# Products and Productivity*

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## Abstract

When firms make decisions about which product to manufacture at a more disaggregated level than observed in the data, measured firm productivity reflects both characteristics of the *firm* and attributes of the *products* that are non-randomly chosen by the firm. This paper develops a model of industry equilibrium in which firms endogenously sort across products and characterizes the resulting bias in measured firm and aggregate productivity. Calibrating the model's parameters, we show that endogenous product selection can have quantitatively important effects on measured firm and aggregate productivity and their response to changes in parameter values.

*Keywords:* Product choice, Productivity measurement, Firm heterogeneity, Industry deregulation

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## 1. Introduction

Measurement of firm productivity is one of the core empirical challenges in both micro and macroeconomics, and one that lies at the heart of an array of policy debates, ranging from the impact of information technology to the consequences of industry deregulation. This paper argues that the endogenous sorting of firms across products is an important and hitherto largely neglected source of bias in productivity measurement. Given that firms typically choose products at a more disaggregate level than is observed in plant and firm-level datasets, measured firm productivity reflects the unobserved attributes of products that are non-randomly chosen by the firm. This bias exists even if a firm produces a single product, because the firm chooses this single product from among several possible products with heterogeneous attributes.[1] To examine the direction and magnitude of the bias in measured firm and aggregate productivity, we develop a model of industry equilibrium in which firms endogenously sort across products. In the model, changes in parameters of technology or demand generate endogenous changes in aggregate productivity as a result of both firm entry and exit and the endogenous re-sorting of firms across products.

The paper is related to the large and growing literature that uses plant or firm-level data to analyze the microeconomics and macroeconomics of productivity. This literature stretches across fields as diverse as macroeconomics (Bloom 2008 and Rossi-Hansberg and Wright 2007), international trade (Pavcnik 2002, Trefler 2004, Tybout 2003), development economics (van Biesebroeck 2005, Banerjee and Munshi 2004) and industrial economics (Dunne *et al.* 1989, Levinsohn and Petrin 2003, Aghion *et al.* 2005 and Griffith *et al.* 2006). The vast majority of the firm and plant-level datasets used in this literature report the main industry of a firm or plant at a relatively aggregated level, but do not report more detailed information on the products within industries supplied by firms. It is precisely this situation that we seek to capture in our model, where the identity of the product supplied by a firm within the industry is not observed.[2]

The paper is also related to existing theories of industry dynamics, which emphasize ei-

[1] For simplicity, we focus in the model below on the case where a firm produces a single product, which is chosen from among alternative possible products. Our focus is therefore quite different from the analysis of multi-product firms in Bernard, Redding and Schott (2009), both because we are concerned with single-product firms, and because we are concerned with the measurement of productivity rather than product adding and dropping.

[2] We use the term "industry" to refer to the level at which output and factor inputs are observed in the data and the term "product" to refer to the *unobserved*, more disaggregated level at which firm decisions are actually made.

ther learning about a true unknown value of productivity (as in Jovanovic 1982) or stochastic realizations of true productivity (as in Hopenhayn 1992, Ericson and Pakes 1995, Melitz 2003 and Bernard, Redding and Schott 2007). Our approach extends Melitz's (2003) model of industry equilibrium. Melitz's framework substantially simplifies industry dynamics by making the following assumptions: a monopolistically competitive industry where firms supply varieties of a single product; firm productivity is a parameter which is drawn from a fixed distribution at the point of entry; and firms face a constant, exogenous probability of death thereafter. We extend this structure to allow a firm to choose one out of two heterogeneous products, which have different production techniques and enter demand asymmetrically, and have a relative price that is determined endogenously in general equilibrium. We believe this to be the simplest framework for understanding the impact of product choice on productivity measurement. It captures the endogenous sorting of firms across products, while remaining tractable enough to quantify the direction and magnitude of the bias in productivity measurement. It also allows analysis of the implications of endogenous product choice for the comparative static properties of the model.

Using the structure of the model, we derive the bias in measured firm productivity as a result of unobserved endogenous product selection following the standard "revenue production function" estimation approach used in recent empirical work to measure productivity in differentiated product markets (see for example Klette and Griliches 1996, Levinsohn and Melitz 2006, De Loecker 2008, Foster, Haltiwanger and Syverson 2008). The bias in measured firm productivity depends on true firm productivity, which determines the product supplied by the firm. The bias in measured firm productivity also depends on the parameters of technology and demand for the two products, which influence relative prices and expenditure shares for the two products, and hence revenue relative to factor inputs. Using the structure of the model, we aggregate across firms and derive the bias in measured aggregate productivity, which is the revenue-share weighted average of measured firm productivity. Calibrating the model's parameters, we show that the bias in measured firm and industry productivity can be quantitatively large and influences the response of both productivity measures to changes in technology and demand.

The remainder of the paper is structured as follows. Section 2 frames the problem and relates the bias in measured productivity from endogenous product selection to other sources of bias in the productivity literature. Section 3 develops the theoretical model, while Section

4 solves for general equilibrium. Section 5 derives expressions for the bias in measured firm and aggregate productivity as a result of endogenous product selection. Section 6 calibrates the model and examines the quantitative magnitude of the bias in measured productivity relative to other sources of bias in the productivity literature. Section 7 concludes. An appendix at the end of the paper collects together proofs and technical derivations.

## 2. Framing the Problem

The existing literature on productivity measurement has focused largely on three broad classes of problems. First, there is the "exit selection problem" of the non-random survival of firms and plants. A large amount of empirical research has demonstrated that exiting plants are systematically less productive than survivors (e.g., Dunne *et al.* 1989, Baily *et al.* 1992, Olley and Pakes 1996 and Foster et al. 2001). Second, there is the "endogeneity problem" that the factor input choices of surviving firms are partly determined in response to firm productivity. Therefore, if a production function is to be estimated, the simultaneity of factor input choices must be controlled for (Marschak and Andrews 1944, Olley and Pakes 1996, and Levinsohn and Petrin 2003). Third, there is a "specification problem" of the correct functional form of the production technology and the market structure assumptions needed to identify productivity separately from the influence of market power. By itself, this third issue encompasses a wide range of research on a variety of issues including Caves *et al.* (1982a,b), Hall (1988), Roeger (1995), Klette and Griliches (1996) and Levinsohn and Melitz (2006) among many others.

The role of these three classes of problems in the measurement of productivity can be illustrated as follows. Denote a standard revenue-based measure of firm productivity, such as labor productivity or total factor productivity (TFP) by $\theta$. The expected value of revenue-based productivity for a firm conditional on a vector of observable characteristics $X$ is:

$$E\left(\theta|X\right) = \underbrace{G\left(I_e = 1|X\right)}_{\text{Term } A}\underbrace{E\left(\theta|X, I_e = 1\right)}_{\text{Term } B} + \underbrace{\left[1 - G\left(I_e = 1|X\right)\right] \cdot 0}_{\text{Term } C} \tag{1}$$

where $I_e$ is an indicator variable that equals one if a firm enters and zero otherwise; $G\left(I_e = 1|X\right)$ is the probability of entry conditional on the observables; $E\left(\theta|X, I_e = 1\right)$ is the expected value of revenue-based productivity conditional on entry and the observables. The selection problem relates to correctly controlling for Terms $A$ and $C$, while the endogeneity and specification problems relate to adequately modelling Term $B$.

Our analysis emphasizes an additional and neglected challenge in measuring productivity. Since firms typically choose products at a more disaggregated level than is observed in the data, there is a "product selection problem" due to the endogenous sorting of firms across products. As firms choose which product to make based on characteristics including firm productivity, measured productivity differences across firms are in general influenced by their non-random product choice. This additional challenge can also be illustrated using the framework above. Taking the simplest case of two products within an industry, the expected value of revenue-based productivity for a firm conditional on the observables can be written as:

$$\begin{aligned} E\left(\theta|X\right) \quad = \quad & \underbrace{G\left(I_e=1|X\right)}_{\text{Term } A} \left[ \begin{array}{c} \underbrace{G\left(I_p=1|X,I_e=1\right)}_{\text{Term } D}\underbrace{E\left(\theta|X,I_e=1,I_p=0\right)}_{\text{Term } E} \\ +\underbrace{G\left(I_p=2|X,I_e=1\right)}_{\text{Term } F}\underbrace{E\left(\theta|X,I_e=1,I_p=1\right)}_{\text{Term } G} \end{array} \right] \\ & +\underbrace{\left[1-G\left(I_e=1|X\right)\right]\cdot 0}_{\text{Term } H}, \end{aligned} \tag{2}$$

where $I_p$ is an indicator variable that takes the values zero or one depending on whether product one or two respectively is supplied. Terms $A$ and $H$ capture the probabilities of entry and exit conditional on observables; Terms $D$ and $F$ capture the probabilities that the firm makes each product conditional on entry and the observables; Terms $E$ and $G$ capture expected revenue-based productivity conditional on making a product, on entering, and on the observables.

The product selection problem relates to the fact that product choice is determined by variables that also affect measured revenue-based productivity. Therefore Terms $D$ and $F$ are correlated with Terms $E$ and $G$. As a result there is not only an aggregation problem, because of the presence of more than one product in the industry, but also a selection problem, because firms with different characteristics are self-selecting into different products. In addition to this product selection problem, there remains the exit selection problem (Terms $A$ and $H$) and the endogeneity and specification problems (Terms $E$ and $G$).

