Advantage

In this section of the Online Appendix, we report the derivation of the results in Section 2.10.1 of the paper. In particular, we derive the decompositions of revealed comparative advantage (RCA) in equations (26) and (27) in the paper. From equation (25) in the paper, log RCA is given by:

$$\ln\left(RCA_{jigt}\right) = \left(1-\sigma^{F}_{g}\right)\left[\ln\left(\mathbb{P}^{E}_{jigt}\right) - \frac{1}{N^{E}_{jgt}}\sum_{h\in\Omega^{E}_{jgt}}\ln\left(\mathbb{P}^{E}_{jhgt}\right)\right] - \frac{1}{N^{T}_{jit}}\sum_{k\in\Omega^{T}_{jit}}\left(1-\sigma^{F}_{k}\right)\left[\ln\left(\mathbb{P}^{E}_{jikt}\right) - \frac{1}{N^{E}_{jkt}}\sum_{h\in\Omega^{E}_{jkt}}\ln\left(\mathbb{P}^{E}_{jhkt}\right)\right]. \tag{A.2.29}$$

where recall that $\Omega^{E}_{jgt} \equiv \left\{\Omega^{I}_{jgt} : i \neq j\right\}$ is the set of foreign exporters that supply importer j within sector g at time t; $N^{E}_{jgt} = \left|\Omega^{E}_{jgt}\right|$ is the number of elements in this set; Ω^{T}_{jit} is the set of tradable sectors that importer j sources from exporter i at time t; and $N^{T}_{jit} = \left|\Omega^{T}_{jit}\right|$ is the number of elements in this set. Using equation (21) in the paper to substitute for the log exporter price index ($\mathbb{P}^{E}_{jigt}$) in equation (A.2.29), we obtain the following exact log-linear decomposition of RCA:

$$\ln\left(RCA_{jigt}\right) = \underbrace{\ln\left(RCA^{P}_{jigt}\right)}_{\text{(i) Average log prices}} + \underbrace{\ln\left(RCA^{\varphi}_{jigt}\right)}_{\text{(ii) Average log appeal}} + \underbrace{\ln\left(RCA^{S}_{jigt}\right)}_{\text{(iii) Dispersion of appeal-adjusted prices}} + \underbrace{\ln\left(RCA^{N}_{jigt}\right)}_{\text{(iv) Variety}}. \tag{A.2.30}$$

The first term in equation (A.2.30) captures average product prices:

$$\ln\left(RCA^{P}_{jigt}\right) \equiv \left\{\begin{array}{l} \left(1-\sigma^{F}_{g}\right)\left[\mathbb{E}^{FU}_{jigt}\left[\ln P^{U}_{ut}\right] - \frac{1}{N^{E}_{jgt}}\sum_{h\in\Omega^{E}_{jgt}}\mathbb{E}^{FU}_{jhgt}\left[\ln P^{U}_{ut}\right]\right] \\ -\frac{1}{N^{T}_{jit}}\sum_{k\in\Omega^{T}_{jit}}\left(1-\sigma^{F}_{k}\right)\left[\mathbb{E}^{FU}_{jikt}\left[\ln P^{U}_{ut}\right] - \frac{1}{N^{E}_{jkt}}\sum_{h\in\Omega^{E}_{jkt}}\mathbb{E}^{FU}_{jhkt}\left[\ln P^{U}_{ut}\right]\right] \end{array}\right\}, \tag{A.2.31}$$

where $\mathbb{E}^{FU}_{jigt}[\cdot]$ denotes an average, first across products within firms (superscript U), and next across firms (superscript F) supplying importer j from exporter i within sector g at time t such that:

$$\mathbb{E}^{FU}_{jigt}\left[\Delta \ln P^{U}_{ut}\right] = \frac{1}{N^{F}_{jigt}}\sum_{f\in\Omega^{F}_{jigt}}\frac{1}{N^{U}_{ft}}\sum_{u\in\Omega^{U}_{ft}}\Delta \ln P^{U}_{ut}. \tag{A.2.32}$$

The second term in equation (A.2.30) incorporates average firm and product appeal:

$$\ln\left(RCA^{\varphi}_{jigt}\right) \equiv \left\{ \begin{array}{l} \left(\sigma^F_g - 1\right)\left[\mathbb{E}^F_{jigt}\left[\ln \varphi^F_{ft}\right] - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \mathbb{E}^F_{jhgt}\left[\ln \varphi^F_{ft}\right]\right] \\ -\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \left(\sigma^F_k - 1\right)\left[\mathbb{E}^F_{jikt}\left[\ln \varphi^F_{ft}\right] - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \mathbb{E}^F_{jhkt}\left[\ln \varphi^F_{ft}\right]\right] \\ +\left(\sigma^F_g - 1\right)\left[\mathbb{E}^{FU}_{jigt}\left[\ln \varphi^U_{ut}\right] - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \mathbb{E}^{FU}_{jhgt}\left[\ln \varphi^U_{ut}\right]\right] \\ -\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \left(\sigma^F_k - 1\right)\left[\mathbb{E}^{FU}_{jikt}\left[\ln \varphi^U_{ut}\right] - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \mathbb{E}^{FU}_{jhkt}\left[\ln \varphi^U_{ut}\right]\right] \end{array} \right\}, \tag{A.2.33}$$

where $\mathbb{E}^F_{jigt}\left[\cdot\right]$ denotes an average across firms (superscript F) supplying importer j from exporter i within sector g at time t such that:

$$\mathbb{E}^F_{jigt}\left[\Delta \ln \varphi^F_{ft}\right] = \frac{1}{N^F_{jigt}} \sum_{f\in\Omega^F_{jigt}} \Delta \ln \varphi^F_{ft}. \tag{A.2.34}$$

The third term in equation (A.2.30) reflects the dispersion of firm and product appeal-adjusted prices, as reflected in the dispersion of firm and product expenditure shares:

$$\ln\left(RCA^{S}_{jigt}\right) \equiv -\left\{ \begin{array}{l} \left[\mathbb{E}^F_{jigt}\left[\ln \mathbb{S}^{EF}_{ft} - \ln \frac{1}{N^F_{jigt}}\right] - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \mathbb{E}^F_{jhgt}\left[\ln \mathbb{S}^{EF}_{ft} - \ln \frac{1}{N^F_{jhgt}}\right]\right] \\ -\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \left[\mathbb{E}^F_{jikt}\left[\ln \mathbb{S}^{EF}_{ft} - \ln \frac{1}{N^F_{jikt}}\right] - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \mathbb{E}^F_{jhkt}\left[\ln \mathbb{S}^{EF}_{ft} - \ln \frac{1}{N^F_{jhkt}}\right]\right] \\ +\frac{\sigma^F_g-1}{\sigma^U_g-1}\left[\mathbb{E}^{FU}_{jigt}\left[\ln S^U_{ut} - \ln \frac{1}{N^U_{ft}}\right] - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \mathbb{E}^{FU}_{jhgt}\left[\ln S^U_{ut} - \ln \frac{1}{N^U_{ft}}\right]\right] \\ -\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \frac{\sigma^F_k-1}{\sigma^U_k-1}\left[\mathbb{E}^{FU}_{jikt}\left[\ln S^U_{ut} - \ln \frac{1}{N^U_{ft}}\right] - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \mathbb{E}^{FU}_{jhkt}\left[\ln S^U_{ut} - \ln \frac{1}{N^U_{ft}}\right]\right] \end{array} \right\}, \tag{A.2.35}$$

where S^U_{ut} is defined in equation (12) in the paper and $\mathbb{S}^{EF}_{ft}$ is defined in equation (A.2.15) of this Online Appendix.

The fourth and final term in equation (A.2.30) comprises firm and product variety:

$$\ln\left(RCA^{N}_{jigt}\right) \equiv \left\{ \begin{array}{l} \left[\ln N^F_{jigt} - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \ln N^F_{jhgt}\right] \\ -\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \left[\ln N^F_{jikt} - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \ln N^F_{jhkt}\right] \\ +\frac{\sigma^F_g-1}{\sigma^U_g-1}\left[\mathbb{E}^F_{jigt}\left[\ln N^U_{ft}\right] - \frac{1}{N^E_{jgt}} \sum_{h\in\Omega^E_{jgt}} \mathbb{E}^F_{jhgt}\left[\ln N^U_{ft}\right]\right] \\ +\frac{1}{N^T_{jit}} \sum_{k\in\Omega^T_{jit}} \frac{\sigma^F_k-1}{\sigma^U_k-1}\left[\mathbb{E}^F_{jikt}\left[\ln N^U_{ft}\right] - \frac{1}{N^E_{jkt}} \sum_{h\in\Omega^E_{jkt}} \mathbb{E}^F_{jhkt}\left[\ln N^U_{ft}\right]\right] \end{array} \right\}, \tag{A.2.36}$$

where N^F_{jigt} is the number of firms that supply importer j from exporting country i within sector g at time t; N^E_{jgt} is the number of exporting countries that supply importer j within sector g at time t; N^T_{jit} is the number of tradable sectors in which exporting country i supplies importer j at time t; and N^U_{ft} is the number of products supplied by firm f at time t.

Taking logarithms and differencing over time in the definition of RCA in equation (25) in the paper, and using the expression for the change in the log exporter price index from equation (A.2.26) of this Online Appendix, the log change in revealed comparative advantage (RCA) over time can be written as:

$$\Delta \ln\left(RCA^{*}_{jigt}\right) = \underbrace{\Delta \ln\left(RCA^{P*}_{jigt}\right)}_{\text{(i) Average log prices}} + \underbrace{\Delta \ln\left(RCA^{\varphi *}_{jigt}\right)}_{\text{(ii) Average log appeal}} + \underbrace{\Delta \ln\left(RCA^{S*}_{jigt}\right)}_{\text{(iii) Dispersion appeal-adjusted prices}} + \underbrace{\Delta \ln\left(RCA^{\lambda}_{jigt}\right)}_{\text{(iv) Variety}}, \tag{A.2.37}$$

where we compute these log changes for all common exporter-sector pairs with positive trade in both periods, as indicated by the asterisks in the superscripts. The first term in equation (A.2.37) captures average log changes in common product prices:

$$\Delta \ln \left(RCA_{jigt}^{P*} \right) \equiv \left\{ \begin{array}{l} \left(1-\sigma_g^F\right) \left[\mathbb{E}_{jigt,t-1}^{FU*} \left[\Delta \ln P_{ut}^U \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{FU*} \left[\Delta \ln P_{ut}^U \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \left(1-\sigma_k^F\right) \left[\mathbb{E}_{jikt,t-1}^{FU*} \left[\Delta \ln P_{ut}^U \right] - \frac{1}{N_{jkt}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{FU*} \left[\Delta \ln P_{ut}^U \right] \right] \end{array} \right\}, \quad \text{(A.2.38)}$$

where $\Omega_{jgt,t-1}^E$ is the set of common foreign exporters that supply importer j within sector g in both periods $t-1$ and t; $N_{jgt,t-1}^E = \left| \Omega_{jgt,t-1}^E \right|$ is the number of elements in this set; $\Omega_{jit,t-1}^T$ is the set of tradable sectors that importer j sources from exporter i in both periods $t-1$ and t; $N_{jit}^T = \left| \Omega_{jit}^T \right|$ is the number of elements in this set; $\mathbb{E}_{jigt}^{FU*}[\cdot]$ denotes an average, first across common products within firms and next across common firms (superscript FU^*), supplying importer j from exporter i within sector g at time t (as defined in equation (A.2.27)). The second term in equation (A.2.37) incorporates average log changes in common firm and product appeal:

$$\Delta \ln \left(RCA_{jigt}^{\varphi*} \right) \equiv \left\{ \begin{array}{l} \left(\sigma_g^F - 1\right) \left[\mathbb{E}_{jigt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \left(\sigma_k^F - 1\right) \left[\mathbb{E}_{jikt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] - \frac{1}{N_{jkt,t-1}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] \right] \\ + \left(\sigma_g^F - 1\right) \left[\mathbb{E}_{jigt,t-1}^{FU*} \left[\Delta \ln \varphi_{ut}^U \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{FU*} \left[\Delta \ln \varphi_{ut}^U \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \left(\sigma_k^F - 1\right) \left[\mathbb{E}_{jikt,t-1}^{FU*} \left[\Delta \ln \varphi_{ut}^U \right] - \frac{1}{N_{jkt,t-1}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{FU*} \left[\Delta \ln \varphi_{ut}^U \right] \right] \end{array} \right\}, \quad \text{(A.2.39)}$$

where $\mathbb{E}_{jigt,t-1}^{F*}[\cdot]$ denotes an average across common firms (superscript F^*) supplying importer j from exporter i within sector g at time t (as defined in equation (A.2.23)). Recall that our normalization of product appeal in equation (A.2.4) implies that $\mathbb{E}_{ft}^{U*}\left[\Delta \ln \varphi_{ut}^U\right] = 0$, which in turn implies that this second term simplifies to:

$$\Delta \ln \left(RCA_{jigt}^{\varphi*} \right) \equiv \left\{ \begin{array}{l} \left(\sigma_g^F - 1\right) \left[\mathbb{E}_{jigt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \left(\sigma_k^F - 1\right) \left[\mathbb{E}_{jikt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] - \frac{1}{N_{jkt,t-1}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{F*} \left[\Delta \ln \varphi_{ft}^F \right] \right] \end{array} \right\}, \quad \text{(A.2.40)}$$

where, in general, $\mathbb{E}_{jigt,t-1}^{F*}\left[\Delta \ln \varphi_{ft}^F\right] \neq \mathbb{E}_{jgt,t-1}^{F*}\left[\Delta \ln \varphi_{ft}^F\right] = 0$ for an individual exporter $i \neq j$. The third term in equation (A.2.37) encapsulates the dispersion in appeal-adjusted prices across common products and firms:

$$\ln \left(RCA_{jigt}^{S*} \right) \equiv - \left\{ \begin{array}{l} \left[\mathbb{E}_{jigt,t-1}^{F*} \left[\Delta \ln \mathbb{S}_{ft}^{EF*} \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{F*} \left[\Delta \ln \mathbb{S}_{ft}^{EF*} \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \left[\mathbb{E}_{jikt,t-1}^{F*} \left[\Delta \ln \mathbb{S}_{ft}^{EF*} \right] - \frac{1}{N_{jkt,t-1}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{F*} \left[\Delta \ln \mathbb{S}_{ft}^{EF*} \right] \right] \\ + \frac{\sigma_g^F - 1}{\sigma_g^U - 1} \left[\mathbb{E}_{jigt,t-1}^{FU*} \left[\Delta \ln S_{ut}^{U*} \right] - \frac{1}{N_{jgt,t-1}^E} \sum_{h \in \Omega_{jgt,t-1}^E} \mathbb{E}_{jhgt,t-1}^{FU*} \left[\Delta \ln S_{ut}^{U*} \right] \right] \\ -\frac{1}{N_{jit,t-1}^T} \sum_{k \in \Omega_{jit,t-1}^T} \frac{\sigma_k^F - 1}{\sigma_k^U - 1} \left[\mathbb{E}_{jikt,t-1}^{FU*} \left[\Delta \ln S_{ut}^{U*} \right] - \frac{1}{N_{jkt,t-1}^E} \sum_{h \in \Omega_{jkt,t-1}^E} \mathbb{E}_{jhkt,t-1}^{FU*} \left[\Delta \ln S_{ut}^{U*} \right] \right] \end{array} \right\}, \quad \text{(A.2.41)}$$

where S_{ut}^{U*} is defined in equation (A.2.13) and $\mathbb{S}_{ft}^{EF*}$ is defined in equation (A.2.21). The fourth and final term in equation (A.2.37) corresponds to the entry and exit of products and firms:

$$\ln\left(RCA_{jigt}^{\lambda}\right) \equiv -\left\{\begin{array}{l} \left[\Delta\ln\lambda_{jigt}^{F} - \frac{1}{N_{jgt,t-1}^{E}}\sum_{h\in\Omega_{jgt,t-1}^{E}}\Delta\ln\lambda_{jhgt}^{F}\right] \\ -\frac{1}{N_{jit,t-1}^{T}}\sum_{k\in\Omega_{jit,t-1}^{T}}\left[\Delta\ln\lambda_{jikt}^{F} - \frac{1}{N_{jkt,t-1}^{E}}\sum_{h\in\Omega_{jkt,t-1}^{E}}\Delta\ln\lambda_{jhkt}^{F}\right] \\ +\frac{\sigma_{g}^{F}-1}{\sigma_{g}^{U}-1}\left[\mathbb{E}_{jigt}^{F*}\left[\Delta\ln\lambda_{ft}^{U}\right] - \frac{1}{N_{jgt,t-1}^{E}}\sum_{h\in\Omega_{jgt,t-1}^{E}}\mathbb{E}_{jhgt}^{F*}\left[\Delta\ln\lambda_{ft}^{U}\right]\right] \\ -\frac{1}{N_{jit,t-1}^{T}}\sum_{k\in\Omega_{jit,t-1}^{T}}\frac{\sigma_{k}^{F}-1}{\sigma_{k}^{U}-1}\left[\mathbb{E}_{jikt}^{F*}\left[\Delta\ln\lambda_{ft}^{U}\right] - \frac{1}{N_{jkt,t-1}^{E}}\sum_{h\in\Omega_{jkt,t-1}^{E}}\mathbb{E}_{jhkt}^{F*}\left[\Delta\ln\lambda_{ft}^{U}\right]\right] \end{array}\right\}, \tag{A.2.42}$$

where λ_{ut}^{U} is defined in equation (19) in the paper and λ_{ft}^{F} is defined in equation (A.2.19) of this Online Appendix.

A.2.10.2 Aggregate Trade

In this section of the Online Appendix, we report additional derivations for Section 2.10.2 of the paper. In particular, we derive the decomposition of countries' shares of aggregate imports in equation (28) in the paper. We begin by rewriting the share of an individual exporter in aggregate imports in terms of a share of common imports (supplied in both periods t and $t-1$) and entry and exit terms. We have the following accounting identity for the share of an individual exporter in aggregate imports:

$$\mathbb{S}_{jit}^{E} \equiv \frac{\mathbb{X}_{jit}^{E}}{\mathbb{X}_{jt}^{T}} = \frac{\mathbb{X}_{jt}^{T*}}{\mathbb{X}_{jt}^{T}}\frac{\mathbb{X}_{jit}^{E}}{\mathbb{X}_{jit}^{E*}}\frac{\mathbb{X}_{jit}^{E*}}{\mathbb{X}_{jt}^{T*}}, \tag{A.2.43}$$

where $\mathbb{X}_{jit}^{E}$ is country j's imports from exporter $i \neq j$ at time t; $\mathbb{X}_{jt}^{T}$ is country j's total imports from all foreign exporters at time t; $\mathbb{X}_{jit}^{E*}$ is country j's imports in common sectors pairs (supplied in both periods $t-1$ and t) from foreign exporter $i \neq j$; $\mathbb{X}_{jt}^{T*}$ is country j's imports in common exporter-sector pairs (supplied in both periods $t-1$ and t) from all foreign exporters.

We now define two terms that capture entry and exit of exporter-sector pairs over time. First, we define λ_{jit}^{E} to be the share of imports in common sectors from an individual foreign exporter $i \neq j$:

$$\lambda_{jit}^{E} \equiv \frac{\mathbb{X}_{jit}^{E*}}{\mathbb{X}_{jit}^{E}} = \frac{\sum_{g\in\Omega_{jit,t-1}^{T}}\mathbb{X}_{jigt}^{E}}{\sum_{g\in\Omega_{jit}^{T}}\mathbb{X}_{jigt}^{E}}, \tag{A.2.44}$$

where Ω_{jit}^{T} is the set of traded sectors in which country j imports from exporter i at time t and $\Omega_{jit,t-1}^{T}$ is the subset of these sectors that are common (supplied in both periods t and $t-1$). Second, we define λ_{jt}^{T} to be the share of imports from common exporter-sector pairs in imports from all foreign exporters:

$$\lambda_{jt}^{T} \equiv \frac{\mathbb{X}_{jt}^{T*}}{\mathbb{X}_{jt}^{T}} = \frac{\sum_{g\in\Omega_{jt,t-1}^{T}}\sum_{i\in\Omega_{jgt,t-1}^{E}}\mathbb{X}_{jigt}^{E}}{\sum_{g\in\Omega_{jt}^{T}}\sum_{i\in\Omega_{jgt}^{E}}\mathbb{X}_{jigt}^{E}}, \tag{A.2.45}$$

where Ω_{jgt}^{E} is the set of foreign exporters $i \neq j$ from which country j imports in sector g at time t and $\Omega_{jgt,t-1}^{E}$ is the subset of these foreign exporters that are common (supplied in both periods t and $t-1$);

Ω^T_{jt} is the set of sectors in which country j imports from foreign exporters at time t; and $\Omega^T_{jt,t-1}$ is the subset of these sectors that are common (supplied in both periods t and $t-1$). Third, we define $\mathbb{S}^{E*}_{jit}$ to be the share of an individual exporter $i \neq j$ in imports from common exporter-sector pairs:

$$\mathbb{S}^{E*}_{jit} \equiv \frac{\mathbb{X}^{E*}_{jit}}{\mathbb{X}^{T*}_{jt}} = \frac{\sum_{g \in \Omega^T_{jit,t-1}} \mathbb{X}^E_{jigt}}{\sum_{g \in \Omega^T_{jt,t-1}} \sum_{m \in \Omega^E_{jgt,t-1}} \mathbb{X}^E_{jmgt}}. \tag{A.2.46}$$

Using equations (A.2.44), (A.2.45) and (A.2.46), we can rewrite the share of an individual foreign exporter $i \neq j$ in country j imports from equation (A.2.43) in terms of its share of common imports ($\mathbb{S}^{E*}_{jit}$), an entry and exit term for that exporter (λ^E_{jit}) and an entry and exit term for imports from all foreign exporters (λ^T_{jt}):

$$\mathbb{S}^E_{jit} = \frac{\lambda^T_{jt}}{\lambda^E_{jit}} \mathbb{S}^{E*}_{jit}. \tag{A.2.47}$$

Using equation (A.2.46) to substitute for $\mathbb{S}^{E*}_{jit}$ in equation (A.2.47), we obtain:

$$\mathbb{S}^E_{jit} = \frac{\lambda^T_{jt}}{\lambda^E_{jit}} \frac{\sum_{g \in \Omega^T_{jit,t-1}} \mathbb{X}^E_{jigt}}{\sum_{g \in \Omega^T_{jt,t-1}} \sum_{m \in \Omega^E_{jgt,t-1}} \mathbb{X}^E_{jmgt}}, \tag{A.2.48}$$

which using CES demand can be further re-written as:

$$\mathbb{S}^E_{jit} = \frac{\lambda^T_{jt}}{\lambda^E_{jit}} \frac{\sum_{g \in \Omega^T_{jit,t-1}} \left(\mathbb{P}^E_{jigt}\right)^{1-\sigma^F_g} \mathbb{X}^G_{jgt} \left(\mathbb{P}^G_{jgt}\right)^{\sigma^F_g - 1}}{\sum_{g \in \Omega^T_{jt,t-1}} \sum_{m \in \Omega^E_{jgt,t-1}} \left(\mathbb{P}^E_{jmgt}\right)^{1-\sigma^F_g} \mathbb{X}^G_{jgt} \left(\mathbb{P}^G_{jgt}\right)^{\sigma^F_g - 1}}, \tag{A.2.49}$$

where $\mathbb{P}^E_{jigt}$ is country j's price index for exporter $i \neq j$ in sector g at time t; $\mathbb{X}^G_{jgt}$ is country j's total expenditure on imports from foreign countries in sector g at time t; and $\mathbb{P}^G_{jgt}$ is country j's import price index for sector g at time t.

To re-write this expression for an exporter's share of imports in a log-linear form, we now define two terms for the importance of imports in a given sector from a given exporter, one as a share of common imports across all sectors from that exporter, and the other as a share of common imports across all sectors from all foreign exporters. First, we define importer j's expenditure on exporter $i \neq j$ in sector g at time t as a share of expenditure on that exporter across all common sectors as:

$$\mathbb{Z}^{E*}_{jigt} \equiv \frac{\mathbb{X}^E_{jigt}}{\sum_{k \in \Omega^T_{jit,t-1}} \mathbb{X}^E_{jikt}} = \frac{\left(\mathbb{P}^E_{jigt}\right)^{1-\sigma^F_g} \mathbb{X}^G_{jgt} \left(\mathbb{P}^G_{jgt}\right)^{\sigma^F_g - 1}}{\sum_{k \in \Omega^T_{jit,t-1}} \left(\mathbb{P}^E_{jikt}\right)^{1-\sigma^F_k} \mathbb{X}^G_{jkt} \left(\mathbb{P}^G_{jkt}\right)^{\sigma^F_k - 1}}, \tag{A.2.50}$$

which can be re-arranged to express the denominator from the right-hand side as follows:

$$\sum_{k \in \Omega^T_{jit,t-1}} \left(\mathbb{P}^E_{jikt}\right)^{1-\sigma^F_k} \mathbb{X}^G_{jkt} \left(\mathbb{P}^G_{jkt}\right)^{\sigma^F_k - 1} = \frac{\left(\mathbb{P}^E_{jigt}\right)^{1-\sigma^F_g} \mathbb{X}^G_{jgt} \left(\mathbb{P}^G_{jgt}\right)^{\sigma^F_g - 1}}{\mathbb{Z}^{E*}_{jigt}}. \tag{A.2.51}$$

Taking geometric means across common sectors $g \in \Omega^{G}_{jit,t-1}$, this becomes:

$$\sum_{k\in\Omega^{T}_{jit,t-1}} \left(\mathbb{P}^{E}_{jikt}\right)^{1-\sigma^{F}_{k}} \mathbb{X}^{G}_{jkt} \left(\mathbb{P}^{G}_{jkt}\right)^{\sigma^{F}_{k}-1} = \frac{\left(\mathbb{M}^{T*}_{jit}\left[\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}}\right]\right) \mathbb{M}^{T*}_{jit}\left[\mathbb{X}^{G}_{jgt}\right] \left(\mathbb{M}^{T*}_{jit}\left[\left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}\right]\right)}{\mathbb{M}^{T*}_{jit}\left[\mathbb{Z}^{E*}_{jigt}\right]}, \tag{A.2.52}$$

where $\mathbb{M}^{T*}_{jit}\left[\mathbb{P}^{E}_{jigt}\right] \equiv \left(\prod_{g\in\Omega^{T}_{jit,t-1}} \mathbb{P}^{E}_{jigt}\right)^{1/N^{T}_{jit,t-1}}$ and $N^{T}_{jit,t-1}$ is the number of common sectors that exporter i supplies to importer j between periods $t-1$ and t. Second, we define importer j's expenditure on exporter $i \neq j$ in sector g at time t as a share of expenditure on common sectors from all foreign exporters as:

$$\mathbb{Y}^{E*}_{jigt} \equiv \frac{\mathbb{X}^{E}_{jigt}}{\sum_{k\in\Omega^{T}_{jt,t-1}} \sum_{m\in\Omega^{E}_{jkt,t-1}} \mathbb{X}^{E}_{jmkt}} = \frac{\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}} \mathbb{X}^{G}_{jgt} \left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}}{\sum_{k\in\Omega^{T}_{jt,t-1}} \sum_{m\in\Omega^{E}_{jkt,t-1}} \left(\mathbb{P}^{E}_{jmkt}\right)^{1-\sigma^{F}_{k}} \mathbb{X}^{G}_{jkt} \left(\mathbb{P}^{G}_{jkt}\right)^{\sigma^{F}_{k}-1}}, \tag{A.2.53}$$

which can be re-arranged to express the denominator from the right-hand side as follows:

$$\sum_{k\in\Omega^{T}_{jt,t-1}} \sum_{m\in\Omega^{E}_{jgt,t-1}} \left(\mathbb{P}^{E}_{jmkt}\right)^{1-\sigma^{F}_{k}} \mathbb{X}^{G}_{jkt} \left(\mathbb{P}^{G}_{jkt}\right)^{\sigma^{F}_{k}-1} = \frac{\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}} \mathbb{X}^{G}_{jgt} \left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}}{\mathbb{Y}^{E*}_{jigt}}. \tag{A.2.54}$$

Taking geometric means across common exporters within each sector and across common sectors, this becomes:

$$\sum_{k\in\Omega^{T}_{jt,t-1}} \sum_{m\in\Omega^{E}_{jgt,t-1}} \left(\mathbb{P}^{E}_{jmkt}\right)^{1-\sigma^{F}_{k}} \mathbb{X}^{G}_{jkt} \left(\mathbb{P}^{G}_{jkt}\right)^{\sigma^{F}_{k}-1} = \frac{\left(\mathbb{M}^{TE*}_{jt}\left[\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}}\right]\right) \mathbb{M}^{TE*}_{jt}\left[\mathbb{X}^{G}_{jgt}\right] \left(\mathbb{M}^{TE*}_{jt}\left[\left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}\right]\right)}{\mathbb{M}^{TE*}_{jt}\left[\mathbb{Y}^{E*}_{jigt}\right]}, \tag{A.2.55}$$

where $\mathbb{M}^{TE*}_{jt}\left[\mathbb{P}^{E}_{jigt}\right] \equiv \left(\prod_{g\in\Omega^{T}_{jt,t-1}} \prod_{i\in\Omega^{E}_{jgt,t-1}} \mathbb{P}^{E}_{jigt}\right)^{1/N^{E}_{jt,t-1}}$ and $N^{E}_{jt,t-1}$ is the number of common exporter-sectors for importer j between periods $t-1$ and t.

