mation $r \to kr$). In case (c) the scale invariance is broken. A massless renormalizable theory leads first to scale invariance (for the scale transformation $Q^2 \to kQ^2$); but, just like the third case, this invariance is broken by renormalization.

[7] $\Gamma(x)$ is the Euler function. It is an analytic function of x except for $x = 0, -1, -2, \ldots$. If $\varepsilon \ll 1$: $\Gamma(1+\varepsilon) = 1 - \varepsilon\gamma + O(\varepsilon^2)$ where γ is the Euler constant $\gamma = 0.5772$.

[8] Wolfgang Gröbner and Nikolaus Hofreiter, *Integraltafel, zweiter teil* (Springer-Verlag, Innsbruch, 1958).

[9] Other interesting cases are the potentials created by a point charge or a uniformly charged plane. These are particular cases of the potential created by a uniformly charged D-dimensional space. We can even generalize to noninteger value of D. We can then show that only when $D = 1$ the potential is renormalizable.

[10] S. Sakata, H. Umezawa, and S. Kamefuchi, Prog. Theor. Phys. 7, 377 (1952).

[11] The following reasoning can be applied to any dimensionless physical quantity depending on Q^2 only. It applies also to $\alpha(Q^2)$ the effective coupling constant in the leading log approximation.

[12] W. Celmaster and R. Gonsalves, Phys. Rev. Lett. **44**, 560 (1979).

[13] K. G. Cheyrkin, A. L. Kataev, and F. V. Tkackov, Phys. Lett. B **85**, 277 (1979).

[14] M. Dine and J. Sapirstein, Phys. Rev. Lett. **43**, 27 (1979).

[15] R. Hollebeek, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies*, Bonn, edited by W. Pfeil (Universität Bonn).

[16] In field theory, μ can be identified to "the subtraction constant" needed to renormalize each order of perturbation. Its appearance is the so-called dimensional transmutation phenomena.

[17] For $R \to \alpha(Q^2)$, it is the Gell-Mann and Low equation.[18]

[18] M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954).

[19] In quantum field theory, the renormalization scale μ necessarily enters in the calculated expression of R. [It plays the role of Λ' in the expression (14) of V_R.] Equation (25) is therefore fundamental. In fact, it implies that the explicit μ dependence in R must be compensated and leads to Eq. (24). We can then deduce Eq. (28).

Photon mass experiment

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A Coulomb null experiment is described that enables physics students to obtain rigorous upper bounds on photon mass. The experimenter searches for subnanovolt signals that would escape a closed shell were photon mass to be positive. The approach can be adapted for several college levels. At the simplest level, a "miniature" low-cost experiment allows a student to verify the exponent " -2" in Coulomb's law to eight or more decimal places. An advanced student given a full-size apparatus (at greater cost) can obtain mass bounds very close to the established laboratory limit.

I. INTRODUCTION

The idea that the photon mass is zero, as assumed in the classical theory of electromagnetism, must always be subject to experimental scrutiny. A particle is not massless until proven so. Indeed, the neutrino now presents laboratory[1] and possibly astronomical[2] evidence of having positive mass. For the photon there are no positive-mass claims at the present time. The history of the photon mass problem is fascinating, however, and is a beautiful introduction to the concept of a null experiment. Previous methods of obtaining mass bounds for the photon include: measurement of the speed of light versus wavelength,[3] measurement of pulsar light dispersion,[4] magnetic methods,[5,6] and laboratory verifications of Coulomb's law. This last method is the most relevant to the present treatment. Williams *et al.* obtained (1971) the mass bound[7]

$$m < 2\times 10^{-47}\ \text{g},$$

while the present author and collaborators have recently improved this to[8]

$$m < 8\times 10^{-48}\ \text{g},$$

by bounding the voltage of a radio-frequency signal that penetrates *into* a closed conducting shell. The present treatment describes a similar experiment having simpler, "inside-out" geometry suitable for student work. Generally speaking, the more advanced student can obtain a tighter mass bound, usually by way of relatively longer signal processing time. The experiment is rich in pedagogical physics and is good for demonstrating precisely what in Maxwell theory is dependent on Coulomb's law.

