

Web Appendix for Heterogeneous Firms and Trade (Not for Publication)

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1 Introduction

This web appendix contains the technical derivations of the relationships reported in the chapter. Each section of the web appendix corresponds to the section with the same name in the chapter.

2 Empirical Evidence

No further derivations required.

3 General Setup

No further derivations required.

4 Closed Economy Equilibrium

No further derivations required.

5 Open Economy with Trade Costs

No further derivations required.

Asymmetric Trade Liberalization

Following Demidova and Rodriguez-Clare (2011), we consider two asymmetric countries with a single differentiated sector and no outside sector. Comparative statics for the effects of import and

export liberalization can be derived from two equilibrium relationships: the “a competitiveness” and “trade balance” conditions. The zero-profit productivity cutoff conditions for country 2 imply:

$$w_2 L_2 P_2^{\sigma-1} \left(\frac{\sigma w_1 \tau_{21}}{(\sigma-1) \varphi_{21}^*} \right)^{1-\sigma} = \sigma w_1 f_{21},$$

$$w_2 L_2 P_2^{\sigma-1} \left(\frac{\sigma w_2}{(\sigma-1) \varphi_{22}^*} \right)^{1-\sigma} = \sigma w_2 f_{22},$$

where φ_{21}^* is the productivity cutoff for serving market 2 from country 1.¹ Combining these two relationships and choosing the wage in country 2 as the numeraire ($w_2 = 1$):

$$\varphi_{21}^* = h_{21}(w_1, \varphi_{22}^*) = \tau_{21} \left(\frac{f_{21}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{\sigma}{\sigma-1}} \varphi_{22}^*. \quad (1)$$

The zero-profit productivity cutoff conditions for country 1 imply:

$$w_1 L_1 P_1^{\sigma-1} \left(\frac{\sigma w_2 \tau_{12}}{(\sigma-1) \varphi_{12}^*} \right)^{1-\sigma} = \sigma w_2 f_{12},$$

$$w_1 L_1 P_1^{\sigma-1} \left(\frac{\sigma w_1}{(\sigma-1) \varphi_{11}^*} \right)^{1-\sigma} = \sigma w_1 f_{11},$$

where φ_{12}^* is the productivity cutoff for serving market 1 from country 2. Combining these two relationships and using our choice of numeraire:

$$\varphi_{12}^* = h_{12}(w_1, \varphi_{11}^*) = \tau_{12} \left(\frac{f_{12}}{f_{11}} \right)^{\frac{1}{\sigma-1}} (w_1)^{-\frac{\sigma}{\sigma-1}} \varphi_{11}^*. \quad (2)$$

The free entry condition for country 2 implies:

$$f_{12} J_2(\varphi_{12}^*) + f_{22} J_2(\varphi_{22}^*) = F_2. \quad (3)$$

The free entry condition for country 1 implies:

$$f_{11} J_1(\varphi_{11}^*) + f_{21} J_1(\varphi_{21}^*) = F_1. \quad (4)$$

¹To ensure consistency of notation with the other sections of the chapter, we use the first subscript to denote the country of consumption and the second subscript to denote the country of production, whereas Demidova and Rodriguez-Clare (2011) use the reverse notation.

From the country 2 cutoff condition (1) and the country 2 free entry condition (3), we obtain:

$$\varphi_{21}^* = \tau_{21} \left(\frac{f_{21}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{\sigma}{\sigma-1}} \varphi_{22}^* (\varphi_{12}^*).$$

Using the country 1 cutoff condition (2), this becomes:

$$\varphi_{21}^* = \tau_{21} \left(\frac{f_{21}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{\sigma}{\sigma-1}} \varphi_{22}^* (h_{12}(w_1, \varphi_{11}^*)).$$

Using the country 1 free entry condition (4), we obtain the “competitiveness” condition in the chapter:

$$\varphi_{21}^* = \tau_{21} \left(\frac{f_{21}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{\sigma}{\sigma-1}} \varphi_{22}^* (h_{12}(w_1, \varphi_{11}^* (\varphi_{21}^*))), \quad (5)$$

which defines an increasing relationship in (w_1, φ_{21}^*) space, as illustrated in Figure 1 and proven in Demidova and Rodriguez-Clare (2011).

Labor market clearing in each country implies:

$$\begin{aligned} R_i &= \sum_n X_{ni} = w_i L_i, \\ &= \sum_n M_{Ei} \left[\int_{\varphi_{ni}^*}^{\infty} \left(\frac{\varphi}{\varphi_{ni}^*} \right)^{\sigma-1} dG_i(\varphi) \right] \sigma w_i f_{ni} = w_i L_i, \\ &= \sum_n M_{Ei} [J_i(\varphi_{ni}^*) + 1 - G_i(\varphi_{ni}^*)] \sigma w_i f_{ni} = w_i L_i, \end{aligned}$$

which implies:

$$M_{Ei} \sigma \sum_n f_{ni} [J_i(\varphi_{ni}^*) + 1 - G_i(\varphi_{ni}^*)] = L_i.$$

Writing out this labor market clearing condition for country 2, we have:

$$M_{E2} \sigma f_{22} [J_2(\varphi_{22}^*) + 1 - G_2(\varphi_{22}^*)] + M_{E2} \sigma f_{12} [J_2(\varphi_{12}^*) + 1 - G_2(\varphi_{12}^*)] = L_2. \quad (6)$$

From the country 2 free entry condition (3), φ_{22}^* can be expressed as a function of φ_{12}^* . From the country 1 cutoff condition (2), φ_{12}^* is a function of w_1 and φ_{11}^* , while from the country 1 free entry condition (4), φ_{11}^* is a function of φ_{21}^* . It follows that the mass of entrants in country 2 can be determined as a function of w_1 and φ_{21}^* : $M_{E2}(w_1, \varphi_{21}^*)$.

Writing out the labor market clearing condition for country 1, we have:

$$M_{E1} \sigma f_{11} [J_1(\varphi_{11}^*) + 1 - G_1(\varphi_{11}^*)] + M_{E1} \sigma f_{21} [J_1(\varphi_{21}^*) + 1 - G_1(\varphi_{21}^*)] = L_1. \quad (7)$$

From free entry (4), φ_{11}^* can be in turn expressed as a function of φ_{21}^* . It follows that the mass of entrants in country 1 also can be determined given w_1 and φ_{21}^* : $M_{E1}(w_1, \varphi_{21}^*)$.

Trade balance requires:

$$\begin{aligned} X_{21} &= X_{12}, \\ M_{E1} \left[\int_{\varphi_{21}^*}^{\infty} \left(\frac{\varphi}{\varphi_{21}^*} \right)^{\sigma-1} dG_i(\varphi) \right] \sigma w_1 f_{21} &= M_{E2} \left[\int_{\varphi_{12}^*}^{\infty} \left(\frac{\varphi}{\varphi_{12}^*} \right)^{\sigma-1} dG_i(\varphi) \right] \sigma w_2 f_{12}, \\ M_{E1} [J_1(\varphi_{21}^*) + 1 - G_1(\varphi_{21}^*)] \sigma w_1 f_{21} &= M_{E2} [J_2(\varphi_{12}^*) + 1 - G_2(\varphi_{12}^*)] \sigma w_2 f_{12}. \end{aligned}$$

Combining trade balance with our results above from labor market clearing, the productivity cutoffs and free entry, we obtain:

$$M_{E1}(w_1, \varphi_{21}^*) w_1 f_{21} [J_1(\varphi_{21}^*) + 1 - G_1(\varphi_{21}^*)] = M_{E2}(w_1, \varphi_{21}^*) f_{12} [J_2(\varphi_{12}^*) + 1 - G_2(\varphi_{12}^*)],$$

where we have used our choice of numeraire ($w_2 = 1$).

