LIFETIME OF THE MUON

Introduction

Muons are unstable particles; otherwise, they are rather like electrons but with much higher masses, approximately 105 MeV. Radioactive nuclear decays do not release enough energy to produce them; however, they are readily available in the laboratory as the dominant component of the cosmic ray flux at the earth’s surface. There are two types of muons, with opposite charge, and they decay into electrons or positrons and two neutrinos according to the rules

\[ \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \]

\[ \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu . \]

The muon decay is a radioactive process which follows the usual exponential law for the probability of survival for a given time \( t \). Be sure that you understand the basis for this law.

The goal of the experiment is to measure the muon lifetime which is roughly 2 \( \mu s \). With care you can make the measurement with an accuracy of a few percent or better. In order to achieve this goal in a conceptually simple way, we look only at those muons that happen to come to rest inside our detector. That is, we first capture a muon and then measure the elapsed time until it decays. As shown in Figure 1, the detector consists of three scintillation counters. At the top and bottom are two thin, \((2' \times 2' \times 0.25'')\) solid plastic scintillators, each viewed by an RCA 6810 photomultiplier tube. Between these is a large tank filled with liquid scintillator and viewed by two Amperex XP1040 tubes.

The experiment is a complicated and subtle one to perform and to analyze. It will yield dividends far outweighing its difficulty, however. You will become familiar with many of the particle detection and measurement techniques widely used in nuclear and particle physics. Some of the instruments, such as the discriminators and PHAs, have a wide range of applications in many fields of physics.

Before doing the experiment you should read all of the references, including the write-up entitled “The Sodium Iodide Gamma Ray Detector” available in the advanced lab office. Briefly stated, such detectors work as follows. As a charged particle passes through a scintillator, it loses energy through collisions with and ionization of the atoms encountered. This causes the scintillator atoms to emit light, mostly in the ultraviolet. The principal difference between the sodium iodide detector and the ones used in the muon experiment is that the latter use organic materials in order to achieve good time resolution, around 5 ns.

You should make generous use of the manuals for the various devices you use in this experiment. They should be found (and replaced when done) above the IBM computer. You will find it helpful to record equipment settings in a notebook (even as you do mini-experiments in preparation for the muon lifetime measurement). Settings are important in interpreting data and troubleshooting problems. Record them!

General Approach

The type of event in which we are interested is labeled “type 1” in Figure 1. A muon passes through the top counter (T) and comes to rest in the large tank, where it subsequently decays. In the caption for Figure 1, \( \overline{T} \) means no muon passed through. The bar in Boolean algebra means “NOT.” The dot denotes
Figure 1: Schematic of the liquid scintillator tank (the box) and fixed plastic scintillators (the hatched areas). **1a.** A muon that stops is recognized by $T \cdot L \cdot R \cdot B$. **1b.** A muon that decays is recognized by $L \cdot R \cdot T \cdot B$. **2.** A misidentified stopping muon $T \cdot L \cdot R \cdot B$. **3.** A through-going muon (or other particle) $T \cdot L \cdot R \cdot B$. **4.** An accidental muon decay signal $L \cdot R \cdot T \cdot B$. 
Muon Decay

Figure 2: Plots of the photomultiplier tube and discriminator outputs. The discriminator output starts about 10 ns after the phototube pulse put into it drops below the $-200 \text{ mV}$ threshold. This delay is a fixed characteristic of the electronic circuit.

We want to arrange things so that the capture of a muon in the tank will start a clock, and the subsequent decay will stop the clock. We will record the clock intervals, make a histogram of the results, and look for the expected exponential decay law. If this sounds simple, it’s because it is! Some fast electronics are required in order to detect automatically the occurrence of the desired events in the tank. (You might want to measure how long a pulse takes to “move” through one of the gates.) The principal components are discriminators and coincidence circuits. A discriminator takes the noisy pulses produced by the phototubes (see Figure 2 for “ideal” pulse shapes) and turns them into square pulses of standard amplitude and duration. The general operation is like that of the Schmidt trigger that you probably made in the Sophomore Lab; only those pulses exceeding a fixed threshold will trigger the discriminator.