The typical bound obtained by methods in keeping with introductory physics classes is

$$m < 10^{-42}\ \text{g},$$

while for third- and fourth-year laboratory students, the bound can be lowered to

$$m < 7\times 10^{-47}\ \text{g},$$

and serious students with research skill can approach the bound of Williams *et al.* stated above.

II. THEORY

When the zero-mass restriction on the photon is lifted, many interesting changes occur in the standard electrodynamical theory. Excellent treatments of the theory exist,[9] some major modifications are lised here:

$m = 0$	becomes	$m > 0$
Maxwell wave equation	"	Proca equation
Coulomb potential	"	Yukawa potential
speed of light, constant	"	dependent on freq.
gauge freedom	"	Lorentz gauge forced.

The wave equation for positive-mass photons is taken to be the scalar component of the Proca equation[10]:

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi + \frac{m^2 c^2}{\hbar^2}\phi = 0, \quad (1)$$

in empty space, where ϕ represents the temporal component of the vector potential. The Compton wavelength of the photon, essentially the scale number:

$$\lambda = \hbar / mc \quad (2)$$

is now finite. In fact the static potential solution to (1) is now the Yukawa potential:

$$\phi(r) = (A/r)\exp[-r/\lambda] \quad (A \text{ constant}), \quad (3)$$

which is valid for $r > 0$, and unlike the Coulomb potential has finite range.

The idea behind Coulomb null techniques is that one can check to what extent the Yukawa form (3) is possible and therefore obtain a bound on λ and in turn on the mass m.

The basic equations required for understanding of the apparatus now being described will now be worked out. First consider Fig. 1, in which is drawn a classical "atom" having equal but oppositely charged nucleus and electron "shell." In Maxwell theory this atom appears neutrally charged to every observer outside the shell. The usual understanding of the electrodynamics student is summarized thus: "both the nucleus and the shell act as if they are sitting at the center, so the charges cancel." This is correct, but depends completely on the special form of Coulomb's law. If one replaces the Coulomb potential $\phi = kQ/r$ with the Yukawa form $\phi = kQ\exp[-r/\lambda]/r$ one finds that the field outside the shell does not vanish. This is an instructive calculation, as is the alternative approach of solving the static version of Eq. (1) with proper boundary conditions. The exact result for the potential outside the atom, with the requirement that this potential vanish at infinity, is

$$\phi(r) = \frac{kQ}{r}\exp\left(\frac{-r}{\lambda}\right)\left(\frac{\sinh(a/\lambda)}{(a/\lambda)} - 1\right), \quad (4)$$

which has the correct limit, namely, $\phi(r) = 0$ everywhere outside the shell, in the limit $m \to 0$.

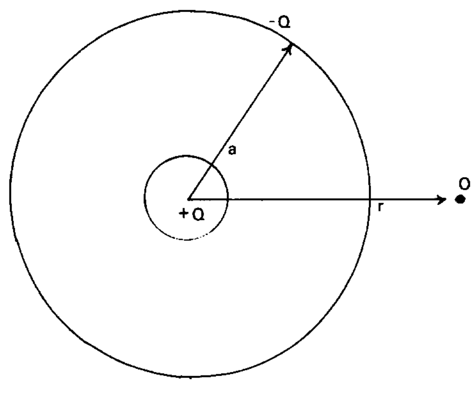

Fig. 1. Classical "atom" formed by two concentric shells of charges $\pm Q$ presents no electric field to any outside observer O, on the basis of Coulomb's law.

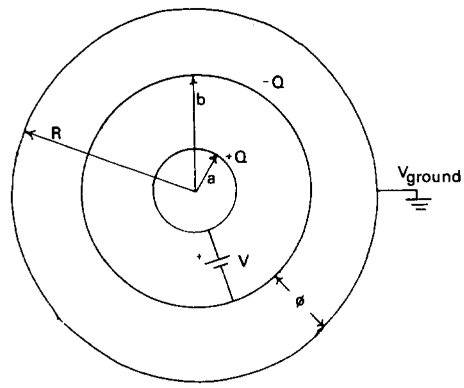

Fig. 2. "Atom" of Fig. 1 is surrounded by a third conductor. If photon mass is positive, there will be a nonzero voltage between the outermost shells.