Using the productivity cutoffs (1)-(2) and free entry (3)-(4), we obtain the “trade balance” condition in the chapter:

$$\begin{aligned} M_{E1}(w_1, \varphi_{21}^*) w_1 f_{21} [J_1(\varphi_{21}^*) + 1 - G_1(\varphi_{21}^*)] \\ = M_{E2}(w_1, \varphi_{21}^*) f_{12} [J_2(h_{12}(w_1, \varphi_{11}^*(\varphi_{21}^*))) + 1 - G_2(h_{12}(h_{12}(w_1, \varphi_{11}^*(\varphi_{21}^*)))]), \end{aligned} \quad (8)$$

which defines a decreasing relationship in (w_1, φ_{21}^*) space, as also illustrated in Figure 1 and proven in Demidova and Rodriguez-Clare (2011).

Pareto Distribution

One special case of the model that has received particular attention in the literature is Pareto distributed productivity, as considered in Helpman et al. (2004), Chaney (2008), Arkolakis et al. (2008,):

$$g(\varphi) = k \varphi_{\min}^k \varphi^{-(k+1)}, \quad G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi} \right)^k, \quad (9)$$

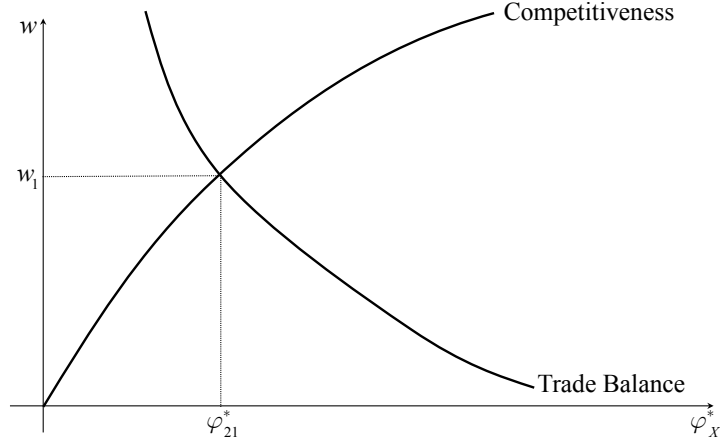


Figure 1: Competitiveness and Trade Balance Conditions

where $\varphi_{\min} > 0$ is the lower bound of the support of the productivity distribution and lower values of the shape parameter k correspond to greater dispersion in productivity.

A key feature of a Pareto distributed random variable is that when truncated the random variable retains a Pareto distribution with the same shape parameter k . Therefore the *ex post* distribution of firm productivity conditional on survival also has a Pareto distribution. Another key feature of a Pareto distributed random variable is that power functions of this random variable are themselves Pareto distributed. Therefore firm size and variable profits are also Pareto distributed with shape parameter $k/(\sigma - 1)$, where we require $k > \sigma - 1$ for average firm size to be finite.²

With Pareto distributed productivity, $J(\varphi^*)$ is a simple power function of the productivity cutoff φ^* , and we obtain the following closed form solutions for the zero-profit cutoff productivities in the closed economy

$$(\varphi^*)^k = \frac{\sigma - 1}{k - (\sigma - 1)} \frac{f}{f_E} \varphi_{\min}^k,$$

and in the symmetric country open economy equilibrium:

$$(\varphi^*)^k = \frac{\sigma - 1}{k - (\sigma - 1)} \left[\frac{f + \tau \left(\frac{f_X}{f} \right)^{\frac{1}{\sigma-1}} f_X}{f_E} \right] \varphi_{\min}^k,$$

²For empirical evidence that the Pareto distribution provides a reasonable approximation to the observed distribution of firm size, see Axtell (2001). The requirement that $k > \sigma - 1$ is needed given that the support for the Pareto distribution is unbounded from above and given the assumption of a continuum of firms. If either of these conditions are relaxed (finite number of firms or a truncated Pareto distribution), then this condition need not be imposed. Empirical estimates violating this condition for some sectors can therefore be explained within this model subject to these modifications.

Gravity and Welfare

Gravity Equation

Bilateral exports from country i to market n in sector j can be decomposed into the number of exporting firms times average firm exports conditional on exporting:

$$X_{nij} = \left(\frac{1 - G_{ij}(\varphi_{nij}^*)}{1 - G_{ij}(\varphi_{iij}^*)} \right) M_{ij} \int_{\varphi_{nij}^*}^{\infty} (\varphi)^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nij} w_{ij} \right)^{1 - \sigma_j} \frac{X_{nj}}{P_{nj}^{1 - \sigma_j}} \frac{dG_{ij}(\varphi)}{1 - G_{ij}(\varphi_{nij}^*)},$$

Under the assumption of a common Pareto distribution for all countries, this becomes:

$$X_{nij} = M_{ij} \int_{\varphi_{nij}^*}^{\infty} (\varphi)^{\sigma_j - 1} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nij} w_{ij} \right)^{1 - \sigma_j} \frac{X_{nj}}{P_{nj}^{1 - \sigma_j}} k_j \varphi_{\min, j}^{k_j} \varphi^{-(k_j + 1)} d\varphi.$$

Evaluating the integral, we obtain:

$$X_{nij} = M_{ij} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nij} w_{ij} \right)^{1 - \sigma_j} \frac{X_{nj}}{P_{nj}^{1 - \sigma_j}} \frac{k_j}{k_j - \sigma_j + 1} \varphi_{\min, j}^{k_j} (\varphi_{nij}^*)^{-(k_j - \sigma_j + 1)}. \quad (10)$$

From the export productivity cutoff condition, we have:

$$(\varphi_{nij}^*)^{\sigma_j - 1} = \frac{\sigma_j w_{ij} f_{nij}}{\left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nij} w_{ij} \right)^{1 - \sigma_j} \frac{X_{nj}}{P_{nj}^{1 - \sigma_j}}}. \quad (11)$$

Using this result in bilateral exports (10), we obtain the decomposition into the extensive and intensive margin in the chapter:

$$X_{nij} = \underbrace{\left(\frac{\varphi_{iij}^*}{\varphi_{nij}^*} \right)^{k_j}}_{\text{extensive}} M_{ij} \underbrace{w_{ij} f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}_{\text{intensive}}, \quad (12)$$

where average firm exports conditional on exporting, $w_{ij} f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}$, are independent of variable trade costs.