Using these two measures of the importance of country imports from an individual exporter in a given sector from equations (A.2.52) and (A.2.55), we can re-write the country import share in equation (A.2.49) in the following log-linear form:

$$\mathbb{S}^{E}_{jit} = \frac{\lambda^{T}_{jt}}{\lambda^{E}_{jit}} \frac{\mathbb{M}^{T*}_{jit}\left[\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}}\right]}{\mathbb{M}^{TE*}_{jt}\left[\left(\mathbb{P}^{E}_{jigt}\right)^{1-\sigma^{F}_{g}}\right]} \frac{\mathbb{M}^{T*}_{jit}\left[\mathbb{X}^{G}_{jgt}\right]\left(\mathbb{M}^{T*}_{jit}\left[\left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}\right]\right)/\mathbb{M}^{T*}_{jit}\left[\mathbb{Z}^{E*}_{jigt}\right]}{\mathbb{M}^{TE*}_{jt}\left[\mathbb{X}^{G}_{jgt}\right]\left(\mathbb{M}^{TE*}_{jt}\left[\left(\mathbb{P}^{G}_{jgt}\right)^{\sigma^{F}_{g}-1}\right]\right)/\mathbb{M}^{TE*}_{jt}\left[\mathbb{Y}^{E*}_{jigt}\right]}. \tag{A.2.56}$$

Taking logarithms, differencing, and re-arranging terms, we obtain the following log-linear decomposition of a country's share of aggregate imports:

$$\Delta \ln \mathbb{S}^{E}_{jit} = \Delta \ln\left(\frac{\lambda^{T}_{jt}}{\lambda^{E}_{jit}}\right) + \mathbb{E}^{T*}_{jit}\left[\left(1-\sigma^{F}_{g}\right)\left[\Delta \ln \mathbb{P}^{E}_{jigt}\right]\right] - \mathbb{E}^{TE*}_{jt}\left[\left(1-\sigma^{F}_{g}\right)\left[\Delta \ln \mathbb{P}^{E}_{jigt}\right]\right] + \Delta \ln \mathbb{K}^{T}_{jit} + \Delta \ln \mathbb{J}^{T}_{jit}, \tag{A.2.57}$$

where $\mathbb{E}^{TE*}_{jt}\left[\mathbb{P}^{E}_{jigt}\right] \equiv \frac{1}{N^{E}_{jt,t-1}} \sum_{g\in\Omega^{T}_{jt,t-1}} \sum_{i\in\Omega^{E}_{jgt,t-1}} \mathbb{P}^{E}_{jigt}$. The penultimate term ($\Delta \ln \mathbb{K}^{T}_{jit}$) captures changes in exporter-sector scale, as measured by the change in the extent to which country j sources imports from exporter i in large sectors (sectors with high sectoral import expenditures $\mathbb{X}_{jgt}^{G}$ and low sectoral import price indexes $\mathbb{P}_{jgt}^{G}$) relative to its overall imports from all exporters:

$$\Delta \ln \mathbb{K}_{jit}^{T} \equiv \Delta \ln \left[\frac{\mathbb{M}_{jit}^{T*}\left[\mathbb{X}_{jgt}^{G}\right]\left(\mathbb{M}_{jit}^{T*}\left[\left(\mathbb{P}_{jgt}^{G}\right)^{\sigma_g^F-1}\right]\right)}{\mathbb{M}_{jt}^{TE*}\left[\mathbb{X}_{jgt}^{G}\right]\left(\mathbb{M}_{jt}^{TE*}\left[\left(\mathbb{P}_{jgt}^{G}\right)^{\sigma_g^F-1}\right]\right)} \right]. \tag{A.2.58}$$

The final term ($\Delta \ln \mathbb{J}_{jit}^{T}$) captures changes in the sectoral concentration of imports, as measured by changes in the importance of country j's imports from exporter i in sector g as a share of common imports from exporter i ($\mathbb{Z}_{jigt}^{E*}$) relative to its share of aggregate common imports ($\mathbb{Y}_{jigt}^{E*}$):

$$\Delta \ln \mathbb{J}_{jit}^{T} \equiv \Delta \ln \left[\frac{\mathbb{M}_{jt}^{TE*}\left[\mathbb{Y}_{jigt}^{E*}\right]}{\mathbb{M}_{jit}^{T*}\left[\mathbb{Z}_{jigt}^{E*}\right]} \right]. \tag{A.2.59}$$

This final term corresponds to an exact Jensen's Inequality correction term that allows us to preserve log linearity in our decompositions of both sectoral and aggregate trade. Using equation (A.2.26) to substitute for the exporter price index ($\mathbb{P}_{jigt}^{E}$) in equation (A.2.57), we obtain the exact log-linear decomposition of changes in country import shares in equation (28) in the paper, as reproduced below:

$$\begin{aligned}
\Delta \ln \mathbb{S}_{jit}^{E} = & -\underbrace{\left\{\mathbb{E}_{jit}^{TFU*}\left[\left(\sigma_g^F-1\right)\Delta \ln P_{ut}^{U}\right] - \mathbb{E}_{jt}^{TEFU*}\left[\left(\sigma_g^F-1\right)\Delta \ln P_{ut}^{U}\right]\right\}}_{\text{(i) Average log prices}} \\
& + \underbrace{\left\{\mathbb{E}_{jit}^{TFU*}\left[\left(\sigma_g^F-1\right)\Delta \ln \varphi_{ut}^{U}\right] - \mathbb{E}_{jt}^{TEFU*}\left[\left(\sigma_g^F-1\right)\Delta \ln \varphi_{ut}^{U}\right]\right\}}_{\text{(ii) Average log product appeal}} \\
& + \underbrace{\left\{\mathbb{E}_{jit}^{TF*}\left[\left(\sigma_g^F-1\right)\Delta \ln \varphi_{ft}^{F}\right] - \mathbb{E}_{jt}^{TEF*}\left[\left(\sigma_g^F-1\right)\Delta \ln \varphi_{ft}^{F}\right]\right\}}_{\text{(iii) Average log firm appeal}} \\
& - \underbrace{\left\{\mathbb{E}_{jit}^{TFU*}\left[\frac{\sigma_g^F-1}{\sigma_g^U-1}\Delta \ln S_{ut}^{U*}\right] - \mathbb{E}_{jt}^{TEFU*}\left[\frac{\sigma_g^F-1}{\sigma_g^U-1}\Delta \ln S_{ut}^{U*}\right]\right\}}_{\text{(iv) Dispersion product appeal-adjusted prces}} - \underbrace{\left\{\mathbb{E}_{jit}^{TF*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] - \mathbb{E}_{jt}^{TEF*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right]\right\}}_{\text{(v) Disperion firm appeal-adjusted prices}} \\
& - \underbrace{\left\{\mathbb{E}_{jit}^{TF*}\left[\frac{\sigma_g^F-1}{\sigma_g^U-1}\Delta \ln \lambda_{ft}^{U}\right] - \mathbb{E}_{jt}^{TEF*}\left[\frac{\sigma_g^F-1}{\sigma_g^U-1}\Delta \ln \lambda_{ft}^{U}\right]\right\}}_{\text{(vi) Product Variety}} - \underbrace{\left\{\mathbb{E}_{jit}^{T}\left[\Delta \ln \lambda_{jigt}^{F}\right] - \mathbb{E}_{jt}^{TE*}\left[\Delta \ln \lambda_{jigt}^{F}\right]\right\}}_{\text{(vii) Firm Variety}} - \underbrace{\Delta \ln \left(\lambda_{jit}^{E}/\lambda_{jt}^{T}\right)}_{\text{(viii) Country-Sector Variety}} \\
& + \underbrace{\Delta \ln \mathbb{K}_{jit}^{T}}_{\text{(ix) Country-sector Scale}} + \underbrace{\Delta \ln \mathbb{J}_{jit}^{T}}_{\text{(x) Country-sector Concentration}},
\end{aligned} \tag{A.2.60}$$

where we have the following definitions:

$$\mathbb{E}_{jit}^{TFU*}\left[\Delta \ln P_{ut}^{U}\right] \equiv \frac{1}{N_{jit,t-1}^{T}} \sum_{g \in \Omega_{jit,t-1}^{T}} \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \frac{1}{N_{ft,t-1}^{U}} \sum_{u \in \Omega_{ft,t-1}^{U}} \Delta \ln P_{ut}^{U}, \tag{A.2.61}$$

$$\mathbb{E}_{jt}^{TEFU*}\left[\Delta \ln P_{ut}^{U}\right] \equiv \frac{1}{N_{jt,t-1}^{E}} \sum_{g \in \Omega_{jt,t-1}^{T}} \sum_{i \in \Omega_{jgt,t-1}^{E}} \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \frac{1}{N_{ft,t-1}^{U}} \sum_{u \in \Omega_{ft,t-1}^{U}} \Delta \ln P_{ut}^{U}, \tag{A.2.62}$$

$$\mathbb{E}_{jit}^{TF*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] \equiv \frac{1}{N_{jit,t-1}^{T}} \sum_{g \in \Omega_{jit,t-1}^{T}} \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \Delta \ln \mathbb{S}_{ft}^{EF*}, \tag{A.2.63}$$

$$\mathbb{E}_{jt}^{TEF*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] \equiv \frac{1}{N_{jt,t-1}^{E}} \sum_{g \in \Omega_{jt,t-1}^{T}} \sum_{i \in \Omega_{jgt,t-1}^{E}} \frac{1}{N_{jigt,t-1}^{F}} \sum_{f \in \Omega_{jigt,t-1}^{F}} \Delta \ln \mathbb{S}_{ft}^{EF*}, \tag{A.2.64}$$

$$\mathbb{E}_{jit}^{T}\left[\Delta\ln\lambda_{jigt}^{F}\right] \equiv \frac{1}{N_{jt,t-1}^{T}} \sum_{g\in\Omega_{jt,t-1}^{T}} \Delta\ln\lambda_{jigt}^{F}, \tag{A.2.65}$$

$$\mathbb{E}_{jt}^{TE*}\left[\Delta\ln\lambda_{jigt}^{F}\right] \equiv \frac{1}{N_{jt,t-1}^{E}} \sum_{g\in\Omega_{jt,t-1}^{T}} \sum_{i\in\Omega_{jgt,t-1}^{E}} \Delta\ln\lambda_{jigt}^{F}. \tag{A.2.66}$$

A.2.11 Aggregate Prices

In this section of the Online Appendix, we report additional derivations for Section 2.11 of the paper. In particular, we derive the decompositions of the aggregate price index (P_{jt}) and aggregate import price indexes ($\mathbb{E}_{jt}^{T}\left[\Delta\ln\mathbb{P}_{jgt}^{G}\right]$) in equations (29) and (30) of the paper respectively.

Aggregate Price Index From equation (4) in the paper, the log aggregate price index (P_{jt}) can be written in terms of the share of expenditure on tradable sectors (μ_{jt}^{T}) and the tradables sector price index ($\mathbb{P}_{jt}^{T}$):

$$\ln P_{jt} = \frac{1}{\sigma^{G}-1}\ln\mu_{jt}^{T} + \ln\mathbb{P}_{jt}^{T}, \tag{A.2.67}$$

Now note that the share of individual tradable sector in expenditure on all tradable sectors is given by:

$$\mathbb{S}_{jgt}^{T} = \frac{\left(P_{jgt}^{G}/\varphi_{jgt}^{G}\right)^{1-\sigma^{G}}}{\sum_{k\in\Omega^{T}}\left(P_{jkt}^{G}/\varphi_{jkt}^{G}\right)^{1-\sigma^{G}}} = \frac{\left(P_{jgt}^{G}/\varphi_{jgt}^{G}\right)^{1-\sigma^{G}}}{\left(\mathbb{P}_{jt}^{T}\right)^{1-\sigma^{G}}}. \tag{A.2.68}$$

Rearranging equation (A.2.68), and taking geometric means across tradable sectors, we obtain the following expression for the tradables sector price index ($\mathbb{P}_{jt}^{T}$):

$$\mathbb{P}_{jt}^{T} = \frac{\mathbb{M}_{jt}^{T}\left[P_{jgt}^{G}\right]}{\mathbb{M}_{jt}^{T}\left[\varphi_{jgt}^{G}\right]}\left(\mathbb{M}_{jt}^{T}\left[\mathbb{S}_{jgt}^{T}\right]\right)^{\frac{1}{\sigma^{G}-1}}, \tag{A.2.69}$$

where $\mathbb{M}_{jt}^{T}[\cdot]$ is the geometric mean across tradable sectors (superscript T) for a given importer (subscript j) and time period (subscript t) such that:

$$\mathbb{M}_{jt}^{T}\left[P_{jgt}^{G}\right] = \left(\prod_{g\in\Omega^{T}} P_{jgt}^{G}\right)^{\frac{1}{N^{T}}}. \tag{A.2.70}$$

Substituting this expression for the tradable sector price index from equation (A.2.69) into the aggregate price index in equation (A.2.67), we obtain:

$$\ln P_{jt} = \frac{1}{\sigma^{G}-1}\ln\mu_{jt}^{T} + \mathbb{E}_{jt}^{T}\left[\ln P_{jgt}^{G}\right] - \mathbb{E}_{jt}^{T}\left[\ln\varphi_{jgt}^{G}\right] + \frac{1}{\sigma^{G}-1}\mathbb{E}_{jt}^{T}\left[\ln\mathbb{S}_{jgt}^{T}\right], \tag{A.2.71}$$

where $\mathbb{E}_{jt}^{T}[\cdot]$ is the mean across tradable sectors (superscript T) for a given importer (subscript j) and time period (subscript t) such that:

$$\mathbb{E}_{jt}^{T}\left[P_{jgt}^{G}\right] = \frac{1}{N^{T}}\sum_{g\in\Omega^{T}}\ln P_{jgt}^{G}. \tag{A.2.72}$$

Now using the expression for the sectoral price index from equation (7) in the paper, the aggregate price index in equation (A.2.71) can be written in the following form:

$$\ln P_{jt} = \frac{1}{\sigma^G - 1} \ln \mu_{jt}^T + \mathbb{E}_{jt}^T \left[\ln \mathbb{P}_{jgt}^G \right] + \mathbb{E}_{jt}^T \left[\frac{1}{\sigma_g^F - 1} \ln \mu_{jgt}^G \right] - \mathbb{E}_{jt}^T \left[\ln \varphi_{jgt}^G \right] + \frac{1}{\sigma^G - 1} \mathbb{E}_{jt}^T \left[\ln \mathbb{S}_{jgt}^T \right], \quad \text{(A.2.73)}$$

where $\mathbb{P}_{jgt}^G$ is the sectoral import price index and μ_{jgt}^G is the share of expenditure on foreign varieties within each sector. Taking differences over time, noting that the set of tradable sectors is constant over time, we obtain:

$$\underbrace{\Delta \ln P_{jt}}_{\substack{\text{Aggregate} \\ \text{Price Index}}} = \underbrace{\frac{1}{\sigma^G - 1} \Delta \ln \mu_{jt}^T}_{\substack{\text{Non-Tradable} \\ \text{Competitiveness}}} + \underbrace{\mathbb{E}_{jt}^T \left[\frac{1}{\sigma_g^F - 1} \Delta \ln \mu_{jgt}^G \right]}_{\substack{\text{Domestic} \\ \text{Competitiveness}}} + \underbrace{\mathbb{E}_{jt}^T \left[\Delta \ln \varphi_{jgt}^G \right]}_{\substack{\text{Average} \\ \text{Appeal}}} + \underbrace{\mathbb{E}_{jt}^T \left[\frac{1}{\sigma^G - 1} \Delta \ln \mathbb{S}_{jgt}^T \right]}_{\substack{\text{Dispersion appeal-} \\ \text{adjusted prices across sectors}}} + \underbrace{\mathbb{E}_{jt}^T \left[\Delta \ln \mathbb{P}_{jgt}^G \right]}_{\substack{\text{Aggregate Import} \\ \text{Price Indexes}}}, \quad \text{(A.2.74)}$$

which corresponds to equation (29) in the paper.

Aggregate Import Price Indexes We next derive the decomposition of the final term in equation (A.2.74) for the average change in sectoral import price indexes ($\mathbb{E}_{jt}^T \left[\Delta \ln \mathbb{P}_{jgt}^G \right]$) that is reported in equation (30) in the paper. From equation (10) in the paper, the change in the import price index over time can be written as:

$$\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G} = \left[\frac{\sum_{i \in \Omega_{jgt}^E} \left(\mathbb{P}_{jigt}^E \right)^{1-\sigma_g^F}}{\sum_{i \in \Omega_{jgt-1}^E} \left(\mathbb{P}_{jigt-1}^E \right)^{1-\sigma_g^F}} \right]^{\frac{1}{1-\sigma_g^F}}, \quad \text{(A.2.75)}$$

where the entry and exit of exporters over time implies that $\Omega_{jgt}^E \neq \Omega_{jgt-1}^E$. We define the share of expenditure on common foreign exporters $i \in \Omega_{jgt,t-1}^E$ that supply importer j within sector g in both periods $t-1$ and t as:

$$\lambda_{jgt}^E \equiv \frac{\sum_{i \in \Omega_{jgt,t-1}^E} \left(\mathbb{P}_{jigt}^E \right)^{1-\sigma_g^F}}{\sum_{i \in \Omega_{jgt}^E} \left(\mathbb{P}_{jigt}^E \right)^{1-\sigma_g^F}}, \qquad \lambda_{jgt-1}^E \equiv \frac{\sum_{i \in \Omega_{jgt,t-1}^E} \left(\mathbb{P}_{jigt-1}^E \right)^{1-\sigma_g^F}}{\sum_{i \in \Omega_{jgt-1}^E} \left(\mathbb{P}_{jigt-1}^E \right)^{1-\sigma_g^F}}, \quad \text{(A.2.76)}$$

where $\Omega_{jgt,t-1}^E$ is the set of common foreign exporters for importer j within sector g and $N_{jgt,t-1}^E = \left| \Omega_{jgt,t-1}^E \right|$ is the number of elements within this set. Using this definition from equation (A.2.76), the change in the import price index in equation (A.2.75) can be re-written in the following form:

$$\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G} = \left(\frac{\lambda_{jgt}^E}{\lambda_{jgt-1}^E} \right)^{\frac{1}{\sigma_g^F - 1}} \left[\frac{\sum_{i \in \Omega_{jigt,t-1}^E} \left(\mathbb{P}_{jigt}^E \right)^{1-\sigma_g^F}}{\sum_{i \in \Omega_{jigt,t-1}^E} \left(\mathbb{P}_{jigt-1}^E \right)^{1-\sigma_g^F}} \right]^{\frac{1}{1-\sigma_g^F}} = \left(\frac{\lambda_{jgt}^E}{\lambda_{jgt-1}^E} \right)^{\frac{1}{\sigma_g^F - 1}} \frac{\mathbb{P}_{jgt}^{G*}}{\mathbb{P}_{jgt-1}^{G*}}, \quad \text{(A.2.77)}$$

where the first term ($\left(\lambda^E_{jgt}/\lambda^E_{jgt-1}\right)^{\frac{1}{\sigma^F_g-1}}$) corrects for the entry and exit of exporters; the second term ($\mathbb{P}^{G*}_{jgt}/\mathbb{P}^{G*}_{jgt-1}$) is the change in the import price index for common exporters; and we again use the superscript asterisk to denote a variable for common varieties. We can also define the share of expenditure on an individual common exporter in overall expenditure on common exporters as:

$$\mathbb{S}^{E*}_{jigt} = \frac{\left(\mathbb{P}^E_{jigt}\right)^{1-\sigma^F_g}}{\sum_{h\in\Omega^E_{jgt,t-1}}\left(\mathbb{P}^E_{jhgt}\right)^{1-\sigma^F_g}} = \frac{\left(\mathbb{P}^E_{jigt}\right)^{1-\sigma^F_g}}{\left(\mathbb{P}^{G*}_{jgt}\right)^{1-\sigma^F_g}}. \tag{A.2.78}$$

Rearranging equation (A.2.78) so that the import price index for common exporters ($\mathbb{P}^{G*}_{jgt}$) is on the left-hand side, dividing by the same expression for period $t-1$, and taking geometric means across the set of common exporters, we have:

$$\frac{\mathbb{P}^{G*}_{jgt}}{\mathbb{P}^{G*}_{jgt-1}} = \mathbb{M}^{E*}_{jgt}\left[\frac{\mathbb{P}^E_{jigt}}{\mathbb{P}^E_{jigt-1}}\right]\left(\mathbb{M}^{E*}_{jgt}\left[\frac{\mathbb{S}^{E*}_{jigt}}{\mathbb{S}^{E*}_{jigt-1}}\right]\right)^{\frac{1}{\sigma^F_g-1}}, \tag{A.2.79}$$

where $\mathbb{M}^{E*}_{jgt}[\cdot]$ is the geometric mean across the common set of foreign exporters (superscript E^*) for a given importer (subscript j), sector (subscript g) and time period (subscript t) such that:

$$\mathbb{M}^{E*}_{jgt}\left[\mathbb{P}^E_{jigt}\right] = \left(\prod_{i\in\Omega^E_{jgt,t-1}}\mathbb{P}^E_{jigt}\right)^{\frac{1}{N^E_{jgt,t-1}}}. \tag{A.2.80}$$

Combining equations (A.2.77) and (A.2.79), the overall change in the import price index can be written as:

$$\frac{\mathbb{P}^G_{jgt}}{\mathbb{P}^G_{jgt-1}} = \left(\frac{\lambda^E_{jgt}}{\lambda^E_{jgt-1}}\right)^{\frac{1}{\sigma^F_g-1}}\mathbb{M}^{E*}_{jgt}\left[\frac{\mathbb{P}^E_{jigt}}{\mathbb{P}^E_{jigt-1}}\right]\left(\mathbb{M}^{E*}_{jgt}\left[\frac{\mathbb{S}^{E*}_{jigt}}{\mathbb{S}^{E*}_{jigt-1}}\right]\right)^{\frac{1}{\sigma^F_g-1}}. \tag{A.2.81}$$

Taking logarithms in equation (A.2.81), we obtain:

$$\Delta\ln\mathbb{P}^G_{jgt} = \frac{1}{\sigma^F_g-1}\Delta\ln\lambda^E_{jgt} + \mathbb{E}^{E*}_{jgt}\left[\Delta\ln\mathbb{P}^E_{jigt}\right] + \frac{1}{\sigma^F_g-1}\mathbb{E}^{E*}_{jgt}\left[\Delta\ln\mathbb{S}^E_{jigt}\right], \tag{A.2.82}$$

where $\mathbb{E}^{E*}_{jgt}[\cdot]$ is the geometric mean across common exporters (superscript E^*) for an importer j within sector g at time t such that:

$$\mathbb{E}^{E*}_{jgt}\left[\Delta\ln\mathbb{P}^E_{jigt}\right] = \frac{1}{N^E_{jgt,t-1}}\sum_{i\in\Omega^E_{jgt,t-1}}\Delta\ln\mathbb{P}^E_{jigt}. \tag{A.2.83}$$

We now derive an expression for the average log change in exporter price indexes ($\mathbb{E}^{E*}_{jgt}\left[\Delta\ln\mathbb{P}^E_{jigt}\right]$) on the right-hand side of equation (A.2.82). Taking the mean across common exporters in equation (A.2.26), we obtain:

$$\mathbb{E}_{jgt}^{E*}\left[\Delta\ln\mathbb{P}_{jigt}^{E}\right]=\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln P_{ut}^{U}\right]-\left\{\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\varphi_{ft}^{F}\right]+\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln\varphi_{ut}^{U}\right]\right\}$$
$$+\left\{\frac{1}{\sigma_g^U-1}\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln S_{ut}^{U*}\right]+\frac{1}{\sigma_g^F-1}\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\mathbb{S}_{ft}^{EF}\right]\right\}$$
$$+\left\{\frac{1}{\sigma_g^U-1}\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\lambda_{ft}^{U}\right]+\frac{1}{\sigma_g^F-1}\mathbb{E}_{jgt}^{E*}\left[\Delta\ln\lambda_{jigt}^{F}\right]\right\}, \quad \text{(A.2.84)}$$

where $\mathbb{E}_{jgt}^{EFU*}[\cdot]$ is the mean, first across common products within firms, next across common firms within each exporter-sector, and then across common exporting countries (superscript EFU^*), for a given importer (subscript j), sector (subscript g) and time period (subscript t), such that:

$$\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln P_{ut}^{U}\right]=\frac{1}{N_{jgt,t-1}^{E}}\sum_{i\in\Omega_{jgt,t-1}^{E}}\frac{1}{N_{jigt,t-1}^{F}}\sum_{f\in\Omega_{jigt,t-1}^{F}}\frac{1}{N_{ft,t-1}^{U}}\sum_{u\in\Omega_{ft,t-1}^{U}}\Delta\ln P_{ut}^{U}; \quad \text{(A.2.85)}$$

recall that $\mathbb{E}_{jgt}^{EF*}[\cdot]$ is the mean, first across common firms (superscript F^*), and next across common exporters (superscript E) for a given importer (subscript j), sector (subscript g) and time period (subscript t), as defined in equation (A.2.27). Substituting equation (A.2.84) into equation (A.2.26), we obtain the following expression for the change in the sectoral import price index ($\Delta\ln\mathbb{P}_{jgt}^{G}$) in equation (A.2.82) above:

$$\Delta\ln\mathbb{P}_{jgt}^{G}=\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln P_{ut}^{U}\right]-\left\{\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\varphi_{ft}^{F}\right]+\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln\varphi_{ut}^{U}\right]\right\} \quad \text{(A.2.86)}$$
$$+\left\{\frac{1}{\sigma_g^U-1}\mathbb{E}_{jgt}^{EFU*}\left[\Delta\ln S_{ut}^{U*}\right]+\frac{1}{\sigma_g^F-1}\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\mathbb{S}_{ft}^{EF}\right]+\frac{1}{\sigma_g^F-1}\mathbb{E}_{jgt}^{E*}\left[\Delta\ln\mathbb{S}_{jigt}^{E}\right]\right\}$$
$$+\left\{\frac{1}{\sigma_g^U-1}\mathbb{E}_{jgt}^{EF*}\left[\Delta\ln\lambda_{ft}^{U}\right]+\frac{1}{\sigma_g^F-1}\mathbb{E}_{jgt}^{E*}\left[\Delta\ln\lambda_{jigt}^{F}\right]+\frac{1}{\sigma_g^F-1}\Delta\ln\lambda_{jgt}^{E}\right\}.$$

Taking averages across tradable sectors in equation (A.2.86), we obtain equation (30) in the paper:

$$\underbrace{\mathbb{E}_{jt}^{T}\left[\Delta\ln\mathbb{P}_{jgt}^{G}\right]}_{\substack{\text{Import}\\\text{Price Indexes}}}=\underbrace{\mathbb{E}_{jt}^{TEFU*}\left[\Delta\ln P_{ut}^{U}\right]}_{\text{(i) Average log prices}}-\underbrace{\mathbb{E}_{jt}^{TEF*}\left[\Delta\ln\varphi_{ft}^{F}\right]}_{\substack{\text{(ii) Average log}\\\text{firm appeal}}}-\underbrace{\mathbb{E}_{jt}^{TEFU*}\left[\ln\varphi_{ut}^{U}\right]}_{\substack{\text{(iii) Average log}\\\text{product appeal}}} \quad \text{(A.2.87)}$$
$$+\underbrace{\mathbb{E}_{jt}^{TE*}\left[\frac{1}{\sigma_g^F-1}\Delta\ln\mathbb{S}_{jigt}^{E}\right]}_{\substack{\text{(iv) Dispersion country-sector}\\\text{appeal-adjusted prices}}}+\underbrace{\mathbb{E}_{jt}^{TEF*}\left[\frac{1}{\sigma_g^F-1}\Delta\ln\mathbb{S}_{ft}^{EF}\right]}_{\substack{\text{(v) Dispersion firm}\\\text{appeal-adjusted prices}}}+\underbrace{\mathbb{E}_{jt}^{TEFU*}\left[\frac{1}{\sigma_g^U-1}\Delta\ln S_{ut}^{U}\right]}_{\substack{\text{(vi) Dispersion product}\\\text{appeal-adjusted prices}}}$$
$$+\underbrace{\mathbb{E}_{jt}^{T}\left[\frac{1}{\sigma_g^F-1}\Delta\ln\lambda_{jgt}^{E}\right]}_{\substack{\text{(vii) Country - Sector}\\\text{Variety}}}+\underbrace{\mathbb{E}_{jt}^{TE*}\left[\frac{1}{\sigma_g^F-1}\Delta\ln\lambda_{jigt}^{F}\right]}_{\substack{\text{(viii) Firm}\\\text{Variety}}}+\underbrace{\mathbb{E}_{jt}^{TEF*}\left[\frac{1}{\sigma_g^U-1}\Delta\ln\lambda_{ft}^{U}\right]}_{\substack{\text{(ix) Product}\\\text{Variety}}},$$

where the means $\mathbb{E}_{jt}^{T}[\cdot]$, $\mathbb{E}_{jt}^{TEFU*}[\cdot]$, $\mathbb{E}_{jt}^{TEF*}[\cdot]$ and $\mathbb{E}_{jt}^{TE*}[\cdot]$ are defined in equations (A.2.65), (A.2.62), (A.2.64), (A.2.66) of this Online Appendix.

Interpretation Together equations (A.2.74) and (A.2.87) (which correspond to equations (29) and (30) in the paper) provide an exact log-linear decomposition of the change in the aggregate cost of living.

Each term in these equations has an intuitive interpretation. In the paper, we discuss the interpretation of each term in equation (A.2.74). In this section of the Online Appendix, we now provide a more detailed discussion of the interpretation of each term in equation (A.2.87).

The first term (i), "Average Prices," captures changes in the average price of common imported products that are supplied in both periods t and $t-1$. Other things equal, a fall in these average prices ($\mathbb{E}_{jt}^{TEFU*}\left[\Delta \ln P_{ut}^{U}\right] < 0$) reduces average import price indexes and hence the cost of living. The second and third terms ((ii) and (iii)) incorporate changes in average firm appeal (φ_{ft}^{F}) across common firms and average product appeal (φ_{ut}^{U}) across common products. Our choice of units for product appeal in equation (A.2.4) implies that the second term for the average log change in appeal across common products within each firm is zero: $\mathbb{E}_{jt}^{TEFU*}\left[\ln \varphi_{ut}^{U}\right] = 0$. Our choice of units for firm appeal in equation (A.2.7) implies that the unweighted average log change in appeal across common foreign firms within each sector is zero: $\mathbb{E}_{jt}^{TF*}\left[\Delta \ln \varphi_{ft}^{F}\right] = 0$. However, the average of firm appeal in the third term ($\mathbb{E}_{jt}^{TEF*}\left[\Delta \ln \varphi_{ft}^{F}\right]$) involves first averaging across firms within a given foreign exporter, and then averaging across foreign exporters, which corresponds to a weighted average across firms. Although in principle the weighted and unweighted averages across firms could differ from one another, we find that in practice they take similar values, which implies that the third term is close to zero.