Consider secondly the arrangement of Fig. 2, involving a triplet of concentric shells; radii a, b, and R. The charges on the inner shells are equal and opposite, but the outer shell is held at some potential V_{ground}. It is important to notice that V_{ground} cannot necessarily be assumed to vanish, as it can when photon mass is zero. This is an example of broken symmetry—one cannot add to each potential solution of the Yukawa theory a constant, because this constant is not generally a solution in itself. We assume that the charges $\pm Q$ on the inside pair of spheres are generated by a voltage source of strength V, and that the voltage ϕ is to be measured between the outer two spheres. Using the same boundary considerations that give Eq. (4) in the atom problem, we find that the outside measurement is related to the inside measurement by

$$\phi \cong \text{const} + \frac{ab(a+b)(1/R - 1/b)}{6\lambda^2} V + O(1/\lambda^4), \quad (5)$$

where the "constant" depends on V_{ground} but not on V. This means that if V is an alternating voltage, the voltage ϕ will be alternating with the same frequency, and we shall have the following bound on the photon mass:

$$m \lesssim \frac{\hbar}{c}\left(\frac{6\phi_{\text{rms}} R}{V_{\text{rms}} a(a+b)(R-b)}\right)^{1/2}. \quad (6)$$

This is the formula relevant to the apparatus we describe next. The idea is that if photon mass is zero, Coulomb's law holds and there should be no received voltage ϕ_{rms} in Eq. (6).

III. APPARATUS

Figure 3 shows the three-sphere arrangement. The geometry is inside-out with respect to that of Williams *et al.*,[7] so the student can work safely outside the closed middle conductor. There is a high-Q resonant tank between the outer spheres. Thinking of the photon mass as a weak capacitive coupling to this tank, the voltage ϕ_{rms} will be improved by a factor equal to the Q of the tank. In practice this is a factor > 100 when tuning is proper, so the improvement is substantial.[11] Second, the voltage generator for V is inside the middle sphere, whereas the Williams *et al.* setup had the driving voltage between the outer two spheres, with the received voltage measured between the inner two. The advantages here is that students can arrange many kinds of signal processing circuits, of varying sophistication, without having to build and test them for use in the small innermost sphere.

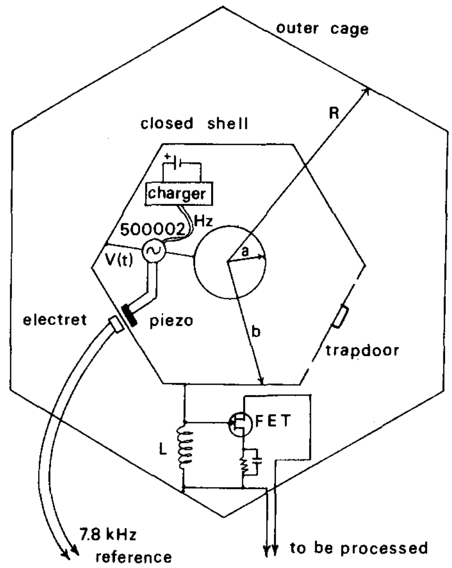

Fig. 3. Actual realization of the three-conductor apparatus. The FET preamplifier will sense massy-photon signal, while the electret microphone senses a reference signal for the signal processing option. Only the $V(t)$ source and an oscilloscope are needed in elementary mode of Sec. IV.

The outer spheres in the model are icosahedra, and it is assumed that no substantial change in the approximate formulas such as Eq. (5) is due to the departure from sphericity. The dimensions are $a = 0.2$ m, $b = 0.5$ m, $R = 1.0$ m. The middle conductor is welded aluminum, the inner is a pair of press-fitted aluminum hemispheres, and the outermost is a mesh screen. It is important that the middle conductor be hole-free, for it is this very surface which causes the null result one obtains under the grace of Coulomb's law. There is a trap door in this surface, having special clamps for best sealing after the door is shut.