Bilateral exports from country i to market n in sector j can also be re-written in another equivalent form. Using the export productivity cutoff condition (11) to substitute for φ_{nij}^* in (10),

we obtain:

$$X_{nij} = M_{ij} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nij} w_{ij} \right)^{-k_j} \left(\frac{X_{nj}}{P_{nj}^{1-\sigma_j}} \right)^{\frac{k_j}{\sigma_j-1}} \frac{k_j}{k_j - \sigma_j + 1} \varphi_{\min j}^{k_j} (\sigma_j w_{ij} f_{nij})^{-\frac{k_j - \sigma_j + 1}{\sigma_j - 1}}. \quad (13)$$

Note that industry revenue in country i in sector j is:

$$R_{ij} = \sum_n X_{nij} = M_{ij} \left(\frac{\sigma_j}{\sigma_j - 1} w_{ij} \right)^{-k_j} \frac{k_j}{k_j - \sigma_j + 1} \varphi_{\min j}^{k_j} (\sigma_j w_{ij})^{-\frac{k_j - \sigma_j + 1}{\sigma_j - 1}} \Xi_{ij}. \quad (14)$$

$$\Xi_{ij} = \sum_n \left(\frac{X_{nj}}{P_{nj}^{1-\sigma_j}} \right)^{\frac{k_j}{\sigma_j-1}} \tau_{nij}^{-k_j} (f_{nij})^{-\frac{k_j - \sigma_j + 1}{\sigma_j - 1}}$$

Using industry revenue (14), bilateral exports from country i to market n in sector j (13) can be re-written as the expression in the chapter:

$$X_{nij} = \frac{R_{ij}}{\Xi_{ij}} \left(\frac{X_{nj}}{P_{nj}^{1-\sigma_j}} \right)^{\frac{k_j}{\sigma_j-1}} \tau_{nij}^{-k_j} f_{nij}^{-\frac{k_j - \sigma_j + 1}{\sigma_j - 1}}. \quad (15)$$

Free entry

The free entry condition is:

$$v_{eij} = [1 - G_{ij}(\varphi_{ij}^*)] \bar{\pi}_{ij} = w_i f_{eij}$$

$$[1 - G_{ij}(\varphi_{ij}^*)] = \left(\frac{\varphi_{\min ij}}{\varphi_{ij}^*} \right)^{k_j}$$

$$\bar{\pi}_{ij} = \sum_v \int_{\varphi_{vij}^*}^{\infty} \left(\frac{1 - G_{ij}(\varphi_{vij}^*)}{1 - G_{ij}(\varphi_{ij}^*)} \right) \bar{\pi}_{vij} \frac{g_{ij}(\varphi) d\varphi}{1 - G_{ij}(\varphi_{vij}^*)} - \sum_v \int_{\varphi_{vij}^*}^{\infty} \left(\frac{1 - G_{ij}(\varphi_{vij}^*)}{1 - G_{ij}(\varphi_{ij}^*)} \right) w_i f_{vij} \frac{g_{ij}(\varphi) d\varphi}{1 - G_{ij}(\varphi_{vij}^*)}$$

Therefore:

$$\begin{aligned} v_{eij} &= \sum_v \int_{\varphi_{vij}^*}^{\infty} \frac{\left(\frac{\sigma_j}{\sigma_j - 1} \frac{\tau_{vij} w_i}{\varphi} \right)^{1-\sigma_j}}{P_{vj}^{1-\sigma_j} \sigma_j} \beta_j w_v L_v \left(\frac{\varphi_{ij}^*}{\varphi_{vij}^*} \right)^{k_j} k_j \frac{(\varphi_{vij}^*)^{k_j}}{\varphi^{k_j+1}} d\varphi \\ &\quad - \sum_v \int_{\varphi_{vij}^*}^{\infty} w_i f_{vij} \left(\frac{\varphi_{ij}^*}{\varphi_{vij}^*} \right)^{k_j} k_j \frac{(\varphi_{vij}^*)^{k_j}}{\varphi^{k_j+1}} d\varphi = \frac{w_i f_{eij}}{(\varphi_{\min ij})^{k_j} / (\varphi_{ij}^*)^{k_j}} \end{aligned}$$

Evaluating the integrals:

$$v_{eij} = \sum_v \left[-\frac{k_j}{k_j - \sigma_j + 1} \frac{\left(\frac{\sigma_j}{\sigma_j - 1} \tau_{vij} w_i\right)^{1-\sigma_j}}{P_{vj}^{1-\sigma_j}} \beta_j w_v L_v (\varphi_{ij}^*)^{k_j} \varphi^{-(k_j - \sigma_j + 1)} \right]_{\varphi_{vij}^*}^{\infty} - \sum_v \left[-w_i f_{vij} (\varphi_{ij}^*)^{k_j} \varphi^{-k_j} \right]_{\varphi_{vij}^*}^{\infty} = \frac{w_i f_{eij}}{(\varphi_{\min ij})^{k_j} / (\varphi_{ij}^*)^{k_j}}$$

Evaluating the terms in square parentheses and using the export cutoff productivity condition (11):

$$v_{eij} = \sum_v w_i f_{vij} \frac{k_j}{k_j - \sigma_j + 1} \left(\frac{\varphi_{ij}^*}{\varphi_{vij}^*}\right)^{k_j} - \sum_v w_i f_{vij} \left(\frac{\varphi_{ij}^*}{\varphi_{vij}^*}\right)^{k_j} = \frac{w_i f_{eij}}{(\varphi_{\min ij})^{k_j} / (\varphi_{ij}^*)^{k_j}},$$

which can be re-arranged to yield:

$$\sum_v f_{vij} \frac{(\varphi_{ij}^*)^{k_j}}{(\varphi_{vij}^*)^{k_j}} \left(\frac{\sigma_j - 1}{k_j - \sigma_j + 1}\right) = \frac{f_{eij}}{(\varphi_{\min ij})^{k_j} / (\varphi_{ij}^*)^{k_j}} \quad (16)$$

Price Index

The price index can be expressed as follows:

$$P_{nj}^{1-\sigma_j} = \sum_v M_{vj} \frac{1 - G_{vj}(\varphi_{nvj}^*)}{1 - G_{vj}(\varphi_{vvj}^*)} \int_{\varphi_{nvj}^*}^{\infty} p_{nvj}(\varphi)^{1-\sigma_j} \frac{g_{vj}(\varphi)}{1 - G_{vj}(\varphi_{nvj}^*)} d\varphi,$$

$$P_{nj}^{1-\sigma_j} = \sum_v M_{vj} \left(\frac{\varphi_{vvj}^*}{\varphi_{nvj}^*}\right)^{k_j} \int_{\varphi_{nvj}^*}^{\infty} \left(\frac{\sigma_j}{\sigma_j - 1} \frac{\tau_{nvj} w_v}{\varphi}\right)^{1-\sigma_j} \frac{k_j (\varphi_{nvj}^*)^{k_j}}{\varphi^{k_j+1}} d\varphi$$

$$P_{nj}^{1-\sigma_j} = \sum_v M_{vj} (\varphi_{vvj}^*)^{k_j} (\varphi_{nvj}^*)^{-(k_j - \sigma_j + 1)} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nvj} w_v\right)^{1-\sigma_j} \frac{k_j}{k_j - \sigma_j + 1}. \quad (17)$$