## 3. Theoretical Model

In this section, we develop a theoretical model of industry equilibrium in which heterogeneous firms endogenously sort across products, thus introducing a bias into measured productivity. We consider a single industry within which consumers and firms choose whether to consume and produce varieties of two distinct products.[3] To keep the analysis as tractable as possible, we assume that consumer preferences between the two products can be represented by the following CES utility function:

$$U = \left[a_1 C_1^{\nu} + a_2 C_2^{\nu}\right]^{1/\nu}. \tag{3}$$

where $a_i > 0$ captures the strength of consumer preferences for product $i$, and we assume that the products are imperfect substitutes with elasticity of substitution $\psi = \frac{1}{1-\nu} > 1$.[4] Firms produce horizontally differentiated varieties of their chosen product. $C_i$ is therefore a consumption index defined over varieties $\omega$ of product $i$:

$$C_i = \left[\int_{\omega \in \Omega_i} q_i(\omega)^{\rho} d\omega\right]^{1/\rho}, \qquad P_i = \left[\int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} d\omega\right]^{1/1-\sigma}. \tag{4}$$

where $\Omega_i$ is the (endogenous) set of available varieties in market $i$, $P_i$ is the price index dual to $C_i$, and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution between varieties of the same product. We make the natural assumption that varieties of the same product are more easily substitutable than different products, so that $\sigma > \psi$.

Consumer expenditure minimization yields the following expression for equilibrium expenditure (equals revenue, $r_i(\omega)$) on a variety:

$$r_i(\omega) = R_i \left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} = \alpha_i(\mathcal{P})\, R \left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} \tag{5}$$

which is increasing in aggregate expenditure (equals aggregate revenue $R = R_1 + R_2 = \int_{\omega \in \Omega_1} r_1(\omega) d\omega + \int_{\omega \in \Omega_2} r_2(\omega) d\omega$), increasing in the share of expenditure allocated to product $i$, $\alpha_i(P_2/P_1) = \alpha_i(\mathcal{P})$, decreasing in own variety price, $p_i(\omega)$, and increasing in the price of competing varieties as summarized in the price index, $P_i$.

With CES utility, the share of expenditure allocated to product 1 is increasing in the relative price of product 2, $\mathcal{P} = P_2/P_1$ (since $\psi > 1$), and increasing in the relative weight of product 1 in consumer utility ($a_1/a_2$):

$$\alpha_1(\mathcal{P}) = \left[1 + \left(\frac{a_2}{a_1}\right)^{\psi} \mathcal{P}^{1-\psi}\right]^{-1}, \qquad \alpha_2(\mathcal{P}) = 1 - \alpha_1(\mathcal{P}). \tag{6}$$

[3] It is straightforward to embed this framework in a multi-industry model or to allow a finite number of distinct products within the industry. The model developed here is the simplest framework within which to demonstrate the importance of firms' choice between heterogeneous products in influencing measured firm and industry outcomes.

[4] One interpretation of the parameter $a_i$ is product quality, though it also captures other more subjective product characteristics that influence the representative consumer's demand for that product.

*3.1. Production*

As well as entering demand in different ways, the products have different production technologies. Labor is the sole factor of production and is supplied inelastically at its aggregate level $L$, which also indexes the size of the economy. The production technology follows Melitz (2003) in that variable cost is assumed to depend on heterogeneous firm productivity. We differ in that we allow for different products and hence endogenous product choice within the industry. To analyze the bias in measured productivity from endogenous product selection in as simple a framework as possible, we assume that firms choose one out of the two products to supply.[5] The labor required to produce $q_i$ units of a variety of product $i \in \{1, 2\}$ is given by:

$$l_i = f_i + \frac{b_i q_i}{\varphi} \tag{7}$$

so that the variable cost of production depends on $b_i$, which is common to all firms that supply product $i$, as well as on the firm-specific productivity, $\varphi$.[6]

Products differ in terms of both their fixed and variable costs of production. We assume that product 2 has a lower variable cost and higher fixed cost than product 1: $b_2 < b_1$ and $f_2 > f_1$. These assumptions are natural if a lower variable cost reflects a higher level of technology and a firm must incur a greater fixed cost in order to manufacture a higher technology product. Nonetheless, it is straightforward to also consider the case where product 2 has both a higher variable and fixed cost than product 1. Even in this case, both products are produced in equilibrium, because from (3) the products are imperfect substitutes in demand and the marginal utility derived from a product approaches infinity as consumption approaches zero. Therefore, the relative price indices for the products, $\mathcal{P}$, adjust to ensure that both products are produced in equilibrium.

Fixed production costs and consumer love of variety imply that each firm supplies a unique variety of its chosen product. Profit maximization yields the standard result that

[5] We assume a fixed cost of supplying more than one product as a result of managerial diseconomies of scope, which is prohibitively large so that no firm supplies more than one product in equilibrium. While it is straightforward to extend the model to allow for multi-product firms, as shown in the working paper version of this paper, the bias in measured productivity from endogenous product selection arises even with single-product firms, and so we do not pursue this extension here. For analyses of managerial diseconomies of scope, see for example Lucas (1978) and Rosen (1982).

[6] The assumption that fixed costs of production are independent of productivity captures the idea that many fixed costs, such as building and equipping a factory with machinery, are unlikely to vary substantially with firm productivity. As long as fixed costs are less sensitive to productivity than variable costs, there is endogenous selection on productivity in firms' exit and product choice decisions.

equilibrium prices are a constant mark-up over marginal cost, with the size of the mark-up depending on the elasticity of substitution between varieties:

$$p_i(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{wb_i}{\varphi}. \tag{8}$$

We choose the wage as the numeraire so that $w = 1$. Using this choice of numeraire and the pricing rule in the expression for revenue above, equilibrium firm revenue and profits are:

$$\begin{aligned} r_i(\varphi) &= \alpha_i(\mathcal{P})R\left(P_i\rho\frac{\varphi}{b_i}\right)^{\sigma-1} \\ \pi_i(\varphi) &= \frac{r_i(\varphi)}{\sigma} - f_i. \end{aligned} \tag{9}$$

One property of equilibrium revenue that will prove useful below is that the relative revenue of two firms with different productivities that supply the same product depends solely on their relative productivities: $r_i(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_i(\varphi')$. Similarly, the relative revenue of two firms with different productivities that supply different products depends on their relative productivities, the relative variable cost of making the two products, the relative expenditure share devoted to the two products, and relative price indices:

$$r_2(\varphi'') = \left(\frac{1-\alpha_1(\mathcal{P})}{\alpha_1(\mathcal{P})}\right)\left(\frac{\varphi''}{\varphi'}\mathcal{P}\frac{b_1}{b_2}\right)^{\sigma-1} r_1(\varphi'). \tag{10}$$

### *3.2. Industry Entry and Exit*

To enter the industry (and supply either product), a firm must pay a fixed entry cost, $f_e > 0$, which is thereafter sunk. After paying the sunk cost, the firm draws its productivity, $\varphi$, from a distribution, $g(\varphi)$, with corresponding cumulative distribution $G(\varphi)$. This formulation captures the idea that there are sunk costs of entering an industry and that, once these costs are incurred, some uncertainty regarding the nature of production and firm profitability is realized. Firm productivity is assumed to remain fixed thereafter, and firms face a constant exogenous probability of death, $\delta$, which we interpret as due to *force majeure* events beyond managers' control.[7]

[7] Firm death ensures steady-state entry into the industry. New entrants make an endogenous exit decision, since their decision whether or not to produce in the industry depends on their productivity draw $\varphi$ from the distribution $g(\varphi)$. Together with fixed production costs, this will generate the result that exiting firms are on average less productive than surviving firms. For incumbent firms, the probability of death $\delta$ is independent of productivity. This assumption can be relaxed by allowing firm productivity to evolve stochastically after entry (e.g. Hopenhayn 1992). While this would achieve greater realism, it would not change the qualitative results below on the importance of endogenous product choice for measured firm and industry productivity, and would come at the cost of a substantial increase in the complexity of the industry dynamics.

A particularly tractable productivity distribution is the Pareto distribution, $g(\varphi) = zk^z\varphi^{-(z+1)}$, where $k > 0$ is the minimum value of productivity in the industry and $z > 0$ determines the skewness of the distribution. Although we develop our results analytically without assuming a particular functional form for the productivity distribution, we consider a Pareto distribution when we calibrate the model and examine the quantitative magnitude of the bias in measured firm and aggregate productivity due to endogenous product selection.

After entry, firms decide whether to begin producing in the industry or exit. If firms decide to produce, they choose which of the two products to supply. Therefore, the value of a firm with productivity $\varphi$ is the maximum of 0 (if the firm exits) or the stream of future profits from producing one of the two products discounted by the probability of firm death:

$$v(\varphi) = \max\left\{0, \frac{1}{\delta}\pi_1(\varphi), \frac{1}{\delta}\pi_2(\varphi)\right\}. \tag{11}$$

### *3.3. Product Choice*

Firms decide which product to make based on their realized productivity, taking as given aggregate variables such as the price indices. From our expression for equilibrium profits above, firms with zero productivity have negative post-entry profits and profits are monotonically increasing in productivity. Fixed production costs mean that there is a positive value for productivity below which negative profits would be made. Firms drawing a productivity below this **zero-profit productivity cutoff**, $\varphi^*$, exit the industry immediately.