The fourth to sixth terms ((iv)-(vi)) summarize the impact of the dispersion in appeal-adjusted prices across common exporter-sector pairs, common firms and common products, respectively. "Country-sector appeal-adjusted prices" reflects the fact that consumers are made better off if exporters improve performance in their most successful sectors. For example, consumers are better off if Japanese car makers and Saudi oil drillers become more relatively more productive (raising dispersion in appeal-adjusted prices) than if Saudi car makers and Japanese oil drillers are the relative winners (lowering dispersion in appeal-adjusted prices). Similarly at the firm-level, consumers benefit more from relative cost reductions or quality improvements for firms with low appeal-adjusted prices (high expenditure shares), which increases the dispersion of appeal-adjusted prices. Since varieties are substitutes ($\sigma_{g}^{U} > 1$ and $\sigma_{g}^{F} > 1$), increases in the dispersion of these appeal-adjusted prices reduce the cost of living, as consumers can substitute away from high-appeal-adjusted-price varieties to low-appeal-adjusted-price varieties.

The seventh to eighth terms ((vii)-(viii)) summarize the effect of the entry/exit of exporter-sector pairs, firms and products respectively. "Firm Variety" accounts for the entry and exit of foreign firms when at least one foreign firm from an exporter and sector exports in both time periods. "Country-Sector Variety" is an extreme form of foreign firm entry and exit that arises when the number of firms from a foreign exporter rises from zero to a positive value or falls to zero. Finally, the last term (ix), "Product Variety," accounts for changes in the set of products within continuing foreign firms. For all three terms, the lower the shares of expenditure on common varieties at time t relative to those at time $t-1$ (the smaller values of $\Delta \ln \lambda_{jgt}^{E}$, $\Delta \ln \lambda_{jigt}^{F}$ and $\Delta \ln \lambda_{ft}^{U}$), the more attractive are entering varieties relative to exiting varieties, and the greater the reduction in the cost of living between the two time periods.

A.3 Structural Estimation

In this section of the Online Appendix, we provide further details on our structural estimation of the elasticities of substitution from Section 3 of the paper. We outline a reverse-weighting (RW) estimator of these elasticities of substitution (σ_g^U, σ_g^F, σ^G), which uses the equality between alternative expressions for the CES unit expenditure function, as developed in Redding and Weinstein (2023).

In Subsection A.3.1, we introduce the RW estimator for the simplest case of a single CES nest with entry and exit. The remaining subsections apply our RW estimator to our nested CES demand structure. In Subsection A.3.2, we begin by estimating the elasticity of substitution across products (σ_g^U) for each sector g. In Subsection A.3.3, we next estimate the elasticity of substitution across firms (σ_g^F) for each sector g. In Subsection A.3.4, we next estimate the elasticity of substitution across sectors (σ^G).

In robustness tests, we also report results using alternative estimates for these elasticities of substitution (σ_g^U, σ_g^F, σ^G), as discussed in Sections 5.1 and 5.3 of the paper. In Subsection A.3.5, we report the results of a Monte Carlo simulation, in which we examine the performance of the RW estimator and compare it to ordinary least squares (OLS). Finally, in Subsection A.3.6, we develop an equivalent representation of the reverse-weighting estimator, which imposes more of the nesting structure of the model.

A.3.1 RW Estimator with Entry and Exit

Consider a single CES nest where the set of goods available in time t (Ω_t) differs from the set of goods available in time $t-1$ (Ω_{t-1}). The change in the unit expenditure function between periods $t-1$ and t is:

$$\frac{P_t}{P_{t-1}} = \frac{\left[\sum_{u \in \Omega_t} \left(\frac{P_{ut}}{\varphi_{ut}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\sum_{u \in \Omega_{t-1}} \left(\frac{P_{ut-1}}{\varphi_{ut-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}. \tag{A.3.1}$$

Followng Feenstra (1994), we define the common set as goods as those that are supplied in both time periods $t-1$ and t: $\Omega_{t,t-1} = \Omega_t \cap \Omega_{t-1}$. Using this definition, we can re-write the change in the unit expenditure between time periods $t-1$ and t in equation (A.3.1) as follows:

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\left[\sum_{u \in \Omega_{t,t-1}} \left(\frac{P_{ut}}{\varphi_{ut}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\sum_{u \in \Omega_{t,t-1}} \left(\frac{P_{ut-1}}{\varphi_{ut-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}, \tag{A.3.2}$$

where λ_t and λ_{t-1} are the shares of common goods in total expenditure in periods t and $t-1$ respectively:

$$\lambda_t \equiv \frac{\sum_{u \in \Omega_{t,t-1}} \left(\frac{P_{ut}}{\varphi_{ut}}\right)^{1-\sigma}}{\sum_{u \in \Omega_t} \left(\frac{P_{ut}}{\varphi_{ut}}\right)^{1-\sigma}}, \tag{A.3.3}$$

$$\lambda_{t-1} \equiv \frac{\sum_{u \in \Omega_{t,t-1}} \left(\frac{P_{ut-1}}{\varphi_{ut-1}} \right)^{1-\sigma}}{\sum_{u \in \Omega_{t-1}} \left(\frac{P_{ut-1}}{\varphi_{ut-1}} \right)^{1-\sigma}}. \tag{A.3.4}$$

The CES demand system implies the following expressions for the share of an individual good in all expenditure on common goods at times t and $t-1$:

$$S^*_{kt} = \frac{\left(\frac{P_{ut}}{\varphi_{ut}} \right)^{1-\sigma}}{\sum_{k \in \Omega_{t,t-1}} \left(\frac{P_{kt}}{\varphi_{kt}} \right)^{1-\sigma}} = \frac{\left(\frac{P_{ut}}{\varphi_{ut}} \right)^{1-\sigma}}{\left(P^*_t \right)^{1-\sigma}}, \tag{A.3.5}$$

$$S^*_{kt-1} = \frac{\left(\frac{P_{ut-1}}{\varphi_{ut-1}} \right)^{1-\sigma}}{\sum_{k \in \Omega_{t,t-1}} \left(\frac{P_{kt-1}}{\varphi_{kt-1}} \right)^{1-\sigma}} = \frac{\left(\frac{P_{ut-1}}{\varphi_{ut-1}} \right)^{1-\sigma}}{\left(P^*_{t-1} \right)^{1-\sigma}}, \tag{A.3.6}$$

where we use an asterisk to denote the value of a variable for common goods; $P^*_t = \left[\sum_{u \in \Omega_{t,t-1}} \left(\frac{P_{ut}}{\varphi_{ut}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$ is the unit expenditure function for common goods; and P^*_{t-1} is defined analogously.

Dividing S^*_{kt} in equation (A.3.5) by its geometric mean across common goods, and dividing S^*_{kt-1} in equation (A.3.6) by its geometric mean across common goods, we also have the following relationships:

$$P^*_t = \frac{\mathbb{M}^*_t \left[P_{ut} \right]}{\mathbb{M}^*_t \left[\varphi_{ut} \right]} \left(\mathbb{M}^*_t \left[S^*_{ut} \right] \right)^{\frac{1}{\sigma-1}}, \tag{A.3.7}$$

$$P^*_{t-1} = \frac{\mathbb{M}^*_{t-1} \left[P_{ut-1} \right]}{\mathbb{M}^*_{t-1} \left[\varphi_{ut-1} \right]} \left(\mathbb{M}^*_{t-1} \left[S^*_{ut-1} \right] \right)^{\frac{1}{\sigma-1}}, \tag{A.3.8}$$

where $\mathbb{M}^*_t \left[S^*_{ut} \right] \equiv \left(\prod_{u \in \Omega_{t,t-1}} S^*_{ut} \right)^{1/N_{t,t-1}}$ is the geometric mean operator across common goods and $N_{t,t-1} = |\Omega_{t,t-1}|$ is the number of goods in the common set.

Using equations (A.3.5)-(A.3.8), we can re-write the change in the unit expenditure function between $t-1$ and t in equation (A.3.2) in the following three equivalent ways:

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{\sigma-1}} \left[\sum_{u \in \Omega_{t,t-1}} S^*_{ut-1} \left(\frac{P_{ut}}{P_{ut-1}} \right)^{1-\sigma} \left(\frac{\varphi_{ut}}{\varphi_{ut-1}} \right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}, \tag{A.3.9}$$

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{\sigma-1}} \left[\sum_{u \in \Omega_{t,t-1}} S^*_{ut} \left(\frac{P_{ut}}{P_{ut-1}} \right)^{-(1-\sigma)} \left(\frac{\varphi_{ut}}{\varphi_{ut-1}} \right)^{-(\sigma-1)} \right]^{-\frac{1}{1-\sigma}}, \tag{A.3.10}$$

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}} \right)^{\frac{1}{\sigma-1}} \frac{\mathbb{M}^*_t \left[P_{ut} \right] / \mathbb{M}^*_t \left[\varphi_{ut} \right]}{\mathbb{M}^*_{t-1} \left[P_{ut-1} \right] / \mathbb{M}^*_t \left[\varphi_{ut-1} \right]} \left(\frac{\mathbb{M}^*_t \left[S^*_{ut} \right]}{\mathbb{M}^*_{t-1} \left[S^*_{ut-1} \right]} \right)^{\frac{1}{\sigma-1}}. \tag{A.3.11}$$

Together these three expressions for the change in the CES unit expenditure function imply the following two equalities:

$$\ln \left\{ \frac{\mathbb{M}^*_t \left[P_{ut} \right]}{\mathbb{M}^*_{t-1} \left[P_{ut-1} \right]} \left(\frac{\mathbb{M}^*_t \left[S^*_{ut} \right]}{\mathbb{M}^*_{t-1} \left[S^*_{ut-1} \right]} \right)^{\frac{1}{\sigma-1}} \right\} = \ln \left\{ \left[\sum_{u \in \Omega_{t,t-1}} S^*_{ut-1} \left(\frac{P_{ut}}{P_{ut-1}} \right)^{1-\sigma} \left(\frac{\varphi_{ut} / \mathbb{M}^*_t \left[\varphi_{ut} \right]}{\varphi_{ut-1} / \mathbb{M}^*_{t-1} \left[\varphi_{ut-1} \right]} \right)^{\sigma-1} \right]^{\frac{1}{1-\sigma}} \right\}, \tag{A.3.12}$$

$$\ln\left\{\frac{\mathbb{M}_t^*\left[P_{ut}\right]}{\mathbb{M}_{t-1}^*\left[P_{ut-1}\right]}\left(\frac{\mathbb{M}_t^*\left[S_{ut}^*\right]}{\mathbb{M}_{t-1}^*\left[S_{ut-1}^*\right]}\right)^{\frac{1}{\sigma-1}}\right\}=\ln\left\{\left[\sum_{u\in\Omega_{t,t-1}}S_{ut}^*\left(\frac{P_{ut}}{P_{ut-1}}\right)^{-(1-\sigma)}\left(\frac{\varphi_{ut}/\mathbb{M}_t^*\left[\varphi_{ut}\right]}{\varphi_{ut-1}/\mathbb{M}_{t-1}^*\left[\varphi_{ut-1}\right]}\right)^{-(\sigma-1)}\right]^{-\frac{1}{1-\sigma}}\right\}, \tag{A.3.13}$$

where the terms in the common goods expenditure shares ($(\lambda_t/\lambda_{t-1})^{\frac{1}{\sigma-1}}$) have cancelled from these two equalities.

Taking the limit as appeal shocks become small for all goods ($\frac{\varphi_{ut}/\mathbb{M}_t^*[\varphi_{ut}]}{\varphi_{ut-1}/\mathbb{M}_{t-1}^*[\varphi_{ut-1}]}\rightarrow 1$ for all u), we obtain the following moment conditions:

$$\ln\left\{\frac{\mathbb{M}_t^*\left[P_{ut}\right]}{\mathbb{M}_{t-1}^*\left[P_{ut-1}\right]}\left(\frac{\mathbb{M}_t^*\left[S_{ut}\right]}{\mathbb{M}_{t-1}^*\left[S_{ut-1}\right]}\right)^{\frac{1}{\sigma-1}}\right\}-\ln\left\{\left[\sum_{u\in\Omega_{t,t-1}}S_{ut-1}^*\left(\frac{P_{ut}}{P_{ut-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}\right\}=0, \tag{A.3.14}$$

$$\ln\left\{\frac{\mathbb{M}_t^*\left[P_{ut}\right]}{\mathbb{M}_{t-1}^*\left[P_{ut-1}\right]}\left(\frac{\mathbb{M}_t^*\left[S_{ut}\right]}{\mathbb{M}_{t-1}^*\left[S_{ut-1}\right]}\right)^{\frac{1}{\sigma-1}}\right\}-\ln\left\{\left[\sum_{u\in\Omega_{t,t-1}}S_{ut}^*\left(\frac{P_{ut}}{P_{ut-1}}\right)^{-(1-\sigma)}\right]^{-\frac{1}{1-\sigma}}\right\}=0, \tag{A.3.15}$$

which depend only on observed data and parameters.

The RW estimator chooses the value of σ to minimize the sum of squared deviations of the moment conditions in equations (A.3.14) and (A.3.15) from zero. As appeal shocks become small for all goods ($\frac{\varphi_{ut}/\mathbb{M}_t^*[\varphi_{ut}]}{\varphi_{ut-1}/\mathbb{M}_{t-1}^*[\varphi_{ut-1}]}\rightarrow 1$ for all u), the RW estimator consistently estimates the elasticity of substitution, as shown in Redding and Weinstein (2023). In Section A.3.5 of this Online Appendix, we provide Monte Carlo evidence on the finite sample performance of the RW estimator and compare it to the OLS estimator.

A.3.2 Elasticity of Substitution Across Products (σ_g^U)

We now apply the RW estimator to our nested CES demand structure in three steps for each tier of utility. In our first step, we estimate the elasticity of substitution across products within firms (σ_g^U). Equating the three equivalent expressions for the change in the CES unit expenditure function for each firm from Subsection A.3.1 above, and taking the limit as appeal shocks becomes small ($\frac{\varphi_{ut}^U/\mathbb{M}_{ft}^{U*}\left[\varphi_{ut}^U\right]}{\varphi_{ut-1}^U/\mathbb{M}_{ft-1}^{U*}\left[\varphi_{ut-1}^U\right]}\rightarrow 1$ for all u), we obtain the following two moment conditions:

$$m_g^U\left(\sigma_g^U\right)=\left(\begin{array}{c}\ln\left\{\left[\sum_{u\in\Omega_{ft,t-1}^U}S_{ut-1}^{U*}\left(\frac{P_{ut}^U}{P_{ut-1}^U}\right)^{1-\sigma_g^U}\right]^{\frac{1}{1-\sigma_g^U}}\right\}-\ln\left\{\mathbb{M}_{ft}^{U*}\left[\frac{P_{ut}^U}{P_{ut-1}^U}\right]\left(\mathbb{M}_{ft}^{U*}\left[\frac{S_{ut}^{U*}}{S_{ut-1}^{U*}}\right]\right)^{\frac{1}{\sigma_g^U-1}}\right\}\\ \ln\left\{\left[\sum_{u\in\Omega_{ft,t-1}^U}S_{ut}^{U*}\left(\frac{P_{ut}^U}{P_{ut-1}^U}\right)^{-(1-\sigma_g^U)}\right]^{-\frac{1}{1-\sigma_g^U}}\right\}-\ln\left\{\mathbb{M}_{ft}^{U*}\left[\frac{P_{ut}^U}{P_{ut-1}^U}\right]\left(\mathbb{M}_{ft}^{U*}\left[\frac{S_{ut}^{U*}}{S_{ut-1}^{U*}}\right]\right)^{\frac{1}{\sigma_g^U-1}}\right\}\end{array}\right)=\left(\begin{array}{c}0\\0\end{array}\right). \tag{A.3.16}$$

We stack these moment conditions for each foreign firm with two or more products and for all time periods within a given sector. We estimate the elasticity of substitution across products within firms (σ_g^U) using the generalized method of moments (GMM):

$$\hat{\sigma}_g^U=\arg\min\left\{m_g^U\left(\sigma_g^U\right)'\times\mathbb{I}\times m_g^U\left(\sigma_g^U\right)\right\}, \tag{A.3.17}$$

where $\mathbb{I}$ is the identity matrix.

A.3.3 Elasticity of Substitution Across Firms (σ_g^F)

Using our estimate of the elasticity of substitution across products (σ_g^U) from the first step, we can recover the appeal shifter for each product (φ_{ut}^U) and compute the firm price index (P_{ft}^F) for all foreign firms:

$$\varphi_{ut}^U = \frac{P_{ut}^U}{\mathbb{M}_{ft}^U\left[P_{ut}^U\right]}\left(\frac{S_{ut}^U}{\mathbb{M}_{ft}^U\left[S_{ut}^U\right]}\right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.3.18}$$

$$P_{ft}^F = \mathbb{M}_{ft}^U\left[P_{ut}^U\right]\left(\mathbb{M}_{ft}^U\left[S_{ut}^U\right]\right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.3.19}$$

where we have used our choice of units for product appeal such that $\mathbb{M}_{ft}^{U*}\left[\varphi_{ut}^U\right] = 1$.

In our second step, we estimate the elasticity of substitution across firms within sectors (σ_g^F). Equating the three equivalent expressions for the change in the CES unit expenditure function for each sector from Subsection A.3.1 above, and taking the limit as appeal shocks becomes small ($\frac{\varphi_{ft}^F/\mathbb{M}_{jgt}^{F*}\left[\varphi_{ft}^F\right]}{\varphi_{ft-1}^F/\mathbb{M}_{jgt-1}^{F*}\left[\varphi_{ft-1}^F\right]} \to 1$ for all f), we obtain the following two moment conditions:

$$m_g^F\left(\sigma_g^F\right) = \begin{pmatrix} \ln\left\{\left[\sum_{i\in\Omega_{jgt,t-1}^E}\sum_{f\in\Omega_{jigt,t-1}^F} \mathbb{S}_{ft}^{F*}\left(\frac{P_{ft-1}^F}{P_{ft}^F}\right)^{1-\sigma_g^F}\right]^{\frac{1}{1-\sigma_g^F}}\right\} - \ln\left\{\mathbb{M}_{jgt}^{F*}\left[\frac{P_{ft}^F}{P_{ft-1}^F}\right]\left(\mathbb{M}_{jgt}^{F*}\left[\frac{\mathbb{S}_{ft}^{F*}}{\mathbb{S}_{ft-1}^{F*}}\right]\right)^{\frac{1}{\sigma_g^F-1}}\right\} \\ \ln\left\{\left[\sum_{i\in\Omega_{jgt,t-1}^E}\sum_{f\in\Omega_{jigt,t-1}^F} \mathbb{S}_{ft}^{F*}\left(\frac{P_{ft}^F}{P_{ft-1}^F}\right)^{-\left(1-\sigma_g^F\right)}\left(\frac{\varphi_{ft}^F}{\varphi_{ft-1}^F}\right)^{-\left(\sigma_g^F-1\right)}\right]^{-\frac{1}{1-\sigma_g^U}}\right\} - \ln\left\{\mathbb{M}_{jgt}^{F*}\left[\frac{P_{ft}^F}{P_{ft-1}^F}\right]\left(\mathbb{M}_{jgt}^{F*}\left[\frac{\mathbb{S}_{ft}^{F*}}{\mathbb{S}_{ft-1}^{F*}}\right]\right)^{\frac{1}{\sigma_g^F-1}}\right\} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{A.3.20}$$

We stack these moment conditions for all time periods for a given sector. We estimate the elasticity of substitution across firms (σ_g^F) using the generalized method of moments (GMM):

$$\hat{\sigma}_g^F = \arg\min\left\{m_g^F\left(\sigma_g^F\right)' \times \mathbb{I} \times m_g^F\left(\sigma_g^F\right)\right\}, \tag{A.3.21}$$

where $\mathbb{I}$ is the identity matrix.

A.3.4 Elasticity of Substitution Across Sectors (σ^G)

Using our estimate of the elasticity of substitution across firms (σ_g^F) from the second step, we can recover the appeal shifter for each foreign firm (φ_{ft}^F) and compute the sector import price index ($\mathbb{P}_{jgt}^G$). Combining this solution for the sector import price index ($\mathbb{P}_{jgt}^G$) with the share of expenditure within each sector on foreign varieties (μ_{jgt}^G), we can also compute the overall sector price index (P_{jgt}^G).

In our third step, we estimate the elasticity of substitution across sectors (σ^G). Equating the three equivalent expressions for the change in the CES unit expenditure function across tradable sectors from Subsection A.3.1 above, and taking the limit as appeal shocks becomes small ($\frac{\varphi_{jgt}^G/\mathbb{M}_{jt}^T\left[\varphi_{jgt}^G\right]}{\varphi_{jgt-1}^G/\mathbb{M}_{jt-1}^T\left[\varphi_{jgt-1}^G\right]} \to 1$ for all g), we obtain the following two moment conditions:

$$m^T\left(\sigma^G\right) = \begin{pmatrix} \ln\left\{\left[\sum_{g\in\Omega^T} S_{jgt-1}^T\left(\left(\frac{\mu_{jgt}^G}{\mu_{jgt-1}^G}\right)^{\frac{1}{\sigma_g^F-1}}\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G}\right)^{1-\sigma^G}\right]^{\frac{1}{1-\sigma^G}}\right\} - \ln\left\{\mathbb{M}_{jt}^T\left[\left(\frac{\mu_{jgt}^G}{\mu_{jgt-1}^G}\right)^{\frac{1}{\sigma_g^F-1}}\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G}\right]\left(\mathbb{M}_{jt}^T\left[\frac{S_{jgt}^T}{S_{jgt-1}^G}\right]\right)^{\frac{1}{\sigma^G-1}}\right\} \\ \ln\left\{\left[\sum_{g\in\Omega^T} S_{jgt}^T\left(\left(\frac{\mu_{jgt}^G}{\mu_{jgt-1}^G}\right)^{\frac{1}{\sigma_g^F-1}}\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G}\right)^{-\left(1-\sigma^G\right)}\right]^{-\frac{1}{1-\sigma^G}}\right\} - \ln\left\{\mathbb{M}_{jt}^T\left[\left(\frac{\mu_{jgt}^G}{\mu_{jgt-1}^G}\right)^{\frac{1}{\sigma_g^F-1}}\frac{\mathbb{P}_{jgt}^G}{\mathbb{P}_{jgt-1}^G}\right]\left(\mathbb{M}_{jt}^T\left[\frac{S_{jgt}^G}{S_{jgt-1}^G}\right]\right)^{\frac{1}{\sigma^G-1}}\right\} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{A.3.22}$$

We stack these moment conditions for all time periods and estimate the elasticity of substitution across sectors (σ^G) using the generalized method of moments (GMM):

$$\hat{\sigma}^G = \arg\min \left\{ m^T \left(\sigma^G\right)' \times \mathbb{I} \times m^T \left(\sigma^G\right) \right\}, \tag{A.3.23}$$

where $\mathbb{I}$ is the identity matrix.

A.3.5 Monte Carlo

We now provide Monte Carlo evidence on the finite-sample performance of the reverse-weighting estimator. Recall that the reverse-weighting estimator uses only the subset of common goods, because the variety correction term cancels from equivalent expressions for the change in the unit expenditure function. Therefore, we focus on this subset of common goods, and are not required to make assumptions about entering and exiting goods. We assume 10,000 common varieties. We assume an elasticity of substitution of $\sigma = 4$. We begin by drawing initial values for appeal (φ_{ut-1}) and prices (P_{ut-1}) in period $t-1$ from a joint log log normal distribution:

$$\begin{pmatrix} \ln\left(\varphi_{ut-1}/\tilde{\varphi}_{t-1}\right) \\ \ln\left(P_{ut-1}/\tilde{P}_{t-1}\right) \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \chi_\varphi^2 & \rho\chi_\varphi\chi_p \\ \rho\chi_\varphi\chi_p & \chi_p^2 \end{bmatrix} \right). \tag{A.3.24}$$

Since initial appeal (φ_{ut-1}) and initial prices (P_{ut-1}) are expressed relative to their geometric means ($\tilde{\varphi}_{t-1}$ and $\tilde{P}_{t-1}$, respectively), they are mean zero in logs by construction. We set the standard deviation for initial prices to one ($\chi_p = 1$); we consider three different values for the correlation between initial prices and appeal ($\rho \in \{-0.5, 0, 0.5\}$); and we examine five different values for the standard deviation for initial appeal ($\chi_\varphi \in \{0.001, 0.01, 0.1, 0.5, 1\}$). We use these initial realizations for appeal (φ_{ut-1}) and prices (P_{ut-1}) to solve for initial equilibrium expenditure shares (S_{ut}^*) in period $t-1$:

$$S_{ut-1}^* = \frac{\left(P_{ut-1}/\varphi_{ut}\right)^{1-\sigma}}{\sum_{\ell\in\Omega}\left(P_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}. \tag{A.3.25}$$

We next draw appeal shocks ($\frac{\varphi_{ut}/\tilde{\varphi}_t}{\varphi_{ut-1}/\tilde{\varphi}_{t-1}}$) and price shocks ($\frac{P_{ut}/\tilde{P}_t}{P_{ut-1}/\tilde{P}_{t-1}}$) from period $t-1$ to period t from the same joint log log normal distribution:

$$\begin{pmatrix} \ln\left(\frac{\varphi_{ut}/\tilde{\varphi}_t}{\varphi_{ut-1}/\tilde{\varphi}_{t-1}}\right) \\ \ln\left(\frac{P_{ut}/\tilde{P}_t}{P_{ut-1}/\tilde{P}_{t-1}}\right) \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \chi_\varphi^2 & \rho\chi_\varphi\chi_p \\ \rho\chi_\varphi\chi_p & \chi_p^2 \end{bmatrix} \right), \tag{A.3.26}$$

where appeal shocks ($\frac{\varphi_{ut}/\tilde{\varphi}_t}{\varphi_{ut-1}/\tilde{\varphi}_{t-1}}$) and price shocks ($\frac{P_{ut}/\tilde{P}_t}{P_{ut-1}/\tilde{P}_{t-1}}$) are expressed relative to their geometric means ($\tilde{\varphi}_t/\tilde{\varphi}_{t-1}$ and $\tilde{P}_t/\tilde{P}_{t-1}$, respectively) and are hence mean zero in logs by construction. We use these realizations for appeal and price shocks to solve for prices (P_{ut}) and expenditure shares (S_{ut}) in period t:

$$P_{ut} = \left(\frac{P_{ut}/\tilde{P}_t}{P_{ut-1}/\tilde{P}_{t-1}}\right) \times P_{ut-1}, \tag{A.3.27}$$

$$S_{ut}^{*}=\frac{\left(\frac{P_{ut}/\tilde{P}_{t}}{P_{ut-1}/\tilde{P}_{t-1}}\right)^{1-\sigma}\times\left(\frac{\varphi_{ut}/\tilde{\varphi}_{t}}{\varphi_{ut-1}/\tilde{\varphi}_{t-1}}\right)^{\sigma-1}\times S_{ut-1}^{*}}{\sum_{\ell\in\Omega}\left(\frac{P_{\ell t}/\tilde{P}_{t}}{P_{\ell t-1}/\tilde{P}_{t-1}}\right)^{1-\sigma}\times\left(\frac{\varphi_{\ell t}/\tilde{\varphi}_{t}}{\varphi_{\ell t-1}/\tilde{\varphi}_{t-1}}\right)^{\sigma-1}\times S_{\ell t-1}^{*}}. \tag{A.3.28}$$

Given the prices and expenditure shares for periods $t-1$ and t (P_{ut-1}, P_{ut}, S_{ut-1}^{*}, S_{ut}^{*}), we implement our reverse-weighting estimator ($\widehat{\sigma}^{RW}$) of the elasticity of substitution (σ) using the following moment conditions:

$$m(\sigma)=\left(\begin{array}{c}\ln\left\{\left[\sum_{u\in\Omega}S_{ut-1}^{*}\left(\frac{P_{ut}}{P_{ut-1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}\right\}-\ln\left\{\mathbb{M}_{t}\left[\frac{P_{ut}}{P_{ut-1}}\right]\left(\mathbb{M}_{t}\left[\frac{S_{ut}^{*}}{S_{ut-1}^{*}}\right]\right)^{\frac{1}{\sigma-1}}\right\}\\ \ln\left\{\left[\sum_{u\in\Omega}S_{ut}^{*}\left(\frac{P_{ut}}{P_{ut-1}}\right)^{-(1-\sigma)}\right]^{-\frac{1}{1-\sigma}}\right\}-\ln\left\{\mathbb{M}_{t}\left[\frac{P_{ut}}{P_{ut-1}}\right]\left(\mathbb{M}_{t}\left[\frac{S_{ut}^{*}}{S_{ut-1}^{*}}\right]\right)^{\frac{1}{\sigma-1}}\right\}\end{array}\right)=\left(\begin{array}{c}0\\0\end{array}\right). \tag{A.3.29}$$

We stack these moment conditions and estimate the elasticity of substitution (σ) using the generalized method of moments (GMM):

$$\hat{\sigma}^{RW}=\arg\min\left\{m(\sigma)'\times\mathbb{I}\times m(\sigma)\right\}, \tag{A.3.30}$$

where $\mathbb{I}$ is the identity matrix.

To provide a point of comparison, we also estimate the elasticity of substitution (σ) using the OLS estimator ($\widehat{\sigma}^{OLS}$). From the CES expenditure shares in periods $t-1$ and t, we have the following log linear demand system relationship:

$$\ln\left(\frac{S_{ut}^{*}/\tilde{S}_{t}^{*}}{S_{ut-1}^{*}/\tilde{S}_{t-1}^{*}}\right)=(1-\sigma)\ln\left(\frac{P_{ut}/\tilde{P}_{t}}{P_{ut-1}/\tilde{P}_{t-1}}\right)+(\sigma-1)\ln\left(\frac{\varphi_{ut}/\tilde{\varphi}_{t}}{\varphi_{ut-1}/\tilde{\varphi}_{t-1}}\right). \tag{A.3.31}$$

Absorbing appeal shocks into the error term ($\ln\left(\frac{\epsilon_{ut}}{\epsilon_{ut-1}}\right)$), we estimate the following OLS regression:

$$\ln\left(\frac{S_{ut}^{*}/\tilde{S}_{t}^{*}}{S_{ut-1}^{*}/\tilde{S}_{t-1}^{*}}\right)=\beta\ln\left(\frac{P_{ut}/\tilde{P}_{t}}{P_{ut-1}/\tilde{P}_{t-1}}\right)+\ln\left(\frac{\epsilon_{ut}}{\epsilon_{ut-1}}\right), \tag{A.3.32}$$

where we recover the implied elasticity of substitution using $\widehat{\sigma}^{OLS}=-\widehat{\beta}^{OLS}+1$.