Using the export cutoff productivity condition (11) in the price index (17), we obtain:

$$P_n^{-k_j} = \sum_v M_{vj} (\varphi_{vvj}^*)^{k_j} \left(\frac{\sigma_j}{\sigma_j - 1} \tau_{nvj} w_v\right)^{-k_j} \left(\frac{\sigma_j w_v f_{nvj}}{\beta_j w_n L_n}\right)^{1-\frac{k_j}{\sigma_j-1}} \frac{k_j}{k_j - \sigma_j + 1}. \quad (18)$$

Labor Demand and Supply

We now use the equality between labor demand and supply to solve for the mass of firms given the allocation of labor across sectors. Equating sectoral labor demand and supply, we obtain:

$$M_{ij} \left(\sum_v \int_{\varphi_{vij}^*}^{\infty} l_{vij}^{var}(\varphi) \frac{1 - G_{ij}(\varphi_{vij}^*)}{1 - G_{ij}(\varphi_{iij}^*)} \frac{g_{ij}(\varphi) d\varphi}{1 - G_{ij}(\varphi_{vij}^*)} \right) + M_{ij} \frac{f_{eij}}{1 - G_{ij}(\varphi_{iij}^*)} + \sum_v M_{ij} \frac{1 - G_{ij}(\varphi_{vij}^*)}{1 - G_{ij}(\varphi_{iij}^*)} f_{vij} = L_{ij},$$

where L_{ij} is the measure of labor allocated to sector j in country i and the destination fixed costs f_{vij} are incurred in terms of source country labor by firms in country i serving market v . Note that

$$x_{vij}(\varphi) = \frac{p_{vij}(\varphi)^{-\sigma_j}}{P_{vj}^{1-\sigma_j}} \beta_j w_v L_v \tau_{vij},$$

and hence variable labor input is:

$$l_{vij}^{var}(\varphi) = \left(\frac{\sigma_j}{\sigma_j - 1} \frac{w_i}{\varphi} \tau_{vij} \right)^{-\sigma_j} \frac{\beta_j w_v L_v \tau_{vij}}{P_{vj}^{1-\sigma_j} \varphi}.$$

Using this expression and the Pareto productivity distribution, the equality between labor demand and supply can be written as:

$$M_{ij} \left(\sum_v \int_{\varphi_{vij}^*}^{\infty} \left(\frac{\sigma_j}{\sigma_j - 1} \frac{w_i}{\varphi} \tau_{vij} \right)^{-\sigma_j} \frac{\beta_j w_v L_v \tau_{vij}}{P_{vj}^{1-\sigma_j} \varphi} \frac{k_j (\varphi_{iij}^*)^{k_j}}{\varphi^{k_j+1}} d\varphi \right) + M_{ij} \frac{f_{eij}}{(\varphi_{\min ij} / \varphi_{iij}^*)^{k_j}} + \sum_v M_{ij} \left(\frac{\varphi_{iij}^*}{\varphi_{vij}^*} \right)^{k_j} f_{vij} = L_{ij}.$$

$$M_{ij} \sum_v \left[-\frac{k_j}{k_j - \sigma_j + 1} \left(\frac{\sigma_j}{\sigma_j - 1} w_i \tau_{vij} \right)^{-\sigma_j} \frac{\beta_j w_v L_v \tau_{vij}}{P_{vj}^{1-\sigma_j}} (\varphi_{iij}^*)^{k_j} \varphi^{-(k_j - \sigma_j + 1)} \right]_{\varphi_{vij}^*}^{\infty} + M_{ij} \frac{f_{eij}}{(\varphi_{\min ij} / \varphi_{iij}^*)^{k_j}} + \sum_v M_{ij} \left(\frac{\varphi_{iij}^*}{\varphi_{vij}^*} \right)^{k_j} f_{vij} = L_{ij}.$$

Evaluating the terms in square parentheses, using the export cutoff productivity condition (11) and simplifying we obtain:

$$M_{ij} \left\{ \sum_v \left\{ f_{vij} \left(\frac{\varphi_{ii}^*}{\varphi_{vij}^*} \right)^{k_j} \left[\frac{k_j(\sigma_j - 1) + k_j - (\sigma_j - 1)}{k_j - \sigma_j + 1} \right] \right\} + \frac{f_{eij}}{\left(\varphi_{\min ij} / \varphi_{ii}^* \right)^{k_j}} \right\} = L_{ij},$$

which using free entry (16) becomes:

$$M_{ij} = \left(\frac{\varphi_{\min ij}}{\varphi_{ii}^*} \right)^{k_j} \frac{(\sigma_j - 1)}{k_j \sigma_j f_{eij}} L_{ij}, \quad (19)$$

$$M_{Ei} = \frac{(\sigma_j - 1)}{k_j \sigma_j f_{eij}} L_{ij}.$$

Trade Shares

The share of total income of country n spent on goods from country i in sector j can be expressed using (12) as:

$$\lambda_{nij} = \frac{X_{nij}}{\sum_v X_{nvj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{nij}^*} \right)^{k_j} M_{ij} w_i f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}{\sum_v \left(\frac{\varphi_{vv}^*}{\varphi_{nvj}^*} \right)^{k_j} M_{vj} w_v f_{nvj} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}.$$

Using the expression for the equilibrium mass of firms (19), this becomes:

$$\lambda_{nij} = \frac{\left(\varphi_{nij}^* \right)^{-k_j} \left(\varphi_{\min ij} \right)^{k_j} \left(L_{ij} / f_{eij} \right) w_i f_{nij}}{\sum_v \left(\varphi_{nvj}^* \right)^{-k_j} \left(\varphi_{\min vj} \right)^{k_j} \left(L_{vj} / f_{evj} \right) w_v f_{nvj}}.$$

Using the export cutoff productivity condition (11), the trade share can be written as:

$$\lambda_{nij} = \frac{\left(L_{ij} / f_{eij} \right) \left(\varphi_{\min ij} \right)^{k_j} \left(w_i \right)^{-\left(\frac{k_j \sigma_j - (\sigma_j - 1)}{\sigma_j - 1} \right)} \left(\tau_{nij} \right)^{-k_j} f_{nij}^{1 - k_j / (\sigma_j - 1)}}{\sum_v \left(L_{vj} / f_{evj} \right) \left(\varphi_{\min vj} \right)^{k_j} \left(w_v \right)^{-\left(\frac{k_j \sigma_j - (\sigma_j - 1)}{\sigma_j - 1} \right)} \left(\tau_{nvj} \right)^{-k_j} f_{nvj}^{1 - k_j / (\sigma_j - 1)}}, \quad (20)$$

where the exponent on wages differs from the exponent on variable trade costs because of the assumption that the fixed exporting costs are incurred in terms of labor in the source country. The above expression determines the trade share as a function of wages in each country, the labor allocated to each sector in each country, and parameters.