Since product 2 has a higher fixed product cost than product 1, firms with zero productivity would make the largest losses from producing product 2:

$$0 > \pi_1(0) = -f_1 > \pi_2(0) = -f_2. \tag{12}$$

Since profits for each product are monotonically increasing in productivity, a necessary condition for both products to be produced is that profits from product 2 increase more rapidly with productivity than those from product 1:

$$\begin{aligned} \left(\frac{1-\alpha_1(\mathcal{P})}{\alpha_1(\mathcal{P})}\right)\left(\mathcal{P}\frac{b_1}{b_2}\right)^{\sigma-1} &> 1 \\ \Rightarrow \left(\frac{a_2}{a_1}\right)^{\psi}\left(\frac{b_1}{b_2}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi} &> 1 \end{aligned} \tag{13}$$

where the *relative* rate at which profits increase with productivity is independent of productivity, and depends instead on parameters, such as the demand-shifter $a_i$ and the variable cost parameter $b_i$, as well as endogenous relative price indices, $\mathcal{P}$.

The *sufficient* condition for both products to be produced is that profits are positive in each product market and exceed those in the other product market over a range of productivities:

$$\begin{aligned} \pi_1(\varphi) &> 0 \quad \text{and} \quad \pi_1(\varphi) > \pi_2(\varphi) \quad \text{for } \varphi \in \Phi_1 \subset (0, \infty) \\ \pi_2(\varphi) &> 0 \quad \text{and} \quad \pi_2(\varphi) > \pi_1(\varphi) \quad \text{for } \varphi \in \Phi_2 \subset (0, \infty) \end{aligned} \tag{14}$$

which requires the profit functions for the two products to intersect at a value for productivity where positive profits are made, as shown graphically in Figure 1. As we show formally when we solve for general equilibrium, consumers' taste for both products implies that relative prices, $\mathcal{P}$, will adjust to ensure that the conditions in equation (14) are satisfied even if product 2 has both a higher fixed and variable cost. The point at which the two profit functions intersect defines the **product-indifference productivity cutoff**, $\varphi^{**}$, at which a firm is exactly indifferent between the two products.

The higher fixed cost for product 2 and the requirement that the two profit functions intersect at a value for productivity where positive profits are made together imply that product 1 will be produced by the lowest productivity firms that are active in the industry and product 2 will be produced by higher productivity firms. The zero-profit productivity cutoff determining the lowest level of productivity where product 1 is produced is given by:

$$r_1(\varphi^*) = \sigma f_1, \tag{15}$$

while the product-indifference productivity cutoff defining the lowest level of productivity where product 2 is produced is defined by:

$$\frac{r_2(\varphi^{**})}{\sigma} - f_2 = \frac{r_1(\varphi^{**})}{\sigma} - f_1. \tag{16}$$

Firms drawing a productivity below $\varphi^{**}$ but above $\varphi^*$ will make product 1, while those drawing a productivity above $\varphi^{**}$ will make product 2.

The special case of our framework where $a_1 = a_2 = 1$, $\psi = \sigma$, $f_1 = f_2 = f$ and $b_1 = b_2 = 1$ corresponds to the Melitz (2003) model. With $a_1 = a_2 = 1$ and $\psi = \sigma$, the two products receive equal weight in consumers' utility, and the elasticity of substitution across products is the same as the elasticity of substitution across varieties within products.

With $f_1 = f_2 = f$ and $b_1 = b_2 = 1$, there are no differences in production technology across products. Therefore, taking these two sets of properties together, the model collapses to the special case of many varieties of a single product within the industry. In contrast to this special case, our framework allows for heterogeneity in both demand and production technology across products, and we discuss below the respective contributions of these sources of heterogeneity to biases in productivity measurement.

### *3.4. Free Entry*

From the characterization of entry and product choice in the previous sections, the *ex ante* probability of successful entry into the industry is $[1 - G(\varphi^*)]$, with the *ex ante* probability of producing product 1 given by $[G(\varphi^{**}) - G(\varphi^*)]$, and the *ex ante* probability of producing product 2 given by $[1 - G(\varphi^{**})]$. The *ex post* productivity distribution for each product, $\mu_i(\varphi)$, is conditional on successful entry and product choice and is a truncation of the *ex ante* productivity distribution, $g(\varphi)$:

$$\begin{aligned} \mu_1(\varphi) &= \begin{cases} \frac{g(\varphi)}{G(\varphi^{**})-G(\varphi^*)} & \text{if } \varphi \in [\varphi^*, \varphi^{**}) \\ 0 & \text{otherwise} \end{cases}, \\ \mu_2(\varphi) &= \begin{cases} \frac{g(\varphi)}{1-G(\varphi^{**})} & \text{if } \varphi \in [\varphi^{**}, \infty) \\ 0 & \text{otherwise} \end{cases}. \end{aligned} \tag{17}$$

In equilibrium we require the expected value of entry in the industry, $v_e$, to equal the sunk entry cost, $f_e$. The expected value of entry is the *ex ante* probability of making product 1 times expected profitability in product 1 until death plus the *ex ante* probability of making product 2 times expected profitability in product 2 until death, and the **free entry condition** is:

$$v_e = \left[\frac{G(\varphi^{**}) - G(\varphi^*)}{\delta}\right]\overline{\pi}_1 + \left[\frac{1 - G(\varphi^{**})}{\delta}\right]\overline{\pi}_2 = f_e, \tag{18}$$

where $\overline{\pi}_i$ is expected or average firm profitability in product market $i$. Equilibrium revenue and profit in each market are constant elasticity functions of firm productivity (equation (9)) and, therefore, average revenue and profit are equal respectively to the revenue and profit of a firm with weighted average productivity, $\bar{r}_i = r_i(\widetilde{\varphi}_i)$ and $\overline{\pi}_i = \pi_i(\widetilde{\varphi}_i)$, where weighted average productivity, $\tilde{\varphi}_1(\varphi^*, \varphi^{**})$ and $\tilde{\varphi}_2(\varphi^{**})$, is determined by the *ex post* productivity distributions, $\mu_i(\varphi)$, and is defined formally in the Appendix.

### *3.5. Product and Labor Markets*

The steady-state equilibrium is characterized by a constant mass of firms entering each period, $M_e$, and a constant mass of firms producing within each product market, $M_i$. In steady-state equilibrium, the mass of firms that enter and draw a productivity sufficiently high to produce in a product market must equal the mass of firms already within that product market who die, yielding the following **steady-state stability conditions** (**SC**):

$$[1 - G(\varphi^{**})]M_e = \delta M_2 \tag{19}$$

$$[G(\varphi^{**}) - G(\varphi^{*})]M_e = \delta M_1. \tag{20}$$

The firms' equilibrium pricing rule implies that the prices charged for individual varieties are inversely related to firm productivity. The price indices are weighted averages of the prices charged by firms with different productivities, with the weights determined by the *ex post* productivity distributions. Exploiting this property of the price indices, we can write them as functions of the mass of firms producing a product, $M_i$, and the price charged by a firm with weighted average productivity within each product market, $p_i(\widetilde{\varphi}_i)$:

$$P_1 = M_1^{1/1-\sigma} p_1(\widetilde{\varphi}_1), \qquad P_2 = M_2^{1/1-\sigma} p_2(\widetilde{\varphi}_2) \tag{21}$$

In equilibrium, we also require that the demand for labor used in production, $L^p$, and entry, $L^e$, equals the economy's supply of labor, $L$:

$$L_p + L_e = L. \tag{22}$$

## 4. Industry Equilibrium

Industry equilibrium is referenced by the sextuple $\{\varphi^*,\ \varphi^{**},\ P_1,\ P_2,\ R_1,\ R_2\}$, in terms of which all other endogenous variables may be written. The equilibrium vector is determined by the following equilibrium conditions: the zero-profit productivity cutoff (equation (15)), the product-indifference productivity cutoff (equation (16)), free entry (18), steady-state stability ((19) and (20)), the values for the equilibrium price indices implied by consumer and producer equilibrium (equation (21)), and labor market clearing (22).

### *4.1. Relative Supply and Relative Prices*

Combining the zero-profit productivity cutoff condition (15), the product indifference cutoff condition (16) and relative variety profitability (10), we obtain a downward-sloping (supply-side) relationship between two key variables: the relative productivity cutoffs, $\varphi^{**}/\varphi^{*}$, and the relative price indices, $\mathcal{P}$:

$$\Lambda \equiv \frac{\varphi^{**}}{\varphi^{*}} = \left[ \frac{\left(\frac{f_2}{f_1} - 1\right)}{\left[\left(\frac{a_2}{a_1}\right)^{\psi}\left(\frac{b_1}{b_2}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi} - 1\right]} \right]^{1/(\sigma-1)}. \tag{23}$$

where this relationship is derived under the assumption that products are asymmetric, so that $f_2 > f_1$ and equation (13) holds. In the special case of symmetric products discussed above, where $a_1 = a_2 = 1$, $\psi = \sigma$, $f_1 = f_2 = f$ and $b_1 = b_2 = 1$, our model collapses to the standard model of industry equilibrium. In this case, there is a single cutoff for productivity, the zero-profit cutoff $\varphi^{*}$, and firms who draw a productivity above $\varphi^{*}$ are indifferent between the two identical products and therefore manufacture a variety of either product.