The discriminator outputs are fed into coincidence circuits that can be configured to make appropriate logical combinations such as AND, OR, and NOT. Each coincidence unit has 4 inputs: 3 for positive (marked “Yes”) coincidence and one used as a veto (marked “No”). Each input has an enable/disable switch; there is an AND/OR switch that determines the logic function to be performed, and a RATE knob to provide coarse control of output pulse widths. Figure 3 provides some examples of the operation of this circuit.

The clock in our experiment is a time-to-amplitude converter, or TAC for short. This module requires two inputs, a start pulse and a stop pulse. Upon receipt of a START signal, the TAC internally generates a linearly rising voltage ramp. When the next STOP arrives, the TAC generates a fixed duration output pulse with amplitude equal to the value of the ramp. The slope of the ramp is controlled by a RANGE knob on the front panel of the TAC.

Pulses from the TAC are fed into a multichannel pulse height analyzer (PHA), implemented as a special purpose card plugged into the motherboard of an IBM PC. The PHA measures the pulses it receives, sorts them into about 100 quantized bins, and counts the number of pulses whose heights fall in the range of each bin. The PC displays a continuously updated histogram of the counts; in our experiment the histogram represents the distribution of time intervals between START and STOP pulses, and thus a graphical picture of the muon decay law. The PHA is a complicated device—but one of the most useful and versatile
As always, read the relevant manuals and learn as much as you can about its operation!

A block diagram of the overall electronics setup is shown in Figure 4. Please note that except for the discriminators, the electronic circuits expect, as input, negative-going pulses of well defined shapes and sizes. When you think a particular unit is not working, the first thing to check is the input signals it is getting. Look at them with the oscilloscope! A unit will not trigger if an input is too small. There are often lower level (and sometimes upper level) discriminators that can be set. Also, keep in mind that you are dealing with very short times and very narrow pulses. The propagation speed down a piece RG-58/U coaxial cable is two-thirds the speed of light in free space, or about 1.5 ns per foot. If you are careless you will get reflections from the ends of cables, causing great confusion. The logic units have internal terminations that match the 50-ohm characteristic impedance of the coax, but the oscilloscope has high input impedance. In general, you should terminate cables that you plug into the scope. Also, in general, you should terminate unused ports of the discriminators and logic circuits. Use the oscilloscope to snoop around in your experiment, and make sure you know what’s happening everywhere!

The NIM bin devices you in the Lab that might prove useful include: Amplifiers (nuclear electronics’ amplifiers have pulse shaping, why?), Delay Gates, Discriminators, Logic Gates, Attenuators (small

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### Figure 3: Operation of coincidence circuit.

<table>
<thead>
<tr>
<th>Input</th>
<th>Switch</th>
<th>Input Signal</th>
<th>Logic</th>
<th>Output Signal</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (inverted)</td>
<td>IN</td>
<td></td>
<td>AND</td>
<td></td>
<td>Proper coincidence operation.</td>
</tr>
<tr>
<td>2</td>
<td>IN</td>
<td></td>
<td>AND</td>
<td></td>
<td>Vetoed coincidence.</td>
</tr>
<tr>
<td>3</td>
<td>IN</td>
<td></td>
<td>AND</td>
<td></td>
<td>Vetoed coincidence.</td>
</tr>
<tr>
<td>1 (inverted)</td>
<td>IN</td>
<td></td>
<td>AND</td>
<td></td>
<td>Veto does not count coincidence. Must be widened.</td>
</tr>
<tr>
<td>2</td>
<td>IN</td>
<td></td>
<td>OR</td>
<td></td>
<td>Proper OR operation.</td>
</tr>
<tr>
<td>3</td>
<td>IN</td>
<td></td>
<td>AND</td>
<td></td>
<td>Veto disabled.</td>
</tr>
</tbody>
</table>

### Figure 4: Electronics Block diagram. Open circles denote the “NOT” operation.
Muon Decay

Figure 5: Photomultiplier tube high voltage distribution and monitoring.

brass BNC connectors with attenuation factor stamped on it), and Terminators (stamped with termination resistance in ohms).