Variety Effects of Trade Liberalization

Note that the trade share λ_{nij} can also be written as:

$$\lambda_{nij} = \frac{X_{nij}}{\beta_j w_n L_n}$$

Using the extensive-intensive margin decomposition of X_{nij} in (12), we obtain:

$$\lambda_{nij} = \frac{\left(\frac{\varphi_{ij}^*}{\varphi_{nij}^*}\right)^{k_j} M_{ij} w_i f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}{\beta_j w_n L_n}.$$

Therefore the measure of firms selling from country i to country n in sector j relative to country i 's share of the market in sector j in country n is:

$$\frac{M_{nij}}{\lambda_{nij}} = \frac{\left(\frac{\varphi_{ij}^*}{\varphi_{nij}^*}\right)^{k_j} M_{ij}}{\lambda_{nij}} = \frac{\left(\frac{\varphi_{ij}^*}{\varphi_{nij}^*}\right)^{k_j} M_{ij}}{\left(\frac{\varphi_{ij}^*}{\varphi_{nij}^*}\right)^{k_j} \frac{M_{ij} w_i f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}{\beta_j w_n L_n}},$$

which yields:

$$M_{nij} = \lambda_{nij} \frac{\beta_j w_n L_n}{w_i f_{nij} \frac{\sigma_j k_j}{k_j - \sigma_j + 1}}. \quad (21)$$

Now consider a world of two countries. From (21), we have:

$$M_{nij} + M_{nnj} = \frac{\beta_j w_n L_n}{\frac{\sigma_j k_j}{k_j + \sigma_j + 1}} \left(\frac{1 - \lambda_{nnj}}{w_i f_{nij}} + \frac{\lambda_{nnj}}{w_n f_{nnj}} \right).$$

Therefore trade liberalization (a reduction in λ_{nnj}) will reduce the measure of varieties available for consumption if $w_i f_{nij} > w_n f_{nnj}$.

Welfare

Assuming no differentiated sector, so that all sectors $j = 1, \dots, J$ are differentiated, the welfare of the representative consumer can be written as:

$$\mathbb{V}_n = \frac{w_n}{P_n} = \frac{w_n}{\prod_{j=1}^J P_{nj}^{\beta_j}}. \quad (22)$$

Using the mass of firms (19) to substitute for M_{vj} , the price index equation (18) can be written as:

$$P_{nj}^{-k_j} = \left[\sum_v (L_{vj}/f_{evj}) (\varphi_{\min vj})^{k_j} (w_v)^{-\left(\frac{k_j \sigma_j - (\sigma_j - 1)}{\sigma_j - 1}\right)} (\tau_{nvj})^{-k_j} (f_{nvj})^{1 - \frac{k_j}{\sigma_j - 1}} \right] (\beta_j w_n L_n)^{-\left(1 - \frac{k_j}{\sigma_j - 1}\right)} \left(\frac{\sigma_j}{\sigma_j - 1}\right)^{-k_j} \sigma_j^{-\frac{k_j}{\sigma_j - 1}} \frac{\sigma_j - 1}{k_j - \sigma_j + 1}. \quad (23)$$

Now using country n 's share of expenditure on itself (20) within sector j in the above expression, we obtain the following solution for the price index as a function of country n 's share of trade with itself (λ_{nnj}), its wage (w_n), the labor allocation (L_{nj}) and parameters:

$$P_{nj} = \left[\frac{\lambda_{nnj} (\beta_j L_n)^{\left(1 - \frac{k_j}{\sigma_j - 1}\right)}}{(L_{nj}/f_{enj}) (f_{nnj})^{1 - \frac{k_j}{\sigma_j - 1}}} \right]^{\frac{1}{k_j}} \frac{w_n}{\varphi_{\min nj}} \left(\frac{\sigma_j}{\sigma_j - 1}\right) \sigma_j^{\frac{1}{\sigma_j - 1}} \left[\frac{k_j - \sigma_j + 1}{\sigma_j - 1}\right]^{\frac{1}{k_j}}. \quad (24)$$

From welfare (22) and the price index (24), relative welfare in country n under autarky and free trade is given by:

$$\hat{V}_n = \prod_{j=0}^J \left(\frac{\hat{\lambda}_{nnj}}{\hat{L}_{nj}} \right)^{-\frac{\beta_j}{k_j}}, \quad (25)$$

where:

$$\hat{\lambda}_{nnj} = \frac{\lambda_{nnj}^{\text{Open}}}{\lambda_{nnj}^{\text{Closed}}} = \frac{\lambda_{nnj}^{\text{Open}}}{1}.$$

6 Structural Estimation

No further derivations required.

7 Factor Abundance and Heterogeneity

No further derivations required.

8 Trade and Market Size

Consumer's Problem

The representative consumer's preferences in country i are defined over consumption of a continuum of differentiated varieties, $q_{\omega i}^c$, and consumption of a homogeneous good, q_{0i}^c :

$$U_i = q_{0i}^c + \alpha \int_{\omega \in \Omega_i} q_{\omega i}^c d\omega - \frac{1}{2}\gamma \int_{\omega \in \Omega_i} (q_{\omega i}^c)^2 d\omega - \frac{1}{2}\eta \left(\int_{\omega \in \Omega_i} q_{\omega i}^c d\omega \right)^2. \quad (26)$$

The representative consumer's budget constraint is:

$$\int_{\omega \in \Omega_i} p_{\omega i} q_{\omega i}^c d\omega + q_{0i}^c = w_i, \quad (27)$$

where we have chosen the homogeneous good as the numeraire and hence $p_{0i}^c = 1$. Labor is the sole factor of production and each country i is endowed with L^i workers. Each country's labor endowment is assumed to be sufficiently large that it both consumes and produces the homogeneous good. Using the budget constraint (27) to substitute for consumption of the homogeneous good, q_{0i}^c , in utility (26), the representative consumer's first-order conditions for utility maximization imply the following inverse demand curve for a differentiated variety:

$$p_{\omega i} = \alpha - \gamma q_{\omega i}^c - \eta Q_i^c, \quad Q_i^c = \int_{\omega \in \Omega_i} q_{\omega i}^c d\omega, \quad (28)$$

where demand for a variety is positive if:

$$p_{\omega i} \leq \alpha - \eta Q_i^c,$$

which defines a "choke price" above which demand is zero. Total output of differentiated varieties, Q^c , can be expressed as follows:

$$\begin{aligned} Q_i^c &= \int_{\omega \in \Omega_i} q_{\omega i}^c d\omega, \\ &= N_i \left(\frac{\alpha - \eta Q_i^c}{\gamma} \right) - \int_{\omega \in \Omega_i} \frac{p_{\omega i}}{\gamma} d\omega, \\ &= \frac{N_i (\alpha - \bar{p}_i)}{\eta N_i + \gamma}, \end{aligned} \quad (29)$$

where:

$$\bar{p}_i = \frac{1}{N_i} \int_{\omega \in \Omega_i} p_{\omega i} d\omega.$$

Substituting for Q_i^c in demand (28) yields:

$$\begin{aligned} q_{\omega i}^c &= \frac{\alpha}{\gamma} - \frac{p_{\omega i}}{\gamma} - \frac{\eta}{\gamma} \left(\frac{N_i \alpha - N_i \bar{p}_i}{\eta N_i + \gamma} \right), \\ &= \frac{\alpha}{\eta N_i + \gamma} - \frac{p_{\omega i}}{\gamma} + \frac{\eta N_i}{\eta N_i + \gamma} \frac{\bar{p}_i}{\gamma}, \end{aligned}$$

where demand for a variety is positive if:

$$p_{\omega i} \leq \frac{1}{\eta N_i + \gamma} (\alpha \gamma + \eta N_i \bar{p}_i). \quad (30)$$