There is a battery-charge circuit and storage battery within the middle icosahedron. This is used to drive a special phase-shift circuit that produces the signals of interest for the student who can do signal processing. Let the crystal frequency be f, in our case $f = 1.000\,000$ MHz. A digital circuit that synthesizes a precise sideband, 2 Hz away from $f/2$, drives a power amplifier which in turn produces the wanted signal $V(t)$ between the two innermost conductors. Thus, we can represent the kilovolt-level signal V as

$$V(t) = 500 \sin[2\pi(500\,002\, t)] \quad \text{(V)}, \tag{7}$$

where t is in seconds. Though the crystal frequency when divided by two may not be precisely 500 000 Hz, the circuitry ensures that the *difference* between this frequency and that of $V(t)$ be exactly 2 Hz. The reference frequency formed by more circuitry that divides f by 2^7, creating about 8 kHz, is sent into a piezoelectric transducer. This reference is to be picked up on the outside of the middle conductor with a microphone. This method of coupling a reference frequency out of the inner region does not involve holes in the middle conductor. Of course, the 8-kHz reference must be multiplied back up to $f/2$ to be successfully heterodyned with the possible massy-photon signal embodied in $V(t)$. This part of the experiment is described below.

Besides the microphone reference of frequency $f/2^7$, the only other signal issuing from the outer conductor is any massy-photon signal itself. This is amplified with a low-noise field effect transistor (FET) for further processing. Figure 4 shows how the signals are reconstructed and combined to form a 2-Hz beat proportional to the square of photon mass. This part is not necessary for the most elementary measurements, as might be taken by a beginning physics student.

IV. ELEMENTARY LOW-COST EXPERIMENT

The most elementary, and naturally lowest-cost (see Sec. VII) approach is to use a simple sinusoidal generator between the innermost conductors, but eliminate all other electronics of Fig. 3. One then measures the raw ϕ_{rms} between the two outermost conductors with an oscilloscope or ac voltmeter. The essential schematic of the apparatus is then Fig. 2, with the battery of that figure "switching" at a periodic rate.

According to Coulomb's law, the oscillating charges on the two innermost conductors should give a null reading for ϕ in Fig. 2. The student is asked to consider the phenomenological formula for point-charge potential:

$$\phi(r) = C/r^{1+\epsilon}, \tag{8}$$

where ϵ represents departure from Coulomb's law. It turns out, fortunately, that the field due to a uniform sphere of charge can be obtained from elementary integrals involving Eq. (8), and the student can compute a value for ϕ of Fig. 2 in terms of ϵ. With only a few hundred volts of generator output (see Sec. VI) and an oscilloscope the student can easily obtain a bound:

$$|\epsilon| < 10^{-8} \quad \text{[miniature apparatus (Sec. VII)]}$$

meaning, in the implied sense, that Coulomb's law is good to 8 places. Each addition of Fig. 3 components, such as the coil L, its FET preamplifier, and finally the phase-reference piezoelectret circuit gives roughly an order of magnitude improvement for ϵ. Recent work puts ϵ in the region of 6×10^{-17}, using traditional geometry.[8]

V. MODERATELY ADVANCED EXPERIMENT

The circuitry of Fig. 4 not only gives substantial signal gain to the massy-photon signal channel, but also recreates the original reference frequency with which the photon signal was generated. These signals are multiplied together

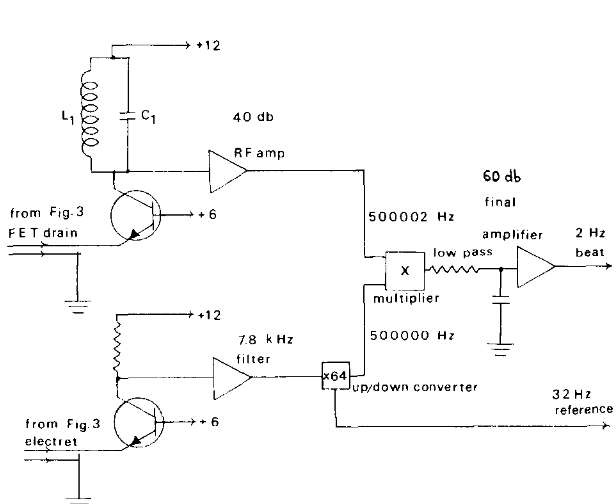

Fig. 4. Typical processing circuitry. A massy-photon signal will appear at the output as a 2-Hz beat signal.

and when fed to a low-pass filter give an extremely small 2-Hz beat signal (if photon mass is zero there is no such signal). If an oscilloscope is connected to the final amplifier output, a good bound on photon mass can be obtained via formula (6). It should be remembered that the LC tank between the outermost shells offers additional gain, at resonance, on the order of 10^2. The bound obtainable with this heterodyne approach is about 10^{-46} g, as mentioned in Sec. I. The only requirement on the oscilloscope is that it have a millivolt capability. The astute experimenter will notice in this experiment a noise component to the 2-Hz beat. This is the Johnson noise in a very small band near 500 kHz, plus some FET noise from the same band.