Equation (23) is the mathematical statement of the relationship between the two productivity cutoffs captured graphically in Figure 1. As $\varphi^{**}$ rises relative to $\varphi^{*}$, the fraction of firms producing product 2 falls, and the fraction of firms producing product 1 increases. Equation (23) therefore yields the following intuitive comparative statics. A higher value for the relative price, $\mathcal{P}$, increases profitability in product 2 relative to product 1 and causes the relative number of firms producing product 2 to rise, i.e. a reduction in $\varphi^{**}$ relative to $\varphi^{*}$, since $\sigma > \psi$. For a given value for the relative price, $\mathcal{P}$, a higher fixed cost for product 2 , $f_2$, reduces profitability in product 2 and causes the relative number of firms producing product 2 to fall, i.e. an increase in $\varphi^{**}$ relative to $\varphi^{*}$.

### *4.2. Relative Demand and Relative Prices*

Combining the equilibrium price indices in (21) and the steady-state stability conditions in (19) and (20), we obtain the following upward-sloping (demand-side) relationship between the relative productivity cutoffs and the relative price indices:

$$\Psi\left(\frac{\varphi^{**}}{\varphi^{*}}\right) \equiv \left[\left(\frac{b_2}{b_1}\right)^{\sigma-1} \frac{\int_{\varphi^{*}}^{\varphi^{**}} \varphi^{\sigma-1} g(\varphi)\, d\varphi}{\int_{\varphi^{**}}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi}\right] = \mathcal{P}^{\sigma-1}. \tag{24}$$

This expression has an intuitive interpretation. An increase in the relative price index for product 2, $\mathcal{P}$, reduces demand for product 2 relative to product 1 and shrinks the range of productivities where product 2 is produced relative to the range where product 1 is produced, i.e. an increase in $\varphi^{**}/\varphi^{*}$. For a given value of $\varphi^{**}/\varphi^{*}$, an increase in $b$, the relative variable cost for product 2, raises the price of product 2 varieties relative to product 1 varieties, i.e. an increase in $\mathcal{P}$.

*4.3. Free Entry*

The free entry condition can be written in a more convenient form using the expression for the zero-profit productivity cutoff (15), the relationship between the revenues of firms producing varieties in the same market with different productivities $(r_i(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_i(\varphi'))$, and the supply-side relationship between the two productivity cutoffs (23):

$$
\begin{aligned}
v_e = {} & \frac{f_1}{\delta} \int_{\varphi^*}^{\Lambda\varphi^*} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi \\
& + \frac{f_1}{\delta} \int_{\Lambda\varphi^*}^{\infty} \left[ \left( \frac{a_2}{a_1} \right)^{\psi} \left( \frac{b_1}{b_2} \right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - \frac{f_2}{f_1} \right] g(\varphi) d\varphi = f_e.
\end{aligned} \tag{25}
$$

where $\Lambda$ is defined in equation (23).

*4.4. Steady-state Stability, Labor Market Clearing and Goods Market Clearing*

Using the steady-state stability conditions to substitute for the *ex ante* probability of producing each product in the free entry condition, total payments to labor used in entry equal total industry profits: $L_e = M_e f_e = M_1 \bar{\pi}_1 + M_2 \bar{\pi}_2 = \Pi$ (by choice of numeraire, $w = 1$). The existence of a competitive fringe of potential entrants means that firms enter until the expected value of entry equals the sunk entry cost, and as a result the entire value of industry profits is paid to labor used in entry.

Total payments to labor used in production equal the difference between industry revenue, $R$, and industry profits, $\Pi$: $L_p = R - \Pi$. Taking these two results together, total payments to labor used in both entry and production equal industry revenue, $L = R$. Substituting for $R$ in the expressions for $L_e$ and $L_p$ above, this establishes that the labor market clears.

In equilibrium we also require the goods market to clear, which implies that the value of expenditure equals the value of revenue for each product. Utility maximization implies that the consumer allocates the expenditure shares $\alpha_1(\mathcal{P})$ and $(1 - \alpha_1(\mathcal{P}))$ to the two products. Imposing expenditure equals revenue for each product, goods market clearing may be expressed as:

$$
R_1 = \alpha_1(\mathcal{P})R, \qquad R_2 = (1 - \alpha_1(\mathcal{P}))R. \tag{26}
$$

*4.5. Existence and Uniqueness of Equilibrium*

**Proposition 1** *There exists a unique value of the equilibrium vector* $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$*. All other endogenous variables of the model can be written as functions of this equilibrium vector.*

**Proof.** See Appendix. ■

Combining the supply-side relationship between the relative productivity cutoffs and relative prices in equation (23) with the demand-side relationship in equation (24) yields a unique equilibrium value of $\varphi^{**}/\varphi^*$ and $\mathcal{P} = P_2/P_1$. In the proof of Proposition 1, we establish that at the unique equilibrium value of $\mathcal{P}$, $\varphi^* > 0$ and $\varphi^{**} > \varphi^*$, so that both products are produced in equilibrium.

*4.6. Properties of Industry Equilibrium*

A key implication of the model is that firms endogenously sort across products depending on their heterogeneous characteristics. As a result of this non-random choice of products, measured productivity reflects both firm characteristics and product attributes.

**Proposition 2** *There is endogenous sorting of firms across products such that higher productivity firms choose the higher fixed cost product.*

**Proof.** This proposition follows immediately from the Proof of Proposition 1 where we have established that $\varphi^* > 0$ and $\varphi^{**} > \varphi^*$. ■

The productivity thresholds $\{\varphi^*, \varphi^{**}\}$ that determine the range of productivities where products 1 and 2 are manufactured depend on both the parameters of the production technology $\{f_1, f_2, b_1, b_2\}$ and those of demand $\{a_1, a_2, \psi, \sigma\}$. Intuitively, technology and demand parameters each influence the slope of the profit functions shown in Figure 1, and so influence $\varphi^*$ and $\varphi^{**}$. The roles of technology and demand can be seen particularly clearly for the case of a Pareto productivity distribution. In this case, the expression for relative demand in equation (24) simplifies, and combining relative demand and relative supply, the ratio of the two productivity cutoffs $\varphi^{**}/\varphi^* > 1$ is implicitly defined as follows:

$$\left[\left(\frac{\varphi^{**}}{\varphi^*}\right)^{\gamma} - 1\right]^{\frac{1}{\sigma-1}} = \left[\left(\frac{\varphi^{**}}{\varphi^*}\right)^{1-\sigma}\left(\frac{f_2}{f_1} - 1\right) + 1\right]^{\frac{1}{\sigma-\psi}}\left(\frac{a_1}{a_2}\right)^{\frac{\psi}{\sigma-\psi}}\left(\frac{b_2}{b_1}\right)^{\frac{\psi-1}{\sigma-\psi}}. \qquad (27)$$

where $\gamma \equiv z - \sigma + 1$ and $z$ is the shape parameter of the Pareto distribution.

Taking the technology parameters in this expression first, an increase in the fixed cost of production for product 2 relative to product 1 $(f_2/f_1)$ increases $\varphi^{**}$ relative to $\varphi^{*}$. Intuitively, as the fixed cost for product 2 rises relative to that for product 1, a higher productivity is required for a firm to charge a low enough price and generate enough revenue to cover the higher fixed cost for product 2. Similarly, an increase in the variable cost of production for product 2 relative to product 1 $(b_2/b_1)$ increases $\varphi^{**}$ relative to $\varphi^{*}$.

Taking the demand parameters in the expression next, an increase in the weight of product 1 relative to product 2 in consumer utility $(a_1/a_2)$ increases $\varphi^{**}$ relative to $\varphi^{*}$. The reason is that, as consumers increase the share of expenditure on product 1 and reduce that on product 2, a higher productivity is required for a firm to charge a low enough price and generate enough revenue to cover the higher fixed cost for product 2. Finally, the elasticities of substitution $\sigma$ and $\psi$ determine the impact of marginal cost differences on relative revenue, and hence also influence the relative range of productivities where products 1 and 2 are produced.

## 5. Implications for Measured Productivity

In this section, we examine the implications of the non-random sorting of firms across products for measured productivity. We suppose that a researcher has firm-level data on revenue and factor inputs for an industry and falsely assumes that all firms produce the same product within the industry. This is the special case of our model that corresponds to the Melitz (2003) model, in which $a_1 = a_2 = 1$, $\psi = \sigma$, $f_1 = f_2 = f$ and $b_1 = b_2 = 1$. While the researcher assumes a single product, the true model involves two products with heterogeneous characteristics within the industry and has firms endogenously sorting across these two products, as analyzed above.

Standard revenue-based measures of productivity (see for example Klette and Griliches 1996, Levinsohn and Melitz 2006, and De Loecker 2008) deflate firm revenue by a common industry price index, $P$:

$$\frac{r(\varphi)}{P} = \frac{p(\varphi)\, q(\varphi)}{P}.$$

In the special case of a single product assumed by the researcher, the CES inverse demand curve can be used to substitute for $p(\varphi)$ as a function of $q(\varphi)$, which together with the production technology implies the following "revenue production function":

$$\log r^s(\varphi) - \log P^s = \frac{\sigma-1}{\sigma}\left[\log\varphi + \log l_v^s(\varphi)\right] + \frac{1}{\sigma}\log\left(\frac{R^s}{P^s}\right), \tag{28}$$

where the superscript $s$ indicates values for a single product; thus $l_v^s(\varphi)$ denotes variable labor input with a single product, while $P^s$ and $R^s$ respectively denote the aggregate price index and aggregate revenue with a single product.