Total demand for each differentiated variety across all consumers is:

$$q_{\omega i} = L_i q_{\omega i}^c = \frac{\alpha L_i}{\eta N_i + \gamma} - \frac{p_{\omega i} L_i}{\gamma} + \frac{\eta N_i}{\eta N_i + \gamma} \frac{\bar{p}_i L_i}{\gamma}. \quad (31)$$

Indirect Utility

To derive the indirect utility function, note that the representative consumer's demand for the outside good is:

$$q_{0i}^c = w_i - \int_{\omega \in \Omega_i} p_{\omega i} q_{\omega i}^c d\omega,$$

Using $p_{\omega i} = \alpha - \gamma q_{\omega i}^c - \eta Q_i^c$, we have:

$$q_{0i}^c = w_i - \alpha \int_{\omega \in \Omega_i} q_{\omega i}^c d\omega + \gamma \int_{\omega \in \Omega_i} (q_{\omega i}^c)^2 d\omega + \eta (Q_i^c)^2$$

Using this result in (26), we obtain:

$$U_i = w_i + \frac{1}{2} \gamma \int_{\omega \in \Omega_i} (q_{\omega i}^c)^2 d\omega + \frac{1}{2} \eta Q_i^2.$$

Now from (29) we have:

$$Q_i^2 = \left(\frac{N_i}{\gamma + N_i \eta} \right)^2 (\alpha - \bar{p}_i)^2.$$

While from (31) and (29), we have:

$$(q_{\omega i}^c)^2 = \left(\frac{1}{N_i}\right)^2 Q_i^2 + \left(\frac{p_{\omega i} - \bar{p}_i}{\gamma}\right)^2 - \frac{2}{N_i} Q_i \left(\frac{p_{\omega i} - \bar{p}_i}{\gamma}\right).$$

Combining these results, we obtain the following expression for indirect utility:

$$U_i = w_i + \frac{1}{2} \left(\eta + \frac{\gamma}{N_i}\right)^{-1} (\alpha - \bar{p}_i)^2 + \frac{1}{2} \frac{N_i}{\gamma} \sigma_{p_i}^2, \quad (32)$$

$$\sigma_{p_i}^2 = \left(\frac{1}{N_i}\right) \int_{\omega \in \Omega_i} (p_{\omega i} - \bar{p}_i)^2 d\omega.$$

Therefore, welfare is higher when (a) there are more varieties (love of variety), (b) when average prices are lower, (c) when the variance of prices σ_p^2 is high.

Firm's Problem

The firm's problem in the closed economy is:

$$\max_{p_i(c)} \{ \pi_i(c) = p_i(c)q_i(c) - cq_i(c) \},$$

where all firms with same cost (c) behave symmetrically and hence firms are indexed from now on by c alone. The first-order condition for profit maximization is:

$$q_i(c) = \frac{L_i}{\gamma} [p_i(c) - c]. \quad (33)$$

The zero-profit cutoff cost above which firms exit is defined by:

$$p_i(c_{Di}) = c_{Di}, \quad q(c_{Di}) = 0.$$

This zero-profit cutoff cost condition can be used to determine average prices \bar{p}_i . Using $q_i(c_{Di}) = 0$ and $p_i(c_{Di}) = c_{Di}$ in (31), we have:

$$\bar{p}_i = \frac{(\eta N_i + \gamma)c_{Di} - \alpha\gamma}{\eta N_i}. \quad (34)$$

Substituting for average prices \bar{p}_i in equilibrium demands (31), we get:

$$q_i(c) = \frac{L_i}{\gamma} (-p_i(c) + c_{Di}).$$

Closed Economy Equilibrium

Using the firm's first-order condition (33) to substitute for $q_i(c)$, we can solve for the closed economy equilibrium values of $p_i(c)$ and hence all other firm variables:

$$\begin{aligned} p_i(c) &= \frac{1}{2} (c_{Di} + c) \\ \mu_i(c) &= \frac{1}{2} (c_{Di} - c) \\ q_i(c) &= \frac{L_i}{2\gamma} (c_{Di} - c) \\ r_i(c) &= \frac{L_i}{4\gamma} [(c_{Di})^2 - c^2] \\ \pi_i(c) &= \frac{L_i}{4\gamma} (c_{Di} - c)^2. \end{aligned} \tag{35}$$

The equilibrium zero-profit cutoff cost c_{Di} is determined by the free entry condition that the expected value of entry equals the sunk entry cost:

$$G(c_{Di}) \int_0^{c_{Di}} \pi_i(c) \frac{dG(c)}{G(c_{Di})} = \frac{L_i}{4\gamma} \int_0^{c_{Di}} (c_{Di} - c)^2 dG(c) = f_E, \tag{36}$$

which determines the zero-profit cost cutoff c_{Di} independently of the other endogenous variables of the model. The zero-profit cutoff cost is *lower* (average productivity is *higher*): (a) when sunk costs (f_E) are lower, (b) when varieties are closer substitutes (lower γ), (c) in larger markets (larger L_i). Having determined c_{Di} , average prices \bar{p}_i are:

$$\begin{aligned} \bar{p}_i &= \int_0^{c_{Di}} p(c) \frac{dG(c)}{G(c_{Di})}, \\ \bar{p}_i &= \frac{1}{2} (c_{Di} + \bar{c}_i), \quad \bar{c}_i = \int_0^{c_{Di}} c \frac{dG(c)}{G(c_{Di})}. \end{aligned} \tag{37}$$

To determine the mass of firms, use the zero-profit cutoff cost in the choke price (30):

$$c_{Di} = \frac{1}{\eta N_i + \gamma} (\gamma \alpha + \eta N_i \bar{p}_i),$$

which implies:

$$N_i = \frac{\gamma(\alpha - c_{Di})}{\eta(c_{Di} - \bar{p}_i)}.$$

Substituting for \bar{p}_i in the expression for the mass of firms above, we obtain the following solution:

$$N_i = \frac{2\gamma(\alpha - c_{Di})}{\eta(c_{Di} - \bar{c}_i)} \quad (38)$$

Therefore we have:

$$\bar{p}_i = \frac{1}{2}(c_{Di} + \bar{c}_i), \quad \bar{c}_i = \int_0^{c_{Di}} c \frac{dG(c)}{G(c_{Di})},$$

$$N_i = \frac{2\gamma(\alpha - c_{Di})}{\eta(c_{Di} - \bar{c}_i)},$$

where the mass of entrants is:

$$N_{Ei} = \frac{N_i}{G(c_{Di})}.$$

from (36), (37) and (38), larger markets are characterized by tougher competition: More firms and lower average prices. Tougher competition implies that firms with a given c set lower mark-ups (mark-ups are increasing in c_{Di}).