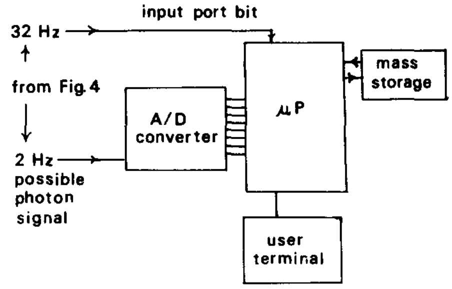

Fig. 5. Block arrangement of a typical microprocessor connection for sharp analysis of possible 2-Hz beats. Advanced students can approach the established bound in this way.

VI. ADVANCED PROCESSING

The bound of 10^{-46} g is still about one order of magnitude worse than the best laboratory results. To meet the established limit, the experimenter must use some form of signal processing. Any massy-photon signal will appear across the LC tank as a component of frequency $f/2 + 2 = 500\,002$ Hz. Even with no FET preamplifier present, there will be Johnson noise arising from the real part of the tank impedance. This noise will be on the order of 1 $\text{nV/Hz}^{1/2}$ in the 500-kHz region. Since the established bound of 2×10^{-47} amounts to about 1 pV across the tank, the signal-to-noise ratio (S/N) at the output of the final amplifier of Fig. 4 should be, in order to approach this established picovolt bound, about

$$\text{S/N} \sim 0.001 T^{1/2}, \quad (9)$$

where T is the signal processing time for the 2-Hz beat signal. Roughly speaking, this means that in 10^6 s one can meet the established bound. Actually, it is possible to meet this bound in only one week, by raising the generator voltage for the two innermost shells, carefully selecting FET's, and adding more shielding.

A concise method of signal processing goes as follows. Let $f_0 = 2$ Hz, and let the final output for Fig. 4 be given by

$$f(t) = S \sin(2\pi f_0 t) + N(t), \quad (10)$$

where $N(t)$ is the noisy component of the whole signal. The signal-to-noise ratio is $S/\sigma\sqrt{2}$, where σ is the rms value of $N(t)$. This ratio is *a priori* about 1/1000 as stated above. Now we sample the data at times $t_1,...,t_n$ such that each sample time t_j occurs right at a peak of the sin function in Eq. (10). Then the sum of these n readings has the mean value

$$\langle \text{sum} \rangle = nS \quad (11)$$

while the rms deviation is $n^{1/2}\sigma$. Thus the effective signal-to-noise ratio is

$$\text{S/N} = (S/\sigma\sqrt{2}) n^{1/2} \quad (12)$$

and so the best possible relation (9) is realized, since measurement real-time is roughly equal to n for the 2-Hz beat.

A typical configuration for this kind of processing is shown in Fig. 5. A microprocessor is used (possibly for many days) to perform a sum such as described above. Since there is one measurement per half second that should be stored, one could alternatively memorize all data points for many hours, for later processing.

One great advantage of this type of signal-processing is that, using the phase-locked 32-Hz reference signal of Fig. 4, one can be virtually certain to sample always near peaks of any photon signal. This is because the 32-Hz signal is absolutely synchronous with the possible photon signal, they being both generated from the same crystal.

VII. APPARATUS COSTS

The basic mechanical construction cost is dominated by the *middle* icosahedral conductor, which is unique in that it must be a good, closed conductor. By contrast, the innermost and outermost conductors need simply to approximate the spherical geometry and can even possess "breaks" or crevasses. We used two metal hemispheres, moderately well electrically coupled for the innermost pseudosphere, and a wire mesh screen for the outermost conductor. For the middle conductor we used nineteen (19) aluminum triangles to make a 1-m-across icosahedron, with a twentieth triangle modified into a trapdoor. The welded aluminum icosahedron thus constructed requires on the order of \$100 metal, and the total cost of materials for the three conductors was about \$160. For the elementary experiment of Sec. IV, it is recommended that a *miniature* system be constructed, with icosahedral dimension on the order of 1 ft. The student still obtains verification of Coulomb's law to eight places, and there is no significant metal cost. In any case, it should be remembered that mass bounds will be poorer for smaller apparatus size.

