1. Consider again the L^1-regression problem:

$$\text{minimize } \|b - Ax\|_1.$$

Complete the following steps to derive the step direction vector Δx associated with the primal-dual affine-scaling method for solving this problem.

(a) Show that the L^1-regression problem is equivalent to the following linear programming problem:

$$\begin{aligned} \text{minimize} \quad & e^T(t_+ + t_-) \\ \text{subject to} \quad & Ax + t_+ - t_- = b \\ & t_+, t_- \geq 0. \end{aligned} \tag{1}$$

(b) Write down the dual of (**??**).

(c) Add slack and/or surplus variables as necessary to reformulate the dual so that all inequalities are simple nonnegativities of variables.

(d) Identify all primal-dual pairs of complementary variables.

(e) Write down the nonlinear system of equations consisting of: (1) the primal equality constraints, (2) the dual equality constraints, (3) all complementarity conditions (using $\mu = 0$ since we are looking for an affine-scaling algorithm).

(f) Apply Newton's method to the nonlinear system to obtain a linear system for step directions for all of the primal and dual variables.

(g) We may assume without loss of generality that both the initial primal solution and the initial dual solution are feasible. Explain why.

(h) The linear system derived above is a 6×6 block matrix system. But it is easy to solve most of it by hand. First eliminate those step direction associated with the nonnegative variables to arrive at a 2×2 block matrix system.

(i) Next, solve the 2×2 system. Give an explicit formula for Δx.

(j) How does this primal-dual affine-scaling algorithm compare with the iteratively reweighted least squares algorithm defined in Section 12.5?

2. *GPS.* The *global positioning system* (GPS) is a satelite-based navigational system. It works as follows. There are approximately 30 satelites orbiting the Earth. They are well-spaced (no pun intended) and they know their exact position at all times. Every few seconds each of these satelites, in precise synchronization, sends out a signal indicating its exact position. The Earth-bound traveler using GPS for navigation has a GPS receiver. This receiver listens to four of the 30 satelites that are roughly overhead at the moment that a position fix is desired. Since these satelites are at varying distances from the traveler and the speed of light is finite (fast, but finite), the four signals arrive at the receiver at slightly different moments. Let $x = (x_1, x_2, x_3)$ denote the traveler's unknown position and let $p(i) = (p_1(i), p_2(i), p_3(i))$, for $i = 0, 1, 2, 3$, denote the known positions of the 4 satelites. By carefully noting the four arrival times of the signals, the receiver can take the difference between the arrival times of the signals from satelites 1 and 0, satelites 2 and 0, and satelites 3 and 0. Multiplying these three time differences by the speed of light, the receiver knows the difference between the distances of the traveler from satelites 1 and 0, etc. Letting $\delta(i)$, $i = 1, 2, 3$, denote these differences, we get the following three equations

$$\begin{aligned} \|x - p(1)\| - \|x - p(0)\| &= \delta(1) \\ \|x - p(2)\| - \|x - p(0)\| &= \delta(2) \\ \|x - p(3)\| - \|x - p(0)\| &= \delta(3) \end{aligned}$$

for the three components of the unknown vector x. These equations are nonlinear. In fact, it is not possible to solve them explicitly. But given a starting guess at the position (that can be off by hundreds of miles), one can use Newton's method to solve this nonlinear system. Write down the 3×3 linear system that defines the step direction vector Δx.