Pareto Distribution

Suppose that productivity draws $1/c$ follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \geq 1$. This implies the following distribution of cost draws:

$$G(c) = \left(\frac{c}{c_M}\right)^k, \quad c \in [0, c_M].$$

Using this cost distribution in the free entry condition:

$$c_{Di} = \left[\frac{\gamma\phi}{L_i}\right]^{\frac{1}{k+2}}.$$

where $\phi = 2(k+1)(k+2)(c_M)^k f_E$ is an (inverse) index of technology. The mass of firms and average prices are:

$$N_i = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_{Di}}{c_{Di}}.$$

$$\bar{p}_i = \frac{2k+1}{2k+2} c_{Di}.$$

9 Endogenous Productivity

Multi-Product

No further derivations required.

Innovation

Innovation Intensity

In this section, we sketch out a static version of the innovation intensity decision used by Atkeson & Burstein (2010). Consider a rescaling of firm productivity $\phi = \varphi^{\sigma-1}$ so that this new productivity measure ϕ is now proportional to firm size.³ We assume that successful innovation increases productivity by a fixed factor $\iota > 1$ (from ϕ to $\iota\phi$). The probability of successful innovation is an endogenous variable α that reflects a firm's innovation intensity choice. The cost of higher innovation intensity is determined by an exogenous convex function $c_I(\alpha) \geq 0$ and scales up proportionally with firm size and productivity ϕ – so the total cost of innovation intensity α is $\phi c_I(\alpha)$. This scaling up of innovation cost with firm size is needed to deliver the prediction of Gibrat's Law that growth rates for large firms are independent of their size.

We first examine the choice of innovation intensity in a closed economy. Consider a firm with productivity ϕ that is high enough such that the exit option is excluded (that is, the firm will produce even if innovation is not successful). This firm will choose innovation intensity α to maximize expected profits

$$E[\pi(\phi)] = [(1 - \alpha) + \alpha\iota] B\phi - \phi c_I(\alpha) - f,$$

where B is the same market demand parameter for the domestic economy as in earlier sections. The first-order condition is given by

$$c'_I(\alpha) = (\iota - 1) B. \tag{39}$$

This implies that, in the closed economy, all firms (above a certain productivity threshold satisfying the no exit restriction) will choose the same innovation intensity α . In a dynamic setting, this delivers Gibrat's Law for those firms, and also generates an ergodic distribution of firm produc-

³Since the rescaling involves the demand side product differentiation parameter σ , caution must be used when interpreting any comparative statics that include this parameter.

tivity (hence firm size) that is Pareto in the upper tail independently of the initial distribution of productivity upon entry. To establish this result, note that firm productivity ($\phi = \varphi^{\sigma-1}$) evolves according to the following law of motion for all firms with productivity above a certain productivity threshold satisfying the no exit restriction ($\phi > \underline{\phi}$):

$$\phi_{t+1} = \gamma\phi_t, \quad \phi_t \geq \underline{\phi} \quad (40)$$

where γ is independently and identically distributed such that

$$\gamma = \begin{cases} \iota > 1 & \text{with probability } \alpha \\ 1 & \text{with probability } (1 - \alpha) \end{cases} \quad (41)$$

Define normalized firm productivity as firm productivity divided by average firm productivity ($\bar{\phi}_t$):

$$\tilde{\phi}_t = \phi_t / \bar{\phi}_t. \quad (42)$$

With a continuum of firms, the growth rate of average firm productivity is:

$$g = \alpha(\iota - 1). \quad (43)$$

Therefore the law of motion for normalized firm productivity is:

$$\tilde{\phi}_{t+1} = \begin{cases} \frac{\iota}{1+g}\tilde{\phi}_t & \text{with probability } \alpha \\ \frac{1}{1+g}\tilde{\phi}_t & \text{with probability } (1 - \alpha) \end{cases}, \quad \tilde{\phi}_t \geq \underline{\phi} / \bar{\phi}_t. \quad (44)$$

Define the countercumulative distribution of normalized firm productivity by $G_t(x) = P(\tilde{\phi}_t > x)$.

The equation of motion for G_t is

$$G_{t+1}(\tilde{\phi}) = \alpha G_t\left(\frac{(1+g)\tilde{\phi}}{\iota}\right) + (1 - \alpha)G_t\left((1+g)\tilde{\phi}\right), \quad \tilde{\phi}_t \geq \underline{\phi} / \bar{\phi}_t. \quad (45)$$

If a steady-state distribution for normalized firm productivity exists it satisfies:

$$G(\tilde{\phi}) = \alpha G\left(\frac{(1+g)\tilde{\phi}}{\iota}\right) + (1 - \alpha)G\left((1+g)\tilde{\phi}\right), \quad \tilde{\phi} \geq \underline{\phi} / \bar{\phi}. \quad (46)$$

Consider the conjecture that the steady-state distribution for normalized productivity takes the

form $G(\tilde{\phi}) = \Gamma \tilde{\phi}^{-k}$. Under this conjecture, the steady-state distribution for normalized firm productivity satisfies:

$$\frac{\Gamma}{\tilde{\phi}^k} = \frac{\alpha \Gamma}{\left(\frac{(1+g)\tilde{\phi}}{\iota}\right)^k} + \frac{(1-\alpha)\Gamma}{\left((1+g)\tilde{\phi}\right)^k}, \quad \tilde{\phi} \geq \underline{\phi}/\bar{\phi}, \quad (47)$$

or equivalently

$$1 = \frac{\alpha}{\left(\frac{(1+g)}{\iota}\right)^k} + \frac{(1-\alpha)}{(1+g)^k}, \quad \tilde{\phi} \geq \underline{\phi}/\bar{\phi}, \quad (48)$$

$$(1+g)^k = \iota^k \alpha + (1-\alpha), \quad \tilde{\phi} \geq \underline{\phi}/\bar{\phi}, \quad (49)$$

which is satisfied for $k = 0$ and $k = 1$, since $g = \alpha(\iota - 1)$.

Note that $k = 0$ cannot be the solution, because it implies that the counter cumulative distribution is given by $G(\tilde{\phi}) = \Gamma$ for all $\tilde{\phi} \geq \underline{\phi}/\bar{\phi}$, which implies that all firms have a normalized productivity equal to zero and violates the hypothesis that a steady-state normalized productivity distribution exists. However the solution $k = 1$ and $\Gamma = \underline{\phi}$ is consistent with the existence of a steady-state normalized productivity distribution. Therefore this steady-state distribution is a Pareto distribution with shape parameter $k = 1$ and scale parameter $\Gamma = \underline{\phi}$:

$$\Pr \left[\tilde{\phi} \leq x \right] = 1 - \underline{\phi}/x. \quad (50)$$

Note the importance of the assumption that firm productivity $\phi_t \geq \underline{\phi}$. Otherwise if the law of motion (40) holds for all values of ϕ_t , we have:

$$\ln \tilde{\phi}_{t+1} = \ln \tilde{\phi}_0 + \sum_{s=1}^t \ln \tilde{\gamma}_s, \quad (51)$$

where $\tilde{\gamma}$ is independently and identically distributed and given by:

$$\tilde{\gamma} = \begin{cases} \frac{\iota}{1+g} & \text{with probability } \alpha \\ \frac{1}{1+g} & \text{with probability } (1-\alpha) \end{cases}. \quad (52)$$

Under these circumstances, the limiting distribution of normalized firm productivity is a log normal distribution and no steady-state distribution exists, since the variance of normalized firm productivity in (51) converges towards infinity as t converges towards infinity.