Checkout of the Counters

The high voltage power supplies should already be connected as shown in Figure 5, using special RED color-coded coaxial cables with high-voltage insulation. Check these connections, and then turn on the power supplies and apply about 2200 volts to each photomultiplier tube. (The digital voltmeter (DVM) is connected through a 10-to-1 divider, so it will read 220 Volts.) The high voltage base voltage is set by the main supply(gray box on floor) and then distributed to a set of 4 potentiometers that allows separate setting of each PMT. Use the oscilloscope to inspect the pulses coming out of the photomultipliers. Observe the range of sizes of the noise pulses, and see what happens when you vary the high voltage. **NOTE: never apply more than 2400 V to a 6810 PMT or 2800 V to an XP1040.**

If any room light reaches the PMTs, they will become very noisy. Check to see that you are not getting excessively large (> 1 V) pulses or large changes in counting rate when the room lights are turned on or off. If you find such evidence of light leaks, black tape and scissors are available to make repairs; in any event, when you are making long data runs, it is best to do it with the lights out and the room locked.

The purpose of the discriminators is to accept real pulses and reject most of the noise pulses. The adjustment of the high voltage to this end is called “plateauing” the counters. Set the width of output pulses from each discriminator to about 100 ns. Then, for each phototube, use the digital scalers in the rack to count the number of noise pulses passing through the discriminator in a fixed time, say 20 seconds. Repeat this as you vary the high voltage through a suitable range (but not exceeding 2400 V (T,B) or 2800 V (L,R)). This procedure will tell you how high you can turn up the voltage before the PMTs become...
too noisy.

The actual plateau of a counter involves the use of the coincidence circuits. As in the actual experiment, you will use cosmic rays to trigger the counters. To maximize the signal-to-noise ratio obtainable from counter $z$, you want to measure the ratio

$$\frac{x \cdot y \cdot z}{x \cdot y}$$

as you vary the high voltage on $z$. (It is not necessary that $x$ and $y$ be at their optimum voltages for this to work.) The measurements should resemble those shown in Figure 6. Set the voltage for the T and B counters in the plateau region, about 50–100 volts above the knee of the curve. Setting the voltage on L and R is somewhat different. Recall that the amount of light produced depends on the amount of energy deposited in the scintillator. A fast cosmic ray muon deposits about 2 MeV per cm of scintillator, or a total of $\approx 120$ MeV in the 2 ft tank. The electron resulting from muon decay has a total energy between 0 and 53 MeV, so it can't possibly deposit as much energy as the cosmic rays do. Think about how to set the high voltage on counters L and R to optimize the efficiency for detection of the muon decay. The following questions may help in thinking about the L,R plateauing. What does the spectrum of the muon decay electron look like? Is the amplitude of the peak one is doing the plateauing with important? The HV power supplies may drift, so you should check the voltages occasionally when you are running the experiment. Drifts will affect both the efficiencies and the relative times of phototube pulses.

**Coincidence and Timing**

Run signal cables to set up the required coincidences $T \cdot L \cdot R \cdot B$ and $L \cdot R \cdot T \cdot B$ (see Figure 1). For this to work properly, the various signals must arrive at the logic circuits in the proper sequence; to check the timing, you can use the same TAC-PHA system that you will use later for the actual data taking. Read the manual that describes the computerized PHA system. It is a very capable and flexible instrument, and you must be familiar with it. To measure the relative times of two pulses, put one into the START and one into the STOP input of the TAC and look at the pulse height spectrum with the PHA. To allow for the possible early arrival of the STOP, delay it by a known amount. You must calibrate the system by feeding in pulses separated by a known delay. In your calibration think about: 1) what time delay range you need (consider the trade-offs involved in choosing to cover a wider range of decay times, versus having a smaller range but better time resolution in your data.) 2) what the beginning and end of the range should be. To bring the signals from a pair of counters into proper coincidence, you can take cosmic ray data for a few minutes until you see a peak on the computer screen. The channel number of the peak tells the relative time delay, and the width of the peak gives the time resolution. This width is important: it tells you how narrow you can set your discriminator output pulses, and still have efficient coincidence detections. The coincidence width can also be used later to calculate the accidental coincidence rate. Set the discriminator output widths to the smallest reasonable values, and put in the appropriate cable lengths. Remember that a veto must arrive a few nanoseconds earlier than the other signals and must last until they are gone (Figure 3).