In Figure A.3.1, we display the mean estimated elasticities of substitution across 1,000 replications for the reverse-weighting estimator ($\widehat{\sigma}^{RW}$) and the OLS estimator ($\widehat{\sigma}^{OLS}$). We also report the 5th and 95th percentiles of the distribution of estimated reverse-weighting elasticities ($\widehat{\sigma}^{RW}$) across these 1,000 replications. In each panel, the vertical axis displays the elasticity of substitution (σ), and the horizontal axis shows the standard deviation of appeal shocks (χ_{φ}) using a log scale. The top panel reports results for a negative correlation of price and appeal shocks ($\rho=-0.5$); the middle panel gives results for orthogonal price and appeal shocks ($\rho=0$); and the bottom panel presents results for a positive correlation between price and appeal shocks ($\rho=0.5$).

Across all three panels, we find that the mean reverse-weighting estimate ($\widehat{\sigma}^{RW}$) converges to the true elasticity of substitution (σ) as appeal shocks become small ($\chi_{\varphi}\rightarrow 0$). We find this pattern regardless of whether we consider negatively correlated, orthogonal or positively correlated price and appeal shocks.

Additionally, we find that the mean OLS estimate ($\widehat{\sigma}^{OLS}$) converges to the true elasticity of substitution (σ) as appeal shocks become small ($\chi_{\varphi} \to 0$), because the conventional omitted variables bias becomes small.

Nevertheless, the two estimates in general differ from one another. When price and appeal shocks are negatively correlated in the top panel, the mean OLS estimate lies above the true parameter value. In contrast, when price and appeal shocks are positively correlated, the mean OLS estimate lies below the true parameter value. Again, this pattern of results is line with conventional omitted variable bias, since appeal shocks in the regression residual have a positive impact on expenditure shares. In the case of positively correlated price and appeal shocks in the bottom panel, we find that the mean RW estimator performs somewhat better than the OLS estimator. This case of positively correlated price and appeal shocks is likely to be the empirically relevant one, if supplying products with higher appeal incurs higher marginal costs, and hence raises prices.

A.3.6 Robustness

As a robustness check, we now show that there is an alternative representation of the reverse-weighting estimator for tiers of utility above the lower tier (i.e. for the firm and sector tiers above the product tier). For brevity, we derive this alternative representation for the elasticity of substitution across firms (σ_g^F), but the same derivation goes through for the elasticity of substitution across sectors (σ^G). We begin with the expressions for expenditure on each product (X_{ut}^U) and expenditure on each firm (X_{ft}^F) from CES demand:

$$X_{ut}^U = \left(\frac{P_{ut}^U}{\varphi_{ut}^U} \right)^{1-\sigma_g^U} X_{ft}^F \left(P_{ft}^F \right)^{\sigma_g^U - 1}, \tag{A.3.33}$$

$$X_{ft}^F = \left(\frac{P_{ft}^F}{\varphi_{ft}^F} \right)^{1-\sigma_g^F} \mathbb{X}_{jgt}^G \left(\mathbb{P}_{jgt}^G \right)^{\sigma_g^F - 1}, \tag{A.3.34}$$

where $\mathbb{X}_{jgt}^G$ is importer j's total expenditure on foreign varieties from exporters $i \neq j$ in sector g at time t, and $\mathbb{P}_{jgt}^G$ is importer j's sectoral import price index for sector g at time t. Combining equations (A.3.33) and (A.3.34), we obtain:

$$X_{ut}^U = \left(\frac{P_{ut}^U}{\varphi_{ut}^U} \right)^{1-\sigma_g^U} \left(\frac{P_{ft}^F}{\varphi_{ft}^F} \right)^{1-\sigma_g^F} \mathbb{X}_{jgt}^G \left(\mathbb{P}_{jgt}^G \right)^{\sigma_g^F - 1} \left(P_{ft}^F \right)^{\sigma_g^U - 1}, \tag{A.3.35}$$

$$X_{ut}^U = \left(\frac{P_{ut}^U}{\varphi_{ut}^U} \right)^{1-\sigma_g^U} \left(\varphi_{ft}^F \right)^{\sigma_g^F - 1} \mathbb{X}_{jgt}^G \left(\mathbb{P}_{jgt}^G \right)^{\sigma_g^F - 1} \left(P_{ft}^F \right)^{\sigma_g^U - \sigma_g^F}. \tag{A.3.36}$$

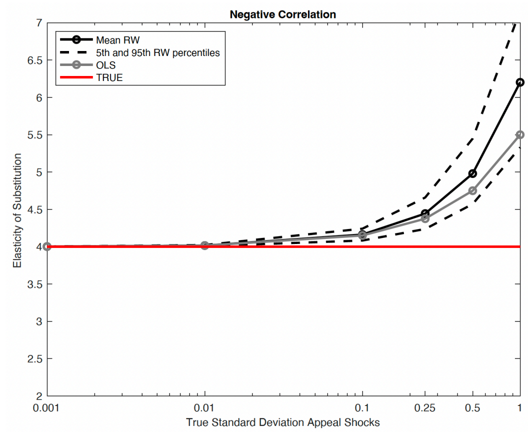
Rearranging equation (A.3.36), we get:

$$\left(P_{ft}^F \right)^{\sigma_g^U - \sigma_g^F} = \frac{X_{ut}^U}{\mathbb{X}_{jgt}^G} \left(\frac{P_{ut}^U}{\varphi_{ut}^U} \right)^{\sigma_g^U - 1} \left(\varphi_{ft}^F \right)^{-\left(\sigma_g^F - 1\right)} \left(\mathbb{P}_{jgt}^G \right)^{-\left(\sigma_g^F - 1\right)}, \tag{A.3.37}$$

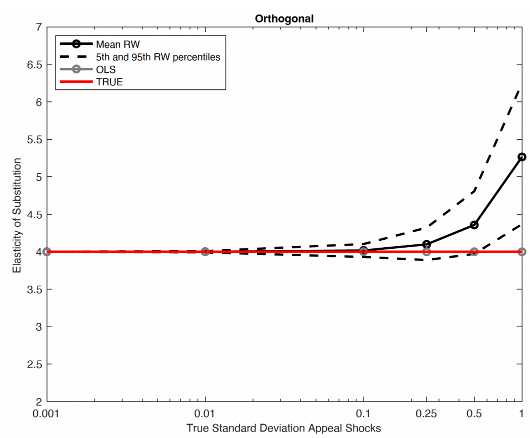
$$P_{ft}^F = \left(\frac{X_{ut}^U}{\mathbb{X}_{jgt}^G} \right)^{\frac{1}{\sigma_g^U - \sigma_g^F}} \left(\frac{P_{ut}^U}{\varphi_{ut}^U} \right)^{\frac{\sigma_g^U - 1}{\sigma_g^U - \sigma_g^F}} \left(\varphi_{ft}^F \right)^{-\frac{\sigma_g^F - 1}{\sigma_g^U - \sigma_g^F}} \left(\mathbb{P}_{jgt}^G \right)^{-\frac{\sigma_g^F - 1}{\sigma_g^U - \sigma_g^F}}. \tag{A.3.38}$$

Figure A.3.1: Mean Estimated Elasticities of Substitution for Alternative Correlations Between Price and Appeal Shocks (ρ) and Standard Deviations of Appeal Shocks (χ_{φ})

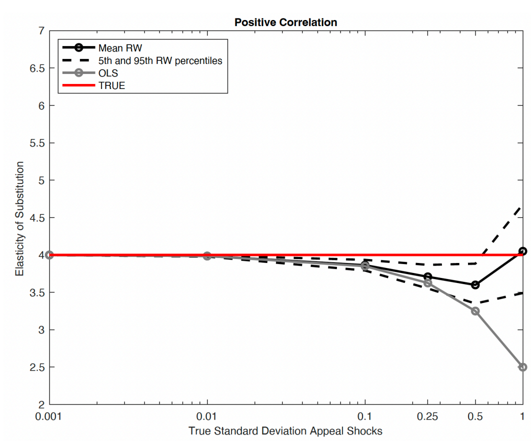
(a) Negative Correlation ($\rho = -0.5$)



(b) Orthogonal ($\rho = 0$)



(c) Positive Correlation ($\rho = 0.5$)



Note: 1,000 Monte Carlo simulations; 10,000 varieties; joint log normal distribution of price and appeal shocks; we set the standard deviation for initial prices to one ($\chi_p = 1$); we consider three different values for the correlation between initial prices and appeal ($\rho \in \{-0.5, 0, 0.5\}$).

Taking geometric means across common products within the firm in equation (A.3.38), we obtain:

$$P_{ft}^{F} = \left(\mathbb{M}_{ft}^{U*}\left[X_{ut}^{U}/\mathbb{X}_{jgt}^{G}\right]\right)^{\frac{1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{M}_{ft}^{U*}\left[P_{ut}^{U}\right]\right)^{\frac{\sigma_g^U-1}{\sigma_g^U-\sigma_g^F}} \left(\varphi_{ft}^{F}\right)^{-\frac{\sigma_g^F-1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{P}_{jgt}^{G}\right)^{-\frac{\sigma_g^F-1}{\sigma_g^U-\sigma_g^F}}, \tag{A.3.39}$$

where we have used our normalization that $\mathbb{M}_{ft}^{U*}\left[\varphi_{ut}^{U}\right]=1$. Now taking geometric means across common foreign firms within a sector in equation (A.3.39), we have:

$$\mathbb{M}_{gt}^{F*}\left[P_{ft}^{F}\right] = \left(\mathbb{M}_{ft}^{FU*}\left[X_{ut}^{U}/\mathbb{X}_{jgt}^{G}\right]\right)^{\frac{1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{M}_{ft}^{FU*}\left[P_{ut}^{U}\right]\right)^{\frac{\sigma_g^U-1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{P}_{jgt}^{G}\right)^{-\frac{\sigma_g^F-1}{\sigma_g^U-\sigma_g^F}}, \tag{A.3.40}$$

where we have used our choice of units that $\mathbb{M}_{jgt}^{F*}\left[\varphi_{ft}^{F}\right]=1$. Finally, using equation (A.3.40) to substitute for $\mathbb{M}_{gt}^{F*}\left[P_{ft}^{F}\right]$, we obtain another equivalent expression for our unified price index that exploits more of the nesting structure of the model:

$$\frac{\mathbb{P}_{jgt}^{G}}{\mathbb{P}_{jgt-1}^{G}} = \left(\frac{\lambda_{gt}^{F}}{\lambda_{gt-1}^{F}}\right)^{\frac{1}{\sigma_g^F-1}} \left(\mathbb{M}_{ft}^{FU*}\left[\frac{X_{ut}^{U}/\mathbb{X}_{jgt}^{G}}{X_{ut-1}^{U}/\mathbb{X}_{jgt-1}^{G}}\right]\right)^{\frac{1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{M}_{ft}^{FU*}\left[\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right]\right)^{\frac{\sigma_g^U-1}{\sigma_g^U-\sigma_g^F}} \left(\frac{P_{gt}^{G}}{P_{gt-1}^{G}}\right)^{-\frac{\sigma_g^F-1}{\sigma_g^U-\sigma_g^F}} \left(\mathbb{M}_{gt}^{F*}\left[\frac{S_{ft}^{F}}{S_{ft-1}^{F}}\right]\right)^{\frac{1}{\sigma_g^F-1}}. \tag{A.3.41}$$

Using equation (A.3.41), we can construct two moment conditions analogous to those in equation (A.3.20) that can be used to estimate the elasticity of substitution across firms (σ_g^F). Following the same line of reasoning, we can also construct two moment conditions analogous to those in equation (A.3.22) that can be used to estimate the elasticity of substitution across sectors (σ^G).

We use these alternative representations of the moment conditions as a robustness check for our estimates of the firm and sector elasticities of substitution (σ_g^F, σ^G) from equations (A.3.20) and (A.3.22). As appeal shocks become small ($\varphi_{ft}^{F}/\varphi_{ft-1}^{F} \to 1$ for all f and $\varphi_{jgt}^{G}/\varphi_{jgt-1}^{G} \to 1$ for all g), these alternative representations of the moment conditions yield the same estimated elasticities of substitution (σ_g^F, σ^G). In our empirical results for the U.S. and Chile, we use our baseline specifications in equations (A.3.16) and (A.3.20) for the firm and product elasticities of substitution. We use the robustness specification based on equation (A.3.41) for our sector elasticity of substitution in order to use more of the model's nesting structure where we have a relatively small number of observations on sectors.

As another robustness check, we use the property of CES that the reverse-weighting estimator can be implemented for any subset of common goods. We now illustrate this property for the firm price index, but it also holds for each of our other tiers of utility. We start by noting that the change in the firm price index for common products ($P_{ft}^{F*}/P_{ft-1}^{F*}$) can be written in terms of the change in the firm price index for a subset of common products ($P_{ft}^{F\#}/P_{ft-1}^{F\#}$) and the change in the expenditure share of this subset in total expenditure on common products ($\lambda_{ft}^{U\#}/\lambda_{ft-1}^{U\#}$) :

$$\frac{P_{ft}^{F*}}{P_{ft-1}^{F*}} = \left(\frac{\lambda_{ft}^{U\#}}{\lambda_{ft-1}^{U\#}}\right)^{\frac{1}{\sigma_g^F-1}} \left[\frac{\sum_{u\in\Omega_{ft,t-1}^{U\#}}\left(P_{ut}^{U\#}\right)^{1-\sigma_g^F}}{\sum_{u\in\Omega_{ft,t-1}^{U\#}}\left(P_{ut-1}^{U\#}\right)^{1-\sigma_g^F}}\right]^{\frac{1}{1-\sigma_g^F}} = \left(\frac{\lambda_{ft}^{U\#}}{\lambda_{ft-1}^{U\#}}\right)^{\frac{1}{\sigma_g^F-1}} \frac{P_{ft}^{F\#}}{P_{ft-1}^{F\#}}, \tag{A.3.42}$$

where the superscript # indicates that a variable is defined for this subset of common goods; we denote this subset of common goods by $\Omega_{ft,t-1}^{U\#} \subset \Omega_{ft,t-1}^{U}$; and the shares of expenditure on this subset of common goods in periods $t-1$ and t are:

$$\lambda_{ft}^{U\#} \equiv \frac{\sum_{u \in \Omega_{ft,t-1}^{U\#}} \left(\frac{P_{ut}^{U}}{\varphi_{ut}^{U}} \right)^{1-\sigma_g^U}}{\sum_{u \in \Omega_{ft,t-1}^{U}} \left(\frac{P_{ut}^{U}}{\varphi_{ut}^{U}} \right)^{1-\sigma_g^U}}, \qquad \lambda_{ft-1}^{U\#} \equiv \frac{\sum_{u \in \Omega_{ft,t-1}^{U\#}} \left(\frac{P_{ut-1}^{U}}{\varphi_{ut-1}^{U}} \right)^{1-\sigma_g^U}}{\sum_{u \in \Omega_{ft,t-1}^{U}} \left(\frac{P_{ut-1}^{U}}{\varphi_{ut-1}^{U}} \right)^{1-\sigma_g^U}}. \tag{A.3.43}$$

Using this property of CES, we obtain the following three equivalent expressions for the change in the firm price index for common products $(P_{ft}^{F*}/P_{ft-1}^{F*})$, which are analogous to those for the change in the overall firm price index $(P_{ft}^{F}/P_{ft-1}^{F})$:

$$\frac{P_{ft}^{F*}}{P_{ft-1}^{F*}} = \left(\frac{\lambda_{ft}^{U\#}}{\lambda_{ft-1}^{U\#}} \right)^{\frac{1}{\sigma_g^U-1}} \left[\sum_{u \in \Omega_{ft,t-1}^{U\#}} S_{ut-1}^{U\#} \left(\frac{P_{ut}^{U}/\varphi_{ut}^{U}}{P_{ut-1}^{U}/\varphi_{ut-1}^{U}} \right)^{1-\sigma_g^U} \right]^{\frac{1}{1-\sigma_g^U}}, \tag{A.3.44}$$

$$\frac{P_{ft}^{F*}}{P_{ft-1}^{F*}} = \left(\frac{\lambda_{ft}^{U\#}}{\lambda_{ft-1}^{U\#}} \right)^{\frac{1}{\sigma_g^U-1}} \left[\sum_{u \in \Omega_{ft,t-1}^{U\#}} S_{ut}^{U\#} \left(\frac{P_{ut}^{U}/\varphi_{ut}^{U}}{P_{ut-1}^{U}/\varphi_{ut-1}^{U}} \right)^{-\left(1-\sigma_g^U\right)} \right]^{-\frac{1}{1-\sigma_g^U}}, \tag{A.3.45}$$

$$\frac{P_{ft}^{F*}}{P_{ft-1}^{F*}} = \left(\frac{\lambda_{ft}^{U\#}}{\lambda_{ft-1}^{U\#}} \right)^{\frac{1}{\sigma_g^U-1}} \mathbb{M}_{ft}^{U\#} \left[\frac{P_{ut}^{U}}{P_{ut-1}^{U}} \right] \left(\mathbb{M}_{ft}^{U\#} \left[\frac{S_{ut}^{U\#}}{S_{ut-1}^{U\#}} \right] \right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.3.46}$$

where $S_{ut-1}^{U\#}$ is the share of an individual product in total expenditure on this subset of common goods:

$$S_{ut-1}^{U\#} \equiv \frac{\left(P_{ut}^{U}/\varphi_{ut}^{U}\right)^{1-\sigma_g^U}}{\sum_{\ell \in \Omega_{ft,t-1}^{U\#}} \left(P_{\ell t}^{U}/\varphi_{\ell t}^{U}\right)^{1-\sigma_g^U}}, \qquad u \in \Omega_{ft,t-1}^{U\#}; \tag{A.3.47}$$

$\mathbb{M}_{ft}^{U\#}[\cdot]$ is the geometric mean across this subset of common goods such that:

$$\mathbb{M}_{ft}^{U\#}\left[P_{ut}^{U}\right] \equiv \left(\prod_{u \in \Omega_{ft,t-1}^{U\#}} P_{ut}^{U} \right)^{\frac{1}{N_{ft,t-1}^{U\#}}} = 1, \tag{A.3.48}$$

where $N_{ft,t-1}^{U\#} = \left|\Omega_{ft,t-1}^{U\#}\right|$ is the number of elements in this subset of common goods; and we now choose units in which to measure product appeal (φ_{ut}^{U}) such that its geometric mean across this subset of common goods is equal to one:

$$\mathbb{M}_{ft}^{U\#}\left[\varphi_{ut}^{U}\right] \equiv \left(\prod_{u \in \Omega_{ft,t-1}^{U\#}} \varphi_{ut}^{U} \right)^{\frac{1}{N_{ft,t-1}^{U\#}}} = 1. \tag{A.3.49}$$

Using the three equivalent expressions for the change in each firm's price index in equations (A.3.44)-(A.3.46), and re-arranging terms, we obtain the following two equalities:

$$\Theta_{ft,t-1}^{U\#+} \left[\sum_{u \in \Omega_{ft,t-1}^{U\#}} S_{ut-1}^{U\#} \left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}} \right)^{1-\sigma_g^U} \right]^{\frac{1}{1-\sigma_g^U}} = \mathbb{M}_{ft}^{U\#} \left[\frac{P_{ut}^{U}}{P_{ut-1}^{U}} \right] \left(\mathbb{M}_{ft}^{U\#} \left[\frac{S_{ut}^{U\#}}{S_{ut-1}^{U\#}} \right] \right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.3.50}$$

$$\left(\Theta_{ft,t-1}^{U\#-}\right)^{-1}\left[\sum_{u\in\Omega_{ft,t-1}^{U\#}} S_{ut}^{U\#}\left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right)^{-\left(1-\sigma_g^U\right)}\right]^{-\frac{1}{1-\sigma_g^U}} = \mathbb{M}_{ft}^{U\#}\left[\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right]\left(\mathbb{M}_{ft}^{U\#}\left[\frac{S_{ut}^{U\#}}{S_{ut-1}^{U\#}}\right]\right)^{\frac{1}{\sigma_g^U-1}}, \tag{A.3.51}$$

where the terms in the share of expenditure on this subset of common products ($\left(\lambda_{ft}^{U\#}/\lambda_{ft-1}^{U\#}\right)^{1/\left(\sigma_g^U-1\right)}$) have cancelled; $\Theta_{ft,t-1}^{U\#+}$ is a forward aggregate demand shifter and $\Theta_{ft,t-1}^{U\#-}$ is a backward aggregate demand shifter such that:

$$\Theta_{ft,t-1}^{U\#+} \equiv \left[\frac{\sum_{u\in\Omega_{ft,t-1}^{U\#}} S_{ut-1}^{U\#}\left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right)^{1-\sigma_g^U}\left(\frac{\varphi_{ut}^{U}}{\varphi_{ut-1}^{U}}\right)^{\sigma_g^U-1}}{\sum_{u\in\Omega_{ft,t-1}^{U\#}} S_{ut-1}^{U\#}\left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right)^{1-\sigma_g^U}}\right]^{\frac{1}{1-\sigma_g^U}}, \tag{A.3.52}$$

$$\Theta_{ft,t-1}^{U\#-} \equiv \left[\frac{\sum_{u\in\Omega_{ft,t-1}^{U\#}} S_{ut}^{U\#}\left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right)^{-\left(1-\sigma_g^U\right)}\left(\frac{\varphi_{ut}^{U}}{\varphi_{ut-1}^{U}}\right)^{-\left(\sigma_g^U-1\right)}}{\sum_{u\in\Omega_{ft,t-1}^{U\#}} S_{ut}^{U\#}\left(\frac{P_{ut}^{U}}{P_{ut-1}^{U}}\right)^{-\left(1-\sigma_g^U\right)}}\right]^{\frac{1}{1-\sigma_g^U}}. \tag{A.3.53}$$

Using the identifying assumption that the appeal shocks cancel out across this subset of common products ($\Theta_{ft,t-1}^{U\#+} = \left(\Theta_{ft,t-1}^{U\#-}\right)^{-1} = 1$), equations (A.3.52) and (A.3.53) can be used to construct moment conditions to estimate the elasticity of substitution across products (σ_g^U) that are analogous to those in equation (A.3.16) above. In estimating the elasticities of substitution for the U.S. and Chile, we focus on the subset of common goods for each tier of utility K that have relative changes in prices (P_{kt}^K/P_{kt-1}^K) and expenditures (X_{kt}^K/X_{kt-1}^K) in between the 10th and 90th percentiles, which enables us to abstract from implausibly large annual changes in prices and expenditures for outlying observations. Given these estimated elasticities of substitution (σ_g^U, σ_g^F, σ^G), we solve for the appeal shifters (φ_{ut}^U, φ_{ft}^F, φ_{jgt}^G) that rationalize the observed data on prices (P_{ut}^U) and expenditures (X_{ut}^U) for all observations.

A.4 Data Description

In this section of the Online Appendix, we report further details on the data sources and definitions for the U.S. trade transactions data and Chilean trade transactions data used in the paper.

A.4.1 U.S. Data

The U.S. trade transactions data comes from the U.S. Census Bureau's Longitudinal Firm Trade Transactions Database (LFTTD). This database covers the universe of U.S.-based firms that import merchandise from abroad. For each import shipment, we observe the freight value of the shipment in U.S. dollars, the quantity shipped, the date of the transaction, the product classification (according to 10-digit Harmonized System (HS) codes), and the Manufacturing ID (MID). The MID is a field that importing firms must record in CBP Form 7501 in order to complete the importation of goods into the United States.

We use the MID to identify the manufacturer of the merchandise. The first two characters of the MID are the two-digit ISO country code for the country of origin. The next three characters are the start of the first word of the exporter's name. The next three characters are the start of the second word of that name. The next four characters are the start of the largest number that appears in the street address of the exporter. The last three characters are the start of the exporter's city.

Kamal, Krizan and Monarch (2015) documents the characteristics of the MID and its ability to identify a foreign supplier. The authors show that simple cleaning procedures, such as removing the city portion of the MID or removing the address-number portion of the MID, result in a close match between the number of exporting firms to the U.S. from each exporting country reported in the LFTTD and that reported in exporting country data.

Guided by these results, we define foreign exporting firms using the MID, after having removed both the address-number and the city, and the NAICS 4-digit code. This procedure enables us to merge together multi-plant firms that operate in different cities. After implementing this procedure, we compared the number of firms per country exporting to the U.S. in the LFTTD and foreign country sources and found that they matched closely. In addition to removing the address from the MID, we also implement the following additional cleaning procedures:

1. Standardize the units in which quantities are reported (e.g., we convert dozens to counts and grams to kilograms).

2. Drop an observation if the unit of quantity does not exist.

3. Drop observations that are indicated to have a high likelihood of input error (as indicated by a "blooper" variable in the data).

4. Drop an observation if the MID is missing.

5. Drop an observation if the ISO code (the first two digits of the MID) is invalid.

6. Drop an observation if the MID does not contain the firm-name portion.

7. Drop an observation if the quantity or value is invalid (negative or missing).

8. If the exporter is from Canada, the first two letters in the MID denotes the Canadian province rather than the ISO code of Canada. We therefore collapse provinces into one Canada.

9. The ISO codes in the MID often separate China and Hong Kong, which we collapse into China.

10. Our transaction data includes imports from U.S. territories and also imports from domestic origin returned to the United States with no change in condition or after having been processed and/or assembled in other countries. We drop these observations, so that we only consider transactions with a foreign country of origin.

A.4.2 Chilean Data

The Chilean trade transactions data come from Datamyne and take a similar form as our U.S. trade transactions data. For each import customs shipment, we observe the cost-inclusive-of-freight value of the shipment in U.S. dollars (converted using market exchange rates), the quantity shipped, the date of the transaction, the product classification (according to 8-digit Harmonized System (HS) codes), the country of origin, and the brand of the exporter (e.g. Nestlé, Toyota).

Using this information on import shipments, we construct a dataset for importer j (Chile) with many exporters i (countries of origin), sectors g (2-digit HS codes), firms f (foreign brands within exporter within sector), and products u (8-digit HS codes within foreign brands within sectors) and time t (year). We drop the small number of HS8 codes that do not use consistent units over time (e.g. we drop any HS8 code that switches from counts to kilograms). We also drop any observations for which countries of origin or brands are missing as well as those where the brand is a major trading company.[1] After several additional cleaning rules, which will be outlined in the next section, we collapse the import shipments data to the annual level by exporting firm and product, weighting by trade value, which yields a total of 6.5 million observations on Chilean imports by exporter-firm-product-year spanning the years 2007-2014.

A.4.3 Data Cleaning Methodology for Chilean Data

In this section, we explain the method used to clean and cluster the firm names in the Chilean import data.

A.4.3.1 Initial Cleaning of Raw Firm Names

We begin by implementing the following basic cleaning procedures to deal with obvious and easily fixable problems with the firm names.

1. Drop trading company names such as “MITSUBISHI CORPORATION”, “MITSUBISHI CORP”, and “SUMITOMO CORP”.

2. Trim company names to have a maximum string length of 50 (this impacted two firm names).

3. Remove substrings such as “-F”, “- F”, “S.A.”.

4. Remove most punctuations and symbols. We remove all of the following: ,.;:()[]{}!%#?/\@^*

5. Drop firm names that consist of only one alphabetical letter (e.g. if the brand name is “A”).

6. Add a space in front of common words. We implement this, because we observe many conjoined words (e.g. APPLEINCORPORATED).

[1] These were taken from the Forbes list of the top 10 trading companies.

7. Remove extra spaces between words (when there is more than one space between words) and remove spaces that come before or after the firm name (e.g. "   APPLE   INCORPORATED   " becomes "APPLE INCORPORATED").

8. Delete companies that are identified only by a Chinese city name (e.g. firm name is simply "BEIJING").

9. After applying these steps, remove firm names that are blank.

A.4.3.2 Standardizing Firm Names

We then use stnd_compname, a user-written Stata package by Wasi and Flaaen primarily to:

1. Remove entity names (e.g. LLC, LTD, INC)

2. Shorten commonly used words (e.g. ELECTRONICS, TECHNOLOGY) that have less distinguishing power, so that they will have less weighting during the string-similarity clustering.

The stnd_compname package comes with 43 standardizations for approximately 104 commonly used words (such as ENTERPRISE, INTERNATIONAL, MANAGEMENT, etc). We add approximately 100 standardizations and 180 words to this list for a total of 150 standardizations and around 300 words based on which words were the most common in the data. In addition to standardizing words, we also implement two more cleaning steps to complement the standardization:

1. Search through and remove a word if the first letter of the word is a numeral and the word is not the first word of the firm name. (MAZDA 4X 7TR turns into MAZDA)

2. If there is numeral within a word that is not the first word in a name, we remove the numeral and the rest of the letters following the numeral in the word. (FUJI F342FDIF turns into FUJI F)

A.4.3.3 Clustering

We then run string-similarity clustering (using strgroup, a user-written Stata package by Julian Reif) on the standardized firm names using a number of different thresholds and groups. These thresholds determine two strings' edit distance below which the two strings (i.e. firm names) will be grouped together. Varying this threshold is useful, because we observe that firm names are more likely to refer to the same firm if they share the same HS category. For example, we would be more comfortable assuming that "Sony Corp" and "Pony Corp" refer to the same company if we were only looking at makers of DVD players than doing cross-sector comparisons (because such cross-sector comparisons could involve assuming that an exporter of DVDs is also an exporter of farm animals). We take advantage of this by implementing clustering multiple times within multiple HS levels (2,4,6 and 8) and choosing stricter clustering thresholds for broader HS levels (i.e. as we cluster within more disaggregated HS-levels, the criterion for grouping firm names are made less strict). Specifically, we set our thresholds at 15 percent, 20 percent, 22 percent, and 30 percent for clustering within 2-digit HS codes, 4-digit HS codes, 6-digit HS codes, and 8-digit HS codes, respectively. After creating 4 different firm identifiers for the various HS levels (HS2, HS4, HS6, and HS8), the groupings are then merged together. If firm name A is matched with firm name B, and firm name B is matched with firm name C, then firm name A is matched with firm name C, and so on.