To obtain consistent estimates of firm revenue productivity from (28), we require appropriate instruments for variable labor input, $l_v(\varphi)$, and controls for aggregate industry characteristics, $R_s$ and $P_s$, which can be captured using time dummies for the industry. The resulting measure of firm revenue productivity in the special case of a single product is:

$$\theta^s(\varphi) = \varphi^{\frac{\sigma-1}{\sigma}}, \tag{29}$$

where the use of revenue data implies that measured firm productivity is influenced by both the firm productivity draw ($\varphi$) and the equilibrium mark-up of price over marginal cost ($\sigma/(\sigma-1)$).

In contrast, in the true model with endogenous product selection, the CES inverse demand curve together with the production technology (7), and equilibrium expenditure shares (6) imply the following "revenue production function":

$$\begin{aligned}\log r_i(\varphi) - \log P &= \frac{\sigma-1}{\sigma}\left[\log\left(\frac{\varphi}{b_i}\frac{P_i}{P}\right) + \frac{1}{\sigma-1}\log\left(\alpha_i(\mathcal{P})\right) + \log l_v(\varphi)\right] \\ &\quad + \frac{1}{\sigma}\log\left(\frac{R}{P}\right),\end{aligned} \tag{30}$$

where $i$ indexes products, $l_v(\varphi)$ denotes variable labor input, $P_i$ is the price index for product $i$, $P$ is the aggregate price index dual to (3), and $R$ is aggregate revenue.

If the true model involves endogenous product selection (equation (30)), but the researcher estimates the revenue production function under the false assumption of a single product (equation (28)), the resulting measure of firm revenue productivity is:

$$\theta_i(\varphi) = \eta_i(\varphi)\,\varphi^{\frac{\sigma-1}{\sigma}}, \qquad \eta_i(\varphi) \equiv \left(\frac{1}{b_i}\frac{P_i}{P}\right)^{\frac{\sigma-1}{\sigma}}\left(\alpha_i(\mathcal{P})\right)^{\frac{1}{\sigma}}, \tag{31}$$

where $\eta_i(\varphi)$ is the bias due to endogenous product selection. This bias is a function of the firm's productivity draw ($\varphi$), which determines the firm's choice of product to supply. Given the firm's choice of product $i$, the bias depends on the product variable cost ($b_i$) and product market conditions, which include the product price index relative to the aggregate price index

$(P_i/P)$ and the expenditure share $\alpha_i(\mathcal{P})$. Both dimensions of product market conditions ($P_i/P$ and $\alpha_i(\mathcal{P})$) are influenced by the demand parameter $a_i$, which incorporates product quality. Indeed, under our assumptions of CES preferences and monopolistic competition, differences in variable production costs ($b_i$) across products have similar effects on equilibrium firm revenue, and hence on measured firm revenue productivity, as differences in the demand parameters ($a_i$).[8]

While the expenditure share for each product ($\alpha_i(\mathcal{P})$ in equation (6)) lies between zero and one, the ratios of the product price indices to the aggregate price index are given by:

$$\frac{P_1}{P} = \frac{1}{\left[a_1^{\psi} + a_2^{\psi}\mathcal{P}^{1-\psi}\right]^{\frac{1}{1-\psi}}}, \quad \frac{P_2}{P} = \frac{1}{\left[a_1^{\psi}\left(1/\mathcal{P}\right)^{1-\psi} + a_2^{\psi}\right]^{\frac{1}{1-\psi}}},$$

which depending on the values of the parameters $\{a_1,\ a_2\}$ and equilibrium relative price indices ($\mathcal{P}$) can be greater than or less than one. Combining the variable cost parameters $\{b_1,\ b_2\}$, expenditure shares $\{\alpha_1(\mathcal{P}),\ \alpha_2(\mathcal{P})\}$ and price indices relative to the aggregate price index $\{P_1/P,\ P_2/P\}$, it follows that the bias ($\eta_i(\varphi)$) is in general different from one, and can be greater than one for both products, less than one for both products, or greater than one for one product and less than one for the other product.

Whenever the bias is not equal to one, measured firm revenue productivity reflects both *firm* characteristics ($\varphi$) and *product* attributes ($\eta_i(\varphi)$). Without separate data on which product is supplied by a firm, the productivity of the firm cannot be separately disentangled from the attributes of the products that it chooses to supply. Furthermore, product attributes are systematically correlated with firm characteristics, since as shown above firms with higher productivity draws ($\varphi$) self-select into the product with the higher fixed cost.

For a given productivity draw ($\varphi$) and choice of product $i$, measured firm revenue productivity in (31) is decreasing in the product variable cost ($b_i$), is increasing in the product expenditure share ($\alpha_i(\mathcal{P})$), and is increasing in the product price index relative to the aggregate price index ($P_i/P$) for a given expenditure share. A low product variable cost, a high product expenditure share and a high product price index relative to the aggregate price index for a given expenditure share each raise revenue relative to variable labor input, and hence raise measured firm revenue productivity.

[8] The equilibrium revenue of a firm supplying a variety of product $i$, can be written as: $r_i(\varphi) = (\rho\varphi/b_i)^{\sigma-1} P_i^{\sigma-1}\alpha_i(\mathcal{P})\,R$, where $\alpha_i(\mathcal{P}) = a_i^{\psi}P_i^{1-\psi}P^{\psi-1}$. Therefore $b_i$ and $a_i$ enter equilibrium revenue in a similar way with different exponents ($b_i^{1-\sigma}$ and $a_i^{\psi}$).

Following the standard approach for productivity aggregation, we define measured aggregate productivity, $\Theta$, as the revenue-share weighted average of measured firm productivity. Therefore, in the special case of a single product, measured aggregate revenue productivity is from (29):

$$\begin{aligned} \Theta^s &= \int_{\varphi^*}^{\infty} \frac{r(\varphi)}{R} \frac{\varphi^{\frac{\sigma-1}{\sigma}} g(\varphi)}{1-G(\varphi^*)} d\varphi, \\ &= \frac{\sigma f}{(\varphi^*)^{\sigma-1} L} \int_{\varphi^*}^{\infty} \frac{\varphi^{\sigma-\frac{1}{\sigma}} g(\varphi)}{1-G(\varphi^*)} d\varphi, \end{aligned} \tag{32}$$

where the second equation uses the fact that the relative revenue of two firms supplying the same product depends on relative productivities, the zero-profit cutoff condition (15), $R = L$, and $f = f_1 = f_2$.

In contrast, if the true model involves endogenous product selection and the researcher falsely assumes a single product, measured aggregate productivity is from (31):

$$\begin{aligned} \Theta &= \int_{\varphi^*}^{\varphi^{**}} \frac{r_1(\varphi)}{R} \frac{\eta_1(\varphi) \varphi^{\frac{\sigma-1}{\sigma}} g(\varphi)}{G(\varphi^{**}) - G(\varphi^*)} d\varphi + \int_{\varphi^{**}}^{\infty} \frac{r_2(\varphi)}{R} \frac{\eta_2(\varphi) \varphi^{\frac{\sigma-1}{\sigma}} g(\varphi)}{1-G(\varphi^{**})} d\varphi, \\ &= \Upsilon \frac{\sigma f_1}{(\varphi^*)^{\sigma-1} L} \int_{\varphi^*}^{\infty} \frac{\varphi^{\sigma-\frac{1}{\sigma}} g(\varphi)}{1-G(\varphi^*)} d\varphi, \end{aligned} \tag{33}$$

where the second equation again uses the expression for the relative revenue of firms with different productivities, the zero-profit cutoff condition (15), the product indifference cutoff condition (16), and $R = L$.

The bias in measured aggregate productivity as a result of the endogenous sorting of firms across products ($\Upsilon$) is a ratio of weighted averages of measured firm productivity:

$$\Upsilon \equiv \frac{\int_{\varphi^*}^{\varphi^{**}} \frac{\eta_1(\varphi)\varphi^{\sigma-\frac{1}{\sigma}} g(\varphi)}{G(\varphi^{**})-G(\varphi^*)} d\varphi + \int_{\varphi^{**}}^{\infty} \left(\frac{\varphi^*}{\varphi^{**}}\right)^{\sigma-1} \frac{(f_2-f_1)}{f_1\left[1-\frac{\alpha_1(\mathcal{P})}{1-\alpha_1(\mathcal{P})}\left(\frac{1}{\mathcal{P}}\frac{b_2}{b_1}\right)^{\sigma-1}\right]} \frac{\eta_2(\varphi)\varphi^{\sigma-\frac{1}{\sigma}} g(\varphi)}{1-G(\varphi^{**})} d\varphi}{\int_{\varphi^*}^{\infty} \frac{\varphi^{\sigma-\frac{1}{\sigma}} g(\varphi)}{1-G(\varphi^*)} d\varphi} > 0. \tag{34}$$

The bias in measured aggregate productivity ($\Upsilon$) therefore has three components: (a) the bias in measured firm productivity ($\eta_1(\varphi)$, $\eta_2(\varphi)$), (b) the production technology ($b_i$, $f_i$) and product market conditions ($P_i$ and $\alpha_i(\mathcal{P})$, where $\alpha_i(\mathcal{P})$ incorporates the demand parameter $a_i$), (c) the productivity cutoffs ($\varphi^*$, $\varphi^{**}$). To examine the quantitative magnitude of these sources of bias in measured aggregate productivity, we turn in the next section to a calibration of the model's parameters and a simulation of its comparative statics.