**Taking Data**

You are now ready to make a data run. You must set the range knob on the TAC appropriately. Select logarithmic scaling for the vertical axis of the computer display. With careful adjustment of the voltages
and timings, you can expect to see some evidence of decaying muons—that is, a straight sloping line on the semi-log plot, plus some background—in an hour or so. However, in order to obtain good statistical accuracy you will have to take data overnight, or even longer. (A 1% measurement requires $\sim 10^4$ events!) To get a full understanding of what’s going on, you should also take a background run in which the “muon capture” and “muon decay” coincidences are intentionally decorrelated, and you should measure or estimate various accidental coincidence rates. Finally, you will have to calibrate the TAC-PHA combination so that you can accurately convert the PHA channel numbers to microseconds.

**Data Analysis**

Several computer programs are provided to help you with data analysis. One called READMUON will convert the binary data recorded by the PHA software to an ordinary ASCII file that you can print, edit, etc. Another program called REDUCE_DATA (a C-program) can then be used to fit your edited data to the relation

$$N(b_i; a_1, a_2, a_3) = a_1 + a_2 e^{-a_3 b_i}.$$ 

Then, with clever reasoning one can recast into the desired form:

$$N(t) = B + N_0 e^{-t/\tau}.$$ 

Here $N(t)$ is the number of muons that lived in the tank for time $t$; $N_0$ is a scaling constant proportional to the total number of events; $B$ is the background counting rate, assumed to be independent of time; and $\tau$ is the desired muon lifetime. With your data as input, the program will determine the best-fit values of these parameters and their (statistical) uncertainties. Finally, a program called PLOTMUON can be used to make a graph of your results with the fitted theoretical curve superimposed on your data. Some documentation for these programs is provided in the laboratory; be sure that you understand the basics of what they are doing for you. You are encouraged to use your own favorite programs if you find this one confining.

**Questions to ponder**

- What are cosmic rays primarily composed of? How are muons formed in the earth’s atmosphere? Given the short muon decay time, why do so many make it to the earth’s surface?
Muon Decay

• What do the PMT output pulses actually look like? How important is it that the pulses look close to that shown in Figure 2?
  • What complications would be introduced by looking at muons in flight, instead of the stationary muons that we deal with here?
    • Is it necessary to know when a muon was produced to measure its lifetime? Why?
    • How does the presence of accidental muon decay signals distort the time histogram?
    • Given two detectors with measured noise rates, write an expression for the rate of accidental coincidences between the two. Assume that you know your time resolution. Generalize this to accidental 3-fold and 4-fold coincidences.
  • What is an approximate upper limit for the noise rate in each phototube for this experiment? Suppose real events come at the rate of one per second.
  • When plateauing a counter $z$ using two probe counters $x$ and $y$, what is the effect of any inefficiency in $x$ and $y$ on your plateau curve? What about noise in $x$ and $y$?
  • What is the statistical counting error if your instrument has counted $N$ events? Suppose you have a set of points in the $(x, y)$ plane. How can you calculate the most likely straight line which these points describe? How is the calculation modified if your confidence in some of these points is less than in others? That is, suppose the statistical (or other) errors are not the same for all your points. All of these matters had to be considered when writing the REDUCE_DATA program, and you should endeavor to understand their implications.

References


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