In parallel with the string-similarity clustering on the standardized firm names, we also implement the string similarity clustering on the firm names prior to standardization. We do this in case the standardization was ineffective (e.g. we missed certain words to be standardized). We run this clustering on a much stricter threshold than in the earlier step, so that we remain conservative about grouping firm names together. If the clustering results are too large (i.e. the threshold is not strict enough), we restrict the size of a cluster to 5 unique firm names (so that a firm name can be spelled in up to 5 different ways while still be identified as the same firm).

After clustering on the two sets of firm names (the firm names prior to standardization and those after standardization) we merge the clusters together. If firm A is matched with firm B in the first step and firm B is matched with firm C in the second step, then these groupings are merged, so that firm A is matched with C as well, implying that firms A, B, and C are all allocated to the same group.

A.4.3.4 Additional Cleaning Steps

After standardizing and clustering, we apply additional cleaning rules:

1. Now that standardization and clustering is complete, we drop the remaining observations with trading companies, blank firm names, and firm names that are only identified by a single alphabetical letter.

2. We observe many firm names in the data of the form “A & W” or “T & W” where the firm names consist of two letters with an “&” in between. The clustering method often clusters these firm names together (depending on the HS level) even if only one of these letters are the same (e.g. “A & W” and “T & W”), because the difference between the two firm names are 1/5 or 20 percent, which is within the threshold in many cases. To address this, we apply a rule such that these firm names are separated into different groups unless there is an exact match.

3. We again restrict the size of a cluster to 5 unique firm names. If a cluster is larger than 5 unique firm names, we cluster again on an ever-stricter threshold until the size of the cluster is five or less.

4. We sometimes encounter observations where the entire firm name is contained exactly at the start of another (e.g. “SONY” and “SONY ELECTRONICS” or “HEWLETT PACKARD” and “HEWLETT PACKARD ENTERPRISE”). Even after standardizing common words, these firm names often fail to be clustered together because their edit distances are too large. We combat this by creating a rule

such that if one firm name appears at the beginning of another, the two firm names are grouped together.

A.4.3.5 Validation

After implementing the above steps, we then checked how well our procedure worked by manually checking the results of this algorithm for the 1,249 raw firm names in the Japanese steel sector (which we had not looked at when developing the procedure). We manually checked the accuracy using two steps. First, we sorted the firm list alphabetically and counted the number of firm names that should have been grouped together (based on our manual inspection) but were not grouped together by our clustering algorithm. Second, we sorted the firm names by our groups and counted the number of firm names that should not have been grouped together (based on our manual inspection) but were grouped together by our clustering algorithm. Summing these type I and type II errors, we found that our cleaning algorithm and manual checking grouped firms in the same way for 99.9 percent of observations. As a final check on the sensitivity of our results to this cleaning algorithm, we replicated our main results of the Chilean import transactions data using the firm names prior to these cleaning steps. Again we find that most of the variation in revealed comparative advantage (RCA) across countries and sectors is explained by variety and appeal. Therefore, while our clustering algorithm improves the allocation of import transactions to firms, our main qualitative and quantitative conclusions hold regardless of whether or not we use this algorithm.

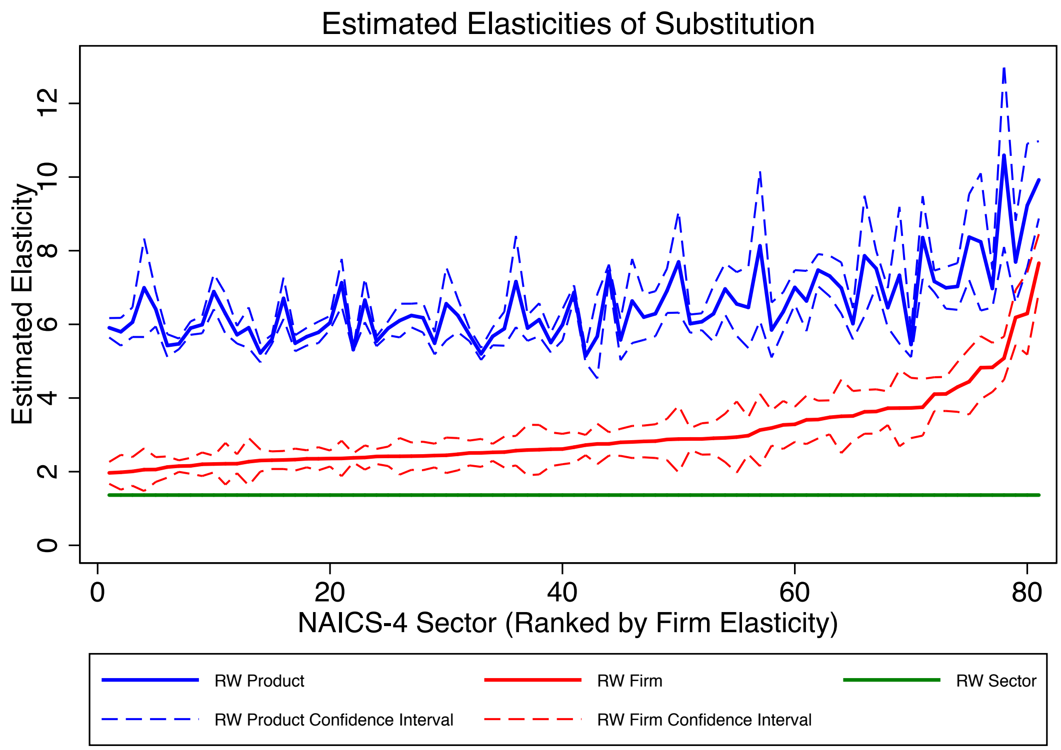
A.5 U.S. Empirical Results

In this section of the Online Appendix, we report additional empirical results using our U.S. data for Section 5 of the paper.

A.5.1 RW Elasticities of Substitution

In Figure A.5.1, we plot our estimated product, firm and sector elasticities of substitution ($\hat{\sigma}_g^U$, $\hat{\sigma}_g^F$, $\hat{\sigma}^G$) across sectors, sorted based on the ranking of the estimated firm elasticity of substitution ($\hat{\sigma}_g^F$). We also show 95% confidence intervals for the estimated product and firm elasticities of substitution ($\hat{\sigma}_g^U$, $\hat{\sigma}_g^F$) based on bootstrapped standard errors. As can be seen in the figure, we find a natural ordering where $\hat{\sigma}_g^U > \hat{\sigma}_g^F > \hat{\sigma}^G$. We also find that the confidence intervals are narrow enough such that the product elasticity is significantly larger than the firm elasticity ($\hat{\sigma}_g^U > \hat{\sigma}_g^F$) at the 5 percent level of significance for all sectors, and the firm elasticity is also significantly larger than the sector elasticity ($\hat{\sigma}_g^F > \hat{\sigma}^G$) at this significance level for all sectors.

Figure A.5.1: Estimated Elasticities of Substitution, Within Firms ($\hat{\sigma}_g^U$), Across Firms ($\hat{\sigma}_g^F$) and Across Sectors ($\hat{\sigma}^G$), sorted based on the ranking of $\hat{\sigma}_g^F$ (U.S. Data)



Note: Estimated elasticities of substitution across products (σ_g^U shown by the solid blue line) and firms (σ_g^F shown by the solid red line) for each NAICS-4 sector; sectors ranked by the elasticity of substitution across firms (σ_g^F); estimated elasticity of substitution across sectors (σ^G) shown by the horizontal green line; dashed lines denote 95 percent bootstrapped confidence intervals.

A.5.2 Variance Decomposition

In this subsection of the Online Appendix, we discuss our regression-based variance decomposition of revealed comparative advantage (RCA) into its components. We also report the results of a robustness test using an alternative variance decomposition suggested by Grömping (2007).

A.5.2.1 Regression-based Variance Decomposition

Our regression-based variance decomposition is relatively common in the macroeconomics and international trade literatures, including, for example, Klenow and Rodriguez-Clare (1997) (see the equation on page 80 of that paper); Eaton, Kortum and Kramarz (2004) (see the equation at the top of page 153 of that paper); Bernard, Jensen, Redding and Schott (2009) (see the discussion on page 488 and the results in Table 1 of that paper); and Bernard, Redding and Schott (2011) (see the equation at the top of page 1305 and the results in Table II of that paper). We now show that this regression-based variance decomposition allocates the covariance terms equally across the components of the decomposition.

Consider the following accounting decomposition, in which the variable y is the sum of the variables x and z:

$$y = x + z. \tag{A.5.1}$$

Our regression-based variance decomposition estimates an OLS regression of each of the individual components x and z on the overall value of y:

$$x = a_x + b_x y + \epsilon_x, \tag{A.5.2}$$

$$z = a_z + b_z y + \epsilon_z. \tag{A.5.3}$$

By the properties of OLS:

$$b_x = \frac{\text{cov}\left(x, y\right)}{\text{var}\left(y\right)} = \frac{\text{cov}\left(x, x+z\right)}{\text{var}\left(y\right)} = \frac{\text{var}\left(x\right) + \text{cov}\left(x, z\right)}{\text{var}\left(y\right)}, \tag{A.5.4}$$

$$b_z = \frac{\text{cov}\left(z, y\right)}{\text{var}\left(y\right)} = \frac{\text{cov}\left(z, x+z\right)}{\text{var}\left(y\right)} = \frac{\text{var}\left(z\right) + \text{cov}\left(x, z\right)}{\text{var}\left(y\right)}, \tag{A.5.5}$$

which shows that the covariance terms are allocated equally across the two coefficients (b_x, b_z) and hence allocated equally across the two components of the variance decomposition. Summing these two coefficients, we have:

$$b_x + b_z = \frac{\text{var}\left(x\right) + \text{var}\left(z\right) + 2\text{cov}\left(x, z\right)}{\text{var}\left(y\right)} = 1, \tag{A.5.6}$$

which shows that the two coefficients sum to one. Therefore, each coefficient (b_x, b_z) provides a measure of the relative importance of each term to the variance decomposition, where the covariance terms are allocated equally across the two components of the variance decomposition. In general, the estimated b_x and/or b_z coefficients in equations (A.5.4) and (A.5.5) can be negative if the covariance terms are negative and large in absolute magnitude relative to the variance terms.

A.5.2.2 Alternative Variance Decomposition

An alternative variance decomposition is proposed in Grömping (2007) based on hierarchical partitioning, which has the desirable property that the contribution of any variable to the explained variance must be positive. Consider again the following accounting decomposition, in which the variable y is the sum of the variables x and z:

$$y = x + z. \tag{A.5.7}$$

Estimate the following three regressions:

$$y = a_x + b_x x + \epsilon_x, \tag{A.5.8}$$

$$y = a_z + b_z z + \epsilon_z, \tag{A.5.9}$$

Table A.1: Variance Decomposition U.S. RCA (Grömping 2007)

	Log Level RCA 2011		Log Change RCA 1998-2011	
	Firm-Level Decomposition	Product-Level Decomposition	Firm-Level Decomposition	Product-Level Decomposition
Firm Price Index	0.129	-	0.178	-
Firm Appeal	0.155	0.110	0.325	0.297
Firm Variety	0.338	0.311	0.394	0.400
Firm Dispersion	0.378	0.351	0.103	0.103
Product Prices	-	0.087	-	0.153
Product Variety	-	0.055	-	0.038
Product Dispersion	-	0.086	-	0.009

Note: Relative importance decomposition for the log level of RCA in 2011 and the log change in RCA from 1998-2011 (from Grömping 2007); dispersion corresponds to the dispersion of appeal-adjusted prices.

$$y = a_{xy} + b_{xy}x + b_{xz}z + \epsilon_{xy}. \tag{A.5.10}$$

The contribution of x to the variance of y is defined as the average of (i) the R-squared from regression (A.5.8) when only x is included and (ii) the increase in the R-squared when both x and z are included in regression (A.5.10). Similarly, the contribution of of z to the variance of y is defined as the average of (i) the R-squared from regression (A.5.9) when only z is included and (ii) the increase in the R-squared when both x and z are included in regression (A.5.10).

Generalizing this approach to $\mathcal{J} > 2$ components, the contribution of each component to the variance of the total is the increase in the R-squared from adding this component to a regression where it is absent, averaged over all possible regressions formed from different permutations of the $\mathcal{J} - 1$ other components.

In Table A.1, we implement the variance decomposition from Table 3 in the paper for U.S. RCA using this alternative procedure. The results are quite similar. For example, the importance of prices, appeal, variety, and dispersion in the decomposition for RCA levels presented in Table 3 are 0.09, 0.22, 0.32, and 0.36 respectively, whereas the Grömping (2007) decomposition gives us 0.13, 0.16, 0.34, and 0.38. Similarly, the importance of prices, appeal, and variety in the decomposition for RCA changes presented in Table 3 are -0.01, 0.42, 0.50, and 0.08 respectively, whereas the Grömping (2007) decomposition gives us 0.18, 0.33, 0.39, 0.10. Thus, while the Grömping (2007) approach does have the benefit of eliminating the small negative contribution of price movements to RCA, the difference in methodologies mostly affects the second significant digit of the estimates. As we discuss in Section A.6.3 below, our results are also robust across different methodologies for the RCA decomposition using Chilean data.

A.5.3 Exporter Price Indexes Across Sectors and Countries

No further results required.

A.5.4 Trade Patterns

As discussed in Sections 5.1 and 5.3 of the paper, we undertake a robustness check, in which we carry out a grid search over the range of plausible values for elasticities of substitution across firms and products. In particular, we consider values of σ_g^F from 2 to 8 (in 0.5 increments) and values of σ_g^U from $(\sigma_g^F + 0.5)$ to 20 in 0.5 increments, while holding σ^G constant at our estimated value, which respects our estimated ranking that $\sigma_g^U > \sigma_g^F > \sigma^G$.

We begin by showing that the percentage contributions from firm variety and firm dispersion are invariant across this parameter grid, because the elasticities of substitution cancel from these expressions. From equation (A.2.42) in Section A.2.10.1 of this Online Appendix, the overall contribution from both firm and product variety to the level of log RCA is,

$$\ln\left(RCA_{jigt}^{\lambda}\right) \equiv - \left\{ \begin{array}{l} \left[\Delta \ln \lambda_{jigt}^{F} - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \Delta \ln \lambda_{jhgt}^{F}\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \left[\Delta \ln \lambda_{jikt}^{F} - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \Delta \ln \lambda_{jhkt}^{F}\right] \\ +\frac{\sigma_g^F-1}{\sigma_g^U-1} \left[\mathbb{E}_{jigt}^{F*}\left[\Delta \ln \lambda_{ft}^{U}\right] - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \mathbb{E}_{jhgt}^{F*}\left[\Delta \ln \lambda_{ft}^{U}\right]\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \frac{\sigma_k^F-1}{\sigma_k^U-1} \left[\mathbb{E}_{jikt}^{F*}\left[\Delta \ln \lambda_{ft}^{U}\right] - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \mathbb{E}_{jhkt}^{F*}\left[\Delta \ln \lambda_{ft}^{U}\right]\right] \end{array} \right\}, \tag{A.5.11}$$

where the component of this contribution that captures firm variety is,

$$\ln\left(RCA_{jigt}^{\lambda F}\right) \equiv - \left\{ \begin{array}{l} \left[\Delta \ln \lambda_{jigt}^{F} - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \Delta \ln \lambda_{jhgt}^{F}\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \left[\Delta \ln \lambda_{jikt}^{F} - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \Delta \ln \lambda_{jhkt}^{F}\right] \end{array} \right\}, \tag{A.5.12}$$

which depends solely on observed moments in the data and is invariant to the assumed elasticities of substitution for finite values of these elasticities ($\sigma_g^U < \infty$ and $\sigma_g^F < \infty$). Taking differences over time in equation (A.5.12), this invariance result also holds for changes in log RCA.

Similarly, from equation (A.2.41) in Section A.2.10.1 of this Online Appendix, the overall contribution from the dispersion of appeal-adjusted prices across common products and firms for the level of log RCA is,

$$\ln\left(RCA_{jigt}^{S*}\right) \equiv - \left\{ \begin{array}{l} \left[\mathbb{E}_{jigt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \mathbb{E}_{jhgt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right]\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \left[\mathbb{E}_{jikt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \mathbb{E}_{jhkt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right]\right] \\ +\frac{\sigma_g^F-1}{\sigma_g^U-1} \left[\mathbb{E}_{jigt,t-1}^{FU*}\left[\Delta \ln S_{ut}^{U*}\right] - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \mathbb{E}_{jhgt,t-1}^{FU*}\left[\Delta \ln S_{ut}^{U*}\right]\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \frac{\sigma_k^F-1}{\sigma_k^U-1} \left[\mathbb{E}_{jikt,t-1}^{FU*}\left[\Delta \ln S_{ut}^{U*}\right] - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \mathbb{E}_{jhkt,t-1}^{FU*}\left[\Delta \ln S_{ut}^{U*}\right]\right] \end{array} \right\}, \tag{A.5.13}$$

where the component of this contribution that captures firm dispersion is,

$$\ln\left(RCA_{jigt}^{SF*}\right) \equiv - \left\{ \begin{array}{l} \left[\mathbb{E}_{jigt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] - \frac{1}{N_{jgt,t-1}^{E}} \sum_{h\in\Omega_{jgt,t-1}^{E}} \mathbb{E}_{jhgt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right]\right] \\ -\frac{1}{N_{jit,t-1}^{T}} \sum_{k\in\Omega_{jit,t-1}^{T}} \left[\mathbb{E}_{jikt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right] - \frac{1}{N_{jkt,t-1}^{E}} \sum_{h\in\Omega_{jkt,t-1}^{E}} \mathbb{E}_{jhkt,t-1}^{F*}\left[\Delta \ln \mathbb{S}_{ft}^{EF*}\right]\right] \end{array} \right\}, \tag{A.5.14}$$

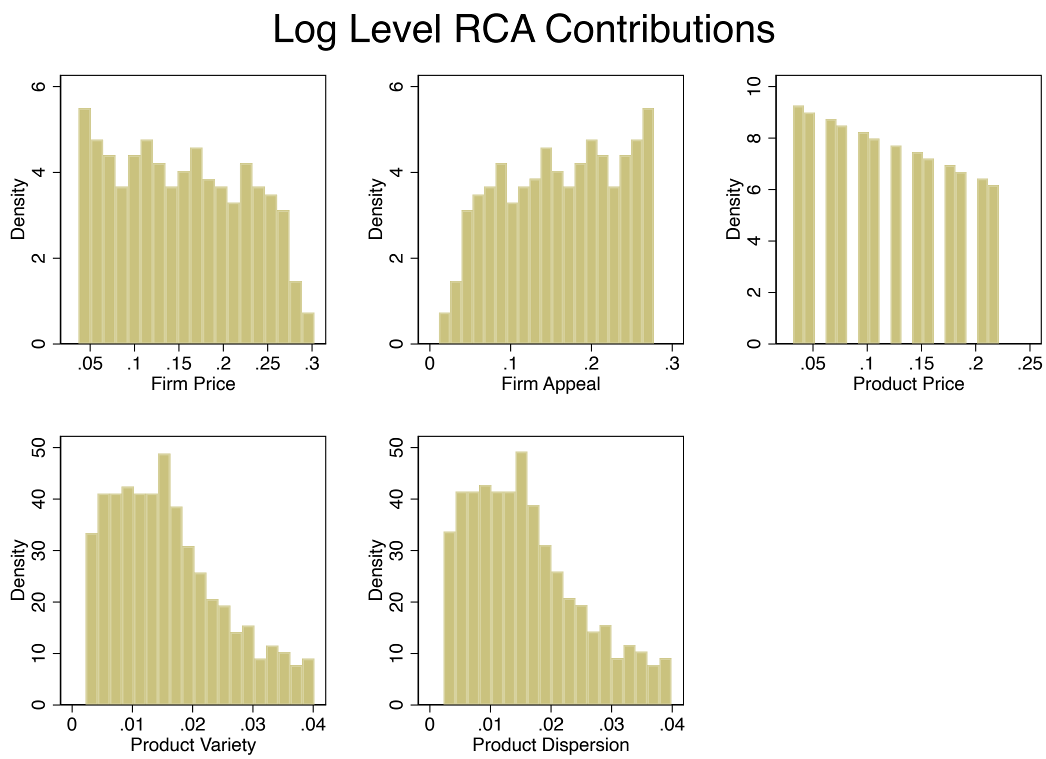
which depends solely on observed moments in the data and is invariant to the assumed elasticities of substitution for finite values of these elasticities ($\sigma_g^U < \infty$ and $\sigma_g^F < \infty$). Taking differences over time in equation (A.5.14), this invariance result also again holds for changes in log RCA.

In Figure A.5.2, we show histograms across the parameter grid for the contribution from each of the remaining terms from our decomposition of the level of RCA in equation (34) in the paper. The contributions from product prices, product variety and product dispersion in the final three panels sum to the contribution from firm prices in the first panel. Additionally, the firm price and firm appeal contributions in the first two panels plus the unreported contributions from firm variety and firm dispersion sum to one. In Figure A.5.3, we display analogous results for our decomposition of changes in RCA over time, where the five panels of the figure have the same relationship with one another as in Figure A.5.2.

In both figures, a higher value for σ_g^F raises the contribution from average prices and reduces the contribution from average appeal. Nonetheless, across the entire grid of parameter values, average prices account for less than 30 percent of the level of the RCA and less than 10 percent of the changes in RCA. In contrast, for all parameter values on the grid, average appeal's contribution to the level of RCA is around as large as that from average prices (from less than 5 percent to over 25 percent in Figure A.5.2). Furthermore, its contribution to changes in RCA is substantially larger than that from average prices (from just over 35 percent to just under 60 percent in Figure A.5.3).

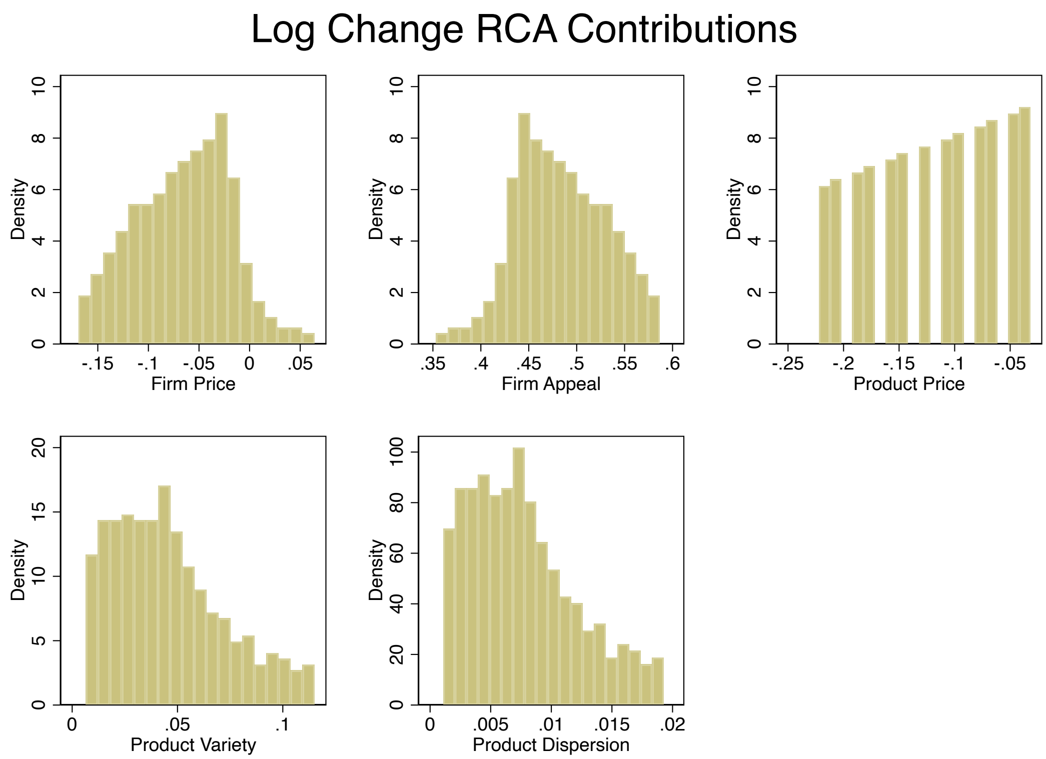
In summary, our findings that most of the variation in patterns of RCA is explained by factors other than average prices is robust to the consideration of alternative elasticities of substitution. In particular, the contributions from firm variety and firm dispersion are invariant to these elasticities of substitution. Furthermore, across the range of plausible values for these elasticities of substitution, the contribution from average appeal remains large relative to that from average prices.

Figure A.5.2: Contributions to the Level of U.S. RCA in 2011 Across the Parameter Grid for the Firm and Product Elasticities of Substitution



Note: Contributions to the level of revealed comparative advantage (RCA) in equation (34) in the paper; the contributions from product prices, product variety and product dispersion in the final three panels sum to the contribution from firm prices in the first panel; the firm price and firm appeal contributions in the first two panels plus the unreported contributions from firm variety and firm dispersion sum to one.

Figure A.5.3: Contributions to the Change in U.S. RCA in from 1998-2011 Across the Parameter Grid for the Firm and Product Elasticities of Substitution



Note: Contributions to changes in revealed comparative advantage (RCA) in equation (34) in the paper; the contributions from product prices, product variety and product dispersion in the final three panels sum to the contribution from firm prices in the first panel; the firm price and firm appeal contributions in the first two panels plus the unreported contributions from firm variety and firm dispersion sum to one.

A.5.5 Additional Theoretical Restrictions

In Section 5.4 of the paper, we compare the observed data for firm sales and our model solutions for the firm price index and firm appeal ($\ln V_{ft}^F \in \left\{\ln \mathbb{X}_{ft}^F, \ln P_{ft}^F, \ln \varphi_{ft}^F\right\}$) with their theoretical predictions under the assumptions of a Pareto distribution or a log normal distribution. In this section of the Online Appendix, we derive these theoretical predictions, as summarized in equations (35) and (36) in the paper.

Empirical Distributions In particular, we use the QQ estimator of Kratz and Resnick (1996), as introduced into the international trade literature by Head, Mayer and Thoenig (2016). We start with the empirical distributions. Ordering firms by the value of a given variable V_{ft}^F for $f \in \left\{1, \ldots, N_{jigt}^F\right\}$ for a given exporter i to importer j in sector g at time t, we observe the empirical quantiles:

$$\mathbb{V}_{ft} = \ln\left(V_{ft}^F\right). \tag{A.5.15}$$

We can use these empirical quantiles to estimate the empirical cumulative distribution function:

$$\widehat{\mathcal{F}}_{jigt}\left(V_{ft}^{F}\right)=\frac{f-b}{N_{jigt}^{F}+1-2b},\qquad b=0.3, \tag{A.5.16}$$

where the plot position of $b=0.3$ can be shown to approximate the median rank of the distribution (see Benard and Boslevenbach 1953). We next turn to the theoretical distributions, first under the assumption of a Pareto distribution, and next under the assumption of a log normal distribution.

Pareto Distribution Under the assumption that the variable V_{ft}^{F} has a Pareto distribution, its cumulative distribution function is given by:

$$\mathcal{F}_{jigt}\left(V_{ft}^{F}\right)=1-\left(\frac{\underline{V}_{jigt}^{F}}{V_{ft}^{F}}\right)^{a_{g}^{V}}, \tag{A.5.17}$$

where $\mathcal{F}_{jigt}\left(\cdot\right)$ is the cumulative distribution function; $\underline{V}_{jigt}^{F}$ is the lower limit of the support of the distribution for variable V_{ft}^{F} for exporter i, importer j, sector g and time t; and a_{g}^{V} is the Pareto shape parameter for variable V_{ft}^{F} for sector g.

Inverting this cumulative distribution function, and taking logarithms, we obtain the following predicted theoretical quantile for each variable:

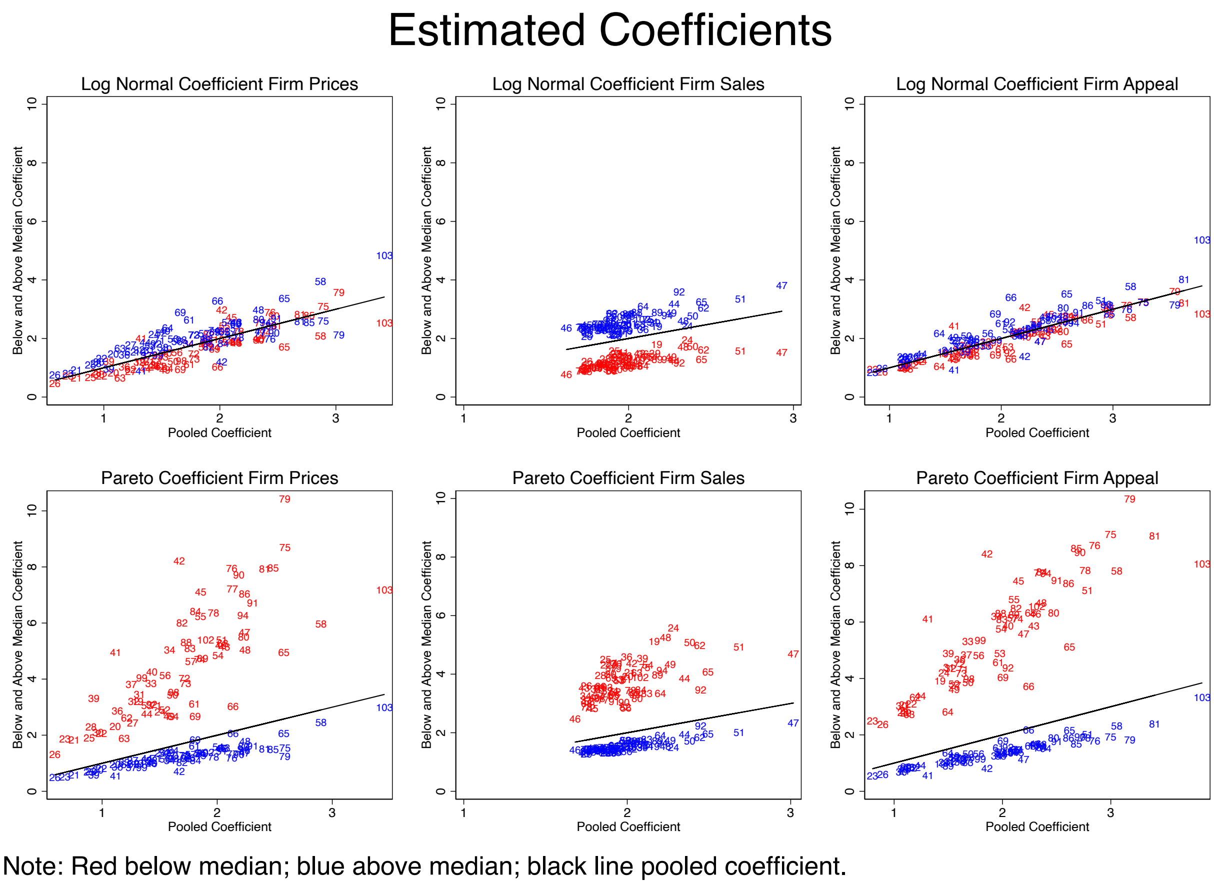
$$\ln\left(V_{ft}^{F}\right)=\ln\underline{V}_{jigt}^{F}-\frac{1}{a_{g}^{V}}\ln\left[1-\mathcal{F}_{jigt}\left(V_{ft}^{F}\right)\right], \tag{A.5.18}$$

which corresponds to equation (35) in the paper.

