1. *The Game of Morra.* Two players simultaneously throw out one or two fingers and call out their guess as to what the total sum of the outstretched fingers will be. If a player guesses right, but his opponent does not, he receives payment equal to his guess. In all other cases, it is a draw.
   (a) List the pure strategies for this game.
   (b) Write down the payoff matrix for this game.
   (c) Formulate the row player's problem as a linear programming problem. (*Hint: Recall that the row player's problem is to minimize the maximum expected payout.*)
   (d) What is the value of this game?
   (e) Find the optimal randomized strategy.
2. *Heads I Win—Tails You Lose.* In the classical coin-tossing game, player A tosses a fair coin. If it comes up heads player B pays player A \$2 but if it comes up tails player A pays player B \$2. As a two-person zero-sum game, this game is rather trivial since neither player has anything to *decide* (after agreeing to play the game). In fact, the matrix for this game is a 1×1 matrix with only a zero in it, which represents the expected payoff from player A to B.

   Now consider the same game with the following twist. Player A is allowed to peek at the outcome and then decide either to stay in the game or to bow out. If player A bows out, then he automatically loses but only has to pay player B \$1. Of course, player A must inform player B of his decision. If his decision is to stay in the game, then player B has the option either to stay in the game or not. If she decides to get out, then she loses \$1 to player A. If both players stay in the game, then the rules are as in the classical game: heads means player A wins, tails means player B wins.
   (a) List the strategies for each player in this game. (Hint: Don't forget that a strategy is something that a player has control over.)
   (b) Write down the payoff matrix.
   (c) A few of player A's strategies are uniformly inferior to others. These strategies can be ruled out. Which of player A's strategies can be ruled out?
   (d) Formulate the row player's problem as a linear programming problem. (*Hints: (1) Recall that the row player's problem is to minimize the maximum expected payout. (2) Don't include rows that you ruled out in the previous part.*)
   (e) Find the optimal randomized strategy.
   (f) Discuss whether this game is interesting or not.