10 Labor Markets

When wages vary with revenue across firms, within-industry reallocations across firms provide a new channel through which trade can affect income distribution. As shown in Helpman et al. (2010), the opening of the closed economy to trade raises wage inequality within industries when the following three conditions are satisfied: (a) wages and employment are power functions of productivity, (b) only some firms export and exporting raises the wage paid by a firm with a given productivity, and (c) productivity is Pareto distributed. Under these conditions, the wage and employment of firms can be expressed in terms of their productivity, φ , a term capturing whether or not a firm exports, $\Upsilon(\varphi)$, the zero-profit cutoff productivity, φ_d^* , and parameters:

$$l(\varphi) = \Upsilon(\varphi)^{\psi_l} l_d \left(\frac{\varphi}{\varphi_d^*} \right)^{\zeta_l}, \quad (53)$$

$$w(\varphi) = \Upsilon(\varphi)^{\psi_w} w_d \left(\frac{\varphi}{\varphi_d^*} \right)^{\zeta_w}, \quad (54)$$

where l_d and w_d are the employment and wage of the least productive firm, respectively, and:

$$\Upsilon(\varphi) = \begin{cases} \Upsilon_x > 1 & \text{for } \varphi \geq \varphi_x^* \\ 1 & \text{for } \varphi < \varphi_x^* \end{cases},$$

where $\Upsilon_x^{\psi_w}$ and $\Upsilon_x^{\psi_l}$ are, respectively, the wage and employment premia from exporting for a firm of a given productivity.

Using the Pareto productivity distribution, Helpman et al. (2010) show that the distribution of wages across workers within the industry, $G_w(w)$, can be evaluated as:

$$G_w(w) = \begin{cases} S_{l,d} G_{w,d}(w) & \text{for } w_d \leq w \leq w_d (\varphi_x^*/\varphi_d^*)^{\zeta_w}, \\ S_{l,d} & \text{for } w_d (\varphi_x^*/\varphi_d^*)^{\zeta_w} \leq w \leq w_x, \\ S_{l,d} + (1 - S_{l,d}) G_{w,x}(w) & \text{for } w \geq w_x, \end{cases} \quad (55)$$

where $w_d (\varphi_x^*/\varphi_d^*)^{\zeta_w}$ is the highest wage paid by a domestic firm; $w_x = w_d \Upsilon_x^{\psi_w} (\varphi_x^*/\varphi_d^*)^{\zeta_w}$ is the lowest wage paid by an exporting firm; $\Upsilon_x^{\psi_w}$ is the increase in wages at the productivity threshold for entry into export markets; $S_{l,d}$ is the employment share of domestic firms.

Given that wages (53) and employment (54) are power functions of productivity and produc-

tivity has a Pareto distribution, the employment share of domestic firms can be evaluated as:

$$\begin{aligned}
S_{l,d} &= 1 - \frac{\int_{\varphi_x^*}^{\infty} \Upsilon_x^{\psi_l} l_d \left(\frac{\varphi}{\varphi_d^*} \right)^{\zeta_l} k (\varphi_d^*)^k \varphi^{-(k+1)} d\varphi}{\int_{\varphi_d^*}^{\varphi_x^*} l_d \left(\frac{\varphi}{\varphi_d^*} \right)^{\zeta_l} k (\varphi_d^*)^k \varphi^{-(k+1)} d\varphi + \int_{\varphi_x^*}^{\infty} \Upsilon_x^{\psi_l} l_d \left(\frac{\varphi}{\varphi_d^*} \right)^{\zeta_l} k (\varphi_d^*)^k \varphi^{-(k+1)} d\varphi}, \quad (56) \\
&= 1 - \frac{\Upsilon_x^{\psi_l} \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^{k-\zeta_l}}{\left[1 - \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^{k-\zeta_l} \right] + \Upsilon_x^{\psi_l} \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^{k-\zeta_l}}, \\
&= \frac{1 - \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^{k-\zeta_l}}{1 + \left[\Upsilon_x^{\psi_l} - 1 \right] \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^{k-\zeta_l}}.
\end{aligned}$$

To characterize the distribution of wages across workers employed by exporters, note that the share of workers employed by exporters whose firm has a productivity less than φ is:

$$\begin{aligned}
Z_x(\varphi) &= 1 - \frac{\int_{\varphi}^{\infty} \Upsilon_x^{\psi_l} l_d \left(\frac{\xi}{\varphi_d^*} \right)^{\zeta_l} k (\varphi_x^*)^k \xi^{-(k+1)} d\xi}{\int_{\varphi_x}^{\infty} \Upsilon_x^{\psi_l} l_d \left(\frac{\xi}{\varphi_d^*} \right)^{\zeta_l} k (\varphi_x^*)^k \xi^{-(k+1)} d\xi}, \\
&= 1 - \left(\frac{\varphi_x^*}{\varphi} \right)^{k-\zeta_l}.
\end{aligned}$$

Now note that relative firm productivities and relative firm wages in (54) are related as follows:

$$\frac{\varphi_x^*}{\varphi} = \left(\frac{w_x}{w} \right)^{\frac{1}{\zeta_w}}.$$

Therefore the share of workers employed by exporters whose firm has a wage less than w – i.e. the cumulative distribution function of wages for workers employed by exporters – is:

$$G_{w,x}(w) = 1 - \left(\frac{w_x}{w} \right)^{\zeta_g}, \quad \zeta_g \equiv \frac{k - \zeta_l}{\zeta_w}, \quad \text{for } w \geq w_x, \quad (57)$$

The distribution of wages across workers employed by domestic firms can be determined by a similar line of reasoning and follows a truncated Pareto distribution with the same shape parameter as the distribution of wages across workers employed by exporters:

$$G_{w,d}(w) = \frac{1 - \left(\frac{w_d}{w} \right)^{\zeta_g}}{1 - \left(\frac{w_d}{w_x} \right)^{\zeta_g}}, \quad \zeta_g \equiv \frac{k - \zeta_l}{\zeta_w}, \quad \text{for } w_d \leq w \leq w_d (\varphi_x^*/\varphi_d^*)^{\zeta_w}. \quad (58)$$

When the three conditions discussed above are satisfied, and hence the wage distribution within

industries is characterized by (55), (56), (57) and (58), Helpman et al. (2010) prove the following results:

Proposition 1 *(i) Sectoral wage inequality in the open economy when some but not all firms export is strictly greater than in the closed economy; and (ii) Sectoral wage inequality in the open economy when all firms export is the same as in the closed economy.*

Proof. See Helpman et al. (2010). ■

Corollary 2 (to Proposition 1) *An increase in the fraction of exporting firms raises sectoral wage inequality when the fraction of exporting firms is sufficiently small and reduces sectoral wage inequality when the fraction of exporting firms is sufficiently large.*

Proof. See Helpman et al. (2010). ■

Both results hold for any measure of inequality that respects second-order stochastic dominance, including all standard measures of inequality, such as the Theil Index and the Gini Coefficient.

11 Conclusions

No further derivations required.

References

All references are included in the chapter itself.