We estimate equation (A.5.18) by OLS using the empirical quantile from equation (A.5.15) for $\ln\left(V_{ft}^{F}\right)$ on the left-hand side and the empirical estimate of the cumulative distribution function from equation (A.5.16) for $\mathcal{F}_{jigt}\left(V_{ft}^{F}\right)$ on the right-hand side. We estimate this regression for each sector across foreign firms (allowing the slope coefficient a_{g}^{V} to vary across sectors) and including fixed effects for each exporter-sector-year combination (allowing the intercept $\ln\underline{V}_{jigt}^{F}$ to vary across exporters, sectors and time). The fitted values from this regression correspond to the predicted theoretical quantiles, which we compare to the empirical quantiles observed in the data. Under the null hypothesis of a Pareto distribution, there should be a linear relationship between the theoretical and empirical quantiles that coincides with the 45-degree line.

To assess this theoretical prediction, we estimate equation (35) in the paper for two separate subsamples: firms with values below the median for each exporter-sector-year cell and firms with values above the median for each exporter-sector-year cell. Under the null hypothesis of a Pareto distribution, the estimated slope coefficient $1/a_{g}^{V}$ should be the same for firms below and above the median. In the bottom three panels of Figure A.5.4, we display the estimated slope coefficients $1/a_{g}^{V}$ for each 4-digit NAICS industry for the log firm price index ($\ln P_{ft}^{F}$ to the left), log firm exports ($\ln \mathbb{X}_{ft}^{F}$ in the middle), and log firm appeal ($\ln \varphi_{ft}^{F}$ to the right). In each panel, we sort industries by the estimated slope coefficient for the full sample for that variable (shown by the black straight line). The red and blue numeric industry codes show the estimates for the subsamples of firms below and above the median respectively. For all three variables, we strongly reject the null hypothesis of a Pareto distribution, with substantial differences in the estimated coefficients below and above the median, which are significant at conventional levels.

Figure A.5.4: Estimated Coefficients from Regressions of the Empirical Quantiles on the Theoretical Quantiles Implied by a Pareto or Log Normal distribution (U.S. data)



Note: Regressions of empirical quantiles on theoretical quantiles for firm observations below (red) and above (blue) the median for each exporter-sector-year cell; numbers corresponds to industry NAICS codes; theoretical quantiles in the top three panels based on a log normal distribution; theoretical quantiles in the bottom three panels based on a Pareto distribution; left column shown results for the firm price index; middle columns shows results for observed firm sales; right column shows results for firm appeal).

Log Normal Distribution In contrast, under the assumption that the variable V_{ft}^{F} has a log normal distribution, its cumulative distribution function is given by:

$$\ln\left(V_{ft}^{F}\right) \sim \mathcal{N}\left(\kappa_{jigt}^{V}, \left(\chi_{g}^{V}\right)^{2}\right), \tag{A.5.19}$$

where κ^V_{jigt} is the mean for $\ln V^F_{ft}$ for exporter i in importer j and sector g at time t and χ^V_g is the standard deviation for $\ln V^F_{ft}$ for sector g. It follows that the standardized value of the log of each variable is drawn from a standard normal distribution:

$$\mathcal{F}_{jigt}\left(V^F_{ft}\right) = \Phi\left(\frac{\ln\left(V^F_{ft}\right) - \kappa^V_{jigt}}{\chi^V_g}\right), \tag{A.5.20}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Inverting this cumulative distribution function, we obtain the following predictions for the theoretical quantiles of each variable:

$$\frac{\ln\left(V^F_{ft}\right) - \kappa^V_{jigt}}{\chi^V_g} = \Phi^{-1}\left(\mathcal{F}_{jigt}\left(V^F_{ft}\right)\right), \tag{A.5.21}$$

which can be re-expressed as:

$$\ln\left(V^F_{ft}\right) = \kappa^V_{jigt} + \chi^V_g \Phi^{-1}\left(\mathcal{F}_{jigt}\left(V^F_{ft}\right)\right), \tag{A.5.22}$$

which corresponds to equation (36) in the paper.

Again we estimate equation (A.5.22) by OLS using the empirical quantile from equation (A.5.15) for $\ln\left(V^F_{ft}\right)$ on the left-hand side and the empirical estimate of the cumulative distribution function from equation (A.5.16) for $\mathcal{F}_{jigt}\left(V^F_{ft}\right)$ on the right-hand side. We estimate this regression for each sector across foreign firms (allowing the slope coefficient χ^V_g to vary across sectors) and including fixed effects for each exporter-sector-year combination (allowing the intercept κ^V_{jigt} to vary across exporters, sectors and time). In the top three panels of Figure A.5.4, we display the estimated slope coefficients χ^V_g for each 4-digit NAICS industry for the log firm price index ($\ln P^F_{ft}$ to the left), log firm exports ($\ln \mathbb{X}^F_{ft}$ in the middle), and log firm appeal ($\ln \varphi^F_{ft}$ to the right), using the same coloring as for the bottom three panels discussed above.

As apparent from the figure, we find that the log normal distributional assumption provides a closer approximation to the data than the Pareto distributional assumption. Consistent with Bas, Mayer and Thoenig (2017), we find smaller departures from the predicted linear relationship between the theoretical and empirical quantiles for a log normal distribution than for a Pareto distribution. Nevertheless, we reject the null hypothesis of a log normal distribution at conventional significance levels for all three variables for the majority of industries, with substantial differences in estimated coefficients above and below the median for a number of industries.

A.5.6 Additional Reduced-Form Evidence

In Figures A.5.5-A.5.8 below, we show that our U.S. trade transactions data exhibit have the same reduced-form properties as found in existing studies in the empirical trade literature (see for example Bernard, Jensen and Schott 2009 and Bernard, Jensen, Redding and Schott 2009 for the U.S.; Mayer, Melitz and Ottaviano 2014 for France; and Manova and Zhang 2012 for China).

First, we find a high concentration of trade across countries and a dramatic increase in Chinese import penetration over time. As shown in Figure A.5.5, the top 20 import source countries account for around 80 percent of U.S. imports; China's import share more than doubles from 7 to 18 percent from 1997-2011; in contrast, Japan's import share more than halves from 14 to 6 percent over this period.

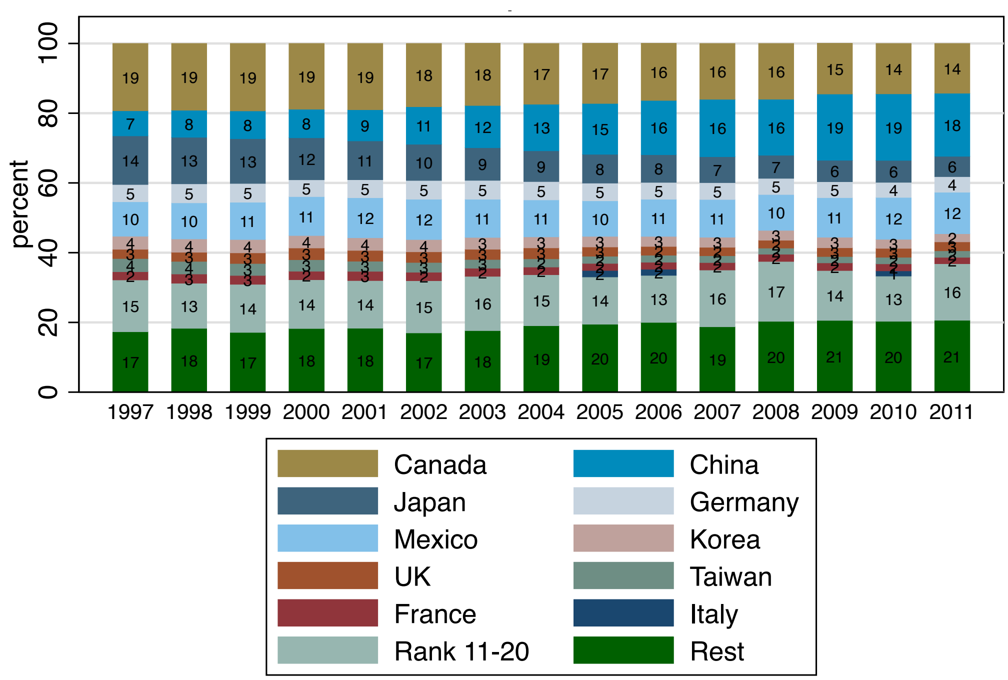
Second, we find high rates of product and firm turnover and evidence of selection conditional on product and firm survival. In Figure A.5.6, we display the fraction of firm-product observations and import value by tenure (measured in years) for 2011, where recall that firms here correspond to foreign *exporting* firms. Around 50 percent of the firm-product observations in 2011 have been present for two years or less, but the less than 5 percent of these observations that have survived for at least fifteen years account for over 20 percent of import value.

Third, we find that international trade is dominated by multi-product firms. In Figure A.5.7, we display the fraction of firm observations and import value in 2011 accounted for by firms exporting different numbers of products. Although less than 40 percent of exporting firms are multi-product, they account for more than 90 percent of import value.

Fourth, we find that the extensive margins of firm and product exporting account for most of the cross-section variation in aggregate trade. In Figure A.5.8, we display the log of the total value of U.S. imports from each foreign country, the log number of firm-product observations with positive trade for that country, and the log of average imports per firm-product observation with positive trade from that country. We display these three variables against the rank of countries in U.S. total import value, with the largest country assigned a rank of one (China). By construction, total import value falls as we consider countries with higher and higher ranks. Substantively, most of this decline in total imports is accounted for by the extensive margin of the number of firm-product observations with positive trade, whereas the intensive margin of average imports per firm-product observation with positive trade remains relatively flat.

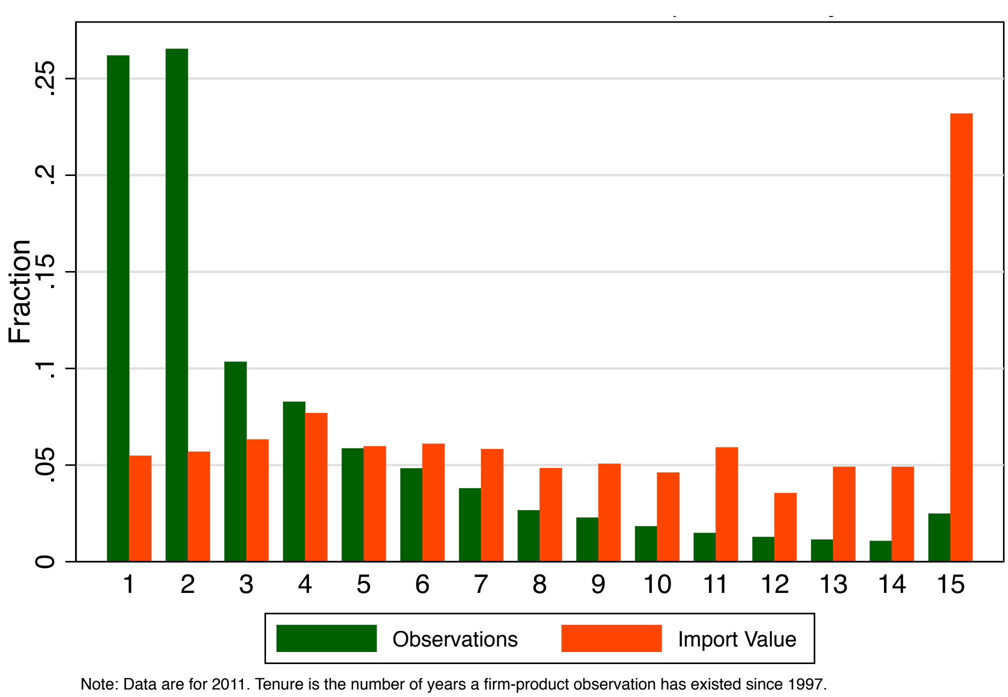
Therefore, across these and a range of other empirical moments, our data are representative of existing empirical findings using international trade transactions data.

Figure A.5.5: Country Shares of U.S. Imports over Time



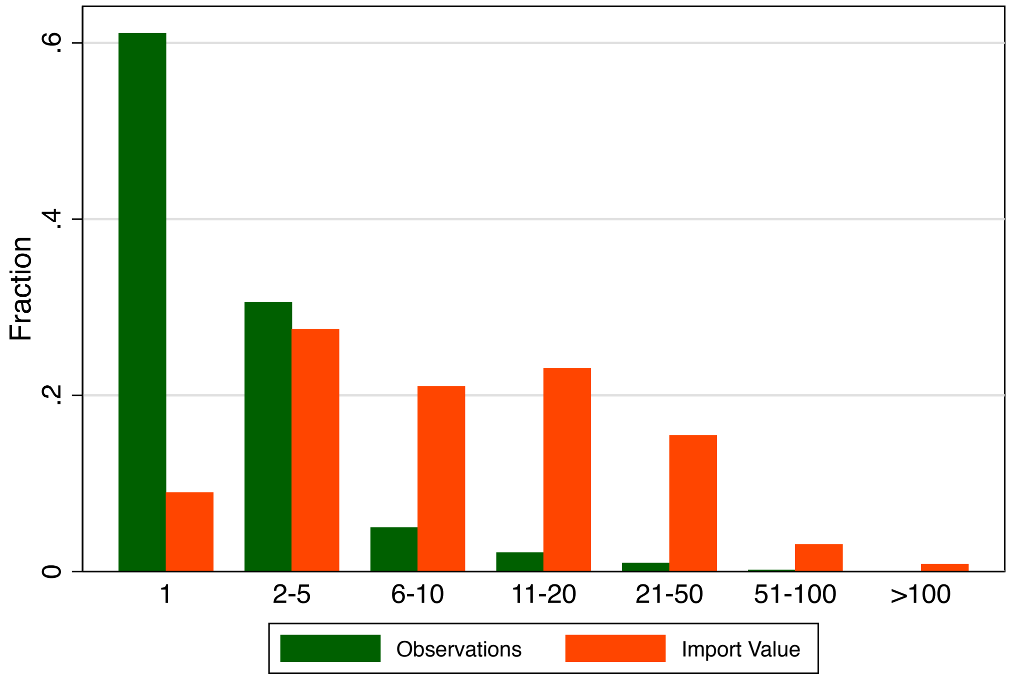
Note: Shares of exporting countries in total U.S. imports over time.

Figure A.5.6: Distribution Firm-Product Observations and Import Value by Tenure 2011 (U.S. data)



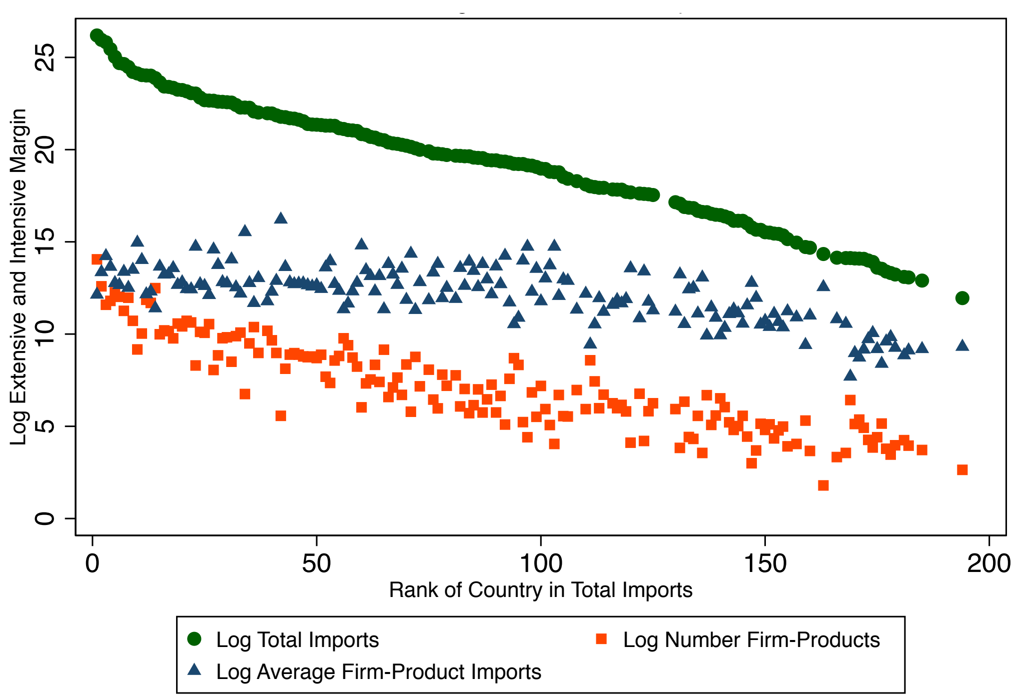
Note: Histogram of the distribution of firm-product observations and import value by the number of years a firm-product observation has existed since 1997.

Figure A.5.7: Distribution of Firm Observations Across Number of Products 2011 (U.S. data)



Note: Histogram of the distribution of firm observations and import value by the number of exported products in 2011.

Figure A.5.8: Extensive and Intensive Margins of Firm-Product Imports Across Countries 2011 (U.S. data)



Note: Log total imports from each exporting country in 2011; log number of imported firm-product observations from each exporting country in 2011; and log average imports per firm-product observation with positive trade for each exporting country in 2011; countries sorted by their rank in total U.S. imports.

A.6 Chilean Empirical Results

In this section of the Online Appendix, we replicate the empirical results from Section 5 of the paper, but using our Chilean data instead of our U.S. data. In Section A.6.1, we report our estimates of the elasticities of substitution (σ_g^U, σ_g^F, σ^G), which we use to invert the model and recover the values of product, firm and sector appeal (φ_{ut}^U, φ_{ft}^F, φ_{jgt}^G). In Section A.6.2, we use these estimates to compute the exporter price indexes that determine the cost of sourcing goods across countries and sectors. In Section A.6.3, we report our main results for the determinants of comparative advantage, aggregate trade and aggregate prices. In Section A.6.4, we compare the results of our framework with special cases that impose additional theoretical restrictions. Finally, in Section A.6.5, we confirm that our Chilean data exhibit the same reduced-form properties as our U.S. data, and as found in other empirical studies using international trade transactions data.

A.6.1 Elasticities of Substitution: RW and HRW Methods Compared

We begin by showing that we find a similar pattern of estimated elasticities of substitution using the Chilean data as using the U.S. data in Section 5.1 of the paper. In Table A.2, we summarize our estimates of the elasticities of substitution (σ_g^U, σ_g^F, σ^G) using the Chilean data. We report quantiles of the distributions of the estimated product and firm elasticities (σ_g^U, σ_g^F) across sectors, as well as the single estimated elasticity of substitution across sectors (σ^G). As for the U.S., we find that the estimated product and firm elasticities are statistically significantly larger than one, and always below eleven. We obtain a median estimated elasticity across products (σ_g^U) of 5.0, a median elasticity across firms (σ_g^F) of 2.7 and an elasticity across sectors (σ^G) of 1.69, which compare closely with our U.S. estimates.

Although we do not impose this restriction on the estimation, we again find a natural ordering, in which varieties are more substitutable within firms than across firms, and firms are more substitutable within industries than across industries: $\hat{\sigma}_g^U > \hat{\sigma}_g^F > \hat{\sigma}^G$. We find that the product elasticity is significantly larger than the firm elasticity at the 5 percent level of significance for 98 percent of sectors, and the firm elasticity is significantly larger than the sector elasticity at this significance level for 88 percent of sectors. Therefore, the Chilean data also rejects the special cases in which consumers only care about firm varieties ($\sigma_g^U = \sigma_g^F = \sigma^G$), in which varieties are perfectly substitutable within sectors ($\sigma_g^U = \sigma_g^F = \infty$), and in which products are equally differentiated within and across firms for a given sector ($\sigma_g^U = \sigma_g^F$). Instead, we find evidence of both firm differentiation within sectors and product differentiation within firms, as for the U.S. in the paper.

Table A.2: Estimated Elasticities of Substitution, Within Firms (σ_g^U), Across Firms (σ_g^F) and Across Sectors (σ^G) using Chilean Data

Percentile	Elasticity Across Products (σ_g^U)	Elasticity Across Firms (σ_g^F)	Elasticity Across Sectors (σ^G)	Product-Firm Difference ($\sigma_g^U - \sigma_g^F$)	Firm-Sector Difference ($\sigma_g^F - \sigma^G$)
Min	4.34	1.80	1.69	1.36	0.11
5th	4.44	2.09	1.69	1.63	0.40
25th	4.63	2.40	1.69	2.06	0.71
50th	5.01	2.68	1.69	2.39	0.99
75th	5.54	3.02	1.69	2.82	1.34
95th	6.88	3.40	1.69	4.33	1.71
Max	8.47	4.14	1.69	4.43	2.45

Note: Estimated elasticities of substitution from the reverse-weighting estimator discussed in Section 3 of the paper and in Section A.3 of this Online Appendix. Sectors are 2-digit Harmonized System (HS) codes; firms correspond to foreign exported brands within each foreign country within each sector; and products u reflect 8-digit HS codes within exported brands within sectors.

Our estimated elasticities of substitution are again broadly consistent with those of other studies that have used similar data but different methodologies and/or nesting structures. Our estimates of the product and firm elasticities (σ_g^F and σ_g^U) are only slightly smaller than those estimated by Hottman et al. (2016) using different data (U.S. barcodes rather than internationally-traded HS products) and a different estimation methodology based on Feenstra (1994).[2] When we apply this alternative methodology to our data as a robustness check, we also obtain similar estimates, with median elasticities of 4.2 at the product level and 1.8 at the firm level, which are close to the 5.0 and 2.7 obtained using the RW method (see Table A.3). Thus, our estimated elasticities typically do not differ substantially from those obtained using other standard methodologies. However, the HRW method generates elasticities with more dispersion in the Chilean data. When we estimate the elasticities using the HRW method, we get one negative elasticity (out of 86). This irregularity does not appear when we implement the RW procedure. Moreover, the differences in the elasticity point estimates do not produce qualitative differences in our decompositions of RCA as we document in Section A.6.3.

[2]Our median estimates for the elasticities of substitution within and across firms of 5.0 and 2.7 respectively compare with those of 6.9 and 3.9 respectively in Hottman et al. (2016).

Table A.3: Estimated Elasticities of Substitution (Hottman et al. (2016)), Within Firms (σ^U) and Across Firms (σ^F) using Chilean Data

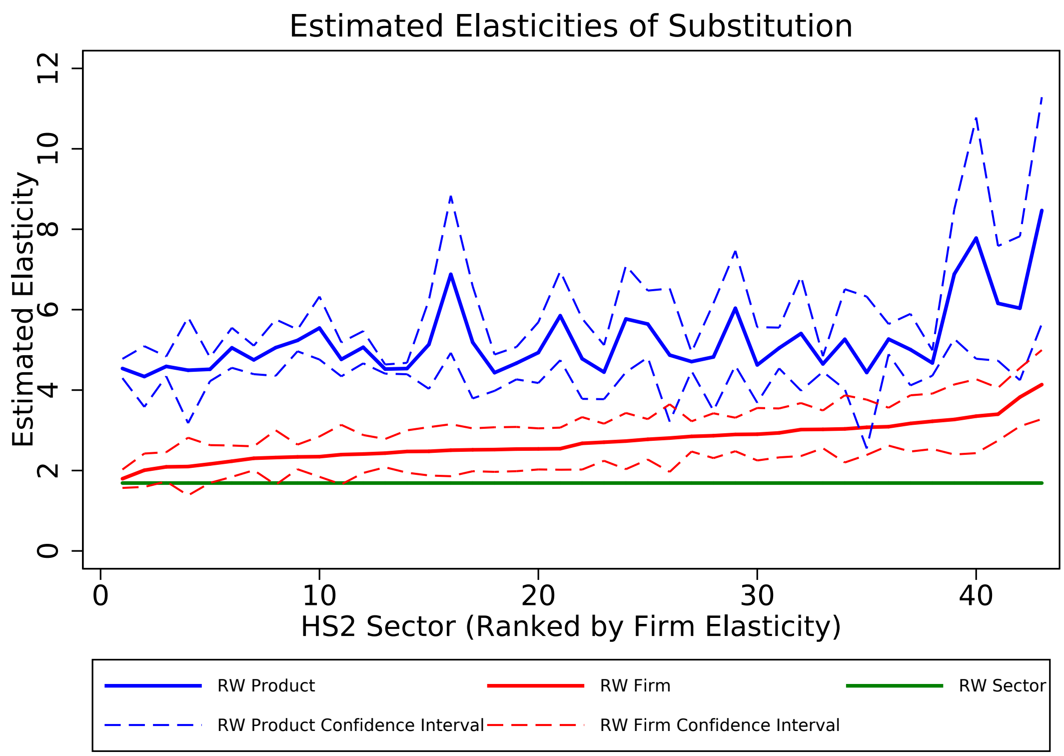
Percentile	Elasticity Across Products (σ^U)	Elasticity Across Firms (σ^F)	Product-Firm Difference ($\sigma^U - \sigma^F$)
Min	1.93	-9.88	0.70
5th	2.34	1.08	1.04
25th	3.67	1.47	1.70
50th	4.16	1.76	2.44
75th	5.19	2.08	3.08
95th	8.87	4.85	5.03
Max	17.22	8.34	23.38

Note: Estimated elasticities of substitution using the Hottman et al. (2016) methodology. Sectors are 2-digit Harmonized System (HS) codes; firms correspond to foreign exported brands within each foreign country within each sector; and products u reflect 8-digit HS codes within exported brands within sectors.

As an additional robustness check, we re-estimated the product, firm and sector elasticities using 4-digit HS categories as our definition of sectors instead of 2-digit HS categories. We find a similar pattern of results, with a somewhat larger median product elasticity of 5.2, a median firm elasticity of 2.6, and a sector elasticity of 1.7. As discussed in Section 5.1 of the paper and reported in further detail in Section A.5.4 of this Online Appendix, we also demonstrate the robustness of our results to undertaking a grid search over the range of plausible values for the elasticity of substitution across firms and products.

In Figure A.5.1, we plot our estimated product, firm and sector elasticities of substitution ($\hat{\sigma}_g^U$, $\hat{\sigma}_g^F$, $\hat{\sigma}^G$), sorted based on the ranking of the estimated firm elasticity of substitution ($\hat{\sigma}_g^F$). We also show 95% confidence intervals for the estimated product and firm elasticities of substitution ($\hat{\sigma}_g^U$, $\hat{\sigma}_g^F$) based on bootstrapped standard errors. As can be seen in the figure, we find a natural ordering where $\hat{\sigma}_g^U > \hat{\sigma}_g^F > \hat{\sigma}^G$. We also find that the confidence intervals are narrow enough such that the product elasticity is significantly larger than the firm elasticity ($\hat{\sigma}_g^U > \hat{\sigma}_g^F$) at the 5 percent level of significance for 98 percent of sectors, and the firm elasticity is significantly larger than the sector elasticity ($\hat{\sigma}_g^F > \hat{\sigma}^G$) at this significance level for 88 percent of sectors.

Figure A.6.1: Estimated Elasticities of Substitution, Within Firms ($\hat{\sigma}_g^U$), Across Firms ($\hat{\sigma}_g^F$) and Across Sectors ($\hat{\sigma}^G$), sorted based on the ranking of $\hat{\sigma}_g^F$ (Chile Data)



Note: Estimated elasticities of substitution across products (σ_g^U shown by the solid blue line) and firms (σ_g^F shown by the solid red line) for each HS2 sector; sectors ranked by the elasticity of substitution across firms (σ_g^F); estimated elasticity of substitution across sectors (σ^G) shown by the horizontal green line; dashed lines denote 95 percent bootstrapped confidence intervals.