## 6. Quantitative Analysis

### *6.1. Model Calibration*

To explore the quantitative magnitude of the bias in measured productivity as a result of endogenous product selection, we follow much of the heterogeneous firm literature in assuming a Pareto productivity distribution.[9] As discussed above, the Pareto distribution is both tractable and provides a reasonable approximation to the observed distribution of firm sizes (see for example Axtell 2001). The distribution of firm productivity is therefore:

$$g(\varphi) = zk^z \varphi^{-(z+1)}, \tag{35}$$

where $k > 0$ is the minimum value for productivity ($\varphi \geq k$) and $z > \sigma - 1$ is a shape parameter that determines the skewness of the distribution.

To calibrate the main parameters of the model, we use estimates from the existing literature. We set the elasticity of substitution between varieties equal to $\sigma = 3.8$ and the elasticity of substitution between products equal to $\psi = 2$, which are in line with the empirical estimates in Bernard, Eaton, Jensen and Kortum (2003) and Broda and Weinstein (2006). We set the shape parameter of the Pareto productivity distribution equal to $z = 4$, which satisfies $z > \sigma - 1$, and hence ensures a finite mean of firm sales. The choice of the lower limit of the Pareto productivity distribution involves a choice of units in which to measure productivity, and without loss of generality we set $k = 0.1$. Similarly, specifying the economy's labor endowment involves choosing units in which to count workers, and without loss of generality, we set $L = 100$. Changing the sunk cost of entry rescales the mass of firms in the industry, and hence we set $f_e = 5$. As the values of the demand parameters $a_i$ have similar effects on equilibrium revenue and measured revenue productivity as the values of the variable cost parameters $b_i$, we set $a_1 = a_2 = 0.5$ and concentrate on differences in production technology between the two products. Finally, the model features endogenous exit (because firms with low productivity draws choose to exit the industry) and also exogenous death as a result of *force majeure* events. Changes in the probability of exogenous firm death, $\delta$, rescale the mass of entrants relative to the mass of firms, and hence without loss of generality we set $\delta = 0.025$.

With a Pareto distribution of firm productivity, the relative value of the productivity cutoffs for producing the two products ($\varphi^{**}/\varphi^{*}$) depends solely on the *relative* value of the

[9] See for example Helpman, Melitz and Yeaple (2004) and Ghironi and Melitz (2005).

variable costs $\{b_1,\ b_2\}$ and fixed costs $\{f_1,\ f_2\}$ (equation (27)). Therefore, without loss of generality, we set $f_2 = 1$ and $b_1 = 1$, and consider values of the fixed costs for product 1 ranging from 45 to 90 percent of those for product 2 and values of the variable cost for product 2 ranging from 45 to 90 percent of those for product 1.[10] In each case, we consider first the bias in measured firm productivity, and next the bias in measured aggregate productivity.

### 6.2. *Bias in Measured Firm Productivity*

To evaluate the quantitative magnitude of the bias in measured firm productivity, we evaluate the expression for the expected revenue productivity of a firm in equation (2) using the Pareto productivity distribution from equation (35):

$$E(\theta) = \underbrace{\left(\frac{k}{\varphi^*}\right)^z}_{\text{Term } A} \left[ \begin{array}{c} \underbrace{\left(1 - \left(\frac{\varphi^*}{\varphi^{**}}\right)^z\right)}_{\text{Term } D} \underbrace{\eta_1\left(\varphi|_{\varphi \in [\varphi^*,\varphi^{**}]}\right)}_{\text{Term } E} \varphi^{\frac{\sigma-1}{\sigma}} \\ + \underbrace{\left(\frac{\varphi^*}{\varphi^{**}}\right)^z}_{\text{Term } F} \underbrace{\eta_2\left(\varphi|_{\varphi \in [\varphi^{**},\infty)}\right)}_{\text{Term } G} \varphi^{\frac{\sigma-1}{\sigma}} \end{array} \right] + \underbrace{\left[1 - \left(\frac{k}{\varphi^*}\right)^z\right]}_{\text{Term } H} .0, \qquad (36)$$

where as before Terms $A$ and $H$ capture the probabilities of entry and exit respectively; Terms $D$ and $F$ capture the probabilities that the firm supplies each product conditional on entry; Terms $E$ and $G$ capture expected revenue productivity conditional on entering and supplying each product.

In Figure 2, we display the six terms that compose expected firm revenue productivity as the product 1 fixed cost varies from 0.45 to 0.90 for a given value of the product 2 variable cost of $b_2 = 0.45$. As the product 1 fixed cost increases, higher productivity is required in order to generate sufficient revenue to cover the product 1 fixed cost, which implies a rise in the zero-profit cutoff productivity ($\varphi^*$) and a decline in the probability of entry (Term $A$ as shown in Panel (i) of the figure). The counterpart of the decline in the probability of entry is a corresponding rise in the probability of exit (Term $H$ as shown in Panel (ii) of the figure). The increase in the product 1 fixed cost also reduces the profitability of product 1 relative to product 2. Therefore there is a fall in the product indifference cutoff productivity ($\varphi^{**}$) relative to the zero-profit cutoff productivity ($\varphi^*$), which leads to a decline in the probability of supplying product 1 (Term $D$ shown in Panel (iii) of the figure) and a rise in the probability of supplying product 2 (Term $F$ shown in Panel (iv) of the figure).

[10]While we concentrate on the case where product 2 has a higher fixed cost but a lower variable cost, it is straightforward to instead consider the case where product 2 has both a higher fixed and variable cost, in which case relative price indices adjust to ensure that both products are produced, as discussed above.

While the change in the product 1 fixed cost does not directly affect the bias in measured firm productivity for each product ($\eta_i(\varphi)$), the resulting general equilibrium changes in expenditure shares and price indices for the two products affect revenue relative to variable factor inputs and hence measured firm revenue productivity for each product. As the product 1 fixed cost increases and the probability of supplying product 1 falls, there is a rise in the product 1 price index relative to the aggregate price index ($P_1/P$) and a decline in the product 1 expenditure share ($\alpha_1(\mathcal{P})$). Conversely, there is a decline in the product 2 price index relative to the aggregate price index ($P_2/P$) and a rise in the product 2 expenditure share ($\alpha_2(\mathcal{P})$). The net effect of these general equilibrium changes is a rise in the bias of measured firm productivity for product 1 (Term $E$ shown in Panel (v) of the figure) and a decline in the bias of measured firm productivity for product 2 (Term $G$ shown in Panel (vi) of the figure).

While existing research on productivity measurement has paid careful attention to the probability of firm exit, we find that the rise in the fixed cost for product 1 has a larger impact on the probability of supplying each product than on the probability of firm entry and exit, as can be seen from comparing Panels (iii)-(iv) and (i)-(ii). Additionally, we find that the bias in measured firm productivity ($\eta_i(\varphi)$) differs substantially from one, as shown in Panels (v)-(vi). As the fixed cost for product 1 increases from 0.45 to 0.90, the bias in measured firm productivity for product 1 rises from around 0.68 to 0.72, while the bias in measured firm productivity for product 2 falls from around 0.75 to 0.73.

Although in the interests of brevity, we only display comparative statics for the product 1 fixed cost, we also find that changes in the product 2 variable cost have quantitatively significant effects on the components of expected firm productivity. As the product 2 variable cost increases, the probability of supplying product 1 increases and the probability of supplying product 2 decreases. Unlike the fixed cost, changes in the product 2 variable cost directly enters the bias in measured firm productivity for product 2 ($\eta_2(\varphi)$) and also has indirect general equilibrium effects through price indices and expenditure shares for both products. Again the change in the probability of supplying each product is larger than the change in the probability of firm entry and exit, and the bias in measured firm productivity differs substantially from one.

### 6.3. *Bias in Measured Aggregate Productivity*

Having examined the quantitative magnitude of the bias for measured firm productivity, we now turn to examine its magnitude for measured aggregate productivity. With a Pareto productivity distribution, the expression for measured aggregate productivity in the special case of a single product in (32) can be evaluated as:

$$\Theta^s = \frac{z\sigma f\left(\varphi^*\right)^{1-\frac{1}{\sigma}}}{L\left(z-\left(\sigma-\frac{1}{\sigma}\right)\right)}. \tag{37}$$

Similarly, if the true model involves endogenous product selection but the researcher falsely assumes a single product, the resulting expression for measured aggregate productivity in (31) can be evaluated as:

$$\Theta = \frac{z\sigma f\left(\varphi^*\right)^{1-\frac{1}{\sigma}}}{L\left(z-\left(\sigma-\frac{1}{\sigma}\right)\right)}\Upsilon, \tag{38}$$

where the bias in measured aggregate productivity as a result of endogenous product selection ($\Upsilon$) is now:

$$\Upsilon \equiv \left[\eta_1\left(\varphi\right)\left(\frac{1-\Lambda^{-\left(z-\left(\sigma-\frac{1}{\sigma}\right)\right)}}{1-\Lambda^{-z}}\right)+\eta_2\left(\varphi\right)\frac{\left(f_2-f_1\right)\Lambda^{\left(1-\frac{1}{\sigma}\right)}}{f_1\left(1-\frac{\alpha_1(\mathcal{P})}{\alpha_2(\mathcal{P})}\left(\frac{1}{\mathcal{P}}\frac{b_2}{b_1}\right)^{\sigma-1}\right)}\right],$$

which depends on the bias in measured firm productivity ($\eta_i\left(\varphi\right)$), equilibrium expenditure shares ($\alpha_i\left(\mathcal{P}\right)$) and relative prices ($\mathcal{P}$), and the ratio of the productivity cutoffs ($\Lambda \equiv \varphi^{**}/\varphi^*$) for the two products.

