Web-Based Technical Appendix to The Uneven Pace of Deindustrialization in the OECD

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Theoretical Framework

This web-based technical appendix outlines the model set-up that leads to equation (1) in the paper. We use as our theoretical framework the neoclassical model of trade, which models cross-country differences in production structure in terms of cross-country differences in preferences, technology and factor endowments (see for example Dixit and Norman 1980 and Woodland 1982).

The market structure is assumed to be perfectly competitive. The production technology in each sector is assumed to be a twice continuously differentiable constant returns to scale function of factor inputs:

$$y_{cj} = \theta_{cj} F_j \left(v_{cj} \right), \qquad \text{(A1)}$$

where $c \in \{1, \ldots, C\}$ denotes countries, $j \in \{1, \ldots, n\}$ denotes industries, y_{cj} denotes output, θ_{cj} denotes technical efficiency and v_{cj} is an $m \times 1$ vector of the amount of each the m factors of production used in sector j in country c. As shown in equation (A1), we allow production technologies to vary across countries for each sector, but assume that these technology differences take the Hicks-neutral form, so that they raise or reduce the productivity of all factors of production by the same proportion.

Firms maximize profits taking prices as given and subject to the production technology (A1), which defines the equilibrium revenue function: $r(\theta_c p_c, v_c)$, where p_c is the $n \times 1$ vector of goods prices, v_c is the $m \times 1$ vector of the *aggregate* factor endowments, and θ_c is a $n \times n$ diagonal matrix of the Hicks-neutral industry technology parameters in country c. We assume that the revenue function is twice continuously differentiable, for which a sufficient condition is that there are at least as many factors of production as goods: $m \geq n$.[1] Using a standard duality result, the derivative of the revenue function with respect to the price of a good j equals the economy's profit-maximizing output of that good:

$$y_{cj} = y_j(\theta_c p_c, v_c) = \frac{\partial r(\theta_c p_c, v_c)}{\partial p_{cj}}, \qquad \text{(A2)}$$

where output y_{cj} is defined in value-added terms.

The function $y_j(\theta_c p_c, v_c)$ is a general equilibrium supply-side relationship derived from producer optimization, which maps factor endowments, goods prices and technology parameters into the supply of value-added for good j. This supply-side relationship includes the economy's *aggregate* endowment of each factor, v_c, and *not* the industry's employment of the factor, v_{cj}, which is determined as part of the general equilibrium.

Following Harrigan (1997) and Kohli (1991), we consider a very general form for the revenue function – the translog specification – which serves as an approximation of any constant returns to scale revenue function:

$$\ln r(\theta_c p_c, v_c) = \alpha_{00} + \sum_j \alpha_{0j} \ln \theta_{cj} p_{cj} + \frac{1}{2} \sum_j \sum_k \alpha_{jk} \ln(\theta_{cj} p_{cj}).\ln(\theta_{ck} p_{ck})$$

$$+ \sum_i \beta_{0i} \ln v_{ci} + \frac{1}{2} \sum_i \sum_h \beta_{ih} \ln v_{ci}.\ln v_{ch}\,, \qquad \text{(A3)}$$

$$+ \sum_j \sum_i \gamma_{ji} \ln(\theta_{cj} p_{cj}).\ln(v_{ci})$$

where c indexes countries, $j,k \in \{1,\dots,N\}$ index sectors; and $i,h \in \{1,\dots,M\}$ index factors.

Differentiating with respect to the price of a good j in the translog revenue function (A3), and using the standard duality result (A2), yields the following expression for a sector's share of GDP, which is equation (1) in the paper:

$$\frac{\partial \ln r(\theta_c p_c, v_c)}{\partial p_{cj}} = \frac{y_j(\theta_c p_c, v_c)}{r(\theta_c p_c, v_c)} \equiv s_{cj} = \alpha_{0j} + \sum_{k=1}^{N} \alpha_{jk} \ln p_{ck} + \sum_{k=1}^{N} \alpha_{jk} \ln \theta_{ck} + \sum_{i=1}^{M} \gamma_{ji} \ln v_{ci}\ .$$

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[1] In the Heckscher-Ohlin model with identical preferences, identical technology and no trade costs, the assumption of more factors than goods ensures that production is determinate and the revenue function is twice continuously differentiable. In the neoclassical model, differences in technology and prices across countries help to make production determinate and the revenue function twice continuously differentiable even with more goods than factors.

References

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