The elementary experiment still requires an oscillator. This is best achieved with a simple transistor circuit driven by any 6- to 12-V battery.[12] Battery charging is not necessary for this mode because long integration times are not used. For the more advanced experimental modes, the signal processing circuitry costs and the necessary increase in oscillator–piezoelectret costs are substantial. Finally, in the most advanced mode (Sec. VI), a microprocessor must be equipped with an analog-to-digital converter and connected to the signal processing circuitry.

The costs of materials are summarized as follows[13]:

miniature elementary experiment: \$100 for all materials and circuits, not including oscilloscope used to analyze null signal; the icosahedral middle conductor is about 1 ft across; Coulomb's law is verified in the exponent to 6 places.

large-size advanced experiment, without signal processing: \$500 for all materials and circuits, not counting meters and oscilloscopes for analyzing signals; icosahedron is 1 m across.

most advanced experiments: Same cost as last, \$500, but extra signal conversion circuitry with microprocessor is brought in.

ACKNOWLEDGMENTS

I would like to thank N. A. Wheeler for bringing to my attention the challenge of bounding photon mass in the laboratory. I am indebted to D. Griffiths for invaluable aid in manuscript preparation. Students D. Fink, M. Weaver, and R. Leavitt have contributed substantially to the apparatus design, and G. Eibel is the creator of the beautiful mechanical structure of our 2-m prototype.

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[13]Many details of design, especially signal-processing circuitry, have been omitted from this treatment for reasons of space and clarity. More information is available from the author on request.

An interdisciplinary science–humanities course

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A course in the intellectual tradition of the West containing a generous amount of the history of science is described. It has been successfully taught by scientist–humanist teams of two for the last seven years. A selected bibliography of books and articles which have proved to be useful is given.

I. SURVEY

There are many successful ways to present science courses to non-technically-oriented students. Teachers in most physics departments sooner or later have the responsibility to undertake such a task. I would like to describe an unusual course that has been given successfully at the University of Utah for the last seven years and which has a format which some physics teachers and their colleagues in the humanities may find attractive. It is not a science course, as such, but a modification and extension of a Western Civilization course of the sort which many colleges and universities offer in various versions to incoming students. Notable examples are the Humanities Program at Reed College and the Contemporary Civilization and Humanities courses at Columbia University. My purpose here is to describe this course briefly, say a little in justification of it, and to give some bibliography and a few hints to anyone who might like to venture on a similar undertaking.

It seems obvious, to a scientist at least, that a course in the intellectual tradition of the West would be widely missing the mark if the development of sciences were not an important component of it. Butterfield[1] has written of the scientific revolution that "...it outshines everything since the rise of Christianity and reduces the Renaissance and Reformation to mere episodes... ." Most of us would concur. The great syntheses of Newton, Maxwell, and Einstein are monuments of our culture, and yet the typical Western Civilization course contains little, if any, history of science. Some exceptions have been described in this Journal,[2–4] but our course seems unique in its scope and in its goal of continuously attempting to integrate contemporaneous scientific, philosophical, and religious thought over a period of 2500 years.

Our course takes five academic quarters of 10 weeks each (1 $\frac{2}{3}$ academic years in all) and is taught jointly by a scientist and someone from the humanities. Ideally the same pair of co-teachers carries through the entire sequence but we have deviated from this ideal with only minor difficulties. Both attend all the classes which meet five days a week, and both read all the assigned reading as well as most of the students' written work. The assigned readings are exclusively original sources. We have found this to be far more stimulating than the use of secondary material, which we have occasionally tried. The reliance on original sources also gives an authenticity to the student's experience which the (necessarily) nonexpert teachers cannot hope to offer in all the subjects covered. To date, the scientists have been either physicists or biologists while the humanities co-teachers have come from English, languages, and classics. Six different teams have taught the course, some of them more than once. The ongoing strength and vitality of the course lies in the fact that each team makes its own course to fit its interests and knowledge. The participation of two teachers with different academic backgrounds contributes diversity and controversy to class discussion. We have frequently found it useful and stimulating for the two teachers to adopt adversary roles. I will describe a version of this course which was taught by a classicist–physicist team.