In Figure 3, we display the bias in measured aggregate productivity and its components for product 1 fixed costs between 0.45 and 0.90 and a product 2 variable cost of $b_2 = 0.45$. As discussed above, the increase in the product 1 fixed cost reduces the probability of supplying product 1, which in turn results in a decline in the relative price of product 2 ($\mathcal{P}$ as shown in Panel (i) of the figure) and a decline in the product 1 expenditure share ($\alpha_1(\mathcal{P})$ as shown in Panel (ii) of the figure). As the product 1 fixed cost increases, and the profitability of product 1 falls relative to product 2, there is also a decline in the product indifference cutoff relative to the zero-profit cutoff ($\Lambda \equiv \varphi^{**}/\varphi^*$), as shown in Panel (iii) of the figure.

The bias in measured aggregate productivity ($\Upsilon$) is substantial and greater than 2 for all values of the product 1 fixed cost between 0.45 and 0.90, as shown in Panel (iv) of the figure. As the product 1 fixed cost increases, the degree of heterogeneity in production technology between the two products diminishes. As a result, there is a decline of more than one third in the bias of measured aggregate productivity, which incorporates the reductions in the relative price of product 2 ($\mathcal{P}$), the expenditure share of product 1 ($\alpha_1(\mathcal{P})$) and the relative productivity cutoff ($\Lambda \equiv \varphi^{**}/\varphi^{*}$).

While in the interests of brevity we again concentrate on comparative statics with respect to the product 1 fixed cost, we also find quantitatively significant effects for the product 2 variable cost. The bias in measured aggregate productivity is again substantial and varies systematically with changes in the product 2 variable cost. Taken together, the quantitative analysis of the model suggests that endogenous product selection can have substantial effects on measured firm and aggregate productivity for a range of parameter values.

## 7. Conclusions

In this paper we argue that endogenous product selection provides a neglected source of bias in measured productivity in addition to the conventional biases of exit self-selection, endogeneity, and mis-specification of the production technology and demand. When firms choose their products at a more disaggregated level than is observed in plant and firm-level datasets, measured productivity reflects both characteristics of the firm and attributes of the products that are non-randomly chosen by the firm. To characterize the resulting bias in measured productivity, we develop a model of industry equilibrium in which firms endogenously sort across products. Following the standard "revenue production function" estimation approach, we use the model to derive the bias in measured firm and aggregate productivity. Calibrating the model's parameters, we show that the bias in measured firm and industry productivity can be quantitatively large and influences the response of both productivity measures to changes in parameter values.

Our analysis points to a number of areas for further research. On the one hand, the estimation of structural models of industry equilibrium that feature endogenous product choice is a promising line of inquiry. On the other hand, the development of census of production datasets containing detailed information on establishments' output, inputs, and prices by product is clearly a priority. While census datasets typically allocate establishments to a "main industry" based on their largest good, our research points to the additional insights to be gained from gathering more detailed information on the set of goods establishments supply. It is important to remember, however, that even when this information is available, the industrial classification used is typically coarse compared to the level at which firms make decisions about products. As a result the bias in measured productivity induced by endogenous product selection is likely to remain a concern.

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# A Appendix: Theoretical Derivations

*A1. Weighted Average Productivity and Average Profitability*

$$\begin{aligned} \widetilde{\varphi}_1(\varphi^*, \varphi^{**}) &= \left[\frac{1}{G(\varphi^{**}) - G(\varphi^*)} \int_{\varphi^*}^{\varphi^{**}} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)} \\ \widetilde{\varphi}_2(\varphi^{**}) &= \left[\frac{1}{1 - G(\varphi^{**})} \int_{\varphi^{**}}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)} \end{aligned} \tag{39}$$

Using the relationship between the revenues of firms producing varieties in the same and in different markets, as well as the expression for the zero-profit productivity cutoff and the CES expenditure share, average profit in the two product markets, $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$ may be written as follows:

$$\overline{\pi}_1(\varphi^*, \varphi^{**}) = \left[\left(\frac{\widetilde{\varphi}_1(\cdot)}{\varphi^*}\right)^{\sigma-1} - 1\right] f_1 \tag{40}$$

$$\overline{\pi}_2(\varphi^*, \varphi^{**}, \mathcal{P}) = \left[\left(\frac{a_2}{a_1}\right)^{\psi} \left(\frac{b_1}{b_2}\frac{\widetilde{\varphi}_2(\cdot)}{\varphi^*}\right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} - \frac{f_2}{f_1}\right] f_1 \tag{41}$$

*A2. Proof of Proposition 1*

**Proof.** We begin by determining the equilibrium sextuple: $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$. First, we use the relative supply and relative demand relationships in equations (23) and (24) to establish that there exist unique equilibrium values of $\varphi^{**}/\varphi^*$ and $\mathcal{P}$. Rearranging the product supply relationship, we obtain:

$$\mathcal{P} = \left(\frac{b_2}{b_1}\right)^{\frac{\sigma-1}{\sigma-\psi}} \left(\frac{a_1}{a_2}\right)^{\frac{\psi}{\sigma-\psi}} \left[\left(\frac{\varphi^{**}}{\varphi^*}\right)^{1-\sigma} \left(\frac{f_2}{f_1} - 1\right) + 1\right]^{\frac{1}{\sigma-\psi}}. \tag{42}$$

Since $\sigma > 1$, the right-hand side is monotonically decreasing in $\varphi^{**}/\varphi^*$. Note that takes the value $(f_2/f_1)^{1/(\sigma-\psi)} (a_1/a_2)^{\psi/(\sigma-\psi)} (b_2/b_1)^{(\sigma-1)/(\sigma-\psi)} > 0$ at $\varphi^{**}/\varphi^* = 1$ and converges to a lower value of $(a_1/a_2)^{\psi/(\sigma-\psi)} (b_2/b_1)^{(\sigma-1)/(\sigma-\psi)} > 0$ as $\varphi^{**}/\varphi^*$ tends to infinity. Turning now to the product demand relationship (equation (24)), the left-hand side is monotonically increasing in $\varphi^{**}/\varphi^*$. Note that as $\varphi^{**}/\varphi^*$ approaches 1, $\mathcal{P}$ converges to 0, while as $\varphi^{**}/\varphi^*$ tends to infinity, $\mathcal{P}$ converges to $\infty$. Therefore, there exists a unique equilibrium value of $(\mathcal{P},\ \varphi^{**}/\varphi^*)$ where both the relative supply and relative demand relationships are satisfied and where $\varphi^{**}/\varphi^* > 1$.

Given values of $\Lambda \equiv \varphi^{**}/\varphi^{*}$ and $\mathcal{P}$, equation (25) is monotonically decreasing in $\varphi^{*}$: $dv_e/d\varphi^{*} < 0$. Furthermore, as $\varphi^{*} \to 0$ in equation (25), $v_e \to \infty$, while as $\varphi^{*} \to \infty$, $v_e \to 0$. As a result equations (23), (24) and (25) together determine unique equilibrium values of the three unknowns $(\varphi^{*}, \varphi^{**}, \mathcal{P})$. Since $\varphi^{*} > 0$ and $\varphi^{**} > \varphi^{*}$ both products are indeed produced in equilibrium.

These three elements of the equilibrium vector are sufficient to determine weighted average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, in equation (39), as well as average revenue and hence average profitability, $\overline{\pi}_1$ and $\overline{\pi}_2$, in equations (40) and (41).

As shown in the main text, the steady-state stability and free entry conditions (equations (19), (20) and (18)) imply that total revenue, $R$, is equal to total payments to labor used in both entry and production, $L$.

Revenue in each product market may be determined from the CES expenditure share (equation (6)) at the equilibrium value of relative prices, $\mathcal{P}$, for which we solved above: $R_1 = \alpha_1(\mathcal{P})L$ and $R_2 = (1 - \alpha_1(\mathcal{P}))L$.