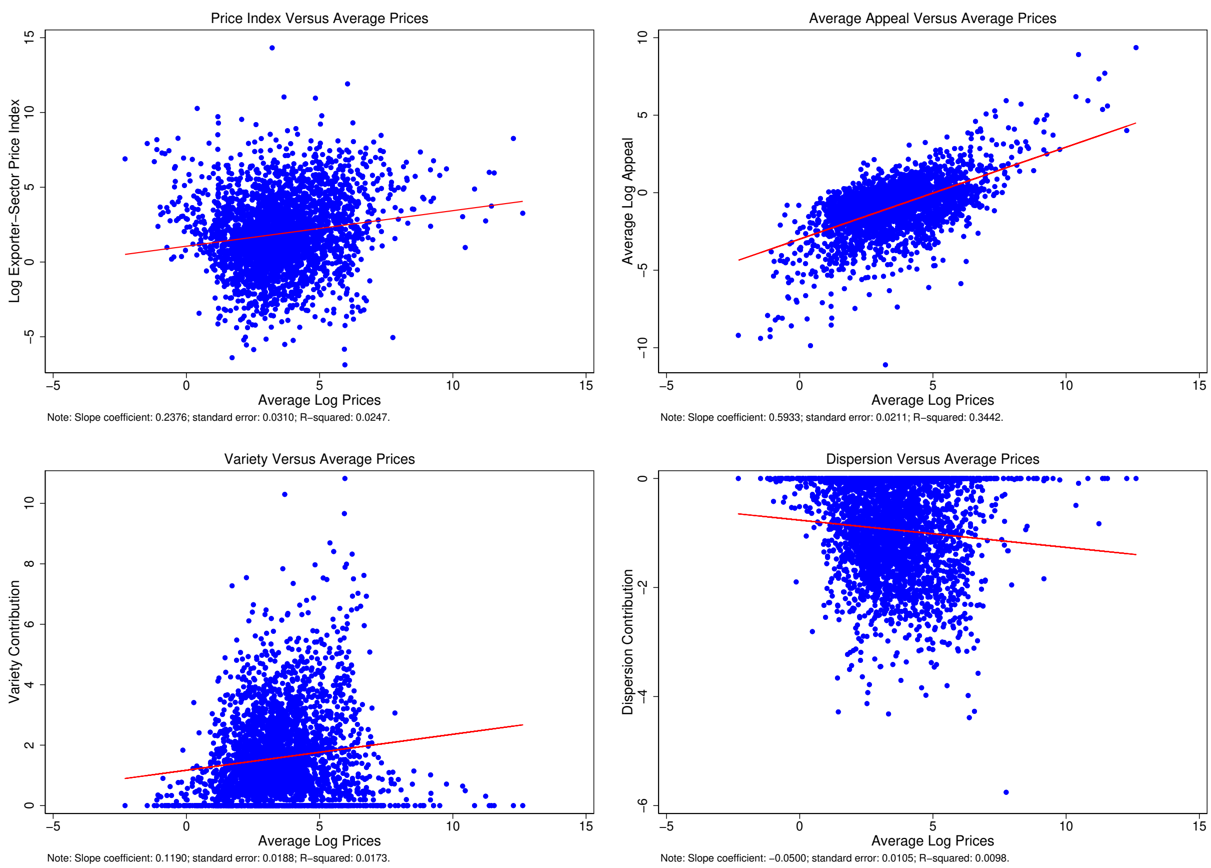
A.6.2 Exporter Price Indexes Across Sectors and Countries

We next show that we find a similar pattern of results for the exporter price indexes across countries and sectors using our Chilean data as using our U.S. data in Section 5.2 of the paper.

In the four panels of Figure A.6.2, we display the log of the exporter price index ($\ln \mathbb{P}_{jigt}^E$) against its components using the Chilean data, where each observation is an exporting country and sector pair. For brevity, we show results for 2014, but find the same pattern for the other years in our sample. Whereas we show bin scatters using the U.S. data in Figure 1 in the paper, we show the observations for each exporting country and sector using our Chilean data in Figure A.6.2. In the top left panel, we compare the log exporter price index ($\ln \mathbb{P}_{jigt}^E$) to average log product prices ($\mathbb{E}_{jigt}^{FU}\left[\ln P_{ut}^U\right]$). In the special case in which firms and products are perfect substitutes within sectors ($\sigma_g^U = \sigma_g^F = \infty$) and there are no differences in appeal ($\varphi_{ft}^F = \varphi_{mt}^F$ for all f, m and $\varphi_{ut}^U = \varphi_{\ell t}^U$ for all u, ℓ), these two variables would be perfectly correlated. In contrast to these predictions, we find a positive but imperfect correlation, with an estimated regression slope of 0.24 and R^2 of essentially zero. In other words, average prices are weakly correlated with the true

CES price index, which underscores the problem of using average prices as a proxy for the CES price index.

Figure A.6.2: Exporter-Sector Price Indexes and their Components Versus Average Log Product Prices, 2014 (Chilean data)



Note: Log exporter-sector price index and its components for Chilean imports in 2014 from equation (21) in the paper; dispersion corresponds to the dispersion of appeal-adjusted prices; Blue dots show exporter-sector observations; Red line shows the linear regression relationship between the variables.

In the remaining panels of Figure A.6.2, we explore the three sources of differences between the exporter price index and average log product prices. As shown in the top-right panel, exporter-sectors with high average prices (horizontal axis) also have high average appeal (vertical axis), so that the impact of higher average prices in raising sourcing costs is partially offset by higher average appeal. This positive relationship between average appeal and prices is strong and statistically significant, with an estimated regression slope of 0.59 and R^2 of 0.34. This finding of a tight connection between higher appeal and higher prices is consistent with the quality interpretation of appeal stressed in Schott (2004), in which producing higher appeal incurs higher production costs.[3]

In the bottom-left panel of Figure A.6.2, we show that the contribution from the number of varieties to

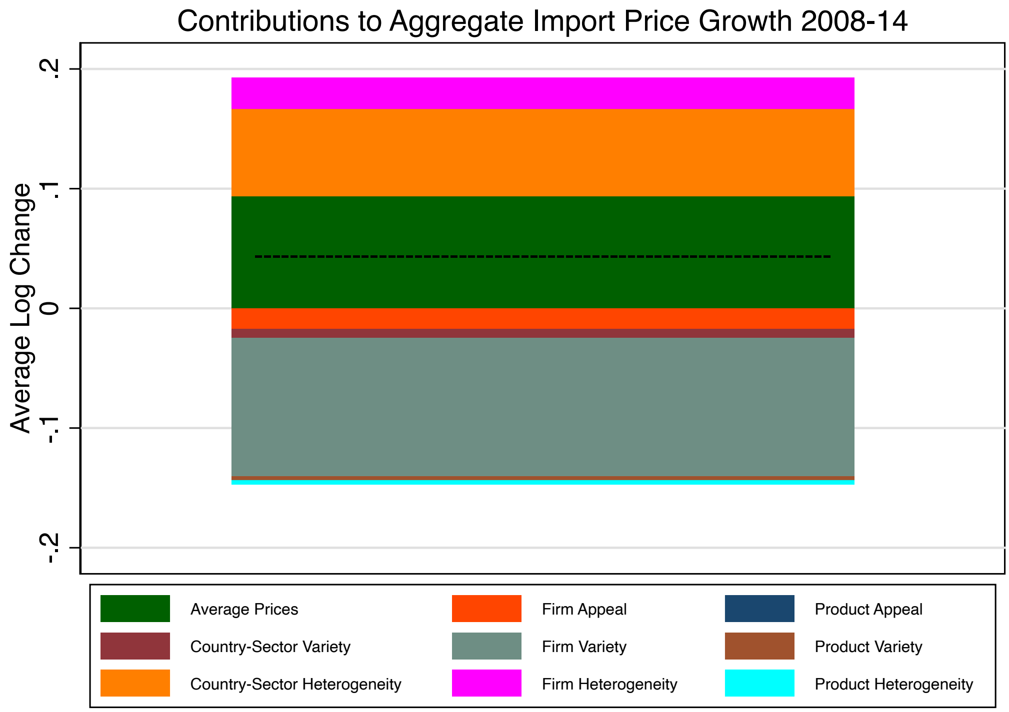
[3]This close relationship between appeal and prices is consistent the findings of a number of studies, including the analysis of U.S. barcode data in Hottman et al. (2016) and the results for Chinese footwear producers in Roberts et al. (2018).

the exporter-sector price index exhibits an inverse U-shape, at first increasing with average prices before later decreasing. This contribution ranges by more than six log points, confirming the empirical relevance of consumer love of variety. In contrast, in the bottom-right panel of Figure 1, we show that the contribution from the dispersion of appeal-adjusted prices displays the opposite pattern of a U-shape, at first decreasing with average prices before later increasing. While the extent of variation is smaller than for the variety contribution, this term still fluctuates by more than four log points between its minimum and maximum value. Therefore, the imperfect substitutability of firms and products implies important contributions from the number of varieties and the dispersion in appeal-adjusted prices across those varieties towards the true cost of sourcing goods across countries and sectors.

These non-conventional determinants are not only important in the cross-section but are also important for changes in the cost of sourcing goods over time. A common empirical question in macroeconomics and international trade is the effect of price shocks in a given sector and country on prices and real economic variables in other countries. However, it is not uncommon to find that measured changes in prices often appear to have relatively small effects on real economic variables, which has stimulated research on "elasticity puzzles" and "exchange rate disconnect." Although duality provides a precise mapping between prices and quantities, the actual price indexes used by researchers often differ in important ways from the formulas for price indexes from theories of consumer behavior. For example, as discussed in the paper, our average price term is the log of the "Jevons Index," which is used by the U.S. Bureau of Labor Statistics (BLS) as part of its calculation of the consumer price index. Except in special cases, however, this average price term will not equal the theoretically-correct measure of the change in the unit expenditure function.

We first demonstrate this point for aggregate import prices. In Figure A.6.3, we use equation (30) in the paper to decompose the log change in aggregate import price indexes ($\mathbb{E}^{T}_{jt}\left[\Delta \ln \mathbb{P}^{G}_{jgt}\right]$) for Chile from 2008-14, where the analogous results for the U.S. are reported in the third column of Table 2 in the paper. This figure provides some important insights into why it is difficult to link import behavior to conventional price measures. If one simply computed the change in the cost of imported goods using a conventional Jevons Index of the prices of those goods (the first term in equation (30) in the paper), one would infer a substantial increase in the cost of imported goods of around 9.2 percent over this time period (prices are measured in current price U.S. dollars). However, this positive contribution from higher prices of imported goods was offset by a substantial negative contribution from firm entry (variety). This expansion in firm variety reduced the cost of imported goods by around 11.7 percent. By contrast, country-sector and firm dispersion fell over this period, which raised the cost of imported goods, and offset some of the variety effects. As a result, the true increase in aggregate import prices from 2008-14 was only 4.4 percent, less than half of the value implied by a conventional Jevons Index. In other words, the true measure of aggregate import prices is strongly affected by factors other than movements in average prices.

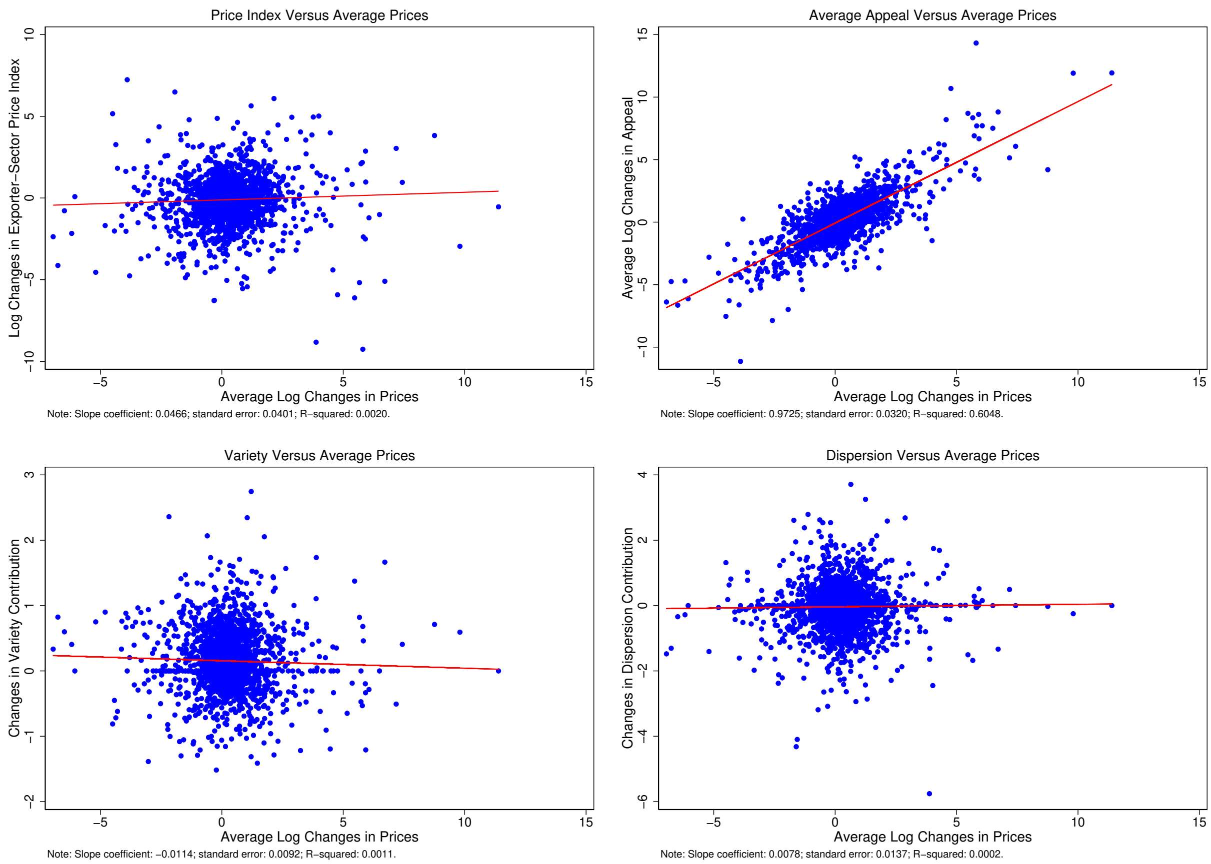
Figure A.6.3: Growth of Aggregate Import Prices 2008-14 (Chilean data)



Note: Decomposition of the growth in the aggregate import price index in Chile from 2008-2014 using equation (30) in the paper; dispersion corresponds to the dispersion of appeal-adjusted prices.

We next show that this point applies not only to aggregate import prices but also to changes in the exporter price indexes $\Delta \ln \mathbb{P}^{E}_{jigt}$ that summarize the cost of sourcing goods across countries and sectors. Figure A.6.4 displays the same information as in Figure A.6.2, but for log changes from 2008-2014 rather than for log levels in 2014 (where the corresponding results using the U.S. data are in Figures 1 and 2 respectively in the paper). Whereas we show bin scatters using the U.S. data in the paper, we again show the observations for each exporting country and sector using our Chilean data in this Online Appendix. In changes, the correlation between average prices and the true model-based measure of the cost of sourcing goods is even weaker and the role for appeal is even greater. Indeed, the slope for the regression of average log changes in appeal on average log changes in prices is almost one, indicating that most price changes are almost completely offset by appeal changes. As in the U.S. data, this result highlights the challenge of rationalizing the observed price and expenditure share data using price indexes such as the Sato-Vartia price index that assume no changes in appeal.

Figure A.6.4: Log Changes in Sector-Exporter Price Indexes and their Components Versus Average Log Changes in Product Prices, 2008-2014 (Chilean data)



Note: Log changes in the exporter-sector price index and its components for Chilean imports from 2008-2014 based on equation (22) in the paper; dispersion corresponds to the dispersion of appeal-adjusted prices; Blue dots show exporter-sector observations; Red line shows the linear regression relationship between the variables.

A.6.3 Trade Patterns

The similarity of our findings for exporter price indexes for Chile and the U.S. suggests that we should also find similar results for patterns of trade, because revealed comparative advantage (RCA) depends on relative price indexes. In this section of the Online Appendix, we confirm that this is indeed the case.

In Table A.4, we present the decompositions of RCA from equation (34) in Section 5.3 of the paper, but using our Chilean data instead of our U.S. data (see Table 3 in the paper for the U.S. results). In Columns (1)-(2), we report results for levels of RCA. In Columns (3) and (4), we present the corresponding results for changes in RCA. While Columns (1) and (3) undertake these decompositions down to the firm level, Columns (2) and (4) undertake them all the way down to the product-level. For brevity, we concentrate on the results of the full decomposition in Columns (2) and (4). We find that average prices make a relatively small contribution to explaining patterns of trade. In the cross-section, average product prices account for

12.6 percent of the variation in RCA. In comparison, in the time-series, we find an even smaller contribution of 9 percent. These results reflect the low correlations between average prices and exporter price indexes seen in the last section. If average prices are weakly correlated with exporter price indexes, they are unlikely to matter much for RCA, because RCA is determined by relative exporter price indexes. By contrast, we find that average appeal is two to three times more important than average prices, with a contribution of 23 percent for the levels of RCA and 36 percent for the changes in RCA. When we repeat this exercise using the Grömping (2007) decomposition in Table A.5, we obtain qualitatively similar results.

Table A.4: Variance Decomposition Chilean RCA

	Log Level RCA 2014		Log Change RCA 2008-14	
	Firm-Level Decomposition	Product-Level Decomposition	Firm-Level Decomposition	Product-Level Decomposition
Firm Price Index	0.126	-	0.091	-
Firm Appeal	0.233	0.233	0.356	0.356
Firm Variety	0.344	0.344	0.464	0.464
Firm Dispersion	0.297	0.297	0.089	0.089
Product Prices	-	0.107	-	0.060
Product Variety	-	0.013	-	0.030
Product Dispersion	-	0.010	-	0.002

Note: Variance decomposition for the log level of RCA in 2014 and the log change in RCA from 2008-14 (from equation (34) in the paper).

Table A.5: Variance Decomposition Chilean RCA (Grömping 2007)

	Log Level RCA 2014		Log Change RCA 2008-14	
	Firm-Level Decomposition	Product-Level Decomposition	Firm-Level Decomposition	Product-Level Decomposition
Firm Price Index	0.142	-	0.170	-
Firm Appeal	0.191	0.170	0.328	0.306
Firm Variety	0.383	0.371	0.398	0.403
Firm Dispersion	0.284	0.282	0.104	0.105
Product Prices	-	0.120	-	0.149
Product Variety	-	0.029	-	0.034
Product Dispersion	-	0.029	-	0.003

Note: Relative importance decomposition for the log level of RCA in 2011 and the log change in RCA from 1998-2011 (from Grömping 2007).

By far the most important of the different mechanisms for trade in Table A.4 is firm variety, which accounts for 34 and 46 percent of the level and change of RCA respectively. These findings for firm variety are consistent with research that emphasizes the role of the extensive margin in understanding patterns of trade, including Hummels and Klenow (2005), Chaney (2008) and Kehoe and Ruhl (2013). But we also find a notable contribution from the dispersion of appeal-adjusted prices across firms, which accounts for 30 percent of the variation in RCA in the cross-section and 9 percent of this variation over time. These results are consistent with a substantial role for producer heterogeneity, as emphasized in the large literature on heterogeneous firms following Melitz (2003), as reviewed in Bernard, Jensen, Redding and Schott (2007) and Melitz and Redding (2014).

As with the U.S. data, we find that switching to the choice of elasticity estimation and method of variance decomposition when using Chilean data has little qualitative impact on the results. For example, the importance of prices, appeal, variety, and dispersion in the decomposition for RCA levels are 0.13, 0.23, 0.34, and 0.30, respectively, when using our methodology to estimate the elasticities (see the first column of Table A.4), and 0.06, 0.30, 0.34, and 0.30 when using the HRW methodology (see the first column of Table A.6). When we use both the HRW estimated elasticities and the Grömping (2007) variance decomposition, we find contributions of 0.13, 0.20, 0.38, and 0.29, respectively (see the first column of Table A.7). In our decomposition for RCA changes, we again find similar contributions of prices, appeal, and variety, whether we use the RW or the HRW estimated elasticities, and regardless of which of the two methods of variance decomposition we use (see the final two columns of Tables A.4, A.6 and A.7).

Table A.6: Variance Decomposition Chilean RCA, HRW Sigmas

	Log Level RCA 2014		Log Change RCA 2008-14	
	Firm-Level Decomposition	Product-Level Decomposition	Firm-Level Decomposition	Product-Level Decomposition
Firm Price Index	0.064	-	0.053	-
Firm Appeal	0.295	0.295	0.391	0.391
Firm Variety	0.344	0.344	0.467	0.467
Firm Dispersion	0.298	0.298	0.089	0.089
Product Prices	-	0.051	-	0.032
Product Variety	-	0.009	-	0.019
Product Dispersion	-	0.007	-	0.002

Note: Variance decomposition for the log level of RCA in 2014 and the log change in RCA from 2008-14 (from equation (34) in the paper), using the HRW elasticities.

Table A.7: Variance Decomposition Chilean RCA, HRW Sigmas (Grömping 2007)

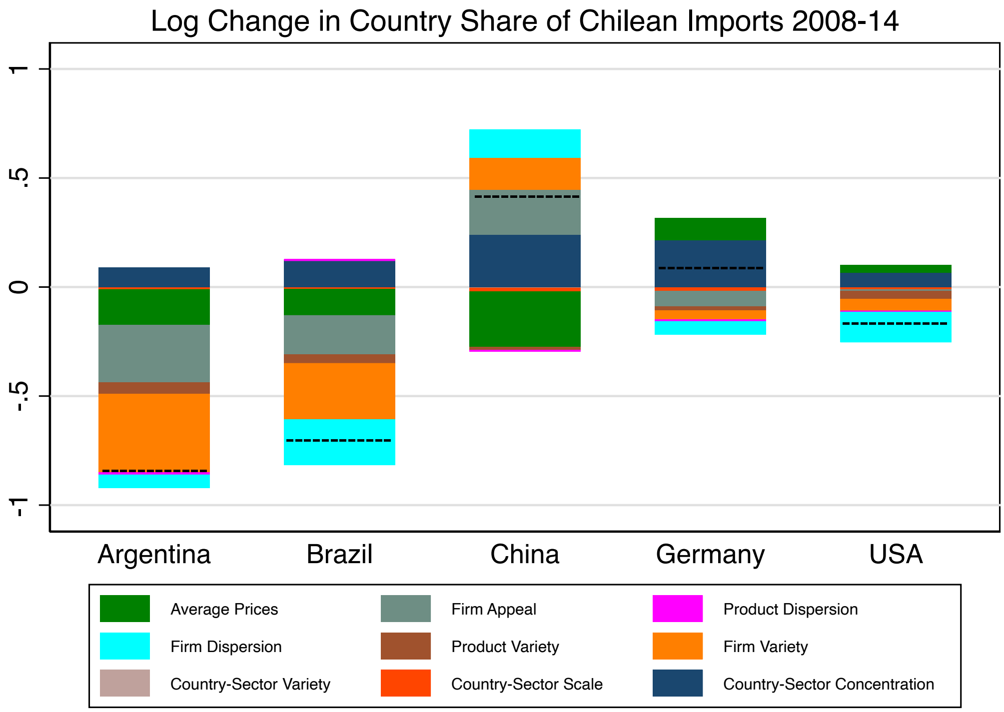
	Log Level RCA 2014		Log Change RCA 2008-14	
	Firm-Level Decomposition	Product-Level Decomposition	Firm-Level Decomposition	Product-Level Decomposition
Firm Price Index	0.132	-	0.201	-
Firm Appeal	0.199	0.175	0.307	0.288
Firm Variety	0.383	0.371	0.390	0.395
Firm Dispersion	0.286	0.283	0.102	0.103
Product Prices	-	0.117	-	0.190
Product Variety	-	0.031	-	0.021
Product Dispersion	-	0.022	-	0.002

Note: Relative importance decomposition for the log level of RCA in 2011 and the log change in RCA from 1998-2011 (from Grömping 2007), using the HRW elasticities.

We now show that the non-conventional forces of variety, average appeal and the dispersion of appeal-adjusted prices are also important for understanding movements in aggregate Chilean imports from its largest trade partners, consistent with our U.S. results in Section 5.3 of the paper. In Figure A.6.5, we show the time-series decompositions of aggregate import shares from equation (28) in the paper for Chile's top-six trade partners. As apparent from the figure, we can account for the substantial increase in China's market share over the sample period by focusing mostly on increases in firm variety (orange), average firm appeal (gray), and the dispersion of appeal-adjusted prices across firms (light blue).[4] In contrast, average product prices (green) increased more rapidly for China than for the other countries in our sample, which worked in the opposite direction to reduce China's market share. In other words, our decomposition indicates that the reason for the explosive growth of Chinese exports was not due to cheaper Chinese exports, but rather substantial firm entry (variety), appeal upgrading, and improvements in the performance of leading firms relative to lagging firms (the dispersion of appeal-adjusted prices). By contrast the dramatic falls in import shares from Argentina and Brazil were driven by a confluence of factors that all pushed in the same direction: higher average product prices, firm exit (variety), a deterioration in the performance of leading firms relative to lagging firms (the dispersion of appeal-adjusted prices), and falls in average appeal relative to other countries.

[4]Our finding of an important role for firm entry for China is consistent with the results for export prices in Amiti, Dai, Feenstra, and Romalis (2020). However, their price index is based on the Sato-Vartia formula, which abstracts from changes in appeal for surviving varieties, and they focus on Chinese export prices rather than trade patterns.

Figure A.6.5: Country Aggregate Shares of Chilean Imports



Note: Decomposition of exporting countries shares of total Chilean imports from equation (28) in the paper; dispersion corresponds to the dispersion of appeal-adjusted prices.

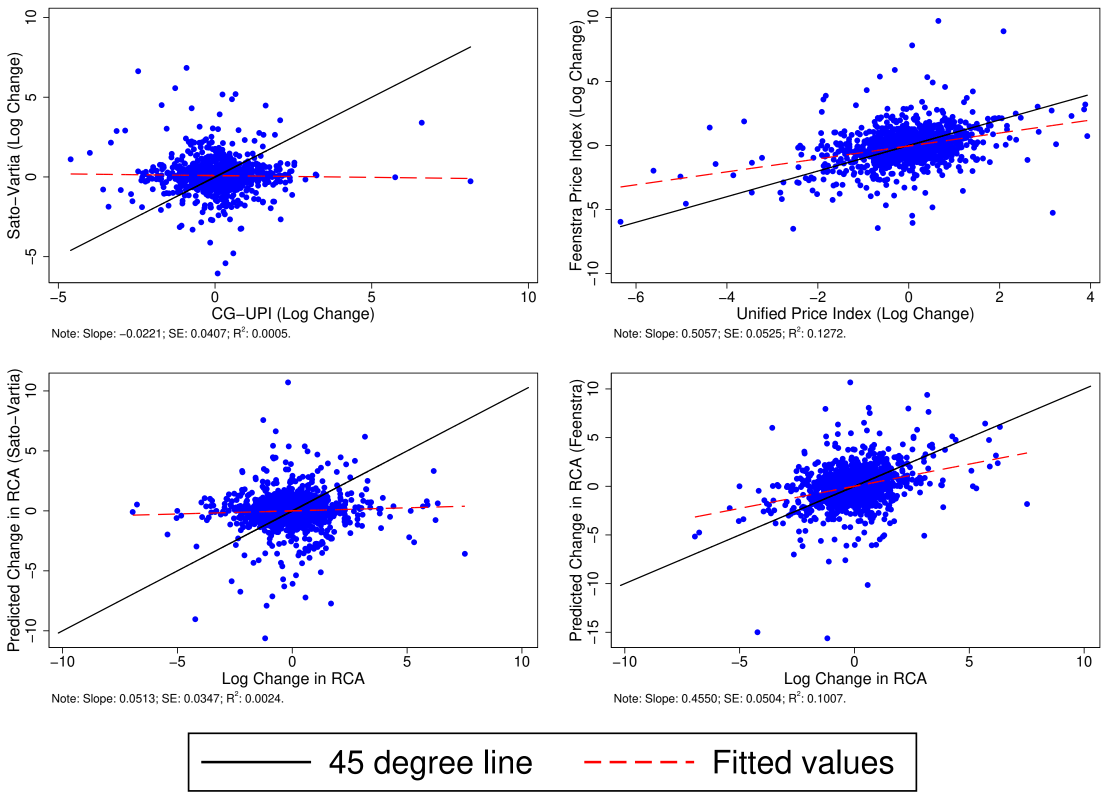
A.6.4 Additional Theoretical Restrictions

We now compare our approach, which exactly rationalizes both micro and macro trade data, with special cases of this approach that impose additional theoretical restrictions. We show that we find a similar pattern of results using the Chilean data as using the U.S. data in Section 5.4 of the paper. As a result of imposing additional theoretical restrictions, these special cases no longer exactly rationalize the micro trade data, and we quantify the implications of these departures from the micro data for macro trade patterns and prices.

No Changes in Appeal Almost all existing theoretical research with CES demand in international trade is encompassed by the Sato-Vartia price index, which assumes no shifts in appeal for common varieties. Duality suggests that there are two ways to assess the importance of this assumption. First, we can work with a price index and examine how a CES price index that allows for appeal shifts (i.e., the UPI in equation (20) in the paper) differs from a CES price index that does not allow for appeal shifts (i.e., the Sato-Vartia index). Since the common goods component of the UPI (CG-UPI) and the Sato-Vartia indexes are identical in the absence of appeal shifts, the difference between the two is a metric for how important appeal shifts are empirically. Second, we can substitute each of these price indexes into our expression for revealed comparative advantage (RCA) in equation (25) in the paper, and examine how important the assumption of no appeal shifts is for understanding patterns of trade. Because we know that the UPI perfectly rationalizes the data, any deviation from the data arising by using a different price index must reflect the effect of the restrictive assumptions used in the index's derivation. In order to make the comparison fair, we need to also adjust the Sato-Vartia index for variety changes, which we do by using the Feenstra (1994) index, which is based on the same no-appeal-shifts assumption for common goods, but adds the variety correction term given in equation (20) in the paper to incorporate entry and exit.

In Figure A.6.6, we report the results of these comparisons using our Chilean data, which corresponds to Figure 4 in the paper using our U.S. data. The top two panels consider exporter price indexes, while the bottom two panels examine RCA. In the top-left panel, we compare the Sato-Vartia exporter price index (on the vertical axis) with our common goods exporter price index (the CG-UPI on the horizontal axis), where each observation is an exporter-sector pair. If the assumption of time-invariant appeal were satisfied in the data, these two indexes would be perfectly correlated with one another and aligned on the 45-degree line. Again, we find little relationship between them. The reason is immediately apparent if one recalls the top-right panel of Figure A.6.4, which shows that price shifts are strongly positively correlated with appeal shifts. The Sato-Vartia price index fails to take into account that higher prices are typically offset by higher appeal. In the top-right panel, we compare the Feenstra exporter price index (on the vertical axis) with our overall exporter price index (the UPI on the horizontal axis), where each observation is again an exporter-sector pair. These two price indexes have exactly the same variety correction term, but use different common goods price indexes (the CG-UPI and Sato-Vartia indexes respectively). The importance of the variety correction term as a share of the overall exporter price index accounts for the improvement in the fit of the relationship. However, the slope of the regression line is only around 0.5, and the regression R^2 is about 0.1. Therefore, the assumption of no shifts in appeal for existing goods results in substantial deviations between the true and measured costs of sourcing goods from an exporter and sector.