From consumer and producer optimization, the price indices, $P_1$ and $P_2$, may be written as functions of the mass of firms, $M_1$ and $M_2$, and the price charged by a firm with weighted average productivity, $p_1(\widetilde{\varphi}_1)$ and $p_2(\widetilde{\varphi}_2)$:

$$
\begin{aligned}
P_1 &= (M_1)^{\frac{1}{1-\sigma}} p_1(\widetilde{\varphi}_1) = \left( \frac{\alpha_1(\mathcal{P})L}{\sigma(\overline{\pi}_1 + f_1)} \right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\widetilde{\varphi}_1} \\
P_2 &= (M_2)^{\frac{1}{1-\sigma}} p_2(\widetilde{\varphi}_2) = \left( \frac{(1 - \alpha_1(\mathcal{P}))L}{\sigma(\overline{\pi}_2 + f_2)} \right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\widetilde{\varphi}_2}
\end{aligned}
$$

where we have used $M_i = R_i/\overline{r}_i$ and ($\overline{\pi}_1$, $\overline{\pi}_2$, $\widetilde{\varphi}_1$, $\widetilde{\varphi}_2$) were determined above. We have thus characterized the equilibrium sextuple $\{\varphi^{*}, \varphi^{**}, P_1, P_2, R_1, R_2\}$. As shown in the previous working paper version of this paper, all other endogenous variables of the model may be derived from the equilibrium sextuple $\{\varphi^{*}, \varphi^{**}, P_1, P_2, R_1, R_2\}$. ■

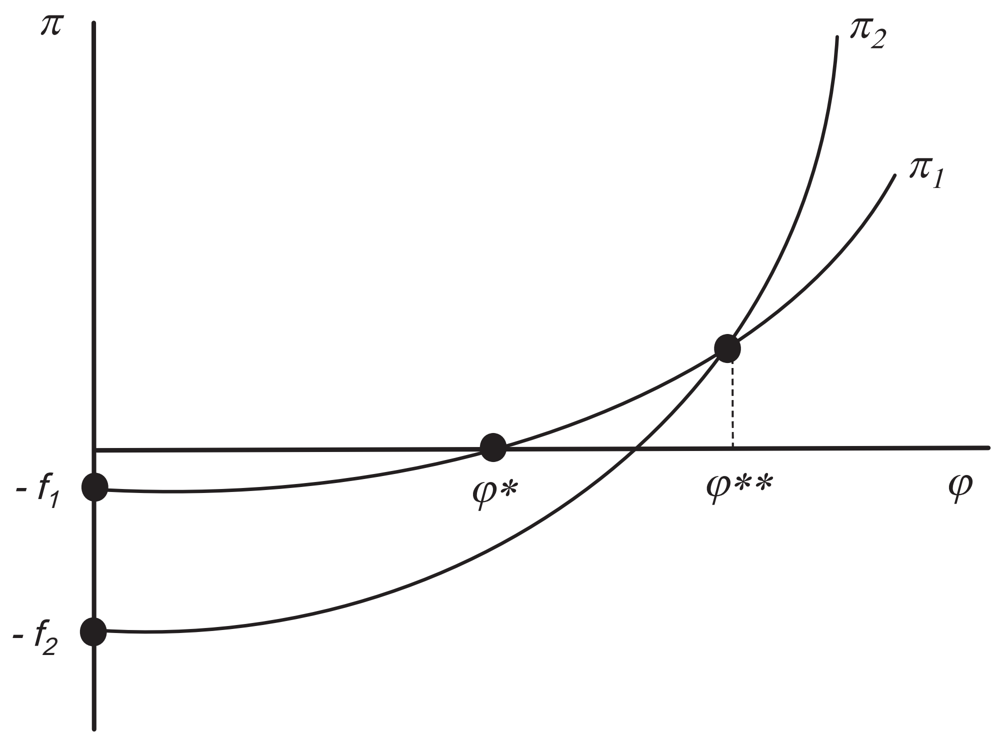


Figure 1: Profit versus Productivity for the Two Products

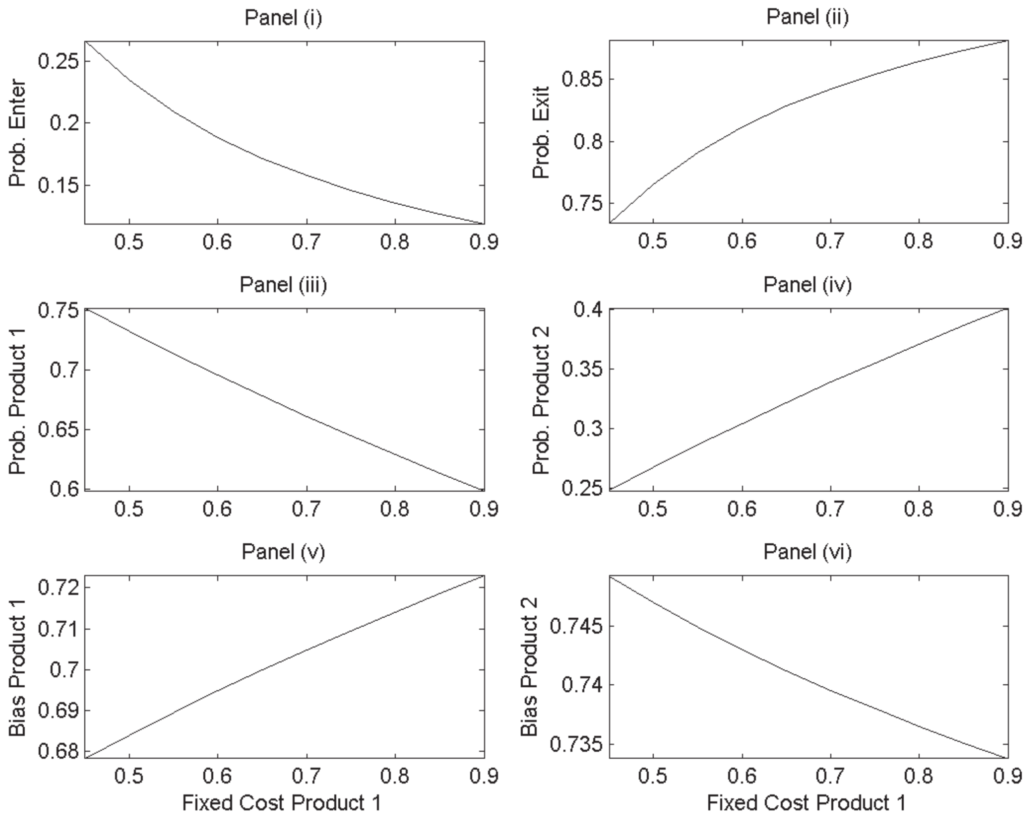


Figure 2: Measured firm revenue productivity

Notes: Each panel displays equilibrium values of a variable as a function of the fixed cost for product 1 for the given values of the other parameters discussed in the main text; Panel (i) shows probability of firm entry; Panel (ii) shows probability of firm exit; Panel (iii) shows the probability of supplying product 1 conditional on entry; Panel (iv) shows the probability of supplying product 2 conditional on entry; Panel (v) shows the bias in firm productivity for firms supplying product 1; Panel (vi) shows the bias in firm productivity for firms supplying product 2.

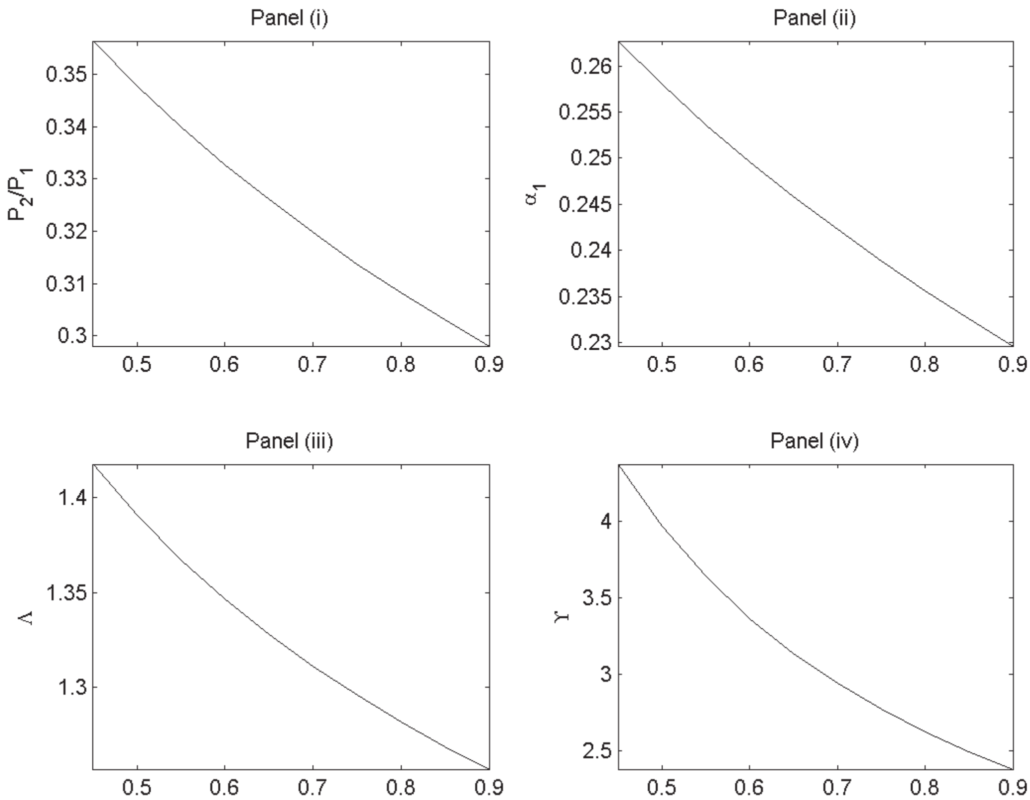


Figure 3: Measured aggregate revenue productivity

Notes: Each panel displays equilibrium values of a variable as a function of the fixed cost for product 1 for the given values of the other parameters discussed in the main text; Panel (i) shows relative price indices; Panel (ii) shows the share of expenditure on product 1; Panel (iii) shows the ratio of the productivity cutoffs; Panel (iv) shows the bias in aggregate productivity.