Figure A.6.6: Sector-exporter Price Indexes with Time-Invariant Appeal (Vertical Axis) Versus Time-Varying Appeal (Horizontal Axis) for Chile



Note: Log changes in sector-exporter price indexes allowing for changes in appeal (common goods unified price index (CG-UPI)) and assuming no changes in appeal (Sato-Vartia price index); Feenstra (1994) price index adjusts the Sato-Vartia price index for changes in variety; unified price index (UPI) adjusts the common goods unified price index (CG-UPI) for changes in variety.

In the bottom left panel, we compare predicted changes in RCA based on relative exporter Sato-Vartia price indexes (on the vertical axis) with actual changes in RCA (on the horizontal axis). As the Sato-Vartia price index has only a weak correlation with the UPI, we find that it has little predictive power for changes in RCA, which are equal to relative changes in the UPI across exporters and sectors. Hence, observed changes in trade patterns are almost uncorrelated with the changes predicted under the assumption of no shifts in appeal and no entry and exit of firms and products. In the bottom right panel, we compare predicted changes in RCA based on relative exporter Feenstra price indexes (on the vertical axis) with actual changes in RCA (on the horizontal axis). The improvement in the fit of the relationship attests to the importance of adjusting for entry and exit. However, again the slope of the regression line is only around 0.5 and the regression R^2 is about 0.1. Therefore, even after adjusting for the shared entry and exit term, the assumption of no appeal shifts for existing goods can generate predictions for changes in trade patterns that diverge substantially from those observed in the data.

Additional Functional Form Restrictions We now examine the implications of imposing additional theoretical restrictions on these cross-sectional distributions. In particular, an important class of existing trade theories assumes not only a constant demand-side elasticity but also a constant supply-side elasticity, as reflected in the assumption of Fréchet or Pareto productivity distributions. As our approach uses only demand-side assumptions, we can examine the extent to which these additional supply-side restrictions are satisfied in the data. In particular, we compare the observed data for firm sales and our model solutions for the firm price index and firm appeal ($\ln V_{ft}^F \in \left\{\ln \mathbb{X}_{ft}^F, \ln P_{ft}^F, \ln \varphi_{ft}^F\right\}$) with their theoretical predictions under alternative supply-side distributional assumptions.

To derive these theoretical predictions, we use the QQ estimator. The QQ estimator compares the empirical quantiles in the data with the theoretical quantiles implied by alternative distributional assumptions. As shown in Section A.5.5 of this Online Appendix, under the assumption that a firm variable V_{ft}^F has a Pareto distribution, we obtain the following theoretical prediction for the quantile of the logarithm of that variable:

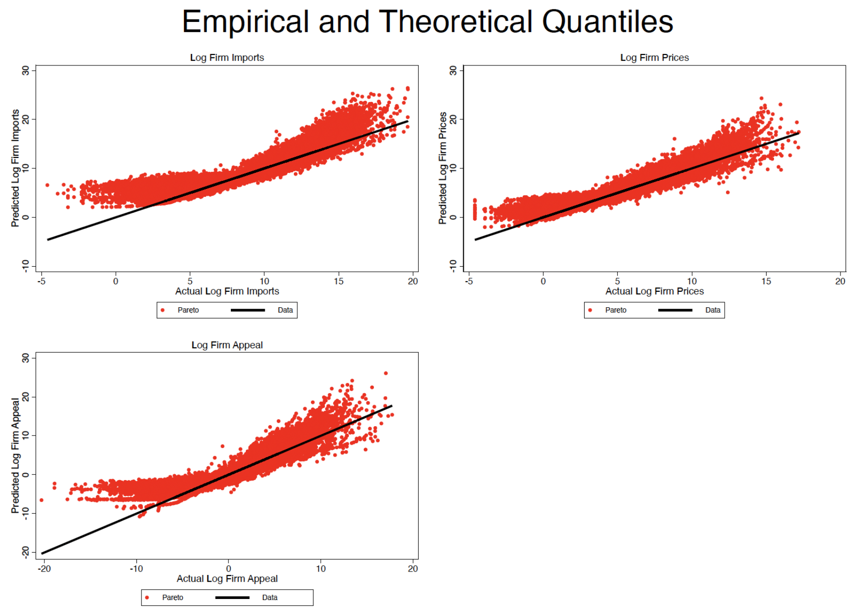
$$\ln\left(V_{ft}^F\right) = \ln \underline{V}_{jigt}^F - \frac{1}{a_g^V} \ln\left[1 - \mathcal{F}_{jigt}\left(V_{ft}^F\right)\right]. \tag{A.6.1}$$

where $\mathcal{F}_{jigt}(\cdot)$ is the cumulative distribution function; $\ln \underline{V}_{jigt}^F$ is the lower limit of the support of the Pareto distribution, which is a constant across firms f for a given importer j, exporter i, sector g and year t; a_g^V is the shape parameter of this distribution, which we allow to vary across sectors g.

We estimate equation (A.6.1) by OLS using the empirical quantile for $\ln\left(V_{ft}^F\right)$ on the left-hand side and the empirical estimate of the cumulative distribution function for $\mathcal{F}_{jigt}\left(V_{ft}^F\right)$ on the right-hand side. We estimate this regression for each sector across foreign firms (allowing the slope coefficient a_g^V to vary across sectors) and including fixed effects for each exporter-sector-year combination (allowing the intercept $\ln \underline{V}_{jigt}^F$ to vary across exporters, sectors and time). The fitted values from this regression correspond to the predicted theoretical quantiles, which we compare to the empirical quantiles observed in the data. Under the null hypothesis of a Pareto distribution, there should be a linear relationship between the theoretical and empirical quantiles that coincides with the 45-degree line.

In Figure A.6.7, we show the predicted theoretical quantiles (vertical axis) against the empirical quantiles (horizontal axis) using our Chilean data. We display results for log firm imports (top left), log firm price indexes (top right) and log firm appeal (bottom left). In each case, we observe sharp departures from the linear relationship implied by a Pareto distribution, with the actual values below the predicted values in both the lower and upper tails. Following the same approach as in Section 5.4 of the paper, we estimate the regression in equation (A.6.1) separately for observations below and above the median, and compare the estimated coefficients. Consistent with the U.S. results in Figure A.5.4 of this Online Appendix, we find substantial departures from linearity using the Chilean data, which are statistically significant at conventional levels.

Figure A.6.7: Theoretical and Empirical Quantiles for Chile (Pareto Distribution)



Note: Relationships between theoretical quantiles predicted by a Pareto distribution and empirical quantiles for log firm imports, the log firm price index and log firm appeal.

As a point of comparison, we also examine the alternative distributional assumption of a log normal distribution. As shown in Section A.5.5 of this Online Appendix, under this distributional assumption, we obtain the following theoretical prediction for the quantile of the logarithm of a variable V^F_{ft}:

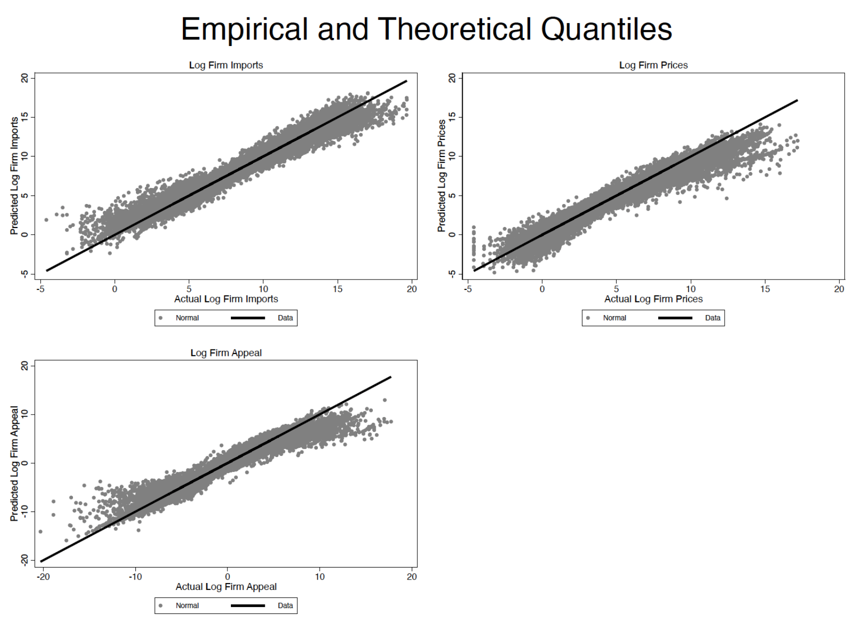
$$\ln\left(V^F_{ft}\right) = \kappa^V_{jigt} + \chi^V_g \Phi^{-1}\left(\mathcal{F}_{jigt}\left(V^F_{ft}\right)\right). \tag{A.6.2}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the normal cumulative distribution function; κ^V_{jigt} and χ^V_g are the mean and standard deviation of the log variable, such that $\ln\left(V^F_{ft}\right) \sim \mathcal{N}\left(\kappa^V_{jigt}, \left(\chi^V_g\right)^2\right)$; we make analogous assumptions about these parameters as for the Pareto distribution above; we allow the parameter controlling the mean (κ^V_{jigt}) to vary across exporters i, sectors g and time t for a given importer j; we allow the parameter controlling dispersion (χ^V_g) to vary across sectors g.

Again we estimate equation (A.6.2) by OLS using the empirical quantile for $\ln\left(V^F_{ft}\right)$ on the left-hand side and the empirical estimate of the cumulative distribution function for $\mathcal{F}_{jigt}\left(V^F_{ft}\right)$ on the right-hand side. We estimate this regression for each sector across foreign firms (allowing the slope coefficient χ^V_g to vary across sectors) and including fixed effects for each exporter-sector-year combination (allowing the intercept κ^V_{jigt} to vary across exporters, sectors and time).

In Figure A.6.8, we show the predicted log normal theoretical quantiles (vertical axis) against the empirical quantiles (horizontal axis) using our Chilean data. Again we display results for log firm imports (top left), log firm price indexes (top right) and log firm appeal (bottom left). In each case, we find that the relationship between the theoretical and empirical quantiles is closer to linearity for a log-normal distribution than for a Pareto distribution, which is consistent with Bas, Mayer and Thoenig (2017) and Fernandes et al. (2021). Nonetheless, we observe substantial departures from the theoretical predictions of a log-normal distribution, and we reject the null hypothesis of normality at conventional levels of significance for the majority of sectors using a Shapiro-Wilk test. Following the same approach as in Section 5.4 of the paper, we also estimate the regression in equation (A.6.2) separately for observations below and above the median, and compare the estimated coefficients. Consistent with the U.S. results in Figure A.5.4 of this Online Appendix, we again find substantial departures from linearity using the Chilean data, which are statistically significant for the majority of sectors at conventional levels.

Figure A.6.8: Theoretical and Empirical Quantiles for Chile (Log Normal Distribution)



Note: Relationships between theoretical quantiles predicted by a log normal distribution and empirical quantiles for log firm imports, the log firm price index and log firm appeal.

A.6.5 Additional Reduced-Form Evidence

In Figures A.6.9-A.6.12, we confirm that our Chilean trade transaction data have the same reduced-form properties as our U.S. data and as found in other empirical studies using international trade transactions data (see for example Bernard, Jensen and Schott 2009 and Bernard, Jensen, Redding and Schott 2009 for the U.S.; Mayer, Melitz and Ottaviano 2014 for France; and Manova and Zhang 2012 for China).

First, Chilean imports are highly concentrated across countries and characterized by a growing role of China over time. As shown in Figure A.6.9, Chile's six largest import sources in 2007 were (in order of size) China, the U.S., Brazil, Germany, Mexico, and Argentina, which together accounted for more than 60 percent of its imports. Between 2007 and 2014, China's import share grew by over 50 percent, with all other major suppliers except Germany experiencing substantial declines in their market shares.

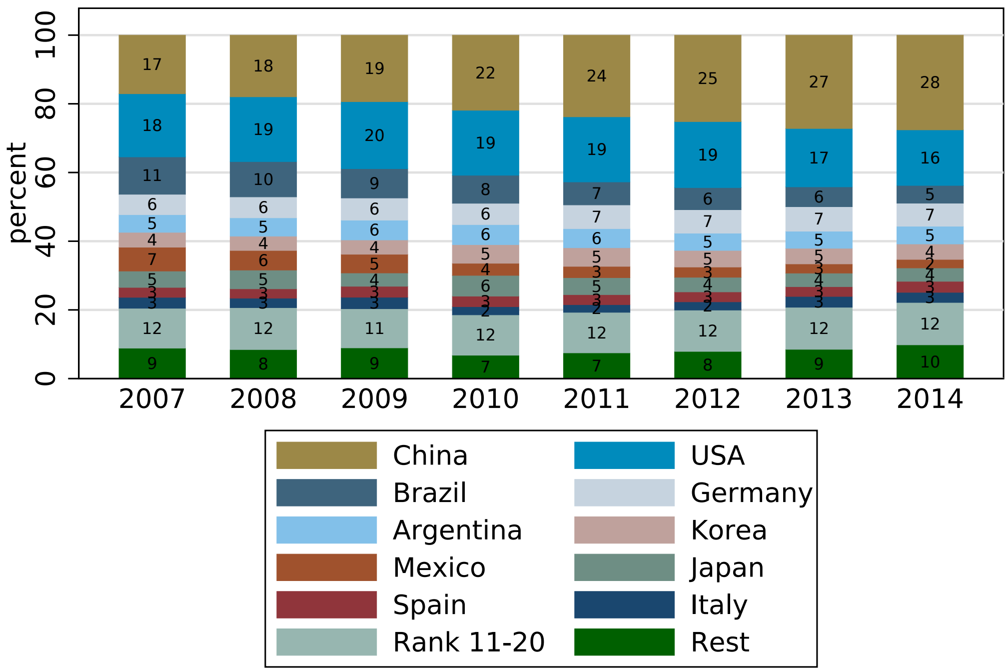
Second, we find high rates of product and firm turnover and evidence of selection conditional on product and firm survival. In Figure A.6.10, we display the fraction of firm-product observations and import value by tenure (measured in years) for 2014, where recall that firms here correspond to foreign *exporting* firms. Around 50 percent of the firm-product observations in 2014 have been present for one year or less, but the just over 10 percent of these observations that have survived for at least seven years account for over 40 percent of import value.

Third, we find that international trade is dominated by multi-product firms. In Figure A.6.11, we display the fraction of firm observations and import value in 2014 accounted for by firms exporting different numbers of products. Although less than 30 percent of exporting firms are multi-product, they account for more than 70 percent of import value.

Fourth, we find that the extensive margins of firm and product exporting account for most of the cross-section variation in aggregate trade. In Figure A.6.12, we display the log of the total value of Chilean imports from each foreign country, the log number of firm-product observations with positive trade for that country, and the log of average imports per firm-product observation with positive trade from that country. We display these three variables against the rank of countries in Chile's total import value, with the largest country assigned a rank of one (China). By construction, total import value falls as we consider countries with higher and higher ranks. Substantively, most of this decline in total imports is accounted for by the extensive margin of the number of firm-product observations with positive trade, whereas the intensive margin of average imports per firm-product observation with positive trade remains relatively flat.

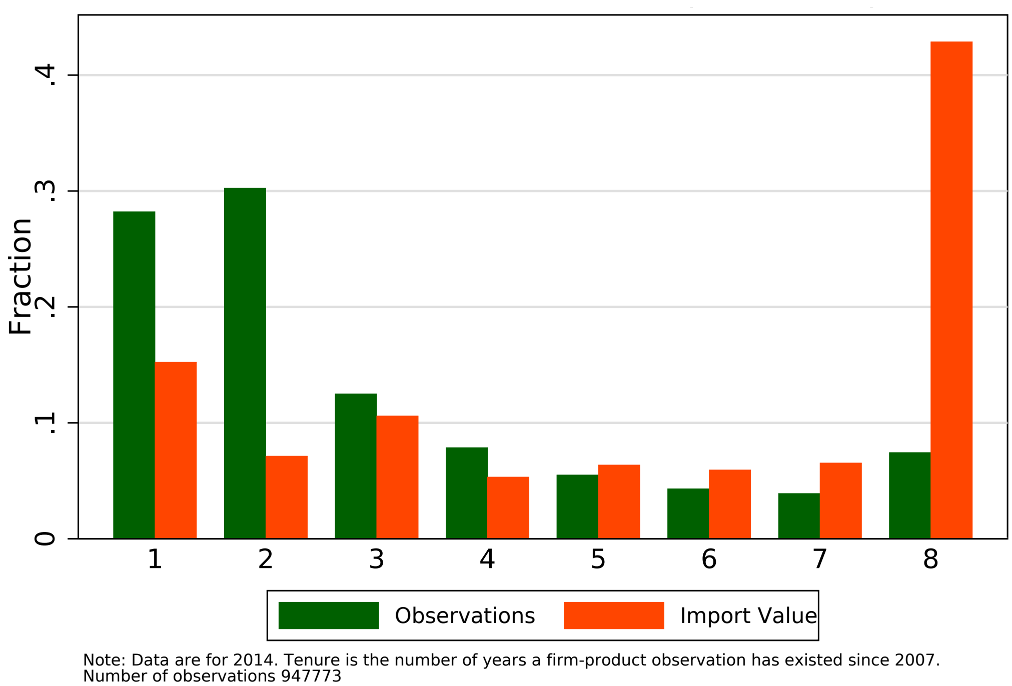
Therefore, across these and a range of other empirical moments, the Chilean data are representative of empirical findings using international trade transactions data for a number of other countries.

Figure A.6.9: Country Shares of Chilean Imports over Time



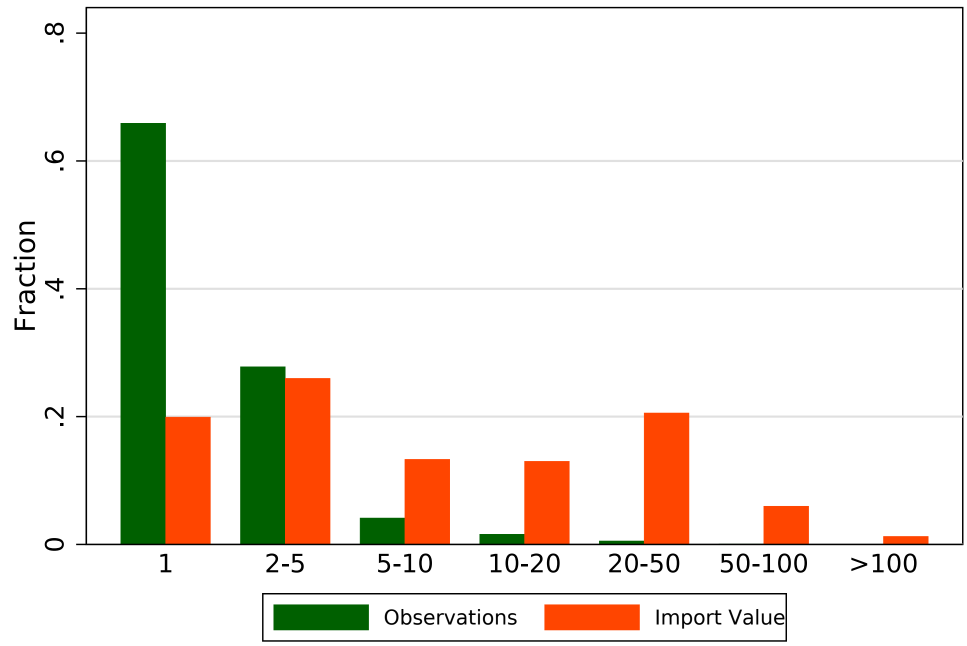
Note: Shares of exporting countries in total Chilean imports over time.

Figure A.6.10: Distribution Firm-Product Observations and Import Value by Tenure 2014 (Chilean Data)



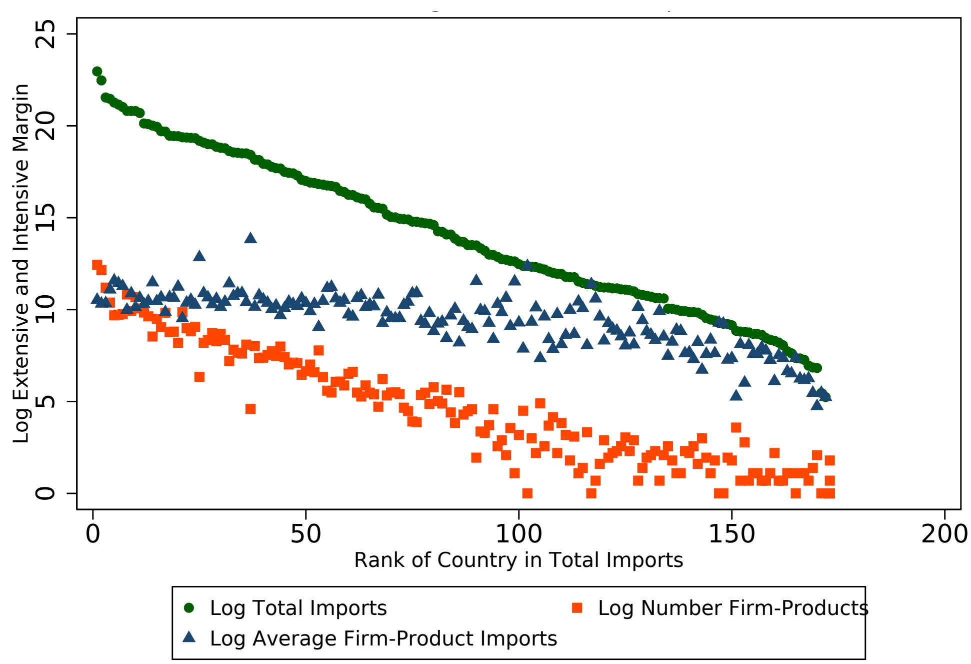
Note: Histogram of the distribution of firm-product observations and import value by the number of years a firm-product observation has existed since 2007.

Figure A.6.11: Distribution of Firm Observations Across Number of Products 2014 (Chilean Data)



Note: Histogram of the distribution of firm observations and import value by the number of exported products in 2014.

Figure A.6.12: Extensive and Intensive Margins of Firm-Product Imports Across Countries 2014 (Chilean Data)



Note: Log total imports from each exporting country in 2014; log number of imported firm-product observations from each exporting country in 2014; and log average imports per firm-product observation with positive trade for each exporting country in 2014; countries sorted by their rank in total Chilean imports.

A.7 Unobserved Differences in Product Composition

In this section of the Online Appendix, we show that our approach allows for unobserved differences in composition within observed product categories, which enter the model in the same way as unobserved differences in appeal for each observed product category. In the paper, we assume for simplicity that the products supplied by firms are the same as those observed in the data, which enables us to abstract from these unobserved differences in product composition. We now generalize our results to the case in which firms supply products at a more disaggregated level (e.g. unobserved barcodes) than the categories observed in the data (Harmonized System (HS) categories).

A.7.1 True Data Generating Process

We suppose that the true data generating process is as follows. At the aggregate level, we have sectors (g); below sectors we have firms (f); below firms we have products (u); and below products we have barcodes (b). Aggregate utility and the consumption index for each sector remain unchanged. The consumption index for each firm (C_{ft}) is defined over an unobserved consumption index for each product (C^U_{ut}):

$$C^F_{ft} = \left[\sum_{u \in \Omega^U_{ft}} \left(\varphi^U_{ut} C^U_{ut} \right)^{\frac{\sigma^U_g - 1}{\sigma^U_g}} \right]^{\frac{\sigma^U_g}{\sigma^U_g - 1}}, \qquad \sigma^U_g > 1, \varphi^U_{ut} > 0, \tag{A.7.1}$$

where σ^U_g is the elasticity of substitution across products within the firm; φ^U_{ut} is the appeal for each product; and Ω^U_{ft} is the set of products supplied by firm f at time t. Each product consumption index (C^U_{ut}) is defined over the unobserved consumption of each barcode (C^B_{bt}):

$$C^U_{ut} = \left[\sum_{b \in \Omega^B_{ut}} \left(\varphi^B_{bt} C^B_{bt} \right)^{\frac{\sigma^B_g - 1}{\sigma^B_g}} \right]^{\frac{\sigma^B_g}{\sigma^B_g - 1}}, \qquad \sigma^B_g > 1, \varphi^B_{bt} > 0. \tag{A.7.2}$$

Similarly, the dual price index for each firm (P^F_{ft}) is defined over an unobserved dual price index for each product (P^U_{ut}):

$$P^F_{ft} = \left[\sum_{u \in \Omega^U_{ft}} \left(\frac{P^U_{ut}}{\varphi^U_{ut}} \right)^{1-\sigma^U_g} \right]^{\frac{1}{1-\sigma^U_g}}, \tag{A.7.3}$$

and this unobserved dual price index for each product (P^U_{ut}) is defined over the unobserved price of each barcode (P^B_{bt}):

$$P^U_{ut} = \left[\sum_{b \in \Omega^B_{ut}} \left(\frac{P^B_{bt}}{\varphi^B_{bt}} \right)^{1-\sigma^B_g} \right]^{\frac{1}{1-\sigma^B_g}}. \tag{A.7.4}$$

A.7.2 Observed Data

Suppose that in the data we observe the total value of sales of each product (E^U_{ut}), which corresponds to the sum of the sales of all the unobserved barcodes ($E^U_{ut} = \sum_{b \in \Omega^B_{ut}} E^B_{bt}$):

$$E^U_{ut} = P^U_{ut} C^U_{ut} = \sum_{b \in \Omega^B_{ut}} E^B_{bt} = \sum_{b \in \Omega^B_{ut}} P^B_{bt} C^B_{bt}. \tag{A.7.5}$$

We also observe the total physical quantity of each product ($\mathcal{Q}^U_{ut}$), which corresponds to the sum of the physical quantities of all barcodes ($\mathcal{Q}^U_{ut} = \sum_{b \in \Omega^B_{ut}} C^B_{bt}$). Dividing sales by quantities for each product, we can compute a unit value for each product ($\mathcal{P}^U_{ut} = E^U_{ut} / \mathcal{Q}^U_{ut}$). Note that observed expenditure on each product equals both (i) observed physical quantities times observed unit values and (ii) unobserved consumption indexes times unobserved price indexes:

$$P^U_{ut} C^U_{ut} = \mathcal{P}^U_{ut} \mathcal{Q}^U_{ut} = E^U_{ut}, \tag{A.7.6}$$

which implies that the ratio of observed unit values to unobserved price indexes is the inverse of the ratio of observed physical quantities to unobserved consumption indexes:

$$\frac{\mathcal{P}^U_{ut}}{P^U_{ut}} = \frac{1}{\mathcal{Q}^U_{ut} / C^U_{ut}}. \tag{A.7.7}$$

A.7.3 Relationship Between Observed and Unobserved Variables

We now use these relationships to connect the observed physical quantities and unit values ($\mathcal{Q}^U_{ut}$, $\mathcal{P}^U_{ut}$) to the true unobserved consumption and price indexes (C^F_{ft}, P^F_{ft}). The firm consumption index (C^F_{ft}) can be re-written in terms of the observed physical quantities of each product ($\mathcal{Q}^U_{ut}$) and a quality-adjustment parameter (θ^U_{ut}) that captures the appeal of each product (φ^U_{ut}) and the discrepancy between the observed quantity of each product ($\mathcal{Q}^U_{ut}$) and the unobserved product consumption index (C^U_{ut}):

$$C^F_{ft} = \left[\sum_{u \in \Omega^U_{ft}} \left(\theta^U_{ut} \mathcal{Q}^U_{ut} \right)^{\frac{\sigma^U_g - 1}{\sigma^U_g}} \right]^{\frac{\sigma^U_g}{\sigma^U_g - 1}}, \tag{A.7.8}$$

where the appeal-adjustment parameter is defined as:

$$\theta^U_{ut} \equiv \varphi^U_{ut} \frac{C^U_{ut}}{\mathcal{Q}^U_{ut}}. \tag{A.7.9}$$

Combining this definition in equation (A.7.9) with the relationship between observed and unobserved variables in equation (A.7.7), the firm price index (P^F_{ft}) also can be re-written in terms of the observed unit values for each product ($\mathcal{P}^U_{ut}$) and this same appeal-adjustment parameter (θ^U_{ut}):

$$P^F_{ft} = \left[\sum_{u \in \Omega^U_{ft}} \left(\frac{\mathcal{P}^U_{ut}}{\theta^U_{ut}} \right)^{1-\sigma^U_g} \right]^{\frac{1}{1-\sigma^U_g}}. \tag{A.7.10}$$

Note that equations (A.7.8) and (A.7.10) are identical to equations (A.7.1) and (A.7.3), except that the unobserved consumption and price indexes (C^F_{ft}, P^F_{ft}) in equations (A.7.1) and (A.7.3) are replaced by the observed quantities and unit values ($\mathcal{Q}^U_{ut}$, $\mathcal{P}^U_{ut}$), and the unobserved appeal parameters (φ^U_{ut}) are replaced by the appeal-adjustment parameter (θ^U_{ut}). Therefore, we can implement our entire analysis using the observed quantities and unit values ($\mathcal{Q}^U_{ut}$, $\mathcal{P}^U_{ut}$) and the appeal-adjustment parameter (θ^U_{ut}). We cannot break out this appeal-adjustment parameter (θ^U_{ut}) into the separate contributions of true product appeal (φ^U_{ut}) and the discrepancy between the true consumption index and observed physical quantities ($C^U_{ut} / \mathcal{Q}^U_{ut}$). But we can use our estimation procedure to estimate the elasticity of substitution across products (σ^U_g), recover the appeal-adjustment parameter for each product (θ^U_{ut}), recover the true firm consumption and price indexes (C^F_{ft}, P^F_{ft}), estimate the elasticity of substitution across firms (σ^F_g), and implement the remainder of our analysis.

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